15.097 Homework 1

due March 3, 2016

1. Consider the central problem of compressed sensing:

$$\begin{aligned} \min \quad & ||\mathbf{x}||_0 \\ s.t. \quad & \mathbf{Ax} = \mathbf{b}, \end{aligned} \tag{1}$$

where $\mathbf{A}$ is an $m \times n$ matrix, $n > m$. Please devise an MIO algorithm for the problem, implement it in JuMP and compare it with

$$\begin{aligned} \min \quad & ||\mathbf{x}||_1 \\ s.t. \quad & \mathbf{Ax} = \mathbf{b}. \end{aligned} \tag{2}$$

for the following instances: Generate $a_{ij} \sim N(0,1)$. Let $\mathbf{x_0}$ be a vector with k nonzero entries and let $\mathbf{b} = \mathbf{Ax_0}$. Please estimate $\mathbf{x_0}$ for Problems (1) and (2). For a fixed (m,n) try different values of $k \leq m$. Please simulate the experiment several times and plot the proportion of coefficients recovered correctly on a graph where the axes are: m/n and k/m.

2. In Lecture 3, we presented an algorithmic framework for regression that involves an MIO approach. Please implement the framework in JuMP that incorporates robustness, sparsity, pairwise collinearity, group sparsity, nonlinear transformations, and statistical significance. Please test the method with the enclosed data set (train, validation and test) lpga2008.csv

3. Write down a first order method following the method presented in Lecture 2–best subset selection.

$$\begin{aligned} \min \quad & \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \Gamma\|\boldsymbol{\beta}\|_1 \\ s.t. \quad & \|\boldsymbol{\beta}\|_0 \leq k. \end{aligned}$$

Hint: majorize the quadratic term by a quadratic separable in β_i's. Leave other terms unchanged.