Test 2

● Graded

Student

Colin Cano

Total Points

88.75 / 100 pts

Question 1

P1 **10** / 10 pts

✔ **+ 10 pts** All Correct

+ 2 pts Correct gcd (5823,354) = 3

+ 2 pts Correctly states that the numbers are not relatively prime for gcd(5823,354)

+ 6 pts Correct Euclidean Algorithm

+ 5 pts Correct Algorithm but not in the form of table, check comments

+ 3 pts Partially correct Euclidean Algorithm

+ 1 pt Wrong but tried

+ 0 pts All wrong or empty

Question 2

P2 **10** / 10 pts

✔ **+ 10 pts** Correct

+ 3 pts a)

$$a_n = a_{n-1} + 6$$

+ 1 pt a) partially correct recursion formula

+ 2 pts a)

$$a_1 = -8$$

+ 5 pts b)

$$a_n = 6n - 14$$

or

$$a_n = -8 + 6(n-1)$$

+ 4 pts b) minor mistake in the explicit formula

+ 1 pt b) tried

+ 0 pts incomplete | incorrect

+ 1 pt see the specific comment

Question 3

P3 **9.5** / 15 pts

+ 15 pts Correct

✔ **+ 2 pts** BC: Correct base case n=1

+ 1 pt BC: One side is correct, other is missing or wrong

✔ **+ 2 pts** IA: Correct assumption/hypothesis

+ 1 pt IA: Partial induction assumption

✔ **+ 2 pts** IS set up: Correct set up of what needs to be proved

+ 1 pt IS set up: Partially correct

+ 3 pts IS: Correct use of IA: clearly stated use of IA has exact IA form on the left to replace with IA)

✔ **+ 1.5 pts** IS: Partially correct use of IA

+ 6 pts IS: Correct mathematical manipulations

+ 5 pts IS: Most of the math us right

+ 4 pts IS: Some of the math is right

✔ **+ 2 pts** IS: Started common denominator correctly

+ 2 pts IS: Correctly opened right side ()

+ 0 pts Empty or totally wrong

Question 4

P4 **12** / 12 pts

✔ **+ 12 pts** Correct a: $\mathcal{O}(x^4)$ b: $\mathcal{O}(x^7)$ c: $\mathcal{O}(3^x)$

+ 4 pts correct answer for a)
$\mathcal{O}(x^4)$

+ 3.5 pts for part a answered x^4 instead of $O(x^4)$

+ 2 pts For part a incorrectly included coefficients. Ex $O(375x^4)$ instead of $O(x^4)$

+ 1.5 pts Answer included x^4 but incorrectly used coefficients and had notational errors.

+ 0.5 pts Correct notation, incorrect answer

+ 4 pts correct answer for b)
$\mathcal{O}(x^7)$

+ 3.5 pts for part b answered x^7 instead of $O(x^7)$

+ 2 pts For part b incorrectly included coefficients.

+ 1.5 pts Answer included x^7 but incorrectly used coefficients and had notational errors.

+ 0.5 pts Correct notation, incorrect answer

+ 4 pts correct answer for c)
$\mathcal{O}(3^x)$

+ 3.5 pts for part c answered 3^x instead of $O(3^x)$ or other small notation error.

+ 1.5 pts Answer included 3^x but incorrectly used coefficients and had notational errors.

+ 0.5 pts Correct notation, incorrect answer

+ 0 pts All incorrect.

Question 5

P5 **8** / 10 pts

✔ **+ 5 pts** (a) Correct number of ways $\binom{1}{1} \cdot \binom{13}{3} \cdot \binom{38}{2}$ with explanation

+ 4 pts Correct $\binom{1}{1} \cdot \binom{13}{3} \cdot \binom{38}{2}$ terms and explanations but added unnecessary terms.

+ 4 pts (a) Had correct answer but no explanations.

+ 4.5 pts (a) Correct terms but used addition instead of multiplication.

+ 4 pts (a) Correct terms but used permutations instead of combinations.

+ 3.5 pts (a) Had correct [$\binom{1}{1} \cdot \binom{13}{3}$] OR [$\binom{13}{3} \cdot \binom{38}{2}$] terms with explanations

+ 2 pts (a) Had correct $\binom{13}{3} \cdot \binom{38}{2}$ terms missing explanation.

+ 2 pts (a) Incorrect answer but correct method with explanations

+ 1 pt Incorrect answer but correct method without explanations

+ 1.5 pts (a) Identified the $\binom{13}{3}$ term

+ 2 pts (a) valid explanation

+ 1 pt (a) Attempted to provide explanations but they do not quite match with the terms of the answer.

+ 5 pts (b) Correct number of ways $\binom{48}{6}$ with explanation

+ 4 pts (b) Almost correct but missing explanation

+ 4 pts (b) Correct term but used permutation instead of combination.

✔ **+ 3 pts** (b) Correct $\binom{48}{6}$ term and explanation but with unnecessary other terms .

+ 2.5 pts (b) Correct $\binom{48}{6}$ term but with invalid explanation and unnecessary other terms .

+ 2 pts (b) Incorrect answer but correct method

+ 2 pts (b) valid explanation

+ 1 pt (b) Correctly identified that we are selecting from a pool of 48 cards.

+ 1 pt (b) Attempted to provide explanations but they do not quite match with the terms of the answer.

+ 0 pts No answer or completely incorrect

Question 6

P6 **5** / 5 pts

✔ **+ 5 pts** Correct answer: 4 and showed all steps

+ 4 pts Correct answer and showed majority of the work

+ 4 pts Showed valid steps for the whole process and reached correct final answer but made small computational error in the process

+ 3.5 pts Showed almost all steps for the whole process and reached correct final answer but made small computational error in the process

+ 3 pts Showed valid steps for the whole process but incorrect final answer (small computational or mod understanding error)

+ 2 pts Correct answer but no work shown

+ 1.5 pts Showed some work and had somewhat valid steps but made multiple errors when evaluating mod functions.

+ 1 pt Made an attempt to solve and showed at least two steps but did not give a final answer.

+ 0 pts All wrong or missing

Question 7

P7 **12.25** / 16 pts

7.1 **a) and b)** **2.75** / 4 pts

+ 4 pts Correct

+ 2 pts (a) Not one-to-one with proof.
f(a) not equal to f(b) hence a not equal to b, correct

+ 1 pt (a) is correct without proof

✔ **+ 2 pts** (b) is correct with proof (no)

+ 1 pt (b) is correct without proof

✔ **+ 0.75 pts** tried but wrong

+ 1.5 pts (a) and (b) both incorrect, but tried

+ 0 pts All wrong or missing

+ 2.75 pts one correct and one wrong

+ 3 pts (d) is correct with proof (yes)

+ 1 pt (d) yes, it is onto.

+ 1 pt (d) has correct setup

+ 1 pt (d) has correct work

+ 3 pts (e) is correct with proof

+ 1 pt (e) Yes, it is invertible

+ 1 pt (e) shows $g(g^{-1}(y)) = y$

+ 1 pt (e) shows $g^{-1}(g(x)) = x$

+ 0 pts All wrong or missing

+ 2 pts (f) is correct

+ 1 pt (f) started correct but wrong simplification

+ 2 pts (g) is correct

+ 1 pt (g) started correct but wrong simplification

+ 0 pts All wrong

2 Not 1-1

7.2 **c), d), and e)** **5.5** / 8 pts

+ **8 pts** All Correct

✔ + **3 pts** g(x) is 1-1

+ **1.5 pts** c is correct without proof

+ **3 pts** g(x) is Onto

✔ + **1.5 pts** d is correct without proof

+ **2 pts** Has an invertible function

✔ + **1 pt** Wrong inverse function

+ **2 pts** Tried but wrong

+ **0 pts** Missing/wrong

1 not the correct inverse function

7.3 **f) and g)** **4** / 4 pts

✔ + **4 pts** All Correct

+ **1.5 pts** Switched the answers for (f) and (g)

+ **0 pts** Missing/Completely wrong

Question 8

P8 **9** / 10 pts

+ **10 pts** All correct

✔ + **2 pts** (a) Showed correct work in terms of adding binary numbers

+ **2 pts** (a) minor mistakes in calculation in final answer

+ **1 pt** (a) slightly correct final answer

✔ + **3 pts** (a) Correct answer (10000111)

✔ + **2 pts** (b) Showed correct work in terms of multiplying binary numbers

✔ + **2 pts** (b) minor mistakes in calculation in final answer

+ **1 pt** (b) slightly correct final answer

+ **3 pts** (b) Correct answer (110000000010)

+ **3 pts** Tried but wrong/not enough work shown

+ **0 pts** All wrong or no answer

Question 9

P9 **12** / 12 pts

+ 12 pts All correct.

✔ **+ 4 pts** Binary is correct. 0011 1101 1010 0111

✔ **+ 4 pts** Octal is correct. 36647

✔ **+ 4 pts** Decimal is correct. 15783

+ 2 pts Method for binary is correct.

+ 2 pts Method for octal is correct.

+ 2 pts Answer for octal is correct but no work shown

+ 2 pts Method for decimal is correct.

+ 2 pts Answer for decimal is correct but no work shown

+ 1 pt Intermediary values for decimal conversion are correct.

+ 1 pt Binary conversion for 3 is correct. 0011

+ 1 pt Binary conversion for D is correct. 1101

+ 1 pt Binary conversion for A is correct. 1010

+ 1 pt Binary conversion for 7 is correct. 0111

+ 0 pts Answer is missing/ incorrect.

Question 10

EX **1** / 0 pts

+ 6 pts Correct B-(A∩B∩C) or equivalent

+ 3 pts Wrong answer used union instead of intersection.

+ 2 pts Tried and used correct symbols

✔ **+ 1 pt** Tried, but not valid expression

+ 0 pts Wrong or empty

P25

CS 2210 Discrete Structures

Test 2A

Name: Colin Cano ID 01573158

- You have 60 minutes to complete this exam.
- If you enter the room, you must turn in an exam before leaving the room.
- You must show your work and have correct final answer to receive full credit.
- You can use a simple calculator.

Problem 1: (10 points) Use Euclidian Algorithm to find gcd(5823,354) and decide whether 5823 and 354 are relatively prime.

a	b	q	r
5823	354	16	159
354	159	2	36
159	36	4	15
36	15	2	6
15	6	2	3
6	3	2	0

it is not relativly prime because $3 \neq 1$

Problem 2: (10 points) Let sequence $\{a_n\}$ be -8, -2, 4, 10, 16, 22, 28,… with n starting at 1.

(a) Find recursive formula for a_n.

$a_n = a_{n-1} + 6 \qquad a_1 = -8$

(b) Find explicit (iterative) formula for a_n.

$a_n = 6n - 14$

n	
1	-8
2	-2
3	4
4	10

Problem 3: (15 points) Prove for all $n \geq 1$ that

$$\sum_{i=1}^{n} i(i+3) = \frac{n(n+1)(n+5)}{3}$$

BC: $n=1$ RHS: $\frac{1(1+1)(1+5)}{3} = 4$ ✓

LHS: $1(1+3) = 4$

IA: Assume $n=K$ is true for $\exists k \in \mathbb{Z}$ s.t. $\sum_{i=1}^{K} i(i+3) = \frac{K(K+1)(K+5)}{3}$

IS: Prove $n=K+1$ for $\sum_{i=1}^{K+1} i(i+3) = \frac{(K+1)((K+1)+1)((K+1)+5)}{3}$

I don't see it

using IA ↓ how

$$\rightarrow \frac{K(K+1)(K+5)}{3} + (K+1)(K+4) =$$

$$\rightarrow \frac{K(K+1)(K+5) + 3(K+1)(K+4)}{3}$$

$\rightarrow K^2+K(K+5$

$K^3+6K^2+5K +$

Problem 4: (12 points) Find closest big-O bound for:

(a) $f(x) = (5x)^3(3x + \log_2 x)$ $125x^3(3x+\log_2 x)$ $375x^4 + \ldots \in O(x^4)$

(b) $f(x) = (8x^4 - 3x)(x + 7x^3)$

$8x^5 + 56x^7 - 3x^2 - 21x^4 \in O(x^7)$

(c) $f(x) = 100x^3 + 8\log_2 x + 3^x$

$100x^3 + 8\log_2 x + 3^x \in O(3^x)$

Problem 5: (10 points) There are 52 cards in the deck. You must explain your answer to get full points.

(a) How many distinct six-card hands can be formed that include eight of hearts and exactly three cards of diamond suit? Ex: 5♦, 10♦, Q♦, 8♥, K♥, 9♠ meets the requirements.

$1 \cdot \binom{13}{3} \cdot \binom{38}{2}$

80

0

remaining cards

(b) How many different six-card hands don't contain any 10s?

$\binom{52}{6} - \binom{48}{6}$

Every card to Choose from

Cards without 10's

Problem 6: (5 points) Find value of $(-134 mod 23 + 165 mod 23)^2 mod 5$. Show every step.

$-134/23 = -6R4$

$165/23 = 7R4$

$8^2 = 64$

$64/16 = 12R4$

$= (4 + 165 \bmod 23)^2 \bmod 5$

$= (4+4)^2 \bmod 5$

$= 64 \bmod 5 = 4$

Problem 7: (16 points) Consider functions $f: \mathcal{R} \to \mathcal{R}$ and $g: \mathcal{R} \to \mathcal{R}$ defined by $f(x) = -2x^2 + 7$ and $g(x) = 5x - 6$.

(a) Is f(x) one-to-one? Prove your answer.

$f(x) = -2x^2+7$
$f(y) = -2y^2+7$

$f(x) = f(y)$
$-2x^2+7 = -2y^2+7$
$-2x^2 = -2y^2$
$x^2 = y^2$
$x = y$

R

yes one to one

(b) Is f(x) onto? Prove your answer.

$y = -2x^2+7$
$y-7 = -2x^2$
$\sqrt{\frac{y-7}{-2}} = x$

Not onto because of square root

(c) Is g(x) one-to-one? Prove your answer.

$g(x) = 5x-6$
$g(y) = 5y-6$

$g(x) = g(y)$
$5x-6 = 5y-6$
$5x = 5y$
$x = y$ ✓

yes one to one

(d) Is g(x) onto? Prove your answer.

$y = 5x-6$
$y+6 = 5x$
$\frac{y+6}{5} = x$ ✓

yes onto

(e) Is g(x) invertible. If yes, then find inverse function.

yes it is invertible

$g^{-1}(x) = 5\left(\frac{y+6}{5}\right) - 6$
$= y+6-6$
$= y$

(f) Find $f \circ g$.

$fog = -2(5x-6)^2+7$

(g) Find $g \circ f$.

$gof = 5(-2x^2+7)-6$

Problem 8: (10 points) Let a = 1101010 and b = 11101. Find sum of a and b and product of a and b without converting to decimal numbers.

$$1101010 + 11101 = 1000111$$

$$1101010 \times 11101 = 1000000000010$$

1101010

$(3DA7)_{16}$

Problem 9: (12 points) Convert 3DA7 to binary, octal, decimal.

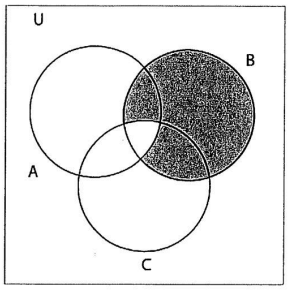
$(3DA7)_{16}$ → 0011 | 1101 | 1010 | 0111

Binary: $(0011\,1101\,1010\,0111)_2$

Decimal: $2^{13}+2^{12}+2^{11}+2^{10}+2^8+2^7+2^5+2^2+2^1+1 = (15783)_{10}$

Octal: $(36647)_8$

Extra Credit: (6 points) Give an expression equivalent to:



$B \cap (A+C)$