MATH:1860 Activity 7 – (Sections 9.6, 11.1-11.2) Mar. 13

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Instructions: Work with others or independently to complete the activity.

1. For each predator-prey system, determine which of the variables, x or y, represents the prey population and which represents the predator population. Is the growth of the prey restricted just by the predators or by other factors as well? Do the predators feed only on the prey or do they have additional food sources? Explain.

   (a) $\frac{dx}{dt} = -0.05x + 0.0001xy$ prey
   $\frac{dy}{dt} = 0.1y - 0.005xy$ predator

   prey does not naturally grow
   predators natural growth.

   (b) $\frac{dx}{dt} = 0.2x - 0.0002x^2 - 0.006xy$ prey
   $\frac{dy}{dt} = -0.015y + 0.00008xy$ predator

   prey has logistic growth
   predator decrease naturally.

2. Determine whether $a_n = n^{1/n}$ converges or diverges. (Hint: Consider the sequence $\ln a_n$, and compute its limit by using L'Hospital's rule.)

   $\lim_{n\to\infty} \frac{\ln n}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{1}{n} = 0$ Converges

3. Find the first 10 terms of the sequence defined by $a_{n+1} = \begin{cases} \frac{1}{2}a_n & \text{if } a_n \text{ is an even number} \\ 3a_n + 1 & \text{if } a_n \text{ is an odd number} \end{cases}$ and $a_1 = 12$. You may do the same for other starting values a_1 and the same thing should happen. Make a conjecture about this type of sequence. This is a famous unproven conjecture called the *Collatz conjecture.*

   $a_1 = 12$ $a_2 = 6$ $a_3 = 3$ $a_4 = 10$ $a_5 = 5$ $a_6 = 16$ $a_7 = 8$
   $a_8 = 4$ $a_9 = 2$ $a_{10} = 1$

4. Define a sequence $\{a_n\}$ recursively by $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{1 + a_n}$ for $n \geq 1$.

   (a) List out the first three terms a_1, a_2, a_3 of the sequence without simplifying. Then simplify them.

   $a_1 = 1$
   $a_2 = 1 + \frac{1}{1+1} = \frac{3}{2}$
   $a_3 = 1 + \frac{1}{1+\frac{3}{2}} = \frac{7}{5}$

(b) By the *Monotonic Sequence Theorem* and a technique out of scope of this course called *Mathematical Induction*, it can be proven that $\lim_{n\to\infty} a_n$ exists. Denote this limit by L. Use the given recurrence relation $a_{n+1} = 1 + \frac{1}{1+a_n}$ to show $L = \sqrt{2}$. This proves $\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2+\cdots}}$, the *continued fraction expansion of* $\sqrt{2}$. (Hint: Take the limit as $n \to \infty$ of both sides of $a_{n+1} = 1 + \frac{1}{1+a_n}$.)

$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} 1 + \frac{1}{1+a_n}$ $\quad L = 1 + \frac{1}{1+L}$ $\quad L - 1 = \frac{1}{1+L}$

$1 = (L-1)(L+1)$ $\quad L^2 - 1 = 1$ $\quad L^2 = 2$ $\quad L = \sqrt{2}$

5. (a) Fibonacci posed the following problem: Suppose that rabbits live forever and that every month each pair produces a new pair which becomes productive at age 2 months. If we start with one newborn pair, how many pairs of rabbits will we have in the nth month? Show that the answer is f_n, where $\{f_n\}$ is the Fibonacci sequence. (Hint: Let f_n represent the number of pairs of rabbits in the nth month. Show $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.)

$f_1 = 1$

$f_2 = 1$

(b) Let $a_n = \frac{f_{n+1}}{f_n}$. Show $a_{n-1} = 1 + \frac{1}{a_{n-2}}$.

$a_2 = \frac{1}{1} = 1$ $\quad a_3 = \frac{2}{1} = 2$ $\quad a_4 = \frac{3}{2} = 1.5$

$L = 1 + \frac{1}{L}$

(c) By the *Monotonic Sequence Theorem* and *Mathematical Induction*, it can be proven that $\{a_n\}$ is convergent. Denote the limit by L. Use the recurrence relation of part(b) to show $L = \frac{1+\sqrt{5}}{2} = 1.61803398\ldots$, the *golden ratio*.

$L = 1 + \frac{1}{L}$

$L^2 = L + 1$ $\quad L^2 - L - 1 = 0$ $\quad L = \frac{1+\sqrt{5}}{2} \approx 1.61803398$

6. The following "proof" claims that $0 = 1$. What is wrong with it?

$$
\begin{aligned}
0 &= 0 + 0 + 0 + \cdots \\
&= (1-1) + (1-1) + (1-1) + \cdots \\
&= 1 + (-1+1) + (-1+1) + (-1+1) + \cdots \\
&= 1 + 0 + 0 + 0 + \cdots \\
&= 1
\end{aligned}
$$

it should not be addition