Homework 3

7.5

14. $\int_0^1 \frac{x}{(2x+1)^3}dx =$ $u = 2x+1$ $du = 2dx$ $dx = \frac{du}{2}$

$x = \frac{u-1}{2}$ $0 = \frac{1-1}{2}$ 1 $1 = \frac{3-1}{2}$ 3

$\int_1^3 \frac{\frac{u-1}{\cancel{2}}}{u^3} \cdot \frac{1}{\cancel{2}} du = \int_1^3 \frac{u-1}{4u^3} du = \frac{1}{4}\int_1^3 \frac{u}{u^3} du - \frac{1}{4}\int_1^3 \frac{1}{u^3} du$

$= \frac{1}{4}\left(\frac{-1}{u}\right) - \frac{1}{4}\left(-\frac{1}{2u^2}\right)\Big|_1^3 = \frac{2}{12} - \frac{8}{72} = \frac{1}{18}$

18. $\int \frac{\cos(\frac{1}{x})}{x^3}dx$ $u = \frac{1}{x}$ $du = -\frac{1}{x^2}dx$ $dx = -x^2 du$

$\int \cos(u) \cdot (-u^2 du) = -\int \cos u \cdot u^2 du$ $v = 2u$ $dv = 2du$ $w = -\cos u$ $dw = \sin u\, du$

$= -u^2 \sin u + 2u\cos - 2\sin u = \boxed{-\left(\frac{1}{x}\right)^2 \sin(\frac{1}{x}) + \frac{2}{x}\cos(\frac{1}{x}) - 2\sin(\frac{1}{x}) + c}$

20. $\int \frac{2x-3}{x^3+3x}dx$ $x^3+3x = x(x^2+3)$

$\int \frac{2x-3}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3} = Ax^2+3A + Bx^2 + Cx$

$= (A+B)x^2 + Cx + 3A = 2x+3$ $A = -1$ $B = 1$

$\int \frac{-1}{x} + \frac{x+2}{x^2+3} dx = \boxed{-\ln|x| + \frac{1}{2}\ln|x^2+3| + \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c}$ $C = 2$

22. $\int \ln(1+x^2)dx$ $u = \ln 1+x^2$ $dv = dx$ $du = \frac{2x}{1+x^2}$ $v = x$

$= x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}dx$ $\int \frac{2x^2}{1+x^2} = \int 2dx - \int \frac{2}{1+x^2}dx = 2x - 2\tan^{-1}x$

$= \boxed{x\ln(1+x^2) - 2x + 2\tan^{-1}x + c}$

$\sin\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ $\sin\theta = 0$

24. $\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}} = \int \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}} = \int \frac{\sin^2\theta}{\sqrt{\cos^2\theta}} = \int \frac{\sin^2\theta}{\cos\theta}$ $x = \sin\theta$ $dx = \cos\theta$ $= \int \sin^2\theta = \int \frac{1}{2}(1-\cos\theta)$

$= \frac{\theta}{2} - \frac{\sin 2\theta}{4}\Big|_0^{\pi/4} = \frac{\pi}{8} - \frac{1}{4} = \boxed{\frac{\pi-2}{8}}$ $u = \sqrt{at}$ $du = \frac{a}{2\sqrt{at}}dt$

36. $\int \sin(\sqrt{at})dt = \int \sin u \cdot \frac{2u}{a}du$ $v = u$ $dv = du$ $dt = \frac{2u}{a}du$ $w = -\cos u$ $dw = \sin u\, du$

$= \frac{2}{a}\left(-u\cos u + \int \cos u\, du\right)$ $\int \cos u\, du = \sin u$

$= \boxed{\frac{2}{a}\left(\sin(\sqrt{at}) - \sqrt{at}\cdot\cos(\sqrt{at})\right) + c}$

48 $\int_0^{\pi} \sin 6x \cos 3x \, dx = \frac{1}{2}\left(\sin(6x+3x) + \sin(6x-3x)\right)$

$= \int \frac{1}{2}(\sin 9x + \sin 3x)$

$\frac{1}{2}\left(\int_0^{\pi} \sin 9x \, dx + \int_0^{\pi} \sin 3x \, dx\right)$

$= \int \sin 9x \, dx = -\frac{1}{9}\cos 9x \qquad \int \sin 3x = -\frac{1}{3}\cos x$

$= -\frac{1}{9}\cos(9\pi) + \frac{1}{3}\cos(9\pi) - 1 - 1 \qquad = \frac{1}{2}\left(\frac{2}{9} + \frac{2}{3}\right) = \boxed{\frac{4}{9}}$

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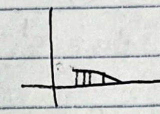
16 $\int_1^3 \frac{\sin t}{t} \, dt, \quad n=4$



$\frac{3-1}{4} = 0.5$

$[1, 1.5][1.5, 2][2, 2.5][2.5, 3]$

Trap:

$\frac{1}{4}\left(\frac{\sin 1}{1} + 2\left(\frac{\sin \frac{3}{2}}{3/2} + \frac{\sin 2}{2} + \frac{\sin 5/2}{5/2}\right) + \frac{\sin 3}{3}\right)$

Midpoint:

$\frac{1}{2}\left(\frac{\sin \frac{5}{4}}{5/4} + \frac{\sin 7/4}{7/4} + \frac{\sin 9/4}{9/4} + \frac{\sin 11/4}{11/4}\right)$

Simpson

$\frac{1}{6}\left(\frac{\sin 1}{1} + 4\left(\frac{\sin 3/2}{3/2} + \frac{\sin 5/2}{5/2}\right) + 2\left(\frac{\sin 2}{2}\right) + \frac{\sin 3}{3}\right)$

7.8

12. $\int_0^{\infty} \frac{1}{\sqrt[4]{1+x}} = \int u^{-1/4} = \frac{4}{3} u^{3/4}\Big|_0^{\infty} = \left(\frac{4}{3}(\infty)^{3/4}\right) - (0) = \infty - 0$ Divergent

$u = 1+x \qquad du = dx$

22 $\int_1^{\infty} \frac{e^{-1/x}}{x^2} \qquad u = \frac{1}{x} \qquad du = -\frac{1}{x^2} dx \qquad \int e^{-u} \cdot -du = -e^{-u}\Big|_1^{\infty} = -e^{\frac{1}{\infty}}$

$= \boxed{1 - \frac{1}{e}}$ convergent

24 $\int_0^{\infty} \sin\theta \, e^{\cos\theta} \, d\theta \qquad u = \cos\theta \qquad du = -\sin\theta \, d\theta$

$\int e^u \cdot -du = -\int e^u du = -e^u\Big|_0^{\infty} \qquad -e^{\cos\infty} + e^{\cos 0}$

$\lim_{x\to\infty} -e^{\cos x} = \text{DNE}$ Divergent

26. $\int_2^\infty \frac{dv}{v^2+2v+3}$ $\quad v^2+2v-3 = (v+3)(v-1)$

$\frac{1}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$

$1 = A(v-1) + B(v+3)$ $\quad B = \frac{1}{4}$ $\quad A = -\frac{1}{4}$

$= \int \frac{-1/4}{v+3} + \frac{1/4}{v-1}\, dv = -\frac{1}{4}\ln|v+3| + \ln|v-1| \Big|_2^\infty = \frac{1}{4}\left(\ln\frac{v-1}{v+3}\right)$

$= \lim_{x\to\infty} \ln\frac{v-1}{v+3} \stackrel{LH}{=} \ln\frac{1}{1} = 0$

$\ln\frac{2-1}{2+3} = \ln\frac{1}{5}$

$= \frac{1}{4}\left(0 - \ln\frac{1}{5}\right) = \boxed{-\frac{\ln 5}{4}}$

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28 $\int_2^\infty ye^{-3y}\, dy$ $\quad u = y \quad du = dy \quad dv = e^{-3y} \quad v = -\frac{1}{3}e^{-3y}$

$= -\frac{y}{3}e^{-3y} + \int \frac{1}{3}e^{-3y} dy$

$= -\frac{y}{3}e^{-3y} - \frac{1}{9}e^{-3y}\Big|_2^\infty = (\infty - \infty) - \left(-\frac{2}{3}e^{-6} - \frac{1}{9}e^{-6}\right)$

$= 0 - \boxed{-\frac{7}{9}e^{-6}}$

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36. $\int_0^5 \frac{1}{\sqrt[3]{5-x}}\, dx$ $\quad u = 5-x \quad du = -dx$

$= \int_0^5 u^{-1/3}\, du = \frac{3}{2}u^{2/3}\Big|_0^5 = \frac{3}{2}\left(5^{2/3} - 0^{2/3}\right)$

$\boxed{= \frac{3}{2}5^{2/3}}$