Name: ———————————————————   **Student ID**: ——————————————

1. Calculate the following limit:

$$\lim_{x \to 1} \frac{x^2 - 1}{\sin(\pi x)}$$

2. Calculate the following limit:

$$\lim_{x \to 0} \frac{e^{2x} - 1}{x}$$

3. Take the derivative of the following function:

$$f(x) = e^{\left(\frac{\sin(x)}{x-1}\right)}$$

4. Take the derivative of the following function:

$$(x^3 + 3x^2 + 7x)e^{(x^2+1)}$$

5. Evaluate the tangent line of the curve given by $x^2 + y^2 = 8$ at $(2, 2)$. Show each step.

6. Evaluate the tangent line of the curve given by $x^2 + xy + y^2 = 4$ at $(0, 2)$. Show each step.

7. A dolphin is repeatedly jumping out of the water and diving back in. Its height above the water is given by $h(t) = 4\sin(\frac{\pi}{2}t)$. At $t = \frac{1}{2}$, is the dolphin above the water or below? Is it moving upward or downward? Is it accelerating or decelerating?

8. A monkey is swinging from branch to branch in the jungle. Its height above the treeline is given by $h(t) = -(x - 3)^2 + x + 1$. At $t = 5$, is the monkey above the treeline or below? Is it moving upward or downward? Is it accelerating or decelerating?

9. Use linear approximation to estimate the value of $f(x) = \ln(1 + x^2)$ at $x = 0.1$.

10. Use linear approximation to estimate the value of $f(x) = \sin(\frac{\pi}{180}x)$ at $x = 46$.

11. A cylindrical bottle with a volume $V = \pi(\frac{D}{2})^2 h = 3\pi$ in^3 (approx. 5 ounces) is being constructed. If the bottle cap costs twice as much as the rest of the bottle, then what diameter D and height h would minimize the cost?

12. A box with a square base and no lid needs to have a volume of 96m³. If the base costs three times as much as the sides, then what is the height of the box that minimizes the cost?

13. Compute the following integral:
$$\int \left(\frac{1}{1+x^2} + 2\sin(x) + e^x \right) dx$$

14. Compute the following integral:
$$\int \left(2x^2 + 5\cos(x) + \frac{3}{x} \right) dx$$

15. Compute the following integral:
$$\int x(5 - 3x^2)^{10} \, dx$$

16. Compute the following integral:
$$\int \frac{\sin(\frac{1}{x})}{x^2} \, dx$$

17. Compute the following integral:
$$\int_{-1}^{1} \frac{3x^2}{1 + x^6} \, dx$$

18. Compute the following integral:
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{-\cos x}{1 + \sin^2 x} \, dx$$

19. Find the area enclosed by the curves $y = 2 - x$ and $y = 2x - x^2$.

20. Find the area enclosed by the curves $y = 1$ and $y = (2 - x)^2$.

21. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ about the x-axis.

22. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{16 - 4x^2}$ and $y = 0$ about the x-axis.

23. Find the volume of the solid obtained by rotating the region bounded by $y = x^5$ and $y = x$ about the y-axis.

24. Find the volume of the solid obtained by rotating the region bounded by $y = 8x^3$ and $y = 2x$ about the y-axis.