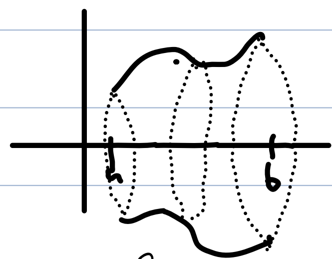
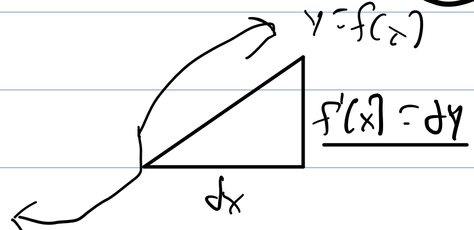
8.2 Area of a surface of revolution formed when a curve is rotated about a line





hypotenuse: $\sqrt{dx^2+dy^2}$

$= \sqrt{dx^2+f'(x)^2\,dx}$

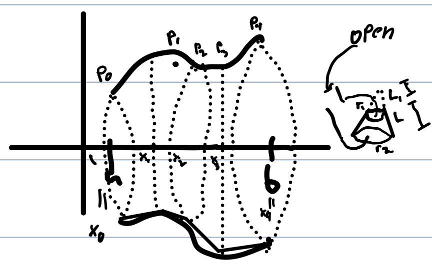
8.1

$= \sqrt{1+(f'(x))^2}\,dx$

$L=\int ds$

$= ds$

Approx surface with sequence of bands then take the limit



It has $SA = 2\pi r l$, where $r=\frac{1}{2}(r_1+r_2)$

Let M denote surface of revolution defined by f on $[a,b]$

then make into sub-intervals into n [illegible]

of equal width Δx_i $[x_0, x_1], \ldots [x_{i-1}, x_i$

Let P be point (x_i, y_i) where $y_i = f(x_i)$

by $(\star)$ area is $\displaystyle\sum_{i=1}^{n} 2\pi\left(\frac{y_0+y_1}{2}\right)(P_{i-1} \cdot P_i)$

8.1 : $P_{i-1}P = \sqrt{1+f'(x_i^*)^2}\ \Delta x$ for x_i^* in $(x_{i-1}, x_i]$

when n is large, Δx is small

$y_{i-1} = f(x_{i-1}) \approx f(x_i^*$ $\quad \frac{y_{i-1}+y_i}{2} \approx f(x_i^*)$

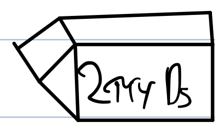
$y_i \approx f(x_i^*)$

SA of M is

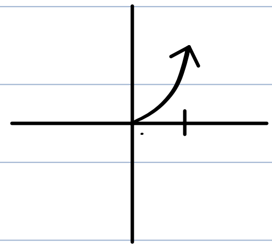
$$S = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi\left(\frac{y_{i-1}+y_i}{2}\right)(P_{i-1}P_i)$$

$$= \lim_{n\to\infty} \sum 2\pi(x_i^*) \cdot \left(\sqrt{1+f'(x_i^*)^2}\ \Delta x\right.$$

$$S = \int_a^b 2\pi f(x) \left(\sqrt{1+f'(x)^2}\ dx\right.$$ can use y, $y = f(x)$



Ex: Find the area of the surface obtained by rotating $y=x^2$, $0 \le x \le 2$, y axis $\quad y'=2x$



$S=\int 2\pi x \, ds = \int_0^2 2\pi x\sqrt{1+(2x)^2}\,dx$

$x' = \frac{1}{2}y^{-1/2}$

$\cancel{x=\sqrt{y},\ 0 \le y \le 4}$

$\cancel{S=\int_0^4 2\pi x \cdot \sqrt{1+\left(\tfrac{1}{2}y^{-1/2}\right)^2}\,dy}$

$\cancel{=\int_0^4 2\pi(\sqrt{y})\sqrt{1+\left(\tfrac{1}{2}y^{-1/2}\right)^2}\,dy}$ another way

$S=\int 2\pi x\,ds = \int_0^2 2\pi x\sqrt{1+(2x)^2}\,dx$

$u = 1+4x^2$

$u(0)=1$

$u(2)=17$

$du = 8x\,dx \qquad \frac{1}{8}du = x\,dx$

$=\int_0^2 2\pi x\sqrt{1+4x^2}\,dx$

$=\frac{2\pi}{8}\int_1^{17}\sqrt{u}\cdot du = \frac{\pi}{4}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{17} = \frac{\pi}{6}\left(17^{3/2}-1\right)$