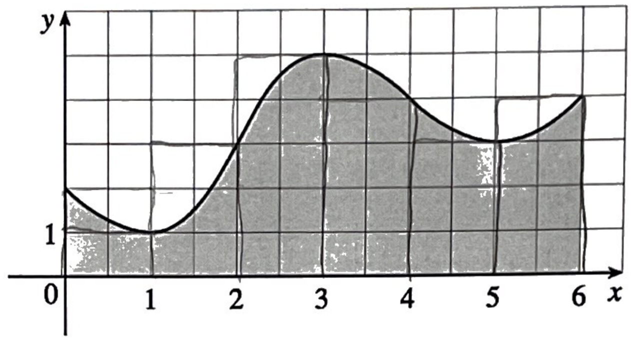
MATH:1860 Activity 3 – (Sections 7.5, 7.7-7.8) Feb. 06

Name:__________________________________

Instructions: Work with others or independently to complete the activity.

1. A radar gun was used to record the speed of a runner during the first 6 seconds of a race, resulting in the following graph where y is the speed of the runner in meters per second (m/s) and x is the time in seconds (s).



(a) Use the Trapezoidal Rule with $n = 6$ to estimate the distance the runner covered during the first 6 seconds.

$\frac{1}{2}((2+1)+(3+1)+(5+3)+(4+5)+(3+4)+(4+3))$

$= 18$

(b) Use the Midpoint Rule with $n = 6$ to estimate the distance the runner covered during the first 6 seconds.

$\int_0^6 f(x)dx \approx \Delta x \sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)$ $[0,1][1,2][2,3][3,4][4,5][5,6]$

$\left(\frac{2+1}{2} + \frac{3+1}{2} + \frac{5+3}{2} + \frac{4+5}{2} + \frac{3+4}{2} + \frac{4+3}{2}\right)$

$= 19$

(c) Do you think this runner could beat a runner who could run 22.5 meters in 6 seconds?

Yes

2. Evaluate the integral $\int_1^{\infty} \frac{1}{x^2+x}\,dx$.

$x^2+x = x(x+1)$ $\quad 1 = Ax + A + Bx$ $\quad A=1 \quad B=-1$

$\int \frac{1}{x} + \frac{-1}{x+1} = \ln|x| - \ln|x+1| \Big|_1^{\infty} =$

$\infty - \infty$

$\ln\left|\frac{x}{x+1}\right| \Big|_1^{\infty} = \ln\left|\lim_{x\to\infty} \frac{x}{x+1}\right| = \ln\left(\frac{1}{1}\right)$

$\lim_{x\to\infty} \ln\left|\frac{x}{x+1}\right|$ $\quad = \ln 1 = \boxed{0}$

$0 - \ln\left|\frac{1}{2}\right| \quad = \ln 2$

3. This problem gives one way to evaluate $\int \frac{1}{(\sin x + \cos x)^2}\,dx$.

(a) Prove $\frac{1}{(\sin x + \cos x)^2} = \frac{1}{1+\sin 2x}$.

(b) Use the trig identities $\cos^2 x = \frac{1+\cos 2x}{2}$ and $\sin x = \cos(x - \frac{\pi}{2})$ (i.e., the graph of $\sin x$ is obtained by shifting the graph of $\cos x$ to the right by $\pi/2$) to derive the identity $2\cos^2\left(x - \frac{\pi}{4}\right) = 1 + \sin(2x)$.

(c) Evaluate $\int \frac{1}{(\sin x + \cos x)^2}\,dx$.