Worksheet 12

CS 2210 Discrete Structures

Due 4/23 9pm. Late submissions get grade 0.

* Teams of 3-4 students (must work in group). Follow direction given during discussion.
** This page is double sided. Make sure to do both sides. Show your work.

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Question 1: Find the solution for $a_n = -2a_{n-1} + a_{n-2} + 2a_{n-3}$, with $a_0 = 0, a_1 = 1$, and $a_2 = 1$. Prove by induction that your solution is indeed a solution to this recursion.

$r^3 = -2r^2 + r + 2$

$r^3 + 2r^2 - r - 2 = 0$

$(r-1)(r^3 + 3r + 2)$

$(r-1)(r+1)(r+2)$

$r = 1, -1, -2$

$a_n = A(1)^n + (-1)^n + (-2)^n$

$a_n = A + (-1)^n + (-2)^n$

a_0: $0 = A + B + C$

a_1: $1 = A - B - 2C$

a_2: $1 = A + B + 4C$

$A = -B - C$

$1 = \cancel{-B} - C \cancel{+B} + 4C$

$C = \frac{1}{3}$

$1 = -2B - 1$

$2 = -2B$

$B = -1$

$0 = A - 1 + \frac{1}{3}$

$A = \frac{2}{3}$

$a_n = \frac{2}{3} + (-1)^{n+1} + \frac{1}{3}(-2)^n$

Proof

BC $a_0 = \frac{2}{3} + (-1)^1 + \frac{1}{3}(-2)^0 = 0$ ✓

$a_1 = \frac{2}{3} + (-1)^2 + \frac{1}{3}(-2)^1 = 1$ ✓

$a_2 = \frac{2}{3} + (-1)^3 + \frac{1}{3}(-2)^2 = 1$ ✓

IA: Assume for $n = k-1, k-2, k-3$ that

$a_{k-i} = \frac{2}{3} + (-1)^{k-i+1} + \frac{1}{3}(-2)^{k-i}$, $i = 1, 2, 3$

IS: Prove $a_k = -2a_{k-1} + a_{k-2} + 2a_{k-3}$

$a_k = -2\left(\frac{2}{3} + (-1)^k + \frac{1}{3}(-2)^{k-1}\right) + \left(\frac{2}{3} + (-1)^{k-1} + \frac{1}{3}(-2)^{k-2}\right) + 2\left(\frac{2}{3} + (-1)^{k-2} + \frac{1}{3}(-2)^{k-3}\right)$

$a_k = -\frac{4}{3} - 2(-1)^k - \frac{2}{3}(-2)^{k-1} + \frac{2}{3} + (-1)^{k-1} + \frac{1}{3}(-2)^{k-2} + \frac{4}{3} + 2(-1)^{k-2} + \frac{2}{3}(-2)^{k-3}$

$a_k = \frac{2}{3} - 2(-1)^k - \frac{1}{-2}\frac{2}{3}(-2)^k + -(-1)^k + \frac{1}{4}\frac{1}{3}(-2)^k + 2(-1)^k + \frac{1}{-8}\frac{2}{3}(-2)^k$

$a_k = \frac{2}{3} + (-1)^{n+1} + \frac{1}{3}(-2)^n$

Proved by induction

Question 2: Solve the following recursive problem: $T(n) = 3T(n-1) + 2, T(0) = 4$. Prove by induction that your solution is indeed a solution to this recursion.

$T(0) = 4 \quad T(1) = 14 \quad T(2) = 44 \quad T(3) = 144$

$T(n) = A \cdot 3^n + B$

$3(A \cdot 3^{n-1} + B) + 2$

$A3^n + 3B + 2$

$A3^n + B = A3^n + 3B + 2$

$B = 3B + 2$

$-2B = 2$

$B = -1$

$T(0) = A3^0 - 1 = 4$

$A = 5$

$T(n)\ 5 \cdot 3^n - 1$

Proof

BC: $T(0) = 5 \cdot 3^0 - 1 = 4 \checkmark$

IA: Assume for $n = k$, $T(k) = 5 \cdot 3^k - 1$

IS: Prove for $n = k+1$, $T(k+1) = 3T(n) + 2$

$3(5 \cdot 3^k - 1) + 2 \quad -1$

$15 \cdot 3^k - 3 + 2$

$15 \cdot 3^k - 1$

$5 \cdot 3^{k-1} - 1$

Proved by induction

Question 3: Decide whether the relation $R = \{(1,1), (1,3), (2,1), (2,2), (2,3), (3,3)\}$ on set $S = \{1,2,3\}$ is reflexive, symmetric, antisymmetric and/or transitive. Explain.

Reflexive: Yes. $(1,1), (2,2), (3,3) \in R$

Symmetric: no. Counterexample: $(1,3) \in R$ but not $(3,1)$

Antisymmetric: yes, There is no (a, b) and (b, a) in R. $a,b \in S$

Transitive: yes, $(1,3)$ and $(3,3) \to (1,3)$ yes

$(2,1)$ and $(1,3) \to (2,3)$ yes

Question 4: Determine whether the relation R where $R = \{(a,b) | a \neq b \text{ and } a, b \in \mathbb{Z}\}$ is reflexive, symmetric, antisymmetric and/or transitive. Explain.

Reflexive: no, it is stated that $a \neq b$ so it will never be reflexive

Symmetric: yes, for all (a, b) exists (b, a)

Antisymmetric: no, (a,b) and (b,a) must exist

Transitive: no. Counterexample: $(1, 2)$ $(2, 1)$: $(1, 1)$ doesn't exist