Exam 2

Graded

Student

Colin Cano

Total Points

45 / 60 pts

Question 1

Q1

10 / 10 pts

- ✔ **+ 10 pts** Correct
- **+ 2 pts** Correct derivatives
- **+ 2 pts** Correct substitution of expressions for y, y'' into the differential equation
- **+ 2 pts** Correct conclusion based on previous work
- **+ 4 pts** Incorrect but reasonable attempt
- **+ 0 pts** Missing / Incorrect

Question 2

Q2

6 / 10 pts

- **+ 10 pts** Correct
- **+ 2 pts** Correctly wrote given differential equation in the form $y' - \frac{2}{x}y = x$
- ✔ **+ 2 pts** Found the correct integrating factor based on previous work
- ✔ **+ 2 pts** Correctly multiplied $y' + P(x)y = Q(x)$ by the integrating factor found
- ✔ **+ 2 pts** Integrated correctly based on previous work
- **+ 1 pt** Simplified correctly to get an equation of the form $y = F(x)$
- ✔ **+ 1 pt** Found the correct value of C based on the initial value
- **+ 4 pts** Incorrect but reasonable attempt
- **− 1 pt** Answer can be simplified further
- ✔ **− 1 pt** Erroneous negative sign after integrating
- **+ 0 pts** Missing / Incorrect

Question 3

Q3

7 / 10 pts

Part(a)

- ✔ **+ 5 pts** Correct and clear explanation
- **+ 4 pts** Correct but unclear explanation
- **+ 3 pts** Incorrect but clear explanation
- **+ 2 pts** Incorrect and unclear explanation
- **+ 1 pt** Incorrect but reasonable attempt
- **+ 0 pts** Missing / Incorrect

Part(b)

- **+ 5 pts** Correct
- ✔ **+ 2 pts** Correctly separated the variables
- **+ 1 pt** Correctly integrated both sides based on previous work
- **+ 1 pt** Correctly took the natural log of both sides of the equation
- **+ 1 pt** Correctly simplified to an equation of the form $y = F(x)$
- **+ 1 pt** Incorrect but reasonable attempt
- **+ 0 pts** Missing / Incorrect

Question 4

Q4

10 / 10 pts

- ✔ **+ 10 pts** Correct
- **+ 2 pts** Correct formula for odd-indexed terms
- **+ 2 pts** Correct formula for even-indexed terms
- **+ 2 pts** Correct limit of odd-indexed terms based on previous work, or conclusion that the series of odd-indexed terms is divergent
- **+ 2 pts** Correct limit of even-indexed terms based on previous work, or conclusion that the series of even-indexed terms is convergent
- **+ 1 pt** Correctly reasoned that the Test for Divergence hypothesis is satisfied, or that the sum of a convergent series and a divergent series is divergent
- **+ 1 pt** Concluded that the series is divergent
- **+ 4 pts** Incorrect or insufficient justification but reasonable attempt
- **+ 0 pts** Missing / Incorrect

Question 5

Q5 **5** / 10 pts

Part(a)

+ 5 pts Correct

✔ **+ 1 pt** Recognized the series as a telescoping series

✔ **+ 2 pts** Correct partial sum formula

+ 1 pt Correct computation of limit of partial sums based on previous work

+ 1 pt Correctly stated the sum of the series based on previous work

+ 2 pts Incorrect but reasonable attempt

+ 0 pts Missing / Incorrect

Part(b)

+ 5 pts Correct

+ 1 pt Recognized the series as a geometric series

+ 2 pts Correctly rewrote the series

+ 1 pt Correctly applied the geometric series formula based on previous work

+ 1 pt Correctly found the sum of the series based on previous work

✔ **+ 2 pts** Incorrect but reasonable attempt

+ 0 pts Missing / Incorrect

Question 6

Q6

7 / 10 pts

Part(a)

✔ **+ 5 pts** Correct

+ 1 pt Attempted to apply a derivative rule to compute the derivative of f

+ 2 pts Correct derivative of f

+ 1 pt Stated that the denominator of f' is always greater than 0 on $[2, \infty)$

+ 1 pt Correctly showed that the numerator of f' is less than 0 on $[2, \infty)$

+ 2 pts Incorrect but reasonable attempt

− 1 pt Mistake simplifying derivative

+ 0 pts Missing / Incorrect

Part(b)

+ 5 pts Correct

+ 2 pts Correctly rewrote integral by u-substitution

+ 1 pt Correct computation of integral based on previous work

+ 1 pt Correct evaluation at the bounds of integration based on previous work

+ 1 pt Stated whether the series is convergent or divergent based on the result of the integral

✔ **+ 2 pts** Incorrect but reasonable attempt

− 1 pt Incorrect bounds of integration

+ 0 pts Missing / Incorrect

MATH:1860 – Exam 2 Apr. 10

Name: Colin Caro

Instructions: There is a total of 6 problems on this exam. Each problem is worth 10 points. Be sure to show all your work, write neatly and legibly, and simplify your final answers. Any problem with a correct answer without work to support it will receive 0 points. If you have any questions about a problem, you can raise your hand or come up and ask.

1. (10 points) Determine whether $y = \sin x + \cos 2x$ is a solution of the differential equation

$$y'' + y + 3\cos 2x = 0$$

$y' = \cos x - 2\sin 2x$

$y'' = -\sin x - 2 \cdot 2\cos 2x$

$= -\sin x - 4\cos 2x$

$= (-\sin x - 4\cos 2x) + (\sin x + \cos 2x) + 3\cos =$

$= -4\cos 2x + 4\cos 2x = 0$

$= 4\cos 2x = 4\cos 2x$

yes they're equal so it's a solution

2. (10 points) Solve the initial-value problem

$$xy' - 2y = x^2,\ x > 0,\ y(1) = 2$$

$\frac{dy}{dx} + P(x)y = Q(x)$

$x\frac{dy}{dx} - 2y = x^2$

$\frac{-2}{x^2}y = -x\ y'$

$\frac{dy}{dx} - 2y = x$

$\frac{dy}{dx} - 2y = \frac{x}{1}$

$\frac{dy}{dx} - \frac{2y}{x} = 1$

$I(x) = e^{\int -\frac{2}{x}dx} = e^{\ln x^{-2}\,dx}$

$= x^{-2}$

$y' - \frac{2}{x}y = 1 \cdot x^{-2}$

$\frac{d}{dx}(x^{-2}y) = x^{-2}$

$\int \frac{d}{dx}(x^{-2}y) = \int x^{-2}$

$x^{-2}y = -\frac{1}{x} + C$

$y = -\frac{x^2}{x} + Cx^2$

$y = -x + Cx^2$

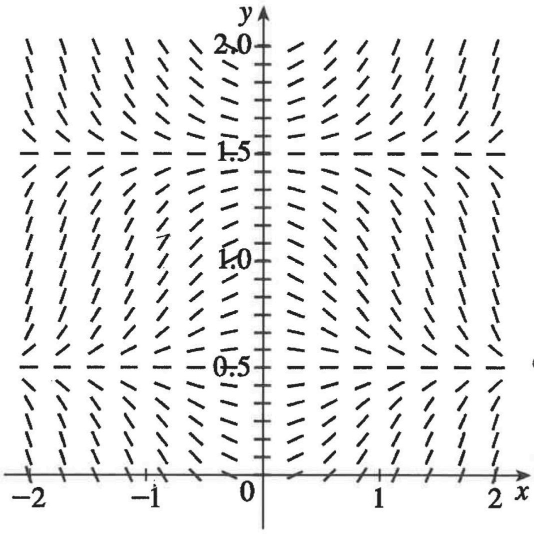
$2 = -1 + C(1)^2$

$3 = C$

$\boxed{y = -x + 3x^2}$

3. (10 points) Consider the differential equation $\dfrac{dy}{dx} = xe^y$.

(a) (5 points) Is this the direction field for the differential equation? Justify your answer.



$(0,1) = 0 \cdot e^1 = 0$ ✓

$(1,0) = 1 \cdot e^0 = 1$ ✓

$(-1,1) = -e^0 = -1$ ✓

$(-2,1) = -2e^1 \approx -5.4$ ✓

$(1,1.5) = e^{1.5} \approx 4.48$ X

at $y=1.5$ it should not be slope of 0

X

(b) (5 points) Solve the differential equation.

$$\frac{dy}{dx} = xe^y$$

$$\frac{dy}{e^y} = x\,dx$$

$$\int \frac{1}{e^y}\,dy = \int x\,dx$$

$$\ln|e^y| = \frac{x^2}{2} + C$$

$$e^{\ln|e^y|} = e^{x^2/2} e^C$$

$$|e^y| = e^{x^2/2} C$$

$$\ln|e^y| = \ln\left[e^{x^2/2} C\right]$$

$$y = \ln\left(e^{x^2/2} \cdot C\right)$$

4. (10 points) Determine whether the following series is convergent or divergent

$$\frac{1}{3}+\frac{9}{4}+\frac{2}{5}+\frac{9}{16}+\frac{3}{7}+\frac{9}{64}+\frac{4}{9}+\frac{9}{256}+\frac{5}{11}+\cdots$$

$$\sum_{n=1}^{\infty} \frac{n}{2n+1} + \sum_{n=1}^{\infty} \frac{9}{4^n}$$

$$\lim_{n\to\infty} \frac{n}{2n+1} \overset{LH}{=} \frac{1}{2} \neq 0$$

So this diverges, meaning the whole series diverges.

5. (10 points) Determine whether the following series are convergent or divergent. If the series is convergent, find its sum. Justify your answers.

(a) (5 points) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) - \ln\left(\frac{n+1}{n+2}\right)$ $= \left(\ln\frac{1}{2} - \ln\frac{2}{3}\right) + \left(\ln\frac{2}{3} - \ln\frac{3}{4}\right) \ldots - \ln\frac{n+1}{n+2}$

$$= \ln\frac{1}{2} - \ln\frac{n+1}{n+2}$$

the series will cancel each other out

Convergent

(b) (5 points) $\sum_{n=1}^{\infty} 2^{n+1}3^{-n}$ $= \sum_{n=1}^{\infty} 2 \cdot 2^n \cdot 3^{-n} = 2\sum_{n=0}^{\infty} 2 \cdot 2^n \cdot 3^{0n} = 4\sum_{n=0}^{\infty} 2^n$

So, $ar^n = \frac{4}{1-2} = -4$ $\quad |-4| > 1$ so it diverges

6. (10 points) Use the Integral Test to determine whether $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is convergent or divergent.

(a) (5 points) Let $f(x) = \frac{1}{x \ln x}$. Then f is continuous and positive on $[2, \infty)$. Prove that f is decreasing on $[2, \infty)$.

$$f'(x) = \frac{-\ln x + 1}{(x \ln x)^2} < 0 \qquad 1 < \ln x \quad \text{So its decreasing}$$

$$\frac{d}{dx}(x \ln x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

always positive

(b) (5 points) Use the Integral Test to determine whether the series is convergent or divergent.

$$\int_2^{\infty} \frac{1}{x \ln x}\, dx = \int_2^{\infty} \frac{1}{x} \cdot \frac{1}{\ln x}\, dx$$

$u = (\ln x)^{-1}$ $dv = \frac{1}{x}\, dx$

$du = \frac{-(\ln x)^{-2}}{x}$ $v = \ln x$

new bounds

$\frac{1}{\infty} = 0$ $\ln 2$

$$= \frac{1}{\ln x} \ln x - \int \ln x \cdot \frac{1}{-x(\ln x)^2}$$

$$= 1 - \frac{\ln x}{-x(\ln x)^2} \Big| = \infty$$

Divergent