Homework 06

● Graded

Student

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Total Points

10 / 10 pts

Question 1

Completion

8 / 8 pts

✔ **– 0 pts** Complete

– 2 pts Mostly complete

– 4 pts Half complete

– 6 pts Mostly incomplete

– 8 pts Incomplete

Question 2

Correctness of 9.5#18

2 / 2 pts

✔ **– 0 pts** Correct

– 1 pt Partially correct

– 2 pts Incomplete or incorrect

Homework #6
9.3

4.) $xy' = y+3 \Rightarrow x\frac{dy}{dx} = y+3 \Rightarrow \frac{dy}{y+3} = \frac{dx}{x}$

$\int \frac{dy}{y+3} = \int \frac{dx}{x}$ 1

$\ln|y+3| = \ln|x| + C \quad y+3 = e^C \cdot x \Rightarrow \boxed{y = Cx - 3}$

6.) $y' + xe^y = 0 \Rightarrow \frac{dy}{e^y} = -x\,dx \Rightarrow \int \frac{dy}{e^y} = \int -x\,dx$

$e^{-y} = -\frac{x^2}{2} + C$

$\boxed{y = -\ln\left(\frac{x^2}{2} - C\right)}$

10.) $\frac{dz}{dt} + e^{t+z} = 0 \Rightarrow \frac{dz}{dt} = -e^t e^z \Rightarrow$

$\frac{dz}{e^z} = -e^t dt \Rightarrow \int \frac{dz}{e^z} = \int -e^t dt$

$= \quad e^{-z} = -e^t + C$

$\boxed{z = -\ln(-e^t + C)}$

18.) $x + 3y^2\sqrt{x^2+1}\frac{dy}{dx} = 0, \quad y(0) = 1$

$3y^2\sqrt{x^2+1}\frac{dy}{dx} = -x \Rightarrow \frac{-x\,dx}{3\sqrt{x^2+1}} = \frac{dy}{y^2}$

$\int \frac{-x\,dx}{3\sqrt{x^2+1}} = \int \frac{dy}{y^2}$

$\int \frac{-x\,dx}{3\sqrt{x^2+1}} \quad u = x^2+1$

$du = 2x$

$= -\frac{1}{6}\int \frac{du}{u^{1/2}} = 2u^{1/2}\left(-\frac{1}{6}\right) = -\frac{1}{3}\sqrt{x^2+1} + C$

$-\frac{1}{3}\sqrt{x^2+1} = -\frac{1}{y}$

$\frac{1}{\frac{1}{3}\sqrt{x^2+1} + C} = y$

$1 = \frac{1}{\frac{1}{3}\sqrt{0^2+1} + C} \Rightarrow \frac{1}{3} + C = 1 \quad C = -\frac{2}{3}$

$\boxed{y = \frac{1}{\frac{1}{3}\sqrt{x^2+1} - \frac{2}{3}}}$

22) $f(x) = xf(x) - x$

$f(1) = 4$

$\frac{dy}{dx} = xy - x$

$= x(y-1)$

$\int \frac{dy}{1} = \int x\,dx$

$\ln|y-1| = \frac{x^2}{2} + C$

$\ln|2-1| = \frac{0^2}{2} + C$

$0 = C$

$\pm y - 1 = e^{x^2/2}$

$y = e^{x^2/2} + 1$

$f(x) = e^{x^2/2} + 1$

9.4 $\frac{dP}{dt} = 0.4P - 0.001P^2$ $P(0) = 50$

$\frac{400}{2} = 200$

6.a) $\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{400}\right)$ Carry Capacity: $\boxed{400}$

6.b) $P'(0) = 0.4P(0)\left(1 - \frac{P(0)}{400}\right) = 20\left(1 - \frac{1}{8}\right) = \boxed{17.5}$

6.c) $P(t) = \frac{M}{1+Ce^{-kt}}$ $C = \frac{M - P_0}{P_0}$ $C = \frac{400 - 50}{50} = 7$

$P(t) = \frac{400}{1+7e^{-0.4t}}$

$200(1+7e^{-0.4t}) = 400 \Rightarrow (1+7e^{-0.4t}) = 2 \Rightarrow 7e^{-0.4t} = 1$

$e^{-0.4t} = \frac{1}{7} \Rightarrow -0.4t = \ln\left|\frac{1}{7}\right| = -\ln 7$ $\boxed{\frac{-\ln 7}{-0.4} \approx 4.86 \text{ years}}$

10.a) $M = 800$ million $P_0 = 282$ million $C = \frac{800 - 282}{282} \approx 1.8376$

$P(t) = \frac{800}{1+1.8736e^{-kt}}$

10.b) $P(10) = 309 \Rightarrow 309 = \frac{800}{1+1.8736e^{-10k}} \Rightarrow (1+1.8736e^{-10k})309 = 800 \Rightarrow$

$\frac{800}{309} - 1 = 1.8736e^{-10k} \Rightarrow \frac{\frac{491}{309}}{1.8376} = e^{-10k} \Rightarrow -10k = \ln\left(\frac{\frac{491}{309}}{1.8376}\right)$

$k = \frac{\ln\left(\frac{\frac{491}{309}}{1.8376}\right)}{-10} \approx 0.015$

10.c) $P(100) = \frac{800}{1+1.8736e^{-0.015(100)}} \approx \boxed{561.7 \text{ million}}$

$P(200) = \frac{800}{1+1.8736e^{-0.015(200)}} \approx \boxed{727.3 \text{ million}}$

10.d $500 = \frac{800}{1+1.8736e^{-0.015t}} \Rightarrow 500(1+1.8736e^{-0.015t}) = 800 \Rightarrow \frac{800}{500} = 1+1.8736e^{-0.015t}$

$\frac{800}{500} - 1 = 1.8736e^{-0.015t} \Rightarrow \frac{0.6}{1.8736} = e^{-0.015t} \Rightarrow \ln\left|\frac{0.6}{1.8736}\right| = -0.015t$

$t = \frac{\ln\frac{0.6}{1.8736}}{-0.015} \approx \boxed{76.7 \text{ years}}$

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12. $M = 10000$ $P_0 = 400$ $C = \frac{10000 - 400}{400} = 24$

12a.) $P(t) = \frac{10000}{1+24e^{-kt}}$ $t = 1$

$1200(1+24e^{-k}) = 10000 \Rightarrow \frac{25}{3} = 1+24e^{-k}$

$\frac{22}{3} = 24e^{-k} \Rightarrow \frac{11}{36} = e^{-k} \Rightarrow \ln\left(\frac{11}{36}\right) = -k$ $k = -\ln\left(\frac{11}{36}\right) \approx 1.186$

$P(t) = \frac{10000}{1+24e^{-1.186t}}$

12.6) $5000 = \frac{10000}{1+24e^{-1.186t}} \Rightarrow 5000(1+24e^{-1.186t}) = 10000$

$= 2 = 1+24e^{-1.186t} \Rightarrow 1 = 24e^{-1.186t} \Rightarrow \frac{1}{24} = e^{-1.186t}$

$\frac{\ln|\frac{1}{24}|}{-1.186} = \frac{-1.186t}{-1.186}$

$t \approx 2.68$ years

9.9

$\frac{dy}{dx} + P(x)y = Q(x)$

2.) $y' - x = y\tan x$ $\quad \frac{dy}{dx} - \tan x\, y = x$ ✓ yes

4.) $\frac{dR}{dt} + t\cos R = e^{-t}$ $\quad$ X non linear

10.) $2x\frac{dy}{dx} + y = 2\sqrt{x} \Rightarrow \frac{dy}{dx} + \frac{y}{2x} = x^{-1/2}$

$\int \frac{d}{dx}(\sqrt{x}y) = \int x^{-1/2} \cdot x^{1/2} = 1$

$I(x) = e^{\int P(x)\,dx}$

$= e^{\ln x^{1/2}} = x^{1/2}$

$\sqrt{x}y = x + C$

$y = \frac{x+C}{\sqrt{x}} = \boxed{\sqrt{x} + Cx^{-1/2}}$

18. $x\frac{dy}{dx} - 2y = 2x$, $y(2) = 0$

$\frac{dy}{dx} - \frac{2}{x}y = 2$

$I(x) = e^{\int -\frac{2}{x}}$

$= x^{-2}$

$\int \frac{d}{dx}(x^{-2}y) = \int 2x^{-2}\,dx$

$x^{-2}y = 2x^{-1} + C$

$y = -2x + Cx^2$ $\quad \boxed{y = -2x + x^2}$

$0 = -2(2) + C(2)^2$

$= -4 + 4C$

$\frac{4}{4} = C \quad C = 1$

Section 9.6

2.a) Cooperation because both positively affect each other.

2.b) Competition because if you increase y it would decrease x and vice versa

4. $\frac{dW}{dt} = r_w W - \alpha WH$

$\frac{dH}{dt} = bWH - d_H H - CHL$

$\frac{dL}{dt} = eHL - d_L L$