Final Test

Graded

Student

Colin Cano

Total Points

73.25 / 150 pts

Question 1

P1 **5** / 7 pts

- ✔ **+ 3 pts** Euler circuit correct: No
- **+ 4 pts** Euler path correct: Yes, for example 7168743548528
- **+ 3 pts** Yes to Euler path, minor mistake
- ✔ **+ 2 pts** Yes to the path, only 2/3 of path are correct
- **+ 1 pt** Yes Euler path, no path or only one or two edges of the path
- **+ 0 pts** No answer/both incorrect

Question 2

P2 **6** / 7 pts

- **+ 7 pts** All are correct.
- ✔ **+ 2 pts** Yes Hamiltonian circuit.
- ✔ **+ 2 pts** Correct Hamiltonian circuit, e.g., 8516234798
- ✔ **+ 2 pts** Yes Hamiltonian path.
- **+ 1 pt** Correct Hamiltonian path. For example, 851623479. Hamiltonian circuit is NOT a Hamiltonian path.
- **+ 2 pts** The path and circuit were correct but flipped.
- **+ 0 pts** Incorrect.

Question 3

P3 **3** / 7 pts

+ 7 pts Graph is bipartite and partitions are specified correctly.
V1 = {1, 2, 4, 5, 7}
V2 = {3, 6, 8, 9} or vice versa

+ 6 pts Partitions are specified correctly, but it is not specified whether graph is bipartite.

+ 5 pts Graph is bipartite and graph is labeled/colored correctly, but partitions are not specified

✔ **+ 3 pts** Graph is bipartite, but partitions are incorrect.

+ 2 pts Only said graph is bipartite.

+ 0 pts Completely incorrect or not answered.

Question 4

P4 **10.75** / 12 pts

- **+ 12 pts** All correct
- **+ 3 pts** (a) All degrees are correct
- ✔ **+ 2.75 pts** (a) All degrees correct, but - flipped with +.
- **+ 2.5 pts** (a) 1-2 degrees incorrect
- **+ 2 pts** (a) 3-4 degrees incorrect
- **+ 1.5 pts** (a) 5-6 degrees incorrect
- **+ 1 pt** (a) 7-8 degrees incorrect
- **+ 1 pt** (b) Correct type of graph (Simple directed)
- ✔ **+ 3 pts** (c) Correct matrix
- **+ 2.75 pts** (c) 1 entry incorrect
- **+ 2.5 pts** (c) 2-3 entries incorrect
- **+ 1.5 pts** (c) 4-6 entries incorrect
- **+ 1 pt** (c) 6-9 entries incorrect
- ✔ **+ 3 pts** (d) Correct adjacency lists
- **+ 2.5 pts** (d) 1 list incorrect
- **+ 2 pts** (d) 2 lists incorrect
- **+ 1.5 pts** (d) 3 lists incorrect
- **+ 1 pt** (d) 4 lists incorrect
- **+ 0.5 pts** (d) 5 lists incorrect
- **+ 1.75 pts** (d) correct lists but not in set notation
- ✔ **+ 2 pts** (e) Correct induced graph
- **+ 0 pts** Answer is missing/ incorrect.

Question 5

P5 **2** / 8 pts

- **+ 8 pts** Yes isomorphic and correct isomorphism function
- **+ 7 pts** isomorphic, correct identification but did not use function notation i.e. f(1) = e
- **+ 7 pts** Correct functions, forgot to mention isomorphic
- **+ 5 pts** Yes isomorphic and partially correct function
- **+ 4 pts** Yes isomorphic, no function
- ✔ **+ 2 pts** No Isomorphic, but work shown
- **+ 0 pts** totally incorrect

Question 6

P6 **2** / 6 pts

- **+ 6 pts** Correct answer with all three cases explained
- **+ 4 pts** Correct answer with some explanations for each row
- ✔ **+ 2 pts** Correct answer (only explained the answer).
- **+ 1 pt** Just correct answer, no explanations.
- **+ 1 pt** explanations given for cases, wrong conclusion
- **+ 1.5 pts** correct explanations but somehow wrong conclusion
- **+ 0 pts** Did not attempt/All incorrect

Question 7

P7 **10** / 10 pts

✔ **+ 5 pts** (a) Correct and work shown

+ 3.5 pts (a) Correct Probability for 2 and 7 and ($\frac{1}{10} \cdot \frac{1}{10}$, correct probabilities for 3 and 6 ($\frac{1}{10} \cdot \frac{2}{10}$ or consistent with probabilities) and correct probabilities for 4 and 5 ($\frac{2}{10} \cdot \frac{1}{10}$. Missing the flip (7 and 2 and so on).

+ 4 pts (a) correct steps, but wrong final answer

+ 2 pts (a) missing a couple of cases to get 9, but has correct/consistent probabilities for the rest.

+ 2 pts (a) correct answer with no explanation

✔ **+ 5 pts** (b) Correct answer ($\frac{39}{10}$) or correct E(X) based on (a) answers.

+ 4 pts (b) correct steps, but wrong final answer

+ 2 pts (b) Correct probabilities for all the possible values of a dice ($P(2) = P(3) = P(5) = P(7) = \frac{1}{10}\ and\ P(1) = P(4) = P(6) = \frac{2}{10}$)

+ 1 pt Correct X values

+ 2 pts (b) Correct setup for equation and work or consistent with probabilities from (a)

+ 2 pts (b) correct answer with no explanation

+ 0 pts All incorrect or missing

+ 1.5 pts attempted (a) and (b), but wrong final answer

+ 2 pts correct computations but missing 7 in (a)

+ 2 pts correct computations but missing 7 in (b)

+ 3.5 pts total probability taken as 7 instead 10, but showed all steps correctly

+ 1 pt attempted (a)

+ 1 pt attempted (b) excluding 7

Question 8

P8 **0** / 8 pts

- **+ 2 pts** Correct first box conditional probability $\left(\frac{1}{3} \cdot \frac{2}{5}\right)$
- **+ 2 pts** Correct second box conditional probability $\left(\frac{1}{3} \cdot \frac{1}{2}\right)$
- **+ 2 pts** Correct third box conditional probability $\left(\frac{1}{3} \cdot \frac{1}{3}\right)$
- **+ 2 pts** Answer is explained
- **+ 1 pt** Explanation has errors or does not match work
- ✔ **+ 0 pts** All wrong or missing
- **+ 0 pts** Click here to replace this description.
- **+ 0 pts** Click here to replace this description.

Question 9

P9 **3.5** / 6 pts

- **+ 6 pts** Correct solution with steps.
- **+ 4 pts** Correct but not enough work shown
- **+ 2 pts** Correct (4), but no work shown.
- ✔ **+ 3.5 pts** Steps shown, but has mistake.
- **+ 0 pts** All wrong or missing

Question 10

P10 **10** / 14 pts

+ 14 pts Correct (a), (b), (c), (d) and (e)

✔ **+ 3 pts** (a) correct
$\{(1,1),(1,2),(1,3),(1,4),(1,5),(2,2),(2,3),(2,4),(2,5),(3,3),(3,4),(3,5),(4,4),(4,5),(5,5)\}$

+ 0 pts (a) partially correct (.2 per each) - added in point adjustment

✔ **+ 3 pts** (b) correct

+ 2 pts (b) missing arrows or self loops

✔ **+ 4 pts** (c) correct
yes, no - not symmetric as $((1,2) \in R, (2,1) \notin R)$, yes, yes, yes, no - not equivalence as R is not symmetric

+ 0 pts (c) partially correct (.5 per for each 'yes' and 1 for each 'no'+explanation) - added in point adjustment

+ 2 pts (d) correct
$(\{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(4,$

+ 1 pt (d) partially correct (missing R)

+ 2 pts (e) correct (R is already transitive)

+ 1 pt (e) partially correct (missing specific explanation as its same as R)

+ 0 pts All wrong or missing

+ 0.5 pts attempted (d) and (e), but wrong final answer

Question 11

P12

1 / 8 pts

- **+ 8 pts** Correct proof
- **+ 2 pts** Correct definition of odd/even
- ✔ **+ 1 pt** Partially correct definition of odd/even
- **+ 4 pts** Correct math
- **+ 3 pts** Majority of math correct but did not end in the form: $2 \cdot (2k^2 + 2k) + 1$ or other minor error
- **+ 2 pts** Minor errors in math and did not end in the form $2 \cdot (2k^2 + 2k) + 1$
- **+ 2 pts** Correctly showed x^2 is odd/even
- **+ 1 pt** Plugged in $z = 2k^2 + 2k$ but did not specify it was an integer **and** why (or similar expression for even case)
- **+ 1 pt** Said $2 \cdot (2k^2 + 2k) + 1$ was odd by definition and that $2k^2 + 2k$ is an integer but did not correctly plug in $z = 2k^2 + 2k$
- **+ 0 pts** Wrong or missing

1 If you were using contraposition the statement to prove would become:
If x^2 is even then x is even. So you would have to start with x^2 to then make some conclusion about x which doesn't really workout.

Question 12

P13

4 / 9 pts

- **+ 9 pts** Click here to replace this description.
- **+ 3 pts** Binary conversion correct (110111110011 or Correct based on other answer)
- **+ 2 pts** Binary showed conversion but has a mistake
- **+ 1 pt** Correct binary, but no work
- **+ 3 pts** Octal conversion correct (6763 or Correct based on other answer)
- ✔ **+ 2 pts** Octal showed conversion but has a mistake
- **+ 1 pt** Correct octal, but no work
- **+ 3 pts** Hex conversion correct ($DF3$ or Correct based on other answer)
- ✔ **+ 2 pts** Hex showed conversion but has a mistake
- **+ 1 pt** Correct hex, but no work
- **+ 0 pts** All wrong or missing

Question 13

P14 **4** / 12 pts

✔ **+ 2 pts** (a) correct $(2^8\ OR\ 256)$

+ 1.75 pts (a) forgot to include the empty set and answered 255 or other minor error

+ 1.5 pts (a) used permutations instead of combinations

+ 2 pts (b) correct $\binom{8}{6}\ OR\ 28$

+ 1.5 pts (b) close with minor error

+ 2 pts (c) correct $\binom{4}{2}\binom{4}{2}\ OR\ 36$

+ 1.5 pts (c) correct terms but added $\binom{4}{2}+\binom{4}{2}$ instead of using multiplication or similar small error

+ 0.75 pts (c) used $\binom{4}{2}+/x\binom{4}{2}$ with additional incorrect terms

+ 2 pts (d) correct $\binom{4}{4}\ OR\ 1$

+ 1.5 pts (d) minor error

+ 2 pts (e) correct $(\{\emptyset,\{2\},\{5\},\{6\},\{2,5\},\{2,6\},\{5,6\},\{2,5,6\}\})$

+ 1.75 pts (e) small error

+ 1 pt (e) incorrectly included all orderings or other major error

✔ **+ 2 pts** (f) correct 5

+ 1 pt (f) found $T \cap R$ but did not find cardinality

+ 1 pt (f) found cardinality of incorrectly evaluated $T \cap R$

+ 0 pts Completely incorrect

Question 14

P15 **0** / 16 pts

- **+ 2 pts** Correct roots
- **+ 1 pt** Wrong roots
- **+ 1 pt** Correct in between equation
- **+ 2 pts** Correct alphas using you equation.
- **+ 1 pt** Wrong alphas
- **+ 2 pts** Correct solution using your alphas
- **+ 1 pt** Solution is missing ()
- **+ 2 pts** BC: Correct base case
- **+ 1 pt** BC: One case instead of two or missing side.
- **+ 1 pt** Correct IA
- **+ 1 pt** Correct IS set up
- **+ 1 pt** Used IA correctly in IS and specified it
- **+ 0.5 pts** Didn't specify that IA is used, but used correctly
- **+ 4 pts** IS: Correct math
- **+ 3 pts** IS: Most steps are correct
- **+ 2 pts** Multiple steps missing
- **+ 1 pt** IS: Only one step was right
- ✔ **+ 0 pts** Wrong or Empty

Question 15

P16 **3** / 6 pts

- **+ 3 pts** (a) correct: $O(n^9)$
- **+ 1.5 pts** *(a) provided k instead*
- ✔ **+ 3 pts** (b) correct: $O(2^n)$
- **+ 5 pts** Correct but needs the proper O notation.
- **+ 4 pts** Correct but time notation needs coefficients removed.
- **+ 0 pts** All wrong or missing

Question 16

P17

4 / 8 pts

- **+ 4 pts** (a) Correct
- **+ 3.25 pts** (a) Minor mistake/ Forgot about the probability
- **+ 2 pts** (a) Wrong but tried
- **+ 4 pts** (b) Correct
- **+ 3.25 pts** (b) Minor mistake
- **+ 2 pts** (b) Wrong but tried
- **+ 0 pts** Wrong or missing
- **+ 8 pts** Both Correct
- ✔ **+ 4 pts** Tried both but wrong

Question 17

P18

1 / 6 pts

- **+ 6 pts** Correct
- **+ 5 pts** Mostly correct.
- **+ 5 pts** Left x.
- **+ 3 pts** Partially correct.
- ✔ **+ 1 pt** Minimally correct.
- **+ 0 pts** Empty or totally wrong.

Question 18

EX

4 / 0 pts

+ 9 pts Correct

✔ **+ 3 pts** Pre-order is correct.

+ 1 pt Pre-order is partially correct.

+ 3 pts In-order is correct.

✔ **+ 1 pt** In-order is partially correct.

+ 3 pts Post-order is correct.

+ 1 pt Post-order is partially correct.

+ 4 pts Pre-order, post-order switched.

+ 2 pts correct pre order but switched with other traversal

+ 2 pts correct in order but switched with other traversal

+ 2 pts correct post order but switched with other traversal

+ 0 pts Answer is missing/ incorrect.

515
P12

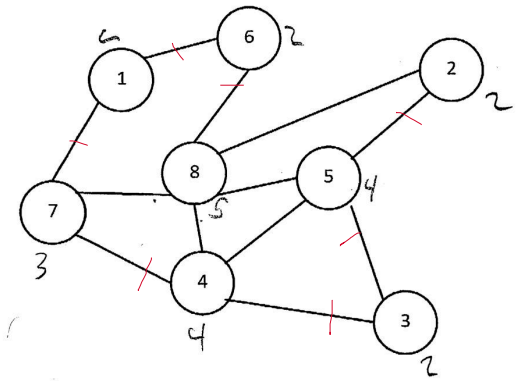
CS 2210 Discrete Structures

Final Test

Name: Colin Cond Student ID 01573158

- You have 120 minutes to complete this exam.
- If you enter the room, you must turn in an exam before leaving the room.
- You must show your work and have the correct final answer to receive full credit.
- You can use a simple calculator.

Problem 1: (7 points) Decide whether the graph below has an Euler circuit, and/or path (state answer for both). If yes, exhibit one by stating vertices in order visited (*2852*) or giving sequence of edges in order used ((*2,8*),(*8,5*),(*5,2*)).



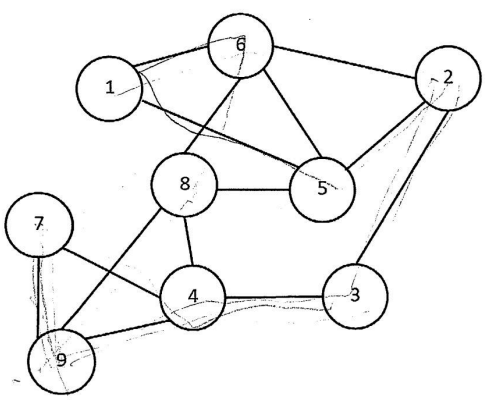
It has an euler's path since exactly 2 vertices have odd degree

No euler circuit.

25347168

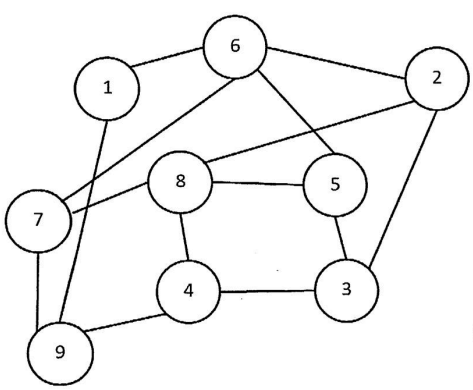
Problem 2: (7 points) Decide whether the below graph has Hamiltonian circuit and/or Hamiltonian path (state answer for both). If yes, exhibit one by stating vertices in order visited (*1561*) or giving sequence of edges in order used ((*1,5*),(*5,6*),(*6,1*)).

9743251689



Yes it has a hamilton path and circuit.

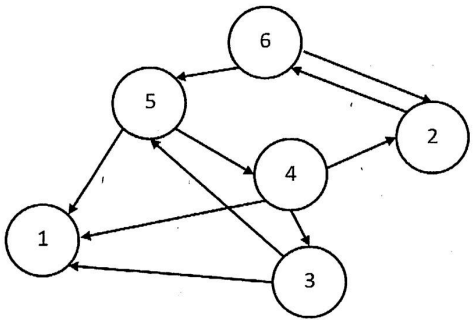
Problem 3: (7 points) Decide whether below graph is a bipartite graph. If yes, what are V_1 and V_2?



V_1	V_2
1	6
	9
4	9
	5
	3
2	6
	7
	3

Yes bipartite since no V_2 edges connect.

Problem 4: (12 points) For the graph below:



(a) Find degree-in and degree out for every vertex.

$deg^-(1) =$ 0 $deg^+(1) =$ 3

$deg^-(2) =$ 1 $deg^+(2) =$ 2

$deg^-(3) =$ 2 $deg^+(3) =$ 1

$deg^-(4) =$ 3 $deg^+(4) =$ 1

$deg^-(5) =$ 2 $deg^+(5) =$ 2

$deg^-(6) =$ 2 $deg^+(6) =$ 1

(b) What type of graph is it?

Direct

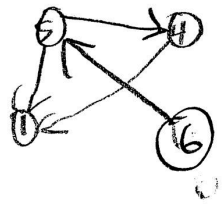
(c) Represent graph using adjacency matrix.

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 & 0 \\ 6 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$

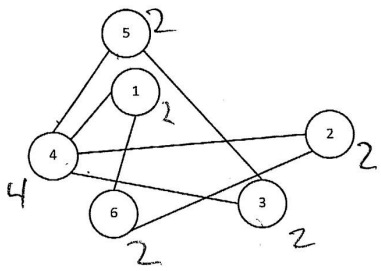
(d) Represent graph using adjacency list.

1	{∅}
2	{6}
3	{1,5}
4	{1,2,3}
5	{1,4}
6	{2,5}

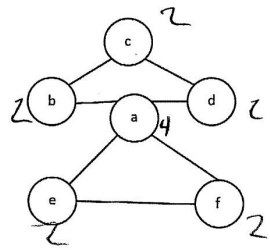
(e) Find the subgraph of G induced by W={1, 4, 5, 6}.



Problem 5: (8 points) Decide whether those two graphs are isomorphic. If yes, give the function for isomorphism.



G1



G2

Not isomorphic since G_2 has bcd isolated with a while G_1 has 53 isolated with 4.

4→5
5→3

a→d
d→c

Problem 6: (6 points) As a reward for saving his daughter from pirates, the King has given you the opportunity to win a treasure hidden inside one of three trunks. The two trunks that do not hold the treasure are empty. To win, you must select the correct trunk. Trunks 1 and 3 are each inscribed with the message "The treasure is in Trunk 2," and Trunk 2 is inscribed with the message "The treasure is in Trunk 3." The Queen, who never lies, tells you that only one of these inscriptions is true, while the other two are wrong. Can you know for sure where the treasure is? Explain your answer in detail. Examine every option. State where the treasure can be.

well since 2 inscriptions have to be false and 2 of the inscriptions are the same thing. The treasure must be in trunk 3

Problem 7: (10 points) Two seven-sided dice are rolled. Each dice is biased so that a 1, 4 and 6 comes up to 2 times as often as each of the other numbers.

(a) What is the probability that the numbers on the dice add up to 9?

Potential combos: (2,7), (3,6), (4,5), (5,4), (6,3), (7,2) $\quad 2,3,5,7 = \frac{1}{10} \quad 1,4,6 = \frac{2}{10}$

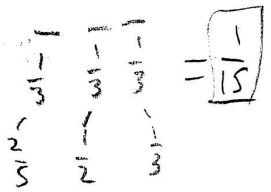
$$= \left(\frac{1}{10}\cdot\frac{1}{10}\right)+\left(\frac{1}{10}\cdot\frac{2}{10}\right)+\left(\frac{2}{10}\cdot\frac{1}{10}\right)+\left(\frac{1}{10}\cdot\frac{2}{10}\right)+\left(\frac{2}{10}\cdot\frac{1}{10}\right)+\left(\frac{1}{10}\cdot\frac{1}{10}\right)$$

$$= \boxed{0.1}$$

(b) What is the expected value of the dots when only one dice is rolled?

$$\frac{2+3+5+7+(1\cdot 2)+(4\cdot 2)+(6\cdot 2)}{18} = \boxed{3.9}$$

Problem 8: (8 points) Consider three boxes, one containing two black and three white marble, second one black and one white marble, third two white and one black marble. A box is selected at random with equal probability, and then a marble is drawn at random with equal probability from the selected box. What is the probability that the marble picked has black color? Explain.



$= \boxed{\frac{1}{15}}$

Since there is equal probality for boxes, it's $\frac{1}{3}$ probality a box gets picked. Then the marbles all equal probality so you can multiply them to get anuser

Problem 9: (6 points) Compute $(4^8 mod 7)^2 mod 11$. Show your steps. You can't use calculator to directly compute 4^8, instead use theorem from the class on distributing *mod*. Show all steps.

$$(4^8 \bmod 7)^2 \bmod 11 = ((4 \bmod 7)^8) \bmod 11 = (4^8)^2 \bmod 11$$

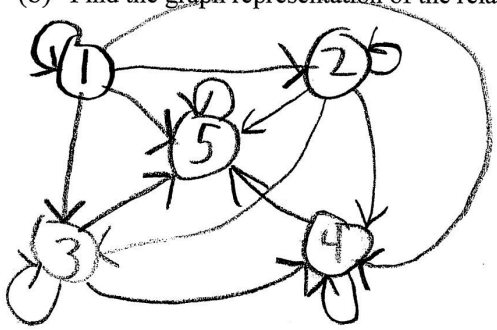
$$= 4^{16} \bmod 11 = (4 \bmod 11)^{16} = \boxed{4^{16}}$$

Problem 10: (14 points) For R={(1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,5)} on set X={1, 2, 3, 4, 5}.

(a) Find the matrix representation of the relation R on X.

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 1 & 1 & 1 \\ 4 & 0 & 0 & 0 & 1 & 1 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}$$

(b) Find the graph representation of the relation R on X.



(c) Determine if R reflexive Yes, symmetric no, antisymmetric Yes, transitive Yes, a partial order Yes, and/or equivalence relation no? If answered no, explain why.

It's not symmetric since $\in (1,2)$, there should be $\in (2,1)$ but there is not. Not equivalence relation since it is not symmetric.

(d) Find the symmetry closure on the relation R.

(4,5)

(e) What is the transitive closure on the relation R

(4,5)

Problem 12: (8 points) Prove that square of any odd number is odd.

Proof by contrapositive: Prove the square of any even number is even

let $\exists b \in \mathbb{Z}$.

Thus, $b^2 = b \cdot b = 2b$

By def of an even number (2b) this means b^2 has to be even.

Proved by contraposition

Problem 13: (9 points) Convert the 3571 to hex, octal and to binary. Show your work.

(3571)10

0011 — 0101 0111 0001

0011 0101 0111 0001

1 1 1

5 7 1

0 011 010 101 110 001

3 2 5 6 1

0		10
1		11
10	2	100
11	3	101
100	4	110
101	5	111
110	6	1000
111	7	
1000	8	
1001	9	

hex 32561

binary 00110101 0111 0001

octal 3571

Problem 14: (12 points) Let T={1,2,3,4,5,6,7,8} and set R={1,3,4,7,8}.

(a) How many subsets of T are there?

$2^8 = 256$

(b) How many 6-element subsets of T are there?

6

(c) How many 4-element subsets of T consist of 2 odd and 2 even numbers?

16

(d) How many 4-element subsets of T consist only of odd numbers?

16

(e) Find power set of S=T-R.

{∅, {1}, {3}, {4}, {7}, {8}, {1,3}, {1,4}, {1,7}, {1,8}, {3,4}, {3,7}, {3,8},
{4,7}, {4,8}, {7,8}, {1,3,4}, {1,3,7}, {1,3,8}, {1,4,7}, {1,4,8}, {1,7,8}
{3,4,7}, {3,4,8}, {4,7,8}, {1,3,4,7}, {1,3,4,8}, {1,4,7,8}, {1,3,7,8}
{3,4,7,8}, {1,3,4,7,8}}

(f) Compute $|T \cap R|$.

$T \cap R = \{1,3,4,7,8\}$

$|T \cap R| = 5$

Problem 15: (16 points) Find solution for s_n, where $s_0 = 4, s_1 = -8$, and $s_n = -4s_{n-1} + 21s_{n-2}$ for $n \geq 2$. Prove your answer is correct using induction.

$S_n = -4S_{n-1} + 21S_{n-2}$

$r = -4r^{n-1} + 21r^{n-2}$

$= -4r^n r^{-1} + 2r^n r^{-2}$

BC: let $n = 2$

$S_2 = -4(-8) + 21(4)$

$= 32 + 84 = 116$ ✓

IA: Assume $n = k$ s.t $\exists k \in \mathbb{Z}$

IS: let $n = k+1$

$S_n = -4S_k + S_{k-1} \cdot 21$

Problem 16: (6 points) Find the big-O (closest bound).

(a) $s(n) = (n^2 + 1)(2n^5 + 7n^7 + 6)$

$2n^{10} + 7n^{14} \ldots \quad \in O(n^{14})$

(b) $s(n) = \sqrt{n^3 - 2n} + 2^n \quad \in O(2^n)$

Problem 17: (8 points) There are 52 cards in the deck.

(a) What is the probability that a six-card poker hand contains exactly two diamond suit card and the queen of hearts? Explain your answer.

$\binom{52}{1}\binom{13}{2}\binom{36}{1}\binom{35}{1}\binom{34}{1} = \binom{52}{1}\binom{13}{2}\binom{36}{1}\binom{35}{1}\binom{34}{1}$

a queen ♡

13 diamonds left and need 2 diamonds

can't be any more diamond cards

(b) What is the probability that a five-card poker hand contain a ten? Explain your answer.

all cards in deck

$\binom{52}{5} - \binom{52}{4}$

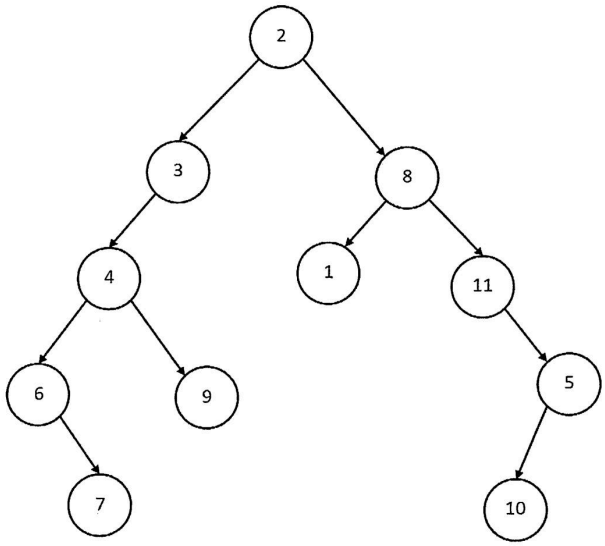
there is 4 tens in deck

Problem 18: (6 points) What is the coefficient of x^9 in $(2 - x)^{19}$?

$2^{19} - x^{19}$

$2^{19} - x^{10} x^{9}$

Extra credit: (9 points) Write the elements of the tree below in the order they would be visited by each kind of traversal:



- Pre-order: 2, 3, 4, 6, 7, 9, 8, 1, 11, 5, 10
- In-order: 6, 7, 4, 9, 3, 2, 1, 8, 11, 5, 11
- Post-order: 6, 7, 9, 4, 3, 1, 10, 5, 11, 8, 2