MATH:1860 Activity 4 – (Sections 8.1-8.2)

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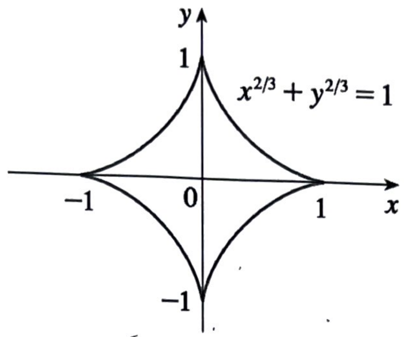
Instructions: Work with others or independently to complete the activity.

1. Find the length of the astroid $x^{2/3} + y^{2/3} = 1$.

$y = (1-x^{2/3})^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}(1-x^{2/3})^{1/2} \cdot -\frac{2}{3}x^{-1/3}$



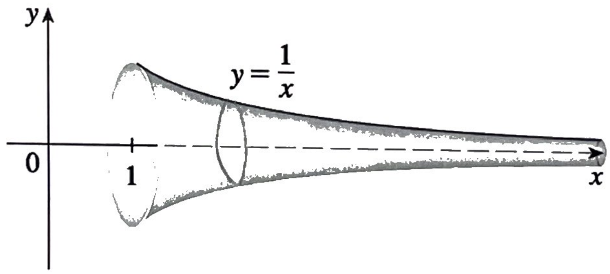
$L = \int_{-1}^{1} \sqrt{1 + \left(\frac{3}{2}(1-x^{2/3})^{1/2} \cdot -\frac{2}{3}x^{-1/3}\right)^2}\, dx$

$= \int_0^1 x^{-1/3}\, dx$

$= \frac{3}{2}x^{2/3}\Big|_0^1 = \frac{3}{2}$

$4 \cdot \frac{3}{2} = 6$!

2. The surface formed by rotating the curve $y = 1/x, x \geq 1$, about the x-axis is known as Gabriel's horn. Show that the surface area is infinite, although the enclosed volume is finite.



(a) Prove that the enclosed volume is finite.

$V = \int A(x)\, dx$ $\quad A(x) = \pi\left(\frac{1}{x}\right)^2$

$V = \int_1^\infty \pi\left(\frac{1}{x}\right)^2 dx$

(b) Prove that the surface area is infinite.

i. Show that it is given by the integral $2\pi \int_1^{\infty} \frac{1}{x^3}\sqrt{x^4+1}\,dx$.

$S = \int 2\pi y \, ds \qquad S = \int 2\pi \frac{1}{x} \sqrt{1+x^4}\, dx$

$\lim_{n \to \infty} \sum_{i=1}^{n}$

ii. Make the u-substitution $u = x^2$ and then the trigonometric substitution $u = \tan\theta$ to rewrite the integral given in (i) as $\pi \int_{\pi/4}^{\pi/2} \csc^2\theta \sec\theta \, d\theta$.

iii. Use the trigonometric identity $\cot^2\theta + 1 = \csc^2\theta$ to finish computing the integral given in (ii).