Math homework 4

8.1

14. $x = \frac{y^4}{8} + \frac{1}{4y^2}$, $1 \le x \le 2$ $\qquad x' = \frac{y^3}{2} - \frac{1}{2y^3}$

$$L = \int_1^2 \sqrt{1 + \left(\frac{y^3}{2} - \frac{1}{2y^3}\right)^2}\, dy \qquad (a-b)^2 = a^2 - 2ab + b^2$$

$$= \sqrt{1 + \left(\frac{y^3}{2}\right)^2 - 2\left(\frac{y^3}{2} \cdot \frac{1}{2y^3}\right) + \left(\frac{1}{2y^3}\right)^2}$$

$$= \sqrt{\frac{y^6}{4} + \frac{1}{2} + \frac{1}{4y^6}} = \frac{y^3}{2} + \frac{1}{2y^3} = \int_1^2 \frac{y^3}{2} + \frac{1}{2y^3}\, dy = \frac{1}{2}\int y^3 + \frac{1}{y^3}$$

$$= \frac{y^4}{4} + \frac{-1}{2y^2}\Big|_1^2 = \left(\frac{15}{8} + \frac{3}{16}\right) = \boxed{\frac{33}{16}}$$

16. $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{3}$ $\qquad y' = \frac{-\sin x}{\cos x}\, dx$

$$L = \int_0^{\pi/3} \sqrt{1 + \left(\frac{-\sin x}{\cos x}\right)^2}\, dx = \int \sqrt{1 + \tan^2 x} = \int \sqrt{\sec^2 x} = \int \sec x$$

$$= \ln|\sec x + \tan x|\Big|_0^{\pi/3} = \ln(2 + \sqrt{3})$$

19. $x = \frac{1}{3}\sqrt{y}(y-3)$, $1 \le y \le 9$

$x = \frac{\sqrt{y}(y-3)}{3}$

$x' = \frac{y-1}{2\sqrt{y}}\, dy$

$$L = \int_1^9 \sqrt{1 + \left(\frac{y-1}{2\sqrt{y}}\right)^2}\, dy$$

$$= \int \sqrt{1 + \left(\frac{(y-1)^2}{4y}\right)}\, dy$$

$$= \int \sqrt{\frac{(y-1)^2 + 4y}{4y}}\, dy$$

$$= \int \sqrt{\frac{y^2 + 2y + 1}{4y}}\, dy$$

$$= \int \frac{y+1}{2\sqrt{y}}\, dy = \int \frac{y^{1/2}}{2} + \frac{1}{2\sqrt{y}}\, dy$$

$$= \frac{y^{3/2}}{3} + \sqrt{y}\Big|_1^9 = \left(\frac{9^{3/2}}{3} + 3\right) - \left(\frac{1^{3/2}}{3} + 1\right)$$

$$\boxed{= \frac{32}{3}}$$

$S(x) = \int_0^x \sqrt{3t+5}\, dt$

44.a) arclength: $1 + f'(t)^2 = 3t + 5$

$f'(t)^2 = 3t + 4$

$f'(t) = \sqrt{3t+4}$ $\quad \int \sqrt{3t+4} = \frac{2}{3}(3t+4)^{3/2} \cdot 3 + C$

$f(0) = 2 = \frac{2}{3}(3(0)+4)^{3/2} \cdot 3 + C$

$2 = \frac{2}{9}(8) + C$ $\quad f(x) = \frac{2}{9}(3x+4)^{3/2}$

$\frac{18}{9} - \frac{16}{9} = C$ $\quad C = \frac{2}{9}$

44.b $\quad 3 = \int_0^x \sqrt{3t+5}\, dt$ $\quad 3 = \frac{2}{3} \cdot \frac{1}{3}(3t+5)^{3/2} \Big|_0^x$

$3 = \frac{2}{9}\left((3x+5)^{3/2} - (0+5)^{3/2}\right)$

$\frac{27}{2} + 5^{3/2} = (3x+5)^{3/2}$

$\left(\frac{27}{2} + 5^{3/2}\right)^{2/3} = 3x + 5$

$x = \frac{1}{3}\left(\left(\frac{27}{2} + 5^{3/2}\right)^{2/3} - 5\right)$

Section 8.2

3.b) $x = \ln(2y+1)$, $0 \le y \le 1$ x axis $x' = \frac{2}{2y+1}$

$S = \int_0^1 2\pi y \sqrt{1 + \left(\frac{2}{2y+1}\right)^2}\, dy$

3.a) $S = \int_0^{\ln 3} 2\pi\left(\frac{1}{2}e^x - \frac{1}{2}\right)\sqrt{1 + \frac{1}{4}e^{2x}}\, dx$ $x = \ln(2y+1)$

$e^x = 2y+1$ $y = \frac{1}{2}e^x - \frac{1}{2}$

$y' = \frac{1}{2}e^x$

5. a) $1 \le x \le 8$ $y = \frac{4}{x}$ $y' = \frac{-4}{x^2}$

$S = \int 2\pi x \sqrt{1 + \frac{16}{x^4}}\, dx$

5.b) $x = \frac{4}{y}$ $x' = \frac{-4}{y^2}$ $S = \int 2\pi\left(\frac{4}{y}\right)\sqrt{1 + \left(\frac{16}{x^4}\right)}\, dy$

12. $y = \sqrt{1+e^x}$, $0 \le x \le 1$ x axis $y' = \frac{1}{2}(1+e^x)^{-1/2} e^x$

$= \frac{e^x}{2\sqrt{1+e^x}}$

$\sqrt{1 + \left(\frac{e^x}{2\sqrt{1+e^x}}\right)^2} = \sqrt{1 + \left(\frac{(e^x)^2}{4(1+e^x)}\right)}$

$= \frac{e^x + 2}{2\sqrt{1+e^x}}$ $S = \int 2\pi\left(\sqrt{1+e^x}\right)\left(\frac{e^x+2}{2\sqrt{1+e^x}}\right) dx$

$= \int_0^1 2\pi\left(1 + \frac{e^x}{2}\right)$

$= 2\pi\left(x + \frac{e^x}{2}\right)\Big|_0^1 = \boxed{2\pi\left(\frac{1}{2} + \frac{e}{2}\right)}$

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16. $x = 1 + 2y^2$, $1 \le y \le 2$ x axis $x' = 4y$

$1 + (y')^2 = 1 + 16y^2$

$S = 2\pi \int_1^2 y\sqrt{1+16y^2}\, dy = \frac{\pi}{16}\int_1^2 (16y^2+1)^{1/2}\, 32y\, dy = \frac{\pi}{16}\left(\frac{2}{3}(16y^2+1)^{3/2}\right)\Big|_1^2 =$

$\boxed{\frac{\pi}{24}\left(65\sqrt{65} - 17\sqrt{17}\right)}$

$u = 16y^2 + 1$ $\frac{1}{2}u^{3/2}\, du =$

$du = 32y\, dy$