$\int u\,dv = uv - \int v\,du$

$* \to \frac{x^2}{2}$

$1e^x \to e^{x}$

$-e$

e^{-y}

Homework #1 Colin Ccano

Section 7.1

6. $\int ye^{-y}\,dy$ $\quad u = y \quad dv = e^{-y}\,dy$

$du = dy \quad v = -e^{-y}$

$\int ye^{-y}\,dy = y(-e^{-y}) - \int -e^{-y}\,d\cancel{x}$

$= y(-e^{-y}) + \int e^{-y}\,dy$ $\qquad \int e^{-y}\,dy = -e^{y}$

$\boxed{\int ye^{-y}\,dy = -ye^{-y} - e^{-y} + C}$

8. $\int (\pi - x)\cos\pi x\,dx$ $\quad u = (\pi - x) \quad dv = \cos\pi x\,dx$

$du = -dx \quad v = \frac{1}{\pi}\sin(\pi x)$

$\int (\pi - x)\cos\pi x\,dx = (\pi - x)\cdot\frac{1}{\pi}\sin(\pi x) - \int \frac{1}{\pi}\sin(\pi x)\cdot -dx$

$= \frac{(\pi - x)\sin\pi x}{\pi} + \frac{1}{\pi}\int \sin(\pi x)\,dx$

$\int \sin(\pi x)\,dx = -\frac{\cos(\pi x)}{\pi}$ $\qquad \int (\pi - x)\cos\pi x\,dx = \boxed{\frac{(\pi - x)\sin\pi x}{\pi} - \frac{\cos(\pi x)}{\pi^2} + C}$

10. $\int \frac{\ln x}{x^2}\,dx = \ln x\, x^{-2}\,dx$ $\quad u = \ln x \quad dv = x^{-2}\,dx$

$du = \frac{1}{x}\,dx \quad v = -x^{-1}$

$\int \ln x\, x^{-2}\,dx = \ln x \cdot (-x^{-1}) - \int -x^{-1}\cdot\frac{1}{x}\,dx$

$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx$ $\qquad \int \frac{1}{x^2}\,dx = -\frac{1}{x}$

$\boxed{\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x + 1}{x} + C}$

12. $\int t^2 \sin(\beta t)\,dt$ $\quad u = t^2 \quad dv = \sin(\beta t)\,dt$

$du = 2t\,dt \quad v = -\frac{\cos(\beta t)}{\beta}$

$\int t^2\sin(\beta t)\,dt = t^2 \cdot -\frac{\cos(\beta t)}{\beta} - \int \frac{-\cos(\beta t)}{\beta}\cdot 2t\,dt$

$= -\frac{t^2\cos(\beta t)}{\beta} + \frac{2}{\beta}\int t\cos(\beta t)\,dt$ $\quad u = t \quad dv = \cos(\beta t)\,dt$

$du = dt \quad v = \frac{\sin\beta t}{\beta}$

$\int t\cos\beta t\,dt = \frac{t\sin\beta t}{\beta} - \frac{1}{\beta}\int \sin\beta t\,dt$ $\qquad \int \sin\beta t\,dt = \frac{-\cos\beta t}{\beta}$

$\int t\cos\beta t\,dt = \frac{t\sin\beta t}{\beta} + \frac{\cos(\beta t)}{\beta^2}$ $\qquad \int t^2\sin\beta t\,dt = -\frac{t^2\cos\beta t}{\beta} + \frac{2}{\beta}\left(\frac{t\sin\beta t}{\beta} + \frac{\cos\beta t}{\beta^2}\right)$

$\boxed{\int t^2\sin\beta t\,dt = -\frac{t^2\cos\beta t}{\beta} + \frac{2t\sin(\beta t)}{\beta^2} + \frac{2\cos(\beta t)}{\beta^3} + C}$

$v\,du$

14. $\int \ln(\sqrt{x})\,dx$

$u = \ln(\cancel{x})$ $\quad dv = dx$

$du = \frac{1}{\cancel{x}}dx$ $\quad v = x$

$\int \ln(\sqrt{x})\,dx =$

$\frac{1}{2}\int \ln(x)\,dx = x\ln(x) - \int x \cdot \frac{1}{x}\,dx$

$= x\ln(x) - \int 1\,dx$

$\int \ln x\,dx = x\ln(x) - x + C$

$\int \ln(\sqrt{x})\,dx = \frac{1}{2}(x\ln x - x) + c$

$\boxed{\int \ln(\sqrt{x})\,dx = \frac{x\ln x}{2} - \frac{x}{2} + c}$

22. $\int e^x \sin(\pi x)\,dx$

$du = \pi\cos(\pi x)\,dx$ $\quad u = \sin(\pi x)$

$v = e^x$ $\quad dv = e^x\,dx$

$\int e^x\sin(\pi x)\,dx = \sin\pi x \cdot e^x - \int e^x \pi\cos(\pi x)\,dx$

$= \sin(\pi x)\,e^x - \pi\int e^x\cos(\pi x)\,dx$

$\int e^x\cos(\pi x)\,dx$ $\quad u = \cos(\pi x)$ $\quad dv = e^x\,dx$

$du = -\pi\sin(\pi x)\,dx$ $\quad v = e^x$

$\int e^x\cos(\pi x)\,dx = \cos\pi x \cdot e^x + \pi\int e^x\sin(\pi x)\,dx$

$\int e^x\sin(\pi x)\,dx = e^x\sin(\pi x) - \pi\left(e^x\cos(\pi x) + \pi\int e^x\sin(\pi x)\,dx\right)$

$\int e^x\sin\pi x\,dx = e^x\sin(\pi x) - \pi e^x\cos(\pi x) - \pi^2\int e^x\sin(\pi x)\,dx$

$\frac{\int e^x\sin\pi x(1+\pi^2)}{1+\pi^2} = \frac{e^x(\sin\pi x) - \pi\cos(\pi x)}{1+\pi^2}$

$\boxed{\int e^x\sin(\pi x)\,dx = \frac{e^x(\sin(\pi x) - \pi\cos(\pi x))}{1+\pi^2} + c}$

32. $\int_1^2 w^2\ln w\,dw$

$u = \ln w$ $\quad dv = w^2\,dw$

$du = \frac{1}{w}dw$ $\quad v = \frac{w^3}{3}$

$\int_1^2 w^2\ln w\,dw = \frac{\ln w \cdot w^3}{3} - \int \frac{w^3}{3}\cdot\frac{1}{w}\,dw$

$\int_1^2 w^2\ln w\,dw = \frac{\ln w \cdot w^3}{3} - \frac{1}{3}\int w^2\,dw$

$\int w^2\,dw = \frac{w^3}{3}$

$\int_1^2 w^2\ln w\,dw = \frac{\ln w\cdot w^3}{3} - \frac{w^3}{9}\Big|_1^2 = \left(\frac{\ln 2\cdot 27}{3} - 3\right) - \left(0 - \frac{1}{9}\right) = \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) + \frac{1}{9}$

$= \boxed{\frac{8}{3}\ln 2 - \frac{7}{9}}$

Section 7.2

6. $\int \cos^3(\frac{t}{2})\sin^2(\frac{t}{2})\,dt = \int \cos^2(\frac{t}{2})\sin^2(\frac{t}{2})\cos(\frac{t}{2})\,dt$

$= \int (1-\sin^2\frac{t}{2})(\sin^2\frac{t}{2}) \cdot \cos(\frac{t}{2})\,dt = \int \sin^2(\frac{t}{2}) - \sin^4(\frac{t}{2}) \cdot \cos(\frac{t}{2})\,dt$

$u = \sin\frac{t}{2}$ $\quad du = \frac{\cos(\frac{t}{2})}{2}dt$

$\int = u^2 - u^4 \cdot 2du = 2\int u^2 - u^4\,du = 2\left(\frac{u^3}{3} - \frac{u^5}{5}\right)$

$= 2\left(\frac{\sin^3(\frac{t}{2})}{3} - \frac{\sin^5(\frac{t}{2})}{5}\right)$

$\boxed{\cos^3(\frac{t}{2})\sin^2(\frac{t}{2})\,dt = \frac{2}{3}\sin^3(\frac{t}{2}) - \frac{2}{5}\sin^5(\frac{t}{2}) + C}$

8. $\int_0^{\frac{\pi}{4}} \sin^2(2\theta)\,d\theta = \int_0^{\frac{\pi}{4}} \frac{1-\cos(4\theta)}{2}\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1-\cos 4\theta)\,d\theta = \frac{1}{2}\int 1\,d\theta - \frac{1}{2}\int \cos(4\theta)\,d\theta$

$= \frac{1}{2}(\theta) - \frac{1}{2}\left(\frac{\sin 4\theta}{4}\right)\Big|_0^{\frac{\pi}{4}} = \left(\frac{\pi}{8} - 0\right) - (0) = \boxed{\frac{\pi}{8}}$

16. $\int \csc^5\theta \cos^3\theta\,d\theta = \int \csc^3\theta \cos^3\theta \csc^2\theta\,d\theta = \int \frac{\cos^3\theta}{\sin^3\theta}\csc^2\theta\,d\theta = \int \cot^3\theta \csc^2\theta\,d\theta =$

$\int \cot^2\theta \csc^2\theta \cot\theta\,d\theta$ $\quad u = \cot\theta$ $\quad du = \csc^2\theta\,d\theta$

$= \int u^3\,du = \frac{u^4}{4} = -\frac{\cot^4\theta}{4} + C$

18. $\int \tan^2 x \cos^3 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}\cos^3 x\,dx = \int \sin^2 x \cos x\,dx$ $\quad u = \sin x$ $\quad du = \cos x$

$= \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{\sin^3 x}{3} + C}$

44. $\int \sin 2\theta \sin 6\theta\,d\theta = \int \frac{1}{2}(\cos(2\theta - 6\theta) - \cos(2\theta + 6\theta))$

$= \int \sin 2\theta \sin 6\theta\,d\theta = \int \frac{1}{2}(\cos(4\theta) - \cos(8\theta)) = \frac{1}{2}\int \cos 4\theta\,d\theta - \frac{1}{2}\int \cos 8\theta\,d\theta =$

$\int \cos 4\theta = \frac{\sin 4\theta}{4}$ $\quad \int \cos 8\theta\,d\theta = \frac{\sin 8\theta}{8}$

$\frac{1}{2}\left(\frac{\sin 4\theta}{4} - \frac{\sin 8\theta}{8}\right) =$

$\boxed{\int \sin 2\theta \sin 6\theta\,d\theta = \frac{\sin 4\theta}{8} - \frac{\sin 8\theta}{16} + C}$