$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Table 6

$\neg(\neg p) \equiv p$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \vee (p \wedge q) \equiv p$ → Absorption
$p \wedge (p \vee q) \equiv p$ ↗

$p \vee \neg p \equiv T$
$p \wedge \neg p \equiv F$

$p \rightarrow q \vee \equiv \neg p \vee q$ Table 7

$\neg(p \rightarrow q) \equiv (p \wedge \neg q)$

Example

Tautology is always true: $p \vee \neg p$

Contradiction is always false: $p \wedge \neg p$

Ex: $\neg p \leftrightarrow q \;?=\; p \leftrightarrow \neg q$

p	q	¬p	① ¬p ↔ q	¬q	② p ↔ ¬q	① ↔ ②
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	T	F	T	T
F	F	T	F	T	F	T

← tautology

Use table from pg 29 to do same

8.2

$$\neg p \leftrightarrow q \equiv \neg(\neg p) \leftrightarrow \neg q \equiv p \leftrightarrow \neg q$$

(double negation; distribute)

Ex: $(\neg p \wedge (p \vee q)) \rightarrow q \equiv (\neg p \wedge p) \vee (\neg p \wedge q) \rightarrow q \equiv F \vee (\neg p \wedge q) \rightarrow q$ (negation)

$\equiv (\neg p \wedge q) \rightarrow q \equiv (\neg p \rightarrow q) \vee (q \rightarrow q) \equiv (\neg p \rightarrow q) \vee T \equiv T$ (True)

Predicates and Quantifiers

Statement that involves variables generally are not T/F

$x = y + 7$ ← not proposition

Propositional function will have variable $P(x, y)$: $x = y + 7$

depending on value of x, y

Generally P.F have 2 parts, variables & predicate

Ex: $y \leq 3$ "less equal 3" predicate

y is variable

$P(y)$: $y \leq 3$
$P(2)$: T
$P(-10)$: T
$P(10)$: F

Universal Quantifier of $P(x)$ is the statement "$P(x)$ for all values of x)

Notation: $\forall x\, P(x)$ (↑ for all)

existential Quantifier of $P(x)$ is "There exist an element x in the domain such that $P(x)$"

Notation: $\exists x\, P(x)$ (↑ exists)

Ex: All students in CS 2210 did hwl

$\forall x P(x)$ $P(x)$: x did hwl

Domain: student correctly enrolled in CS 2210

$\neg \exists x\, P(x)$: There is no student in CS 2210 such that student did hwl

Order of Precidence: $\forall$, $\exists$, take priority over logical statements

$\neg(\exists x\, P(x)) \equiv \forall x\, \neg P(x)$

$\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$

$P(x)$: x did hwl

$\exists x \neg P(x)$ exist student who didn't do hwl