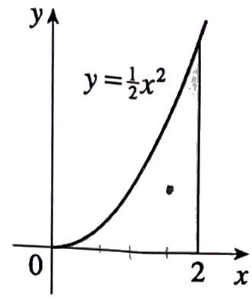
# MATH:1860 Activity 5 – (Sections 8.3-8.5) Feb. 20

**Name:** Colin Cano

**Instructions:** Work with others or independently to complete the activity.

1. Visually estimate the location of the centroid of the region shown. Then find the exact coordinates of the centroid.




$\bar{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))\,dx$

$= \frac{3}{4}\int_0^2 x\left(\frac{1}{2}x^2\right)dx = \frac{3}{4}\int \frac{1}{2}x^3$

$= \frac{3}{4}\left(\frac{x^4}{8}\right)\Big|_0^2 = \frac{3}{4}(2) = \frac{6}{4}$

$\bar{y} = \frac{3}{4}\int_0^2 \frac{1}{2}\left(\frac{1}{2}x^2\right)^2 dx = \int \frac{1}{8}x^4$

$= \frac{x^5}{40}\Big|_0^2 = \frac{3}{4}\left(\frac{32}{40}\right)$

$= \frac{3}{4}\left(\frac{4}{5}\right) = \frac{12}{20} = \frac{3}{5}$

$A = \int_0^2 \frac{1}{2}x^2\,dx = \frac{x^3}{6}$

$A = \frac{8}{6}$

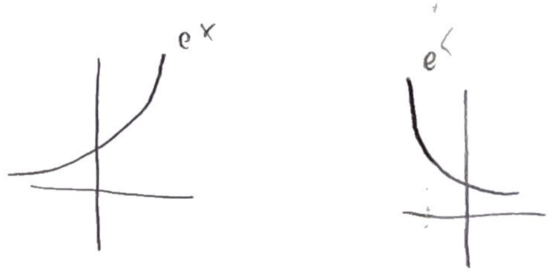
Centroid is $\left(\frac{3}{2}, \frac{3}{5}\right)$

2. An exponential density function $f(t)$ has the form

$$f(t) = \begin{cases} ce^{-ct} & \text{if } t \geq 0, \\ 0 & \text{if } t < 0 \end{cases}$$

for some $c > 0$.

(a) Verify that $f(t)$ is indeed a probability density function, that is, show $f(t) \geq 0$ for all $t$ and $\int_{-\infty}^{\infty} f(t)\,dt = 1$.




exponential functions always positive

(b) An online retailer has determined that the average time for credit card transactions to be electronically approved is 1.6 seconds. Let $T$ be a random variable representing the time a customer must wait for a transaction to be approved, and let $f(t)$ be the corresponding probability density function. Assume that $f(t)$ is given by an exponential density function as above. Find the value of c.

$\mu = 1.6$ $T$

$$1{,}6 = \int_{-\infty}^{\infty} x f(t)\,dt = \int_0^{\infty} t c e^{-ct}\,dt = c\int_0^{\infty} x e^{-cx}\,dx$$

$u = x$ $dv = e^{-cx}dx$

$du = dx$ $v = -\frac{1}{c}e^{-cx}$

$$x \cdot \frac{1}{c}e^{-cx} - \int e^{-cx}\,dx$$

$$= x \cdot \frac{1}{c}e^{-cx} - \left(-\frac{1}{c}e^{-cx}\right)\Big|_0^{\infty}$$

(c) Using the value of $c$ from part(b), find the probability that a customer waits less than a second for credit card approval.

$.5$