MATH:1860 Activity 1 – Calculus I Review **Jan. 21**

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Instructions: Work with others or independently to complete the activity.

1. Solve the following integrals.

$u = 4x^2+9 \qquad du = 8x\,dx \qquad \frac{du}{8} = dx$

(a) $\int \frac{x}{\sqrt{4x^2+9}}\,dx = \int \frac{1}{\sqrt{u}} \cdot \frac{1}{8} du = \frac{1}{8}\int u^{-1/2}\,du =$

$= \frac{1}{8} \cdot 2u^{1/2} = \frac{\sqrt{u}}{4} = \frac{\sqrt{4x^2+9}}{4} + C = \boxed{\frac{1}{4}(4x^2+9)^{1/2} + C}$

(b) $\int (4-t)\sqrt{t}\,dt = \int 4\sqrt{t} - t^{3/2}\,dt = \boxed{\frac{8}{3}t^{3/2} - \frac{2}{5}t^{5/2} + C}$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

(c) $\int_1^2 \frac{\sqrt{\ln x}}{x}\,dx = \int_1^2 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_1^2 \quad \frac{2}{3}(\ln(2))^{3/2} - \frac{2}{3}(\ln(1))^{3/2}$

$\boxed{\frac{2}{3}(\ln(2))^{3/2}}$

(d) $\int (x^e + e^x)\,dx = \boxed{\frac{x^{e+1}}{e+1} + e^x + C}$

(e) $\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int \frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \int_0^{\frac{\pi}{4}} \sec^2\theta + 1\,d\theta = (\tan\theta + \theta)\Big|_0^{\frac{\pi}{4}}$

$= \left(\tan\left(\frac{\pi}{4}\right) + \frac{\pi}{4}\right) - (\tan 0 + 0)$

$= \boxed{1 + \frac{\pi}{4}}$

2. Recall $\sec x = \dfrac{1}{\cos x}$. Prove $\dfrac{d}{dx}\sec x = \sec x \tan x$.

$$\frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

3. Let $f(x) = x^2 \cos 2x$.

(a) Find the derivative $f'(x)$.

$$f'(x) = 2x(\cos 2x) + \left(x^2 \cdot 2\cos(2x)\right)$$
$$= 2x(\cos(2x) - x\sin 2x)$$

(b) What is the slope of the line tangent to the graph of $f(x)$ at $x = \pi/4$?

$$f'\left(\frac{\pi}{4}\right) = 2\left(\frac{\pi}{4}\right)\left(\cos\left(\frac{\pi}{2}\right) + \left(\frac{\pi^2}{4} \cdot 2\cos\left(\frac{\pi}{2}\right)\right)\right)$$
$$= \frac{-\pi^2}{8}$$

(c) What is the equation of the line tangent to the graph of $f(x)$ at $x = \pi/4$?

$$y - 0 = \frac{-\pi^2}{8}\left(x - \frac{\pi}{4}\right)$$

(d) Find the net area under the graph of $f'(x)$ from $x = 0$ to $x = \pi$.

$$\int_0^{\pi} f'(x)\,dx = f(\pi) - f(0) = \pi^2$$