Worksheet 11

CS 2210 Discrete Structures

Due 4/16 9pm. Late submissions get grade 0.

* Teams of 3-4 students (must work in group). Follow direction given during discussion.
** This page is double sided. Make sure to do both sides. Show your work.

Name1: Cobin Bliss Name 2: Colin Cano

Name3: Hongwei Zhao Name 4: ______

Question 1: One hundred people line up to board an airplane. Each has a boarding pass with assigned seat. However, the first person to board has lost his boarding pass and takes a random seat. After that, each person takes the assigned seat if it is unoccupied, and one of unoccupied seats at random otherwise. What is the probability that the last person to board gets to sit in his assigned seat?

if first person sits in 1, everyone sits in the right seat
if sits in last seat, last person loses seat
every other case is .50

Therefore, the chance is $\frac{1}{2}$

Question 2: We roll the same six-sided dice twice. The dice is biased so that a 4 comes up to 3 times as often as each of the other numbers.

a. What is the probability that 4?

$P(4) = \frac{3}{6} = 3P(x)$

$\frac{1}{6} = P(1) = P(2) = P(3) = P(5) = P(6) = P(x)$

$P(1)+P(2)+P(3)+P(4)+P(5)+P(6) = 1$

$5P(x) + P(4) = 1$

$5P(x) + 3P(x) = 1$

$P(x) = \frac{1}{8} \Rightarrow 3P(x) = \boxed{\frac{3}{8}}$

b. Let the random variable be the number of dots on the dice rolled. What is the expected value of the dots when the dice is rolled once?

$E(X) = \sum_{s \in S} P(s)X(s) = 1 \cdot \frac{1}{8} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} + 4 \cdot \frac{3}{8} + 5 \cdot \frac{1}{8} + 6 \cdot \frac{1}{8} = \frac{29}{8}$

$\frac{29}{8}$

Question 3: Find the solution for $a_n = 3a_{n-1} + 4a_{n-2}$, with $a_0 = 2$ and $a_1 = 4$, use induction to prove it is a correct solution.

$r^2 - 3r - 4 = 0$

$(r-4)(r+1) = 0$

$r = 4, -1$

$a_n = \alpha_1 4^n + \alpha_2 (-1)^n$

$a_0 = \alpha_1 4^0 + \alpha_2 (-1)^0 \qquad 4 = \alpha_1 4 + \alpha_2 (-1)$

$2 = \alpha_1 + \alpha_2$

$2 + 4 = \alpha_1 4 + \alpha_2 (-1) + \alpha_1 + \alpha_2$

$6 = 5\alpha_1 + 0$

$\alpha_1 = \frac{6}{5}$

$2 = \frac{6}{5} + \alpha_2$

$\alpha_2 = \frac{4}{5}$

$a_n = \frac{6}{5}4^n + \frac{4}{5}(-1)^n$

Base Case: $a_0 = 2,\ a_1 = 4$

IA: Assume for $n = k,\ n = k-1$

$a_k = \frac{6}{5}4^k + \frac{4}{5}(-1)^k, \quad a_{k-1} = \frac{6}{5}4^{k-1} + \frac{4}{5}(-1)^{k-1}$

IS: Prove for $n = k+1$

$a_{k+1} = 3a_k + 4a_{k-1}$

$a_{k+1} = 3\left(\frac{6}{5}4^k + \frac{4}{5}(-1)^k\right) + 4\left(\frac{6}{5}4^{k-1} + \frac{4}{5}(-1)^{k-1}\right)$

$a_{k+1} = \frac{18}{5}4^k + \frac{12}{5}(-1)^k + \frac{24}{5}4^{k-1} + \frac{16}{5}(-1)^{k-1}$

$a_{k+1} = \frac{72}{5}4^{k-1} - \frac{12}{5}(-1)^{k-1} + \frac{24}{5}4^{k-1} + \frac{16}{5}(-1)^{k-1}$

$a_{k+1} = \frac{96}{5}4^{k-1} + \frac{4}{5}(-1)^{k-1}$

$a_{k+1} = \frac{6}{5}4^{k+1} + \frac{4}{5}(-1)^{k+1}$

Proved by induction