# Test 1

● Graded

**Student**

Colin Cano

**Total Points**

**78 / 100 pts**

**Question 1**

**Q1** **16** / 16 pts

✔ **+ 16 pts** All correct

**+ 2 pts** (a) correct
D = {a,g}

**+ 2 pts** (b) correct
B-A = {b,e,f,l}

**+ 2 pts** (c) correct
A U B = {a,b,c,d,e,f,g,j,k,l}

**+ 2 pts** (d) correct
6 + 6 - 2 = 10

**+ 1 pt** (d) is partially correct

**+ 2 pts** (e) correct
B' = {c,d,h,i,j,k}

**+ 3 pts** (f) correct
$\mathcal{P}(D)$ = {∅,{a},{g},{a,g}}

**+ 1.5 pts** (f) is partially correct

**+ 1 pt** (f) is partially correct - have few elements

**+ 3 pts** (g) correct
D X C = { (a,a), (a,b), (a,c), (g,a), (g,b), (g,c)}

**+ 2 pts** (g) is partially correct or missing the curly braces/ comma-separated pairs

**+ 1 pt** (g) contains additional elements

**+ 0 pts** All Wrong

**Question 2**

## Q2

**6** / 6 pts

✔ **+ 6 pts** All correct.

**+ 2 pts** Converse is correct. If September is rainy, then May is sunny.

**+ 2 pts** Inverse is correct. If May is **not** sunny, then September is **not** rainy.

**+ 2 pts** Contrapositive is correct. If September is **not** rainy, then May is **not** sunny

**+ 3 pts** Converse and inverse are switched, but both are correct. Partial grades given for both.

**+ 3 pts** Partial grades, sentence not written.

**+ 1.5 pts** Partial grades given, sentence formation incorrect (Please check comments).

**+ 0 pts** Answer is missing/incorrect.

**Question 3**

## Q3

6 / 10 pts

+ 10 pts All Correct with work shown

✔ + 1 pt Row 1 truth values T T T : **X** Cody cannot be knight and knave

+ 0.5 pts Row 1 truth values T T T : **X** No explanation provided | incorrect explanation

+ 1 pt Row 2 truth values T T F : ✓ Alex knight, Brook spy, Cody knave

✔ + 0.5 pts Row 2 truth values T T F : ✓ No explanation provided | incorrect explanation | Did not specify who's what

✔ + 1 pt Row 3 truth values T F T : **X** Cody cannot be knight and knave

+ 0.5 pts Row 3 truth values T F T : **X** No explanation provided | incorrect explanation

+ 1 pt Row 4 truth values T F F : **X** Brook is knight but lying

+ 0.5 pts Row 4 truth values T F F : **X** No explanation provided | incorrect explanation

✔ + 1 pt Row 5 truth values F T T : **X** Alex is knight but lying

+ 0.5 pts Row 5 truth values F T T : **X** No explanation provided | incorrect explanation

✔ + 1 pt Row 6 truth values F T F : **X** Alex is knight but lying

+ 0.5 pts Row 6 truth values F T F : **X** No explanation provided | incorrect explanation

+ 1 pt Row 7 truth values F F T : ✓ Alex knave, Brook spy, Cody knight or Alex spy, Brook knave, Cody knight

✔ + 0.5 pts Row 7 truth values F F T : ✓ No explanation provided | incorrect explanation | Did not specify who's what

✔ + 1 pt Row 8 truth values F F F : **X** all can't lie (need a knight)

+ 0.5 pts Row 8 truth values F F F : **X** No explanation provided | incorrect explanation

+ 2 pts Stated "you cannot decide for certain"

+ 5 pts Has right result for each row / Did not provide explanation for each row in the truth table (It says "examine all possibilities and show your work." You have to state which possibility leads to your answer. You have to state the contradictions too.)

+ 4 pts Has wrong result for each row / Did not provide explanation for each row in the truth table (It says "examine all possibilities and show your work." You have to state which possibility leads to your answer. You have to state the contradictions too.)

+ 2 pts No truth table shown, and only explanation given.

+ 2 pts Tried but wrong

1 Explanation not given for every instance

**Question 4**

**Q4** **6** / 8 pts

**+ 8 pts** All Correct

✔ **+ 1 pt** (A) is True

✔ **+ 1 pt** (A) explanation is correct

✔ **+ 1 pt** (B) is True

✔ **+ 1 pt** (B) explanation is correct

**+ 1 pt** (D) is False

**+ 1 pt** (D) explanation is correct

✔ **+ 1 pt** (E) is True

✔ **+ 1 pt** (E) explanation is correct

**+ 1 pt** Tried but wrong

**+ 0 pts** All Incorrect or not answered

**Question 5**

## Q5

**10.75** / 15 pts

+ **15 pts** All correct

+ **3 pts** (a) is correct $\neg\forall x(M(x) \vee C(x))$

+ **2.75 pts** (a) is logically equivalent but not a direct translation

+ **2 pts** (a) used $\wedge$ instead of $\vee$ but otherwise correct

+ **1.5 pts** (a) correct $(M(x) \vee C(x))$ portion

+ **1.5 pts** (a) correct negation and x quantifier

✔ + **0.75 pts** (a) correct x quantifier

✔ + **3 pts** (b) is correct $\exists x \forall y\,(M(x) \wedge \neg W(x, y))$

+ **2.5 pts** (b) is correct except missing or incorrect quantifier

+ **2 pts** (b) correct $(M(x) \wedge \neg W(x, y))$ portion

+ **2 pts** (b) incorrectly used $\rightarrow$ and incorrect y quantifier

+ **2.5 pts** (b) correct except missing $\neg$ on $W(x, y)$

+ **0.5 pts** (b) correct x quantifier

+ **1 pt** (b) correct $\neg W(x, y) portion$

✔ + **3 pts** (c) is correct $M(Tom) \rightarrow W(Anna, Tom)$

+ **2.5 pts** (c) is close but flipped Tom and Anna in $W(Anna, Tom)$

+ **2.5 pts** (c) correct except gave Tom and Anna quantifiers

+ **2 pts** (c) incorrectly gave Tom and Anna quantifiers and flipped Tom and Anna in $W(Anna, Tom)$

+ **1 pt** (c) correct $W(Anna, Tom)$ portion.

+ **0.5 pts** (c) correctly identified needed to use $\rightarrow$

+ **3 pts** (d) is correct $\forall x(M(x) \wedge C(x) \rightarrow W(x, ProfessorSmith))$

✔ + **2.5 pts** (d) is correct except missing or incorrect quantifier

+ **1 pt** (d) correct $M(x) \wedge C(x)$ portion

+ **0.5 pts** (d) correctly identified needed to use $\rightarrow$

+ **0.5 pts** (d) correctly plugged in Professor Smith for y

+ **3 pts** (e) is correct $M(Tom) \wedge \forall y(M(y) \rightarrow W(Tom, y))$

+ **2 pts** (e) correct $\forall y(M(y) \rightarrow W(Tom, y))$ portion

✔ + **0.5 pts** (e) correctly plugged in Tom for x / correct $M(Tom)$ portion

✔ + **0.5 pts** (e) correct y quantifier

+ **0.5 pts** (e) correctly identified needed to use $\rightarrow$

✔ + **0.5 pts** (e) used $\wedge$ after $M(Tom)$

+ **0.25 pts** (e) correct y quantifier but invalid placement

+ **0 pts** All wrong or missing

**Question 6**

**Q6** **7** / 14 pts

+ **14 pts** Correct proof - good job!

✔ + **2 pts** Correct Contraposition statement

+ **2 pts** Set up: Correct assumption for contraposition: n is even

+ **4 pts** Set up: Correct assumption for contradiction

+ **3 pts** Even def: Had correct def of even in set up

✔ + **2 pts** Even def: Definition of even misses integer part or definition part

+ **1 pt** Even def: Definition of even misses integer part and definition part

+ **3 pts** Math: Correct math manipulations

✔ + **2 pts** Math: Mostly correct math

+ **1 pt** Math: Some math is correct

+ **2 pts** Put in 2*s+1 form

✔ + **1 pt** Put is 2*s or 2*s-1 form

+ **1 pt** Used def to explain why odd

+ **1 pt** Explained contradiction or stated proved by contrapositive (in case prove was done right)

+ **0 pts** Wrong or missing

**Question 7**

**Q7** **8** / 8 pts

✔ **+ 8 pts** All correct.

**+ 4 pts** Logic gates are correct.

**+ 2 pts** gates correct but connections wrong/incomplete

**+ 2 pts** logic gate drawn for simplified expression $\neg q \wedge (\neg p \wedge \neg q)$

**+ 1 pt** Gate for simplified expression is partially correct

**+ 4 pts** Expressions evaluated correctly. Final answer is 0010100.

**+ 1 pt** $\neg q$ done correctly 0010101

**+ 1 pt** $p \vee q$ done correctly. 1101011

**+ 1 pt** $\neg(p \vee q)$ done correctly 0010100

**+ 1 pt** evaluated $(p \wedge q)$ 1001010

**+ 2 pts** Correct answer, work not shown.

**+ 2 pts** correct final answer for $\neg q \wedge \neg(p \vee q)$
0010100 (using simplified expression)

**+ 1 pt** partially correct final answer for $\neg q \wedge \neg(p \vee q)$ (using simplified expression)

**+ 4.5 pts** Final answer written in T/F.

**+ 0 pts** Answer is missing/incorrect.

**Question 8**

**Q8** **0** / 10 pts

**+ 10 pts** Correct - disprove using example x=1, y=1/2.

**+ 9 pts** Correct - missing steps

**+ 5 pts** Mentioned it is False

**+ 2 pts** Correct def of rational number (by def of rational number, $\exists p, q \in \mathbb{Z}, q \neq 0, x = p/q$

✔ **+ 0 pts** Wrong

**+ 0 pts** Empty

**Question 9**

## Q9

**12.5** / 13 pts

**+ 13 pts** Correct.

✔ **+ 1.5 pts** $p \rightarrow \neg q$ column is correct

✔ **+ 1.5 pts** $p \rightarrow \neg r$ column is correct

✔ **+ 1.5 pts** $(p \rightarrow \neg q) \wedge (p \rightarrow \neg r)$ column is correct

✔ **+ 1.5 pts** $q \vee r$ column is correct

✔ **+ 1.5 pts** $p \wedge (q \vee r)$ column is correct

✔ **+ 1.5 pts** $\neg(p \wedge (q \vee r))$ column is correct

✔ **+ 1 pt** $((p \rightarrow \neg q) \wedge (p \rightarrow \neg r)) \Leftrightarrow \neg(p \wedge (q \vee r))$ column correct.

**+ 3 pts** Explanation/ Stated $((p \rightarrow \neg q) \wedge (p \rightarrow \neg r)) \Leftrightarrow \neg(p \wedge (q \vee r))$ is a tautology therefore expressions are logically equivalent

✔ **+ 2.5 pts** $((p \rightarrow \neg q) \wedge (p \rightarrow \neg r)) \Leftrightarrow \neg(p \wedge (q \vee r))$ column but missing conclusion that tautology implies logical equivalence.
Shows: $(p \rightarrow \neg q) \wedge (p \rightarrow \neg r)$ and $\neg(p \wedge (q \vee r))$ have identical values.

**+ 2 pts** Missing only $((p \rightarrow \neg q) \wedge (p \rightarrow \neg r)) \Leftrightarrow \neg(p \wedge (q \vee r))$ column but correctly concluded logically equivalent

**+ 2 pts** attempted

**+ 1 pt** showed some sort of equivalency by pointing to the truth values and indicating that they are the same

**+ 0 pts** All wrong or no answer

2 bidirectional implication

**Question 10**

**EX** **5.75** / 0 pts

**+ 6 pts** Correct

✔ **+ 3 pts** a is correct: $\forall x \exists y \forall z((x \leq 3) \vee (y > -9) \wedge (z \leq 8))$

**+ 1 pt** a is correct but did not show work

**+ 1 pt** a correctly negated quantifiers

**+ 1 pt** a correctly negated inequalities

**+ 1 pt** a correctly had $\vee$ between first two clauses and $\wedge$ beween second and third clauses

✔ **– 0.25 pts** missing at least one row of work in part a

✔ **+ 3 pts** b is correct: $\forall x \exists y(P(x, y) \wedge Q(x, y))$

**+ 1.5 pts** b made rule application error but still reached correct solution

**+ 1 pt** b is correct did not show work

**+ 1 pt** b correctly negated quantifiers

**+ 1 pt** b correct use of $\wedge$

**+ 1 pt** b correct P(x,y) , Q(x,y) terms

**– 0.25 pts** missing at least one row of work in part b

**+ 0 pts** Wrong or empty

L 22

# CS 2210 Discrete Structures

## Test 1A

Name: Colin Cano ID: 01573158

- You have 60 minutes to complete this exam. **Closed book, closed notes.**
- If you enter the room, you must turn in an exam before leaving the room.
- You **must show your work** and have correct final answer to receive full credit.

**Problem 1:** (16 points) Let U = {a, b, c, d, e, f, g, h, i, j, k, l} and let sets A = {a, c, d, g, j, k}, B = {a, b, e, f, g, l}, and C={a, b, c}.

(a) Determine the set $D = A \cap B$.

$D = \{a, g\}$

(b) Determine the set $B - A$. $= \{b, e, f, l\}$

(c) Determine the set $A \cup B$. $= \{a, b, c, d, e, f, g, j, k, l\}$

(d) Find the value of $|A| + |B| - |A \cap B|$. $= 10$ (6, 6, 2)

(e) Determine $\overline{B}$. $= \{c, d, h, i, j, k\}$

(f) $\mathcal{P}(A \cap B)$. $= \{\phi, \{a\}, \{g\}, \{a, g\}\}$

(g) D×C. $= \{(a,a), (a,b), (a,c), (g,a), (g,b), (g,c)\}$

**Problem 2:** (6 points) Proposition: If May is sunny, then September is rainy.

(a) Find converse: If September is rainy, then May is sunny. ($q \to p$)

(b) Find inverse: If May isn't sunny, then September isn't rainy. ($\neg p \to \neg q$)

(c) Find contrapositive: If September isn't rainy, then May isn't sunny. ($\neg q \to \neg p$)

**Problem 3:** (10 points) On the fabled Island of Knights and Knaves, we meet three people Alex, Brook and Cody, one of whom is a knight, one a knave, and one a spy. Recall: The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth. Alex says: "Cody is a knave.", Brook says: "Alex is a knight." and Cody says: "I am a knight." Who is the knight, who the knave, and who the spy? Examine all possible options. Show your work. Can you decide for certain? Explain.

Let a represent "Cody is a knave"
let b represent "Alex is a Knight"
let c represent "I am a knight"

| a | b | c | |
|---|---|---|---|
| T | T | T | F. They can't all being telling truth |
| T | T | F | |
| T | F | T | F Cody can't be 2 things |
| T | F | F | |
| F | T | T | F. Alex can't lie if he's the knight (1) |
| F | T | F | F. (1) |
| F | F | T | |
| F | F | F | F. Not everyone can be lying. |

It is inconclusive on who the Knight, Knave and the spy because it lacks a hint on who the spy is which makes it impossible to figure out.

**Problem 4:** (8 points) Let Q(x,y) be a statement "x·y=5". The domain x, y are positive real numbers. What are the truth values of the below? Explain.

(a) Q(1,5) $x \cdot y = 5$
$1 \cdot 5 = 5$ ✓ True

(b) $\exists x \exists y \neg Q(x,y)$ $x \cdot y = 5$ let $x = 5$ let $y = 1$
$5 \cdot 1 = 5$ ✓ True

(d) $\exists x \forall y Q(x,y)$ let $x = 5$ $5 \cdot y = 5$
$y = 1$ True ✓

(e) $\forall x \exists y Q(x,y)$ $x \cdot y = 5$ $y = \frac{5}{x}$ True ✓ because for every x you could find a y since $y \in \mathbb{R}$.

**Problem 5:** (15 points) Let M(x): "x likes Math", C(x): "x likes programming", and W(x,y): " x works with y". Domain students in CS 2210 class. Use quantifiers to express each of those statements:

(a) Not all student in CS 2210 like Math or programming.

$\forall x(\neg M(x) \vee \neg C(x))$

(b) There is a student in CS 2210 who didn't work with other students in CS 2210 and likes Math.

$\exists x \forall y(\neg W(x,y) \wedge M(x))$

(c) Tom and Anna are students in CS 2210. If Tom likes Math, then Anna works with Tom.

$M(\text{Tom}) \rightarrow W(\text{anna}, \text{Tom})$

(d) If a student likes math and programming, then this student work with Professor Smith.

$\exists x(M(x) \wedge C(x) \rightarrow (W(x, \text{professor smith})))$

(e) Tom likes math and worked with all the students in class who like Math.

$M(\text{tom}) \wedge (\forall y(W(\text{tom}, y))$

**Problem 6:** (14 points) Prove or disprove: Let n be integer number. If $n^2 + 2n - 1$ is even, then $n$ is odd.

Contrapositive: if n is even, then $n^2+2n-1$ is odd.

Proof: Assume n is even and ~~$n^2+2n-1$ is odd~~ show where ~~$\exists n \in \mathbb{Z}$~~

let $n = 2k$ where $\exists k \in \mathbb{Z}$. ← can't assume

by def of even

$n^2+2n-1 = (2k)^2 + 2(2k) - 1$

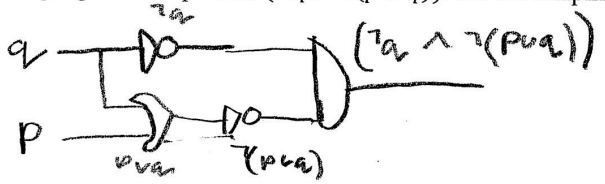
$= 4k^2 + 4k - 1$

$= 2(2k^2+2k) - 1$ . let $s = 2k^2+2k$ where $\exists s \in \mathbb{Z}$

$= 2s - 1$ ← odd by definition

Proved by contradiction

**Problem 7:** (8 points) Use logic gates to represent $(\neg q \wedge \neg(p \vee q))$. Do not simplify expression.




Let p=1001011 and q=1101010, evaluate the expression above. Do not simplify expression.

¬q = 0010101

1001011 p∨q
1101010
1101011 OR

¬p∨q = 0010100

0010101 ¬q
0010100 ¬(p∨q)
0010100 AND

**Problem 8:** (10 points) Prove or disprove: If x and y are rational numbers, then $(2x)^y$ is rational number.

p∧¬q

Assume x and y are rational numbers and $(2x)^y$ is irrational

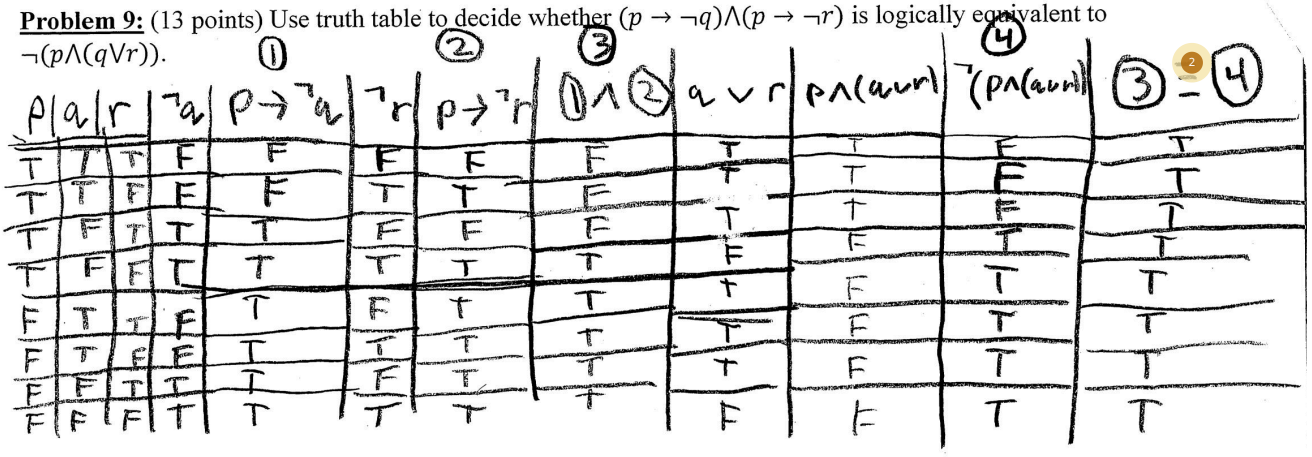
let $x, x = \frac{p}{q}$ where $p, q \in \mathbb{Z}$, $q \neq 0$

$(2x)^y = (2\frac{p}{q})^{\frac{p}{q}}$ . Let $s = \frac{p}{q}$ where $s \in \mathbb{Z}$ because $p, q \in \mathbb{Z}$

$(2x) = (2s)^s$ ← rational by definition

Proved by contradiction.

**Problem 9:** (13 points) Use truth table to decide whether $(p \to \neg q) \wedge (p \to \neg r)$ is logically equivalent to $\neg(p \wedge (q \vee r))$.



| p | q | r | ¬q | p→¬q | ¬r | p→¬r | ① ∧ ② | q ∨ r | p∧(q∨r) | ¬(p∧(q∨r)) | ③ = ④ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | F | T | T | F | T |
| T | T | F | F | F | T | T | F | T | T | F | T |
| T | F | T | T | T | F | F | F | T | T | F | T |
| T | F | F | T | T | T | T | T | F | F | T | T |
| F | T | T | F | T | F | T | T | T | F | T | T |
| F | T | F | F | T | T | T | T | T | F | T | T |
| F | F | T | T | T | F | T | T | T | F | T | T |
| F | F | F | T | T | T | T | T | F | F | T | T |

yes they're logically equivalent

**Extra Credit:** (6 points) Negate following statements, so that negation appears only in predicates or immediately precedes predicates. Show your work.

a. $\exists x \forall y \exists z(((x > 3) \wedge (y \leq -9)) \vee (z > 8))$

$\neg(\exists x \forall y \exists z(((x>3) \wedge (y \leq -9)) \vee (z>8))$

$\forall x \exists y \forall z \neg(((x>3) \wedge (y \leq -9)) \vee (z>8))$

$\forall x \exists y \forall z((x \leq 3 \vee y > -9) \wedge (z \leq 8))$

b. $\neg(\exists x \forall y(P(x,y) \to \neg Q(x,y)))$

$\forall x \exists y \neg(P(x,y) \to \neg Q(x,y))$

$\forall x \exists y(P(x,y) \wedge Q(x,y))$