Discrete Hw #6

2.4

30.a) $\sum_{j\in S} j = 1+3+5+7 = 16$

30.c) $\sum_{j\in S} 1/j = \frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7} = \frac{176}{105}$

32.b) $\sum_{j=0}^{8}(3^j-2^j) = 0+1+5+(19)+(81-16)+(3^5-32)+(3^6-64)+(3^7-128)+(3^8-256) = 9330$

32.c) $\sum_{j=0}^{8} 2\cdot 3^j+3\cdot 2^j - \sum_{j=0}^{8} 2\cdot 3^j + \sum_{j=0}^{8} 3\cdot 2^j$

$= \frac{2\cdot 3^9-2}{3-1} + \frac{3\cdot 2^9-3}{1}$

$= 19682 + 1533 = 21215$

34.b $\sum_{i=0}^{2}\sum_{j=0}^{3}(2i+3j) = \sum^{2}((2i+0)+(2i+3)+(2i+6)+(2i+9))$

$=((0+0)+(0+3)+(0+6)+(0+9))+((2+0)+(2+3)+(2+6)+(2+9))+(4+0)+(4+3)+(4+6)+(4+9))$

$= 18+26+34 = 78$

2.5

4.a) Countable The formula $f(n) = 3(\frac{n}{4}-1)+1$ gives all positive integers not divisible by 3.

4.b) Countably infinite $f(n) = \begin{cases} 5n, & 1\le n\le 6 \\ -5(n-6), & 7\le n\le 12 \end{cases}$ for positive / for negative

4.c Countably infinite because it has ∞ elements and 1-1 correspondence

$1+\frac{1^{a-1}}{10^a} = 1, 1.1, 1.11, \ldots$ $111+\frac{1^{a-1}}{10^a} = 111, 111.1, 111.11, \ldots$

$11+\frac{1^{a-1}}{10^a} = 11, 11.1, 11.11, \ldots$

4.d) Uncountable

6. $f(n) = 2n-1$. We move every guest from room n to room $2n-1$.

$f(1)=1$ $f(2)=3$ $f(3)=5$ $f(4)=7$ .....

3.2

2.a) $|f(x)| = |17x+1| <= |17x| + |1| = |17x+1| \le 18x \le x \cdot x = x^2 = |x^2|$

$\boxed{\text{yes}}$ $O(x^2)$ ✓ $k=18$ $c=1$

2.b) $|f(x)| = |x^2+100| \le$ $\boxed{\text{yes}}$ ✓

2.c) $\boxed{\text{yes}}$ $O(x^2)$ ✓ 2.d) $\boxed{\text{NO}}$ X 2.e) $\boxed{\text{no}}$ X

2.f) $\boxed{\text{yes}}$ $O(x^2)$ ✓

6. $\frac{x^3+2x}{2x+1}$

$$2x+1 \overline{\smash{)}x^3+2x}$$ quotient $\frac{1}{2}x^2 - \frac{1}{4}x + \frac{9}{8}$

$-x^3+\frac{1}{2}x^2$

$-\frac{1}{2}x^2+2x$

$-\frac{9}{8}$

$= \frac{1}{2}x^2 - \frac{1}{4}x + \frac{9}{8} - \frac{9}{8(2x+1)}$

$\boxed{\text{thes ids } O(x^2)}$

8a) $f(x) = 2x^2 + x^3 \log x \le 2x^2 + x^4 \le |2x^2| + |x^4| \le x^4 + x^4 = 2x^4 = 2|x^4|$

$n=4$ $K=2$ $C=2$

8.b) $f(x) = 3x^5 + (\log x)^4 \le |3x^5| + |(\log x)^4| < 3x^5 + x^4 \le 3x^5 + x^5 = 4x^5 = 4|x^5|$ $n=5$

$\boxed{f(x) = O(x^5)}$ $\boxed{k=1, c=4}$

8.c) $f(x) = \frac{(x^4+x^2+1)}{(x^4+1)}$

$$x^4+1 \overline{\smash{)}x^4+x^2+1}$$ quotient $1 + \frac{x^2}{x^4+1}$

x^4+0x^2+1

x^2

$= |1| + \left|\frac{x^2}{x^4+1}\right| \le |1| + |1| < 2$

$= 2 \cdot |1|$

$= 2|x^0|$

$n=0$ $\boxed{O(1), k=0, c=2}$

8.d) $f(x) = \frac{x^3+5\log x}{x^4+1} \le \frac{x^3+5x}{x^4+1} \le \frac{x^3+9x}{x^4} \le \frac{1}{x} + \frac{5}{x^3} \le \frac{1}{x} + \frac{5}{x} \le \frac{6}{x} = 6\left|\frac{1}{x}\right|$

$n = \frac{1}{x}$ $\boxed{f(x) = O(\frac{1}{x})}$ $k=?$ $C=6$

14.a) $g(x)=x^2$ $f(x)=x^3$, $|x^3| \le C|x^2| = |Cx^2|$

Contradiction because $|x^3| > |cx^2|$ when $x > C$, Not $O(x^3)$

~~14.b) $g(x)=x^3$ $f(x)=x^3$~~

~~$|f(x)| = |x^3| = |g(x)|$ yes $g(x)$ is $O(x^3)$~~

14.c) $g(x)=x^2+\boxed{x^3}$ yes ✓

14.d) $g(x)=x^2+x^4$ yes ✓ $x^4 \ge x^3$

~~14.e) $g(x)=3^x$ $|f(x)| = |x^3| = x^3 < 3^x = |3^x|$ $C=1$~~

~~$f(x)=x^3$ is $O(3^x)$ with $k=3, C=1$ yes~~

~~14.f) $|x^3| = x^3 = 2\left(\frac{x^3}{2}\right) = 2\left|\frac{x^3}{2}\right| = 2|g(x)|$ $C=2$~~

~~yes $f(x)$ is $O\left(\frac{x^3}{2}\right)$ $k=0$ $C=2$~~

22.

$(\log n)^3 < \sqrt{n}\log n < n^{99}+n^{98} < n^{100} < 1.5^n < 10^n < n!^2$

30.a) $f(x)=3x+7$ $g(x)=x$ let $k=7$ $x>7$

$|f(x)| = |3x+7| \le 3x+x = 4x = 4|x|$ $C=4$ $O(x)$, $k=7$, $C=4$

$f(x)=3x+7$ let $k=0$ $x>0$

$= 3x+7 > 3x = 3|x|$ $f(x)$ $O(x)$, $k=0$, $C=3$