MATH:1860 Activity 6 – (Sections 9.1-9.5) Mar. 06

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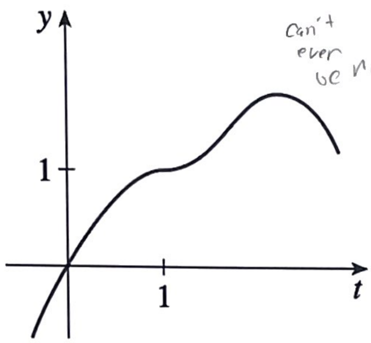
Instructions: Work with others or independently to complete the activity.

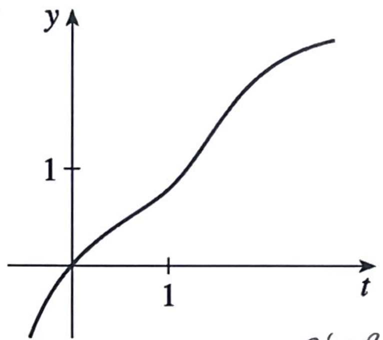
1. Explain why the functions with the given graphs *can't* be solutions of the differential equation $\frac{dy}{dt} = e^t(y-1)^2$.

   Can't ever be negative

   (a)

   (b)

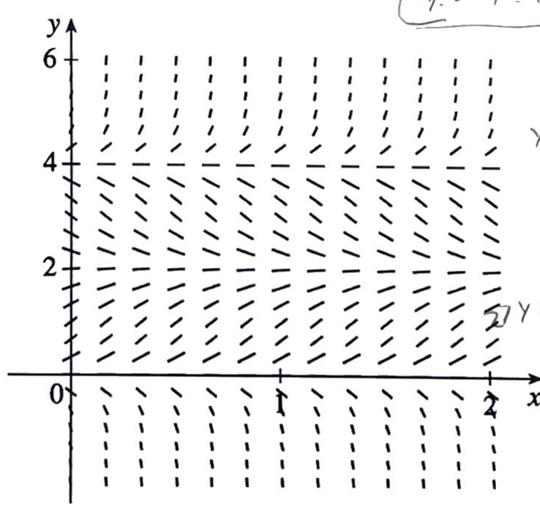
   positive slope $x=1$

   $y=1$

   $\frac{dy}{dx}=0$ so can't work

2. A direction field for the differential equation $y' = y(y-2)(y-4)$ is shown. If the initial condition is $y(0) = c$, for what values of c is $\lim_{x\to\infty} y(x)$ finite? What are the equilibrium solutions?

   $y=0$ $y=4$ $y=2$

   $y>4$ increasing

   $2>y>0$ decreasing

   $\boxed{0 \le c \le 4}$

3. A tank contains 1000 L of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. Let $y(t)$ be the amount of salt (in kg) in the tank at time t.

(a) Show that $\frac{dy}{dt} = -\frac{y}{100}$.

(Hint: $\frac{dy}{dt}$ is the rate at which the amount of salt in the tank changes and $\frac{dy}{dt}$ = (rate of salt in) − (rate of salt out). The units of $\frac{dy}{dt}$ are kg/min).

0

$ROSO = \frac{y(\ )}{1000} \cdot 10 = \frac{y}{100}$

$\frac{dy}{dt} = ROSi - \frac{y}{100}$

$= -\frac{y}{100}$

(b) How much salt is in the tank after t minutes?

$y(0)=15$

$\frac{dy}{y} = -\frac{dt}{100}$

$\int \frac{dy}{y} = \int \frac{dt}{100}$

$\ln|y| = \frac{-t}{100} + c$

$\pm y = e^c e^{-t/100}$

$y = ce^{-t/100}$

$15 = ce^0$

$15 = c$

$y = 15e^{-t/100}$ kg

4. Find an equation of the curve that passes through the point $(0, 2)$ and whose slope at (x, y) is x/y.

$\frac{dy}{dx} = \frac{x}{y}$

$dy\,y = x\,dx$

$\int dy\,y = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

$y^2 = x^2 + c$

$2^2 = 0^2 + c$

$4 = c$

$y = \sqrt{x^2+4}$

5. Solve the initial-value problem $t\frac{du}{dt} = t^2 + 3u$, $t > 0$, $u(2) = 4$.

$I(t) = e^{\int -\frac{3}{t}\,dt}$

$= e^{-3\ln|t|}$

$= e^{\ln|t|^{-3}}$

$= t^{-3}$

$\frac{du}{dt} - \frac{3}{t}u = t$

$\int \frac{d}{dt}(t^{-3}u) = \int \frac{1}{t^2}$

$t^{-3}u = \frac{-1}{t} + c$

$u = -\frac{1}{t} + c(t^3)$

$u = t^2 + ct^3$

$4 = 4 + c8$

$0 = c8$

$c = 0$

$u = t^2$

2