Exam 2 corrections

2.) Solve the IVP $xy' - 2y = x^2,\ x>0,\ y(1)=2$

$$y' - \frac{2}{x}y = x \qquad I(x) = e^{\int -\frac{2}{x}} = e^{-2\ln x} = x^{-2}$$

$$x^{-2}\left(y' - \frac{2}{x}y\right) = x \cdot x^{-2}$$

$$\int \frac{d}{dx}\left(x^{-2}y\right)dx = \int x^{-1}dx$$

$$x^{-2}y = \ln x + C$$

$$y = x^2(\ln x + c). \qquad y(1)=2$$

$$2 = 1^2(\ln(1)+c)$$

$$2 = 0 + c \qquad c=2$$

$$\boxed{y = x^2\ln x + 2x^2}$$

Explain: I rewrote problem wrong originally, which led to the answer being wrong.

3.b) Solve the D.E $\frac{dy}{dx} = xe^y$

$$\frac{dy}{e^y} = x\,dx$$

$$\int \frac{dy}{e^y} = \int x\,dx$$

$$-e^{-y} = \frac{x^2}{2} + C$$

$$e^{-y} = -\left(\frac{x^2}{2} + C\right)$$

$$-y = \ln\left(-\left(\frac{x^2}{2} + C\right)\right)$$

$$y = -\ln\left(-\frac{1}{2}x^2 + C\right)$$

Explain: I incorrectly integrated $\frac{1}{e^y}$ to $\ln|e^y|$ instead of $-e^{-y} + C$. This led to incorrect answer.

5. a) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) - \ln\left(\frac{n+1}{n+2}\right) = \left(\ln\frac{1}{2} - \cancel{\ln\frac{2}{3}}\right) + \left(\cancel{\ln\frac{2}{3}} - \cancel{\ln\frac{3}{4}}\right) + \left(\cancel{\ln\frac{3}{4}} - \ln\frac{4}{5}\right)$

<u>Telescoping Series</u> $\ldots\ldots - \ln\frac{n+1}{n+2}$

$$= \ln\frac{1}{2} - \ln\frac{n+1}{n+2}$$

$$= \lim_{n\to\infty} \ln\frac{1}{2} - \ln\frac{n+1}{n+2}$$

$$= \ln\frac{1}{2} \cdot \lim_{n\to\infty} -\left(\ln\frac{n+1}{n+2}\right) \overset{LH}{=} \boxed{\ln\frac{1}{2} - \ln(1)}$$

finite number

So it converges

Explain: I did not find exact sum and explain why it is convergent.

5.b) $\sum_{n=1}^{\infty} 2^{n+1} \cdot 3^{-n} = \sum_{n=1}^{\infty} 2 \cdot 2^n \cdot \left(\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} 2 \cdot \left(\frac{2}{3}\right)^n$

geometric series

$= 2 \cdot \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \frac{\frac{2}{3}}{1-\frac{2}{3}} = 2$

So, $2 \cdot 2 = \boxed{4}$. Converges and sum is 4

Explain: I did not re write 3^{-n} as $\left(\frac{1}{3}\right)^n$ and combine it with 2^n. This would've helped me find right answer

6.b) $\int_2^{\infty} \frac{1}{x \ln x} dx$.

$u = \ln x$
$du = \frac{1}{x} dx$

$\ln(\infty) = \infty$
$\ln(2) = \ln(2)$

$$\int_2^{\infty} \frac{1}{x \ln x} dx = \int_{\ln(2)}^{\infty} \frac{1}{u} du = \ln|u| \Big|_{\ln(2)}^{\infty} = \infty$$

Since the integral diverges the series also diverges by the integral test.

Explain: I incorrectly used integration by parts instead of basic u-sub. This over complicated a simple integral.

Final Questions

1.) I made a good cheat sheet and reviewed old homework assignment, and did the review.

2.) Honestly, I need to additional problems from the textbook instead of redoing old Problems,