Homework 13

Section 9.1

3.) $\{1,2,3,4\}$

3.e) $\{(1,1),(2,2),(3,3),(4,4)\}$ Reflexive?: Yes. Symmetric?: Yes.
Antisymmetric?: yes Transitive?: yes

3.f) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$ Reflexive?: No
Symmetric?: no (4,1) Antisymmetric?: No, $\exists$ (1,3)∧(3,1). Transitive?: No (1,1) $\notin R$

6.c) Reflexive?: Yes, $x-x=0$, always rational
Symmetric?: Yes, since $x-y$ is rational, $y-x=-(x-y)$ which is rational
Antisymmetric?: No Transitive?: Yes

6.d) Reflexive?: no, would only be true if $x=0$. Symmetric?: No.
Antisymetric?: yes. Transitive?: no, $x=2y$ $y=2z$, then $x=4z$, which not true

12.) Only 4(a) is irreflexive

19.) only 4(a) is asymmetric

34.a) $R_1 \cup R_3 = \{(a,b) \in R^2 \mid a>b \vee a<b\} \Rightarrow = \{(a,b) \in R^2 \mid a \neq b\}$
$= R_6$

34.c) $R_2 \cap R_4 = \{(a,b) \in R^2 \mid a \geq b \vee a \leq b\}$
$= \{(a,b) \in R^2 \mid a=b\} = R_5$

34.f) $R_2 - R_1 = \{(a,b) \in R^2 \mid a>b \wedge a \neq b\} = \{(a,b) \in R^2 \mid a>b \text{ and } a<b\}$
$= \{\emptyset\}$ empty

34.h) $R_2 \oplus R_4 = \{(a,b) \in R^2 \mid (a \geq b \text{ or } b \geq a) \wedge \neg(a \geq b \wedge a \leq b)\}$

$= \{(a,b) \in R^2 \mid T \wedge \neg(a = b)\}$

$= \{(a,b) \in R^2 \mid T \wedge a \neq b\} = R_6$

$a \geq b$ or $b \geq a$ is always true

36.e) $R_1 \circ R_5 = \{(a,c) \in R^2 \mid \exists b \in R \text{ s.t. } (a,b) \in R_5 \text{ and } (b,c) \in R_1\}$

$= \{(a,c) \in R^2 \mid \exists b \in R \text{ s.t. } a = b \wedge b > c\}$

$\{(a,c) \in R^2 \mid \exists b \in R \text{ s.t. } a > c\}$

$\{(a,b) \in R^2 \mid \exists b \in R \text{ s.t. } a > b\} = R_1$

36.f) $R_1 \circ R_6 = \{(a,c) \in R^2 \mid \exists b \in R \text{ s.t. } (a,b) \in R_6 \text{ and } (b,c) \in R_1\}$

$= \{(a,c) \in R^2 \mid \exists b \in R \text{ s.t. } a \neq b \wedge b > c\}$ always true

$\{(a,c) \in R^2 \mid \} = R^2$

Section 9.3:

2.c) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ 2.d) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

4.a) $R = \{(1,1),(1,2),(1,4),(2,1),(2,3),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4)\}$

4.b) $R = \{(1,1),(1,2),(1,3),(2,2),(3,3),(3,4),(4,1),(4,4)\}$

4.c) $R = \{(1,2),(1,4),(2,1),(2,3),(3,2),(3,4),(4,1),(4,3)\}$

8.a) Not Reflexive, Not irreflexive, Symmetric, not antisymmetric, not transitive.

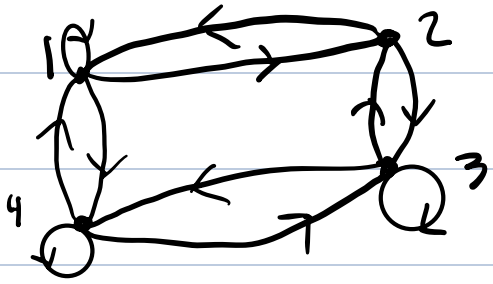
8.b) Reflexive, Not irreflexive, not Symmetric, antisymmetric not transitive

8.c) Not Reflexive, irreflexive, Symmetric, not antisymmetric, not transitive.

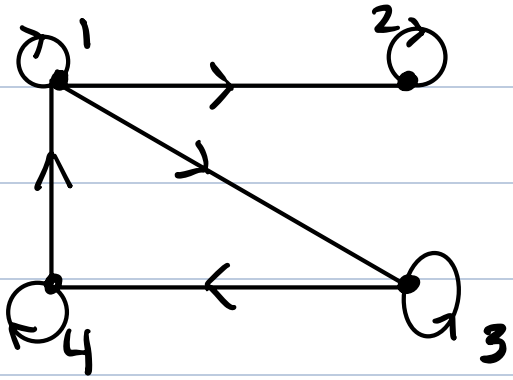
9.c) 99 nonzero entries 9.d) 100 nonzero entries

9.e) 1 nonzero entry

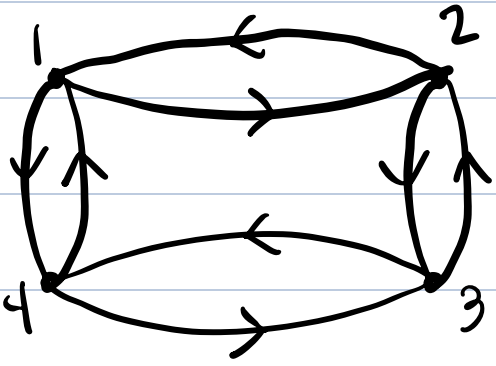
21.a)



21.b)



21.c)

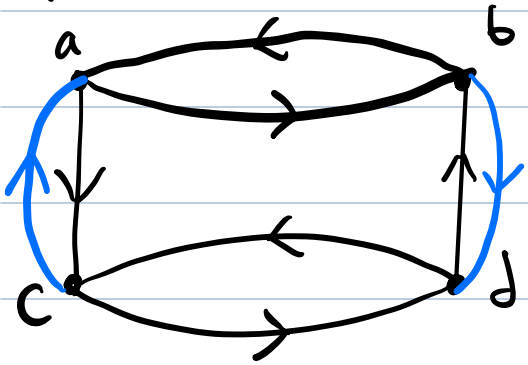


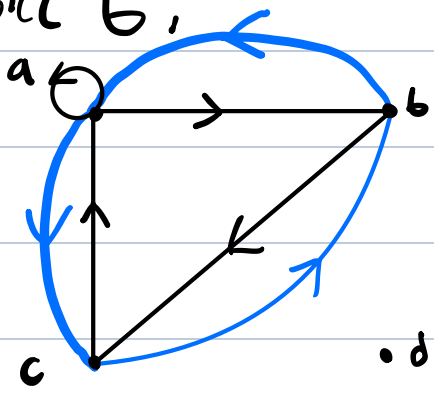
24.) $R = \{(a,a),(a,c),(b,a),(b,b),(b,c),(c,c)\}$

26.) $R = \{(a,a),(a,b),(b,a),(b,b),(c,a),(c,c),(c,d),(d,d)\}$

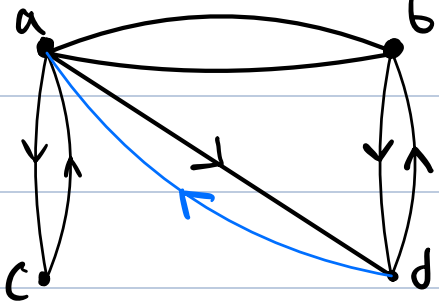
Section 9.4

9.a Exercise 5: 9.b Exercise 6:





9.C Exercise 7:



16.a) Yes it's a path
16.b no
16.c) Yes
16.d) not path
16.e) Yes
16.f) not path

Section 9.5

1.d) Not transitive
1.e) not symmetric or transitive
2.a) It is reflexive, symmetrical, and transitive.
2.c) Not transitive
8. $|S|=|S| \Rightarrow R$ is reflexive. $|S|=|T| \Rightarrow |T|=|S| \Rightarrow R$ is symmetrical
Yes transitive. Thus, R is equivalence relation
$[\{1,2,3\}]_R$ = Set of all ~~sets~~ of $\mathbb{R}$ with 3 different elements
$[\mathbb{Z}]$ = Set of non finite real numbers

22. Reflexive: yes Symetric: yes Transitive: yes
Yes it shows equivlence relation
23. Not transitive since no edge from d to b

24.b) Reflexive: yes Symetric: yes Transitive: yes
Yes it shows equivlence relation

42.a) It is a partition since union operator unites it S

42.b) NO!!!