

## Quiz 8

● Graded

Student

Colin Cano

Total Points

8.5 / 10 pts

Question 1

Linear approximations

4 / 4 pts

✓ - 0 pts Correct: Uses the formula  $f(x + dx) \approx f(x) + f'(x) \cdot dx$  with  $f(x) = \sqrt{x}$ ,  $x = 25$  and  $dx = 1$

- 2 pts The formula used is incorrect: neither  $f(x + dx) \approx f(x) + f'(x) \cdot dx$  nor  $f(a) \approx f(x) + f'(x) \cdot (a - x)$  (or variations)

- 1 pt The function is not  $f(x) = \sqrt{x}$  (or a correct variation as  $f(x) = \sqrt{x + 25}$ )

- 1 pt The chosen values of  $x$  and  $dx$  do not give  $x + dx = 26$  (or  $a = 26$ )

- 0.5 pts The chosen value of  $x$  is not 25 (so it is not good for the approximation)

- 0.5 pts Algebra mistake

- 4 pts No correct work

Great!

## Question 2

### Finding extremum points

3 / 4 pts

– 0 pts Correct:  $t = 1$  or after 1 day

---

### Finding critical numbers incomplete

– 1 pt Does not apply quotient rule correctly to obtain the derivative of  $E$

✓ – 1 pt Does not solve correctly for the zeroes of  $E'$

1 This is not how you simplify this expression

– 1 pt Takes the derivative correctly, but does not find critical numbers

– 3 pts Does not attempt to find the critical numbers (not even taking the derivative)

---

### Evaluating to find maximum incomplete

– 1 pt If they found the critical numbers but did not evaluate  $E$  on the critical numbers (can skip critical numbers not in the interval)

– 1 pt They did not evaluate the function on the endpoints and provided no justification (for example using second derivative to know 1 is where a local maximum is attained)

---

– 4 pts No correct work

– 0.5 pts Algebraic mistakes

---

## Question 3

### Rolle's Theorem

1.5 / 2 pts

– 0 pts Correct: (a) True (or Yes), (b) False (or No), (c) the function is not differentiable (or even continuous) on the whole interval (precisely at 0)

– 0.5 pts Part (a) Incorrect: Anything other than **True** or **Yes**

✓ – 0.5 pts Part (b) Incorrect: Anything else than **False** or **No**

– 1 pt Part (c) Incorrect: Anything other than  $f(x)$  not being differentiable or not being continuous

$$f(x+dx) \approx f(x) + f'(x) \cdot dx$$

$$dy \approx f'(x) \cdot dx$$

MATH:1850

Quiz 8

Fall 2024

Name: Colin Cano Student ID: \_\_\_\_\_

1. Use a linear approximation to show that  $\sqrt{26} \approx 5.1$ .

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

$$dx = 1$$

$$f(5) = \sqrt{25} = 5$$

$$\begin{aligned} f(26) &= \sqrt{25} + \left( \frac{1}{2} \cdot \frac{1}{\sqrt{25}} \right) \cdot 1 \\ &= 5 + \left( \frac{1}{50} \right) \cdot 1 \\ &= \frac{251}{50} \approx 5.1 \end{aligned}$$

2. Studies have shown the energy boost a vampire gets after drinking blood is not immediate, their energy is given by the function

$$E(t) = \frac{2t}{1+t^2}$$

where  $t$  is the number of days since their last meal. We know after a week our vampire will be starved to death. When in the first 7 days after eating ( $t$  in the interval  $[0, 7]$ ) will our vampire have the most energy to go hunt again?

$$E(t)$$

$$E'(t) = \frac{2(1+t^2) - 2t(2t)}{(1+t^2)^2} = 0$$

$$\begin{aligned} &= \frac{2t \cdot (1+t^2)^{-1}}{(1+t^2)^2} \\ &= \frac{2(1+t^2)^{-1} + -2(1+t^2)^{-2}}{(1+t^2)^2} = 0 \\ &= \frac{2(1+t^2)^{-1} - (1+t^2)^{-2}}{(1+t^2)^2} = 0 \end{aligned}$$

$$\frac{1}{1+t^2} = \frac{1}{(1+t^2)^2} \quad 2t = 2 \quad t = 1$$

$$E(0) = \frac{2(0)}{1+0^2} = 0$$

$$E(7) = \frac{14}{50} = \frac{7}{25}$$

$$E(1) = \frac{2}{2} = 1$$

the max is 1 at  $x=1$   
the min is 0 at  $x=0$   
 $-2x^{-3}$

3. Consider  $f(x) = \frac{1}{x^2}$

(a) True or False) Is  $f(1) = f(-1)$ ? True

(b) True or False) Is there  $c$  in  $(-1, 1)$  such that  $f'(c) = 0$ ? True

(c) Why does this not contradict Rolle's Theorem?

$x$  is not defined on  $x=0$

SCRATCH PAPER