Worksheet 2

CS 2210 Discrete Structures

Due 2/5 9pm. Late submissions get grade 0.

* Teams of 3-4 students (must work in a group). Follow directions given during discussion.
** This page is double sided. Make sure to do both sides. Show your work!

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Question 1: Decide whether $(p \wedge r) \rightarrow q$ and $(p \rightarrow q) \vee (r \rightarrow q)$ are logically equivalent.

$(p \wedge r) \rightarrow q \equiv \neg p \vee \neg r \vee q$

$(p \rightarrow q) \vee (r \rightarrow q) \equiv (\neg p \vee q) \vee (\neg r \vee q) \equiv \neg p \vee q \vee \neg r \vee q$

$\equiv \neg p \vee q \vee \neg r$

$\neg p \vee \neg r \vee q \equiv \neg p \vee \neg r \vee q$ Yes, they are equivalent

Question 2: Express the negation of $\exists x \forall y((x < -1) \vee (y > 2))$ without using negation symbol. Show each step.

$\neg(\exists x \forall y((x<-1) \vee (y>2)))$

$\forall x \neg(\forall y((x<-1) \vee (y>2)))$

$\forall x \exists y \neg((x<-1) \vee (y>2))$ ← $\forall x \exists y (\neg(x<-1) \wedge \neg(y>2))$

$\forall x \exists y((x \geq -1) \wedge (y \leq 2))$

Question 3: Let P(x) be statement "x has internet connection" and C(x,y) be statement " x and y have chatted over the internet". Domain of x and y are all students in our class. Use quantifiers to express each of those statements:

(a) Anna has not chatted over internet with John.

$\neg C(\text{Anna}, \text{John})$

(b) Not everyone in our class has internet connection.

$\neg \forall x\, P(x)$

(c) Someone in our class has an internet connection, but never chatted with anyone.

$\exists x \forall y\, P(x) \neg C(x, y)$

(d) There is a student in our class who chatted with everyone in our class.

$\exists x \forall y\, C(x, y)$

Question 4: Let Q(x,y) be a statement "x-y=4x+2y". The domain x, y are integers. What are the truth values of the below? Explain. Give example when relevant.

(a) Q(1,-1) $1-(-1) = 4(1) + 2(-1)$ $2=2$ True

(b) $\exists x \exists y Q(x,y)$ $1-(-1) = 4(1) + 2(-1)$ $2=2$ True

(c)) $\exists y \forall x Q(x,y)$ False. There is not a value y which all x would satisfy $Q(x,y)$

(d) $\exists x \forall y Q(x,y)$ False. There is not a value x which all y would satisfy $Q(x,y)$

(e) $\forall x \exists y Q(x,y)$ True. For all x exists y which satisfies $Q(x,y)$

(f) $\forall y \exists x Q(x,y)$ True. For all y, exists x which satisfies $Q(x,y)$

Question 5: Definition: 5|n means 5 divides n, that is exists x ∈ Z, such that n=5x. $5 \nmid n$ means 5 does not divide n. Suppose n ∈ Z. Prove that if $5 \mid n$, then $5 \mid n^2$.

Assume $5 \mid n$, then by definition of divisibility, $\exists k \in \mathbb{Z}$ s.t. $n = 5k$

by definition $5 \mid n^2$

$n^2 = (5k)^2 = 25k^2 = \underbrace{5(5k^2)}_{t} = 5t \qquad t = 5k^2,\ t \in \mathbb{Z}$

Since $k \in \mathbb{Z}$

Proved that $5 \mid n^2$