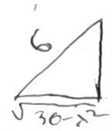
Colin Cano

7.3 Section

12. $\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx \quad x = 6\sin\theta \qquad dx = 6\cos\theta\, d\theta$

$\sqrt{36-(6\sin\theta)^2} = \sqrt{36(1-\sin^2\theta)} = \sqrt{36\cos^2\theta} = 6\cos\theta$

$\int_0^3 \frac{6\sin\theta}{6\cos\theta}\cdot 6\cos\theta\, d\theta = \int 6\sin\theta\, d\theta = 6\int \sin\theta\, d\theta = -6\cos\theta + C$



$\sin\theta = \frac{x}{6} \qquad x^2+b^2=36 \qquad b = \sqrt{36-x^2}$

$\cos\theta = \left(\frac{\sqrt{36-x^2}}{6}\right)\cdot -6 = -\sqrt{36-x^2}\Big|_0^3 = (-\sqrt{36-9}) - (-\sqrt{36})$

$-3\sqrt{3}+6 =$

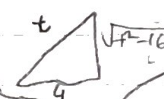
$\boxed{6-3\sqrt{3}}$

14. $\int \frac{dt}{t^2\sqrt{t^2-16}} \qquad t = 4\sec\theta \qquad dt = 4\tan\theta\sec\theta\, d\theta$

$t^2\sqrt{t^2-16} = 16\sec^2\theta\sqrt{16\sec^2\theta-16} = 16\sec^2\theta\sqrt{4\tan\theta}$

$\int \frac{4\cancel{\sec\theta\tan\theta}}{16\sec^2\theta\cdot 4\tan\theta} = \int \frac{4\cancel{\sec\theta}\cancel{\tan\theta}}{64\sec^2\theta\tan\theta} = \int \frac{4\sec\theta}{64\sec^2\theta} = \int \frac{1}{16\sec^2\theta} = \int \frac{16}{\cos\theta}\,d\theta = \frac{1}{16}\int \cos\theta\, d\theta$

$= \frac{1}{16}(\sin\theta) + C \qquad \sec\theta = \frac{t}{4}$



$\sin\theta = \frac{\sqrt{t^2-16}}{t}\cdot\frac{1}{16} = \frac{\sqrt{t^2-16}}{16t} + C$

16. $\int_0^{2/3} \sqrt{4-9x^2}\,dx \qquad x = \frac{2}{3}\sin\theta \qquad dx = \frac{2}{3}\cos\theta\, d\theta \qquad \int\sqrt{4-9\left(\frac{4}{9}\sin^2\theta\right)} = \int\sqrt{4-4\sin^2\theta}$

$= \int\sqrt{4(1-\sin^2\theta} = \int 4\cos^2\theta = 2\cos\theta \qquad \sin(0)=0 \qquad \sin\frac{2}{3} = \frac{\pi}{2}$

$\int_0^{\pi/2} 2\cos\theta\cdot\frac{2}{3}\cos\theta\, d\theta = \frac{4}{3}\int\cos^2\theta\, d\theta = \frac{4}{3}\int\frac{1+\cos 2\theta}{2}\,d\theta = \frac{2}{3}\int 1+\cos 2\theta\, d\theta = \frac{\sin 2\theta}{2} + \theta$

$\frac{2}{3}\cdot\frac{\sin 2\theta}{2} + \theta\Big|_0^{\pi/2} = \frac{2}{3}\left(\frac{\pi}{2}\cdot\frac{\sin\pi}{2} - \left(0 + \frac{\sin 0}{2}\right)\right) = \frac{2}{3}\cdot\frac{\pi}{2} = \frac{\pi}{3}$

18. $\int_0^2 \frac{dt}{\sqrt{4+t^2}} \qquad t = 2\tan\theta \qquad dt = 2\sec^2 x\, dx$

$\sqrt{4+(2\tan\theta)^2} = \sqrt{4(1+\tan^2\theta} = 2\sec\theta \qquad \tan 0 = 0 \qquad \tan 2 = \frac{\pi}{4}$

$\int \frac{2\sec^2 x}{2\sec\theta}\,dx = \int \sec\theta = \ln|\sec\theta + \tan\theta|\Big|_0^{\pi/4} = (\ln|\sqrt{2}+1| - \ln|0+1|)$

$= \boxed{\ln(\sqrt{2}+1)}$

22. $\int_{1/4}^{\sqrt{3}/4} \sqrt{1-4x^2}\,dx$ $\quad x = \frac{1}{2}\sin\theta \quad dx = \frac{1}{2}\cos\theta\,d\theta$

$\sqrt{1-4(\frac{1}{2}\sin\theta)^2} = \sqrt{1-4\cdot\frac{1}{4}\sin^2\theta} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$

$\sin(\sqrt{3}/4) = \frac{\pi}{3}$
$\sin(\frac{1}{4}) = \frac{\pi}{6}$

$\int_{\pi/6}^{\pi/3} \cos\theta \cdot \frac{1}{2}\cos\theta\,d\theta = \frac{1}{2}\int \cos^2\theta\,d\theta = \frac{1}{4}\int 1+\cos 2\theta\,d\theta$

$= \frac{1}{4}\left(\theta + \frac{\sin 2\theta}{2}\right)\Big|_{\pi/6}^{\pi/3} = \frac{1}{4}\left(\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right) - \left(\frac{\pi}{6}+\frac{\sqrt{3}}{4}\right)\right) = \frac{1}{4}\left(\frac{\pi}{6}\right)$

$\boxed{= \frac{\pi}{24}}$

24. $\int \frac{x}{\sqrt{1+x^2}}\,dx$ $\quad x = \tan\theta \quad dx = \sec^2\theta\,d\theta$

$\sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta}$ $\quad = \int \frac{\tan\theta}{\sec\theta} \cdot \sec^2\theta = \int \tan\theta\sec\theta$

$= \sec\theta + C$

$\sec\theta = \sqrt{1+x^2}$

$\boxed{\sqrt{1+x^2} + C}$

Section 7.4

1.a) $\frac{1}{(x-3)(x+5)} = \frac{A}{x-3} + \frac{B}{x+5}$ $\qquad$ 1.B) $\frac{2x+5}{(x-2)^2(x^2+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+2}$

3.a) $\frac{x^2+4}{x^3-3x^2+2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$ $\qquad$ 3.B) $\frac{x^3+x}{x(2x-1)^2(x^2+3)^2} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2} + \frac{Dx+E}{x^2+3} + \frac{Fx+G}{(x^2+3)^2}$

5.a) $\frac{x^5+1}{(x^2-x)(x^4+2x^2+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$ $\qquad$ 5.B) $\frac{x^2}{x^2+x-6} = 1 + \frac{A}{x-2} + \frac{B}{x+3}$

8. $\int \frac{x-12}{x^2-4x}\,dx$ $\quad x^2-4x = x(x-4)$ $\quad \int \frac{x-12}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4} =$

$x-12 = A(x-4) + Bx$ $\qquad x-12 = (A+B)x - 4A$

$A=3 \quad B=-2$ $\quad \int \frac{3}{x} - \frac{2}{x-4}\,dx = \boxed{3\ln|x| - 2\ln|x-4| + C}$

10 $\int \frac{y}{(y+4)(2y-1)}\,dy = \frac{A}{y+4} + \frac{B}{2y-1}$ $\qquad y = A(2y-1) + B(y+4)$

$y = 2Ay - A + By + 4B$

$y = (2A+B)y + (-A+4B)$ $\qquad A = \frac{4}{9} \quad B = \frac{1}{9}$

$\int \frac{4/9}{y+4} + \frac{1/9}{2y-1} = \boxed{\frac{4}{9}\ln|y+4| + \frac{1}{18}\ln|2y-1| + C}$

16. $\int \frac{3t-2}{t+1}\,dt$ Long division

$$\begin{array}{r} 3-\frac{1}{t+1} \\ t+1 \overline{)\,3t-2} \\ \underline{3t+3} \\ 1 \end{array}$$

$\int 3-\frac{1}{t+1}\,dt = \boxed{3t-\ln|t+1|+C}$

20. $\int_2^3 \frac{x(3-5x)}{(3x-1)(x-1)^2}\,dx = \frac{A}{3x-1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$

$x(3-5x) = A(x-1)^2 + B(3x-1)(x-1) + C(3x-1)$

$A=1 \quad C=-1 \quad B=-2$

$\int_2^3 \frac{x(3-5x)}{(3x-1)(x-1)^2} = \int_2^3 \frac{1}{3x-1} - \frac{2}{x-1} - \frac{1}{(x-1)^2} = \frac{1}{3}\ln|3x-1| - 2\ln|x-1| + \frac{1}{x-1}\Big|_2^3 =$

$\boxed{\left(\frac{1}{3}\ln|8| - 2\ln|2| + \frac{1}{2}\right) - \left(\frac{1}{3}\ln|5| - 2\ln|1| + 1\right) =}$

68 $\int_1^2 \frac{1}{x^3+x}\,dx =$ $x^3+x = x(x^2+1)$

$\frac{1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$1 = Ax^2 + A + Bx^3 + Cx \qquad B=0 \quad A=1 \quad C=0$

$\int \frac{1}{x(x^2+1)}\ \int \frac{1}{x} + \frac{0}{x^2+1}\,dx = \ln|x| - \frac{1}{2}\ln|x^2+1|\Big|_1^2 =$

$= \left(\ln 2 - \frac{1}{2}\ln 5\right) - \left(0 - \frac{1}{2}\ln 2\right)$

$= \boxed{\frac{3}{2}\ln 2 - \frac{1}{2}\ln 5}$