Direct Proof

Example: Contraposition

$p \to q$

C.P: $\neg q \to \neg p$

Contraposition
proof by contraposition:
Step1: write contrapositive of $p \to q$
$$\neg q \to \neg p$$
Step2: use direct proof to prove
that $\neg q \to \neg p$. Assume $\neg q$ and   show
$\neg p$ is true, "start left, prove right"
works because. . . . .

Prove if $x+y \geq 2$, then $x \geq 1$ or $y \geq 1$

Proof by contraposition

Proof by contraposition

Step 1: if $x<1$ and $y<1$, then $x+y<2$

Step 2: direct proof: Assume $x<1$ and $y<1$

$$x+y \qquad 1+y < 1+1 = 2 \qquad \text{so} \qquad x+y<2$$
$$x<1 \qquad y<1$$

Def: Real number r is rational if $\exists p, q \in \mathbb{Z}$, $q \neq 0$, s.t

$$r = \frac{p}{q}$$

Proof difference of 2 rational numbers is also a rational number.

If $x, y \in \mathbb{Q}$ are rational, then $x-y$ is also rational

Use direct proof:

Assume x, y are rational, by def of rational numbers $\exists p_1, q_1 \in \mathbb{Z}$, with $q \neq 0$ s.t $x = \frac{p_1}{q_1}$ also $\exists p_2, q_2 \in \mathbb{Z}$ $\qquad y = \frac{p_2}{q_2}$

$$x-y = \frac{p_1}{q_1} - \frac{p_2}{q_2} = \frac{p_1 q_2 - p_2 q_1}{q_1 p_2} = \frac{s}{t}$$

$$s = p_1 q_2 - p_2 q_1 \qquad s \in \mathbb{Z}$$
$$t = q_1 q_2 \qquad t \in \mathbb{Z}$$

negation of $p \to q$
$$p \wedge \neg q$$

Given a Statement p and if you need to prove p is always true. Take p and Show always false.

Contradiction for $p \to q$

Step 1: Assume $p \wedge \neg q$ and Show ....

Prove $\sqrt{2}$ is irrational

proof by contradiction: Assume $\sqrt{2}$ is not irrational

$\sqrt{2}$ is rational

$\exists p, q \in \mathbb{Z}, q \neq 0$ s.t. $\sqrt{2} = \frac{p}{q}$ Let $\frac{p}{q}$ te

$$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$$

q^2

$$2 = \frac{r}{q^2} \quad q \neq 0, \quad /\cdot q^2$$

$$2q^2 = p^2 \implies C1: p^2 \text{ must be divisbe by 2}$$

$$C2: p \text{ must be even}$$

$$\exists$$

$$2q^2 = 2k^2$$

$$2q^2 =$$

Hello there

Colin Cano
Colin Woods
Cade Chism
Parker Chyma
Jeremiah Peiffer