Exam 1

Graded

Student

Colin Cano

Total Points

49 / 70 pts

Question 1

Q1

9 / 10 pts

Integration by parts

- ✔ **+ 2 pts** Correct u and dv
- ✔ **+ 2 pts** Correct du and v
- **− 1 pt** v off by a constant
- ✔ **+ 2 pts** Correct application of integration by parts formula
- ✔ **+ 2 pts** Correct integration
- ✔ **+ 2 pts** Correct evaluation at bounds of integration
- **− 1 pt** Did u-substitution incorrectly
- ✔ **− 1 pt** Incorrect simplification of final answer
- **− 1 pt** Final answer off by a constant
- **− 1 pt** Off by a sign when plugging in the bounds of integration

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- **+ 4 pts** Incorrect but reasonable attempt
- **+ 0 pts** Missing / Incorrect

Question 2

Q2

8 / 10 pts

Trig integral

- ✔ **+ 2 pts** Used $\sin^2 x + \cos^2 x = 1$ to convert factors of one trig function to factors of the other
- **+ 2 pts** Correctly rewrote integrand
- ✔ **+ 2 pts** Correct u-substitution
- **− 1 pt** Mistake computing du
- ✔ **+ 2 pts** Correct integration
- **− 1 pt** Minor integration mistake
- ✔ **+ 2 pts** Correct evaluation at the bounds of integration
- **− 1 pt** Flipped bounds

- **+ 4 pts** Incorrect but reasonable attempt
- **+ 0 pts** Missing / Incorrect

incorrect integrand

Question 3

Q3

10 / 10 pts

Improper integral

- ✔ **+ 2 pts** Correct u-substitution
- ✔ **+ 2 pts** Correct du (based on u)
- ✔ **+ 2 pts** Correct integration
- ✔ **+ 2 pts** Correct evaluation at bounds of integration
- ✔ **+ 2 pts** Correct conclusion (based on previous answer)

- **+ 10 pts** Correct steps to deduce whether the integral converges or diverges
- **+ 4 pts** Incorrect but reasonable attempt
- **+ 0 pts** Missing / Incorrect

Question 4

Q4

4 / 10 pts

Trig substitution

+ 2 pts Preliminary substitution $u = x^2$

+ 2 pts Trig sub $u = \sin\theta$

+ 2 pts Correct integrand

+ 2 pts Correct antiderivative in terms of θ (based on previous answer)

+ 2 pts Correct antiderivative in terms of x (based on previous answer)

✔ **+ 4 pts** Incorrect but reasonable attempt

+ 0 pts Missing / Incorrect

Question 5

Q5

8 / 10 pts

+ 10 pts Correct

✔ **+ 8 pts** Correct application of arclength formula but the derivative y' used is wrong

+ 6 pts Mistake applying arclength formula and computing the derivative

+ 4 pts Stated correct arclength formula

− 2 pts Missing bounds of integration

− 1 pt Incorrect simplification

+ 0 pts Missing / Incorrect

Question 6

Q6

6 / 10 pts

- ✔ + **10 pts** Correct
- + **8 pts** Correct application of surface area formula but the derivative used is wrong
- + **8 pts** Correct application of a surface area formula but it is the one for rotation about the y-axis
- + **6 pts** Mistake applying surface area formula
- ✔ + **6 pts** Mixed x and y under the integral sign
- + **4 pts** Stated correct surface area formula
- − **2 pts** Missing / Incorrect bounds of integration
- − **2 pts** Incorrect derivative
- − **2 pts** Did not express y in terms of x under the integral sign
- − **1 pt** Incorrect simplification
- + **0 pts** Missing / Incorrect

Question 7

Q7

4 / 10 pts

Partial fraction decomposition

- + **2 pts** Did long division
- − **1 pt** Incorrect long division
- + **2 pts** Correct factorization of denominator
- + **2 pts** Correct form of partial fraction decomposition (based on previous answer)
- + **2 pts** Correct constants in partial fraction decomposition (based on previous answer)
- − **1 pt** Flipped the partial fraction decomposition constants
- + **2 pts** Correct antiderivative (based on previous answer)
- − **1 pt** Off by some signs or constants in final answer

- ✔ + **4 pts** Incorrect but reasonable attempt
- + **0 pts** Missing / Incorrect

MATH:1860 – Exam 1 **Feb. 27**

Name: Colin Cuno

Instructions: There is a total of 7 problems on this exam. Each problem is worth 10 points. Be sure to show all your work, write neatly and legibly, and simplify your final answers. Any problem with a correct answer without work to support it will receive 0 points. If you have any questions about a problem, you can raise your hand or come up and ask.

1. (10 points) Evaluate the integral $\int_0^1 xe^{7x}\,dx$

$u = x \quad dv = e^{7x}dx$

$du = dx \quad v = \frac{e^{7x}}{7}$

$\int = x\cdot\frac{e^{7x}}{7} - \int \frac{e^{7x}}{7}dx$

$= \frac{1}{7}xe^{7x} - \frac{1}{7}\int e^{7x}dx$

$= \frac{1}{7}xe^{7x} - \frac{e^{7x}}{49}\Big|_0^1 = \left(\frac{1}{7}e^7 - \frac{e^7}{49}\right) - \left(0 - \frac{1}{49}\right)$

$= \frac{1}{343}\left(e^7 - e^7\right) + \frac{1}{49}$

$= \frac{1}{343}(0) + \frac{1}{49}$

$= \frac{1}{49}$

2. (10 points) Evaluate the integral $\int_0^{\pi/2} \sin^3 x \cos^5 x\,dx$ $\sin^3 x(1-\sin^2 x)\cos^3 x\,dx$

$= \int \sin^3 x - \sin^3 x(\cos^3 x)dx$ $= \int -\sin^2 x\cdot(1-\sin^2 x)\cos x\,dx = \int \sin^4 x - \sin^2 x(\cos x)dx$

$u = \sin x \quad du = \cos x\,dx$

$\sin(\frac{\pi}{2}) = 1$

$\sin(0) = 0$

$= \int_0^{\pi/2} u^4\,dx - \int_0^{\pi/2} u\,dx$

$= \frac{u^5}{5} - \frac{u^3}{3}\Big|_0^1 = \frac{1}{5} - \frac{1}{3} = \boxed{-\frac{2}{15}}$

3. (10 points) Does $\int_{-\infty}^{0} \frac{x}{(x^2+1)^3}\,dx$ converge or diverge?

$u = x^2+1 \quad du = 2x\,dx \quad \frac{du}{2} = x\,dx \quad u = 0^2+1 = 0 \quad u = t^2+1$

$$\int_{-\infty}^{0} \frac{x}{(x^2+1)^3} = \lim_{t\to\infty} \frac{1}{(x^2+1)^3}\,x\,dx = \lim_{t\to\infty} \int_{t^2+1}^{1} \frac{1}{u^3}\,\frac{du}{2} = \lim_{t\to\infty} \frac{1}{2}\int_{t^2+1}^{1} \frac{1}{u^3}\,du$$

$$= \lim_{t\to-\infty} \frac{1}{2}\left(-\frac{1}{2}u^{-2}\right)\Big|_{t^2+1}^{0} = \lim_{t\to-\infty} -\frac{1}{4} + \frac{1}{4(t^2+1)^2} = -\frac{1}{4} + \frac{1}{\infty} = \boxed{-\frac{1}{4}}$$

Converges.

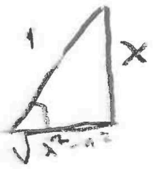
4. (10 points) Evaluate the integral $\int x\sqrt{1-x^4}\,dx = \int x\sqrt{1^2-(x^2)^2}$ $\quad x = \sin\theta \; \ldots\ldots \; x^2 = \sin^2\theta$ (with 4 written above the 2)

$dx = \cos\theta\, d\theta$

$$= \int \sin\theta\sqrt{1-(\sin^2\theta)^2}\,\cos\theta\,d\theta = \int \sin\theta\,(1-\sin^2\theta)\cos\theta\,d\theta$$

$$= \int \sin\theta - \int \sin^3\theta\cdot\cos\theta\,d\theta \qquad u = \sin\theta \qquad du = \cos\theta\,d\theta$$

$\theta = \sin^{-1}(x)$



$\sin\theta = \frac{x}{1}$

$\cos\theta = \frac{\sqrt{1-x^2}}{1}$

$$= -\cancel{\cos\theta} - \int u^3\,du$$

$$= -\cos\theta - \frac{1}{4}\left(\frac{\sin^4\theta}{4}\right)$$

$$\boxed{= -\sqrt{1-x^2} - \frac{\sin^4\left(\frac{x}{4}\right)}{16} + C}$$

$\frac{1+L}{L}$ $-x$ $\int \frac{3}{2} = \int 1.5^1 = \frac{1.5^2}{2}$

5. (10 points) Set up, but do not evaluate, an integral to find the exact arclength L of $y = \cos\left(\frac{x}{2}\right)$ over the interval $0 \le x \le \pi$.

$y' = -\sin\left(\frac{x}{2}\right)$

$$L = \int_0^{\pi} \sqrt{1 + \left(-\sin\frac{x}{2}\right)^2}\, dx$$

6. (10 points) Set up, but do not evaluate, an integral to find the exact area S of the surface obtained by rotating the curve $y = (x+1)^4$, $0 \le x \le 2$ about the x-axis.

$y = (x+1)^4$ $\sqrt[4]{y} = x+1$

$\frac{dy}{d} = 4(x+1)^3$ $x = \sqrt[4]{y} - 1$

$\frac{dx}{dy} = \frac{y^{-3/4}}{4}\, dy$

$$\int_0^2 2\pi y \sqrt{1 + \frac{dx}{dy}^2}\, dy$$

$$= 2\pi \int_0^2 (x+1)^4 \sqrt{1 + \left(\frac{y^{-3/4}}{4}\right)^2} \cdot 4(x+1)^3$$

7. (10 points) Evaluate the integral $\int \frac{x^2}{x^2 - x - 6}\, dx$ $= \int \frac{x^2}{(x-3)(x+2)}$

$$\int \frac{x^2}{x^2-x-6} = \int x^2\left(x^{-2}-x^{-1}-\frac{1}{6}\right) = \int \frac{x^2}{x^2} - \frac{x^2}{x} - \frac{x^2}{6} = \int 1 - x - \frac{x^2}{6}$$

$$= \frac{1}{6}\int 1 - x - x^2 = \frac{1}{6}\left(x - x^2 - \frac{x^3}{3} + C\right)$$

$$= \frac{x}{6} - \frac{x^2}{6} - \frac{x^3}{18} + C$$