Section 8.2

2.e) homogeneous, not linear since no constents

2.g) linear homogeneous since RHS is a linear combination of previous terms and has no constnts. The degree is $\boxed{7}$

3.d) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 8$

$r^n = 4r^{n-1} - 4r^{n-2}$

$r^2 = 4r - 4 \Rightarrow r^2 - 4r + 4 = 0 \Rightarrow (r-2)^2 = 0 \Rightarrow r = 2$

$a_n = \alpha_1 2^n + \alpha_2 n 2^n$

$= (\alpha_1 + \alpha_2 n) \cdot 2^n$

$a_0 = 6$: $(\alpha_1 + \alpha_2 \cdot 0) \cdot 2^0 = 6$

$\alpha_1 = 6$

$a_1 = 8$: $(\alpha_1 + \alpha_2 \cdot 1) \cdot 2 = 8$

$\alpha_1 + \alpha_2 = 4$

$6 + \alpha_2 = 4$

$\alpha_2 = -2$

$\boxed{a_n = (6 - 2n) \cdot 2^n}$

3.f) $a_n = 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 4$

$a_n = r^2$ $a_{n-1} = r$ $a_{n-2} = 1$

$r^2 = 4$

$r_1 = 2$, $r_2 = -2$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$= \alpha_1 2^n + \alpha_2 (-2)^n$

$a_0 = 0$: $= \alpha_1 2^0 + \alpha_2 (-2)^0$

$0 = \alpha_1 + \alpha_2$

$a_1 = 4$: $4 = 2\alpha_1 - 2\alpha_2$

$-4 = 4\alpha_2$

$-1 = \alpha_2$

$0 = \alpha_1 - 1$

$1 = \alpha_1$

$\boxed{a_n = 2^n - (-2)^n, \ n \geq 0}$

4.c) $a_n = 6a_{n-1} - 8a_{n-2},\ n \geq 2,\ a_0 = 4,\ a_1 = 10$

$a_n = r^2 \quad a_{n-1} = r \quad a_{n-2} = 1$

$r^2 = 6r - 8$

$r^2 - 6r + 8 = 0$

8, -2, -4 $\Rightarrow (r-4)(r-2) = 0 \quad r_1 = 4,\ r_2 = 2$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$a_n = 4^n \alpha_1 + 2^n \alpha_2$

$a_0 = 4 \begin{cases} 4 = \alpha_1 4^0 + \alpha_2 2^0 \\ 4 = \alpha_1 + \alpha_2 \end{cases}$

$a_1 = 10 \begin{cases} 10 = 4\alpha_1 + 2\alpha_2 \\ 10 = 4(4 - \alpha_2) + 2\alpha_2 \end{cases}$

$10 = 16 - 4\alpha_2 + 2\alpha_2$

$10 = 16 - 2\alpha_1$

$-6 = -2\alpha_1$

$3 = \alpha_1$

$4 = 3 + \alpha_2$

$1 = \alpha_2$

$\boxed{a_n = 3 \cdot 2^n + 4^n}$

4.d) $a_n = 2a_{n-1} - a_{n-2}$, for $n \geq 2,\ a_0 = 4,\ a_1 = 1$

$r^2 = a_n \quad r = a_{n-1} \quad 1 = a_{n-2}$

$r^2 = 2r - 1 \Rightarrow r^2 - 2r + 1 \quad r = 1 \ldots \quad a_n = \alpha_1 \cdot 1^n + \alpha_2 n \cdot 1^n = \alpha_1 + \alpha_2 n$

1, -1, -1

$a_0 = 4 \begin{cases} 4 = \alpha_1 + 0 \\ 1 = 4 + \alpha_2 \end{cases} a_1 = 1 \qquad 4 = \alpha_1 \quad \alpha_2 = -3$

$\boxed{a_n = 4 - 3n}$

12. $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3},\ n \geq 3,\ a_0 = 3 \quad a_1 = 6 \quad a_2 = 0$

$a_n = r^3 \quad a_{n-1} = r^2 \quad a_{n-2} = r \quad a_{n-3} = 1$

$r^3 = 2r^2 + r - 2 \Rightarrow r^3 - 2r^2 - r + 2 = 0 \Rightarrow (r-2)(r-1)(r+1) = 0$

$r = 2,\ r = 1,\ r = -1$

$a_n = \alpha_1 \cdot 2^n + \alpha_2 \cdot (-1)^n + \alpha_3 \cdot 1^n$

$a_0 = 3 = \alpha_1 + \alpha_2 + \alpha_3$

$a_1 = 6 = \alpha_1 \cdot 2 + \alpha_2 \cdot (-1) + \alpha_3$

$a_2 = 0 = \alpha_1 \cdot 4 + \alpha_2 + \alpha_3$

$0 = 4\alpha_1 + 3 - \alpha_1$

$-3 = 3\alpha_1$

$-1 = \alpha_1$

$4 = -(4 - \alpha_3) + \alpha_3$

$4 = -4 + \alpha_3 + \alpha_3$

$8 = \alpha_3 + 2$

$6 = \alpha_3$

$3 = -1 + \alpha_2 + 6$

$-2 = \alpha_2$

$\boxed{a_n = -2^n - 2(-1)^n + 6}$

$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}, \quad a_0 = 5, \quad a_1 = -9 \quad a_2 = 15$

19. $r^3 = a_n \quad r^2 = a_{n-1} \quad r = a_{n-2} \quad 1 = a_{n-3}$

$r^3 = -3r^2 - 3r - 1$

$r^3 + 3r^2 + 3r + 1 = 0$

$(r+1)(r+1)(r+1) = (r+1)^3 \Rightarrow r = -1, r = -1, r = -1$

$a_n = \alpha_1 \cdot (-1)^n + \alpha_2 \cdot n(-1)^n + \alpha_3 \cdot n^2 (-1)^n$

$a_0 = 5 = \alpha_1$

$a_1 = -9 = \alpha_1 \cdot (-1) + \alpha_2 \cdot (-1) + \alpha_3 \cdot (-1)$

$-9 = -1(\alpha_1 + \alpha_2 + \alpha_3)$

$4 = \alpha_2 + \alpha_3 \Rightarrow 4 - 1 = \alpha_2$

$3 = \alpha_2$

$15 = \alpha_1 + \alpha_2 \cdot 2 + \alpha_3 \cdot 4$

$10 = 2\alpha_2 + 4\alpha_3$

$10 = 2(4 - \alpha_3) + 4\alpha_3 \Rightarrow 10 = 8 + 2\alpha_3$

$2 = 2\alpha_3$

$1 = \alpha_3$

$\boxed{a_n = 5 \cdot (-1)^n + 3n(-1)^n + n^2(-1)^n}$

24.a) $a_n = 2a_{n-1} + 2^n \quad \ldots \quad a_n = n2^n. \quad a_{n-1} = (n-1)2^{n-1}$

$2a_{n-1} + 2^n = 2((n-1)2^{n-1}) + 2^n$

$= (n-1)2^n + (1)2^n$

$= (n-1+1)2^n$

$= n2^n = a_n$ ✓

24.b) $a_n = r\, a_{n-1} = 1$

$r = 2 \qquad a_n^{(h)} = \alpha \cdot 2^n$

$a_n = \alpha \cdot 2^n + n2^n$

24.c) $a_0 = 2$

$2 = \alpha \cdot 1 \qquad a_n = 2 \cdot 2^n + n2^n$

$a_n = (n+2)2^n$

Section 8.3

2. $f(n) = 2f(\frac{n}{2}) + 2 \qquad f(1) = 0$

$f(128) = 2f(64) + 2$

$= 2(2f(32) + 2)$

$\vdots$

$= 2(2(2(2(2(2(2f(1)+2)+2)+2)+2)+2)+2)+2$

$= 254$ comparisons

7.a) $f(n) = f(\frac{n}{3}) + 1 \qquad f(1) = 1$

$f(3) = f(1) + 1 = 2$

7.b) $f(27) = f(9) + 1$

$= f(3) + 2$

$= f(1) + 3$

$= 1 + 3 = 4$

7.c) $f(729) = f(243) + 1$

$= f(81) + 2$

$= f(27) + 3$. we know $f(27) = 4$

$= 4 + 3 = 7$

Section 8.5

2.) $|C| = 345 \quad |D| = 212 \quad |C \cap D| = 188$

$|C \cup D| = 345 + 212 - 188 = \boxed{369}$

5.b) $100 + 100 + 100 - 50 - 50 - 50 + 0 = \boxed{150}$

5.c) $100 + 100 + 100 - 50 - 50 - 50 + 25 = \boxed{175}$

7.) $|J| = 1876 \quad |L| = 999 \quad |C| = 345 \quad |J \cap L| = 876$

$|L \cap C| = 231 \quad |J \cap C| = 290 \quad |J \cap L \cap C| = 189$

$|J \cup L \cup C| = 1876 + 999 + 345 - 876 - 290 - 231 + 189$

$= 2012$

$2504 - 2012 = \boxed{492 \text{ students}}$

8. $|S| = 64 \quad |B| = 94 \quad |C| = 58 \quad |S \cap B| = 26$

$|S \cap C| = 28 \quad |B \cap C| = 22 \quad |S \cap B \cap C| = 14$

$|S \cup B \cup C| = 64 + 94 + 58 - 26 - 28 - 22 + 14 = 154$

$270 - 154 = \boxed{116 \text{ students}}$

10.) divisible by 5? 20

divisible by 7? 14

divisible by 35? 2

$20 + 14 - 2 = 32$

$100 - 32 = \boxed{68}$

12.) divisible by 3 | divisible by 4 | divisible by 7 | divisible by 11

1 3333 2500 1428 909

$\frac{10000}{3\cdot4} \approx 833$ $\frac{10000}{3\cdot7} \approx 476$ $\frac{10000}{3\cdot11} \approx 303$ $\frac{10000}{4\cdot7} \approx 357$ $\frac{10000}{4\cdot11} \approx 227$

$\frac{10000}{7\cdot11} \approx 129$ $\frac{10000}{3\cdot4\cdot7} \approx 119$ $\frac{10000}{3\cdot4\cdot11} \approx 75$ $\frac{10000}{3\cdot7\cdot11} \approx 43$ $\frac{10000}{4\cdot7\cdot11} \approx 32$ $\frac{10000}{3\cdot4\cdot7\cdot11} \approx 10$

$= 3333 + 2500 + 1428 + 909 - 833 - 476 - 303 - 357 - 227 - 129 + 119 + 75$
$+ 43 + 32 - 10 = 6104$

$10000 - 6104 = 3896$ integers