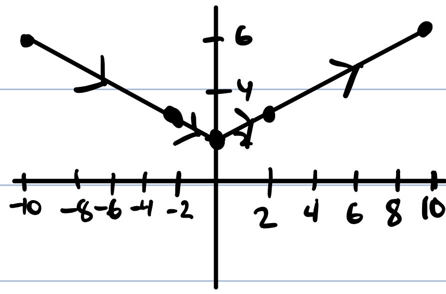
Homework #12 Section 10.1:

2.)

t	x	y
-2	2	$\frac{1}{3}$
-1	0	1
0	0	3
1	2	9
2	6	27

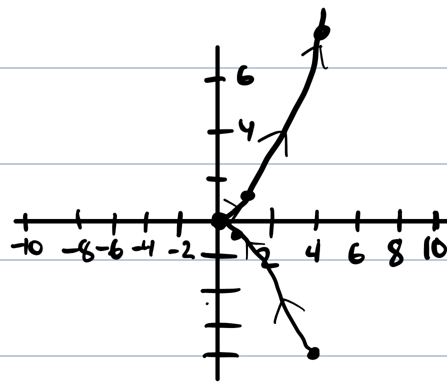
4.) $x = t^3 + t \quad y = t^2 + 2$

t	x	y
-2	-10	6
-1	-2	3
0	0	2
1	2	3
2	10	6



12.a) $x = t^2 \quad y = t^3$

t	x	y
-2	4	-8
-1	1	-1
0	0	0
1	1	1
2	4	8



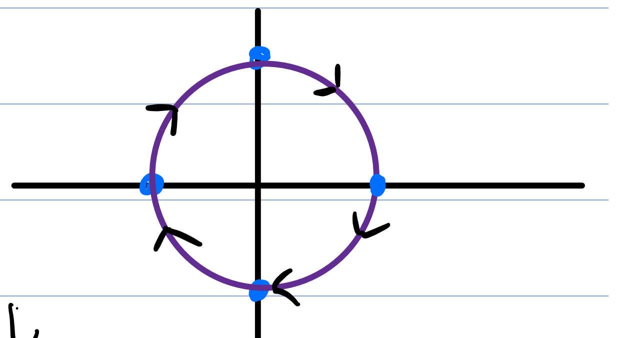
12.b) $y = t^3$

$y^{1/3} = t, \quad x = (y^{1/3})^2$

$= y^{2/3}, \quad x \geq 0$

14) $x = \sin 4\theta, \quad y = \cos 4\theta \quad 0 \leq t \leq \pi$

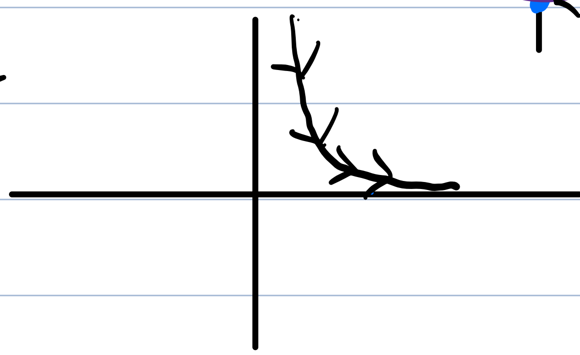
$\sin^2 4\theta + \cos^2 4\theta = 1 \Rightarrow x^2 + y^2 = 1$



18.) $x = t+2, \quad y = \frac{1}{t}, \quad x - 2 = t$

$y = \frac{1}{x-2}$

t	x	y
1	3	1
2	4	$\frac{1}{2}$
3	5	$\frac{1}{3}$



34.a) $x = t^4 - t + 1$, $y = t^2$. Only IV and V have x-intercept $(1,0)$
$t^4 - t + 1$ will never approach ∞. So V

34.b) $x = t^2 - 2t$, $y = \sqrt{t}$. Goes through origin. $\sqrt{t} > 0$
So I is answer

34.c) $x = t^3 - 2t$, $y = t^2 - t$. Go through origin. and only one I have remaining is graph II

34.d) $x = \cos 5t$, $y = \sin t$ Graph VI only one with correct behavior since x, y never approach ∞ & $-\infty$

34.e) $x = t + \sin 4t$, $y = t^2 + \cos 3t$. graph IV only one I have remaining

34.f) $x = t + \sin 2t$, $y = t + \sin 3t$. Graph III since only one that spirals.

Section 10.2

4.) $X = t + \sin(t^2+2)$, $y = \tan(t^2+2)$

$$\frac{dx}{dt} = 1 + 2t\cos(t^2+2), \quad \frac{dy}{dt} = 2t\sec^2(t^2+2)$$

$$\frac{dy}{dx} = \frac{2t \cdot \frac{1}{\cos(t^2+2)}}{1+2t\cos(t^2+2)} = \frac{2t \cdot \frac{1}{\cos(t^2+2)}}{1+2t\cos(t^2+2)} \cdot \frac{\cos^2(t^2+2)}{\cos^2(t^2+2)}$$

$$= \frac{2t}{\cos^2(t^2+2)\left(1+2t\cos(t^2+2)\right)}$$

34.) $X=3t^2+1,\ y=2t^3+1,$ through $(4,3)$.

$\frac{dx}{dt}=6t \qquad \frac{dy}{dt}=6t^2 \qquad \frac{dy}{dx}=\frac{6t^2}{6t}=t$

$3t^2+1=4 \qquad 2t^3+1=3$

$t=\pm 1 \qquad t=1$

$$\boxed{y-3=1(x-4) \qquad y=x-1}$$

36.) $A=\int_0^{\pi/2} \sin t\cdot\cos t(\cos t)\,dt=\int_0^{\pi/2}\cos^2 t\cdot\sin t$ $\quad u=\cos t \quad du=-\sin t\,dt$

$\cos(\frac{\pi}{2})=0$

$\cos(0)=1$

$$=-\int_1^0 u^2\,du=-\frac{u^3}{3}\Big|_1^0=-\frac{0^3}{3}+\frac{1^3}{3}=\boxed{\frac{1}{3}}$$

48.) $x=e^t-t,\ y=4e^{t/2},\ 0\le t\le 2$

$\frac{dx}{dt}=e^t-1 \quad \frac{dy}{dt}=2e^{t/2}$

$$L=\int_0^2\sqrt{(e^t-1)^2+(2e^{t/2})^2}\,dt=\int_0^2\sqrt{e^{2t}-2e^t+1+4e^t}\,dt$$

$$=\int_0^2\sqrt{(e^t+1)^2}\,dt=\int_0^2 e^t+1\,dt=e^t+t\Big|_0^2=e^2+2-e^0-0$$

$$=\boxed{e^2+1}$$

72.) $X=2t^2+\frac{1}{t},\ Y=8\sqrt{t},\ 1\le t\le 3$

$\frac{dx}{dt}=4t-\frac{1}{t^2} \quad \frac{dy}{dt}=\frac{4}{\sqrt{t}}$

$$S=\int_1^3 2\pi(8\sqrt{t})\sqrt{\left(4t-\frac{1}{t^2}\right)^2+\left(\frac{4}{\sqrt{t}}\right)^2}\,dt$$

$$=16\pi\int_1^3\sqrt{t}\sqrt{16t^2-\frac{8}{t}+\frac{1}{t^4}+\frac{16}{t}}\,dt$$

$$= 16\pi \int_1^3 \sqrt{16t^3 - 8 + \frac{1}{t^3} + 16}\, dt$$

$$= 16\pi \int_1^3 \sqrt{16t^3 + \frac{1}{t^3} + 8}\, dt$$

$$= 16\pi \int_1^3 \sqrt{\frac{4(t^3)^2 + 1}{t^{3/2}}} = 16\pi \int_1^3 \frac{4t^3+1}{t^{3/2}} = \int_1^3 4t^{3/2} + \frac{1}{t^{3/2}}$$

$$= 16\pi \left(\frac{8t^{5/2}}{5} - \frac{2}{\sqrt{t}} \right) \Big|_1^3 = \frac{16\pi(2\sqrt{3}+206)}{5\sqrt{3}}$$

Section 10.3:

4.a) $x = 4\cos\frac{4\pi}{3} = 4 \cdot -\frac{1}{2} = -2$ $\Rightarrow (-2, -2\sqrt{3})$

$y = 4\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \cdot 4 = -2\sqrt{3}$

4.b) $x = -2\cos\frac{3\pi}{4} = \sqrt{2}$ $\Rightarrow (\sqrt{2}, \sqrt{2})$

$y = -2\sin\frac{3\pi}{4} = -\sqrt{2}$

4.c) $x = -3\cos -\frac{\pi}{3} = -\frac{3}{2}$ $\Rightarrow \left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$y = -3\sin -\frac{\pi}{3} = \frac{3\sqrt{3}}{2}$

6.a) $(\sqrt{3}, -1)$ $|r| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \pm 2$, $r = 2$, $r = -2$

$2\cos\theta = \sqrt{3}$ $2\sin\theta = -1$ $-2\cos\theta = \sqrt{3}$ $-2\sin\theta = -1$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\sin\theta = -\frac{1}{2}$ $\cos\theta = -\frac{\sqrt{3}}{2}$ $\sin\theta = \frac{1}{2}$

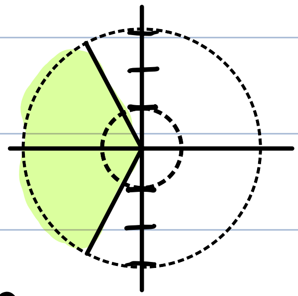
$\theta = \frac{5\pi}{6}$ $\left(-2, \frac{5\pi}{6}\right)$, $r > 0$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ is $\frac{11\pi}{6}$, $r > 0$

6.b) $|r| = \sqrt{(-6)^2 + 0} = \pm 6$, $r = 6$, $r = -6$

$6\cos\theta = -6$ $6\sin\theta = 0$ $(6, \pi)$ $-6\cos\theta = -6$ $\sin\theta = 0$ $(-6, 0)$

$\cos\theta = -1$ $\sin\theta = 0$ $\cos\theta = 1$

10.) 

20.) $r^2\sin 2\theta = 1$

$r^2\cdot 2\sin\theta\cos\theta = 1$, $r\sin\theta\cdot r\cos\theta = \frac{1}{2}$, $xy = \frac{1}{2}$

24.) $y = -2x^2$. $r\sin\theta = -2(r\cos\theta)^2$ $r\sin\theta = -2r^2\cos^2\theta$

$$\frac{r\sin\theta}{-2r\cos^2\theta} = \frac{-2r^2\cos^2\theta}{-2r\cos^2\theta} \Rightarrow -\frac{1}{2}\cdot\frac{\sin\theta}{\cos^2\theta} = r$$

$$-\frac{1}{2}\cdot\frac{\sin\theta}{\cos^2\theta}\cdot\frac{1}{\cos^2\theta} = r$$

$$-\frac{\tan\theta\cdot\sec\theta}{2} = r$$

$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos\theta$

Section 10.4:

6.) $r = 2 + \cos\theta$

$$A = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}(2+\cos\theta)^2 = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} 4 + \cos^2\theta + 4\cos\theta \, d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} 4 + \frac{1}{2} + \frac{1}{2}\cos\theta + 4\cos\theta \, d\theta = \frac{1}{2}\left(\frac{9}{2}\theta + \frac{1}{4}\sin 2\theta + 4\sin\theta\right)\Big|_{\frac{\pi}{2}}^{\pi} =$$

$$= \boxed{\frac{9\pi}{8} - 2}$$

50.) $r = e^{\theta/2}$, $0 \le \theta \le \frac{\pi}{2}$ $L = \int_0^{\pi/2}\sqrt{\left(e^{\theta/2}\right)^2 + \left(\frac{e^{\theta/2}}{2}\right)^2}\, d\theta$

$$= \int_0^{\pi/2}\sqrt{\left(1+\frac{1}{4}\right)\left(e^{\theta/2}\right)^2}\, d\theta = \int_0^{\pi/2}\frac{\sqrt{5}}{\sqrt{4}}\cdot\sqrt{\left(e^{\theta/2}\right)^2} = \frac{\sqrt{5}}{2}\int_0^{\pi/2} e^{\theta/2}\, d\theta$$

$u = \frac{\theta}{2}$ $\theta = 2u$

$d\theta = 2du$

$\theta\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$, $\theta(0) = 0$

$$= \sqrt{5}\int_0^{\pi/4} e^u \, du = \sqrt{5}\left(e^u\right)\Big|_0^{\pi/4}$$

$$\boxed{L = \sqrt{5}\left(e^{\pi/4} - 1\right)}$$