MATH:1860 Activity 11 – (Sections 11.11-10.2) Apr. 24

Name:________________________________

Instructions: Work with others or independently to complete the activity.

1. Find the Taylor polynomial $T_3(x)$ for $f(x) = \cos x$ centered at $a = \pi/2$.

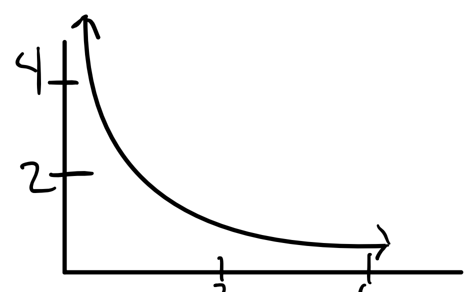
$$f(x) = \cos x \qquad f(\pi/2) = 0$$
$$f'(x) = -\sin x \qquad f'(\pi/2) = -1$$
$$f''(x) = -\cos x \qquad f''(\pi/2) = 0$$
$$f'''(x) = \sin x \qquad f'''(\pi/2) = 1.$$

$$T_3(x) = \sum_{i=0}^{3} \frac{f^{(i)}(\pi/2)}{i!}(x-\pi/2)^i = -(x-\pi/2) + \frac{1}{6}(x-\pi/2)^3$$

2. Let $x = e^{-t}$, $y = e^t$ be parametric equations. Eliminate the parameter to find a Cartesian equation of the curve. Sketch the curve and indicate the direction in which the curve is traced as the parameter increases.

$\ln(y) = \ln(e^t) \Rightarrow \ln(y) = t$

$x = e^{-\ln y}$
$x = y^{-1} \Rightarrow xy = 1 \Rightarrow y = \frac{1}{x}$



3. At what point(s) on the curve $x = 3t^2 + 1$, $y = t^3 - 1$ does the tangent line have slope $\frac{1}{2}$?

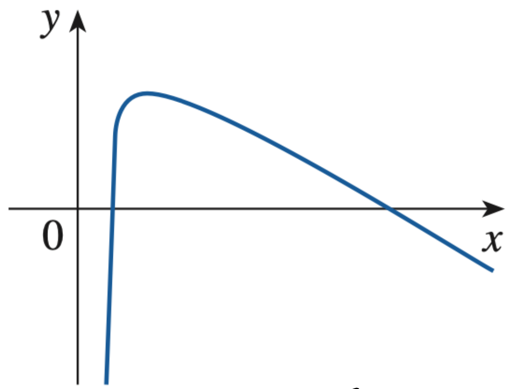
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{6t} = \frac{t}{2}$$

When $t = 1$, slope is $\frac{1}{2}$. So point is $(4, 0)$

$x = 3(1) + 1 = 4$

4. Find the area enclosed by $x = t^3 + 1,\ y = 2t - t^2$ and the x-axis:

$A = \int y\,dx$



$$
\begin{aligned}
A &= \int_0^2 (2t - t^2) 3t^2 \, dt \\
&= \int_0^2 6t^3 - 3t^4 \, dt \\
&= \frac{3}{2}t^4 - \frac{3}{5}t^5 \Big|_0^2 \\
&= \frac{3}{2}2^4 - \frac{3}{5}2^5 \\
&= 24 - \frac{96}{5} \\
&= \frac{24}{5}.
\end{aligned}
$$

5. Find the exact length of the curve $x = t\sin t,\ y = t\cos t,\ 0 \le t \le 1.$