MATH:1860 Activity 2 – (Sections 7.1-7.4) Jan. 30

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Instructions: Work with others or independently to complete the activity.

1. Solve the following integrals.

(a) $\int \sin^{-1} t\, dt$

(Hint: Integration by parts; recall $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$)

$u = \sin^{-1} t \qquad du = \frac{1}{\sqrt{1-t^2}}\, dt$

$dv = dt \quad v = t$

$u = 1-t^2$
$du = -2t\,dt$

$\int \sin^{-1} t\, dt = \sin^{-1} t \cdot t - \int \left(\frac{t}{\sqrt{1-t^2}}\right.$

$\int \frac{t}{\sqrt{1-t^2}} = -\frac{1}{2}\int \frac{-du}{u^{1/2}} = -\frac{1}{2}\int u^{-1/2}\, du = -\frac{1}{2}(2u^{1/2}) = -\sqrt{1-t^2}$

$\int \sin^{-1} t\, dt = \boxed{t\sin^{-1} t + \sqrt{1-t^2} + C}$

(b) $\int \sin x \sec^5 x\, dx = \int \sin x \cdot \frac{1}{\cos^5 x} = \int \frac{\sin x}{\cos^5 x} = \int \tan x \sec^4 x\, dx = \int \tan x(\tan^2 x + 1)\sec^2 x\, dx$

$u = \tan x$
$du = \sec^2 x\, dx$

$= \int (u^3 + u)\, du = \frac{u^4}{4} + \frac{u^2}{2} = \boxed{\frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} + C}$

$\tan^{-1} x = \tan^{-1}\tan\theta \qquad \tan^{-1} x = \theta$

(c) $\int \frac{1}{(1+x^2)^2}\, dx$ (make sure final answer is in terms of x)

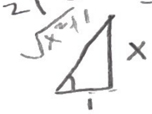
$\sqrt{a^2+x^2}$
$x = \tan\theta$
$dx = \sec^2\theta\, d\theta$

$(1+x^2)^2 = (1+\tan^2\theta)^2 = (\sec^2\theta)^2 = \sec^4\theta$

$\int \frac{1}{\sec^4\theta} \cdot \sec^2\theta\, d\theta = \int \frac{1}{\sec^2\theta}\, d\theta = \int \cos^2\theta\, d\theta = \frac{1}{2}\int(1+\cos 2\theta)$

$= \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + C = \frac{1}{2}\theta + \frac{\sin 2\theta}{4} + C$

$\tan\theta = \frac{x}{1}$



$\sin\theta = \frac{x}{\sqrt{x^2+1}}$

$= \boxed{\frac{1}{2}(\tan^{-1} x) + 2\left(\frac{x}{\sqrt{x^2+1}}\right) + C}$

(d) $\int \frac{2x-1}{x^3+4x}\, dx = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$x^3+4x = x(x^2+4)$

$2x-1 = A(x^2+4) + (Bx+C)x$

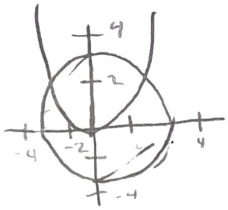
$A = -\frac{1}{4}$
$B = \frac{1}{4}$
$C = 2$

$\frac{2x-1}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$-\frac{1}{4}\ln|x| + \frac{1}{8}\ln(x^2+4) + \arctan\left(\frac{x}{2}\right) + C$

2. (Challenge) The parabola $y = \frac{1}{2}x^2$ divides the disk $x^2 + y^2 \leq 8$ into two parts. Find the areas of both parts.

(a) Graph the two functions.



(b) What is the area of the disk $x^2 + y^2 \leq 8$?

$A = \pi r^2$ $\quad r^2 = 8$

$A = 8\pi$

(c) At what x-values do the two functions intersect?

$(\sqrt{8-x^2})^2 = (\frac{1}{2}x^2)^2 = 8 - x^2 = \frac{1}{4}x^4 = 32 - 4x^2 = x^4 = x^4 + 4x^2 - 32 = 0$

$u = x^2$

$u^2 + 4u - 32 = 0$

$(u+8)(u+4) = 0$

$u = \cancel{-8}$ $\quad u = 4$

$x^2 = 4$ $\quad x = \pm 2$

(d) Find the area of the top part.

$$\int_{-2}^{2} \sqrt{8-x^2}\,dx - \int_{-2}^{2} \frac{1}{2}x^2\,dx$$

$$\int_{-2}^{2} \frac{1}{2}x^2 = \frac{1}{2}\int_{-2}^{2} x^2\,dx = \left(\frac{x^3}{3}\right)\frac{1}{2}\Big|_{-2}^{2} = \frac{8}{3}$$

$$\int_{-2}^{2} \sqrt{8-x^2} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(8) = 4\pi$$

$$A = 4\pi - \frac{8}{3}$$

(e) Find the area of the bottom part.

$$8\pi - \left(4\pi - \frac{8}{3}\right) = 6\pi - \frac{4}{3}$$