Sets

Def:
unordered collection of unique elements

Ex: $\mathbb{N}$ natural numbers $\{0, 1, 2 \ldots\ldots\}$

$\mathbb{Z}$ - integers

$\mathbb{Z}^{+}$ - positive integers

$\mathbb{Q}$ - rational $\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \}$

$\mathbb{I}$ - irrational π $\sqrt{2}$

$\mathbb{C}$ - complex

Notation: $a \in A$ "element a belongs to set A"

$a \notin A$ "element a does not belong to set A"

Intervals: $[a, b]$ (a, b)

closed inclusive open exclude

Closed - include   Open, exclude

Equal Sets:

$$\forall x (x \in A \Leftrightarrow x \in B) \quad A = B$$

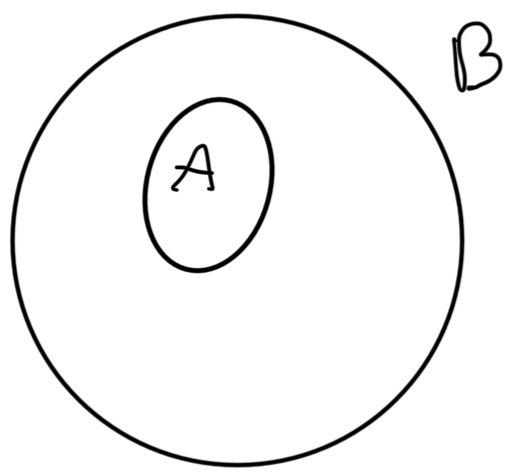
to prove:

$\Rightarrow$ $\forall x \in A$, show $x \in B$

$\Leftarrow$ $\forall x \in B$, show $x \in A$

Def: a set A is a subset of Set B if $\forall x \in A \rightarrow x \in B$

$A \subseteq B$



Def: An empty set is a set with no elements in it.

Notation: $\emptyset$

Ex: Set $A = \{x \in \mathbb{Z} \mid x = 2k \text{ for some } k \in \mathbb{Z}\} =$ set of even numbers

$A \subseteq \mathbb{Z}$ $\mathbb{Q} \subseteq \mathbb{R}$
$\mathbb{N} \subseteq \mathbb{Z}$
$\mathbb{Z} \subseteq \mathbb{R}$ note: $\emptyset \subseteq A$ for any A also $A \subseteq A$ for all A

Ex: $\mathbb{Z} \subseteq \mathbb{Q}$
Proof: $\mathbb{Z} \subseteq \mathbb{Q}$: Let $x \in \mathbb{Z}$, then $\frac{x}{1} = x$, $x, 1 \in \mathbb{Z}$ so x is rational by definition of rational number.

Cardinalality: of set A is a number of eements in A
$|A|$

Ex: $A = \{a, b, c, d, e, f\}$ $|A| = 6$
L = english alphabet $|L| = 26$

Ex: How many elements has set of old numbers between 0 and 16 inclusive? 8

Powerset: Given a set S the powerset of S, denoted $P(S)$ is the set of all subsets of S

EX: $S = \{a, b, c\}$
$P(S) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

note: $|P(S)| = 2^{|S|} = 2^3 = 8$

Cartesian Product

Def: The ordered n-tuple $(a_1, a_2, \ldots\ldots, a_n)$ is ordered collection where a_1 comes before a_2

Denote: A and B be set

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Ex: $A = \{a, b\}$ $B = \{1, 2, 3\}$

$$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$$

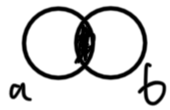
note: $|A \times B| = |A| \cdot |B| = 6$

$$B \times A = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$$

Set operations

1. A union B, $A \cup B$, $\{x \mid x \in A \text{ or } x \in B\}$

2. A intersection B, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



3. A difference B $\quad A - B$

4. A compliment B $\quad \overline{A}$

EX: $A = \{a, b, c, d\} \quad B = \{a, d, e\}$

$A \cup B = \{a, b, c, d, e\}$

$A \cap B = \{a, d\}$

$A - B = \{b, c\}$

let $U = \{a, b, c, d, e, f, g\}$

$\overline{A} = \{e, f, g\}$

$P(A \cap B) = \{\phi, \{a\}, \{d\}, \{a, d\}\}$

$|A \cup B| = 5$

$$A = \{$$