Worksheet 4

CS 2210 Discrete Structures

Due 2/19 9pm. Late submissions get grade 0.

* Teams of 3-4 students (must work in group)
** This page is double sided. Make sure to do both sides.
*** Show and explain your work

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Question 1: Suppose g: A→B and f: B→C where A = {1, 2, 3, 4}, B = {a, b, c}, C = {3, 5, 7} and f and g are defined by

g = {(1, c), (2, a), (3, b), (4, a)}

f = {(a, 5), (b, 7), (c, 3)}.

a. Find $f \circ g$

$\{(1,3), (2,5), (3,7), (4,5)\}$

b. Find f^{-1}

$\{(5,a), (7,b), (3,c)\}$

Question 2: Suppose a and b are odd integers and $a \neq b$. Show that there is a unique integer c such that $|a - c| = |b - c|$.

$a - c = b - c \longrightarrow a = b$ not possible

$a - c = -(b - c) \longrightarrow a - c = -b + c$

$a = -b + 2c$

$a + b = 2c$

$c = \frac{a+b}{2}$ Ex. $a = 3, b = 5$ $\frac{3+5}{2} = c = 4$

$|3 - 4| = |5 - 4|$

$|-1| = |1|$

$1 = 1$

$c = 4$ when $a = 3$ and $b = 5$

Question 3: Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. You can't use Venn Diagrams and/or membership table.

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Let $x \in A \wedge x \in (B \cup C)$

$x \in A \wedge (x \in B \vee x \in C)$

$x \in A \cap B \vee x \in A \cap C$

Thus, $x \in (A \cap B) \cup (A \cap C)$ ①

Let $x \in (A \cap B) \cup (A \cap C)$

$x \in (A \cap B) \vee x \in (A \cap C)$

$x \in A$ and $x \in B$

$x \in A$ and $x \in C$

Thus, $x \in A \cap (B \cup C)$ ②

① $\subseteq$ ② and ② $\subseteq$ ①

Question 4: Let $f(x)\colon \mathbb{R} \to \mathbb{R}, f(x) = \frac{-4x+3}{8}$ and $g(x) = \sqrt{x-1}$

a. Is f(x) one-to-one? Prove your answer.

$\frac{8}{1} \cdot \frac{-4(a)+3}{8} = \frac{-4(b)+3}{8} \cdot 8$

$-4a+3 = -4b+3$

$-3 \quad -3$

$\frac{-4a}{-4} = \frac{-4b}{-4}$

$a = b$ 1-1 ✓

b. Is f(x) onto? Prove your answer.

There is a $x \in \mathbb{R}$ s.t $f(x) = y$

$y = \frac{-4x+3}{8}$ $\quad x = -\frac{8y-3}{4}$

$8y = -4x+3$

$8y-3 = -4x$

$f(x) = f\left(-\frac{8y-3}{4}\right) = \frac{-4\left(-\frac{8y-3}{4}\right)+3}{8}$

$= \frac{8y-3+3}{8}$

$= y$ yes onto!

c. Does f(x) have inverse function? If so, find $f^{-1}(x)$ and prove it is inverse function.

$x = \frac{-4y+3}{8}$

$8x = -4y+3$

$8x-3 = -4y$

$-\frac{8x-3}{4} = y$

yes, $f^{-1}(x) = -\frac{8x-3}{4}$

$f(f^{-1}(x)) = \frac{-4\left(-\frac{8x-3}{4}\right)-3}{8} = \frac{8x}{8} = x$

$f^{-1}(f(x)) = \frac{-8\left(\frac{-4x+3}{8}\right)+3}{4} = \frac{4x}{4} = x$

$x = x$, yes, inverse

d. Find $f \circ g$

$\frac{-4\left(\sqrt{x-1}\right)+3}{8}$

e. Find $g \circ f$

$\sqrt{\left(\frac{-4x+3}{8}\right)-1}$