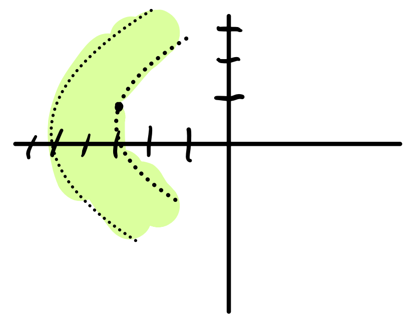
MATH:1860 Activity 12 – (Sections 10.2-10.4) May. 01

Name:__________________________________

Instructions: Work with others or independently to complete the activity.

1. Sketch the region in the plane consisting of points whose polar coordinates satisfy the given condition: $3 < r < 5,\ 2\pi/3 \le \theta \le 4\pi/3$.



2. Find the exact length of the polar curve $r = 2\cos\theta,\ 0 \le \theta \le \pi$.

$$L = \int_0^{\pi} \sqrt{4\cos^2\theta + 4\sin^2\theta}\, d\theta = \int \sqrt{4(1)}\, d\theta$$

$$= \int_0^{\pi} 2\, d\theta = 2\pi$$

3. (a) The point $(-1, -\pi/6)$ is in polar coordinates. Convert it to Cartesian coordinates.

$$(x, y) = \left(-\cos\left(-\frac{\pi}{6}\right), -\sin\left(-\frac{\pi}{6}\right)\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

(b) The point $(3, 3\sqrt{3})$ is in Cartesian coordinates. Convert it to polar coordinates in two different ways.

$r^2 = x^2 + y^2$ $\quad r^2 = 36$

$\tan\theta = \sqrt{3}$. So $r = \pm 6$ $\quad \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

So $(r, \theta) = \left(6, \frac{\pi}{3}\right), \left(-6, \frac{4\pi}{3}\right)$

4. Identify the curve by finding a Cartesian equation for the curve: $r^2 \cos 2\theta = 1$. (Hint: $\cos 2\theta = \cos^2\theta - \sin^2\theta$.)

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$x^2 - y^2 = 1$

$x = r\cos\theta$

$y = r\sin\theta$

Parabola

5. The curve shown in the figure is the astroid $x = a\cos^3\theta,\ y = a\sin^3\theta$. Find the area of the region enclosed by the astroid.