# Midterm 2

Graded

**Student**

Colin Cano

**Total Points**

**37 / 50 pts**

**Question 1**

## Particle Velocity

**4** / 5 pts

Elements are incorrect, select all that apply.

✔ **− 1 pt** Student fails to correctly determine if the particle is speeding up or slowing down based on their evaluations of v(2) and a(2)

**Question 2**

## Linear Approximation

**5** / 5 pts

✔ **− 0 pts** Correct: 2.025

**Question 3**

## Half-Life

**4** / 5 pts

Elements are incorrect, select all that apply.

✔ **− 1 pt** Student does not attempt to use the $Pe^{rt}$ formula.

**Question 4**

## Tangent Line

**4** / 5 pts

Elements are incorrect, select all that apply.

✔ **− 1 pt** Student makes an error in finding the derivative due to a mistake with implicit differentiation.

**Question 5**

## Box

**0** / 5 pts

✔ **− 5 pts** No correct work.

**Question 6**

## L'Hospital 1

**5** / 5 pts

✔ **− 0 pts** Correct: $-\frac{1}{2}$

**Question 7**

## L'Hospital 2

**1** / 5 pts

Elements are incorrect, select all that apply.

✔ **− 4 pts** some correct element but not solving the problem

**Question 8**

## Ladder

**5** / 5 pts

✔ **– 0 pts** Correct: $-\frac{9}{4}$

**Question 9**

## Function Description

**5** / 5 pts

✔ **– 0 pts** Correct: Note, due to the limited space, this question does not require any work to be shown.

Critical point: $(1, 0)$, neither min nor max. Interval of increase: $(-\infty, \infty)$ or $(-\infty, 1) \cup (1, \infty)$. Interval of decrease: none. Inflection point: $(1, 0)$. Interval of concave up: $(1, \infty)$. Interval of concave down: $(-\infty, 1)$.

**Question 10**

## Fence

**4** / 5 pts

Elements are incorrect, select all that apply.

✔ **– 1 pt** Student incorrectly calculates the second dimension based on the first.

MATH:1850 **Midterm II** Fall 2024
TA:

**Name:** Colin Cano **Student ID:** _____

1. A particle moves along the x-axis so that its position at any time $t$ is given by $x(t) = t^3 - 6t^2 + 9t + 2$. Find the velocity and acceleration of the particle at $t = 2$. Determine if the particle is speeding up or slowing down at $t = 2$.

$x(t) = t^3 - 6t^2 + 9t + 2$
$x'(t) = 3t^2 - 12t + 9$
$x''(t) = 6t - 12$

$x'(2) = 3(2)^2 - 12(2) + 9 = -3$
$x''(2) = 6(2) - 12 = 0$

Since the velocity is negative and the acceleration is 0, the particle is slowing down at a constant speed

2. Use linear approximation to estimate the value of $f(x) = \sqrt{x}$ at $x = 4.1$.

$f'(x) = \frac{1}{2}x^{-1/2}$ $f(4) = \sqrt{4} = 2$
$f'(4) = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} = \frac{1}{4}$

$f(4.1 - 0.1) \approx f(x) + f'(x) \cdot dx$
$\approx 2 + \frac{1}{4}(.1)$
$\approx 2.025$

$\frac{1}{4} \cdot \frac{1}{10} = \frac{1}{40}$

3. A certain radioactive material has a half-life of 10 years. If you start with 50 grams of this material, how much will remain after 25 years?

$P(t) = 50e^{rt}$
$50 = e^{10r}$
$\ln(50) = \ln(e^{10r})$
$\frac{\ln(50)}{10} = \frac{10r}{10}$
$\frac{\ln(50)}{10} = r$

$P(25) = 50e^{\left(\frac{\ln(50)}{10}\right) \cdot 25}$

4. Given the equation $x^2y^2 + \sin(\pi y) = 1$, find the equation of the tangent line at the point $(1, 1)$.

$2xy^2 + x^2 2yy' + \cos(\pi y) \cdot \pi y' = 0$
$2(1)(1)^2 + (1)^2 \cdot 2(1)y' + \cos(\pi) \cdot \pi = 0$
$2 + 1 \cdot 2y' - \pi = 0$
$2y' = -1$
$y' = -\frac{1}{2}$

$y - 1 = x - 1\left(-\frac{1}{2}\right)$

5. A closed rectangular box with a square base is to be made. The volume of the box is 32 cubic units. Find the height of the box that will minimize the surface area.

$V = 32 = lwh$
$h = \frac{32}{lw}$
$0 = L \cdot w\left(\frac{32}{lw}\right)$

6. Evaluate the limit using L'Hospital's rule:

$$\lim_{x\to 0}\frac{\ln(1+x)-\sin(x)}{x^2} \overset{LH}{=} \lim_{x\to 0}\frac{\frac{1}{1+x}-\cos(x)}{2x} \overset{LH}{=} \lim_{x\to 0}\frac{\frac{-1}{(1+x)^2}+\sin x}{2} =$$

$$= \frac{\frac{-1}{(1+0)^2}+\sin(0)}{2} = \frac{-1+0}{2} = -\frac{1}{2}$$

7. Use L'Hospital's rule to find:

$$\lim_{x\to 0^+}\left(\frac{1}{x}\right)^{1/x} = \lim_{x\to 0^+} e^{\ln\left(\frac{1}{x}\right)^{1/x}} = e^{\lim_{x\to 0^+}\ln\left(\frac{1}{x}\right)^{1/x}} = 0$$

$$= e^0 = 1$$

8. A 10-foot ladder is leaning against a wall. If the bottom of the ladder is being pulled away from the wall at a rate of 3 feet per second, how fast is the top of the ladder sliding down the wall when the bottom is 6 feet from the wall?

$x^2+y^2=L^2 \quad 6^2+y^2=10^2 \quad x=6 \quad L=10$

$y=8$

$2xx'+2yy'=0$

$2(6)(3)+2(8)y'=0$

$y'=\frac{36}{16}=\frac{9}{4}$ ft/s

9. For function $f(x)=x^3-3x^2+3x-1$, identify and label all critical points and inflection points. Mark all intervals in which the function is increasing and decreasing. Mark all intervals in which the funtion is concave up (CONVEX) or concave down (CONCAVE).

critical point: $x=1$
inflection point: $x=1$
is increasing $(-\infty,\infty)$
conc. up $(1,\infty)$
conc. down $(-\infty,1)$

$f(x)=x^3-3x^2+3x-1$
$f'(x)=3x^2-6x+3$
$f''(x)=6x-6$

$3x^2-6x+3=0$
$3(x-1)(x-1)=0$

$6x-6=0$
$x=1$

$f'(-2)>0$
$f'(0)>0$
$f''(0)<0$
$f''(2)>0$

10. A farmer wants to fence off a rectangular field along a river. If the farmer has 500 meters of fencing and needs no fencing along the river, what dimensions will maximize the area of the field?

$2b+a=200$

$A=ba$

$b=x$

$a=\frac{200}{2x}$

$f(x)\ 2x+\frac{200}{x}=200$

$f'(x)\ 2+\frac{-200}{x^2}=0$

$a=\sqrt{200}$

$2b+\sqrt{200}=200$

$b=\frac{200-\sqrt{200}}{2}$

$A=\frac{200-\sqrt{200}}{2}\cdot\sqrt{200}$