Section 7.3

3.) $P(F) = \frac{1}{2} = P(\bar{F})$ $\quad P(E|F) = \frac{3}{5}$ $\quad P(E|\bar{F}) = \frac{1}{5}$

$$\frac{P(E|F)\cdot P(F)}{P(E|F)\cdot P(F) + P(E|\bar{F})\, P(\bar{F})} = \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2} + \frac{1}{5}\cdot\frac{1}{2}} = \frac{\frac{3}{10}}{\frac{3}{10}+\frac{1}{10}} = \frac{3}{10}\cdot\frac{10}{4} = \boxed{\frac{3}{4}}$$

4.) $P(F) = \frac{1}{2} = P(\bar{F})$ $\quad P(E|F) = \frac{3}{7}$ $\quad P(E|\bar{F}) = \frac{9}{11}$

$$\frac{P(E|F)\cdot P(F)}{P(E|F)\cdot P(F) + P(E|\bar{F})\, P(\bar{F})} = \frac{\frac{3}{7}\cdot\frac{1}{2}}{\frac{3}{7}\cdot\frac{1}{2} + \frac{5}{11}\cdot\frac{1}{2}} = \frac{\frac{3}{14}}{\frac{3}{14}+\frac{5}{22}} = \frac{\frac{66}{308}}{\frac{66}{308}+\frac{70}{308}} = \frac{66}{308}\cdot\frac{308}{136} = \frac{66}{136} = \boxed{\frac{33}{68}}$$

6.) $P(F) = 0.05$ $\quad P(\bar{F}) = 1 - 0.05 = .95$ $\quad P(E|F) = .98$

$P(E|\bar{F}) = .12$

$$\frac{P(E|F)\cdot P(F)}{P(E|F)\cdot P(F) + P(E|\bar{F})\, P(\bar{F})} = \frac{.98\cdot 0.05}{.98\cdot 0.05 + .12\cdot .95} = \frac{\frac{98}{100}\cdot\frac{5}{100}}{\frac{98}{100}\cdot\frac{5}{100} + \frac{12}{100}\cdot\frac{95}{100}} = \frac{\frac{490}{10000}}{\frac{490}{10000}+\frac{1140}{10000}}$$

$$= \frac{490}{10000}\cdot\frac{10000}{1630} = \frac{490}{1630} = \boxed{\frac{49}{163}}$$

8.a) $P(F) = \frac{1}{10000}$ $\quad P(E|F) = .999$ $\quad P(E|\bar{F}) = 0.0002$

$P(\bar{F}) = \frac{9999}{10000}$

$$\frac{P(E|F)\cdot P(F)}{P(E|F)\cdot P(F) + P(E|\bar{F})\, P(\bar{F})} = \frac{.999\cdot\frac{1}{10000}}{.999\cdot\frac{1}{10000} + 0.0002\cdot\frac{9999}{10000}} = \boxed{\frac{555}{1666}}$$

8.b) need to find $P(\bar{F}|\bar{E})$ $\quad P(F) = \frac{1}{10000}$ $\quad P(\bar{E}|\bar{F}) = 1 - 0.0002 = .9998$

$P(\bar{F}) = \frac{9999}{10000}$ $\quad P(E|F) = 1 - .999 = .0001$

$$\frac{P(\bar{E}|\bar{F})\cdot P(\bar{F})}{P(\bar{E}|\bar{F})\cdot P(\bar{F}) + P(\bar{E}|F)\cdot P(F)} = \frac{.9998\cdot .9999}{.9998\cdot .9999 + .0001\cdot .0001} = \boxed{0.\overline{999}}$$

Section 7.4

2.) $P(1) = \frac{1}{2} = P(2) \ldots$

$E(x) = 10 \cdot \frac{1}{2} = \boxed{5}$

5.) $p(1) = p(2) = p(4) = p(5) = p(6) = \frac{1}{7}$, $p(3) = \frac{2}{7}$

$E(x) = 2 \cdot \frac{1}{49} + 3 \cdot \frac{2}{49} + 4 \cdot \frac{5}{49} + 5 \cdot \frac{6}{49} + 6 \cdot \frac{8}{49} + 7 \cdot \frac{8}{49} + 8 \cdot \frac{7}{49} + 9 \cdot \frac{6}{49} + 10 \cdot \frac{3}{49} + 11 \cdot \frac{2}{49} + 12 \cdot \frac{1}{49} = \boxed{\frac{48}{7}}$

8.) $E(x) = p(x=r) \cdot 3$

$p(x=r) = \frac{1+2+3+4+5+6}{6} = \frac{7}{2}$ $\quad E(x) = 3 \cdot \frac{7}{2} = \boxed{\frac{21}{2}}$

10.) $P(H) = P(T) = \frac{1}{2}$

Case 1: $p(x=2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Case 2: $P(x=3) = 2 \cdot \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} = \frac{1}{4}$

Case 3: $p(x=4) = 3 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2 = \frac{3}{16}$

Case 4: $P(x=5) = 4 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Case 5: $P(x=6) = 5 \cdot \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 = \frac{3}{16}$

$E(x) = 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{3}{16} + 5 \cdot \frac{1}{8} + 6 \cdot \frac{3}{16} = \boxed{\frac{15}{4} = 3.75}$

12.a) $p(6) = \frac{1}{6}$ $\quad p(\bar{6}) = \frac{5}{6}$

$P(x=n) = \left(\frac{5}{6}\right)^{n-1} \cdot \frac{1}{6}$

12.b) $E(x) = \frac{1}{p(6)} = \frac{1}{\frac{1}{6}} = \boxed{6}$

Section 8.1

2.a) BS: $P_1 = 1$

RS: let P_n be # of permutations of a set with n elements

$P_{n+1} = P_n + P_n + P_n + \ldots + P_n + P_n = (n+1)P_n$

$P_1 = 1$, $P_{n+1} = (n+1)P_n$ when $n \geq 1$

2.b) $P_n = nP_{n-1}$

$= n(n-1)P_{n-2}$

$= n(n-1)(n-2)P_{n-3}$

$= \ldots$

$= n(n-1)(n-2)\ldots 3 \cdot 2 \cdot P_1$

$= n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1 = n!$

$P_n = n!$

7.a) Case 1: $\underset{n-1}{\underline{a_{n-1}}}\ \underline{1}$

Case 2: $\underset{n-2}{\underline{a_{n-2}}}\ \underline{1}\ \underline{0}$

Case 3: $\underset{2^{n-2}}{\underline{2^{n-2}}}\ \underline{0}\ \underline{0}$

$a_n = a_{n-1} + a_{n-2} + 2^{n-2}$

7.b) when $n=0$ there is 0 bit strings so $a_0 = 0$

when $n=1$ there is 0 bit strings so $a_1 = 0$

7.c) $a_0 = 0$ $a_1 = 0$ $a_2 = 1$ $a_3 = 3$ $a_4 = 8$ $a_5 = 19$

$a_6 = 43$ $a_7 = 94$ 94 Bit Strings