

# 12-2 Lecture:

## Lecture Notes: Key Topics and Concepts

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### 1. Administrivia

- Lab12 is last lab, next week is just for questions
- Final Exam is December 20th from 10-12 in MB AUD
- QOTD24 will be released december 6th, QOTD25 will be released December 9th, and QOTD26(Survey) will be released December 11th
- 2 weeks left and bunch of topics we have not covered that he has in past:

Regular expressions, GUIs, Client/server systems, Cryptography, Tractability/intractability and computability

- In addition, he often gives one general lecture about Computational Epidemiology, the focus of his research for the past 15 years
  - Since we have done enough coding, we will spend a few lectures on tractability/intractability and computability
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### 2. XYZ Sequence

- **Problem Description:**
  - Assume you would like to compute a completely made up sequence called  $xyz(n)$ .
  - This sequence is defined for positive  $n$  where  $xyz(0) = 0$  and  $xyz(1)=1$  and  $xyz(n) = 3 * xyz(n-1) - xyz(n-2)$
  - Q: How would you approach this problem?
  - A: recursively
- **Problem Solution:**

```
def xyz(n):  
    if n in {0,1}:  
        return(1)  
    return(3*xyz(n-1) - xyz(n-2))
```

- Q: how many times does  $xyz(n)$  invoke itself?(runtime)
- A: It is complicated. Clearly many times. Solving for how many times, will wait till CS:3330
- But we can get an operational value by implementing an “invocation counting” of  $xyz(n)$

```
def xyz_count(n, c=0):
    c = c+1
    if n in {0,1}:
        return(c,1)
    c,r1 = xyz_count(n-1, c)
    c,r2 = xyz_count(n-2, c)
    return(c,3*r1 - r2)
```

```
>>>xyz_count(1)
(1, 1)
>>>xyz_count(2)
(3, 2)
>>>xyz_count(10)
(177, 4181)
>>>xyz_count(20)
(21891, 63245986)
>>>xyz_count(30)
(2692537, 956722026041)
```

- Not linear! It is exponential,  $O(2^N)$

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### 3. Tractable and Intractable Problems

- **Primary Goals:**
- Computer science theory tells us that some problems are **Tractable**, while others are **intractable**, that is, they are difficult to solve outright
- A problem is **intractable** in solving a larger version of the problem that takes exponentially more time **regardless of the algorithm you use**.
- Sorting is an example of a *Tractable* problem; as the number of records you want to sort increases, the cost of sorting grows polynomially (e.g.  $O(N^2)$  or  $N\log N$ ) with the number of records,  $N$ .
- The traveling salesperson problem is an example of an intractable program; as the number of cities increases the cost of finding the optimal solution grows exponentially, **regardless of the algorithm used**.
  - Is our xyz problem truly intractable?
- **Key Points:**
  1. Truly intractable problems have no known better algorithmic solutions
  - In some cases, it has been shown or proven mathematically that no such better solution can exist
  - In other cases, no such proof has been found

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## 4. Non deterministic Polynomial Time

- **Definition:**
  - For many of these seemingly intractable problems, the cost of checking a solution for correctness is polynomial and not exponential time (these problems are fundamental in cryptography)
- **More:**
  - These problems are said to be solvable in nondeterministic polynomial time, meaning no better than exponential time solution algorithms, but polynomial time checkable.:
  - Thus the great unsolved problem in theoretical Computer Science: does  $P=NP$ ?
  - If it is, then we just have not found good solutions for these problems
  - If it isn't, then we have not found a proof that they cannot exist
- **Output:**
  - Returns a set of hard constraint patterns.

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## 5. Is XYZ Intractable

- It is exponential but it is not, and there is a better algorithm

```
def xyzmemo(n, M={0:1, 1:1}):  
    try:  
        return(M[n])  
    except:  
        M[n]=3*xyzmemo(n-1, M) - xyzmemo(n-2, M)  
        return(M[n])
```

Dictionary gets bigger and bigger.

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