Cobin Bliss

# Worksheet 7

## CS 2210 Discrete Structures

**Due 10/9 9pm. Late submissions get grade 0.**

* Teams of 3-4 students (must work in group). Follow directions given during discussion.
** This page is double sided. Make sure to do both sides. **Show your work.**

Name1: Cobin Bliss Name 2: Colin Cano

Name3: ________ Name 4: ________

Question 1: Compute $(57^{13} mod 8)^7 mod 11$. Show your work.

$((57 \bmod 8)^{13} \bmod 8)^7 \bmod 11$

$(1 \bmod 8)^7 \bmod 11$

$1^7 \bmod 11$

$1 \bmod 11$

1

Question 2: Use Euclidian Algorithm to decide whether 92927 and 123552 are relatively prime. Show your work. Use table with a, b, q and r for every step.

| a | b | q | r |
|---|---|---|---|
| 123552 | 92927 | 1 | 30,625 |
| 92927 | 30625 | 3 | 1,052 |
| 30625 | 1052 | 29 | 117 |
| 1052 | 117 | 8 | 116 |
| 117 | 116 | 1 | 1 |
| 116 | 1 | 116 | 0 |

GCD of the two numbers is 1, so they are relatively prime

Question 3: Use mathematical induction to prove that for all positive integers n:

$$\sum_{i=1}^{n} i \cdot 2^i = (n-1) \cdot 2^{n+1} + 2$$

Hint: Write in the form of 1·2+…+?

$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + i \cdot 2^i + i2^i$

BC: $n=1$, LHS: $1 \cdot 2^1$ RHS: $(1-1) \cdot 2^{1+1} + 2$

LHS $= 2$ RHS $= 2$ $2 = 2$ ✓

IA: Assume for $n=k$, $\sum_{i=1}^{k} i \cdot 2^i = (k-1) \cdot 2^{k+1} + 2$

IS: Prove for $n=k+1$ $\sum_{i=1}^{k+1} i 2^i = ((k+1)-1) \cdot 2^{(k+1)+1} + 2$

$\sum_{i=1}^{k+1} i 2^i = \sum_{i=1}^{k} i 2^i + (k+1) \cdot 2^{k+1} + 2$

$(k-1) \cdot 2^{k+1} + 2 + (k+1) \cdot 2^{k+1}$

$(k-1+k+1) \cdot 2^{k+1} + 2 = (2k) \cdot 2^{k+1} + 2$

$(k+1-1) \cdot 2^{(k+1)+1} + 2 = (n-1) \cdot 2^{n+1} + 2$

Proved by PMI

Question 4: Prove that 21 divides $4^{n+1} + 5^{2n-1}$, $for\ \forall n > 0$.

BC: let $n=1$, $4^{1+1} + 5^{2-1} = 16 + 5 = 21 \cdot 1$, $1 \in \mathbb{Z}$.

IA: Assume for $n=k$, $4^{k+1} + 5^{2k-1}$ is divisible by 57.

By definition of divisibility, $\exists_t \in \mathbb{Z}$ s.t $4^{k+1} + 5^{2k-1} = 21t$.

IS: Prove for $n = k+1$, $4^{k+1+1} + 5^{2(k+1)-1} =$

$4^{k+2} + 5^{2k+1} = 4 \cdot 4^{k+1} + 5^2 + 5^{2k-1} = 4 \cdot 4^{k+1} + 25 + 5^{2k-1} =$

$4 \cdot 4^{k+1} + (4+21) 5^{2k-1} = 4 \cdot 4^{k+1} + 4 \cdot 5^{2k-1} + 21 \cdot 5^{2k-1} =$

$4(4^{k+1} + 5^{2k-1}) + 21 \cdot 5^{2k-1} =$

using IA $21 \cdot t$

$= 4 \cdot 21 \cdot t + 21^{2k-1} = 21(4t + 21^{2k-1})$

let $l = 4t + 21^{2k-1}$, $l \in \mathbb{Z}$ because $t, k \in \mathbb{Z}$

$= 21 \cdot l$

proved by PMI

Question 5: Convert $(53481)_{10}$ to octal. Show your work.

53481 / 8 = 6685 R 1

6685 / 8 = 835 R 5

835 / 8 = 104 R 3

104 / 8 = 13 R 0

13 / 8 = 1 R 5

1 / 8 = 0 R 1

$(150351)_8$