Final Exam

Graded

Student

Colin Cano

Total Points

55.5 / 90 pts

Question 1

Q1

8 / 10 pts

+ 10 pts Correct: $(x-1)+\frac{1}{2}(x-1)^2-\frac{1}{6}(x-1)^3$

- ✔ **+ 2 pts** Attempt to find derivatives of f
- ✔ **+ 2 pts** Attempt to find values of the derivatives of f at $a=1$
- ✔ **+ 2 pts** Correct form of coefficients in Taylor polynomial
- ✔ **+ 2 pts** Correct center in Taylor polynomial

+ 2 pts Correct final Taylor polynomial

− 0.5 pts Minor mistake in final Taylor polynomial

+ 2 pts Incorrect but reasonable attempt

Question 2

Q2

8 / 10 pts

+ 10 pts Correct: $\int_0^{\pi/2} 2\pi \cos t\sqrt{(2\sin t+2t\cos t)^2+(-\sin t)^2}\,dt$

- ✔ **+ 2 pts** Attempt to find derivatives of $x(t), y(t)$
- ✔ **+ 2 pts** Correct derivatives of $x(t), y(t)$
- ✔ **+ 2 pts** Attempt to get the arclength differential $ds=\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2}\,dt$

+ 2 pts Correct form of integral: $\int 2\pi y\,ds$

- ✔ **+ 2 pts** Correct bounds of integration

− 0.5 pts Minor derivative mistake

− 0.5 pts Minor mistake in integrand

+ 2 pts Incorrect but reasonable attempt

Question 3

Q3

8 / 10 pts

+ 10 pts Correct: $R = \frac{1}{2}$, $I = [\frac{9}{2}, \frac{11}{2})$

✔ **+ 2 pts** Attempt to compute $\left|\frac{a_{n+1}}{a_n}\right|$ or $\sqrt[n]{|a_n|}$

✔ **+ 2 pts** Correct computation for $\left|\frac{a_{n+1}}{a_n}\right|$ or $\sqrt[n]{|a_n|}$

✔ **+ 2 pts** Attempt to compute $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ or $\lim_{n\to\infty}\sqrt[n]{|a_n|}$

+ 2 pts Correct radius of convergence based on previous work

✔ **+ 2 pts** Correct interval of convergence based on previous work

− 1 pt Interval of convergence incorrect at endpoints

+ 2 pts Incorrect but reasonable attempt

Question 4

Q4

4 / 10 pts

+ 10 pts Correct:

(a) $(x, y) = (2, -2\sqrt{3})$;

(b) $(r, \theta) = (2\sqrt{2}, 3\pi/4)$ and $(r, \theta) = (-2\sqrt{2}, 7\pi/4)$

Part(a)

+ 5 pts Correct

+ 1 pt Plot of point lies in the correct quadrant

+ 1 pt Plot of point is accurate

✔ **+ 1 pt** Applied correct conversion formulas $x = r\cos\theta,\ y = r\sin\theta$

+ 2 pts x, y coordinates are correct

✔ **+ 1 pt** x, y coordinates are partially correct

Part(b)

+ 5 pts Correct

✔ **+ 1 pt** Plot of point lies in the correct quadrant

+ 1 pt Plot of point is accurate

+ 1 pt Applied correct conversion formulas $r^2 = x^2 + y^2,\ \tan\theta = y/x$

+ 2 pts The two pairs of r, θ coordinates are correct

✔ **+ 1 pt** The two pairs of r, θ coordinates are partially correct

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+ 2 pts Incorrect but reasonable attempt

Question 5

Q5

10 / 10 pts

✔ **+ 10 pts** Correct: converges by LCT

+ 2 pts Attempt to use LCT or another test

+ 2 pts Attempt to compare the terms of the series to the terms of the geometric series with $r = 2/3$

+ 2 pts Attempt limit computation in LCT

+ 2 pts Correct limit computation in LCT

+ 2 pts Concluded that the series is convergent, or a conclusion of divergent is consistent based on previous work

− 1 pt Justification discusses convergence of sequences rather than convergence of series

+ 2 pts Incorrect but reasonable attempt

Question 6

Q6

2 / 10 pts

+ 10 pts Correct: $(\overline{x}, \overline{y}) = (\frac{3}{4}, \frac{8}{5})$

+ 2 pts Applied correct formula for the centroid

+ 2 pts Attempt to compute the integrals in the centroid formula

+ 2 pts Correct bounds of integration

+ 2 pts Correct area of region based on bounds of integration

+ 2 pts Correct x, y-coordinates of centroid based on previous work

− 0.5 pts Minor mistake computing centroid

− 1 pt Mistakes computing centroid

− 1 pt One bound of integration is incorrect

− 0.5 pts Minor mistake computing area

✔ **+ 2 pts** Incorrect but reasonable attempt

Question 7

Q7

9.5 / 10 pts

+ 10 pts Correct: $\frac{1}{5}e^{2x}(\sin x + 2\cos x) + C$

✔ + 2 pts Attempt integration by parts

✔ + 2 pts Valid choice of u, dv

✔ + 2 pts Correct du, v based on choice of u, dv

✔ + 2 pts Correct application of integration by parts formula based on u, dv, du, v

✔ + 2 pts Correct final answer based on u, dv, du, v

− 1 pt Incorrect du or v

− 1 pt Mistake in one application of integration by parts formula

− 0.5 pts Missing constant of integration

✔ − 0.5 pts Minor algebra mistake

− 1 pt Algebra mistakes

+ 2 pts Incorrect but reasonable attempt

Question 8

Q8

4 / 10 pts

+ 10 pts Correct: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{8^{n+1}}x^{3n-1}, |x| < 2$, or equivalent

✔ + 2 pts Attempt to use power series representation for $\frac{1}{1-x}$ to get another power series representation $\left(\text{should be for } \frac{1}{8+x^3}\right)$

+ 2 pts Attempt to take the derivative of a function $\left(\text{should be for } \frac{1}{8+x^3}\right)$

+ 2 pts Attempt to take the derivative of a power series representation for a suitable function $\left(\text{should be for } \frac{1}{8+x^3}\right)$

+ 2 pts Correct power series representation for $f(x)$

✔ + 2 pts Correct radius of convergence based on power series representation

− 0.5 pts Minor algebra mistake for radius of convergence

+ 2 pts Incorrect but reasonable attempt

Question 9

Q9

2 / 10 pts

- **+ 10 pts** Correct: $y = \ln\left(\frac{4}{1+\cos x} - 1\right)$
- **+ 2 pts** Attempt to separate the variables
- **+ 2 pts** Correct separation of variables
- **+ 2 pts** Correctly integrated LHS showing work
- **+ 2 pts** Correctly integrated RHS showing work
- **+ 2 pts** Obtained correct solution of the initial-value problem based on previous work
- **− 1 pt** Factored both sides of differential equation correctly but did not separate $\frac{dy}{dx}$
- **− 1 pt** Solved differential equation based on previous work but not initial-value problem
- **+ 2 pts** Tried solving a linear differential equation or another approach which makes mathematical sense
- ✔ **+ 2 pts** Incorrect but reasonable attempt

MATH:1860 – Final Exam May. 15

Name: Colin Cano

Instructions: There is a total of 9 problems on this exam. Each problem is worth 10 points. Be sure to show all your work, write neatly and legibly, and simplify your final answers. Any problem with a correct answer without work to support it will receive 0 points. If you have any questions about a problem, you can raise your hand or come up and ask.

1. (10 points) Find the 3rd degree Taylor polynomial $T_3(x)$ for $f(x) = x \ln x$ centered at $a = 1$.

$f(x) =$

$T_1(x) = \frac{f'(1)}{1!} \cdot (x-1)^1 = x-1$

$T_2(x) = \frac{f''(1)}{2!}(x-1)^2 = \frac{(x-1)^2}{2}$

$T_3(x) = \frac{f'''(1)}{3!}(x-1)^3 = \frac{-(x-1)^3}{6}$

$f'(x) = \ln x + 1$

$f''(x) = \frac{1}{x}$

$f'''(x) = -\frac{1}{x^2}$

$f'(1) = 1$

$f''(1) = 1$

$f'''(1) = -1$

2. (10 points) Setup, but do not evaluate, an integral to find the exact area S of the surface obtained by rotating the curve $x(t) = 2t \sin t$, $y(t) = \cos t$, $0 \le t \le \pi/2$ about the x-axis.

$L = \int_0^{\pi/2} \sqrt{(2\sin t + 2t\cos t)^2 + (-\sin t)^2}\, dt$

$x'(t) = 2\sin t + 2t\cos t$

$y'(t) = -\sin t$

$S = \int_0^{\pi/2} 2\pi \cdot (-\sin t) \sqrt{(2\sin t + 2t\cos t)^2 + (-\sin t)^2}\, dt$

3. (10 points) Find the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{2^n(x-5)^n}{\sqrt{n+1}}$. $= \sum_{n=0}^{\infty} \frac{(2x-10)^n}{\sqrt{n+1}}$

$\lim_{n\to\infty} \left| \frac{(2x-10)^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{(2x-10)^n} \right| = \lim_{n\to\infty} \left| \frac{2x-10\sqrt{n+1}}{\sqrt{n+2}} \right| = |2x-10| \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n+2}}$

$(n+1)^{1/2}$ $\quad \frac{\frac{n+1}{2}}{\frac{n+2}{2}} = \frac{n+1}{n+2} = 1$ LH

$\lim_{n\to\infty} \frac{n+1}{n+2} = |2x-10| \cdot 1 < 1$

$|2x| < 11$

$|x| < \frac{11}{2}$

$I = \left(-\frac{11}{2}, \frac{11}{2}\right)$

$R = \frac{11}{2}$

4. (10 points) Convert between Cartesian and polar coordinates.

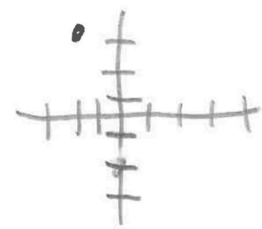
(a) (5 points) The polar coordinates of a point are $(4, -\pi/3)$. Plot the point in the xy-plane and convert it to Cartesian coordinates.

(b) (5 points) The Cartesian coordinates of a point are $(-2, 2)$. Plot the point in the xy-plane and convert it to polar coordinates in two different ways: one for when $r > 0$ and the other for when $r < 0$.

a.) $x = 4\cos -\frac{\pi}{3}$ $\quad Y = 4\sin -\frac{\pi}{3}$

$= 4 \cdot -\frac{1}{2} = -2$ $\quad = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

$(-2, 2\sqrt{3})$



b.) $-2 = r\cos\theta$ when $r = \sqrt{8}$, $\cos\theta = \frac{-2}{\sqrt{8}} = \frac{3\pi}{4}$

$r^2 = x^2 + y^2$

$r^2 = 4 + 4 = 8$ $\quad \pm\sqrt{8} = r$

$\sqrt{10}$ when $r = -\sqrt{8}$, $\cos\theta = \frac{2}{\sqrt{8}} = \frac{\pi}{4}$

$\frac{\pi}{2}$ $\quad \frac{2\pi}{3}$

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

SO. when $r > 0$: $\left(\sqrt{8}, \frac{-\sqrt{2}}{2}\right)$

$r < 0$: $\left(-\sqrt{8}, \frac{\sqrt{2}}{2}\right)$

5. (10 points) Determine whether $\sum_{n=1}^{\infty} \frac{n+2^n}{n+3^n}$ converges or diverges.

LCT: $\frac{a_n}{b_n}$ $b_n = \frac{2^n}{3^n}$

$\lim_{n\to\infty} \frac{n+\cancel{2^n}}{n+\cancel{3^n}} \cdot \frac{\cancel{3^n}}{\cancel{2^n}} = \lim_{n\to\infty} \frac{n}{n} \overset{LH}{=} 1$

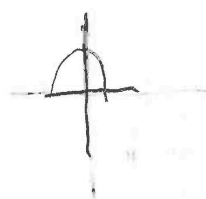
$\sum b_n = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{n+1}$

$= \frac{1}{1-\frac{2}{3}}$

$|r| = \frac{2}{3} < 1$

So converges by using LCT and geometric series.

6. (10 points) Find the centroid of the region bounded by the curves $y = 4 - x^2$, $x = 0$, $y = 0$.



$y' = -2x\,dx$

$y - 0 = -2x(x)$

$\boxed{y = -2x^2}$

centroid

7. (10 points) Evaluate the indefinite integral $\int e^{2x} \cos x \, dx$.

$u = e^{2x}$ $dv = \cos x \, dx$

$du = 2e^{2x}$ $v = \sin x$

$\int e^{2x} \cos x \, dx = e^{2x} \sin x - 2\int e^{2x} \sin x \, dx$

$u = e^{2x}$ $dv = \sin x \, dx$

$du = 2e^{2x}$ $v = -\cos x$

$\int e^{2x} \cos x \, dx = e^{2x} \sin x + 2e^{2x} \cos x + 2\int \cos x \, e^{2x} dx$

$-2\int \cos x \, e^{2x} dx$ $\quad -2\int \cos x \, e^{2x} dx$

$-\int e^{2x} \cos x \, dx = e^{2x} \sin x + 2e^{2x} \cos x + C$

$\int e^{2x} \cos x \, dx = -\left(e^{2x} \sin x + 2e^{2x} \cos x\right) + C$

8. (10 points) Find a power series representation for $f(x) = \left(\frac{x}{8+x^3}\right)^2$ and state the radius of convergence.

$\frac{x}{8+x^3} = \frac{x}{8} \cdot \frac{1}{1-\left(\frac{-x^3}{8}\right)}$

$\left(\frac{-x^3}{8}\right)^n = \frac{(-1)^n (x^n, x^n, x^n)}{8^n}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$

$\frac{1}{1-\left(\frac{-x^3}{8}\right)} = \sum_{n=0}^{\infty} \left(\frac{-x^3}{8}\right)^n, \quad \left|\frac{x}{8}\right| < 1$

$\frac{x}{8} \cdot \frac{1}{1-\left(\frac{-x^3}{8}\right)} = \sum_{n=0}^{\infty} \left(\frac{-x^3}{8}\right)^n \cdot \frac{x}{8}, \quad \left|\frac{-x^3}{8}\right| < 1$

$\frac{x}{8+x^3} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n x^n x^n \cdot x}{8^{n+1}}, \quad \left|-\frac{x^3}{8}\right| < 1$

$\frac{x}{8+x^3} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{3n+1}}{8^{n+1}}$

$|-x^3| < 8$

$|-x| < 2$

$|x| < -2$

$\left(\frac{x}{8+x^3}\right)^2 = \left(\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{3n+1}}{8^{n+1}}\right)^2$

$R = -2$

0.00233763

9. (10 points) Solve the initial-value problem $y' + \cos(x)y' = \sin x + e^{-y}\sin(x)$, $y(0) = 0$.

$0 = \sin x + \frac{1}{e^0}\sin x - \cos x y' - y'$

$0 = \sin x + \sin x - \cos x \cdot y' - y'$

$0 = 1 - \cos^2 x - \cos x y' - y'$

$-1 = -\cos^2 x - \cos x y' - y'$

$-1 = -\frac{1 + \cos 2x}{2} - \cos x\, y' - y'$

$y' =$

$0 = \sin x + e^{-y}\sin x - \cos x$

$0 = \sin x + \sin x - \cos x$

$0 = \sin 2x - \cos x$

$I(x) = e^{\int \cos x} = \sin x$

$\frac{dy}{dx} + \cos x\, y = \sin x + e^{-y}\sin x$

$\int \frac{d}{dx}(\sin x\, y) = \int \sin^2 x + e^{-y}\sin^2 x$

$\sin x\, y = \frac{2\sin^2 x}{2} - \frac{2e^{-y}\sin^2 x}{2} + C$

$y = \frac{\frac{2\sin^2 x}{2} - \frac{2e^{-y}\sin^2 x}{2} + C}{\sin x}$