Homework 8

Section 11.2

15.a) $\{a_n\} = \frac{2n}{3n+1}$ $\quad \lim_{n\to\infty} \frac{2n}{3n+1} \stackrel{LH}{=} \lim_{n\to\infty} \frac{2}{3}$. Converges at $\frac{2}{3}$

15.b) $a_n = \frac{2n}{3n+1}$ $\quad \lim_{n\to\infty} \frac{2n}{3n+1} \stackrel{LH}{=} \lim_{n\to\infty} \frac{2}{3} \neq 0$. Diverges

18. $\sum_{n=4}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = \lim_{N\to\infty} \sum_{n=4}^{N} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$

$= \left(\frac{1}{\sqrt{4}} - \cancel{\frac{1}{\sqrt{5}}}\right) + \left(\cancel{\frac{1}{\sqrt{5}}} - \cancel{\frac{1}{\sqrt{6}}}\right) + \left(\cancel{\frac{1}{\sqrt{6}}} - \cancel{\frac{1}{\sqrt{7}}}\right) \ldots = \lim_{n\to\infty} \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{n+1}} = \frac{1}{2} - 0 = \frac{1}{2}$

Convergent and sum $= \frac{1}{2}$

30. $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n} = \sum_{n=0}^{\infty} \frac{3 \cdot 3^n}{(-2)^n} = \sum_{n=0}^{\infty} 3 \cdot \frac{3^n}{(-2)^n} = \sum_{n=0}^{\infty} 3 \cdot \left(\frac{3}{-2}\right)^n$

$= 3\sum_{n=0}^{\infty}\left(-\frac{3}{2}\right)^n$. $\quad r = -\frac{3}{2}$ $\quad$ since $|r| > 1$, $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n}$ diverges

32.) $\sum_{n=1}^{\infty} \frac{6 \cdot 2^{2n-1}}{3^n} = \sum_{n=1}^{\infty} \frac{6 \cdot 2^{2n} \cdot 2^{-1}}{3^n} = \sum_{n=1}^{\infty} \frac{6 \cdot 2^n \cdot 2^n}{2 \cdot 3^n} = \left(\frac{6}{2}\right)\left(\frac{4}{3}\right)^n$

$= \sum_{n=1}^{\infty} 3\left(\frac{4}{3}\right)^n$ $\quad r = \frac{4}{3}$ $\quad \frac{4}{3} > 1$ so it diverges

36 $\sum_{n=0}^{\infty} \frac{1}{3}\cdot\left(\frac{1}{9}\right)^n + \sum_{n=1}^{\infty} 2\cdot\left(\frac{1}{9}\right)^n =$

$\left(\frac{1}{3}+\frac{2}{9}\right)+\left(\frac{1}{27}+\frac{2}{81}\right)$

$\frac{\frac{1}{3}}{1-\frac{1}{9}} = \frac{3}{8}$ $\quad \frac{2}{1-\frac{1}{9}} = \frac{2}{8} \Rightarrow \frac{3}{8}+\frac{2}{8} = \boxed{\frac{5}{8}}$ convergent

43. $\sum_{k=1}^{\infty} (\sin 100)^k = \sum_{k=0}^{\infty} (\sin 100)^{k+1} = \sum_{k=0}^{\infty} \sin 100 \cdot \frac{1}{1-\sin 100}$. Convergent

46. $\sum_{k=0}^{\infty} (\sqrt{2})^{-k} = \sum_{k=0}^{\infty} \frac{1}{\sqrt{2}^k} = \frac{1}{1-\frac{1}{\sqrt{2}}} =$ convergent since $\frac{1}{\sqrt{2}} < 1$

50. $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$. divergent sinc $e^n \geq n^2$ for all $n \geq 1$

11.3

16. $s_n = \frac{1}{n\sqrt{n}}$. let $f(x) = \frac{1}{x\sqrt{x}} = \int x^{-3/2} = -2x^{-1/2} \Big|$

$\left.\frac{-2}{\sqrt{x}}\right|_1^{\infty} = 0+2 = 2$ Convergent

22. $\sum_{n=3}^{\infty} \frac{3n-4}{n^2-2n}$ $\quad f(x) = \int \frac{3x-4}{x^2-2x} = 2\ln|x| + \ln|x-2| \Big|_3^{\infty}$

$= \infty$. diverges

24.) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2} = \int \frac{\ln x}{x^2} = \ln x \cdot \frac{-1}{x} - \int \frac{1}{x} \cdot \frac{-1}{x} dx$

$u = \ln x \quad v = -\frac{1}{x}$

$du = \frac{1}{x} \quad dv = -\frac{1}{x}$

$-\frac{\ln x}{x} - \frac{-1}{x} dx$

$-\frac{\ln x}{x} - \frac{1}{x}\Big|_2^{\infty} = \left(-\frac{\ln \infty}{\infty} - \frac{1}{\infty}\right) + \frac{\ln 2}{2} + \frac{1}{2} = \frac{1 + \ln 2}{2}$ converges

26.) $\sum_{k=1}^{\infty} k e^{-k^2}$ $f(x) = \int x e^{-x^2} = -\frac{1}{2}\int e^u du = \frac{-e^u}{2} = \frac{-e^{-x^2}}{2}$

$u = -x^2 \quad du = -2x\,dx \quad \frac{dx}{2} =$

$\frac{-e^{-x^2}}{2} = \frac{1}{2(-e^{x^2})}\Big|_1^{\infty} = 0 + \frac{1}{2e}$ Converges.

30. $f(x) = \frac{\cos^2 x}{1+x^2}$ is not monotonically decreasing

34. $\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$ always if $P > 1$