$u = \cos x$
$du = -\sin x$

2. $\int_0^{\pi/2} \sin^3 x \cos^5 x \, dx = \int (1-\cos^2 x)\sin x \cos^5 x \, dx$

$= \int_1^0 (1-u^2)u^5(-du) = \int_1^0 u^7 - u^5 \, du = \int_0^1 u^5 - u^7 \, du$

$= \frac{u^6}{6} - \frac{u^8}{8}\Big|_0^1 = \frac{1}{6} - \frac{1}{8} = \boxed{\frac{1}{24}}$

I should've took from the sin instead of cosine.

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1. $\int_0^1 xe^{7x} dx$ $\quad u = x \quad du = e^{7x}dx$
$v = \frac{1}{7}e^{7x} \quad dv = e^{7x}dx$

$= x \cdot \frac{1}{7}e^{7x}\Big|_0^1 - \int_0^1 \frac{1}{7}e^{7x} dx$

$= \frac{1}{7}e^7 - \frac{1}{7}\int_0^1 e^{7x} dx$

$= \frac{1}{7}e^7 - \frac{1}{7}e^{7x}\Big|_0^1 = \frac{1}{49}(e^7 - 1)$

$= \frac{1}{7}e^7 - \frac{1}{49}(e^7 - 1)$

$= \boxed{\frac{6e^7 + 1}{49}}$

I messed up simplication originally

4. $\int x\sqrt{1-x^4}$ $\quad u=x^2 \quad du=2x\,dx$

$\frac{1}{2}\int \sqrt{1-u^2}\,du \quad u=\sin\theta \quad du=\cos\theta$

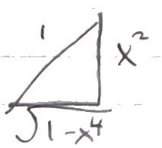
$=\frac{1}{2}\int \sqrt{1-\sin^2\theta}\cos\theta\,d\theta = \frac{1}{2}\int \cos^2\theta\,d\theta$

$=\frac{1}{2}\int \frac{1+\cos 2\theta}{2}\,d\theta = \frac{1}{4}\int 1+\cos 2\theta\,d\theta$

$=\frac{1}{4}\left(\theta+\frac{\sin 2\theta}{2}\right) \quad \theta=\sin^{-1}(x^2)$

$\sin 2\theta = 2x^2\cos\theta$

$\cos\theta=\sqrt{1-x^4}$




$= \frac{1}{4}\sin^{-1}(x^2)+\frac{1}{4}x^2\sqrt{1-x^4}+C$

Should've made $u=x^2$ before did trig sub.

5.

$$L=\int_0^{\pi} \sqrt{1+\left(-\tfrac{1}{2}\sin\left(\tfrac{x}{2}\right)\right)^2}\,dx$$

I got the wrong derivative of $\cos\left(\frac{x}{2}\right)$

6. $y=(x+1)^4 \quad y'=4(x+1)^3$

$ds=\sqrt{1+(4(x+1)^3)^2}\,dx$

$$S=\int_0^2 2\pi\,(x+1)^4\sqrt{1+(4(x+1)^3)^2}\,dx$$

I got it wrong because I thought I had to fill in $dx$.

7. $\int \frac{x^2}{x^2-x-6}$

$$x^2 \overline{)\, x^2-x-6 \;\big|\; 1 + \frac{x+6}{x^2-x-6}}$$

$$\int 1 + \frac{x+6}{x^2-x-6}\, dx$$

$$\int 1\,dx + \int \frac{x+6}{x^2-x-6}\, dx$$

$$= \frac{x+6}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$x+6 = Ax+2A+Bx-3B$

$x+6 = (A+B)x + (2A-3B)$

$A+B=1$

$2-2B-3B=6$

$B=-\frac{4}{5}$

$A=\frac{9}{5}$

$$\int 1\,dx + \int \frac{9/5}{x-3}\, dx + \int \frac{-4/5}{x+2}\, dx$$

$$= x + \frac{9}{5}\ln|x-3| - \frac{4}{5}\ln|x+2| + C$$

I got it wrong initially because i did not know to do long divison if $\deg P(x) = \deg Q(x)$

Next exam, I'm gonna study more HW problems because i did not this time.