## Worksheet 9

## CS 2210 Discrete Structures

**Due 4/2 9pm. Late submissions get grade 0.**

* Teams of 3-4 students (must work in group). Follow direction given during discussion.
** This page is double sided. Make sure to do both sides. Show your work.

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Question 1: Prove by induction that if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets such that $A_i \subseteq B_i$, for $i = 1,2,..,n$, then $\cap_{i=1}^{n} A_i \subseteq \cap_{i=1}^{n} B_i$. Note: The notation is for intersection. The other way to write it is: $\cap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$.

BC: For $n=1$ $\cap_{i=1}^{1} A_i \subseteq \cap_{i=1}^{1} B_i$ $A_1 \subseteq B_1$ true

IA: Assume for $n=k$, $\cap_{i=1}^{k} A_i \subseteq \cap_{i=1}^{k} B_i$

IS: Prove for $n=k+1$ $\cap_{i=1}^{k+1} A_i \subseteq \cap_{i=1}^{k+1} B_i$

By definition of intersection:

$\cap_{i=1}^{k+1} A_i = (\cap_{i=1}^{k} A_i) \cap A_{k+1}$

$\cap_{i=1}^{k+1} B_i = (\cap_{i=1}^{k} B_i) \cap B_{k+1}$

By IA, $(\cap_{i=1}^{k} A_i) \cap A_{k+1} \subseteq (\cap_{i=1}^{k} B_i) \cap B_{k+1}$

$\cap_{i=1}^{k} A_i \subseteq \cap_{i=1}^{k} B_i$

Proved by strong induction

Question 2: How many positive integers between 750 and 7999 inclusive are divisible by 5 but not by 7?

$1 + \frac{7995 - 750}{5} = 1450$ $1450 - 207 = 1243$ integers

$\frac{7980}{35} = 228$

$1 + \frac{228 - 22}{1} = 207$

$\frac{770}{35} = 22$

Question 3: We have 8 t-shirts, 6 pants, 3 hats and 4 watches.
a. In how many ways can we select an outfit that has a t-shirt, pants, hat and watch?

$8 \cdot 6 \cdot 3 \cdot 4 = 576$ ways

b. In how many ways can we select an outfit that has t-shirt, pants and either hat or watch, but not both?

Case1: No Hat $8 \cdot 6 \cdot 4 = 192$

Case2: No watch $8 \cdot 6 \cdot 3 = 144$

$192 + 144 = 336$ ways

Question 4: A ball contains 17 red balls, 5 yellow balls and 9 blue balls. A child selects a ball at random without looking. How many balls must he select to make sure he has at least one of each? 27

$17 + 9 + 1 = 27$

Question 5: What is the coefficient of $x^{10}$ in $(2x - 3)^{15}$.

$$\binom{15}{10} \cdot (2x)^{10} \cdot (-3)^{10}$$

$$= \binom{15}{10} \cdot 2^{10} \cdot (-3)^{10} \cdot x^{10}$$

A: $\binom{15}{10} 2^{10} \cdot (-3)^{10}$

Question 6: How many passwords composed of digits and upper-case English letter start with vowel, have exactly three digit and have length seven. Letters and digits can repeat. Explain.

$5 \cdot \binom{6}{3} \cdot 10^3 \cdot 26^3$

5 vowels

pick digit spots

3 spots for digits

3 spots for letters