

Homework 2 Section 1.3

6. $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$(1) \equiv (2)$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

tautology

8a.) p : Kwame will take a job in industry.

q : Kwame will go to graduate school.

$p \vee q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg p$: Kwame will not take a job in industry.

$\neg q$: Kwame will not go to graduate school.

Kwame will neither take a job in industry or go to graduate school.

12.b)

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$(1) \rightarrow (2)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

tautology!

12.c)

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(1) \rightarrow (2)$	
T	T	T	T	T	
T	F	F	F	T	
F	T	T	F	T	
F	F	T	F	T	

tautology!

	①				②		
18.	p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \wedge (p \rightarrow q)$	$② \rightarrow ①$
	T	T	F	F	T	F	T
	T	F	F	T	F	F	T
	F	T	T	F	T	T	F
	F	F	T	T	T	T	T

not
a tautology!

24. $\neg(p \oplus q)$ and $p \leftrightarrow q$

	p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$	$\neg(p \oplus q) \equiv (p \leftrightarrow q)$
	T	T	F	T	T	T
	T	F	T	F	F	T
	F	T	T	F	F	T
	F	F	F	T	T	T

30. $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$$q \rightarrow r \equiv \neg q \vee r$$

$$\neg p \rightarrow (\neg q \vee r) \equiv \neg(\neg p) \vee (\neg q \vee r) \equiv p \vee \neg q \vee r$$

$$q \rightarrow p \vee r \equiv \neg q \vee (p \vee r) \equiv p \vee \neg q \vee r$$

$$p \vee \neg q \vee r \equiv p \vee \neg q \vee r$$

Section 1.4

6. a.) There is at least one student in the school who has visited North Dakota.

6. B) Every student in the school has visited North Dakota

6. C) No student in the school has visited North Dakota

6. d) There is at least one student in the school who has not visited North Dakota

6. E) Not every student in the school has visited North Dakota

6. F) Every student in the school has not visited North Dakota

$$10a) \exists x (C(x) \wedge D(x) \wedge F(x))$$

$$10.c) \exists x (C(x) \wedge (F(x) \wedge \neg D(x)))$$

$$11a) \exists x (x^3 = -1)$$

$$x^3 = -1 \quad x = -1 \quad -1 \in \mathbb{R} \quad \boxed{\text{True}}$$

$$11.B) \exists x (x^4 < x^2) \equiv x^4 - x^2 < 0 = x^2(x^2 - 1) < 0 = x^2(x-1)(x+1) < 0$$

$$x^2 > 0$$

$$\exists x \left(\frac{1}{2} < \frac{1}{2} \right) \quad x = \frac{1}{2} \quad \boxed{\text{True}}$$

$$11.c) \forall x ((-x)^2 = x^2) \quad x^2 \text{ always positive}$$

$$11.D) \forall x (2x > x) \quad \boxed{\text{True}}$$

if $x \leq 0$ it fails

False

$$36.a) \forall x (-2 < x < 3)$$

$$\neg (\forall x (-2 < x < 3))$$

$$\boxed{\exists x (x < -2 \vee x \geq 3)}$$

$$36.b) \neg \forall x (0 \leq x < 5)$$

$$\neg (\forall x (0 \leq x < 5))$$

$$\boxed{\exists x (x < 0 \vee x \geq 5)}$$

$$36.c) \exists x (-4 \leq x \leq 1)$$

$$\neg (\exists x (-4 \leq x \leq 1))$$

$$\boxed{\forall x (x < -4 \vee x > 1)}$$

$$36.D) \exists x (-5 < x < -1)$$

$$\neg (\exists x (-5 < x < -1))$$

$$\boxed{\forall x (x \leq -5 \vee x \geq -1)}$$