P&Z

Homework #3 Colin Cano

Section 1.5:

10.c) $\forall x \exists y\, F(x,y)$ 10.e) $\forall x \exists y\, F(y,x)$

28.c) $\exists x \forall y\, (xy=0)$ $x=0$ $y \in \mathbb{R}$ $\boxed{\text{True}}$

28.d) $\exists x \exists y (x+y \neq y+x)$ False

28.h) $\exists x \exists y (x+2y=2 \wedge 2x+4y=5)$ $2(x+2y)=2(2)$

$2x+4y=4 \neq 2x+4y=5$

$\boxed{\text{False}}$

32.b) $\exists x \exists y\, P(x,y) \wedge \forall x \forall y\, Q(x,y)$

$\neg(\exists x \exists y\, P(x,y) \wedge \forall x \forall y\, Q(x,y))$

$\forall x \forall y\, \neg P(x,y) \vee \exists x \exists y\, \neg Q(x,y)$

32.d) $\forall y \exists x \exists z (T(x,y,z) \vee Q(x,y))$

$\neg \exists y (\exists x \exists z (T(x,y,z)) \vee Q(x,y))$

$\exists y \forall x \forall z (\neg T(x,y,z) \wedge \neg Q(x,y))$

40.a) $\forall x \exists y (x = \frac{1}{y})$ $x=2$

$2=\frac{1}{y}$ $y=\frac{1}{2}$ $\frac{1}{2} \notin \mathbb{Z}$ False!

40.b) $\forall x \exists y (y^2 - x < 100)$

$y^2 < 100 + x$ $x=10$ $y^2 < 110$ $y=4$

no counter example $16 < 110$ ✓ True

40.c) $\forall x \forall y (x^2 \neq y^2)$ $x \neq 1$

$1 \neq y^2$ $y=1$

$1 \neq 1$ False!

Section 1.6

8, p: is a man $p \to \neg q$

q: is an island q

$\therefore \neg p$ Modus

10.a) $p \to q$ p: I played hockey q: I am sore

$q \to r$ r: I used the whirl pool

$\neg r$

$\therefore \neg p$ Conclusion: I did not play hockey

10.b) $\forall x\, P(x) \to (Q(x) \vee R(x))$ P(x): I worked on x Q(x): x was sunny R(x): x was partially sunny

P(monday) $\vee$ P(Friday) $\to$ P(Friday) $\to$ Q(Friday) $\vee$ R(Friday)

$\neg$Q(Tuesday) F

$\neg$R(Friday) P(Friday) $\to$ Q(Friday)

Friday was sunny

10.C

1 $\forall x(P(x) \to Q(x))$ $P(x)$: x are insects $Q(x)$: x has six legs

2 $P(\text{dragonflies})$ $R(x,y)$: x eats y

3 $\neg Q(\text{spiders})$

4 $P(\text{dragonflies}) \to Q(\text{dragonflies})$ universal instantiation ①

5 $P(\text{spiders}) \to Q(\text{spiders})$

$\therefore Q(\text{dragonflies})$ Dragonflies have six legs

$\therefore \neg P(\text{spiders})$ Spiders are not insects

10.D $P(x)$: x is a student $Q(x)$: x has an internet account

1 $\forall x(P(x) \to Q(x))$

2 $\neg Q(\text{homer})$

3 $\neg Q(\text{Maggie})$

4 $P(\text{homer}) \to Q(\text{homer})$ universal instantiation ①

$\therefore \neg P(\text{homer})$ Homer is not a student

10.e) $P(x)$: x is healthy to eat $Q(x)$: x tastes good

$R(x)$: you eat x

1 $\forall x(P(x) \to \neg Q(x))$

2 $P(\text{tofu})$

3 $\forall x(R(x) \to Q(x))$

4 $\neg R(\text{tofu})$

5. $\neg P(\text{cheeseburgers})$

6. $P(\text{tofu}) \to \neg Q(\text{tofu})$ universal instantiation ①

$\therefore \neg Q(\text{tofu})$ modus ponen ②+⑥

Conclusion: Tofu does not taste good

10.f) p: I'm dreaming q: I am hallucinating r: I see elephants running down the road

1 $p \vee q$

2 $\neg p$

3 $q \to r$

4 $\therefore q$ disjunctive syllogism ①:②

5 $\therefore r$ modus ponens ③:④

Conclusion: I am Hallucinating
I see elephants running down the road.

Section 1.7

2. let $n=2k$ and $m=2l$ be even integers.
Then $n+m=2k+2l=2(k+l)$. Let $s=k+l$ ∴
$n+m=2s$ ← even by definition   since $k \in \mathbb{Z}$, $s \in \mathbb{Z}$
Proof by direct

6. Assume a and b are odd, by definiton $a=2k+1$ $b=2i+1$
$a \cdot b=(2k+1)(2i+1)=4ki+2k+2i+1$ $a,b \in \mathbb{Z}$
$2k+1=2(\underline{2ki+k+i})+1$
$k=2ki+k+i$ ← odd by definition

8. let $n=y^2$ $y \in \mathbb{Z}$ $x \in \mathbb{Z}$
Proof by contradiction
let $n+2=x^2$, so $y^2+2=x^2$, then $x^2-y^2=2$
then $2=(x-y)(x+y)$, since $2 \cdot 1=2$, $x-y=2$ and $x+y=1$
then $x-y+x+y=1+2$ then $2x=3$ $x=\frac{3}{2}$
$\frac{3}{2} \notin \mathbb{Z}$

16 Proof by contradiction. Assume x,y,z are all even,
by definition $x,y,z=2k$ $k \in \mathbb{Z}$
$x+y+z=2k+2k+2k=2(3k)$ let $s=3k$ since $k \in \mathbb{Z}$ let
$=2s$ ← even by definition $s \in \mathbb{Z}$
The assumption x,y,z are all even is false
meaning at least one needs to be odd.

20a) Proof by contraposition: if n is odd, then $3n+2$ is odd
Assume n is odd, by definition $n=2k+1$ where $k \in \mathbb{Z}$.
$3n+2=3(2k+1)+2=6k+3+2=6k+4+1=2(3k+2)+1$. Let $3k+2=j$
$3n+2=2j+1$ ← Odd by definition   where $j \in \mathbb{Z}$

20.b) Proof by contradiction: p: $3n+2$ is even q: n is odd
$r: p \to q$ $\neg r: p \wedge \neg q$ Proof: Assume $3n+2$ is even and n is odd. by def
$\exists k \in \mathbb{Z}$ s.t $n=2k+1$ $3n+2=3(2k+1)+2=6k+4+1=2(3k+2)+1$,
Let $s=3k+2$ where $s \in \mathbb{Z}$, $3n+2=2s+1$ ← odd by definition

30.) $m^2=n^2$ $m^2-n^2=0$ $(m-n)(m+n)=0$ $\Rightarrow m=n$ or $m=-n$

Case1: if $m=n$, then $m^2=n^2$
Case2: if $m=-n$ then $m^2=n^2$

6. Case1: $a\leq b \wedge a\leq c$ Section 1.8
$\min(a,\min(b,c))=a$
$\min(\min(a,b),c)=a$ thus $\min(a,\min(b,c))=\min(\min(a,b),c)$
Case2: $b\leq a \wedge b\leq c$
$\min(a,\min(b,c))=b$
$\min(\min(a,b),c)=b$ Thus $\min(a,\min(b,c))=\min(\min(a,b),c)$
Case3: $c\leq a \wedge c\leq b$
$\min(a,\min(b,c))=c$
$\min(\min(a,b),c)=c$ Thus $\min(a,\min(b,c))=\min(\min(a,b),c)$

8. let $x=2k+1$ let $y=2s$ where $k,s\in\mathbb{Z}$
$5x+5y=5(2k+1)+5(2s)=10k+5+10s=10k+10s+4+1=$
$2(5k+5s+2)+1$ Let $J=5k+5s+2$ where $J\in\mathbb{Z}$
$5x+5y=2J+1$ $\leftarrow$ odd by definition

16. Counter example let $a=2$ and $b=\frac{1}{2}$ where $a,b\in\mathbb{N}$
$a^b=2^{1/2}=\sqrt{2}$ $\leftarrow$ irrational by definition.
False!