Static Harmonic Grammar: Constraint Conflict without Candidate Comparison

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1. Introduction

Harmonic Grammar (HG; Legendre et al., 1990) and Optimality Theory (OT; Prince & Smolensky, 1993/2004) have three main components, as shown in (1). GEN determines a set of possible representations (typically, the set of candidate outputs for a given input, or in serial HG/OT the set of possible derivational steps). The constraints in CON are functions that map representations to violations (or *marks*). EVAL defines grammatical outputs to be those that are maximally harmonic under a given HG weighting or OT ranking of the constraints.

(1) Architecture of HG/OT
input → GEN → CON → EVAL → *output*

As applied to phonology, the first two components of (1) are not specific to HG/OT but shared by many other approaches. There is a long tradition of studying the structure of phonological representations (e.g., Goldsmith, 1990; Hayes, 1995) and the nature of phonological derivations (e.g., Chomsky & Halle, 1968; Mascaró, 1987), all of which can in principle inform GEN (e.g., McCarthy, 2008). There is also a large body of research on constraints in phonology that can contribute to CON, including Licensing Theory (e.g., Itô, 1989; Goldsmith, 1990; Lombardi, 1991; Steriade, 1995), Declarative Phonology (e.g., Bird et al., 1992; Scobbie, 1993; Scobbie et al., 1996), Constraints and Repair Strategies (e.g., Paradis, 1988), and Government Phonology (e.g., Charette, 1988).

HG/OT are distinguished from other theories of grammar by the assumptions, embodied in EVAL, that (i) constraints are *violable* and (ii) well-formed representations are those that *optimally* satisfy the constraints, as determined by candidate comparison under a given weighting or ranking. This paper develops variants of HG/OT that decouple these two assumptions. In Static HG (HGStat), constraints are violable and weighted but constraint conflict plays out internally to each representation rather than through optimization over alternatives. A representation as a whole is well-formed in the static sense iff each of its parts has maximal harmony (as defined in §2). Static OT (OTStat) is identical except that strict domination ranking takes the place of constraint weighting.

The assumption of violability is important to maintain because it allows language-specific phenomena and typological patterns to be reduced to a relatively small set of more general constraints. HG/OT has achieved considerable success in factoring attested phonological patterns into multiple conflicting pressures that, when weighted or ranked differently, yield a restricted typology of other predicted patterns (i.e., a *factorial typology*; Prince & Smolensky, 1993/2004). Furthermore, phonological constraints in HG/OT are often phonetically grounded (e.g., Hayes, 1999; Hayes & Steriade, 2004) and formally simple (e.g., Eisner, 1998; Potts & Pullum, 2002; McCarthy, 2003; Riggle, 2004). The same kind of factorization into grounded, simple statements of well-formedness is not available in theories with inviolable constraints, in which the only mode of 'interaction' is conjunction.

There are computational motivations for eliminating the assumption that grammaticality is determined by optimization over candidates. Optimization is a powerful non-linear operation that can

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be difficult to compute, because determining whether a given representation is well-formed involves (implicitly) comparing it against all other candidates. While optimization itself may be tractable when the set of constraints is fixed and all of the constraints are 'compiled' into a single finite-state machine (Riggle, 2004; Heinz et al., 2009), compilation for HG/OT can produce exponentially large machines and the general conditions under which it will terminate are not known. Moreover, optimization with respect to even elementary constraints can give rise to patterns that lie beyond well-established formal limits on phonological computation (such as non-rational transductions; e.g., Frank & Satta, 1998; Gerdemann & van Noord, 2000; Buccola, 2013; Hao, 2019; Koser & Jardine, 2020).

Because optimization seems to be too complex and expressive as the operation for resolving constraint conflict in phonology, it is replaced in HG/OTStat with a much simpler threshold non-linearity. The formal definitions of the static theories are given in section 2 along with example analyses of local and unbounded phonological patterns. Section 3 shows that an HG/OTStat grammar is only as expressive, in the sense of mathematical logic, as the constraints that it contains. This establishes static constraint interaction as formally very different from optimization, for which no such logical equivalance between constraints and grammars has been or could be shown. Section 4 provides a summary and briefly compares HG/OTStat to previous proposals in connectionism and generative grammar.

2. Static Harmonic Grammar

In HGStat, violable constraints interact within individual constituents (or *nodes*) of a given representation and the non-linear operation of global optimization is replaced by a local threshold.

(2) HGStat

a. **Representation**. Each representation permitted by GEN is a relational-functional structure consisting of a finite nonempty set of nodes ($\{n_1, n_2, \ldots\}$), unary relations that label (or 'sort') the nodes, relations that hold among the nodes, and functions on the same domain.

b. **Evaluation**. A constraint c_k in CON evaluates each node n in the context of its representation M, assigning the node a signed unit mark or zero:

$$c_k(n; M) \in \{+1, -1, 0\}$$

c. **Harmony**. Given constraint weights $\mathbf{w}$, the harmony of a node is determined by summing its weighted constraint violations and applying an upper threshold of zero:

$$h(n; M) = \min(\sum_k w_k\, c_k(n; M), 0) = \min_0 \sum_k w_k\, c_k(n; M)$$

and the harmony of an entire representation is the sum of the harmonies of its nodes:

$$h(M) = \sum_{n \in M} h(n; M)$$

d. **Well-formedness**. A node is well-formed iff its harmony is nonnegative (i.e., zero). A representation is well-formed iff all of its nodes are (i.e., iff its own harmony is zero).

As in HG, applications of HGStat are free to make any desired assumptions about the structure of representations (2a). Nodes can be labeled as segments, features or autosegments, moras or syllables, grid marks or metrical feet, and orthogonally as part of the input or output (e.g., Potts & Pullum, 2002). Relations among nodes can include (immediate) precedence, autosegmental association, tree dominance, and correspondence; functions that map nodes to nodes can express similar representational concepts. The labeling and other relations/functions are specified in GEN, as are inviolable constraints on them (e.g., transitivity of precedence, nontangling in trees; Partee et al., 1993: 16.3).

Constraint evaluation in HGStat is potentially global, because a constraint could inspect the entirety of a representation when determining how to evaluate a given node (2b); the appropriate formal and substantive restrictions on constraints are part of the definition of CON. As shown in §3 below, the definitions of harmony and well-formedness in HGStat guarantee that grammars (or parts of grammars) are only as expressive as the constraints that they contain. This follows from the principle that constraints interact locally, that is within each node (2c), rather than through global optimization. Each node sums up its own weighted marks and is well-formed iff the sum meets or exceeds the threshold at zero (2d).[1]

The well-formedness of a representation does not depend on comparison with alternatives, instead being a simple function of its internal structure. The harmony of a representation is the sum of the harmonies of its nodes. This sum will be exactly zero iff all of the nodes have zero harmonies (i.e., are well-formed) and negative otherwise. It is important that, because the threshold function is applied at each node, positive marks that are assigned to one part of a representation cannot 'leak out' and cancel negative marks assigned elsewhere. Each node must be internally stable, offsetting any negative harmony with equal or greater positive harmony, in order for the structure as a whole to be grammatical.

Static OT (OTStat) is defined in the same way as HGStat, except that node harmony refers to strict domination rather than weighted summation.

(3) OTStat

c. **Harmony**. Given a constraint ranking **r**, the harmony of a node is:

$$h(n; M) = \begin{cases} 0 & \text{if every constraint that assigns } -1 \text{ to } n \text{ is dominated} \\ & \text{by some constraint that assigns } +1 \text{ to } n \\ -1 & \text{otherwise} \end{cases}$$

This implies that $|h(M)|$ counts the number of ill-formed nodes in representation M. More fine-grained information can be obtained by identifying the constraints that assign undominated negative marks.

2.1. Local segmental phonology

To illustrate HGStat, consider the distribution of non-low vowels in native words of Cochabamba Quechua (e.g., Gallagher, 2016). As illustrated by the examples in (4) and simplifying slightly, mid vowels occur only when immediately preceded or followed by a uvular consonant and high vowels occur elsewhere.

(4) Vowel height allophony in Cochabamba Quechua (examples from Wilson & Gallagher, 2018)

a. Mid vowels before/after uvulars

q'epij (*q'ipij) 'to carry'	q'oɲi (*q'uɲi) 'hot'
peqaj (*piqaj) 'to grind'	noqa (*nuqa) 'I'

b. High vowels elsewhere

misi (*mesi, *mise, *mese) 'cat'	kuʎku (*koʎku, *kuʎko, *koʎko) 'bird (sp.)'

The three constraints in (5) are sufficient to account for the Quechua pattern and, under different weightings, typological alternatives. The constraints assume that representations include nodes labeled as segments and others labeled as valued features (e.g., [−high]). The latter could be subnodes of segments, as in classic distinctive feature matrices, or autosegments linked to segments by association lines. Nodes are also labeled for membership in the input or output part of a representation. The definitions also assume that representations include the relations of segment adjacency (which could be defined in terms of a more basic precedence relation) and input-output correspondence among segments.

[1] The $\min_0$ ("minnow") non-linearity in the definition of HGStat harmony was inspired by the highly successful $ReLU$ function of deep neural networks (Nair & Hinton, 2010). The two are related by $\min_0(x) = -ReLU(-x)$.

(5) Constraints on vowel height

a. Lower. Assign $-\alpha$ to the [αhigh] feature of a vowel that is adjacent to a uvular.

b. NoMid. Assign α to the [αhigh] feature of a [$-$low] vowel.

c. Ident(high). Assign $\alpha\beta$ to the [αhigh] feature of a segment that has an input correspondent with a [βhigh] feature.

The Lower constraint penalizes high vowels, and rewards mid and low vowels, in the uvular context. Specifically, it assigns negative marks to [+high] features of vowels that are adjacent on either side to uvulars, and assigns positive marks to [$-$high] features of vowels in the same environment.[2] The constraint NoMid penalizes [$-$high] features and rewards [+high] features of non-low vowels. Finally, the Ident(high) constraint penalizes mismatching specifications, and rewards matching specifications, of [high] features in corresponding segments. (For example, the [+high] feature of an output segment that corresponds to a [$-$high] input segment is assigned a mark of $(+1) \cdot (-1) = -1$.)

Constraint evaluation and conflict in HGStat can be displayed in a *non-comparative* tableau (6). Each representation in the first column is evaluated independently, not as one candidate competing against the others. In principle the tableau would display the marks of all nodes, but this quickly becomes cumbersome and alternative methods of verifying and visualizing analyses will be preferable (see §4). Here we focus on the marks assigned to the [high] feature of the first vowel in each representation.

If this feature is [$-$high], as in the legal form [peqaj], it is rewarded by Lower because of the following uvular, penalized by NoMid because it belongs to a non-low vowel, and either rewarded or penalized by Ident(high) depending on the value of the segment's input correspondent. The condition [aLower $>$ (bNoMid $+$ cIdent(high))] is sufficient to ensure that the sum of the feature's weighted marks, its *node score*, is greater than zero regardless of the input specification. Under these conditions, the harmony of the feature is $\min_0(+a - b \pm c) = 0$ and it is well-formed. The minimally different representation [piqaj], illegal in Quechua, contains an ill-formed [+high]$_2$ node with harmony $\min_0(-a+b\pm c) < 0$. In both cases, Lower and NoMid have opposing assessments of the [high] feature and their weighted conflict is resolved node-internally. The structure [peqaj] is well-formed not because it bests [piqaj], but because all of its nodes are well-formed (assuming no other constraints are operative. The structure [piqaj] does not lose a global competition, it contains within itself a fatal flaw.

(6) Analysis of vowel height allophony

	aLower	bNoMid	cIdent(high)	Node score
$p_1e_2q_3a_4j_5$	$+1$ [$-$high]$_2$	-1 [$-$high]$_2$	±1 [$-$high]$_2$	[$-$high]$_2$: $+a - b \pm c > 0$
$p_1i_2q_3a_4j_5$	-1 [+high]$_2$	$+1$ [+high]$_2$	±1 [+high]$_2$	[+high]$_2$: $-a + b \pm c < 0$!
$m_1i_2s_3i_4$		$+1$ [+high]$_2$	±1 [+high]$_2$	[+high]$_2$: $+b \pm c > 0$
$m_1e_2s_3i_4$		-1 [$-$high]$_2$	±1 [$-$high]$_2$	[$-$high]$_2$: $-b \pm c < 0$!

The second two forms illustrate how the analysis permits high (and low) but not mid vowels outside of the uvular context. The required weighting relationship is [bNoMid $>$ cIdent(high)], so that in legal [misi] the positive NoMid mark assigned to [+high]$_2$ suffices to cancel out any negative Ident(high) mark and in illegal [mesi] the penalty assigned by NoMid guarantees that the feature's harmony is negative.

This analysis is identical to one cast in the original version of HG except for the way in which constraint conflicts are resolved. It compares favorably with the inviolable constraint analysis of Wilson & Gallagher (2018), which breaks NoMid up into several constraints that collectively penalize mid vowels in 'non-uvular' contexts (i.e., all of the contexts in which Lower applies vacuously). Instead of building the lowering exception into the prohibition on mid vowels, both HG and HGStat derive the exception by weighting Lower higher than NoMid, the latter a general constraint that applies in all contexts. HGStat differs from HG in settling the conflict between these two constraints and Ident(high)

[2] As a notational convenience, variables such as α stand for both signed integers $(+1, -1)$ and feature coefficients $(+, -)$ depending on context. For example, 'Assign $-\alpha$ to [αhigh] ...' abbreviates 'Assign -1 $(= -+1)$ to [+high] ...' and 'Assign $+1$ $(= --1)$ to [$-$high] ...'. Marks are assigned to output nodes only, and a constraint implicitly assigns 0 to all nodes other than those that it explicitly penalizes or rewards; see (11) below for the formalities.

locally, at the feature node in question, rather than with comparison of candidates. Clearly, violable constraints are useful for analyzing patterns like this one but global optimization is not needed.

Nothing substantial changes if the analysis is set instead within OTStat. Under the ranking [Lower $\gg$ NoMid $\gg$ Ident(high)], the $+1$ mark that Lower assigns to [$-$high]$_2$ in [peqaj] trumps -1 marks from NoMid and Ident(high), and the -1 mark that Lower assigns to the feature in [piqaj] cannot be redeemed by positive marks from either or both of the lower-ranked constraints. The remaining two structures establish that NoMid, while dominated and violable, is nevertheless active in the language.

Reweighting or reranking the constraints in HG/OTStat predicts the same small typology of vowel height patterns as in HG/OT. For example, under the alternative ranking [NoMid $\gg$ Lower $\gg$ Ident(high)] mid vowels are banned in all contexts and under [Ident(high) $\gg$ Lower, NoMid] a height contrast is permitted in all contexts. There is no weighting/ranking of these constraints in which mid vowels are allowed only when *not* adjacent to uvulars, or in which vowels lower in the *absence* of a uvular trigger, etc. As the original and static theories are analytically and typologically equivalent in this case, local constraint resolution is preferred over optimization on computational grounds.

2.2. Unbounded spreading

A potential concern for HG/OTStat is that, because marks are assigned to individual nodes rather than entire candidates, these theories will be unable to account for non-local interactions that are observed in unbounded feature spreading, stress placement, and other areas of phonology. Perhaps local mark assignment will prevent information from flowing throughout a structure, so that a segment cannot correctly determine whether it should be nasal or oral if it is too distant from a trigger of nasal spreading or a syllable cannot receive the correct stress value if it lies too far from a word edge. This concern turns out to be largely if not entirely misplaced, because phonological representations are sufficiently rich to accommodate non-local interaction directly — as when markedness constraints apply on a tier or grid layer from which otherwise intervening elements are absent — and because apparently long-distance interactions typically can be reduced to chains of more local interactions. As illustrated here with unbounded spreading, global optimization is not needed to achieve non-local effects.

In a typical unbounded spreading pattern, a feature propogates from a trigger segment to any number of undergoers until it reaches a blocking segment or the edge of a word. For example, in Johore Malay (Walker, 1998), a potentially unbounded sequence of vowels and glides becomes [+nasal] after a nasal consonant trigger. Nasal spreading is arrested by obstruents and other segments, in accordance with a restricted cross-linguistic typology of blockers (Walker, 1995; Pulleyblank, 1989; Piggott, 1992; Cohn, 1993). The HGStat account of forms such as [pəŋãw̃ãsan] 'supervision', which contains unbounded but incomplete spreading, requires assumptions about both GEN and CON.

Assume that each segment node x_i is associated one-to-one with a feature node [αF]$_i$ for each feature [F]. In place of autosegmental spreading, which would disrupt one-to-one association, all of the [F] nodes in a word are exhaustively grouped into homogeneous, contiguous, non-overlapping *spans* (McCarthy, 2004). Spans have a minimally articulated internal structure: exactly one node in a span is designated as its *head* and all other nodes are *dependents* (i.e., non-heads). Representations that satisfy these conditions can be given a compact notation by delineating feature spans with parentheses around associated segments and indicating span heads with a leading dot, as in (7).[3]

(7) Feature span notation
[nasal] spans: (.p$_1$) (.ə$_2$) (.ŋ$_3$ ã$_4$ w̃$_5$ ã$_6$) (.s$_7$) (.a$_8$) (.n$_9$)

[3] See McCarthy (2004) on previous representations of feature spreading in OT that are similar to feature spans; the version of span theory adopted here differs in minor respects from that paper. In the representation given in the text, segment nodes x_3, x_4, x_5, x_6 are associated to feature nodes [+nasal]$_3$, [+nasal]$_4$, [+nasal]$_5$, [+nasal]$_6$ and these feature nodes are grouped into a single left-headed span (.[+nasal]$_3$ [+nasal]$_4$ [+nasal]$_5$ [+nasal]$_6$). All other feature spans are degenerate; for example, segment x_1 is associated with [$-$nasal]$_1$ which heads the span (.[$-$nasal]$_1$). In the interest of legibility segment and feature nodes are packaged throughout as IPA symbols.

Constraints for unbounded spreading evaluate local configurations of segment nodes, feature nodes, and spans. The following constraints are sufficient to account for the Johore Malay pattern and can easily be extended to embrace a more complete typology of nasal spreading patterns.

(8) Constraints on [nasal] features

a. Spread-R(+nasal). In the configuration $\begin{array}{cc} x_{i-1} & x_i \\ [+\text{nasal}]_{i-1} & [\alpha\text{nasal}]_i \end{array}$
assign $+1$ to $[\alpha\text{nasal}]_i$ if it is a dependent member of the same span as $[+\text{nasal}]_{i-1}$, else -1.

b. *NasObs. Assign $-\alpha$ to an [αnasal] feature that is associated to an obstruent.

c. *NasVoc. Assign $-\alpha$ to an [αnasal] feature that is associated to a vocoid (glide or vowel).

d. Ident(nasal). Assign $\alpha\beta$ to the [αnasal] feature of a segment that has an input correspondent with a [βnasal] feature.

Evaluation of individual [nasal] nodes by the constraint Spread-R(+nasal) does not require global inspection of entire spans or larger structures. A positive mark is assigned to each dependent member of a [+nasal] span that is immediately preceded by a head or dependent member of the same span, regardless of what else the span may contain; a negative mark is assigned to a node that is immediately preceded by a span-terminating [+nasal] node. The anticipatory spreading constraint Spread-L(+nasal) is defined analogously, with a negative mark assigned to each node that is immediately followed by a span-initiating [+nasal] node. Feature cooccurrence constraints like *NasObs and *NasVoc are also local, because the mark assigned to a feature node depends only on that node and others associated to the same segment, as are Faithfulness constraints such as Ident(nasal), which reference individual elements of the input-output segment correspondence relation and associated features.

The weighting [a*NasObs > bSpread-R(+nasal) >> c*NasVoc > dIdent(nasal) > eSpread-L(+nasal)] accounts for unbounded perseveratory spreading to vocoids initiated by nasal consonants and blocked by obstruents.[4] The following non-comparative tableau illustrates the relevant harmony computations.

(9) Analysis of rightward nasal spreading

/$ŋ_3$ a_4/	a*NasObs	bSpread-R	c*NasVoc	dIdent	Node score
(.$ŋ_3$) (.a_4)		-1 $[-\text{n}]_4$	$+1$ $[-\text{n}]_4$	$+1$ $[-\text{n}]_4$	$[-\text{n}]_4$: $-b+c+d<0$!
(.$ŋ_3$ $ã_4$)		$+1$ $[+\text{n}]_4$	-1 $[+\text{n}]_4$	-1 $[+\text{n}]_4$	$[+\text{n}]_4$: $+b-c-d>0$
(.$ŋ_3$) (.$ã_4$)		-1 $[+\text{n}]_4$	-1 $[-\text{n}]_4$	-1 $[+\text{n}]_4$	$[+\text{n}]_4$: $-b-c-d<0$!

The only well-formed output structure over these two segments is the one in which [+nasal] has spread. Because the spreading constraint is indifferent to whether a preceding [+nasal] node is a head or dependent, the harmony calculations for the output correspondents of /w_5/ and /a_6/ are identical to those of /a_4/, therefore only [(.$ŋ_3$ $ã_4$ $\tilde{w}_5$ $ã_6$)] with a single span is legal. Spreading further as in *[(.$ŋ_3$ $ã_4$ $\tilde{w}_5$ $ã_6$ $\tilde{s}_7$)] is prevented by *NasObs; this constraint rewards the $[-\text{nasal}]_7$ node of [(.$ŋ_3$ $ã_4$ $\tilde{w}_5$ $ã_6$) s_7], cancelling out the negative mark assigned by Spread-R(+nasal). Importantly, the Spread-R(+nasal) violation is entirely localized to the [$-$nasal] node of the obstruent and therefore cannot affect the harmony calculations for the preceding nodes. Furthermore, segments following [s_7] are under no pressure to undergo perseveratory spreading because they are separated by the obstruent from preceding [+nasal] segments. (Nasalization as in *[s_7 ($ã_8$ .n_9)] is prevented by *NasVoc, which assigns a negative mark for which there can be no compensating reward, given that Spread-L(+nasal) is weighted lowest.)

A similar combination of carefully structured representations and locally evaluated constraints accounts for other descriptively unbounded effects. For example, many quantity-insensitive stress patterns (Hayes, 1995; Gordon, 2002) can be analyzed with a distinction between metrical grid marks and their absence (representationalized as grid *holes*), together with constraints such as those in (10).

[4] As McCarthy (2004) discusses, an additional constraint is needed to protect nasal consonants from denasalizing.

(10) Constraints on metrical grids

a. Alternate$_{\mathrm{LR}}$. Assign $-\alpha\beta$ to a level-1 grid element β in the context α __.

b. Alternate$_{\mathrm{RL}}$. Assign $-\alpha\beta$ to a level-1 grid element α in the context __ β.

c. InitialStress. Assign α to a level-1 grid element α in the context $[_{\mathrm{PrWd}}$ __.

d. NonFinality. Assign $-\alpha$ to a level-1 grid element α in the context __ $]_{\mathrm{PrWd}}$.

where for convenience of notation grid marks (x) are interpreted as $+1$ and grid holes ($\circ$) are interpreted as -1. (For example, Alternate$_{\mathrm{LR}}$ assigns the mark $-(+1)\cdot(-1) = +1$ to the second node of a level-1 alternating sequence x $\circ$ and assigns $-(-1)\cdot(-1) = -1$ to the second member of a level-1 lapse $\circ\,\circ$.) Under the weighting [aAlternate$_{\mathrm{LR}}$, bInitialStress $>$ cNonFinality], the last syllable of an odd-syllable word will be stressed (i.e., the final level-1 element will be a grid mark) because that is the well-formed termination of an alternating sequence initiated by the first syllable, an unbounded distance away.

3. Logical structure

Constraint evaluation and interaction in static HG/OT can be given a simple logical formalization with model theory (e.g., Enderton, 2001: chapter 2 and Partee et al., 1993: chapters 7, 13). The relational-functional structures of (2a) are now considered as *models*: the domain of a structure is its finite non-empty set of nodes, the unary and other relations in a structure serve as denotations of predicates in a fixed logical language determined by GEN, and the functions are treated similarly.

A constraint c_k is defined by three formulas $\{\phi_k^+(x), \phi_k^-(x), \phi_k^0(x)\}$ in the logical language, each having a single unbound variable that stands in for a node under evaluation (i.e., each of the formulas is a propositional function). The formula $\phi_k^+(x)$ identifies all and only the nodes in a structure that receive a $+1$ mark from c_k, and similarly for the other two formulas; examples are given in (12) and (13) below. The technical definition of constraint evaluation (11) is in terms of *satisfaction* (denoted by $\models$), where $M, n \models \phi(x)$ iff formula $\phi(x)$ is true relative to model M when $n \in M$ is substituted for every occurrence of the variable x. ("Node n satisfies $\phi(x)$," with the model left implicit, is a convenient shorthand for $M, n \models \phi(x)$.)

(11) Model-theoretic definition of constraint evaluation

Constraint c_k assigns $+1$ to node n in model M iff $M, n \models \phi_k^+(x)$
-1 to node n in model M iff $M, n \models \phi_k^-(x)$
0 to node n in model M iff $M, n \models \phi_k^0(x) = \neg(\phi_k^+(x) \vee \phi_k^-(x))$

where it is assumed that no node can satisfy both $\phi_k^+(x)$ and $\phi_k^-(x)$. More precisely, it is assumed that there is no model M and node $n \in M$ permitted by GEN such that $M, n \models \phi_k^+(x)$ and $M, n \models \phi_k^-(x)$.

(12) Definition of Spread-R(+nasal)

$\phi^+(x) = \texttt{[+nas]}(x) \wedge \neg\texttt{nas\_head}(x) \wedge \exists y(\texttt{[+nas]}(y) \wedge y \lhd x \wedge \texttt{nas\_span}(x, y))$
$\phi^-(x) = \texttt{[−nas]}(x) \wedge \exists y(\texttt{[+nas]}(y) \wedge y \lhd x)$

where `[+nas]` and `[−nas]` are predicates that identify [+nasal] and [−nasal] nodes, respectively, `nas_head` identifies nodes that are heads of nasal spans, `nas_span` identifies pairs of nodes that are members of the same nasal span, and $\lhd$ is the immediate precedence relation on nodes. A model assigns denotations to these predicates — for example, the denotation of `[+nas]` is the unary relation (set) of [+nasal] nodes in the domain of M, and $\lhd$ denotes the set of node pairs related by immediate precedence in M — subject to the usual conditions of exclusivity, transitivity, etc. (e.g., Partee et al., 1993).

(13) Definition of *NasObs

$\phi^+(x) = \texttt{[−nas]}(x) \wedge \exists y, z(\texttt{seg}(y) \wedge \texttt{[−son]}(z) \wedge \texttt{assoc}(y, x) \wedge \texttt{assoc}(y, z))$
$\phi^-(x) = \texttt{[+nas]}(x) \wedge \exists y, z(\texttt{seg}(y) \wedge \texttt{[−son]}(z) \wedge \texttt{assoc}(y, x) \wedge \texttt{assoc}(y, z))$

where `assoc` identifies associated segment and feature nodes. The definition of *NasVoc is parallel but makes use of the predicate or predicates that identify vocoids instead of obstruents.

The preceding formalizes evaluation by a single constraint. Evaluation by a set of constraints can be defined in terms of satisfaction of conjunctions. Given a finite constraint set $\textsc{Con} = \{c_1, \ldots, c_K\}$, let Φ be the set of all conjunctions over $\{\phi_k^+(x), \phi_k^-(x), \phi_k^0(x)\}_{k=1}^K$ that contain *exactly one conjunct* for each constraint $c_k \in C$. The generic form of a member of Φ is $\bigwedge_{k=1}^K \phi_k^{\alpha_k}(x)$, where each $\alpha_k \in \{+, -, 0\}$. Each node in a model satisfies exactly one formula in Φ, and that formula determines the *total marking* of the node: the marks that it receives from all of the constraints.[5] For example, if $M, n \models (\phi^+_{\text{Spread-R(+nasal)}}(x) \wedge \phi^0_{\text{*NasObs}}(x) \wedge \phi^-_{\text{*NasVoc}}(x))$ then node $n \in M$ is assigned $+1$ by Spread-R(+nasal), 0 by *NasObs, and -1 by *NasVoc; it is the [+nasal] feature of a vocoid that has undergone spreading.

Given a fixed weight vector $\mathbf{w} = (w_1, w_2, \ldots, w_K)$, the conjunctions in Φ also determine node harmonies. If $M, n \models \phi$, where $\phi = \bigwedge_{k=1}^K \phi_k^{\alpha_k}(x)$, then the harmony of node n in M is equal to $h_{\mathbf{w}}(\phi) = \min_0 \sum_{k=1}^K w_k \; \alpha_k$. Because the set Φ of conjunctions is finite (bounded by 3^K), a weighted grammar specifies a finite set of node harmonies. In other words, the grammar partitions the nodes within and across models into a finite number of harmony levels, the highest of which is zero.

Let $\Phi_{\mathbf{w}}$ be the subset of Φ such that $h_{\mathbf{w}}(\phi)$ is maxinal (i.e., $\Phi_{\mathbf{w}} = \{\phi \in \Phi \mid h_{\mathbf{w}}(\phi) = 0\}$). A node is well-formed iff it satisfies some member of $\Phi_{\mathbf{w}}$ and a structure (model) is well-formed iff this is the case for each of its nodes. Therefore, the well-formedness of a structure relative to a grammar can be defined with a single formula, the disjunction over $\Phi_{\mathbf{w}}$. Taking advantage of the standard universal quantification over nodes in a model (equivalently, over variable assignment functions as in Partee et al., 1993: chapter 13), M is well-formed according to the grammar iff $M \models \bigvee \Phi_{\mathbf{w}}$.

This result has the important implication that weighted constraint conflict in HGstat conserves logical expressivity. Because all logics of interest are closed under conjunction and disjunction, the logic needed to formalize a weighted grammar is no more complex than that needed to define $\textsc{Con}$ (assuming that $\textsc{Gen}$ is also stated in the same logic). For the examples discussed in this paper, first-order logic with unary and binary relations and no constants or functions is sufficient to state the constraints.

Constraint conflict in OTStat preserves logical expressivity in an analogous way. For a fixed constraint ranking $\mathbf{r}$, let $\Phi_{\mathbf{r}}$ be the set of all conjunctions of the form $\phi = \bigwedge_{k=1}^K \phi_k^{\alpha_k}(x)$ that satisfy the following condition: for each $\phi_j^-(x)$ in ϕ there is some ϕ_i^+ in ϕ such that $r_i \gg r_j$ (compare to (3)). $\Phi_{\mathbf{r}}$ partitions the set of nodes into two classes: nodes that satisfy a member of $\Phi_{\mathbf{r}}$ have harmonies of zero and are well-formed; all other nodes have harmony -1 and are ill-formed. Model M is well-formed according to the grammar iff every one of its nodes is well-formed, that is iff $M \models \bigvee \Phi_{\mathbf{r}}$.

The formalization above extends a long and fruitful tradition of logical approaches to phonology (Bird & Blackburn, 1991; Bird & Klein, 1994; Bird, 1995; Potts & Pullum, 2002; Jardine, 2019; Bhaskar et al., 2020). It is also an endpoint, in the sense that there is nothing more to be said or discovered about the logical expressivity of constraint interaction in HG/OTStat. This allows further research to focus on the formal characterization of individual constraints, constraint families, and the structures (models) that they evaluate (e.g., Bird, 1995; Potts & Pullum, 2002; Jardine, 2014, 2017; Strother-Garcia, 2019) as well as on connections among logic, automata, and processes of generation and recognition.

4. Discussion

In this paper, I have introduced HGStat and OTStat as new theories of violable constraint interaction that do not make use of candidate comparison. The analyses in §2 show that, in representative cases, simple harmony calculations within individual nodes can successfully replace global optimization over entire structures. Whether this will carry over to all aspects of phonology is currently unknown; it may be that HG/OTStat necessitate representations and constraint systems of a sort that are currently unfamiliar, and that are in some instances less appealing than those of HG/OT. However, any loss of elegance must be weighed against the relative formal and computational simplicity of the static theories.

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[5] Satisfaction of conjunctions and disjunctions is defined in the usual way: $M, n \models (\psi_1(x) \wedge \psi_2(x))$ iff $M, n \models \psi_1(x)$ and $M, n \models \psi_2(x)$; $M, n \models (\psi_1(x) \vee \psi_2(x))$ iff $M, n \models \psi_1(x)$ or $M, n \models \psi_2(x)$. The fact that the same node n replaces the unbound variable in each conjunct or disjunct correctly formalizes local mark assignment.

Static grammars are no more logically expressive than the constraints they contain, as established in §3. Relatedly, many static grammars have elementary associated automata. For example, the unbounded spreading grammar of §2.2 can be implemented with a finite-state transducer over a modest vocabulary in which each state represents a one-segment history. If marks are assigned to transitions (arcs) on the basis of their labels and originating states, and all transitions with harmony less than zero are pruned, the resulting machine represents all and only the words in which the desired unbounded spreading pattern is obeyed. This kind of compilation is possible for some but not all OT grammars (Riggle, 2004), and even when it is possible the computations and resulting machines are considerably more complex. Static grammars offer a streamlined approach to implementing and verifying phonological analyses without sacrificing the core insight of conflicting, violable constraints.

HGStat was directly inspired by the connectionist parser of Hale & Smolensky (2001), which employs local accumulation of positive and negative values to formalize context-free grammars. The application of a threshold non-linearity such as $\min_0$ subsequent to weighted summation, while not part of the proposal in Hale & Smolensky (2001), is ubiquitous in both early connectionist and contemporary deep neural network models (e.g., Rosenblatt, 1958; Nair & Hinton, 2010). Indeed, it is tempting to identify nodes with simple neural processing units connected to one another in highly systematic ways that mirror the relations of symbolic structures. In a network of this type, a pattern of activity over a subset of the nodes is stable iff each node in the pattern receives sufficient incoming activation to remain 'on' (i.e., to meet the harmony threshold of zero).

Within phonology, other approaches have made use of both positive and negative constraints (e.g., Kimper, 2016; Kaplan, 2018). Unlike some previous attempts, there is no possibility in HG/OTStat of positive marks resulting in a preference for 'infinite words'. The threshold non-linearity ensures that the most a node can hope to contribute to the harmony of its structure is absolutely zero.

HG/OTStat might also seem to be related to theories in which optimization is performed over substructures rather than entire outputs (e.g., Frank & Satta, 1998; Heck & Müller, 2007). However, there is no subconstituent that can comprehend all known phonological interactions — features spread across syllable and foot boundaries, consonants agree and dissimilate at a distance, stress assignment is calculated over whole words — and for that reason the static theories define grammaticality as a global property. Optimization is applied neither locally nor over entire structures, being replaced by the simpler condition that every node of a well-formed structure must have maximal harmony in its context.

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