

Static Grammar is an approach to phonology and other parts of language that adopts much of Harmonic Grammar (Legendre et al., 1990) and Optimality Theory (Prince & Smolensky 1993/2004), with one key difference. The well-formedness of a representation is determined by its internal structure — a representation is well-formed if and only if each of its parts has maximal harmony when evaluated in the context of the other parts — rather than through comparison with alternative representations.

### Static Harmonic Grammar (HGStat)

- **Representation.** Each representation permitted by **Gen** is a *relational structure* consisting of a finite non-empty set of nodes and relations on those nodes (e.g., unary relations that label the nodes as segments, valued features, or prosodic units; binary relations of precedence, association, and constituency).
- **Evaluation.** A constraint in **Con** evaluates each node within its structure, assigning the node a *signed unit mark* or zero:

Two constraints conflict at a node if they assign it marks of opposite sign.

- **Harmony.** Given constraint weights, the harmony of a node is determined by summing its weighted marks and applying an upper threshold of zero:

and the harmony of a structure is the sum of the harmonies of its nodes:

- **Well-formedness.** A node is well-formed iff its harmony is non-negative (i.e., zero). A structure is well-formed iff all of its nodes are — equivalently, when its harmony is zero.

Static OT (OTStat) is defined in the same way, except that node harmony is determined by a strict-dominance ranking.

- **Harmony.** Given constraint ranking  $\prec$ , the harmony of a node is:

Because harmony and well-formedness are defined with elementary functions, static grammars have no logical expressivity or computational complexity beyond that needed to define the set of possible structures (**Gen**) and state the constraints (**Con**).

- If **Gen** and **Con** can be formalized in a particular logic (e.g., predicate logic), then an entire weighted/ranked grammar corresponds to a single conjunctive normal form statement in the same logic and the well-formed structures are the models of that statement.
- If **Gen** and **Con** can be implemented with a single finite-state machine, in which states encode local or tier-based contexts and nodes live on arcs, then a particular weighted/ranked grammar is derived by pruning some of the arcs (those representing nodes with negative harmony). The pruned result is a finite-state machine that represents the regular language or rational relation defined by the grammar. Each path through the machine specifies a well-formed structure.

Static Grammar was directly inspired by Smolensky (1993) and Hale & Smolensky (2001, 2006), which use local accumulation of positive and negative harmony contributions to formalize context-free grammars (see also Smolensky 2012). Application of a threshold non-linearity subsequent to weighted summation, while not part of those proposals, is the basic building block of both early connectionist model (e.g., Rosenblatt 1958) and contemporary deep neural networks. In particular, the ("minnow") non-linearity is identical to the  $\tanh$  activation function (Nair & Hinton 2010) with a change of sign,  $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ .

The term *static* evokes electrical charges at rest, like the positive and negative marks assigned to structural nodes, and forces in equilibrium, as positive and negative marks cancel out to zero under minnow. While optimization in HG and OT can be thought of as a dynamic harmony-ascending search through a representational space, Static OT/HG emphasizes the internal stability of individual structures.