

A Thesis

entitled

Development of Computational Algorithms for Localization in Wireless Sensor
Networks through Incorporation of Dempster-Shafer Evidence Theory

by

Colin P. Elkin

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the
Masters of Science Degree in Engineering

Dr. Vijaya Devabhaktuni, Committee Chair

Dr. Hong Wang, Committee Member

Dr. Mansoor Alam, Committee Member

Dr. Patricia R. Komuniecki, Dean
College of Graduate Studies

The University of Toledo
August 2015

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Wireless sensor networks are a collection of small, disposable, low-power devices that monitor vital sensory data for a variety of civil, military, and navigational applications. For instance, some cities have a network of emergency phones scattered across walkways so that citizens in distress can immediately reach emergency services. Using effective localization techniques that are both highly accurate and of low computational cost, 911 services can dispatch police, fire, or medical services to a caller's location as quickly as humanly possible. Hence, as far as locating a node in a network is concerned, every percent of accuracy achieved and every second of time saved can be the difference between life and death.

This thesis presents two novel algorithms for wireless sensor network localization through the incorporation of Dempster-Shafer Evidence Theory. The first technique follows a verbose methodology for node positioning that fuses multiple types of signal measurements, such as received signal strength and angle of arrival, and utilizes the expected value property of DS Theory to geo-locate a node with moderate accuracy of 78-87%, thereby providing an introductory approach to the previously untapped fusion of WSN localization and DS Theory.

The second approach consists of a low cost, highly accurate data fusion technique that incorporates the plausibility property of DS theory to establish a high level of

accuracy. Due to this unique approach to data fusion and predictive data modelling, this second algorithm achieves an optimal accuracy range of 83-98% in a flexible multitude of simulation scenarios at a fraction of the runtime required under prior established localization techniques. Overall, these two algorithms provide a groundbreaking new application of Dempster-Shafer Theory as well as fast, accurate, and informative new approaches to wireless sensor network localization that can improve a wide range of vital applications.

To the memories of my late grandparents, John Alan Elkin and Sylvia Urang, who left us over the past year.

Acknowledgments

First and foremost, I would to express my sheer gratitude and appreciation for my advisor and thesis committee chair~~man~~ Dr. Vijay Devabhaktuni for all of the guidance, leadership, patience, motivation, and support with which he has provided me over the course of my graduate studies. I would also like to thank Dr. Hong Wang and Dr. Mansoor Alam for ~~graciously serving~~ as members of my thesis committee.

Secondly, I would like to thank The University of Toledo's Department of Electrical Engineering and Computer Science as well as the Department of ~~engineering technol-~~ogy for providing me with financial support in the forms of graduate assistantships and tuition waivers. I especially wish to extend gratitude towards Dr. Richard Molyet, Prof. Dan Solarek, Mr. Jacob Phillips, Dr. Allen Duncan, and Mrs. Nicole Kamm for enriching my graduate experience with many teaching assistant opportunities. I would also like to express my appreciation to the National Science Foundation (NSF) for providing the EECS department with the funding for the research portion of my assistantships. I also want to thank the university's athletic department, most notably David Sankovich and Oliver Jay, for providing me with part-time employment during the first spring and summer semesters of my graduate studies.

In addition, I want to thank my colleague Rajika Kumarasiri for introducing me to the vast and exciting world of graduate research as well as my lab mate Hem Regmi for the encouragement, support, and friendship he has provided me over the past several months. My current and former colleagues of labs 2033 and 2042 are also greatly acknowledged.

Most importantly, I would like to thank my parents John and Rebecca Elkin for assisting me with the initial move to Ohio to begin graduate studies as well as for providing me with consistent love and support throughout.

Contents

Abstract	iii
Acknowledgments	vi
Contents	viii
List of Tables	xii
List of Figures	xiii
List of Abbreviations	xiv
List of Symbols	xvi
1 Introduction	1
1.1 Overview of Localization	1
1.2 Overview of Data Fusion	3
1.3 Overview of Dempster-Shafer Theory	3
1.4 DS Theory Example	4
1.5 Proposed Localization Techniques	6
1.6 Publications and Contributions to Thesis	7
1.7 Thesis Organization	8
2 Literature Acquisition and Analysis	9
2.1 Localization Techniques	9

2.1.1	High Cost Localization Methods	9
2.1.2	Low Cost Methods and Terminologies	11
2.1.2.1	Time Difference of Arrival	11
2.1.2.2	Time of Arrival	13
2.1.2.3	Angle Of Arrival	13
2.1.2.4	Received Signal Strength	15
2.1.2.5	Hop Count	16
2.1.3	Biologically Inspired Techniques	17
2.1.4	Maximum Likelihood Estimation	18
2.2	Data Fusion	18
2.3	Dempster-Shafer Theory	20
2.3.1	Bayesian Origins	20
2.3.2	Theoretical Foundation	21
2.3.2.1	Basic Probability Assignment	21
2.3.2.2	Aggregation	22
2.3.2.3	Belief and Plausibility	23
2.3.2.4	Prediction of Distance by Expected Value	23
2.3.3	Applications	24

3 Introductory DS-Based Localization Technique Using Expected Value

	Outputs of Basic Probability Assignments	25
3.1	Introduction	26
3.2	Dempster-Shafer Theory	27
3.2.1	Linear Representation	28
3.3	Localization Algorithm	29
3.3.1	Measurement Types	29
3.3.2	Data Fusion	30

3.4	Methods and Materials	33
3.4.1	WSN Setup	34
3.4.1.1	Data Value Ranges	35
3.4.2	Data Generation	36
3.4.3	IPP Toolbox	37
3.4.4	Algorithm Execution	37
3.4.5	Desired Inputs and Output	38
3.4.6	Accuracy Calculation	38
3.5	Experimental Results	40
3.5.1	Accuracy	41
3.5.2	Computational Runtime	41
3.5.3	Comparisons to Other Established Algorithms	42
3.5.3.1	Accuracy	42
3.5.3.2	Computational Runtime	43
3.6	Conclusions and Future Work	43
4	Novel Low Cost, Highly Accurate Method of Localization Using DS- Based Plausibility	44
4.1	Dempster-Shafer Theory	45
4.1.1	Localization Algorithm	48
4.2	Material and Methods	49
4.2.1	Algorithm Execution	50
4.2.1.1	Determination of Acceptable Distance Values	51
4.2.2	Desired Outputs	52
4.2.3	Accuracy Calculation	53
4.3	Results and Discussion	55
4.3.1	Results Under Distance Range Weight Training	55

4.3.2	Training-Independent Results	57
4.3.2.1	Accuracy	57
4.3.2.2	Computational Runtime	59
4.3.3	Comparisons to Other Established Algorithms	59
4.3.3.1	Accuracy	60
4.3.3.2	Computational Runtime	60
4.4	Conclusions	60
5	Conclusive Remarks	62
5.1	Future Work	63
5.1.1	Enhanced Training of Upper and Lower Bound Coefficients . .	64
5.1.2	Fusion of DS Theory With Support Vector Machines	64
5.1.3	Role of DS Theory Within Data Mining and Big Data Analytics	64
	References	66
	A Source Code	74

List of Tables

1.1	Resultant evidence factors for the three mechanics.	5
1.2	Publications and contributions to thesis.	7
2.1	Summary of common high cost localization techniques.	11
2.2	Summary of common high cost localization techniques.	12
2.3	Summary of global, biologically inspired localization techniques.	17
3.1	Simulation results.	41
3.2	Comparison of accuracy among different localization techniques.	42
4.1	Simulation results for two-monitor and three-monitor WSNs.	57
4.2	Runtime results for two-monitor and three-monitor WSNs.	59
4.3	Comparison of accuracy among different localization techniques.	59

List of Figures

2-1	Overview of common localization algorithms	10
2-2	Example of AOA-based localization	14
2-3	Example of RSS Trilateration in a three-monitor WSN Setup	15
3-1	Flowchart of DS localization algorithm.	31
3-2	WSN of 100 sensor nodes, 3 anchor nodes, standby node, and fusion center.	35
3-3	Example of expected-value-based DS algorithm in execution.	39
3-4	First proposed classifier.	40
4-1	Flowchart of DS localization algorithm	48
4-2	Second proposed classifier.	53
4-3	Plots of accuracy versus upper bound weight versus lower bound weight.	56

List of Abbreviations

2-D	Two-dimensional
3-D	Three-dimensional
AOA	Angle of Arrival
ANN(s)	Artificial Neural Network(s)
BPA(s)	Basic Probability Assignment(s)
BT	Bluetooth
CDF	Cumulative Density Function
CPU	Central Processing Unit
CSV	Comma Separated Value
DS	Dempster-Shafer
GA	Genetic Algorithm
GB	Gigabytes
GHz	Gigahertz
GNU	GNU is Not UNIX
GPL	GNU Public License
GPS	Global Positioning System
INS	Inertial Navigation System
IPP	Imprecise Probability Propagation
MATLAB	Matrix Laboratory
MLE	Maximum-Likelihood Estimation
NLOS	Non Line of Sight
PSO	Particle Swarm Optimization
RAM	Random Access Memory

RSS	Received Signal Strength
SB	Standby
SVM(s)	Support Vector Machine(s)
TDOA	Time Difference of Arrival
TOA	Time of Arrival
Wi-Fi	Wireless Fidelity
WLAN(s)	Wireless Local Area Network(s)
WSLA	Weighted Search-Based Refinement Algorithm
WSRA	Weighted Search-Based Refined Algorithm
WSN(s)	Wireless Sensor Network(s)

List of Symbols

\mathbf{x}	vector \mathbf{x}
$P(x)$	probability of x
$f(x)$	function of x
$Y X$	outcome Y given X
Θ	frame of discernment
\oplus	combination
Bel	belief
Pl	plausibility
Val	value
Ex	expected
\subset	subset
\cap	intersection
\emptyset	empty set

Chapter 1

Introduction

Wireless sensor networks are a collection of small, disposable, and interchangeable low-power devices that monitor and report vital sensory data for a variety of civil, military, and navigational applications. As the demand for and scope of wireless sensor networks (or WSNs) continue to grow, the need for identifying a node's location quickly and accurately within such a network becomes one of great importance. For instance, some cities have a network of emergency phones scattered across walkways so that citizens in distress can immediately reach emergency services. Using effective localization techniques, 911 services can dispatch police, fire, or medical services to a caller's location as quickly as humanly possible. Hence, as far as locating a node in a network is concerned, every percent of accuracy achieved and every second of time saved can be the difference between life and death. Achieving such a successful method of localization, however, with a steady balance of minimal power, low overall cost, and high accuracy, remains one of the greatest challenges in the area of WSNs.

1.1 Overview of Localization

WSNs consist of a variety of nodes, ~~the majority of which are~~ sensor nodes, which serve to sense a particular type of measurement such as temperature or vibrations [25]. Anchor nodes, also known as cluster heads or monitors, function as a higher

powered variation of sensor nodes in that they serve to sense the measurements of other nodes, such as distance or position. Other node types that will be included in the scope of this research include the standby node, which serves specifically to sense which nodes are in a low power (standby) state in a method similar to that of anchor nodes, and the fusion center, which is essentially an embedded system that executes vital tasks for the network as a whole based on the sensor measurements reported by the anchor nodes.

Some of the most vital of such techniques include received signal strength (RSS), angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and hop count. In the context of this research, RSS is defined as the electric field at the receiving node divided by the distance between said node and the transmitting antenna. Due to RSS's inverse square relationship with distance [12], highly accurate locations can be determined with three or more RSS measurements from three unique points, as can be seen in Fig. 2-3, by process of trilateration [45]. AOA is the arrival angle of the emitted source signal observed at an anchor node (as shown in Fig. 3) and is a measurement dependent on TDOA of multiple elements in an array. More specifically, AOA is calculated using the differences between arrival times of a transmitted signal [30]. TDOA is dependent on time differences between nodes [26] and can be another effective means of localization by process of multilateration [36]. TOA, on the other hand, is simply the amount of time required for a signal to move from a transmitting node to a receiving node [30]. Hop count is a range-independent method that measures the number of intermediate, or neighbor, nodes required for a signal to travel from a sensor node to an anchor node [43].

Most of the aforementioned data types, such as RSS and AOA, function well enough on their own, but when an algorithm is needed that can benefit from having multiple types of data. A unique challenge becomes apparent in that each measurement type can be vastly different from one another. To solve this problem of uniquely

different data, the concept of data fusion becomes one of particular importance.

1.2 Overview of Data Fusion

Data fusion is a valuable and streamlined process that involves obtaining many different types of data from a single common application and integrating them into a unified, consistent representation. While an array of similar data is relatively simple to implement in the context of data fusion, more relevant aspects thereof are the result of widely varying types of data that derive from a multitude of different sensor types. In the event of such a large magnitude of complexity within the field of data fusion, a significant amount of creativity and elaboration is needed to solve such a problem. To the benefit of this research, one problem solving technique exists to provide a powerful and straightforward approach to fusing dissimilar data types, although its application of the field of wireless sensor networks, as well as that of sensors in general, is one of recent development, whose potential therefore remains largely untapped. As the title of this thesis suggests, this technique is none other than Dempster-Shafer Evidence Theory.

1.3 Overview of Dempster-Shafer Theory

Dempster-Shafer Theory, ~~herein~~ referred as DS Theory ~~or DST~~, is a statistical framework ~~that is~~ based on Bayesian combinational probability but undertakes a drastically different foundation with regards to combinational factors. While Bayesian statistics rely primarily upon combinations of internal probability factors within a system, usually through the use of propositions or random variables, DS Theory is contingent upon external evidence factors, each one typically consisting of a range of data in which the desired output value can be found and a probability of confidence

that the data range is in fact reliable. Because DST is still a relatively new concept in the scope of this research, a brief example that illustrates a fundamental contrast between DS-based probability and its traditional Bayesian counterpart is provided for the benefit of the reader.

1.4 DS Theory Example

To simplify both methods in the context of a practical example, suppose there exists a car that is due to fail at some given moment and has three factors that are the most likely to result in the vehicle's failure, which will only occur if all three of these components fail individually. In the context of this example, the three components are the car's worn out spark plugs, depleting oil, and ageing transmission fluid, herein referred as Components 1, 2, and 3, respectively. The objective of this problem is now to find the overall probability of the car failing within one week. The Bayesian approach would involve measuring the failure rates of each individual component and combining the factors to determine the overall system failure probability. Now suppose the probabilities of Components 1, 2, and 3 respectively failing within one week are .6, .4, and .2, respectively. By use of combinational logic in the form of OR gates (as each component's failure rate is independent of one another), conclusions can be drawn that the combined probability of the car failing within one week shall be $.6 * .4 * .2 = .048$, or 4.8 percent. Thus, the probability of system failure for a fixed length of time has been obtained, but suppose a greater extent of information, such as the amount of time expected to elapse before reaching 100 percent failure, is desired. In such a case, an entirely different approach would be needed.

Applying the same example with DS Theory now requires external evidence factors, which in the context of this problem will be: three mechanics, herein referred as Mechanics A, B, and C. Each mechanic inspects the vehicle as a whole, including but

Table 1.1: Resultant evidence factors for the three mechanics.

Mechanic	Lower Bound	Upper Bound	Confidence Value
A	11	18	.4
B	14	21	.4
C	10	20	.2

not limited to all three aforementioned components, and diagnose an interval of time at which the vehicle is expected to fail as well as the degree of confidence in which the mechanic is certain of these time values. Suppose the vehicle is first taken to its original dealership, which has two mechanics, A and B, that specialize in the exact make and model of this vehicle and hence are highly confident in their predicted time range in which the car will fail, despite the fact that A's time range differs from that of B. Mechanic A concludes that total failure will occur between 11 and 18 days in the future, while Mechanic B is certain that failure will happen between 14 and 21 days from the present. Next, the car is taken to a more general purpose, non-dealer specific repair shop, whose expert, Mechanic C, is less familiar with the specific make and model of this car when compared to the expertise of Mechanics A and B and thus concludes at half the confidence level of the other experts that vehicle-level failure will occur between 10 and 20 days into the future.

Now that all confidence levels have been compared, deductions can be made that A and B have equal confidence levels of .4 each and that C has a confidence level of .2 (half that of the other mechanics). With all data intact, the resultant evidence factors are provided in [1.1](#). By the conditions associated with DS Theory, conclusions can now be drawn that the plausibility of vehicular failure to begin after 14 days and that the belief of total failure to begin after 21 days. In other words, while Bayesian probability was favorable for determining the probability of a car failing after the exact elapsed time of one week, DS Theory allowed more elegant solutions

by obtaining the amount of time at which ~~one hundred percent~~ failure was imminent, which in this case was three weeks.

1.5 Proposed Localization Techniques

Fault or failure prediction is just one of many possible applications that can be enhanced with DS Theory. Other relevant applications, that is, ones that are more closely related with wireless sensor networks and thus with the aim and scope of this research, include the use of dissimilar node measurements (RSS, AOA, etc.) to predict the distance from a node to a monitoring station. The localization of a node, or identification of a node's location within a wireless sensor network, is a challenge of both great importance and great difficulty. Thus, the objective of this research is to utilize Dempster-Shafer Theory as an innovative statistical mechanism for developing highly accurate and low cost algorithms for localization in wireless sensor networks. Resultantly, two localization schemes are proposed in the body of this thesis.

The first technique is based on the expected value function of DS Theory and uses basic probability assignments (BPAs) of distance ranges to predict an expected range of radii between a given base station and the potential location of a sensor node. To enhance accuracy, all BPAs are used as distributions for a normalized inverse sample of 10 BPAs, which are then aggregated into a single data structure to be used in all further evaluation. When these radii are obtained for all working monitors, an area of feasible positions is obtained, which can be used to determine whether a particular node in a particular county matches the given set of measurements (RSS, AOA, operating condition, or a combination thereof) corresponding to aforementioned distance BPAs.

The second method revolves around the plausibility function of DS Theory. Like the previous technique, BPAs are formed from distance values corresponding to var-

Table 1.2: Publications and contributions to thesis.

Type	Publication/Contribution
Journal Paper	A Novel Approach to Localization in Wireless Sensor Networks Using Dempster-Shafer Evidence Theory.
Source Code	A re-usable MATLAB library containing the source code of the algorithm implementations

ious types of measurements similar to those mentioned above. The fundamental difference, however, is that instead of using an expected value function to predict a distance range, a plausibility function is utilized to determine the most plausible radius from a monitor to a node's location. Because this is a single value rather than a range, the value is used as a coefficient for a data range that is compared against a county's actual value. If the actual distance is between a given upper and lower percent of the predicted distance for every respective monitor, then the decision is made that the given set of measurements belongs to **the given county**. The upper and lower ~~percent values~~ are determined by an iterative training stage, which is meant only to be run upon the initialization of a WSN and can be skipped after subsequent localization attempts and even after minor network regenerations.

The first method is a preliminary approach, intended primarily for exploration of DS concepts in WSN localization, due to modest accuracy results and a high time-based computational cost. The second is a novel, refined approach, taking ~~the~~ concepts acquired from the previous **technique** to achieve both high accuracy and low computational cost.

1.6 Publications and Contributions to Thesis

The major publications and contributions of this research are presented in Table 1.2.

1.7 Thesis Organization

This thesis unfolds as follows:

Chapter 2 provides a review of the literature that forms the foundation of this research. It covers a variety of theoretical backgrounds pertaining to prior established research methods in the areas of wireless sensor networks, localization techniques ~~thereof~~, data fusion, ~~predictive data modelling~~, and Dempster-Shafer Theory.

Chapter 3 describes the first proposed localization scheme, a preliminary method that focuses on the expected value property in DS Theory.

Chapter 4 **elaborates** upon the second proposed localization algorithm, a low cost, highly accurate technique based primarily upon the plausibility concept of DS Theory.

Chapter 5 draws conclusive remarks and discusses possible future directions in which this research could advance.

Finally, the thesis ends with an appendix, providing information on how to obtain the MATLAB source code.

Chapter 2

Literature Acquisition and Analysis

This chapter provides an extensive background of prior established research methods in the many fundamental concepts that form the foundation of this thesis research. Because this is the first time that Dempster-Shafer Theory has crossed ~~all of~~ the research realms of WSN localization, each of the three topics of localization techniques, data fusion, and DS Theory are addressed as three separate sections of the same names.

2.1 Localization Techniques

A brief overview of the most common unimodal localization methods of both low cost and high cost categories are presented in Fig. 2-1. A summary of comparisons among high cost techniques is presented in Table 2.1.

2.1.1 High Cost Localization Methods

Some of the most common modern localization techniques are well-known even outside of scientific research areas due to their popularities in modern consumer electronics. These localization methods come at a high computational cost but in exchange are more technologically robust and require less mathematical planning. Such

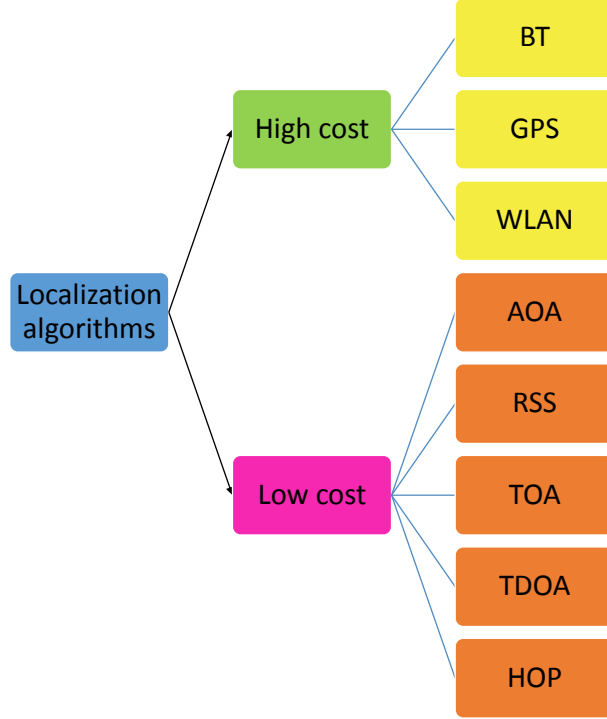


Figure 2-1: Overview of common localization algorithms

techniques include global positioning systems (GPS), wireless local area networks (WLAN), and bluetooth (BT).

GPS is perhaps the most widely known method of localization, whether in a WSN or in a more general navigational context, due to its frequent use in navigational assistance through standalone devices and through embedded navigational capabilities in smartphones and tablet devices. Bluetooth is primarily appealing for short range solutions and thus can be ideal specifically for local positioning in a WSN [58]. WLAN, or Wi-Fi, is another short range solution whose significance lies within its localization accuracy specific to indoor WSN environments [58].

All three of these techniques share the common characteristic of requiring a high computational cost to achieve a desirable level of accuracy. As a result, many scenarios exist in which the desired amount of resources required for proper functionality of RSS, BT, or WLAN are not readily available. In addition, GPS measurements can fall victim to signal blockages or multipath effects [47]. One potential resolution for

Table 2.1: Summary of common high cost localization techniques.

Technique	Overview	Advantages	Disadvantages
Global Positioning Systems (GPS)	Location based on satellite tracking of device position	Highly accurate when signal is available	Susceptible to signal blockage and multipath effects
Wireless Local Area Networks (WLAN)	Location based on Wi-Fi signals in a short-range environment	Ideal for indoor environments	Ineffective in long range or outdoor environments
Bluetooth (BT)	Based on connections between devices	Ideal for small scale node positioning	Even more ineffective than WLAN in long range environments

such a ~~conundrum~~ involves restricting measurement and monitoring capabilities to a small number of nodes known as cluster heads [2, 17], or anchor nodes. A vital further step, however, is to engage in lower cost methods of a more ~~simplistic~~ nature in order to ~~to~~ achieve the full potential of effective localization.

2.1.2 Low Cost Methods and Terminologies

Many established localization techniques exist with a low computational cost compares to the previously discussed methods. ~~However,~~ they also require a more significant amount of creativity and mathematical determination. The methods to be discussed in this subsection consist of received signal strength (RSS), angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and hop count. A brief comparison of such techniques is presented in Table 2.2.

2.1.2.1 Time Difference of Arrival

TDOA is a measurement type that calculates the time of arrival between any two nodes on the condition that each pair of nodes uses the same frame of time in

Table 2.2: Summary of common high cost localization techniques.

Technique	Overview	Advantage	Disadvantage
Time Difference of Arrival (TDOA)	Time between any two nodes calculated within same frame of time	Better accuracy and performance than TOA in cellular communication systems	Fewer range estimates and increased variance of range sampling error
Time of Arrival (TOA)	Amount of time required for a signal to travel from transmitter to receiver	Only measurement required is time needed to arrive at a node	More accurate than TDOA in more general scenarios
Angle of Arrival (AOA)	Angle at which signal arrives at receiver from transmitter	Does not require synchronization	Requires expensive antenna arrays at each receiver
Received Signal Strength (RSS)	Quotient of electric field and distance to some power	Hardware requirements are of low cost	Accuracy can be diminished by environmental factors
Hop Count	Number of neighbor nodes required for signal to pass between transmitter and receiver	Range free	Accuracy is compromised when low cost, small size, and high power efficiency are desired

such calculations. The main disadvantage of TDOA measurements is the tendency for subpar accuracy due to technical constraints in the areas of propagation environments and communication systems [35]. This is the case when evaluating TDOA solely on its own. When comparing this type of measurement to others, particularly in a specific application, benefits become more noticeable. For instance, in a certain aspect of cellular communication systems, when utilized as localization methods, TDOA holds more accuracy and better performance than TOA measurements, although it is at the cost of having fewer range estimates and increased variance of range sampling error [42]. TDOA is dependent on time differences between nodes [26] and thus is another effective means of localization by process of multilateration [36].

2.1.2.2 Time of Arrival

TOA, on the other hand, is simply the amount of time required for a signal to move from one transmitter to a receiving node [30]. This differs from TDOA in that only the time required to arrive at a node is measured, rather than the difference between leaving from one node and arriving at another. TOA estimation in a general context is known to be more accurate than TDOA, as opposed to in a specific application as given before, although calculating TOA can be more difficult than doing so for TDOA, [18]. ~~It should also be noted~~ that the performance of a system that utilizes TOA can be significantly lowered by undetected direct path (UDP) conditions as well as bandwidth limitations, [40]. In general, time-difference-of-arrival (TDOA) and time-of-arrival (TOA) depend on signal time taken to arrive at the receiver for the source location estimation, [20, 49].

2.1.2.3 Angle Of Arrival

Angle-of-arrival (AOA) measures the angles of arrival (AOAs) at two or more receivers. Location estimation is then done using triangulation, [4, 13]. AOA is de-

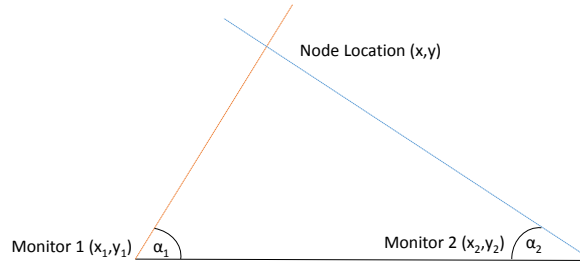


Figure 2-2: Example of AOA-based localization

pendant on the time difference of arrival (TDOA) of multiple elements in an array. TDOA, in turn, is obtained from the difference between phases received by two elements in said array. A practical external example of AOA estimation is the area of smart antenna systems, as the information generated by this type of estimation can be used to enhance the efficiency of transmission and reception of said antenna systems, [53]. ~~It should also be noted that~~ AOA thrives primarily on a dependence upon the Gaussian distribution, ~~especially~~ in the area of geo-location systems, [46]. In other words, AOA is the arrival angle of the emitted source signal observed at a monitor (as shown in Fig. 3.8) and is a measurement dependent on TDOA of multiple elements in an array. More specifically, AOA is calculated using the differences between arrival times of a transmitted signal [30].

~~However, the previously mentioned localization schemes have their own advantages~~ and limitations ~~as well~~. For example, although AOA does not require any synchronization and produces fairly good results even with as few as two serving nodes. However, this measurement type also requires expensive antenna arrays at each receiver, which might not be a scalable solution. Time-of-arrival based schemes, on the other hand, require at least three properly located receivers for two dimensional

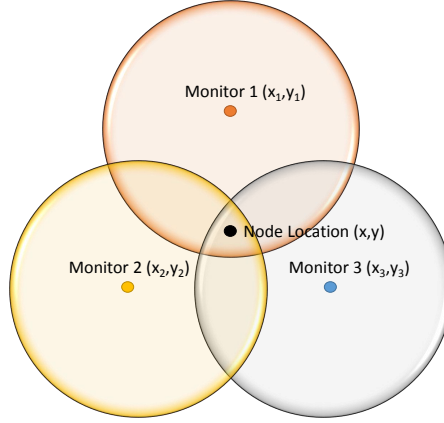


Figure 2-3: Example of RSS Trilateration in a three-monitor WSN Setup

(2-D) source locations and generally have better accuracy than AOA schemes. ~~It is noteworthy to point out that TDOA~~ technique requires synchronization between the serving receivers, whereas TOA requires synchronization not only between the serving receivers but also at the source side. ~~Also,~~ both techniques are highly affected by non-line-of-site (NLOS) conditions, [58]. Moreover, traditional localization schemes can fail or perform poorly under some conditions. For example, if only two TDOA measurements are available in a 2-D scenario, there will be two possible location estimations. Similarly, a single AOA measurement will not be sufficient to estimate the location of the unknown node, [58].

2.1.2.4 Received Signal Strength

~~In general,~~ RSS is the electric field at the receiving node divided by the distance between said node and the transmitting antenna. The key advantage of RSS measurement is that the hardware requirements for such a measurement are inexpensive and simple, [28]. The main disadvantage, however, is that RSS accuracy can suffer from external environmental factors in domains of both temporal and spatial natures, [46]. In the context of this problem, RSS is the average power received at the cluster head

where the transmitted power is emitted by the source of the unknown location. ~~It is then assumed that~~ the received signal is given by [6]

$$P^r = K \frac{P^t}{d^a} \quad (2.1)$$

where P^t is the transmitted power at the source, d is the distance from the source to the receiver, K accounts for all factors that affect the received power, including antenna heights and gains, and a is the path loss constant. [30] defines RSS as the equivalent to measured power, which is the squared magnitude of signal strength. Furthermore, signal power in free space is said to decay proportionally to the square of the distance between a transmitter and receiver [30]. [12] follows a similar approach by defining RSS as the relationship among transmitted power, received power, and distance among nodes in a WSN. This results in the equation

$$P_r = P_t \frac{1}{d^n} \quad (2.2)$$

where P_r and P_t represent the received and transmitted power, respectively, d denotes the distance between a sending node and a receiving node, and n is a propagation-based transmission factor, which is determined to be ~~2~~ 2 in a free space environment [12]. Due to RSS's inverse square relationship with distance [12], highly accurate locations can be determined with three or more RSS measurements from three unique points by process of trilateration [45, 29, 1]. An example of trilateration can be seen in Fig. 2-3.

2.1.2.5 Hop Count

Hop count is a range-free method that measures the number of intermediate, or neighbor, nodes required for a signal to travel from a sensor node to an anchor node [43] and can be extended into a more refined localization technique using weighted

Table 2.3: Summary of global, biologically inspired localization techniques.

Technique	Overview	Advantages	Disadvantages
Genetic Algorithm (GA)	Evolutionary population of candidate solutions	Ideal for large-scale search problems	Premature convergence
Particle Swarm Optimization (PSO)	Based on particles and movements	Ideal for problems with real values	Expensive computational runtime

search algorithms [14]. This method is primarily ideal for networks with low cost, power efficient sensor nodes [52]. Range independence is the primary benefit of hop count as a method of localization while the main drawback is that the intended context of low power WSNs typically results in a compromise of accuracy in exchange for low computational cost of the sensor nodes [52].

2.1.3 Biologically Inspired Techniques

Table 2.3 gives an overview of the two most common biologically inspired localization techniques. Genetic algorithms (GAs) are evolutionary algorithms that are given a population of potential solutions and converge to a single candidate. In a process akin to natural election, GAs act iteratively, with each iteration serving to evaluate the generation of a data population, based on the fitness of each potential solution. These algorithms are often ideal for large scale search-related problems, but they also have their share of limitations, the most notable of which being premature convergence [44, 50]. Particle swarm optimization is similar to GAs in that PSO is an iterative process that is given a population of possible solutions. Unlike GA's, however, PSO involves a convergence of particles, each of which generates movement based on the most desirable positions available in the spaces that are to be searched. The overall method attempts to improve a solution with respect to a predetermined level

of quality [7]. This is mostly ideal for solving optimization problems with real values but comes at the cost of high computational runtime [39]. These global techniques serve as a unique method for WSN localization, either as a standalone technique [32] or in combination with other established methods [20, 33].

2.1.4 Maximum Likelihood Estimation

One of such methods that has been included with global optimization is the statistical technique of maximum likelihood estimation (MLE) [33], although MLE can also function on its own to achieve robust estimation ~~through~~ thorough utilization of low cost techniques such as RSS [28]. This novel category of estimator searches for the x value that maximizes the likelihood function [24], defined as

$$\hat{x}(k) = \arg \max P(\mathbf{z}|x) \quad (2.3)$$

In the context of WSNs, MLE is most commonly used to estimate distance values to aid in the geolocation of sensor nodes [24].

2.2 Data Fusion

Although most of the aforementioned data types, such as RSS and AOA, function well enough on their own, it is when an algorithm can potentially benefit from having multiple types of data that a unique challenge becomes apparent in that each measurement type can be vastly different from one another. To solve this problem of uniquely different data, the concept of data fusion becomes one of particular importance.

Data fusion is a valuable and streamlined process that involves obtaining many different types of data from a single common application and integrating them into a unified, consistent representation, under the primary goal of improving the accuracy

of a particular algorithm or technique [37]. In the event of such a large magnitude of complexity within the field of data fusion, a significant amount of creativity and elaboration is needed to solve such a problem. In the context of WSNs, many novel approaches to data fusion have been established, most notably the fusion of RSS and TDOA measurements [40], ~~including with the addition of wireless~~ networks [23].

[23] proposed the following matrix-based set of equations to determine a measurement vector based on a combination of RSS and TDOA measurements:

$$r = f(\mathbf{x}) + n \quad (2.4)$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, r = \begin{bmatrix} r_{rss,1} \\ \vdots \\ r_{rss,N} \\ r_{rss,w,1} \\ \vdots \\ r_{rss,w,M} \\ r_{tdoa,2} \\ \vdots \\ r_{tdoa,N} \end{bmatrix}, n = \begin{bmatrix} n_{rss,1} \\ \vdots \\ n_{rss,N} \\ n_{rss,w,1} \\ \vdots \\ n_{rss,w,M} \\ n_{tdoa,2} \\ \vdots \\ n_{tdoa,N} \end{bmatrix}, f(\mathbf{x}) = \begin{bmatrix} -aln(d_1) \\ \vdots \\ -aln(d_N) \\ -aln(d_{w,1}) \\ \vdots \\ -aln(d_{w,M}) \\ d_2 - d_1 \\ \vdots \\ d_N - d_1 \end{bmatrix} \quad (2.5)$$

For Equation 2.4, r denotes the measurement vector, $f(\mathbf{x})$ is a known nonlinear function of the source position \mathbf{x} , and n is the measurement error vector. In Equation 2.5, x and y are position coordinates, M and N are the numbers of Wi-Fi-based and non-Wi-Fi-based measurements, respectively, w denotes a Wi-Fi measurement, and

d represents distance. Let distance $d_i = d_1, d_2, \dots d_n$ be represented by

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (2.6)$$

The use of matrix-based notation becomes particularly useful in later proposed data fusion techniques.

2.3 Dempster-Shafer Theory

Dempster-Shafer (DS) Theory is an innovatively different method of data fusion that can be best summed as a divergence of Bayesian combinational probability, particularly with regards to combinational factors. While Bayesian statistics rely primarily upon combinations of internal probability factors within a system, usually through the use of propositions or random variables, DS Theory is contingent upon external evidence factors, each one consisting of a range of data in which the desired output value can be found and a probability of confidence that the data range is in fact reliable. DS Theory is meant to model information that, under more traditional Bayesian methods, would be considered incomplete [10].

2.3.1 Bayesian Origins

Bayesian probability is a technique in which two or more internal probabilistic factors are combined ~~form~~ a single hypothesis or inference, often using random variables to fulfil unknown quantities in the context of a problem [16]. If only two factors, X and Y , are considered, then Bayesian inference can be defined as

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} \quad (2.7)$$

where $P(X|Y)$ denotes the probability of Y occurring given X , $P(Y)$ represents the probability of Y alone, $P(x)$ is the probability of X individually, and $P(Y|X)$ is the probability of X given Y .

Bayesian estimation has been established in a variety of applications, including wind speed probability distribution and battery lifetime analysis [15].

2.3.2 Theoretical Foundation

From a theoretical context, Dempster-Shafer Theory can be best described as a generalization of Bayesian probability theory, forming a divergence based on the concept of ignorance. Typical Bayesian approaches assume an ignorance quantity of zero, or equivalently 100% confidence, for any probabilistic value, which is usually appropriate when a probability factor is internal. Because DS Theory is based upon external factors, however, this property can no longer be unconditionally assumed, and thus a plethora of evidence factors are used to determine all possible outcomes in a given scenario (e.g. the distance between a sensor node and an anchor node in a WSN), each with its own degree of confidence that serves as a weight of evidence [41]. The finite set of all possible mutually exclusive values is known as the frame of discernment and is denoted by Θ . From there, the union of all subsets is given by the power set 2^Θ .

2.3.2.1 Basic Probability Assignment

One of the most significant fundamental concepts of DS Theory is the basic probability assignment (BPA) function, which is essentially the equivalent of the random variable in Bayesian probability theory and is also known as the belief variable [57], denoted as m . Suppose A_1, \dots, A_n are all possible sets within a frame of discernment,

noting that $A_i \in 2^\Theta$. Then

$$m : 2^\Theta \rightarrow [0, 1], \sum_{i=1}^n m(A_i) = 1, m(\emptyset) = 0 \quad (2.8)$$

where \emptyset denotes the empty set. Each set A_i takes the form of the interval $([\underline{x}, \bar{x}])$, denoting the lower and upper bounds of an evidence factor, respectively [57]. Each BPA m consists of two vital properties: a range, as established by a set A , and a degree of confidence μ that lies within the range of $[0, 1]$ [55]. Thus, ignorance can be defined as simply $1 - \mu$. At certain times, manually adding new BPAs can be an exhaustive process, in which case sampling from a distribution becomes an ideal solution. This process, also known as discretization, involves generating n discrete samples based on a lesser amount of initialized BPAs, which collectively form a cumulative density function (CDF) of BPA structures [56].

2.3.2.2 Aggregation

Once two or more evidence factors from different sources exist in the same problem, then typically the aggregation of BPAs becomes essential [57]. ~~There are many~~ possible methods ~~with~~ which this task can be accomplished, but the scope of this research focuses primarily upon Dempster's rule of combination. Suppose m_1 and m_2 denote two BPA functions that are to be aggregated. Under Dempster's rule, a new BPA function $m_1 \oplus m_2 : 2^\Theta \rightarrow [0, 1]$ is defined as [38]

$$[m_1 \oplus m_2] = \frac{\sum_{A \cap B = y} m_1(A) m_2(B)}{1 - \sum_{A \cap B = y} m_1(A) m_2(B)} \quad (2.9)$$

on the condition that $y \neq \emptyset$. Otherwise $[m_1 \oplus m_2] = 0$.

2.3.2.3 Belief and Plausibility

Belief and Plausibility are two of the most vital core functions of an aggregate BPA, as they represent the best and worst case scenarios, respectively, in a frame of discernment and are defined as

$$Bel(B) = \sum_{A_i \subset B} m(A_i) \quad (2.10)$$

$$Pl(B) = 1 - \sum_{A_i \cap B = \emptyset} m(A_i) \quad (2.11)$$

where $m(A_i)$ denotes the BPA mass function for a set A_i .

2.3.2.4 Prediction of Distance by Expected Value

In the general area of statistics, expected value is used as a method of predicting the anticipated outcome of an event. Let x_i be a potential outcome in the set of n possible outcomes x_1, x_2, \dots, x_n and let p_i be the probability of x_i occurring. Then the expected value is defined as

$$E(x) = \sum_{i=1}^n x_i p_i \quad (2.12)$$

DS Theory contains a similar method of calculating expected value but has a fundamental difference in that the output is a range of values rather than a single value. This is because the lower and upper bounds in a BPA are meant to be addressed separately. Recalling that μ refers to the degree of confidence, which essentially functions as a probability value, and that the lower and upper bounds of a distance BPA are d_{min} and d_{max} , respectively, the DS expected value is thus defined as [27]

$$Val_{Ex} = \sum_{BPAs} \begin{bmatrix} \mu d_{min} & \mu d_{max} \end{bmatrix} \quad (2.13)$$

The resultant pair of outputs are then utilized to predict the most likely range of distance between a node's county and a monitor to which the originally given input distance values belong.

2.3.3 Applications

This framework has already been established in a variety of scientific applications, including fault diagnosis [22], fault tree analysis [19], decision analysis [11], text categorization [9], ~~and~~ localization of robot sensors [48]. ~~Applications that have combined DS Theory with neural network based modelling include~~ multi-sensor information fusion [21] ~~and~~ fusion of GPS and Inertial Navigational System (INS) data [47]. The general field of WSNs has also explored the potential of DS Theory, such as classification fusion of moving vehicles with a WSN [8] and network security by means of trust evaluation [54]. Prior to the validation of this research, however, the use of DS Theory for the specific topic of node positioning in WSNs has remained widely untapped.

Chapter 3

Introductory DS-Based Localization

Technique Using Expected Value

Outputs of Basic Probability

Assignments

This verbose, introductory technique provides what can be regarded as a hand holding approach to the previously untapped, unexplored terrain that forms when merging ~~DST~~ with WSN localization. Because the overall goal of this study is to predict an evidence-based location, this study is based on the expected value function of DS Theory and uses basic probability assignments (BPAs) of distance ranges to predict an expected range of radii between a given base station and the potential location of a sensor node. To enhance accuracy, all BPAs are used as distributions for a normalized inverse sample of BPAs, which are then aggregated into a single data structure to be used in all further evaluation. When these radii are obtained for all working monitors, an area of feasible positions is obtained, which can be used to determine whether a particular node in a particular county matches the given set of measurements (RSS, AOA, operating condition, or a combination thereof) corre-

sponding to aforementioned distance BPAs. This approach produces a moderately high range of accuracy, with results ranging from 78-87%, this approach serves as an adequate start to exploring DS Theory and an easily understandable visual approach to location prediction, but a relatively high computational cost leaves this method to be just that, a starter method.

3.1 Introduction

Many methods of predictive data modelling exist in the area of WSN localization, including artificial neural networks (ANNs) and data mining. ANNs are advantageous in their inherent ease of use in simultaneously processing multiple inputs and outputs and modelling highly accurate connection weights thereof using extensive training methods, but they also have their share of drawbacks, as some applications may require significant amounts of training time or fall victim to over-learning and under-learning [51]. In other words, ANN-based data modelling is ideal for cases in which high accuracy is of greatest importance while high speed and low computational cost are often regarded as lower priority.

Data mining tends to follow a reversed trend, in which speed and efficiency are regarded as optimally important even at the cost of accuracy. In the area of WSN localization, prior established research methods typically utilize data mining with the end goal of fast data collection [3]. While both methods may appear to be vastly different from one another due to their opposing polarities in the balance of speed and accuracy, each approach holds a vital common characteristic in that they each require dedicated stages for training and validation.

Dempster-Shafer Theory differs greatly from typical data prediction algorithm in two fundamental ways. One is that no training stage is needed to achieve valid and complete results, even when the data that is to be modelled is in itself considered to be

incomplete. The second is that DS Theory contains the potential to achieve results of both high accuracy and low cost, meaning that one attribute can be achieved ~~without taking away from~~ the other.

The remainder of this chapter is organized as follows. ~~Section 3.2~~ expands upon ~~many of the previous subsections~~ by providing an brief theoretical background of DS Theory, and WSN localization **techniques**. Section 3.3 provides an overview of how the research study was conducted, including generation of the relevant WSN data, the properties and parameters of the sensor network, and the determination of the method's accuracy. Section 3.4 presents the end results, and lastly, conclusions are drawn in Section 3.5.

3.2 Dempster-Shafer Theory

Recall from the literature in Section 2.3.2.1 that one of the most significant fundamental concepts of DS Theory is the basic probability assignment (BPA) function, which is essentially the equivalent of the random variable in Bayesian probability theory and is also known as the belief variable [57], denoted as m . Suppose A_1, \dots, A_n are all possible sets within a frame of discernment, noting that $A_i \in 2^\Theta$. Then

$$m : 2^\Theta \rightarrow [0, 1], \sum_{i=1}^n m(A_i) = 1, m(\emptyset) = 0 \quad (3.1)$$

where \emptyset denotes the empty set. Also recall that if two or more evidence factors from different sources exist in the same problem, then the BPAs are required to be aggregated [57]. Under Dempster's rule of combination, suppose m_1 and m_2 denote two BPA functions that are to be aggregated. Then a new BPA function $m_1 \oplus m_2 :$

$2^\Theta \rightarrow [0, 1]$ is defined as

$$[m_1 \oplus m_2] = \frac{\sum_{A \cap B = y} m_1(A)m_2(B)}{1 - \sum_{A \cap B = y} m_1(A)m_2(B)} \quad (3.2)$$

on the condition that $y \neq 0$. Otherwise $[m_1 \oplus m_2] = 0$. Resultantly, belief and plausibility can be defined as

$$Bel(B) = \sum_{A_i \subset B} m(A_i) \quad (3.3)$$

$$Pl(B) = 1 - \sum_{A_i \cap B = \emptyset} m(A_i) \quad (3.4)$$

where $m(A_i)$ denotes the BPA mass function for a set A_i .

3.2.1 Linear Representation

A BPA contains three vital properties: a lower bound \underline{x} , an upper bound \bar{x} , and a degree of confidence μ . Hence, a linear representation of a BPA as a vector of these three properties becomes a highly convenient organizational method. In such a case, the resultant BPA m would be defined as

$$m = \begin{bmatrix} \underline{x} & \bar{x} & \mu \end{bmatrix} \quad (3.5)$$

Likewise, in order to effectively represent an aggregate BPA, representation of the individual assignments as a matrix is also beneficial. Suppose $m_1^l, m_2^l, \dots, m_n^l$ represent

all BPAs in Θ such that

$$m = \begin{bmatrix} m_1^l \\ m_2^l \\ \vdots \\ m_n^l \end{bmatrix} = \begin{bmatrix} \underline{x}_1^l & \bar{x}_1^l & \mu_1^l \\ \underline{x}_2^l & \bar{x}_2^l & \mu_2^l \\ \vdots & \vdots & \vdots \\ m_n^l & \bar{x}_n^l & \mu_n^l \end{bmatrix} \quad (3.6)$$

and $\bar{x}_1^l < \bar{x}_2^l < \dots < \bar{x}_n^l$.

3.3 Localization Algorithm

Many aspects of the localization technique are based upon a similar approach proposed in [31], in terms of selection and measurement of various measurement types, particularly received signal strength (RSS), angle of arrival (AOA), and operating condition (standby). Using the data from said features for the particular process of node positioning, however, follows a significantly different approach, based upon DS expected value, as defined in Subsubsection 2.3.2.4..

3.3.1 Measurement Types

Before RSS, AOA, and operating condition can be mathematically defined, the distance between any sensor node and any anchor node must also be evident. Suppose the coordinate (x_n, y_n) refers to the position of a sensor node and (x_m, y_m) is the position of a monitor, or anchor node. Then, the distance d can be defined as

$$d_{actual} = \sqrt{(x_m^2 - x_n^2) + (y_m^2 - y_n^2)} \quad (3.7)$$

where all aforementioned variables are measured in meters. For RSS calculation, many formulae exist for exact calculation, but all share the property of having an inverse square proportionality with distance. Hence, in the context of this research, RSS is simply defined as the inverse square of distance, or $RSS = 1/d_{actual}^2$.

Due to the generic nature of this interpretation, no specific unit is needed, as long as the generic unit contains the factor of m^{-1} . AOA, however, is defined in a more specific context as

$$AOA = \begin{cases} \tan^{-1}\left(\frac{y_n - y_m}{x_n - x_m}\right) & \text{if } y_n > y_m \text{ and } x_n > x_m \\ \tan^{-1}\left(\frac{y_n - y_m}{x_n - x_m}\right) + 180 & \text{if } y_n \neq y_m \text{ and } x_n < x_m \\ \tan^{-1}\left(\frac{y_n - y_m}{x_n - x_m}\right) + 360 & \text{if } y_n < y_m \text{ and } x_n > x_m \end{cases} \quad (3.8)$$

with resultant units in degrees. Standby is a unique type of distance measurement in which a sensor node with this feature enabled is detected by a dedicated standby node, which measures the distance thereof by inverse square root RSS calculation. Let (x_{sb}, y_{sb}) be the location of the standby node. Then the standby distance is defined mathematically as

3.3.2 Data Fusion

In order to fuse any of the varied measurement types (RSS, AOA, standby) together, each measurement must be converted into a common value, which in this case is best represented as the distance from a measurement set to a monitor, as determined by inverse square root RSS calculation. Due to this method of distance calculation, the experimental setup will require RSS to be enabled at all times. If a reputable distance or range of distances can be found for all available monitors, then a node's location or respective county can be estimated by an enhanced process of trilateration, which enhancement is based on the effects of. The challenge associ-

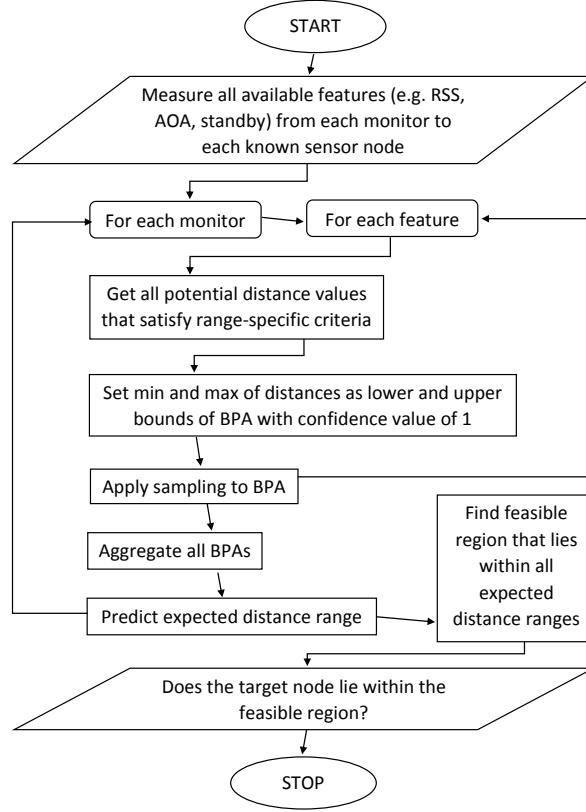


Figure 3-1: Flowchart of DS localization algorithm.

ated with reaching such a goal becomes the task of estimating such a distance range for each monitor. Thus, the need for effective organization and evaluation of optimal distance ranges is where Dempster-Shafer Theory becomes a mechanism of primary interest.

Because external evidence factors are the primary means by which DS-based estimation can occur, the necessity arises to obtain distances and measurement sets that are independent of those given as functional inputs. This distinction will be explained at greater length in Subsection 3.4.2 by means of test data and full data sets. Because such data is external, the amounts and ranges of measurements must be

reduced based on feature-specific criteria. Suppose these range values, r , are defined as

$$r = \begin{bmatrix} r_{RSS} \\ r_{AOA} \\ r_{SB} \end{bmatrix} \quad (3.9)$$

where $r_{RSS} > 1$, $0 < r_{AOA} < 1$, and $r_{SB} = -1$, the latter value indicating that there is no range for standby. Then, if the inverse of a given feature measurement is less than the range r_i , the lower and upper limits of such ranges are defined as $[0, 1/r_i]$. Otherwise, if the value is greater than or equal to r_i , the limits are $[\frac{s_i}{r_i s_i + 1}, s_i]$, where s_i is the particular feature measurement and i is the gesture type. The next step is to find external distance values from external feature measurements that lie within the corresponding range. Once all of such distance values have been filtered, the minimum and maximum of the remaining values become the lower and upper bounds of the resultant BPA. Because all BPAs will aggregate in a later step, the confidence value can simply be 1 for the time being. Thus

$$m_i = \begin{bmatrix} d_{min} & d_{max} & 1 \end{bmatrix} \quad (3.10)$$

Once a BPA is formed for each active feature, aggregation can then occur. To smoothen the transition between BPAs however, sampling each m_i by a consistent function handle, such as inverse normalization, is recommended. The resultant aggregate BPA for n samples thus becomes

$$\lambda = \begin{bmatrix} \lambda_{RSS} \\ \lambda_{AOA} \\ \lambda_{SB} \end{bmatrix} \quad (3.11)$$

where

$$\lambda_{RSS} = \begin{bmatrix} m_{1,RSS} \\ m_{2,RSS} \\ \vdots \\ m_{n,RSS} \end{bmatrix}, \lambda_{AOA} = \begin{bmatrix} m_{1,AOA} \\ m_{2,AOA} \\ \vdots \\ m_{n,AOA} \end{bmatrix}, \lambda_{SB} = \begin{bmatrix} m_{1,SB} \\ m_{2,SB} \\ \vdots \\ m_{n,SB} \end{bmatrix} \quad (3.12)$$

This is under the assumption that all three features are enabled. If one or more features are unavailable, then their corresponding λ values are simply omitted. In the context of this setup, all confidence values are equal, and hence all μ values are simply $\frac{1}{3n}$. If fewer than three features are enabled, then μ is $\frac{1}{fn}$, where f is the number of features. With all aggregation intact, the next step is the actual data prediction process.

3.4 Methods and Materials

A varied plethora of WSN simulation scenarios was created and analysed using the MATLAB scientific programming environment [34]. The resultant software package includes all facets needed for experimental simulation, including selectively randomized generation of data, complete implementation of the algorithm proposed in the previous section, and extensive calculation of the algorithm's accuracy.

An overview of the default simulation parameters in the experimental setup is as follows:

1. 100 nodes deployed throughout a rectangular region of 1000-by-1000 meters
2. Three anchor nodes (both options tested separately in identical networks)
3. Anchor nodes in two-monitor setups located at [150,100] ~~meters~~, [500,100] ~~me-~~
~~ters~~, and [850,100] meters

4. Standby node placed at [500,800] meters; location of fusion center is negligible
5. One-to-one mapping of counties and nodes
6. Range criteria of 20, 100, and -1 for RSS, AOA, and standby, respectively
7. Setup tested for RSS+standby, RSS+AOA, and RSS+AOA+standby
8. Discretization consisting of 10 samples as an inverse normalized distribution
9. Experimental results as the average of ten independent experiments in which the WSN is regenerated upon every trial
10. Zero noise factor and zero anchor node positioning error assumed

Details of the variables and terminologies above are explained in the proceeding subsections.

3.4.1 WSN Setup

The sensor network in this simulation consists of three vital components: the sensor nodes, the anchor nodes, and the fusion center. The sensor nodes comprise the vast majority of the components in this setup, totalling 100 out of 105 nodes in a three-monitor setup. The anchor nodes, also known as monitors or cluster heads, are placed strategically throughout the WSN to measure the available properties of each and every sensor node, such as RSS, AOA, and operating condition (standby), and transmit said data to a fusion center node, which processes all information using the algorithms and methods given in the following subsections. In other words, the fusion center assumes the role of the entire localization software, minus the data generation stage. In addition, a standby node measures the distance of any sensor nodes that are in standby mode. The complete WSN setup is presented in [Figure 3-2](#).

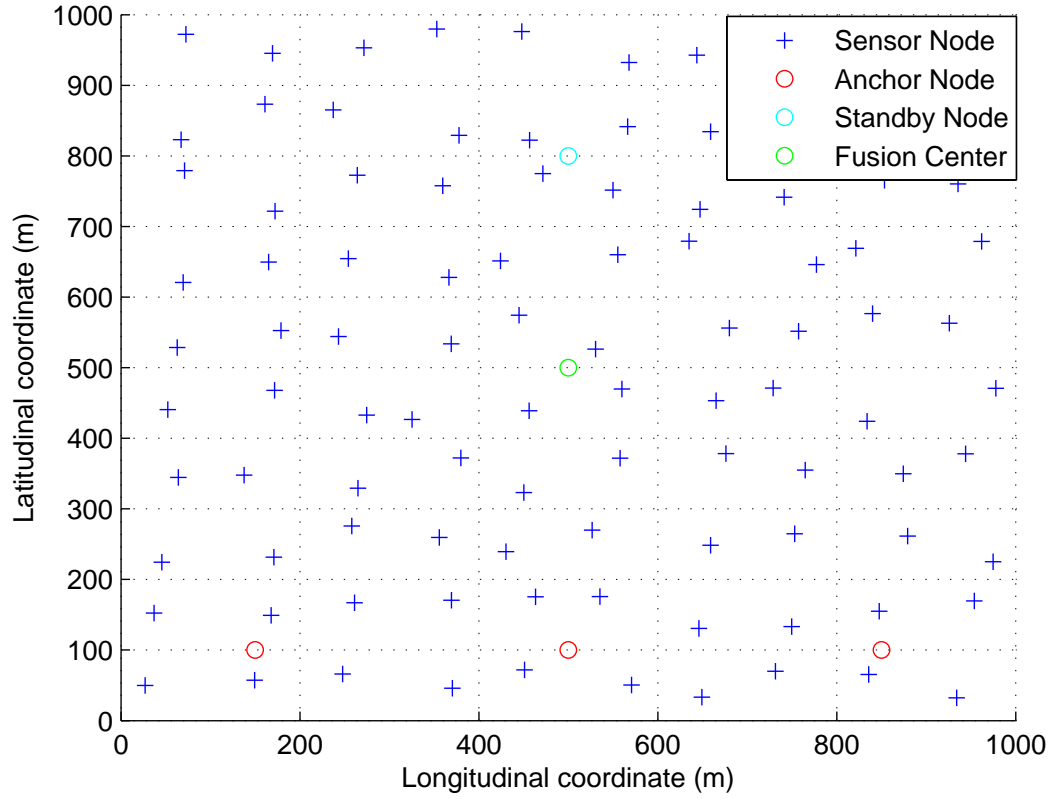


Figure 3-2: WSN of 100 sensor nodes, 3 anchor nodes, standby node, and fusion center.

3.4.1.1 Data Value Ranges

In this network setup, the origin of the coordinate system is positioned at the lower left hand corner of the map, as indicated in Fig. 3-2. County names and monitor names are given as numbers ranging from one to the total number of counties and monitors available, respectively. All monitors are horizontally aligned by having a common latitudinal coordinate of 100 meters as well as longitudinal coordinates from 150 to 850 meters. Ranges of sensor node coordinates are based on the size of the WSN and can be placed anywhere more than 30 meters from any border. Based on the criteria of the latter two data types, distances between sensor nodes and monitors can vary from 40 to 1360 meters. ~~Because RSS is merely the inverse square root of distance, it can range from 1/2000000 to 1/1600.~~ AOA can be any positive number of

degrees up to 360, and the operating condition, or standby distance, can range from 0 to 920 meters.

3.4.2 Data Generation

A multitude of sensor nodes, counties to which each node belongs, and various measurements of nodes relative to various monitors were written to and read from a single comma separated value (CSV) file known as the full data set. Also included in the file are monitor locations, county locations, and distances from any sensor node to any monitor, calculated from the latter two locations. Each sensor node encompasses multiple rows of data, in which each row corresponds to a particular monitor observing the node and its measurements. Each category of data, such as county number, distance, or a type of measurement, takes the form of a column. The full data set contains one hundred samples of sensor node data for every county in the WSN.

A second CSV file known as the test data set contains the county locations and measurements that are to be tested against the full data set. The test data contains only one sensor node per county. The location and measurement values of the array are different from any of the nodes of the corresponding county in the full data. The test data set contains 300 total rows of data while the full data set contains 30,000 rows.

Each sample of data in each file contains ten columns, determined as follows:

1. Anchor node number
2. X coordinate of anchor node
3. Y coordinate of anchor node
4. County number

5. County x coordinate
6. County y coordinate
7. Distance from county to anchor node
8. RSS value
9. AOA value
10. Standby value

Worth noting is that the first three values are intended to be treated independently of measurement data in accordance with the algorithm objectives provided in Subsection 3.4.4. In addition, the seventh column is optional to the algorithm, as any distance value can be calculated directly from the corresponding RSS value due to a consistent inverse proportionality between RSS and squared distance.

3.4.3 IPP Toolbox

To aid in the MATLAB implementation of DS Theory, the open source Imprecise Probability Propagation (IPP) Toolbox [27] was obtained, released under the GNU Public License (GPL), and used to execute all DS-specific functions in the simulation, such as formation of BPAs, aggregation and sampling of BPAs, and calculation of plausibility.

3.4.4 Algorithm Execution

The algorithm was programmed by following the flowchart provided in Figure 4-1 and refining the method in the form of the pseudocode provided in Algorithm 1. The pseudocode was then converted into MATLAB-specific syntax and thoroughly tested

```

1: test_data  $\leftarrow$  read measured feature values, county name and location, and mon-
   itor locations
2: full_data  $\leftarrow$  read potential feature values for the county and monitors
3: range  $\leftarrow$  feature-specific range values for each active feature
4: for all monitors do
5:   for all active features do
6:     distances  $\leftarrow$  all distances corresponding to feature measurements in
       full_data that satisfy range
7:     BPA_per_feat  $\leftarrow$  [min_distance, max_distance, 1]
8:     apply sampling to BPA
9:   end for
10:  aggregate BPAs
11:  [min_dist, max_dist]  $\leftarrow$  expected value range of BPA
12: end for
13: feasible_points  $\leftarrow$  all points that lie within all distance range constraints
14: return whether feasible_points contains county location

```

Algorithm 1: Pseudocode of first proposed DS localization algorithm

to ensure proper functionality and accurate representation of the proposed method.

A visual example of the technique in action is presented in Figure 3-3.

An important distinction to note is that of the two data files presented in Subsection 3.4.2. In the context of the localization algorithm, the full data set is used only for obtaining distance values for corresponding measurements that fit given range criteria in the distance collection stage, which helps to establish the lower and upper bounds of a BPA. The rest of the algorithm utilizes the test data set, particularly for establishing input values for county locations and measurement sets.

3.4.5 Desired Inputs and Output

A summary of the intended inputs and outputs are given in Fig. 3-4. This algorithm seeks to answer the following question: given a set of different measurement types and the location of a given county, do the measurements belong to said county?

3.4.6 Accuracy Calculation

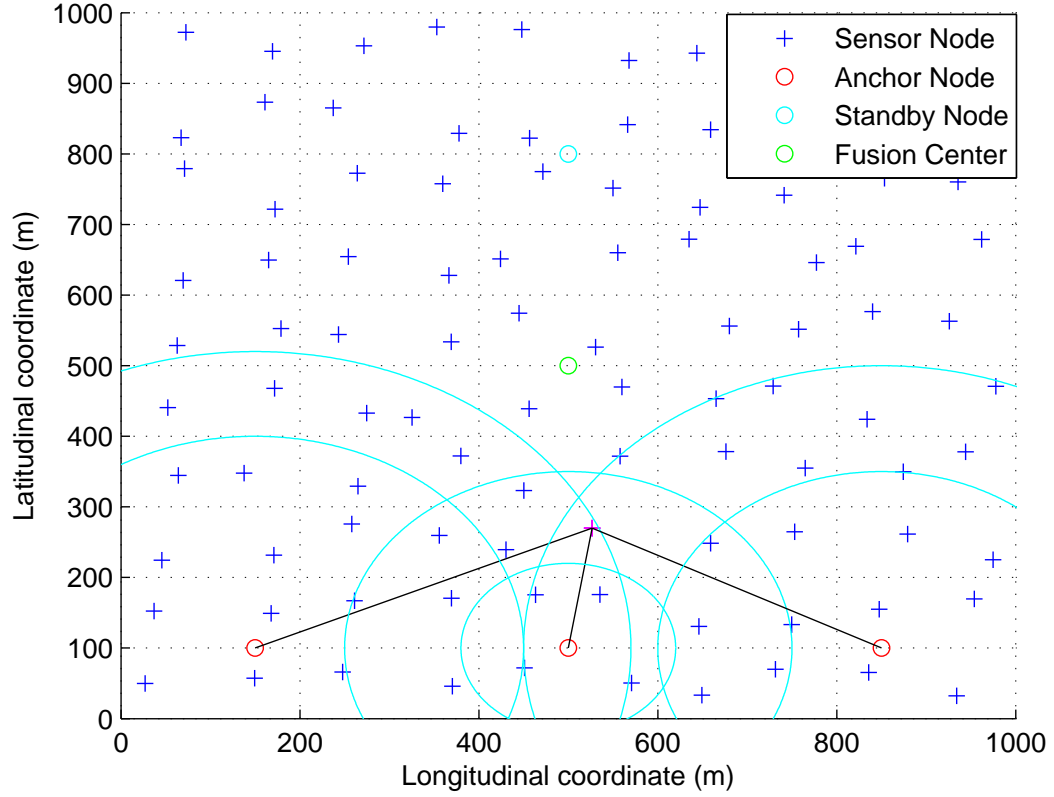


Figure 3-3: Example of expected-value-based DS algorithm in execution.

To determine the effectiveness of this technique addressing the previous subsection, accuracy is calculated as follows. Inputs consist of a single county location and a single list of measurements (as well as their respective distances from a monitor), and a binary decision of one for yes or zero for no serves as the sole output. To test this method of accuracy, every combination of county and measurement set is run through the algorithm as the two inputs, while each output is given in a square matrix whose side length is equal to both the number of counties and the number of measurement sets, where each row corresponds to each measurement set and each column corresponds to each county. This approach is outlined as pseudocode presented in Algorithm 2. Finally, the percent accuracy is calculated by comparing the resultant matrix against an identity matrix of the same dimensions. The resultant

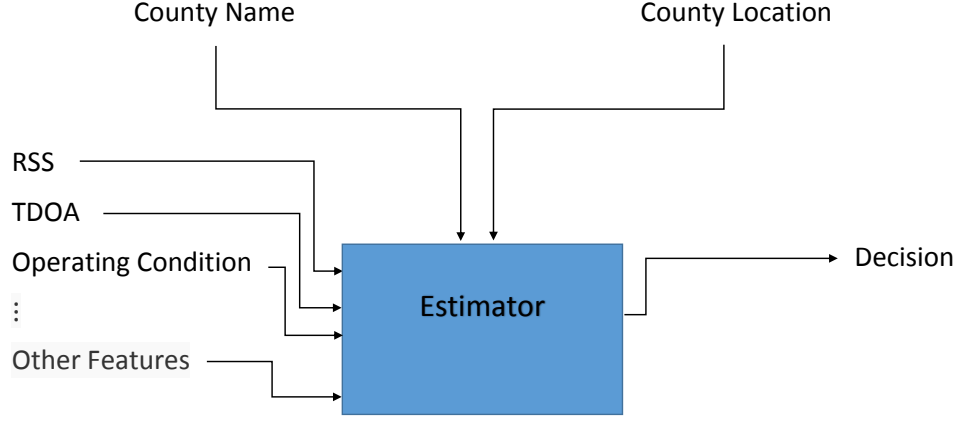


Figure 3-4: First proposed classifier.

formula is as follows:

$$Accuracy = 100 * \frac{cells_{matching}}{cells_{total}} \quad (3.13)$$

3.5 Experimental Results

The number of possible features was determined by addressing the three features (RSS, AOA, operating condition) as a three bit binary combination. Typically, this would result in $2^3 = 8$ possibilities, but because the algorithm is dependent upon RSS to reverse engineer a measured distance, the RSS bit must always be on, thereby resulting in $2^3 - 4 = 4$ possible combinations. In addition, the algorithm proved to be entirely ineffective for single-feature setups, with exactly zero percent accuracy,


```

1:  $test\_data \leftarrow$  read measured feature values, county name and location, and mon-
   itor locations
2:  $full\_data \leftarrow$  read potential feature values for the county and monitors
3:  $range \leftarrow$  feature-specific range values for each active feature
4: for all measurement sets do
5:   for all counties do
6:      $decision_{per\_county\_per\_mon} \leftarrow$  outputs of Algorithm 1 using  $test\_data$ ,
        $full\_data$ , and  $range$  as inputs
7:   end for
8: end for
9: return  $decision$ 

```

Algorithm 2: Pseudocode of DS testing algorithm

Table 3.1: Simulation results.

Feature Description	Test Accuracy (%)	Runtime (s)
RSS, SB	79.97	12736.30
RSS, AOA	78.41	12731.71
RSS, AOA, SB	87.36	12731.49

and hence RSS as a sole feature were omitted from the table, thereby leaving only $2^3 - 4 - 1 = 3$ possible combinations.

3.5.1 Accuracy

The second column of table 3.1 shows the average accuracy values for all ten generations. As expected, the availability of all three features resulted in the highest accuracy. Consequently, feature sets without RSS were removed because RSS is required to get a corresponding distance value, due to the inverse relationship between the latitude and the square of the former.

3.5.2 Computational Runtime

All trials were run on consistent computer hardware and software, including a standard Windows version of MATLAB on a quad core 2.60 GHz processor and 8.0

Table 3.2: Comparison of accuracy among different localization techniques.

Algorithm	PSO	WSLA	WSRA	MLE	DS1
Accuracy (%)	71	90	92	93	87
Runtime (μs)	114570	7800	9700		12733

GB of RAM. The third column of Table 3.1 shows the average runtime results for all ten generations. The significance of these values are addressed in the following subsection.

3.5.3 Comparisons to Other Established Algorithms

To gain further perspective, comparisons were made to four prior established localization methods, including Particle Swarm Optimization (PSO), maximum-likelihood estimation (MLE), a weighted search-based localization algorithm (WSLA), and a weighted search-based refinement algorithm combined with the latter technique (referred by [14] as WSLA+WSRA but reported here as simply WSRA). For the compared methods, the sensor network that was simulated consisted of 100 sensor nodes with eight anchor nodes, and each experiment consisted of 30 independent trials averaged together.

3.5.3.1 Accuracy

The results are based on those presented in [14] under the assumptions of no positioning error on anchor nodes and no noise factor. The accuracy was determined by reported error values under aforementioned conditions, which were then subtracted from 1 and converted to a percentage. Because error values were reported as a plot rather than in exact numerical form, estimation based on eyeing and measuring the graphs was required to obtain error values; hence, to minimize any potential reporting error, accuracy of each algorithm was reported as a whole number percentage. For the

proposed technique (labeled as DST1), accuracy was determined by averaging results for all four feature combinations, rounded to the nearest whole number percentage for consistency. The second row of 3.2 shows all reported accuracies.

3.5.3.2 Computational Runtime

Average run time per node for a three-monitor WSN setup was compared with three of the four aforementioned techniques, based on data given in [14]. The competing data was provided in a table of average CPU time per node with no anchor node positioning error and no noise factor. Hardware specifications and MLE runtime were not reported. All method runtimes, sans that of MLE, are reported in the third row of Table 3.2. Results for both accuracy and runtime indicate that the proposed method contains significant advantages over PSO but still pale in comparison to the other reported methods.

3.6 Conclusions and Future Work

This chapter presented a new approach to wireless sensor network localization through incorporation of Dempster-Shafer Evidence Theory and the use of its ~~re-~~
~~sultant~~ expected value function. This new approach serves as an adequate start to exploring DS Theory by providing a moderately high accuracy and an easily understandable visual approach to location prediction, but a modest accuracy and a high computational cost ~~this~~ method to be just that, a starter method. Luckily, the following approach presented in the next chapter addresses and resolves all shortcomings incurred by this method at the ~~moderate~~ expense of less verbosity.

Chapter 4

Novel Low Cost, Highly Accurate Method of Localization Using DS-Based Plausibility

While the expected value function provided an adequate, introductory approach to WSN localization within the realms of Dempster-Shafer theory, many other DS functions served as potential contenders to drive the core of the localization technique. Upon preliminary testing of other functions and their outputs' correlations with measured distances, the concept of plausibility was proven to be the most effective function in terms of both high accuracy and low cost. Leaving most other components of the algorithm to be the same as in the previous chapter, a rigorous set of testing determined this new method to be up to 98% accurate in a fraction of the time required for mere completion of the previous technique.

The remainder of this chapter is organized as follows. Section 4.1 expands upon the theoretical background of DS Theory established in Section 3.2 as well as an overview of the revised algorithm. Section 4.2 provides an overview of how the research study was conducted, emphasizing both the similarities to and differences from the experimental setup in Section 3.3. Section 4.3 presents the end results of the ex-

periment in a variety of scenarios as well as a brief discussion of the impact thereof. Lastly, conclusions are drawn in Section 4.4.

4.1 Dempster-Shafer Theory

Recall from the previous chapter that

$$m = \begin{bmatrix} m_1^l \\ m_2^l \\ \vdots \\ m_n^l \end{bmatrix} = \begin{bmatrix} \underline{x}_1^l & \bar{x}_1^l & \mu_1^l \\ \underline{x}_2^l & \bar{x}_2^l & \mu_2^l \\ \vdots & \vdots & \vdots \\ m_n^l & \bar{x}_n^l & \mu_n^l \end{bmatrix} \quad (4.1)$$

The addition of the superscript l now indicates that $\bar{x}_1^l < \bar{x}_2^l < \dots < \bar{x}_n^l$. This is of particular importance, as this sorted order of lower bounds can be used to define belief from a more specific, matrix-oriented context. Let the superscript u indicate that contents of m are arranged in order of increasing upper bound. Recall that μ is the degree of confidence in each BPA and thus can be defined as $\mu_1^u, \mu_2^u, \dots, \mu_n^u$. Then belief can be represented as

$$m = \begin{bmatrix} m_1^u \\ m_2^u \\ \vdots \\ m_n^u \end{bmatrix} = \begin{bmatrix} \underline{x}_1^u & \bar{x}_1^u & \mu_1^u \\ \underline{x}_2^u & \bar{x}_2^u & \mu_2^u \\ \vdots & \vdots & \vdots \\ m_n^u & \bar{x}_n^u & \mu_n^u \end{bmatrix} \quad (4.2)$$

and $\bar{x}_1^u < \bar{x}_2^u < \dots < \bar{x}_n^u$. Let the superscript u indicate that contents of m are arranged in order of increasing upper bound. Recall that μ is the degree of confidence in each BPA and thus can be defined as $\mu_1^u, \mu_2^u, \dots, \mu_n^u$. Then belief can be represented

as

$$Bel = \begin{bmatrix} \bar{x}_1^u & \mu_{1,new}^u \\ \bar{x}_2^u & \mu_{2,new}^u \\ \bar{x}_3^u & \mu_{3,new}^u \\ \vdots & \vdots \\ \bar{x}_n^u & \mu_{n,new}^u \end{bmatrix} \quad (4.3)$$

where

$$\mu_{1,new}^u = \mu_1^u \quad (4.4)$$

$$\mu_{2,new}^u = \mu_2^l + \mu_{1,new}^u \quad (4.5)$$

$$\mu_{3,new}^u = \mu_3^u + \mu_{2,new}^u \quad (4.6)$$

and

$$\mu_{n,new}^u = \mu_n^u + \mu_{(n-1),new}^u \quad (4.7)$$

Now suppose m is rearranged and $m_1^u, m_2^u, \dots, m_n^u$ renamed such that

$$m = \begin{bmatrix} m_1^l \\ m_2^l \\ \vdots \\ m_n^l \end{bmatrix} = \begin{bmatrix} \underline{x}_1^l & \bar{x}_1^l & \mu_1^l \\ \underline{x}_2^l & \bar{x}_2^l & \mu_2^l \\ \vdots & \vdots & \vdots \\ m_n^l & \bar{x}_n^l & \mu_n^l \end{bmatrix} \quad (4.8)$$

where $\underline{x}_1^l < \underline{x}_2^l < \dots < \underline{x}_n^l$, where the superscript l indicates that contents of m are now arranged in order of increasing lower bound. Hence, all conditions are available

to define plausibility as

$$Pl = \begin{bmatrix} \underline{x}_1^l & \mu_{1,new}^l \\ \underline{x}_2^l & \mu_{2,new}^l \\ \underline{x}_3^l & \mu_{3,new}^l \\ \vdots & \vdots \\ \underline{x}_n^l & \mu_{n,new}^l \end{bmatrix} \quad (4.9)$$

where

$$\mu_{1,new}^l = \mu_1^l \quad (4.10)$$

$$\mu_{2,new}^l = \mu_2^l + \mu_{1,new}^l \quad (4.11)$$

$$\mu_{3,new}^l = \mu_3^l + \mu_{2,new}^l \quad (4.12)$$

and

$$\mu_{n,new}^l = \mu_n^l + \mu_{(n-1),new}^l \quad (4.13)$$

In other words, the top and bottom rows of the belief matrix represent the least and most believable outcomes, respectively. The latter is the row of greatest importance, where \bar{x}_n^u is the value of the most believable outcome and $\mu_{n,new}^u$ is the belief probability, which in this case is always 100% or 1 since it is the sum of all confidence values. Similarly, the top and bottom rows of the belief matrix respectively represent the least and most plausible outcomes. Also likewise to belief, the most believable row is of most significance, as \underline{x}_n^l signifies the most plausible outcome and $\mu_{n,new}^l$ is another 100% probability. Due to these powerful methods of outcome prediction, development of the proposed localization technique is based extensively on the experimental exploration of these two functions.

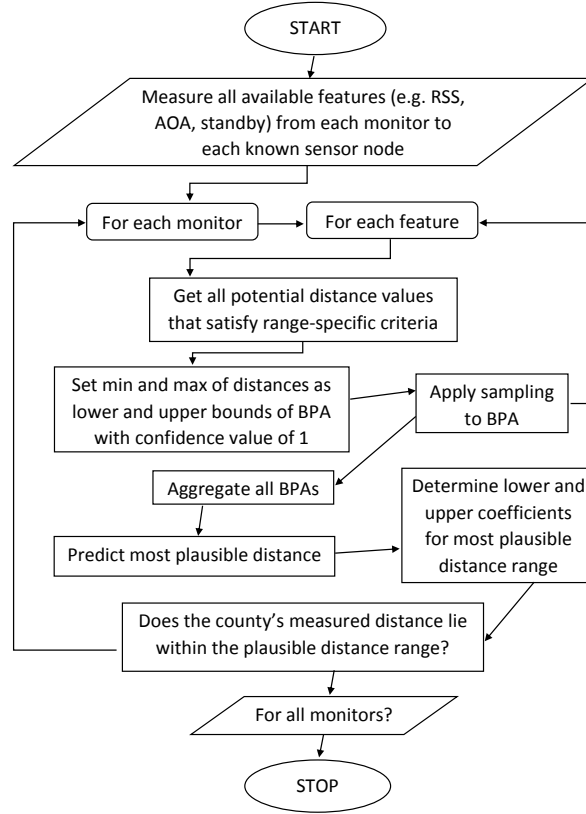


Figure 4-1: Flowchart of DS localization algorithm

4.1.1 Localization Algorithm

The localization algorithm works mostly in the same way as in the previous chapter but with expected value replaced with plausibility. Because Dempster-Shafer Theory is a new concept in the area of WSN localization, a clear relation between actual distance and believed or plausible distance was not available in any found literature. Thus, some preliminary experimentation was needed to understand how *Bel* and *Pl* functions could potentially correlate measured values with calculated values. The resultant analysis from belief and plausibility calculations showed that the former method contained little to no effect in predicting a given distance while the latter

showed encouragingly high correlation. Hence, the localization algorithm becomes complete and is presented in Fig. 4-1.

4.2 Material and Methods

An overview of the default parameters in the experimental setup is as follows:

1. 100 nodes deployed throughout a rectangular region of 1000-by-1000 meters
2. Two or three anchor nodes (~~both options~~ tested separately in identical networks)
3. Anchor nodes in two-monitor setups located at [150,100] ~~meters~~ and [850,100] meters; third anchor node in three-monitor setups located at [500,100]
4. ~~Region divided into 5 by 10 subregions of 200 by 100 meter area with up to two nodes placed randomly per subregion~~
5. Standby node placed at [500,800] meters; location of fusion center is negligible
6. One-to-one mapping of counties and nodes
7. Range criteria of 20, 0.002, and -1 for RSS, AOA, and standby, respectively
8. Setup tested for every combination of features in which RSS is active (as RSS is needed to calculate distance from node to monitor)
9. Discretization consisting of 10 samples as an inverse normalized distribution
10. Differences of -1.5 meters or less for actual distance minus predicted distance ~~are not considered~~ in determining a minimum difference between distances
11. Distance range weights determined through iterative training (see Subsubsection 4.2.1.1) to be 1.3 and 0.95

```

1: test_data  $\leftarrow$  read measured feature values, county name and location, and mon-
   itor locations
2: full_data  $\leftarrow$  read potential feature values for the county and monitors
3: range  $\leftarrow$  feature-specific range values for each active feature
4: for all monitors do
5:   for all active features do
6:     distances  $\leftarrow$  all distances corresponding to feature measurements in
       full_data that satisfy range
7:     BPA_per_feat  $\leftarrow$  [min_distance, max_distance, 1]
8:     apply sampling to BPA
9:   end for
10:  aggregate BPAs
11:  pl_dist  $\leftarrow$  most plausible distance value
12:  act_dist  $\leftarrow$   $\sqrt{(county_{xcrd} - mon_{xcrd})^2 + (county_{ycrd} - mon_{ycrd})^2}$ 
13:  [min_accept, max_accept]  $\leftarrow$  coefficients of acceptable dist values
14:  meas_dist  $\leftarrow$  measured distance from county to monitor
15:  decision_per_mon  $\leftarrow$  min_accept < meas_dist < max_accept
16:  difference_per_mon  $\leftarrow$  actual_dist - pl_dist
17: end for
18: if decision_per_mon == all ones then
19:   decision  $\leftarrow$  1
20: else
21:   decision  $\leftarrow$  0
22: end if
23: difference  $\leftarrow$  minimum difference_per_mon
24: return [decision, difference]

```

Algorithm 3: Pseudocode of second proposed DS localization algorithm.

12. Experimental results as the average of ten independent experiments in which the WSN is regenerated upon every trial

13. Zero noise factor and zero anchor node positioning error assumed

Details of the variables and terminologies above that differ from those given in the previous chapter are explained in the proceeding subsections.

4.2.1 Algorithm Execution

The algorithm was coded by following the flowchart provided in Figure 4-1, which evolved into the pseudocode provided in Algorithm 3, which was then revised to

MATLAB-specific syntax and thoroughly tested to ensure proper functionality and accurate representation of the proposed method.

4.2.1.1 Determination of Acceptable Distance Values

As stated in Line 13 of Algorithm 3, a major factor in determining the desired outputs in the next subsection is the formation of acceptable distance values. One of the fundamental strengths of DS Theory is the ability to forego an elaborate training stage, as required in neural network and data mining techniques. Conversely, however, care must be taken to determine a precise distance range to ensure optimal localization accuracy; hence, a simplistic iterative approach is taken to find the most effective weights for the lower and upper bounds of an acceptable distance range. The resultant approach is detailed in Algorithm 4. For the sake of preserving the training-independent property of DS Theory, and due to the highly intensive time requirement when compared to Algorithm 3, this method is not intended to be executed upon every run of the core algorithm but instead is only to be run upon the initial formation of the WSN as well as highly significant changes thereof. In other words, if a WSN undergoes minor changes, such as the addition or removal of one anchor node or the relocation of sensor nodes across short distances, the impact of the acceptable distance ranges is too low to noticeably alter the accuracy of the core algorithm. The three types of accuracy portrayed in Algorithm 4 as well as the details of Algorithm 3 are provided in Subsection 4.2.3.

In typical data modelling training techniques, the data used for training is intended to contain several times as many samples as those used in validation. Hence, in this approach, WSN data generation and Algorithm 3 execution are run repeatedly and independently three times as much as in validation, or the training-independent stage. Due to the high computational time and cost required for training, only a limited number of weight combinations are considered. As will be indicated in the

```

1: test_data  $\leftarrow$  read measured feature values, county name and location, and mon-
   itor locations
2: full_data  $\leftarrow$  read potential feature values for the county and monitors
3: for all candidate upper bounds do
4:   for all candidate lower bounds do
5:     for all combinations of available features do
6:       generate WSN data and run Algorithm 2 set number of times using corre-
       sponding bounds and feature set as inputs
7:       [decision_matrix, matching_county_vector]  $\leftarrow$  average outputs of Algo-
       rithm 2
8:       [longAcc, shortAcc, matchAcc]  $\leftarrow$  long accuracy, short accuracy, and
       matching accuracy, respectively
9:     end for
10:    avgAcc  $\leftarrow \frac{1}{3} \frac{1}{no.feats.} \sum_{features} (longAcc + shortAcc + matchAcc)$ 
11:  end for
12: end for
13: bestAcc  $\leftarrow$  maximum avgAcc
14: [best_lower, best_upper]  $\leftarrow$  combination of upper and lower bounds that result
   in bestAcc
15: for all combinations of available features do
16:   generate WSN data and evaluate Algorithm 2 using best_lower, best_upper,
   and corresponding feature set
17: end for

```

Algorithm 4: Pseudocode of distance range training algorithm.

experimental results in Subsection 4.3.1, the resultant lower and upper bound coefficients of the maximum plausibility value are ultimately chosen to be 1.5 and 0.95, respectively.

4.2.2 Desired Outputs

A summary of the intended inputs and outputs are given in Fig. 4-2. In order to address two distinctly different styles of application, the proposed method delivers two separate outputs that respectively address the two following questions:

- Given a set of different measurement types and the location of a given county, do the measurements belong to said county?

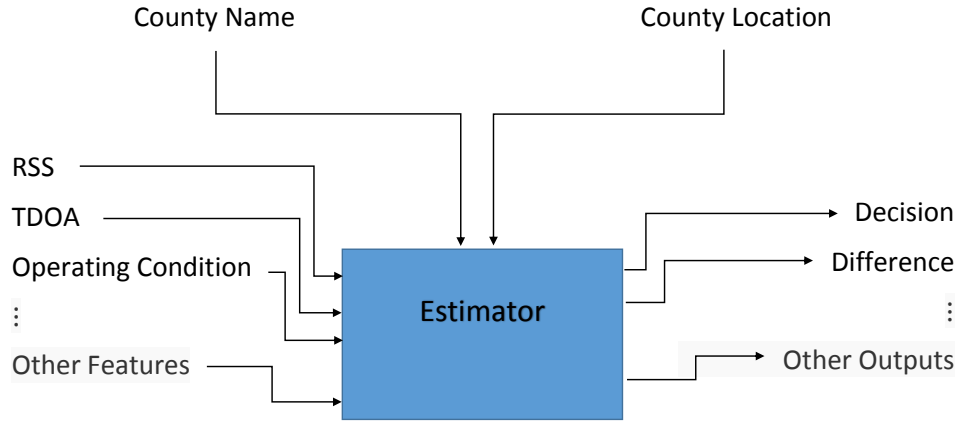


Figure 4-2: Second proposed classifier.

- Given a set of different measurement types and the locations of all available counties, which county most closely matches the measurements?

4.2.3 Accuracy Calculation

Three different types of accuracy are established in calculating the effectiveness of the two questions posed in the previous subsection: full accuracy, short accuracy, and matching accuracy. Full accuracy refers to the method presented in Subsection 3.4.6 and is based on the first question in the previous subsection. A second accuracy technique, known as short accuracy, is also based on the first question and follows a similar process except that only the matching counties and measurement sets are tested, e.g. county 1 with measurement set 1, county 2 with measurement set 2,

```

1:  $test\_data \leftarrow$  read measured feature values, county name and location, and mon-
   itor locations
2:  $full\_data \leftarrow$  read potential feature values for the county and monitors
3:  $range \leftarrow$  feature-specific range values for each active feature
4: for all measurement sets do
5:   for all counties do
6:      $[decision, difference] \leftarrow$  outputs of Algorithm 3 using  $test\_data$ ,
        $full\_data$ , and  $range$  as inputs
7:   end for
8:    $index \leftarrow$  minimum  $difference$ 
9:    $matching\_county \leftarrow$  county at  $index$ 
10: end for

```

Algorithm 5: Pseudocode of DS testing algorithm

etc. The results are recorded in an output vector of length equal to the number of counties, rather than a two-dimensional square matrix of the same side length, and compared against a vector of equal length with values consisting entirely of ones. The two vectors are compared by the same means as before with the same formula as that in Equation 3.13.

The third accuracy type is based on the second question of the previous subsection, in which a single list of measurements and a full list of all counties are the two inputs and the number corresponding to the most likely county match is the output. To test matching accuracy, every measurement set (in order of their matching county numbers) is tested against the list of counties to form a resultant output vector of size similar to that in short accuracy. The contents of said array take the form of county numbers. This vector is then compared against the ideal vector, which is simply every county number in numerical order, on the basis that the measurement sets are given in the same order. The two matrices are then compared similarly to before with Equation 3.13 for calculating accuracy.

In order to thoroughly evaluate all three types of accuracy, a simple testing algorithm is formed as portrayed in the pseudocode of Algorithm 5. This technique consists of two *for* loops, which evaluate the core localization algorithm and extract

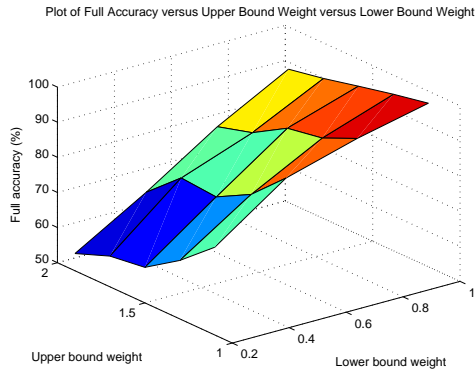
all desired ~~distance difference~~ and county match values required for any of the three forms of accuracy calculation.

4.3 Results and Discussion

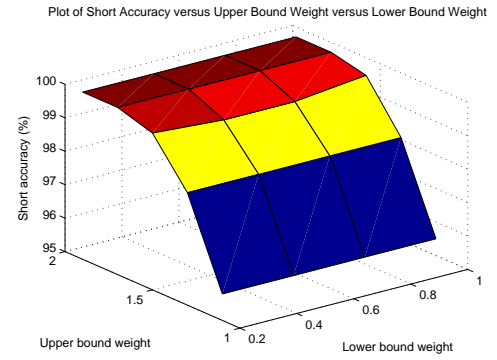
This section provides the results for the experimental setup and parameters detailed in the previous section, including the preliminary training-based approach intended only for initial WSN implementations as well as the core training-independent approach. The latter of the two serves as the primary focus of discussion due to its elegant combination of flexibly high accuracy and low computational cost.

4.3.1 Results Under Distance Range Weight Training

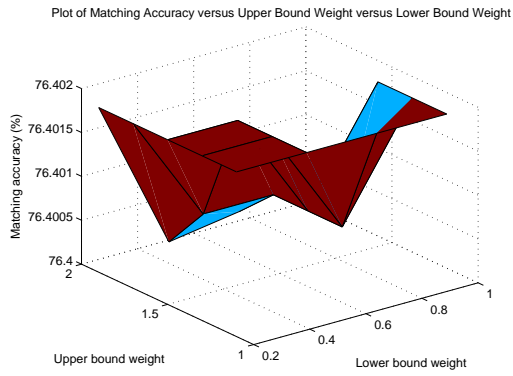
The training-based portion of the simulation was trained with 20 possible combinations of desired values, as detailed in ~~Section 3.4~~. The results for all three accuracy types as well as the averages thereof are presented in Fig. 4-3. Each result is the average of 30 independent trials, that is, three times the amount of trials run in the proceeding subsection. As the surface plots indicate, full accuracy reaches optimal levels when the lower bound weight is maximized and the upper bound weight is minimized. Short accuracy appears to be almost independent of lower bound weight values and optimizes at the maximum upper bound. Matching accuracy follows the most uniform distribution of accuracy values, varying from 76.000% to 76.002%, optimizing at multiple possible points, effectively making this form of accuracy negligible ~~in the attempt~~ to find a single best weight combination. The average accuracy is the primary decision factor of weight selection and optimizes upon a maximum lower bound of 0.95 and an upper bound of 1.3, located in a tightly contained region in which average accuracy narrowly exceeds 90%. Hence, the respective upper and lower weights of 1.3 and 0.95 have been verified to produce the most accurate data and are



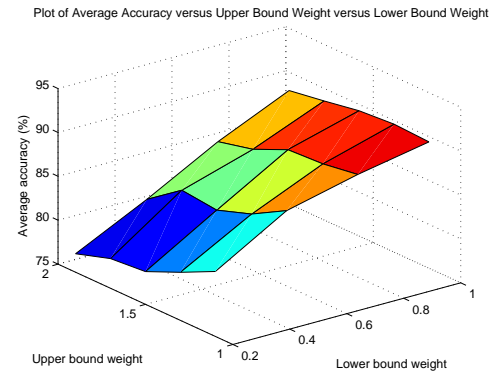
(a) Full Accuracy



(b) Short Accuracy



(c) Matching Accuracy



(d) Average Accuracy

Figure 4-3: Plots of accuracy versus upper bound weight versus lower bound weight.

Table 4.1: Simulation results for two-monitor and three-monitor WSNs.

Accuracy Monitors	Full (%)		Short (%)		Match (%)		Average (%)	
	2	3	2	3	2	3	2	3
RSS	97.04	97.49	97.20	98.50	56.17	66.64	83.47	87.54
RSS, SB	97.00	97.43	100.0	100.0	94.58	96.07	97.19	97.83
RSS, AOA	97.03	97.47	98.13	98.97	57.81	67.29	84.33	87.92
RSS, AOA, SB	97.01	97.44	100.0	100.0	92.52	94.21	96.51	97.22

thus used consistently under the implementation detailed in the following subsection.

4.3.2 Training-Independent Results

The training-independent portion of the simulation was tested in 24 different scenarios resultant from four independent variables: the set of active features, the number of monitors, the type of accuracy, and the overall state of the WSN. The number of monitors used in the simulation were restricted to two and three, as these have been established minimums under unimodal RSS and AOA based localization methods. For reliability and redundancy purposes, the WSN involved in the simulator was re-generated prior to each of the ten experimental trials for each of the above combinations of scenarios. The accuracy results for all test cases are given in Table 4.1, and the runtime results are provided in Table 4.2, where each value for each table is the average run time for all trials.

4.3.2.1 Accuracy

As Table 4.1 indicates, the full accuracy test generates consistently high results in the steady area of 97.0-97.5% accuracy for all eight scenarios, thereby proving the algorithm to be consistently effective for both two-monitor and three-monitor setups under all feasible combinations of active features. Results of the short accuracy test

climb even higher with values range at 97-100%. Under the matching accuracy test, results span a wider range of accuracy results from 56.1% to 94.6%. The average of all three accuracy types indicate a somewhat less diverse range of 83.5% to 97.8%. The results ultimately indicate that the matching of counties works most effectively when only RSS and operating condition are enabled, as the absence of operating condition results in the only accuracy values lower than 94% as well as the only short accuracy values that are below 100%.

The full accuracy test is the only evaluation that does not appear to fully benefit from data fusion. While fusing multiple types of data is intended to enhance an accuracy relative to that of a single type of data, this test shows that full accuracy is most effective when only RSS data is available. The other evaluation methods, however, clearly show that fusion of multiple measurement types do in fact enhance accuracy. Combining this observation with the fact that differences in full accuracies are minimal (all values for each monitor are within 0.06% of one another) can easily infer that data fusion in this context is still an overall **success**.

Another successful outcome is the high accuracy involved in two-monitor WSNs. Most established localization methods require three or more monitors to observe a sensor node's measurements, but in this algorithm, two-monitor WSNs can localize reliably with a less-than-0.5% decrease in accuracy. From the standpoint of having RSS as the only enabled feature, achieving a high accuracy with three monitors is trivial, as this can be achieved purely through trilateration. Under two monitors, however, only bilateration is available, meaning that if distance values calculated from RSS measurements are exact, there is still only a 50% chance of obtaining the correct location. Thus, with DS Theory, highly accurate localization can be achieved just as easily with only two monitors as it can with three or more.

Table 4.2: Runtime results for two-monitor and three-monitor WSNs.

Runtime (ms)	2Mon	3Mon
Total	787478.00	1375261.00
Per Feature Set	196869.00	343815.20
Per Iteration	17.20	30.03
Per Node	0.160	0.281

Table 4.3: Comparison of accuracy among different localization techniques.

Algorithm	PSO	WSLA	WSRA	MLE	DS1	DS2
Accuracy (%)	71	90	92	93	87	97
Runtime (μ s)	114570	7800	9700		12733	281

4.3.2.2 Computational Runtime

Total run time for an entire simulation for all four available feature sets combined was recorded for each trial. From there, the calculated time was averaged per feature mode, then divided by all $107^2 = 11449$ possible input combinations to find average runtime per iteration. Finally, the latter runtime was divided by all 107 nodes to find the average run time per node. All four types of runtimes were averaged for all ten trials and then recorded in Table 4.2 for both two-monitor and three-monitor WSNs. All trials were run on consistent computer hardware and software, including a standard Windows version of MATLAB on a quad core 2.60 GHz processor and 8.0 GB of RAM. The significance of these results will be explained in greater detail upon the next subsection.

4.3.3 Comparisons to Other Established Algorithms

To gain further perspective into this technique’s effectiveness, comparisons were made to the four prior established localization methods presented in the previous chapter as well as the proposed DS-based technique of said chapter. In Table 4.3,

the previous method is still referred as DS1 while the currently presented technique is labeled as DS2.

4.3.3.1 Accuracy

As the second row of Table 4.3 indicates, the proposed technique contains a noticeable edge in accuracy in comparison to the other methods as well as a significant lead over PSO and DS1. Although some parameters had to be tweaked between this method and the others, a fair conclusion can be made that this DS-based algorithm has a distinct advantage on par with, if not over, multiple established state-of-the-art localization techniques.

4.3.3.2 Computational Runtime

Average run time per node for a three-monitor WSN setup was compared with three of the four aforementioned techniques, based on data given in [14]. All method runtimes, sans that of MLE, are reported in the third row of Table 4.3. As the table plainly indicates, the proposed method has an overwhelmingly shorter CPU run time than all reported competing methods, hence solidifying such an algorithm as a novel low cost technique.

4.4 Conclusions

In this chapter, a novel approach to localization in wireless sensor networks was presented. This approach was based heavily upon the innovative statistical framework known as Dempster-Shafer Evidence Theory. While DS Theory contains many unique properties that can aid in data prediction and analysis, the concept of prediction by plausibility is of the most vital levels of importance and interest, due to its consistently high accuracy under a variety of network setups and feature set availabilities as well

as its consistently low computational cost requirements.

Chapter 5

Conclusive Remarks

The end goal of this research was to investigate the concepts and applications of Dempster-Shafer Theory and incorporate such a method into the development of novel, computationally attractive algorithms for wireless sensor network localization. Hence, two new localization schemes were presented in Chapters 3 and 4.

The first technique, proposed in Chapter 3, served as a verbose, introductory method of moderate accuracy with the primary objective of merely wading in the untapped waters that form Dempster-Shafer-based WSN localization. This method of node positioning functioned by fusing multiple types of signal measurements, such as received signal strength and angle of arrival, and executed the expected value function of DS Theory to predict a ranges distances for multiple monitors, through which a feasible region could be obtained, detailing a list of coordinates among which the targeted node could lie. The algorithm produced an accuracy of 78-87% across a multitude of feature selection scenarios, thereby providing an adequate introductory approach to the previously untapped fusion of WSN localization and DS Theory. This new approach is primarily intended to aid in future research and development of Dempster-Shafer-based localization techniques, such as the following one. ~~However,~~ it can also be utilized in practicality for applications in which obtaining a desired area of geo-location is more strongly desired than estimating a specific point.

In Chapter 4, a second method was presented with a significantly higher accuracy and exponentially lower computational cost. This method used the plausibility function of DS theory, rather than expected value, to generate a most plausible distance from node to monitor and use an optimal range of values proportional to the plausible value to determine whether a set of measurements belong to a given county. This technique produced 83-98% accuracy for a multitude of feature selection criteria as well as multiple anchor node setups.

Time-critical localization of WSNs is a topic of increasing concern in today's world, as a variety of civil and military applications can thrive on low-cost, high-accuracy localization techniques. For instance, some universities have a network of emergency phones scattered across campus so that students in distress can immediately reach emergency services. Using effective localization techniques such as that proposed in Chapter 4, campus security can dispatch police, fire, or medical services to a caller's location as quickly as humanly possible. Hence, the uniqueness and effectiveness of this novel algorithm can have a crucial impact on today's world, as every percent of accuracy achieved and every second of time saved can be the difference between life and death.

5.1 Future Work

Due to the groundbreaking, cutting edge research potential of Dempster-Shafer Theory, as demonstrated by the previous chapter of this thesis, the resultant research thereof has the potential to diverge into many possible future directions.

5.1.1 Enhanced Training of Upper and Lower Bound Coefficients

Training of the upper and lower bound weights was done from a limited iterative approach. Although this brief technique resulted in multiple high levels of accuracy above 90%, accuracy could be even further enhanced through more elaborate optimization methods, similar to those utilized in neural network training. Possibilities include local methods, such as Steepest Descent or Conjugate Gradient, as well as global, biologically inspired methods, such as Genetic Algorithm or Ant Colony Optimization.

5.1.2 Fusion of DS Theory With Support Vector Machines

Support vector machines (SVMs) have been of a high level of interest in the field of WSN localization, almost to the same extent as DS Theory, due to its fast localization potential and efficient use of processing resources. Because the end goal of the NSF grant supporting this research involves the fusion of DS Theory with SVMs, the most logical next step of this research will be to develop WSN localization schemes based upon such a combination of techniques.

5.1.3 Role of DS Theory Within Data Mining and Big Data Analytics

Data mining, or big data, is another method of predictive data analysis, as previously addressed in Section 3.1 and is of overwhelmingly significant interest in the modern technological world, primarily due to the growing need for the handling and processing of immense quantities of data. In many applications, however, the necessity arises to supplement data mining paradigms with additional predictive data models by the process of meta-learning. Many typical methods that are included in

meta-learning include tree classifiers, linear analysis, and neural networks. Because Dempster-Shafer Theory is a predictive model that is still inching its way into new and innovative applications (as is clearly the case with WSN localization), the inclusion of DS Theory as a meta-learning mechanism could be highly beneficial to the effectiveness of many big data methods.

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Appendix A

Source Code

The MATLAB source code is released under Apache 2.0 license and can be obtained by contacting the author at Colin.Elkin@utoledo.edu.