Money Matters

MAT 1630 Project

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A NYC Apartment Loan

Suppose you take out a $230,000 loan to buy an apartment. The bank offers you a loan at a 3.1% annual interest rate compounded monthly that must be paid off in 30 years. Round all answers to the nearest cent.

- What is your monthly mortgage payment? **Hint:** You can use either the formula for the monthly payment c from the course notes, or a while loop using the principal balance function, as we have done in class. **Answer**: $982.14
- What is the total interest for this loan? **Hint:** The total interest paid you get by subtracting the original loan principal from the total payments you make each month for the duration of the loan. **Answer**: $123,569.58

Suppose that after owning the apartment for 10 years, you want to refinance the loan with a new 30-year loan at a 2.5% annual interest rate and a closing cost of $6,000.

- What is your new monthly mortgage payment? **Hint**: First, find the principal balance from the original loan after 10 years, and use it as the initial principal for the new loan. Then, use the formula for the monthly payment we give in the course notes to find the monthly payment for the new loan terms. **Answer**: $693.44
- (**Bonus**) Compute the total interest paid for the first 10 years of the old loan and the entire new loan. Compare this total interest, if you refinance your loan, with the total interest you would have to pay for the original loan, if you don't refinance. How much more would you have to pay in interest and closing costs if you do refinance your loan? **Answer**: $19,926.89 **Hint**: To compute the interest paid for the first 10 years of the old loan, define a Python function that returns the total interest paid after t time periods, for a loan with principal balance P_0, annual interest rate r, monthly payment c and compounding frequency $n = 12$. You can use the principal balance function we defined in class with an extra line inside the while loop for updating a variable that accumulates the interest paid each time period.

Solution

```python
#Initial Loan
P0 = 230000
r = 3.1/100
n = 12
t= 30
N = n*t

#Refinancing
t1 = 10 #new time period
N1 = n*t1
r1 = 2.5/100

#Monthly payment function
def monthly_payment(principal,rate,periods,timeframe):
  c = principal*(rate/periods)*(1+rate/periods)**(periods*timeframe) / ((1+rate/periods)**
  return c

c = monthly_payment(P0,r,n,t)

#Total interest function
def total_interest(principal,rate,periods,timeframe):
  c = P0*(rate/periods)*(1+rate/periods)**(periods*timeframe) / ((1+rate/periods)**(period
  total_payment = c*periods*timeframe
  return total_payment - principal

#Balance function
def balance(P0,r1,N,c1):
  P=P0
  for months in range(N1):
    P=P*(1+r/n)-c
  return P

bal = balance(P0,r1,N1,c)

#Show answers
ans1 = round(monthly_payment(P0,r,n,t),2) #Monthly payment of original loan
print(f"Original Monthly Payment: ${ans1:.2f}")
ans2 = round(total_interest(P0,r,n,t),2) #Total interest from original loan
print(f"Total Interest Amount: ${ans2:.2f}")
```

```
ans3 = round(monthly_payment(bal,r1,n,t),2) #Monthly payment of original loan
print(f"New Monthly Payment: ${ans3:.2f}")
```

```
Original Monthly Payment: $982.14
Total Interest Amount: $123569.58
New Monthly Payment: $693.44
```

College Savings

Suppose that your parents had started saving for your college education when you were born. How much would they have had to save each month over 18 years in order to accumulate \$30,000 to pay for the in-state tuition at City Tech for 4 years? Assume that the savings account is earning a steady 4% interest per year, compounded monthly. For your computed monthly contribution, how much money would you have after 18 years if you did not put them in a saving account? **Answer:** around \$95 for the monthly payment.

Solution

```
#Intialize
savings = 30000 #Total savings goal
rate = 0.04 #Interest rate
t = 18 #Time period
n = 12 #Compounding period
N = n*t #Total compounding periods

#Compute payments
c2 = savings / (((1 + rate/n)**N - 1) / (rate/n)) #monthly payment amount
no_interest = c2*N #amount saved with no savings account

#Show answers
print(f"Monthly savings needed: ${c2:.2f}")
print(f"Savings without savings account: ${no_interest:.2f}")
```

```
Monthly savings needed: $95.06
Savings without savings account: $20532.81
```

A Car Loan

Helen is in the market for a new car. She sees an advertisement for a car whose price is \$15548.89 that offers either a \$1,500 discount for paying cash or 0.9% annual interest rate on a 60-month loan. Which is the better deal if savings accounts are currently earning 5% per year? What if everything is the same but the car price is \$16000? What if the price of \$15000?

Solution

```
price = 15548.89 #Car price
discount = 1500 #Cash discount
t = 60 #Length of loan
r = 0.9/100 #Dealership rate
rc = 5/100 #Savings account rate

#Function to find the loan balance
def loan_balance(c):
  balance = price
  for month in range(t):
    balance = balance*(1+r/n)-c
  return balance

c=0
while loan_balance(c) > 0:
  c+=0.01

print(c,loan_balance(c))
```

```
265.1199999999039 -0.007225190158919759
```

```
#Finding the difference in costs between the two options
principal = price-discount
for month in range(t):
  principal=principal*(1+rc/n)-c

#Show the answer
principal
```

```
-0.007777916591635403
```