CSCI 4100 Homework 4

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1. Exercise 2.4:

2. Problem 2.3:

   (a) Positive or Negative Ray:
   $M_{\mathcal{H}}(N) = 2N$, because point a could be placed to the positive direction of any of the N points, pointing either positively or negatively.

   (b) Positive or Negative Interval:
   $M_{\mathcal{H}}(N) = \binom{N+1}{2} + \binom{N-1}{2} + 1$. This is the case because it includes all positive intervals, as well as negative intervals which contain unique points in the interior of the range. This leads to $\binom{N-1}{2}$ new intervals.

   (c) Concentric Spheres:
   $M_{\mathcal{H}}(N) = \binom{N+1}{2} + 1$. This problem is exactly identical to the positive intervals problem, converting each point in $\mathbb{R}^d$ to $\mathbb{R}$ using its radius. Concentric spheres then represent a positive interval over these radii.

3. Problem 2.8: The following are possible growth functions $M_{\mathcal{H}}(N)$:
   $1 + N, 1 + N + \frac{N(N-1)}{2}, 2^N, 1 + N + \frac{N(N-1)(N-2)}{6}$

4. Problem 2.10: If there is not a break point, $M_{\mathcal{H}}(N)^2 = (2^N)^2 = 2^{2N} = M_{\mathcal{H}}(2N)$. Otherwise, it is polynomial bounded, so it is less than $M_{\mathcal{H}}(N)^2$.
   This lets you establish a new generalization bound of $\sqrt{\frac{8}{N} \ln\left(\frac{4M_{\mathcal{H}}(N)^2}{\delta}\right)}$

5. Problem 2.12:
   $0.05 = \sqrt{\frac{8}{N} \ln\left(\frac{4((2N)^{10}+1)}{0.05}\right)}$. $N = 452957$