

CS166 HW1

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Problem 3

- i We can construct hybrid structures of depth $k - 1$ by breaking them up into block sizes $b_1, b_2, b_3, \dots, b_{k-1}$ and use a sparse table RMQ on each level to achieve $< O(n \log n), O(1) >$ for that level. The preprocessing time of the hybrid structure is then

$$O(n + p(\frac{n}{b_1}) + (\frac{n}{b_1})p(\frac{b_1}{b_2}) + (\frac{n}{b_1})(\frac{b_1}{b_2})p(\frac{b_2}{b_3}) + \dots + (\frac{n}{b_{k-2}})p(\frac{b_{k-2}}{b_{k-1}}) + (\frac{n}{b_{k-1}})p(b_{k-1}))$$

If we let

$$b_1 = \log n$$

$$b_2 = \log b_1$$

$$\dots$$

$$b_{k-1} = \log b_{k-2}$$

Since $p(x) = x \log x$ using a sparse table RMQ, we can use the trick mentioned in lecture to reduce every term but the last as follows:

$$\begin{aligned} O((\frac{n}{b_{i-1}})p(\frac{b_{i-1}}{b_i})) &= O((\frac{n}{b_{i-1}})\frac{b_{i-1}}{b_i} \log(\frac{b_{i-1}}{b_i})) \\ &= O((\frac{n}{b_{i-1}})\frac{b_{i-1}}{\log b_{i-1}} \log b_{i-1}) \\ &= O((\frac{n}{b_{i-1}})b_{i-1}) \\ &= O(n) \end{aligned}$$

Upon reduction, we have a preprocessing time of:

$$\begin{aligned} O(n + (\frac{n}{b_{k-1}})(b_{k-1}) \log b_{k-1}) &= O(n \log b_{k-1}) \\ &= O(n \log \log b_{k-2}) \\ &= O(n \log^{(k-1)} b_1) \\ &= O(n \log^{(k-1)} \log n) \\ &= O(n \log^{(k)} n) \end{aligned}$$

Lookup is performed similar to a two level hybrid query: at each of the k levels, we perform RMQ queries one level deeper on the blocks that contain the two boundary indices and perform RMQ on the blocks between them (at the current level). This gives a query time of $O(k) = O(1)$ since k is constant.

Thus, the time complexity of our hybrid structure is $< O(n \log^{(k)} n), O(1) >$ as required.

- ii The increased query time arises from the fact that the query time is $O(k)$ as shown in part (i). Thus, as k increases, the runtime increases. However, since k is a constant, all the hybrid structures still have a query time of $O(1)$, which does not contradict our result in (i).