Creation and analysis of a 3-dimensional boat

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Introduction

The creation of this boat stemmed from a challenge to create a boat that floats flat when undisturbed, has an angle of vanishing stability between 120° and 140°, and is as fast as possible when towed with .1 N of force. This last requirement, being the most open ended, was the main focus of our experiments as it was the hardest to quantify and also the only one that was not a simple pass/fail test. In line with these objectives, we split our experiments between stability and speed, designing a fast hull and then adapting it to have favorable tilt characteristics.

Process

The effective righting moment of a boat is the net torque generated by the misalignment of its center of mass and its center of buoyancy. This torque is dependent on the angle of the boat with respect to the water, and often changes direction somewhere between 0° and 180° at a point called the Angle of Vanishing Stability (AVS). The point at which the torque changes direction corresponds with an unstable equilibrium position, where the net torque is zero but a small change in angle will cause the boat to either right itself or flip completely over. Some shapes, such as rectangles, have several AVSs, though for the most part it is advantageous to choose shapes that have only one AVS as they will tend to float flat. In the case of a three dimensional boat, however, both the side to side and fore and aft tilt angles must be considered on the analysis, as boats that are pointier at the front will tend to lean forward as they are rolled. This is because the larger width at the back begins to dig deeper into the water and provide more floatation than the relatively thin front, requiring a forward roll to maintain equilibrium.

Definitions and Assumptions

Table 0. Variable Names and Definitions

Variable	*Meaning*
θ	Side to side heel (x,z) plane
ϕ	Fore to aft heel (y,z) plane
d	draft at center of boat
M	Total Mass

In this model we assume that the boat is a solid body and can not twist, that the water is fresh and at room temperature, and that the mass can be assumed to act in one point along the centerline of the mast.

Geometry Definition

In order to analyze a boat, it must first be defined in terms of a set of equations that create a 3D volume to be analyzed. We decided to use a parabolic cross section that varied as a function of the length along the keel and offset vertically by another function of length. These equations were determined by performing a free surface flow analysis on several different boat shapes and determining their drag coefficients. The initial design goals were to minimize weight and wetted surface and maximize length to minimize the effect of periodic waves on maximum speed. These tests created a basic design that was then tweaked by the CFD results to yield the following equations:

$$y < 10. \,\&\&\, y > 0.00384615\,(-29. + z)^{2.} + \frac{8.\,x^{2.}}{z^{1.2}} \,\&\&\, 0. < z < 57.$$

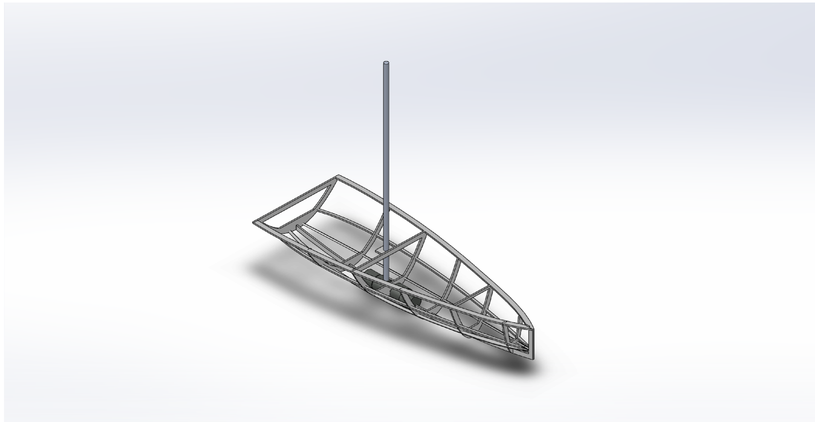

Figure 1. Full boat modelled in SolidWorks

In total, five different boat shapes were analyzed. Our design was compared against a basic extrusion, a symmetric 3D boat, an IMOCA 60 (a very fast ocean boat), and a boat similar to the IMOCA 60 but fitted with hydrofoils to determine how to manage its hydrodynamic properties given the small size and low speed it operates at. Below these tests are pictured in the order described:

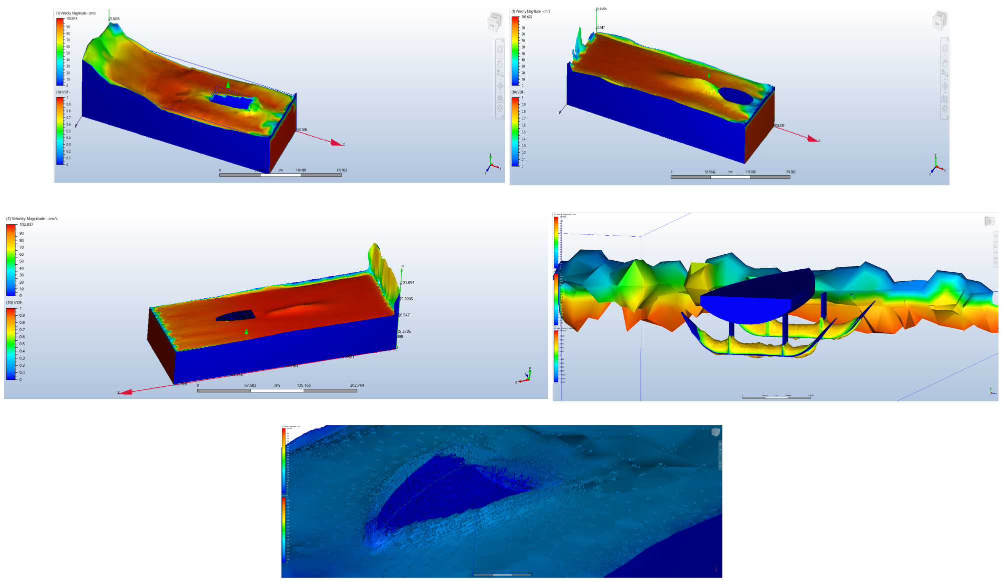

Figure 2. A diagram of the five CFD analyses performed

In order: Extrusion, Simple Parabola, IMOCA 60, Hydrofoils, Current Boat

The drag analysis was performed by summing up the total forces on all wetted surfaces and dividing by the wetted area to find a suitable metric to compare these slightly different sized boats. The results are as follows, with all percentage savings relative to the IMOCA 60:

	IMOCA 60	Foiling	Our Boat	Normal 3D	Basic Extrusion
Approximate Wetted Surface:	600	395	1000	856	1712
Total Drag:	-1.19011	-0.80003	-1.528	-2.74963	-8.0095
Drag per total surface:	-0.00127	-0.00202	-0.00068	-0.001571	-0.002649284
Drag per wetted surface:	-0.00198	-0.00202	-0.00153	-0.003212	-0.004678446
For foiling boats drag per normally wetted surface		-0.0008			
Percentage Drag Savings	NA	-59.4771	-0.22965	61.943917	135.866244

This means that our boat should be about 84% faster than a normal 3D boat and about 158% faster than a basic extrusion when given enough room to accelerate (which may not be possible in the pool).

Center of Mass

The first of the two forces that affect the righting moment is the force of Gravity, which for our purposes can be generalized to act on a single point at the objects center of mass. The center of mass (COM) is a mass weighted average of position across the shape, and can be represented by the formula:

$$\text{COM} = 1/M * \int\int\int \rho \, dx \, dy \, dz$$

Where ρ is an equation of the object's density with respect to x, y, and z. For simple shapes with uniform densities, this point will be at the centroid of the geometry. For our more complicated boat, however, the center of mass is affected by the placement of lead masses and was placed in such a way that it yielded the correct AVS. In our case this is anywhere between 4 and 5.5 centimeters from the keel at the mast.

Figure 4. A cross section with its COM labeled

As one of our chief design concerns was to reduce weight, we decided to use about 700 grams of ballast, the minimum allowed amount. The placement of the ballast was adjusted along with the fiberboard mass until a suitable combination of low weight, high max torque, and correct AVS was determined. The total mass of the system along with the pole is 970 grams, which was determined by assembling all of the components in SolidWorks and using its mass properties feature.

Displaced water and Draft

In order for the force of gravity to be equal to the buoyant force, the total mass of the displaced water must equal the total mass of the vessel. This means that if the density of the displaced water is known, the total displaced water can be calculated by simply dividing total water weight by water density. Once the total volume displaced is known it can be used to find displacement by solving for the distance above the water line for which the submerged volume is equal to its weight in water. For cases where the boat is tilted, this equation defines a plane that is sloped with respect to θ and ϕ and offset in the z axis by a parameter d which represents the centerline draft. Solving the following equation for d will define the upper bound, represented by a function of theta and phi plus a d offset, and the lower bound, the boat hull, therefore fully defining a submerged volume.

$$\text{Displaced Volume} = \int\int\int \text{Region}[g(x, y) < z < f(\theta, \phi, d)]$$

In order to determine if a boat will float, the total density of the boat must be less than 1 g/cm^3. For our boat, the total mass is 970 grams and the total volume is 5163.52 cubic centimeters. This yields a density of .188 g/cm^3 which is much less than 1.

Center of Buoyancy

Once a submerged volume is defined, finding the center of buoyancy only requires doing one more center of mass equation, this time on the water region. Also, because water can be assumed to have a constant density, the integral of its density over all there axes will equal its mass, therefore removing the need for density or mass. Therefore, the center of buoyancy (COB) can be found with the equation:

$$\text{COB} = \int\int\int 1 \, dx \, dy \, dz$$

Figure 5. The underwater region with the boat's COM on the left and the current COB on the right

Torque Calculation

With the equations for the center of mass and buoyancy determined, the torque about the center of mass can then be determined by the cross product of the force of buoyancy and the effective radius (the distance between COM and COB) of the moment.

$$\tau = \vec{r} \times \vec{f}$$

Because the AVS is an equilibrium point it must occur where the net torque is zero. By analyzing the points at which this occurs in terms of θ and ϕ, the likely AVS positions can be found. By removing the stable conditions (upright, upside-down, and any other stable equilibria), only unstable equilibria which represent AVS points remain. If the torque is plotted with respect to heel angle a surface can be created for both the x and y components. The point at which x torque is zero and y torque is zero is the AVS.

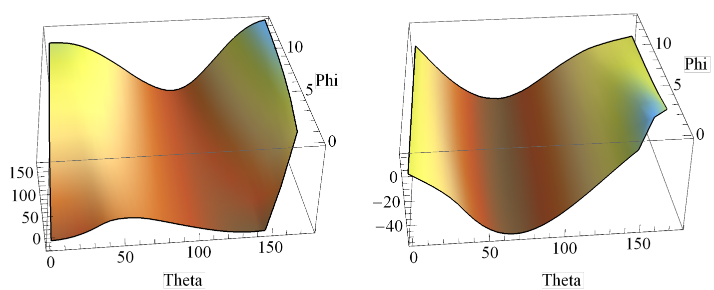

Figure 6. The graph of torque(Nm^-2) versus heel (Degrees) for pitch and roll respectively

(horizontal heel is on the horizontal axis, vertical heel is on the vertical axis)

In order to visualize this curve, these two graphs can be merged together while excluding values that are far from zero to create the following graph, where the AVS is a point (θ,ϕ) where the two surfaces intersect:

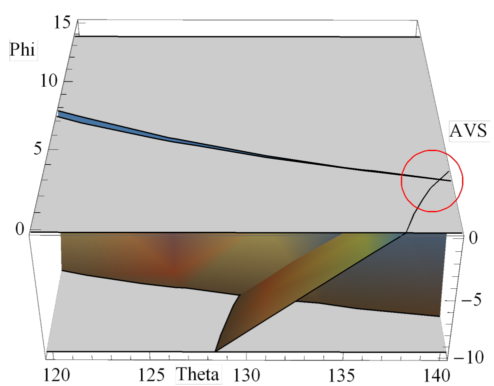

Figure 8. The graph of torque=0 versus heel (Degrees) with AVS at the intersection

This diagram was created by limiting the range from (-10,10) to create near vertical segments

By taking the square root of theta and phi squared, the net heeling angle at the AVS pint can be found, which in our case was about 136.71°, assuming perfect placement of ballast. To account for the possibility of imperfectly placed ballast, we calculated the range of possible center of mass values to yield an AVS between 120° and 140°, which requires the COM to be between 4.5 and 5.5 cm above the keel. If it is outside of this range, the AVS may be seriously affected, either causing it to not float flat or to have an incorrect AVS. The other easily determinable factor is the maximum torque, which is simply the maximum of the x value graph, or .6 Nm.