Simulate the dispersion of a broadband signal in a waveguide

Step 1: imagine a real signal between 2 and 4 GHz, with a Gaussian spectrum centered on 3 GHz, for instance. Use equation (3.1.3) to estimate E(z,t) as a function of z for different times t. Do it first for $k=\omega/c$.

Hint 1: start with defining a vector that provides the span of z: say 1e4 points, covering a few meters.

Hint 2: since the signal is real, its Fourier transform must be conjugate symmetrical: use this property to systemically add up contributions from positive and negative frequencies (so the explicit integration then only takes place over positive frequencies).

Hint 3: you will need to carefully choose the frequency samples (a few tens), the time steps (say a few tens of pico-seconds). Define variables allowing you to tune those quantities. No need to use an FFT at this point, just use explicit superposition of results obtained at different frequencies.

Step 2: now consider a dispersion relation $k(\omega)$ typical of a waveguide with $\omega_c=2\pi*2$ GHz and do the same. Tune the parameters used in Step 1 to see a clear dispersion effect. Roughly speaking, at a given time, the two waveforms may look a bit as in the figure below (that of course strongly depends on the chosen parameters).

Hint 4: please write things down on paper before programming. Your code should not take more than 25 lines.