① HW Due Tuesdays

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1/4/18

de Bruijn
Sequence of 0's and 1's which is
cyclic and s.t. each sequence of
length k occurs exactly once.

1) For which k do de Bruijn sequences
exist? For every k. proof by construction.
How can we construct them?

de Bruijn sequence of order 3:

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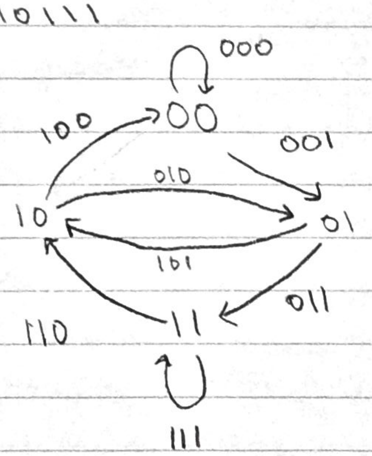
deBruijn graph
of order 2:



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Eulerian circuit in a directed graph is a sequence
of edges s.t.
1) if e_i follows e_j then the end of
e_i is the start of e_j
2) each edge occurs exactly once
3) the end of the last edge is the start of the first edge

Observation: A de Bruijn sequence of order k corresponds
to an Eulerian circuit in the de Bruijn graph
of order k-1

Degree of a vertex v:
# of edges that include v

In-degree of a vertex v:
# of edges that end at v

Out-degree of a vertex v:
# of edges that start at v

Observation: the out and in degrees of each vertex in a de Bruijn graph are 2 (for any k)

Theorem: A directed graph has a Eulerian circuit if and only if:

1) Each vertex has the same in- and out-degrees
2) There is a path from any vertex to any other vertex (connected)

∴ de Bruijn graphs have Eulerian circuits

★ card trick used a linear shift register
we won't go over that

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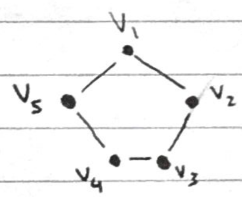
A (simple, undirected) graph is a pair (V, E) where V is a set and $E \subseteq \binom{V}{2}$

An isomorphism between graphs (V_1, E_1) and (V_2, E_2) is a bijection f such that $v \sim u \iff f(v) \sim f(u)$.

$v \sim u$ means that there is an edge $\{v, u\} \in E$.

graph properties are preserved by isomorphisms



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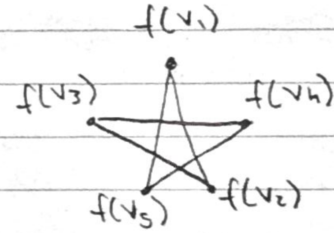

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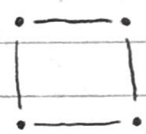

Examples of graph properties:

1) # of vertices
2) # of vertices
3) degree sequence

The degree sequence for a graph G is a list $d_1 \geq d_2 \geq \ldots \geq d_n$ where the d_i's are the degrees of the vertices of G