MLE:

1. 1. B_0: $P(B_0|A_0) = 0.5 > P(B_0|A_1) = 0.1$, decide A_0

B_1: $P(B_1|A_0) = 0.25 < P(B_1|A_1) = 0.3$, decide A_1

B_2: $P(B_2|A_0) = 0.25 < P(B_2|A_1) = 0.6$, decide A_1

If B_0, decide A_0, if B_1, decide A_1, if B_2, decide A_1

MAP:

B_0: $P(B_0) = (0.5)(0.6) + (0.1)(0.4) = 0.3 + 0.04 = 0.34$

$P(A_0|B_0) = \frac{(0.5)(0.6)}{0.34} = 0.8824 > P(A_1|B_0) = \frac{(0.1)(0.4)}{0.34} = 0.1176$

B_1: $P(B_1) = (0.25)(0.6) + (0.3)(0.4) = 0.27$

$P(A_0|B_1) = \frac{(0.25)(0.6)}{0.27} = 0.5556$ $P(A_1|B_1) = \frac{(0.3)(0.4)}{0.27} = 0.4444$

B_2: $P(B_2) = (0.25)(0.6) + (0.6)(0.4) = 0.39$

$P(A_0|B_2) = \frac{(0.25)(0.6)}{0.39} = 0.3846 < P(A_1|B_2) = \frac{(0.6)(0.4)}{0.39} = 0.6154$

If B_0, decide A_0, if B_1, decide A_0, if B_2, decide A_1

2. MLE Error $= P(A_0)P(B_1|A_0) + P(A_0)P(B_2|A_0) + P(A_1)P(B_0|A_1)$

$= (0.6)(0.25) + (0.6)(0.25) + (0.4)(0.1)$

$= 0.34$

MAP Error $= P(A_0)P(B_2|A_0) + P(A_1)P(B_1|A_1) + P(A_1)P(B_1|A_1)$

$= (0.6)(0.25) + (0.4)(0.1) + (0.4)(0.3)$

$= 0.31$

2. Girls in family = $Y = X + 1$

$P(Y = z) = P(z = X + 1)$ $z = 1, 2, 3, 4, 5$

1 girl $P(Y=0) = \binom{4}{0}(0.5)^4 = (1)(0.0625) = 0.0625$

2 girls $P(Y=1) = \binom{4}{1}(0.5)^4 = (4)(0.0625) = 0.25$

3 girls $P(Y=2) = \binom{4}{2}(0.5)^4 = (6)(0.0625) = 0.375$

4 girls $P(Y=3) = \binom{4}{3}(0.5)^4 = (4)(0.0625) = 0.25$

5 girls $P(Y=4) = \binom{4}{4}(0.5)^4 = (1)(0.0625) = 0.0625$

3. See notebook

4. See notebook