Homework Assignment:

- Decision Rules for MAP and MLE
- Discrete R.V.
- MAP and discrete R.V.
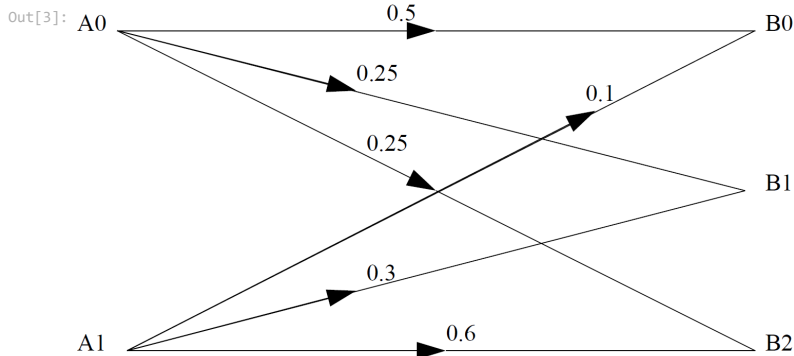
```
In [2]: from IPython.display import Image
```

Problem 1 (20pt, 10pt each)

For the digital communication system shown below, where $P(A_0) = 0.6$ and $P(A_1) = 0.4$.

1. Completely specify the MLE decision rule and the MAP decision rule.

2. Calculate the overall probability of error under that rule.

```
In [3]: Image('figures/2to3.png',width=600)
```

Out[3]:

Problem 2 (10 pt)

A family has 4 natural children and has adopted 1 boy and 1 girl. Each natural child has equal probability of being a boy or a girl, independent of other children.

- Find the probability mass function of the number of girls out of the 6 children.

```
In [ ]:
```

Problem 3 (10pt)

A pharmaceutical company is testing a new vaccine against a virus. Based on previous studies, the vaccine is known to be 70% effective, meaning that each vaccinated person has a 70% chance of developing immunity.

In a trial, 50 people receive the vaccine.

Questions: p.s. you can use python code or handwritten answers.

1. What is the probability that exactly 40 people develop immunity?
2. What is the probability that at least 35 people develop immunity?

```
In [10]: import math
         # 1
         n = 50
         p = 0.7
         x = 40

         print("Probability of exactly 40:", (math.factorial(n) / (math.factorial(x) * math.factorial(n - x)))  * (p ** x) * ((1 - p) ** (n - x)))

         # 2
         sum = 0
         for x in range(35, 51):
             sum += (math.factorial(n) / (math.factorial(x) * math.factorial(n - x)))  * (p ** x) * ((1 - p) ** (n - x))

         print("Probability of at least 35:", sum)
```
```
Probability of exactly 40: 0.03861899068386233
Probability of at least 35: 0.5691784360934438
```

Problem 4 (10 pt, 5 pt each)

A tech support call center receives on average 5 customer calls per hour. The number of calls received follows a Poisson distribution.

Questions:

1. The support center opens from 8am to 5pm. On a typical day, what is the probability that the center received more than 50 customer calls?

2. The tech support call center is considered a day to be "busy day" if it receives more than 75 customer calls. In a week of five weekdays, What is the probability that the tech suport call center have more than three busy days?

p.s. you can use python scipy.stats library. If you choose to do so, please include the code in the submission for this problem 4.

```
In [27]: import scipy.stats as stats

         # 1

         lmd = 5 * 9

         prob50 = 1 - stats.poisson.cdf(50, lmd)

         print("Probability of more than 50:", prob50)

         # 2

         probBusy = 1 - stats.poisson.cdf(75, lmd)

         prob3Busy = 1 - stats.binom.cdf(3, 5, probBusy)
         print("Probability of more than three busy days: ", prob3Busy)
```

```
Probability of more than 50: 0.2037197095925818
Probability of more than three busy days:  0.0
```

In [ ]: