

# Lecture 1

*<2016-03-28 Mon>*

## Contents

<b>1</b>	<b>Bits, Bytes, and Integers</b>	<b>1</b>
1.1	Boolean Algebra . . . . .	2
1.1.1	example . . . . .	2
1.1.2	Contrast to Logic Operation . . . . .	2
1.1.3	Shift Operation . . . . .	2
1.2	Integer . . . . .	3
1.2.1	example . . . . .	3
1.2.2	Numerical Range . . . . .	4
1.2.3	Signed vs Unsigned in C . . . . .	5
1.2.4	example . . . . .	6
1.2.5	Sign Extension . . . . .	6

## 1 Bits, Bytes, and Integers

- Bit is 0 or 1
- Electronic Implementation
  - High voltage: 1, low voltage: 0
- Byte is 8 bits
  - hex: 00 - FF
  - write 0x123f in C for hex number

## 1.1 Boolean Algebra

- And  $\&$
- Or  $|$
- Not  $\sim$
- Xor  $\wedge$

### 1.1.1 example

76543210

01101001: {0, 3, 5, 6} in bit set

01010101: {0, 2, 4, 6} in bit set

- operate on bit vectors
- Representing and Manipulation of Sets
  - $\&$  intersection {0, 6}
  - $|$  union {0, 2, 3, 4, 5, 6}
  - $\wedge$  symmetric difference {2, 3, 4, 5}
  - $\sim$  complement

### 1.1.2 Contrast to Logic Operation

- logical operator  $\&\&$  and,  $||$  or,  $!$  nor
  - 0 as False
  - return 0 or 1
  - early termination
  - example: `p && *p` avoids null pointer access

### 1.1.3 Shift Operation

- left shift  $x \ll y$
- right shift  $x \gg y$ 
  - logical shift: pad with 0
  - arithmetic shift: pad with left most bit

- undefined behavior
  - shift amount  $< 0$  or  $\geq$  word size

11100010	operation
00010000	$\ll 3$
00111000	Log. $\gg 2$
11111000	Arith. $\gg 2$

## 1.2 Integer

- Unsigned
- Signed
  - 2's complement
  - sign bit: most significant bit
    - \* 0: nonnegative
    - \* 1: negative

### 1.2.1 example

```

15213
00111011 01101101
-15213
11000100 10010011

```

-15213 in binary	weight	value	
1	1	1	least significant bit
1	2	2	
0	4	0	
0	8	0	
1	16	16	
0	32	0	
0	64	0	
1	128	128	
0	256	0	
0	512	0	
1	1024	1024	
0	2048	0	
0	4096	0	
0	8192	0	
1	16384	16384	
1	-32768	-32768	most significant bit
		-15213	

### 1.2.2 Numerical Range

- Numerical Range
  - Unsigned
    - \*  $umin: 0$
    - \*  $umax: 2^w - 1$
  - Signed
    - \*  $tmin: -2^{w-1}$
    - \*  $tmax: 2^{w-1} - 1$

X	B2U(X) unsigned	B2T(X) signed
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

#### • 2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive

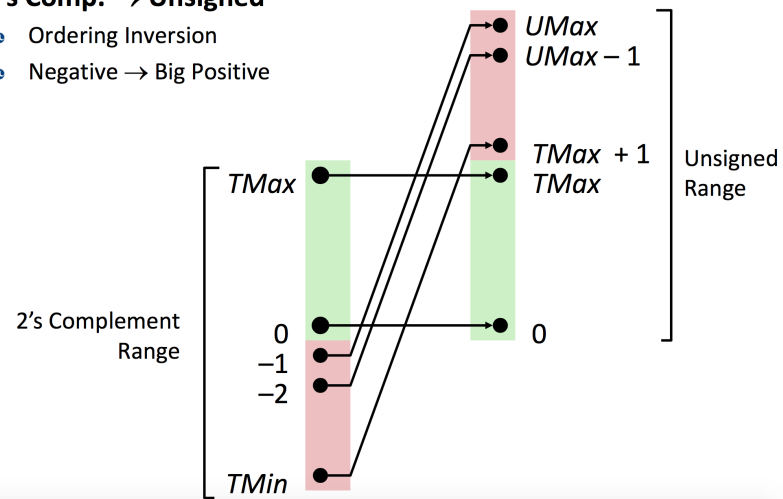


Figure 1: 2's complement

### 1.2.3 Signed vs Unsigned in C

- Constants

- defaults to signed
- unsigned: 4294967295U

- Casting

- mix of unsigned and signed: signed are implicitly cast to unsigned

#### 1. Observation

- $|TMIN| = TMAX + 1$
- $UMAX = 2 * TMAX + 1$
- For C programming

```
#include <limits.h>
```

```
#define ULONG_MAX
```

```
#define LONG_MIN
```

#### 1.2.4 example

W = 32 (word size)

TMIN = -2147483648

TMAX = 2147483647

const1	const2	result	
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int)2147483648U	>	signed

#### 1.2.5 Sign Extension

- w-bit signed integer
- convert to w+k-bit integer with same value
- make k copies of signed bit

word size	decimal	hex	bin
2	-15213	C4 93	11000100 10010011
4	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Make  $k$  copies of sign bit:
- $X' = \underbrace{x_{w-1}, \dots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \dots, x_0$

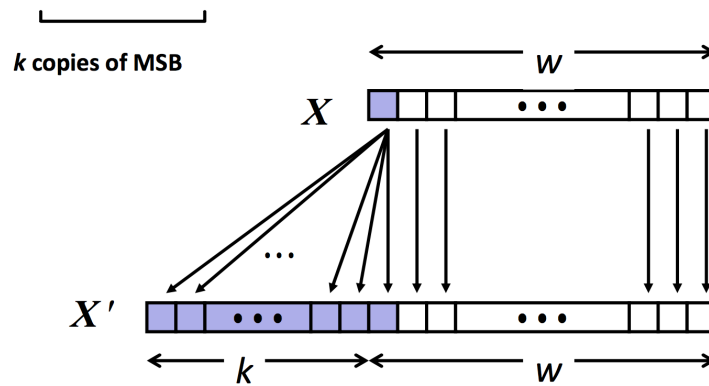


Figure 2: sign extension