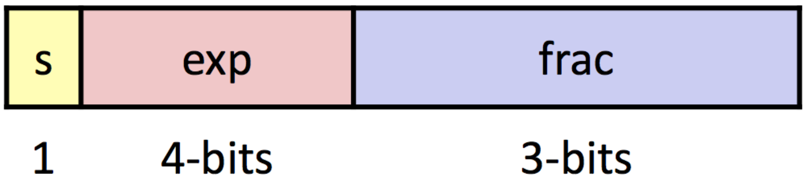
# Lecture 9

*<2016-05-02 Mon>*

## Contents

# 1 Floating Point

### 1.0.1 Tiny Floating Point Example



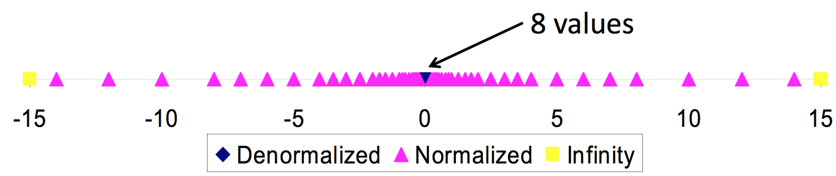

- 8-bit floating point representation
  - the sign bit is in the most significant bit
  - the next four bits are the exponent, with a bias of 7
  - the last 3 bits are the frac

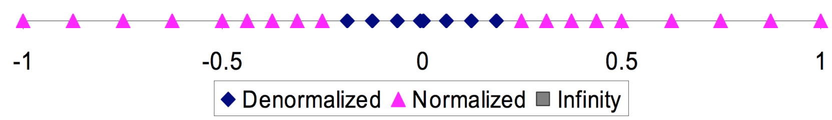
| state | s | exp | frac | E | value | explanation |
|---|---|---|---|---|---|---|
| | 0 | 0000 | 000 | -6 | 0 | |
| denormalized | 0 | 0000 | 001 | -6 | 1/512 | closest to 0 |
| numbers | 0 | 0000 | 010 | -6 | 2/512 | |
| | ... | | | | | |
| | 0 | 0000 | 110 | -6 | 6/512 | |
| | 0 | 0000 | 111 | -6 | 7/512 | largest denorm |
| | 0 | 0001 | 000 | -6 | 8/512 | smallest norm |
| | 0 | 0001 | 001 | -6 | 9/512 | |
| normalized | ... | | | | | |
| numbers | 0 | 0110 | 110 | -1 | 14/16 | |
| | 0 | 0110 | 111 | -1 | 15/16 | closest to 1 below |
| | 0 | 0111 | 000 | 0 | 1 | |
| | 0 | 0111 | 001 | 0 | 9/8 | closest to 1 above |
| | ... | | | | | |
| | 0 | 1110 | 110 | 7 | 224 | |
| | 0 | 1110 | 111 | 7 | 240 | largest norm |
| | 0 | 1111 | 000 | n/a | inf | |

## 1.1 Distribution of Values

- 6-bits IEEE-like format
  - e = 3 exponent bits

  - f = 2 fraction bits
  - bias is $2^{3\text{-}1}$ - 1 = 3
  - ditribution gets denser toward 0




## 1.2 Special Properties of the IEEE Encoding

- floating point 0 is the same as integer 0
  - all bits 0
- can almost use unsigned integer comparison
  - must first compare sign bits
  - must consider -0 = 0
  - `NaN` (s) problematic
    * will be greater than any other values
    * what should comparison yield
  - otherwise OK
    * denormalized vs normalized
    * normalized vs infinity

## 1.3 Floating Point Operation

- basic idea
  - first compute exact result
  - make it fit into desired precision
    * possibily overflow if exponent too large
    * possibily round to fit into fraction part of float

### 1.3.1 Round to Even

1. Rounding Modes

| mode | 1.40 | 1.60 | 1.50 | 2.50 | -1.50 |
|---|---|---|---|---|---|
| towards 0 | 1 | 1 | 1 | 2 | -1 |
| round down -$\infty$ | 1 | 1 | 1 | 2 | -2 |
| round up $+\infty$ | 2 | 2 | 2 | 3 | -1 |
| nearest even | 1 | 2 | 2 | 2 | -2 |

- round-to-even: default rounding mode
  - hard to get any other kind wothout dropping into assembly
  - all other are statistically biased
    - sum of set of positive numbers will consistently be over or under estimated
- applying to other decimal places / bit positions
  - when exactly halfway between 2 possible values
    - **round so that least significant digit is even**

2. rounding binary numbers

- binary fractional numbers
  - "even" when least significant bit is 0
  - "half" way when bits to right of rounding position = $100\ldots_2$

Table 1: round to nearest 1/4 (2 bits right of binary point)

| value | binary | rounded | action | bounded |
|---|---|---|---|---|
| 3/32 | `0.00011` | `0.00` | down | 0 |
| 3/16 | `0.00110` | `0.01` | up | 1/4 |
| 7/8 | `0.11100` | `1.00` | up | 1 |
| 5/8 | `0.10100` | `0.10` | down | 1/2 |

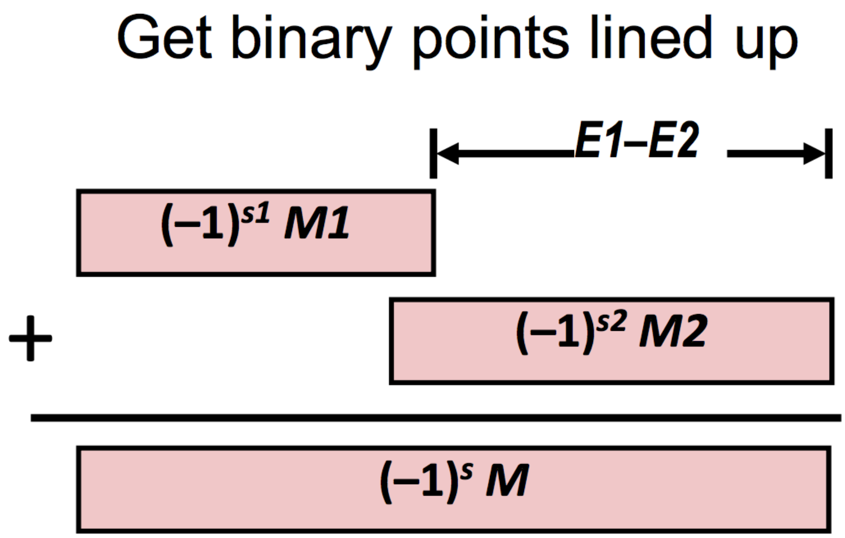
### 1.3.2 Floating Point Multiplication

- $(-1)^{s1}M2^{E1} \times (-1)^{s2}M2^{E2}$
- exact result (-1)$^{\mathrm{s}}$ M 2$^{\mathrm{E}}$
  - sign `s`: `s1 ^ s2`

  - significand `M`: `M1 * M2`
  - exponent `E`: `E1 + E2`
- fixing
  - if `M` $>=$ 2, shift `M` right, increment `E`
  - if `E` out of range, overflow
  - round `M` to fit `frac` precision
- implementation
  - biggest chore is multiplying significand

### 1.3.3 Floating Point Addition

- $(-1)^{s1}M2^{E1} + (-1)^{s2}M2^{E2}$
- exact result: (-1)$^{s}$ M $2^2$
  - sign `s`, significand `M`:
    - result of signed align & add
  - exponent `E`: `E1`
- fixing
  - if M $>=$ 2, shift `M` right, increment `E`
  - if M $<$ 1, shift `M` left `k` positions, decrement `E` by `k`
  - overflow if `E` out of range
  - round `M` to fit `frac` range




### 1.3.4 Floating Point in C

- C guarantees two levels
  - `float` single precision
  - `double` double precision
- conversion/casting
  - casting between int, float, and double changes bit representation
  - `double/float => =int`
    - truncates fractional part
    - like rounding towards 0
    - not defined when out of range or NaN: generally sets to `TMIN`
  - `int => =double`
    - exact conversion, as long as `int` has $<=$ 53 word size
  - `int => =float`
    - will round according to rounding mode

1. Floating Point Puzzle

```
int x;
float f;
double d;
```

| condition | expression | result | explanation |
|---|---|---|---|
| `f` | `-(-f)` | true | |
| `d < 0.0` | `(d * 2 < 0.0)` | true | |
| `d > f` | `-f < -d` | true | |
| | `d*d >= 0.0` | true | |
| | `(d+f) - d == f` | false | add a large number d to a small number f, precision over |

## 1.4 Summary

- IEEE Floating Point has clear mathematical properties
- represents numbers of form $M \times 2^E$
- can reason about operations independent of implementation
  - as if computed with perfect precision then rounded
- not the same as real arithmetic

# 2 Optimization

## 2.1 Optimizing Compilers

- provide efficient mapping of program to machine
  - register allocation
  - code selection and ordering (scheduling)
  - dead code elimination
  - eliminating minor inefficiencies
- don't improve asymptotic efficiency
  - up to programmers to select best overall algorithms
  - big-O savings are (often) more important than constant factors
- have difficulty overcoming "optimization blockers"
  - potiential memory aliasing
  - potiential procedure side-effect

## 2.2 Generally Useful Optimizations

optimizations that you or the compiler should do regardless of processor/compiler

### 2.2.1 Procedure Calls

- procedure may have side effects
  - alter global state each time called
- function may not return same value for given arguments
  - depends on other parts of global state
  - procedure `lower` could interact with `strlen`

```
void lower(char *s) {
    size_t i;
    for (i = 0; i < strlen(s); i++)
        if (s[i] >= 'A' && s[i] <= 'Z')
            s[i] -= 'A' - 'a';
}
```

- `strlen` called multiple times
- move `strlen` outside of loop, since result does not change from 1 iteration to another

### 2.2.2 Example of Memory Aliasing

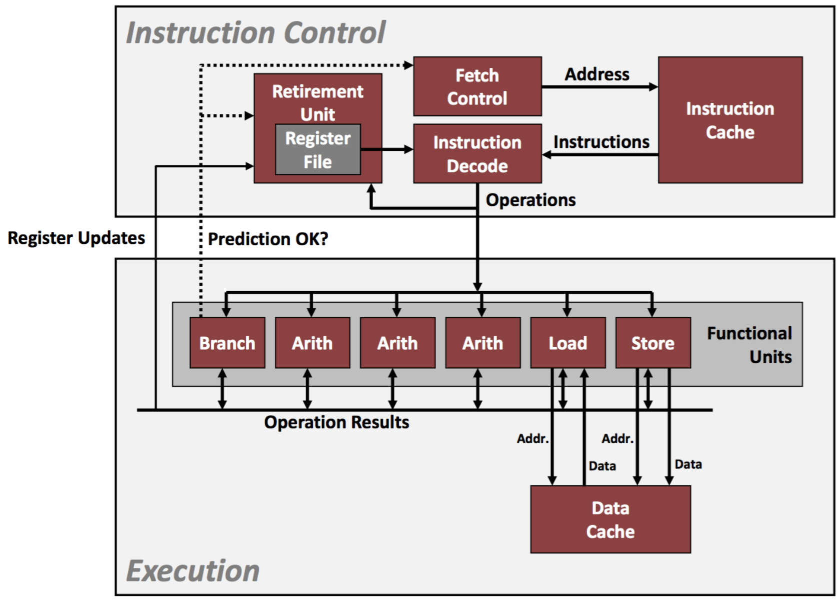
- aliasing
  - 2 different memory references specify single location
  - easy to happen in C
    - since allowed to do address arithmetic
    - direct access to storage structures
  - git in habit of introducing local variables
    - accumulating within loops
    - your way of telling compiler not to check for aliasing

```
void sum_row2(double *a, doubke *b, long n) {
  long i, j;
  for (int i = 0; i < n; ++i) {
    b[i] = 0;
    for (int j = 0; j < n; ++j)
      b[i] += a[i*n + j];
  }
}

void sum_row2(double *a, doubke *b, long n) {
  long i, j;
  for (int i = 0; i < n; ++i) {
    double val = 0;             /* remove memory aliasing */
    for (int j = 0; j < n; ++j)
      val += a[i*n + j];
    b[i] = val;
  }
}
```

- code updates `b[i]` on every iteration
- must consider possiblities that these updates will affect program behavior

### 2.3 Instruction-Level Parallelism

- need general understanding of modern processor design
  - hardware can execute multiple instructions in parallel
- performance limited by data dependencies
- simple transformations can yield dramatic performance improvement
  - compilers often cannot make these transformations
  - lack of associativity and distributivity in floating point arithmetic




### 2.3.1 Superscalar Processor

- Def: a superscalar processor can issue and execute multiple instructions in one cycle. Instructions are retrieved from a sequential instruction stream and are usually scheduled dynamically
- Benefit: without programming effort, superscalar processor can take advantage of the instruction level parallelism that most programs have
- most modern CPUs are superscalar

### 2.3.2 Pipelined Functional Units

```
long mult_eg(long a, long b, long c) {
  long p1 = a*b;
  long p2 = a*c;
  long p3 = p1*p2;
  return p3;
}
```

- divide computations into stages
- pass partial computations from stage to stage

- stage i can start on new computation once values passed to i+1
- e.g. complete 3 multiplications in 7 cycles, even though each requires 3 cycles (9 cycles in total)

| Time | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Stage 1 | a*b | a*c | | | p1*p2 | | |
| Stage 2 | | a*b | a*c | | | p1*p2 | |
| Stage 3 | | | a*b | a*c | | | p1*p2 |

### 2.3.3 Loop Unrolling

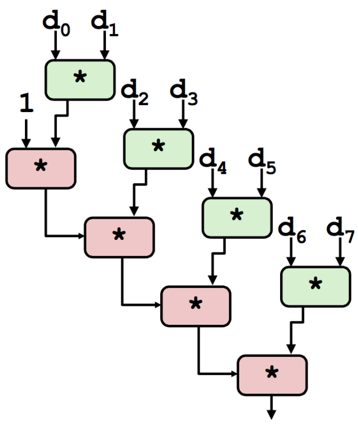
- loop unrolling: perform more useful work per iteration

```
for (i = 0; i < limit; i += 2)
    x = (x OP d[i]) OP d[i+1];

/* loop unrolling with reassociation */
for (i = 0; i < limit; i += 2)
    x = x OP (d[i] OP d[i+1]);

/* loop unrolling with separate accumulators */
for (i = 0; i < limit; i += 2) {
    x0 = x0 OP d[i];
    x1 = x1 OP d[i+1];
}
```

- loop unrolling
  - helps integers add
  - still sequential dependency
- loop unrolling with reassociation
  - faster for floating point
  - nearly 2X speedup for `int *`, `float +`, `float *`
  - breaks sequential dependency
- loop unrolling with separate accumulators

- int + makes use of 2 load units
- 2X speedup (over loop unrolling) for `int *`, `float +`, `float *`

```
x = x OP (d[i] OP d[i+1]);
```




```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```