Analysis 1 Blatt 2 Lösung

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October 22, 2018

Aufgabe 4

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a) $\forall n \in \mathbb{N}$ gilt $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

INDUKTION ANFANG: $n=1$

$$\sum_{k=1}^{1} 1^2 = 1 = \frac{1(1+1)(2\cdot 1+1)}{6} = \frac{1\cdot 2\cdot 3}{6} = \frac{6}{6} = 1 \Rightarrow \text{Wahr}$$

INDUKTION ANNAHME: $n = n+1$

$$\sum_{k=1}^{n+1} (n+1)^2 = \sum_{k=1}^{n} (n+1)^2 + (n+1)^2 = \sum_{k=1}^{n} (n+1)^2 + n^2+2n+1$$

$$= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{n(n+1)(2n+1)+6(n^2+2n+1)}{6} = \frac{(n^2+n)(2n+1)+6n^2+12n+6}{6}$$

$$= \frac{2n^3+2n^2+n^2+n+6n^2+12n+6}{6} = \frac{2n^3+9n^2+13n+6}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$