



# Haskell tc ucs 0 - Zusammenfassung Programmierung und Modellierung

Programmierung und Modellierung (Ludwig-Maximilians-Universität München)

# Haskell Typeclasses Cheat Sheet

## Function Synonyms

Functor	Applicative	Monad	List ([])	Fn. ((->) r)
fmap/(<\$>)	liftA	liftM	map	(.)
	pure	return	(:[])	const
	(<*>)	ap	(<*>)	(<*>)
		(>=)	(>=)	(>=)
Functor	Applicative	Monad	List ([])	Fn. ((->) r)
	(>*)	(>)	(>)	(>)
		join	concat	join
		(=<)	concatMap	(=<)
	liftA2	liftM2	liftA2	liftA2
	liftA3	liftM3	liftA3	liftA3
	sequenceA	sequence	sequence	sequence
	traverse	mapM	traverse	traverse
	for	forM	for	for

## Functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

### Laws

```
identity      fmap id == id
distributivity  fmap (f . g) == fmap f . fmap g
```

### instances – fmap examples

```
[]          fmap f [x,y,z] == [f x, f y, f z]
Maybe      fmap f (Just x) == Just (f x)
IO          fmap length getLine == 4 - input: "test"
((->) r)     fmap f g == f . g
(Either a)  fmap f (Left x) == (Left x)
            fmap f (Right x) == (Right (f x))
((,) a)     fmap f (x,y) == (x,fy)
```

### Functions

```
<$> :: Functor f          f <$> x == fmap f x
=> (a -> b)              f <$> Just x == Just (f x)
-> f a -> f b            f <$> Left x == Left x

<$ :: Functor f          (x <$) == fmap (const x)
=> a -> f b -> f a      x <$ [y,z] == [x,x]

$> :: Functor f          ($> x) == fmap (const x)
=> f a -> b -> f b      [y,z] $> x == [x,x]

void :: Functor f          void f == () <$ f
=> f a -> f ()          void [x,y] == [(),()]
```

## Applicative

```
class Applicative f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

### Laws

```
identity      pure id <*> v == v
composition   pure (.) <*> u <*> v <*> w == u <*> (v <*> w)
homomorphism   pure f <*> pure x == pure (f x)
interchange     u <*> pure y == pure ($ y) <*> u
```

### Instances – pure examples

```
[]          pure x == [x]
Maybe      pure x == Just x
IO          pure x == x "inside" IO
((->) r)     pure x == const x
(Either a)  pure x == Right x
```

### Instances – apply (<\*>) examples

```
[]          [f,g] <*> [x,y] == [f x, g x, f y, g y]
            [(+),(*)] <*> [x,y] <*> [z] == [x+z,y+z,x*z,y*z]
Maybe      Just f <*> Just x == Just (f x)
            Just f <*> Nothing == Nothing
IO          pure (++) <*> getLine <*> getLine
((->) r)     (f <*> g) x == f x (g x)
            (f <*> g <*> h) x == f x (g x) (h x)
(Either a)  Right f <*> Right x == Right (f x)
            Left e1 <*> Right x == Left e1
            Right f <*> Left e2 == Left e2
            Left e1 <*> Left e2 == Left e1
```

### Functions

```
(>*) :: Applicative f          (>*) == flip (<*)
=> f a -> f b -> f b          [x] > [y,z] == [y,z]

(<*) :: Applicative f          (<*) == liftA2 const
=> f a -> f b -> f a          [x] <* [y,z] == [x,x]

liftA :: Applicative f          liftA f x == fmap f x
=> (a -> b)                    liftA f [x,y] == [f x, f y]
-> f a -> f b                    liftA f Nothing == Nothing

liftA2 :: Applicative f          liftA2 (+) (Just x) (Just y)
=> (a -> b -> c)                ==
-> f a -> f b -> f c              Just (x + y)

liftA3 :: Applicative f          liftA3 f [x,y] [z] [w,u]
=> (a -> b -> c -> d)            ==
-> f a -> f b                    [ f x z w, f x z u
-> f c -> f d                    , f y z w, f y z u ]
```

### Functor and Applicative common idioms

```
f <$> x <*> y == pure f <*> x <*> y == fmap f x <*> y
f <$> g <*> x == pure f <*> g <*> x
x <*> v $> z == x <*> v >*> pure z
```

## Monad

```
class Applicative m => Monad m where
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b
```

### Laws

```
left identity   return x >= f == f x
right identity   m >= return == m
associativity    (m >= f) >= g == m >= (\x -> f x >= g)
```

### Instances – return examples

```
[]          return x == [x]
Maybe      return x == Just x
IO          return x == x "inside" IO
((->) r)     return x == const x
(Either a)  return x == Right x
```

### Instances – bind (>=) examples

```
[]          xs >= f == concatMap f xs
            [x,y] >= replicate 3 == [x,x,x,y,y,y]
Maybe      Just x >= f == f x
            Just [x] >= listToMaybe == Just x
IO          getLine >= putStrLn
((->) r)     (f >= g) x == g (f x) x
            (tail >= (++)) [x,y] == [y,x,y]
(Either a)  Right x >= f == f x
            Left e1 >= f == Left e1
```

### Instances – then (>) examples

```
[]          [x,y] > [z,w,u] == [z,w,u,z,w,u]
Maybe      Nothing > Just x == Nothing
            Just x > Just y == Just y
IO          putStrLn "Username:" > getLine
((->) r)     (f > g) x == g x
(Either a)  Right x > Right y == Right y
            Left e1 > Right y == Left e1
            Right x > Left e2 == Left e2
            Left e1 > Left e2 == Left e1
```

### Do notation

do notation	≈	desugared do notation
do patA <- action1		action1 >= \patA ->
action2		action2 >
patB <- action3		action3 >= \patB ->
action4		action4

## Functions

```
mapM :: Monad m          mapM f ≡ sequence . map f
    => (a -> m b)
    -> [a] -> m [b]

sequence :: Monad m      sequence [Just x, Just y]
    => [m a]              ≡
    -> m [a]              Just [x,y]

(>=>) :: Monad m          (f >=> g) x ≡ f x >= g
    => (a -> m b)
    -> (b -> m c)         iterate (+1) >=> replicate 2 $ 0
    -> a -> m c            ≡ [0,0,1,1,2,2,3,3,4,4,5,5...]

foldM :: Monad m          foldM f a [x, y, z]
    => (a -> b -> m a)     ≡ do a1 <- f a x
    -> a -> [b]            a2 <- f a1 y
    -> m a                 f a2 z

replicateM :: Monad m     replicateM 2 [x,y]
    => Int -> m a          ≡
    -> m [a]              [[x,x],[x,y],[y,x],[y,y]]

when, unless :: Monad m   when verbose (putStrLn "msg")
    => Bool
    -> m ()               unless quiet (putStrLn "msg")
    -> m ()

forever :: Monad m        forever (getLine >= putStrLn
    -> m a -> m b         (⌊ poor man's cat)

<fn>_ :: Monad m          sequence_ ≡ void sequence
    => ... -> m ()         mapM_ ≡ void mapM
```

## Monoid

```
class Monoid a where
    mempty :: a
    mappend :: a -> a -> a    -- NOTE: (<>) infix synonym
```

### Laws

```
identity      mempty <> x ≡ x ≡ x <> mempty
associativity x <> (y <> z) ≡ (x <> y) <> z
```

### Instances – mempty and mappend (<>) examples

```
Ordering mempty ≡ EQ      EQ <> GT ≡ GT ≡ GT <> LT
()        mempty ≡ ()      () <> () ≡ ()
[a]       mempty ≡ []      xs <> ys ≡ xs ++ ys
Maybe a  mempty ≡ Nothing Just m <> Just n
                                ≡ Just (m <> n)
```

### Functions

```
(<>) :: Monoid a => a -> a -> a    (<>) ≡ mappend

mconcat :: Monoid a      mconcat ≡ foldr (<>) mempty
    => [a] -> a           mconcat [[x],[y]] ≡ [x,y]
```

## Foldable

```
class Foldable t where
    foldr :: (a -> b -> b) -> b -> t a -> b
```

NOTE: Foldable instances can also be defined by foldMap

### Instances – foldr examples

```
[]          foldr f z [x,y] ≡ x ‘f’ (y ‘f’ z)
Maybe      foldr f x (Just y) ≡ x ‘f’ y
(Either a)   foldr f x (Left y) ≡ x
((,) a)      foldr f x (y,z) ≡ x ‘f’ z
```

### Functions

```
foldMap :: ( Foldable t          foldMap f ≡ fold . map f
               , Monoid m )       foldMap f [x,y] ≡ f x <> f y
    => (a -> m)                   foldMap f (Just x) ≡ f x
    -> t a -> m                 foldMap Nothing ≡ mempty

fold :: ( Foldable t          fold ≡ foldMap id
          , Monoid m )        fold [x,y,z] ≡ x <> (y <> z)
    -> t m -> m                fold Nothing ≡ mempty

foldl :: (Foldable t)         foldl f z [x,y] ≡ (x ‘f’ y) ‘f’ z
    => (b -> a -> b)
    -> b -> t a -> b

toList :: (Foldable t) => t a -> [a]   toList (x,y) ≡ [y]
null :: (Foldable t) => t a -> Bool     null Nothing ≡ True
length :: (Foldable t) => t a -> Int    length [x,y] ≡ 2
concat :: Foldable t => t [a] -> [a]    concat ≡ fold

elem,notElem :: (Foldable t, Eq a) => a -> t a -> Bool
maximum, minimum :: (Foldable t, Ord a) => t a -> a
sum, product :: (Foldable t, Num a) => t a -> a
asum :: (Foldable t, Alternative f) => t (f a) -> f a
concatMap :: Foldable t => (a -> [b]) -> t a -> [b]
and, or :: Foldable t => t Bool -> Bool
any, all :: Foldable t => (a -> Bool) -> t a -> Bool
find :: Foldable t => (a -> Bool) -> t a -> Maybe a
```

## Operators (grouped by precedence)

6(++): mappend	<>
4(&&): fmap, repl. value apply, apply discard	<\$>, <\$, \$> <*>, **>, <*>
3(  ): choice	< >
1(\$): bind, then rv. bind, kleisli c.	(>=), (>) (=≪), (>=>), (<=<)

NOTE: left and right alignments indicate left- and right- associativity

## Traversable

```
class (Functor t, Foldable t) => Traversable t
    traverse :: Applicative f => (a -> f b) -> t a -> f (t b)
    sequenceA :: Applicative f => t (f a) -> f (t a)
```

NOTE: It is only necessary to define either traverse or sequenceA.

### Equivalences (laws are omitted here)

```
traverse f ≡ sequenceA . fmap f
sequenceA ≡ traverse id
```

### Instances – traverse examples

```
[]          traverse f [x,y] ≡
    (:) <$> f x <*> ((:) <$> f y <*> [])
Maybe      traverse f (Just x) ≡ Just <$> f x
(Either a)   traverse f (Right x) ≡ Right <$> f x
             traverse f (Left el) ≡ pure (Left el)
((,) a)      traverse f (x,y) ≡ (,) x <$> f y
```

### Instances – sequenceA examples

```
[]          sequenceA [x,y] ≡ (:) <$> x <*> ((:) <$> y <*> [])
Maybe      sequenceA (Just x) ≡ Just <*> x
Either      sequenceA (Right x) ≡ Right <*> x
((,) a)     sequenceA (x,y) ≡ (,) x <$> y
```

### Functions

```
for :: (Traversable t, Applicative f) flip for ≡ traverse
    => t a -> (a -> f b) -> f (t b)

mapM :: (Traversable t, Monad m)
    => (a -> m b) -> t a -> m (t b)

sequence :: (Traversable t, Monad m)
    => t (m a) -> m (t a)
```

## Alternative

```
class Applicative f => Alternative f where
    empty :: f a
    (<|>) :: f a -> f a -> f a
```

### Laws

```
identity      mempty <|> x ≡ x ≡ x <|> mempty
associativity x <|> (y <|> z) ≡ (x <|> y) <|> z
```

### Instances – empty and choice (<|>) examples

```
[a]          empty ≡ []              xs <|> ys ≡ xs ++ ys
Maybe a     empty ≡ Nothing         Just x <|> Just y ≡ Just x
                                         Nothing <|> Just y ≡ Just y
```