

1 Tutorium 3

1.1 Aufgabe 1

i) A, E, F, G, C

ii) $B := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \xrightarrow{I \leftrightarrow II} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
 $D := \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \xrightarrow{I - II} \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \xrightarrow{II - I} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

2) $\left(\begin{array}{cccc|cccc} 1 & 4 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{2+I \\ II \leftrightarrow III}} \left(\begin{array}{cccc|cccc} 1 & 4 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \downarrow$
b) $\left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right) \checkmark$ d) $\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right) \xrightarrow{I - 2II} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 0 & 1 & 0 & 1 \end{array} \right)$
c) $\left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right) \downarrow$ $D' = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \checkmark$
e) $\left(\begin{array}{cccc|cccc} 1 & 5 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{array} \right) \xrightarrow{\substack{I - 5II \\ II \leftrightarrow I}} \left(\begin{array}{cccc|cccc} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 5 & 0 & 2 & 1 & 0 & 0 & 0 \end{array} \right) \downarrow$
f) $\left(\begin{array}{cccc|cccc} 0 & 1 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \downarrow$
Beobachtung: Es scheint eine Abhängigkeit zu sein zwischen Rang der Matrix und Anzahl der Variablen
 \Rightarrow Wenn $\text{Rang} < n$ sind n nicht invertierbar.

1.2 Aufgabe 2

Name / Matrikelnr. / Tutorium

Aufgabe 2 $A \in \mathbb{R}^{3 \times 3}, a \in \mathbb{R}$

$$\bar{A} := \left(\begin{array}{ccc|c} a-1 & 0 & a+1 & 5a-8 \\ 2(a-1) & 1 & 2(a+1) & 10a-9 \\ (a-1) & -2 & 0 & 2a-6 \end{array} \right) \xrightarrow{\substack{I \cdot III \\ II \cdot 2}} \left(\begin{array}{ccc|c} 0 & 2 & a+1 & 3a+8 \\ 0 & 1 & 0 & a+9 \\ a-1 & -2 & 0 & 2a-6 \end{array} \right) \begin{array}{l} \text{Zeilen} \\ \Rightarrow \\ \text{unsortiert} \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} a-1 & -2 & 0 & 2a-6 \\ 0 & 2 & a+1 & 3a+8 \\ 0 & 1 & 0 & 9 \end{array} \right) \xrightarrow{\substack{I \cdot (a+1) \\ I \cdot 2 \cdot III}} \left(\begin{array}{ccc|c} 1 & -\frac{2}{a+1} & 0 & \frac{2a-6}{a+1} \\ 0 & 0 & a+1 & 3a+10 \\ 0 & 1 & 0 & 9 \end{array} \right) \Rightarrow \begin{array}{l} \text{Zeilen} \\ \text{unsortiert} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -\frac{2}{a+1} & 0 & \frac{2a-6}{a+1} \\ 0 & 1 & 0 & 9 \\ 0 & 0 & a+1 & 3a-10 \end{array} \right) \xrightarrow{\substack{I + 2 \cdot II \\ III \cdot \frac{1}{a+1}}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2(a-8)}{a+1} + \frac{18}{a+1} \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & \frac{3a-10}{a+1} \end{array} \right) \begin{array}{l} = \frac{2a+34}{a+1} \\ = \frac{3a+16+18}{a+1} \end{array}$$

$\text{rang}(A) = \text{rang}(\bar{A}) = n$ $\Gamma_{h_i} = 3$

\Rightarrow Also hat \bar{A} nur eine eindeutige Lösung:

~~$L := \{v \in \mathbb{R}^3 \mid Av = b\}$~~ $L := \{v \in \mathbb{R}^3 \mid Av = b\}$

\Rightarrow

$$L = \left\{ \left(\frac{2(a+17)}{a+1}, 9, \frac{3a-10}{a+1} \right) \in \mathbb{R}^3 \mid a \neq -1 \right\}$$

1.3 Aufgabe 4

Sei $A \in \mathbb{K}^{3 \times 3}$, $A := \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

$$\Rightarrow AA = \begin{pmatrix} a_{11}^2 + a_{12}^2 + a_{13}^2 & a_{11}a_{12} + a_{12}a_{22} + a_{32}a_{13} & a_{13}a_{11} + a_{23}a_{12} + a_{33}a_{13} \\ a_{11}a_{21} + a_{21}a_{22} + a_{31}a_{23} & a_{21}^2 + a_{22}^2 + a_{32}^2 & a_{21}a_{13} + a_{22}a_{23} + a_{23}a_{33} \\ a_{11}a_{31} + a_{21}a_{32} + a_{31}a_{33} & a_{31}a_{12} + a_{32}a_{22} + a_{33}a_{23} & a_{31}a_{13} + a_{23}a_{32} + a_{33}^2 \end{pmatrix}$$

Da $A = A^T \Rightarrow a_{12} = a_{21} \wedge a_{23} = a_{32} \wedge a_{13} = a_{31}$

$$\Rightarrow A^T A = \begin{pmatrix} a_{11}^2 + a_{22}^2 + a_{33}^2 & a_{11}a_{12} + a_{12}a_{22} + a_{23}a_{13} & a_{13}a_{11} + a_{23}a_{12} + a_{33}a_{13} \\ a_{11}a_{12} + a_{12}a_{22} + a_{23}a_{13} & a_{12}^2 + a_{22}^2 + a_{23}^2 & a_{12}a_{13} + a_{22}a_{23} + a_{23}a_{33} \\ a_{11}a_{13} + a_{12}a_{23} + a_{23}a_{33} & a_{13}a_{12} + a_{23}a_{22} + a_{23}a_{33} & a_{13}^2 + a_{23}^2 + a_{33}^2 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} a_{11} \cdot (a_{11} + a_{12} + a_{13}) & a_{21} \cdot (a_{11} + a_{12} + a_{13}) & a_{31} \cdot (a_{11} + a_{12} + a_{13}) \\ a_{11} \cdot (a_{12} + a_{22} + a_{23}) & a_{21} \cdot (a_{12} + a_{22} + a_{23}) & a_{31} \cdot (a_{12} + a_{22} + a_{23}) \\ a_{11} \cdot (a_{13} + a_{23} + a_{33}) & a_{21} \cdot (a_{13} + a_{23} + a_{33}) & a_{31} \cdot (a_{13} + a_{23} + a_{33}) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a_{11} \cdot (a_{11}) & a_{21} \cdot (a_{11}) & a_{31} \cdot (a_{11}) \\ a_{11} \cdot (a_{12}) & a_{21} \cdot (a_{12}) & a_{31} \cdot (a_{12}) \\ a_{11} \cdot (a_{13}) & a_{21} \cdot (a_{13}) & a_{31} \cdot (a_{13}) \end{pmatrix} \quad \checkmark$$

Also können wir verallgemeinern.

\Rightarrow Sei $A \in \mathbb{K}^n$ definiert durch a_{ij} und A^T seine transponierte Form mit $a_{ij} = a_{ji}$

$$\begin{pmatrix}
 \boxed{a_1(a_1)} & \boxed{a_2(a_1)} & \dots & \boxed{a_{i-1}(a_1)} & \boxed{a_j(a_1)} & \dots & \boxed{a_i(a_1)} \\
 \boxed{a_1(a_2)} & \boxed{a_2(a_2)} & \dots & \boxed{a_{i-1}(a_2)} & \boxed{a_j(a_2)} & \dots & \boxed{a_i(a_2)} \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
 \boxed{a_1(a_{j-1})} & \boxed{a_2(a_{j-1})} & \dots & \boxed{a_{i-1}(a_{j-1})} & \boxed{a_j(a_{j-1})} & \dots & \boxed{a_i(a_{j-1})} \\
 \boxed{a_1(a_j)} & \boxed{a_2(a_j)} & \dots & \boxed{a_{i-1}(a_j)} & \boxed{a_j(a_j)} & \dots & \boxed{a_i(a_j)} \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots
 \end{pmatrix}$$

Symmetrie ist dank der Multiplikation immer
 gegeben. \square