# Grassmann log model: a neural network layer to process subspaces



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# (1) Motivation and objective

- We propose the **Log module** to map data from a Grassmann manifold to a vector space.
- The **Grassmann manifold** is the set of subspaces of a vector space:
- It is a foundation for various types of machine learning tools using subspace representation.
- - Most standard machine learning methods cannot be promptly utilized on the Grassmann manifold, since they are constructed on Euclidean space.
- It is hard to directly link the Grassmann manifold to deep neural network architectures.

- Motivation:
- a subspace is well known as a practical and robust representation, being applied to numerous problems such as image set recognition, fine-grained classification and action recognition.
- Abundance of well-established end-to-end network layers which work in Euclidean space, e.g. fully-connected, batch normalization, dropout.
- We attempt to fill these gaps by **connecting** those two representations.
- The key idea is to formulate the manifold logarithmic map (log) as an end-to-end learnable model.

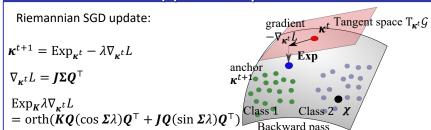
(4) Qualitative results

#### (2) Proposed Log model Tangent space TG(d, m) $X \in \mathbb{R}^{d \times m}$ Tangency vector h **PCA** centeri subspace x Euclidean network Output Grassmann manifold G(d, m)modules vector input image -fully-connected Proposed interface from $\chi$ to h -batch normalization Definition of the Log module: Grassmann Log:

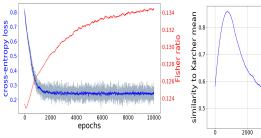
$$h = \text{vec}(\text{Log}_K X) \qquad \text{Log}_K X = W_{:m} \arctan(\Theta_{:m}) Z_{:m}$$

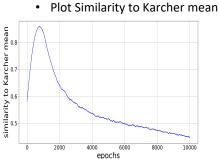
$$(\mathbf{K}^{\mathsf{T}}\mathbf{X})^{-1}(\mathbf{K}^{\mathsf{T}} - \mathbf{K}^{\mathsf{T}}\mathbf{X}\mathbf{X}^{\mathsf{T}}) = \mathbf{W}\mathbf{\Theta}\mathbf{Z}^{\mathsf{T}}$$

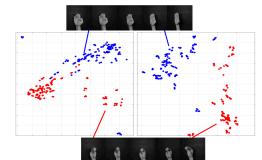
# (3) Backward pass



## • Plot of Loss and Fisher ratio







 tSNE visualizations of 2 classes of hand shapes. The left plot denotes the tangent vectors at the Karcher mean, while the right plot shows the tangent vectors at the log model learned tangent space.

# (5) Experimental results

 To evaluate the proposed method, we use the following databases and achieved the following recognition rates:

Hand Shape dataset		
/lethod	Accuracy (%)	
Carcher Log model	70.65	
og model	81.90	
Conv+Log model	91.90	
Resnet18+Log model	99.40	

AFEW dataset	
Method	Accuracy (%)
RSR-SPDML	30.12%
DCC	25.78%
GDA	29.11%
GGDA	29.45%
PML	28.98%
DeepO2P	28.54%
SPDNet	34.23%
GrNet-1	32.08%

34.23%

32.61%

GrNet-2

Log Model

### (6) Conclusions

- We proposed in this paper the Grassmann log model to map subspace data to vector spaces while maximizing discrimination capability for classification.
- The Log model can be learned with RSGD along with other NN layers.
- Future works include the extension of this idea to other Riemannian manifolds; and to other applications, such as video retrieval, modeling of matrices in signal processing, and text modeling.

#### (7) Publications

- [1] Lincon Souza, Naoya Sogi, Bernardo Gatto, Takumi Kobayashi, Kazuhiro Fukui, "An Interface between Grassmann manifolds and vector spaces", CVPRW 2020.
- [2] Lincon Souza, Bernardo Gatto, Jing-Hao Xue, Kazuhiro Fukui, "Enhanced Grassmann Discriminant Analysis with Randomized Time Warping for Motion Recognition", Pattern Recognition, 2020.
- [3] Lincon Souza, Bernardo Gatto, Kazuhiro Fukui, "Classification of Bioacoustic Signals with Tangent Singular Spectrum Analysis", ICASSP 2019.