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(1) Introduction

- **Image-set recognition:**
 - Efficient approach for object recognition.
 - Utilizing multiple images (multi-view, video) as an input.
- **Subspace-method:**
 - General approach for the image-set recognition.
 - 1. Representing an image-set by a low-dimensional subspace.
 - 2. Classifying an input image-set by the subspace-similarity
- **Our objective:** improving performance of subspace-based methods
- **Approach:** constructing a metric learning method with subspaces

(2) Proposed method

- Subspace similarity with a metric
 - $\text{sim}(\mathbf{S}_1, \mathbf{S}_2) = \|\hat{\mathbf{S}}_1^T \mathbf{A} \hat{\mathbf{S}}_2\|_F^2 = \sum_j \cos^2 \theta_j$, $\hat{\mathbf{S}}_i = \mathbf{S}_i \mathbf{U}_i \Sigma_i^{-1/2}$, $\mathbf{U}_i \Sigma_i \mathbf{U}_i^T = \mathbf{S}_i^T \mathbf{A} \mathbf{S}_i$.
 - θ_j : j -th canonical angle
 - \mathbf{S}_i : Basis of a subspace generated by PCA

- **Basic idea: Learn a suitable metric space**



- Cost function (**Margin maximization**)

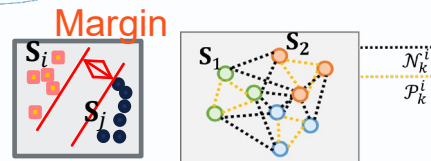
- $\mathbf{A}^* = \min_{\mathbf{A}} J(\mathbf{A}) = J_b(\mathbf{A}) + J_w(\mathbf{A}) + \lambda(1 - \|\mathbf{A}\|_F^2)^2$,

- $J_b(\mathbf{A}) = \frac{1}{kN} \sum_{i=1}^N \sum_{j \in \mathcal{N}_k^i} \text{sim}(\mathbf{S}_i, \mathbf{S}_j)$,

- $J_w(\mathbf{A}) = \frac{1}{kN} \sum_{i=1}^N \sum_{j \in \mathcal{P}_k^i} d^2(\mathbf{S}_i, \mathbf{S}_j) = \frac{1}{kN} \sum_{i=1}^N \sum_{j \in \mathcal{P}_k^i} (r - \text{sim}(\mathbf{S}_i, \mathbf{S}_j))^2$

- $\mathcal{N}_k^i / \mathcal{P}_k^i$: k -neighborhood of the i -th subspace

- Optimization:



- Use Riemannian conjugate gradient, RCG. (\mathbf{A} must be a SPD matrix)

- Need a gradient of $\text{sim}()$. $\text{sim}(\mathbf{S}_i, \mathbf{S}_j) = \|\hat{\mathbf{S}}_i^T \mathbf{A} \hat{\mathbf{S}}_j\|_F^2 = \text{tr}(\hat{\mathbf{S}}_j^T \mathbf{A} \hat{\mathbf{S}}_i \hat{\mathbf{S}}_i^T \mathbf{A} \hat{\mathbf{S}}_j) = \text{tr}(\hat{\mathbf{S}}_j \hat{\mathbf{S}}_j^T \mathbf{A} \hat{\mathbf{S}}_i \hat{\mathbf{S}}_i^T \mathbf{A})$

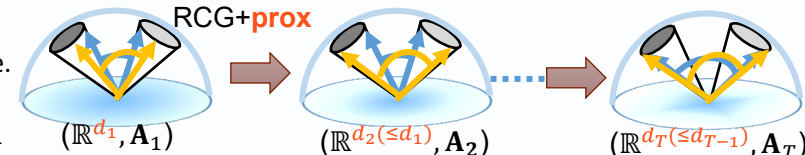
- Reformulate

$$= \text{tr}(\mathbf{S}_j \mathbf{U}_j \Sigma_j^{-1/2} \Sigma_j^{-1/2} \mathbf{U}_j^T \mathbf{S}_j^T \mathbf{A} \mathbf{S}_i \mathbf{U}_i \Sigma_i^{-1/2} \Sigma_i^{-1/2} \mathbf{U}_i^T \mathbf{S}_i^T \mathbf{A})$$

$$= \text{tr}(\mathbf{S}_i^T \mathbf{A} \mathbf{S}_j (\mathbf{S}_j^T \mathbf{A} \mathbf{S}_j)^{-1} \mathbf{S}_j^T \mathbf{A} \mathbf{S}_i (\mathbf{S}_i^T \mathbf{A} \mathbf{S}_i)^{-1}).$$

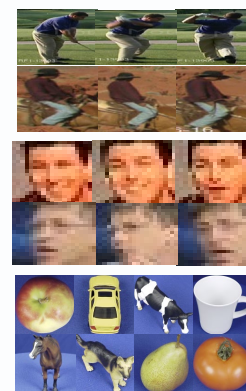
(3) Extension method

- $\text{Rank}(\mathbf{A}) = \text{dim. of the metric space} \rightarrow$ can **estimate the optimal dim automatically** by low-rank constraint
- Cost function: $\min_{\mathbf{A}} J(\mathbf{A}) + \eta \|\mathbf{A}\|_*, \|\cdot\|_*$ trace norm.
- Optimization: **RCG + proximal gradient method**
 - The trace norm constraint is not differentiable.
 - Apply the following operation after RCG step
 - $\text{prox}_{\eta}^{\text{tr}}(\mathbf{A}) = \mathbf{U} \max(\Sigma - \eta \mathbf{I}, \mathbf{0}) \mathbf{U}^T, \mathbf{U} \Sigma \mathbf{U}^T = \mathbf{A}$



(4) Results

- Datasets: 1. UCF sport(10 actions) 2. YTC(47 faces) 3. ETH(8 objects)



| model | method | UCF | YTC | ETH |
|-------------|----------------------|--------------|-------------------|-------------------|
| Affine hull | AHISD[1] | 53.33 | 56.31±4.61 | 70.25±4.78 |
| | GGDA[2] | 47.33 | 47.45±8.06 | 93.75±4.75 |
| subspace | PML[3] | 72.67 | 54.82±4.63 | 93.75±3.17 |
| | RMML[4] | 59.33 | 54.96±4.96 | 90.25±4.16 |
| | Proposed | 71.33 | 59.29±2.72 | 95.28±3.21 |
| | + $\ \mathbf{A}\ _*$ | 74.00 | 59.85±3.53 | 95.25±2.82 |

(5) Conclusion

- We introduced a metric space into a subspace-method and constructed metric learning frameworks by efficiently handling subspaces.
- We showed that our methods can achieve competitive results compared with various previous methods.

[1] Cevikalp, H. and Triggs, B. "Face recognition based on image sets", CVPR 2010.

[2] Harandi, M.T. et al, "Graph embedding discriminant analysis on Grassmannian manifolds for improved image set matching", CVPR 2011.

[3] Huang, Z. et al, "Projection metric learning on Grassmann manifold with application to video based face recognition", CVPR 2015.

[4] Zhu, P. et al, "Towards Generalized and Efficient Metric Learning on Riemannian Manifold. IJCAI 2018.