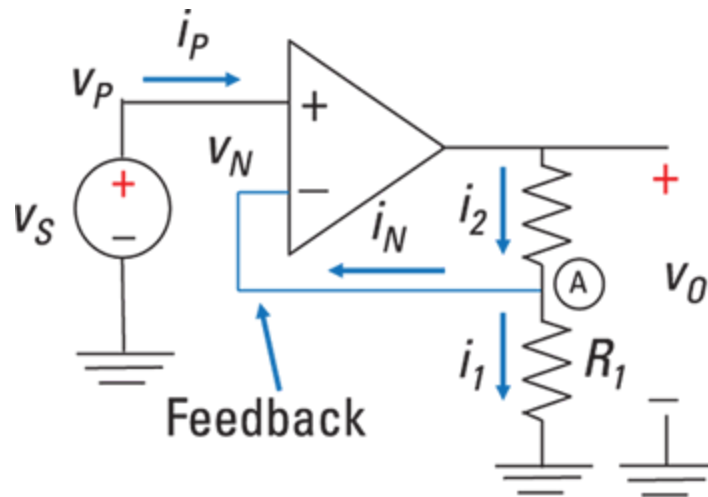
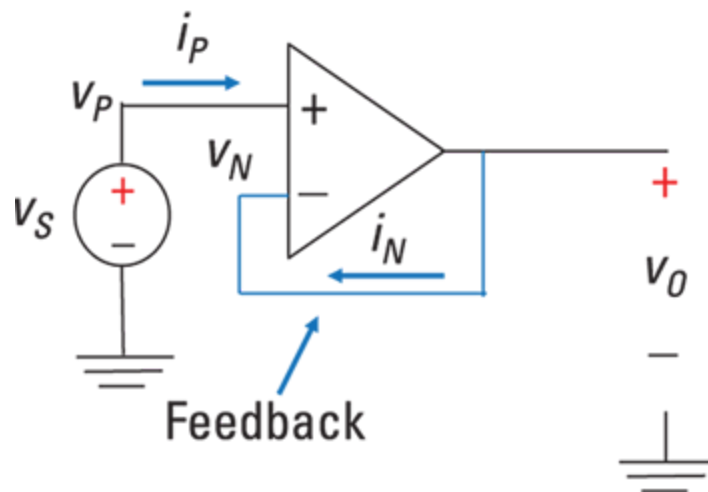


## Part III

# Understanding Circuits with Transistors and Operational Amplifiers



Noninverting Amplifier



Voltage Follower



Go to [www.dummies.com/extras/circuitanalysis](http://www.dummies.com/extras/circuitanalysis) to solve a real-world problem on using a photoresistor and an op-amp circuit to convert light into a voltage with a desired output range.

## ***In this part . . .***

- ✓ Amplify current with transistors.
- ✓ Amplify voltage with operational amplifiers (also known as op amps).

# Chapter 9

## Dependent Sources and the Transistors That Involve Them

---

### ***In This Chapter***

- ▶ Working with linear dependent sources
  - ▶ Analyzing circuits that have dependent sources
  - ▶ Taking control with transistors
- 

Resistors, capacitors, and inductors are interesting, but they're merely passive devices. What makes circuits great is the ability to perform as an electronic switch or amplify signals. Such switching and amplification functions are derived from transistors — *transfer resistors* — named for the fact that the resistance can be electronically tuned.

Most portable electronic devices, such as smartphones and tablets, use integrated circuits (ICs) to drive many system and circuit functions — making it possible for you to watch the latest YouTube sensation on the go. An IC is usually made on a small wafer of silicon or other semiconductor material holding hundreds to millions of transistors, resistors, and capacitors. In the future, gazillions of transistors, capacitors, and resistors could be jammed into a piece of silicon to perform other functions, like making coffee, getting your favorite newspaper, driving you to work, and waking you up to the reality of doing circuit analysis.

In this chapter, I introduce you to dependent sources, which you can use to model transistors and the operational amplifier IC, both of which require power to work. You analyze circuits with dependent sources using a variety of techniques from earlier chapters, and you explore some key types of dependent sources: JFET and bipolar transistors. As for operational amplifiers, I cover them in detail in [Chapter 10](#).

## ***Understanding Linear Dependent Sources: Who Controls What***

A *dependent source* is a voltage or current source controlled by either a voltage or a current at the input side of the device model. The dependent source drives the output side of the circuit. Dependent sources are usually associated with components (or devices) requiring power to operate correctly. These components are considered *active devices* because they require power to work; circuits using these devices are called *active circuits*. Active devices such as transistors perform amplification, allowing you to do things like crank up the volume of your music.



When you're dealing with active devices operating in a linear mode, the relationship between the input and output behavior is directly proportional. That is, the bigger the input, the bigger the output. Mathematically for a given input  $x$ , you have an output  $y$  with a gain amplification of  $G$ :  $y = Gx$ .

The constant or gain  $G$  is greater than 1 for active circuits (think steroids) and less than 1 for passive circuits (think wimpy). In other engineering applications, technical terms for  $G$  include *scale factor*, *scalar multiplier*, *proportionality constant*, and *weight factor*.

The following sections introduce you to the four types of dependent sources and help you recognize the connection between dependent sources and their independent counterparts.

## ***Classifying the types of dependent sources***

Modeling active devices requires the use of dependent sources, and four types of dependent sources exist (see [Figure 9-1](#)):

- ✓ **Voltage-controlled voltage source (VCVS):** A voltage across the input terminals controls a dependent voltage source at the output port.
- ✓ **Current-controlled voltage source (CCVS):** A current flowing through the input terminals controls a dependent voltage source.
- ✓ **Voltage-controlled current source (VCCS):** Now account for a dependent current source at the output terminals. With a voltage across the input, you can control the amount of current output.
- ✓ **Current-controlled current source (CCCS):** Can you guess the last type of dependent source? That's right — with a current flowing through the input port, you can control a dependent current source at the output port.

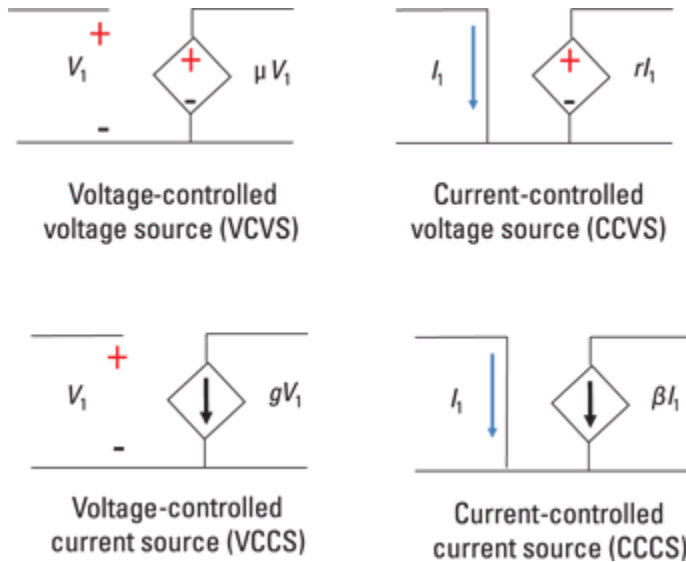


Illustration by Wiley, Composition Services Graphics

**Figure 9-1:** Circuit symbols of dependent sources.



Diagrams use the diamond shape for dependent sources to distinguish them from independent sources, which use a circle. Some books may use circles to denote both dependent and independent sources. After a dependent source reaches 18 years old and leaves home, it becomes an independent source (just checking if you made it this far).

The output of a linear dependent source is proportional to the input voltage or current controlling the source output. In [Figure 9-1](#), the proportionality constants or gains are given as  $\mu$ ,  $r$ ,  $g$ , and  $\beta$ :

- ✓ You can think of  $\mu$  in the VCVS dependent source as voltage gain because it's the ratio of the voltage output to the voltage input.
- ✓ In the CCVS dependent source, the proportionality constant  $r$  is called the *transresistance* because its

input-output relationship takes the form of Ohm's law:  
 $v = iR$ .

- ✓ Similarly, the VCCS dependent source has a proportionality constant  $g$ , called the *transconductance*, following a variation of Ohm's law:  $i = Gv$  (where the conductance  $G = 1/R$ ).
- ✓ For the CCCS dependent source, you can think of the proportionality constant  $\beta$  as the current gain because it's the ratio of current output to current input.



So when the type of input matches the type of output, the proportionality constant gives you the current or voltage gain. When they differ, the input-output relationship stems from Ohm's law.

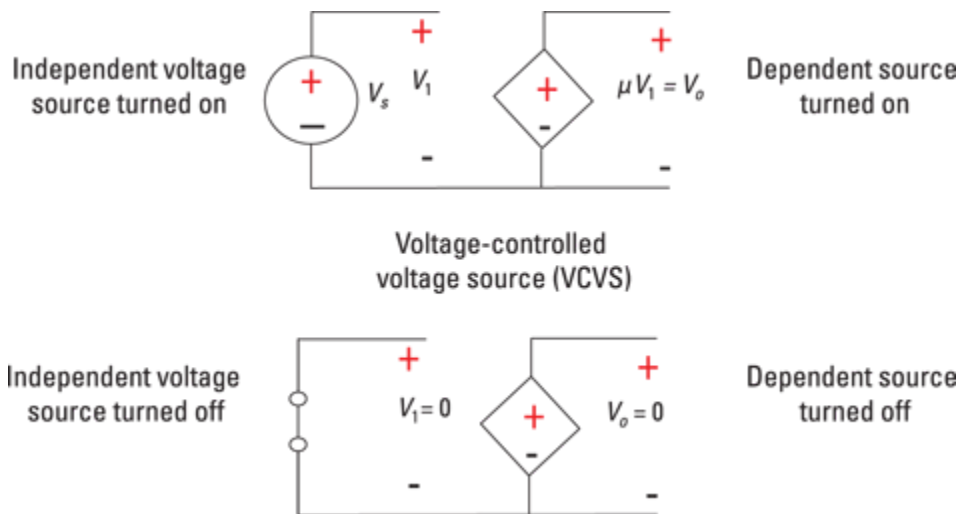
## ***Recognizing the relationship between dependent and independent sources***

You can turn off an independent voltage source by replacing it with a short circuit having zero resistance, and you can turn off an independent current source by replacing it with an open circuit — I show you how in [Chapter 7](#). But you can't just turn dependent sources on or off. Because dependent sources rely on voltage or current from an independent source on the input side, turning off an independent source turns off a dependent source.

[Figure 9-2](#) illustrates the interplay between the independent source and the dependent source. The top diagram shows that when an independent source is turned *on*, the dependent source is turned *on*. The



bottom diagram shows that when an independent source is turned *off*, the dependent source is turned *off*.



*Illustration by Wiley, Composition Services Graphics*

**Figure 9-2:** Interplay between independent and dependent sources.

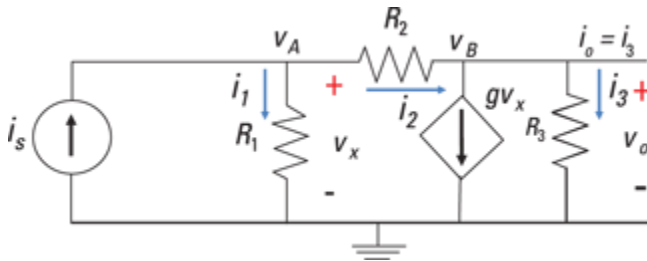
## ***Analyzing Circuits with Dependent Sources***

This section shows how to analyze example circuits that have dependent sources by using the techniques I describe in Part II of this book. Get ready to use node-voltage analysis ([Chapter 5](#)), mesh-current analysis ([Chapter 6](#)), superposition ([Chapter 7](#)), and the Thévenin technique ([Chapter 8](#)) as you work with dependent sources. That's an impressive series of topics, so if you need an energy boost, feel free to grab a snack before you begin.

### ***Applying node-voltage analysis***

Using node voltage methods to analyze circuits with dependent sources follows much the same approach as for independent sources, which I cover in [Chapter 5](#).

Consider the circuit in [Figure 9-3](#). What is the relationship between the output voltage  $v_o$  and  $i_s$ ?



*Illustration by Wiley, Composition Services Graphics*

**Figure 9-3:** Node-voltage analysis for a circuit with a dependent source.

The first step is to label the nodes. Here, the bottom node is your reference node, and you have Node A (with voltage  $v_A$ ) at the upper left and Node B (with voltage  $v_B$ ) at the upper right. Now you can formulate the node voltage equations.

Using node-voltage analysis involves Kirchhoff's current law (KCL), which says the sum of the incoming currents is equal to the sum of the outgoing currents. At Node A, use KCL and substitute in the current expressions from Ohm's law ( $i = v/R$ ). The voltage of each device is the difference in node voltages, so you get the following:

$$i_s = i_1 + i_2$$

$$i_s = \frac{v_A - 0}{R_1} + \frac{v_A - v_B}{R_2}$$

Rearranging gives you the node voltage equation:

$$\text{Node A: } \left( \frac{1}{R_1} + \frac{1}{R_2} \right) v_A + \left( -\frac{1}{R_2} \right) v_B = i_s$$

At Node B, again apply KCL and plug in the current expressions from Ohm's law:

$$i_2 = gv_x + i_3 \quad (\text{where } v_x = v_A - 0 = v_A)$$

$$\frac{v_A - v_B}{R_2} = gv_A + \frac{v_B - 0}{R_3}$$

Rearranging the preceding equation gives you the following node voltage equation at Node B:

$$\text{Node B: } \left(-\frac{1}{R_2} + g\right)v_A + \left(\frac{1}{R_3} + \frac{1}{R_2}\right)v_B = 0$$

The two node voltage equations give you a system of linear equations. Put the node voltage equations in matrix form:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} + g & \frac{1}{R_3} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

You can solve for the unknown node voltages  $v_A$  and  $v_B$  using matrix software. After you have the node voltages, you can set the output voltage  $v_o$  equal to  $v_B$ . You can then use the ever-faithful Ohm's law to find the output current  $i_o$ :

$$v_o = v_B - 0 = v_B \quad (\text{voltage across } R_3)$$

$$i_o = \frac{v_B}{R_3} = \frac{v_o}{R_3}$$

## Using source transformation

To see the source transformation technique for circuits with dependent circuits, consider Circuit A in [Figure 9-4](#). Suppose you want to find the voltage across resistor  $R_3$ . To do so, you can perform a source transformation, changing Circuit A (with an independent voltage source) to Circuit B (with an independent current source). You now have all the devices connected in parallel, including the dependent and independent current sources.



Don't use source transformation for dependent sources, because you may end up changing or losing the dependency. You need to make sure the

dependent source is a function of the independent source.

Here's the equation for the voltage source and current source transformation:

$$i_s = \frac{v_s}{R_1}$$

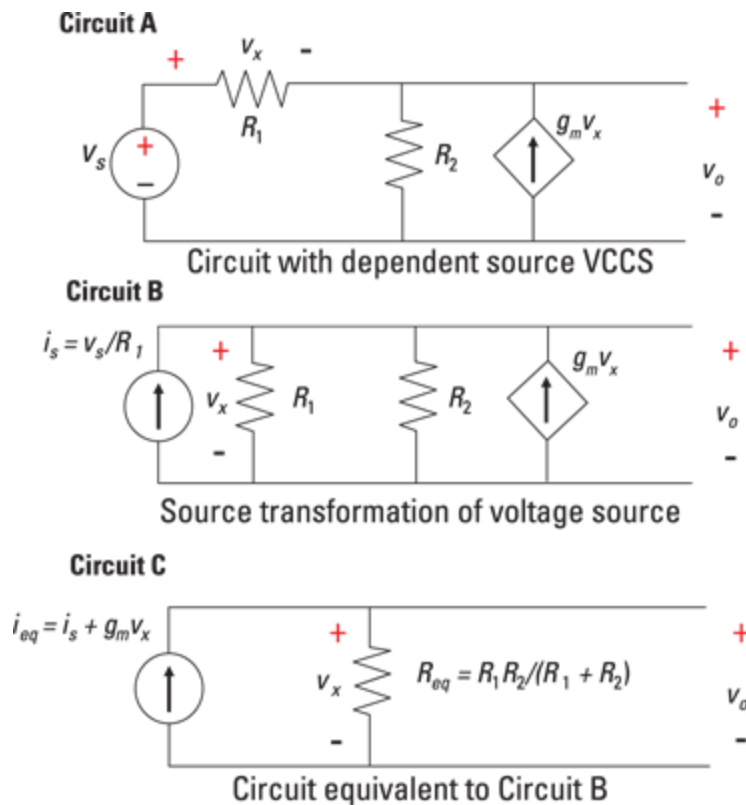


Illustration by Wiley, Composition Services Graphics

**Figure 9-4:** Source transformation for a circuit with a dependent source.

The independent current source  $i_s$  and the dependent current source  $g v_x$  point in the same direction, so you can add these two current sources to get the total current  $i_{eq}$  going through the resistor combination  $R_1$  and  $R_2$ . The

total current  $i_{eq}$  is  $i_{eq} = i_s + g_m v_x$ . Because  $v_x$  is the voltage across  $R_2$ ,  $v_x$  is also

equal to  $v_o$  in Circuit B:  $v_o = v_x$ .

Resistors  $R_1$  and  $R_2$  are connected in parallel, giving you an equivalent resistance  $R_{eq}$ :

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The output voltage is equal to the voltage across  $R_{eq}$ , using Ohm's law and  $i_{eq}$ . You see the equivalent circuit with  $i_{eq}$  and  $R_{eq}$  in Circuit C. Because the dependent current source is dependent on  $v_x$ , you need to replace the voltage  $v_x$  with  $v_o$ :

$$v_o = i_{eq} R_{eq} = (i_s + g_m v_x) R_{eq}$$
$$v_o = (i_s + g_m v_o) \left( \frac{R_1 R_2}{R_1 + R_2} \right) \quad (\text{substitute } v_o = v_x)$$

Solving for the output voltage  $v_o$  gives you

$$v_o = i_s \left( \frac{R_{eq}}{1 - g_m R_{eq}} \right)$$

See how the output voltage is a function of the input source? The final expression of the output should not have a dependent variable.

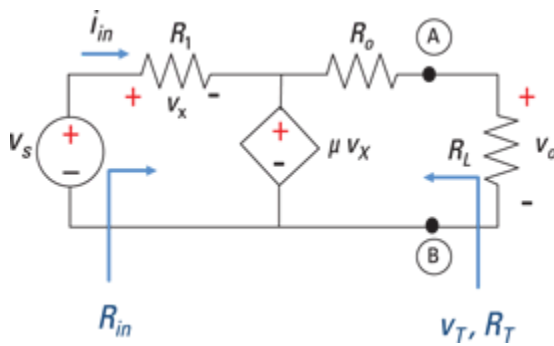
## ***Using the Thévenin technique***

The Thévenin approach reduces a complex circuit to one with a single voltage source and a single resistor.

Independent sources must be turned on because the dependent source relies on the excitation due to an independent source.

As I note in [Chapter 8](#), to find the Thévenin equivalent for a circuit, you need to find the open-circuit voltage and the short-circuit current at the interface. In other words, you need to find the  $i$ - $v$  relationship at the interface.

To see how to get the Thévenin equivalent for a circuit having a dependent source, look at [Figure 9-5](#). This example shows how to find the input resistance and the output Thévenin equivalent circuit at interface points A and B.



*Illustration by Wiley, Composition Services Graphics*

**Figure 9-5:** Thévenin equivalent for a circuit with a dependent source.

The input resistance is

$$R_{in} = \frac{v_s}{i_{in}}$$

Using Ohm's law, the current  $i_{in}$  through  $R_1$  is

$$i_{in} = \frac{v_s - \mu v_x}{R_1}$$

$$i_{in} = \frac{v_s - \mu(i_{in} R_1)}{R_1}$$

Solving for  $i_{in}$ , you wind up with

$$i_{in} = \frac{v_s}{R_1(1 + \mu)}$$

Substituting  $i_{in}$  into the input-resistance equation gives you

$$R_{in} = \frac{v_s}{i_{in}} = \frac{v_s}{\left( \frac{v_s}{R_1(1 + \mu)} \right)}$$

$$R_{in} = R_1(1 + \mu)$$

Here, the dependent source increases the input resistance by approximately multiplying the resistor  $R_1$  by the dependent parameter  $\mu$ .  $R_1$  is the input resistance without the dependent source. To find the Thévenin voltage  $v_T$  and the Thévenin resistance  $R_T$ , you have to find the open-circuit voltage  $v_{oc}$  and short-circuit current  $i_{sc}$ . The resistance  $R_T$  is given by the following relationship:

$$R_T = \frac{v_{oc}}{i_{sc}}$$

Based on [Figure 9-5](#), the open-circuit voltage is  $v_{oc} = \mu v_x$ . You find that the short-circuit current gives you

$$i_{sc} = \frac{\mu v_x}{R_o}$$

After finding  $v_{oc}$  and  $i_{sc}$ , you find the Thévenin resistance:

$$R_T = \frac{v_{oc}}{i_{sc}}$$

$$R_T = \frac{\mu v_x}{\left(\frac{\mu v_x}{R_o}\right)} = R_o$$

The output resistance  $R_o$  and Thévenin resistance  $R_T$  are equal. Based on Kirchhoff's voltage law (KVL), you have the following expression for  $v_x$ :

$$v_s = v_x + \mu v_x$$

$$v_x = \frac{v_s}{1 + \mu}$$

Substituting  $v_x$  into the equation for the open-circuit voltage  $v_{oc}$ , you wind up with

$$v_{oc} = \mu v_x$$

$$v_{oc} = v_s \left( \frac{\mu}{(1 + \mu)} \right)$$

The open-circuit voltage,  $v_{oc}$ , equals the Thévenin voltage,  $v_T$ . The nitty-gritty analysis leaves you with Thévenin voltage  $v_T$  and Thévenin resistance  $R_T$ , entailing a dependent voltage gain of  $\mu$ :

$$v_T = v_s \left( \frac{\mu}{1 + \mu} \right)$$
$$R_T = R_o$$

When  $\mu$  is very large, the Thévenin voltage  $v_T$  equals the source voltage  $v_s$ .

## ***Describing a JFET Transistor with a Dependent Source***

Transistors are amplifiers in which a small signal controls a larger signal. Just picture a Chihuahua taking its hefty owner for a daily walk, and you can imagine what a transistor is capable of.

The two primary types of transistors are bipolar transistors and field-effect transistors. The field-effect transistor is a little simpler than the bipolar kind. You can classify field-effect transistors, or FETs, in two ways: junction field-effect transistors (JFETs) and metal-oxide-semiconductor field-effect transistors (MOSFETs).

Because JFETs provide a good picture of how transistor circuits work, this section concentrates on this type of FET. (I cover bipolar transistors in the later section [“Examining the Three Personalities of Bipolar Transistors.”](#))

Typical transistors have three leads. In the case of a JFET, a voltage on one lead (called the *gate*) is used to



control a current between the two other leads (called the *source* and the *drain*). The gate voltage needs to be referenced to some other voltage, and by convention, it's referenced to the source terminal. [Figure 9-6](#) shows the JFET symbol and its corresponding dependence model. The gate, drain, and source labels ( $G$ ,  $D$ , and  $S$ , respectively) are normally omitted, but I include them here for reference.

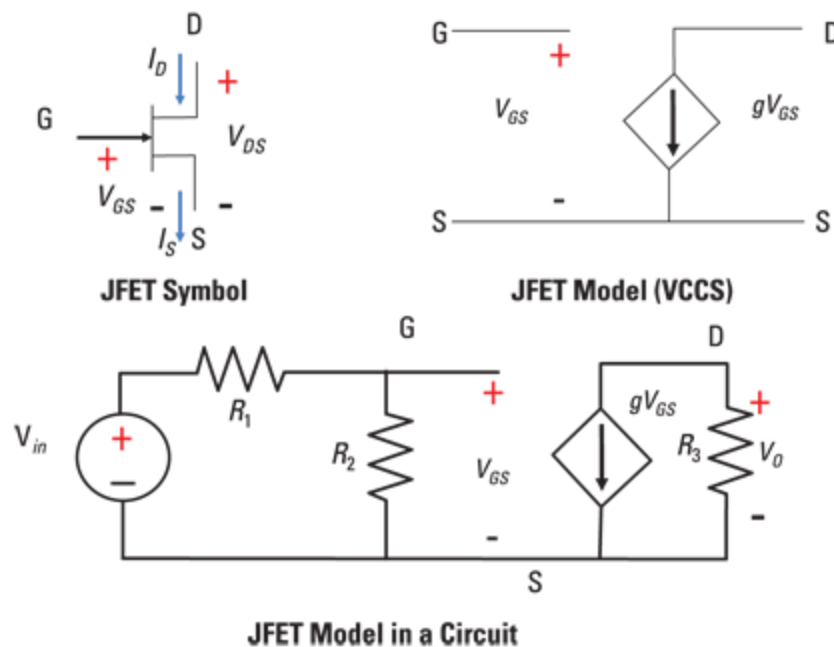


Illustration by Wiley, Composition Services Graphics

**Figure 9-6:** JFET transistor symbol and model and the JFET dependent source model in a circuit.

In the figure,  $V_{GS}$  refers to the voltage between the gate and the source,  $I_D$  is the current into the drain, and  $I_S$  is the current out of the source. No current flows into the gate when it's operating under normal conditions, implying that the drain current  $I_D$  is equal to the source

current  $I_S$ . A useful JFET model, which you see on the upper right of [Figure 9-6](#), uses a voltage-controlled current source (VCCS). The model is part of the circuit at the bottom of the figure.

For the circuit in [Figure 9-6](#), you need to find the ratio between the output voltage  $V_O$  and the input voltage  $V_{in}$ . The dependent source is a voltage-controlled current source, so its current is  $gV_{GS}$  (see the earlier section “[Classifying the types of dependent sources](#)” for details). So at the output terminals of the dependent source model, the output voltage  $V_O$  is a result of the following equation using Ohm’s law ( $v = iR$ ):

$$V_O = (-gV_{GS})R_3$$

The minus sign appears because the current through resistor  $R_3$  flows in the opposite direction of the voltage polarities of the output voltage  $V_O$ . You can find the voltage  $V_{GS}$  on the input terminals of the dependent VCCS model.

Because the devices are connected in series on the input side of the circuit, you can use the voltage divider technique, as follows (see [Chapter 4](#) for details on voltage division):

$$V_{GS} = V_{in} \left( \frac{R_2}{R_1 + R_2} \right)$$

Now substitute this expression for  $V_{GS}$  into the Ohm’s law equation for output voltage  $V_O$ . You get the following input-output relationship:

$$V_O = V_{in} \underbrace{\left( \frac{R_2}{R_1 + R_2} \right)}_{=V_{GS}} (-gR_3)$$

$$V_O = \left( \frac{-gR_2R_3}{R_1 + R_2} \right) V_{in}$$

To see the amount of amplification using this circuit, try plugging in some numbers. Suppose  $g = 1.8$  milliamps per volt,  $R_1 = R_2 = 1 \text{ k}\Omega$ ,  $R_3 = 10 \text{ k}\Omega$ , and  $V_{in} = 1 \text{ volt}$ . The amplifier output is

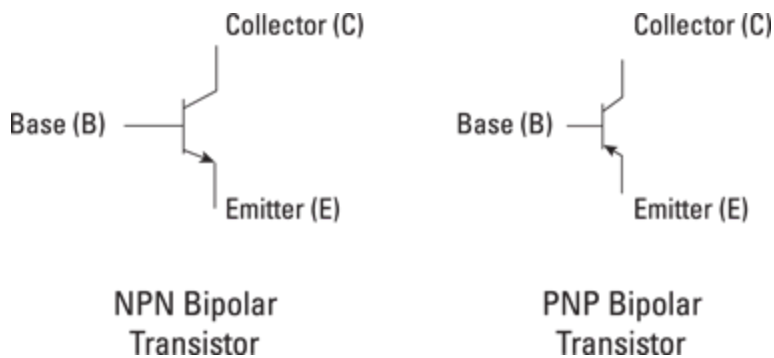
$$V_o = \left( \frac{-gR_2R_3}{R_1 + R_2} \right) V_{in}$$
$$V_o = \left( \frac{(-1.8 \text{ mA/V})(1 \text{ k}\Omega)(10 \text{ k}\Omega)}{1 \text{ k}\Omega + 1 \text{ k}\Omega} \right) (1 \text{ V}) = -9 \text{ V}$$

The input is amplified by -9 at the output of the dependent source. Awesome! The signal is bigger because an external voltage source made this JFET transistor work as an amplifier. The minus sign means that the signal is inverted or upside down, which is no problem because it doesn't change the sound quality of your music.

## ***Examining the Three Personalities of Bipolar Transistors***

Along with FETs (field-effect transistors — see the preceding section), bipolar transistors stand as a cornerstone of modern microelectronics. The word *bipolar* comes from the flow of both electrons and holes (where a *hole* is a positively charged particle). Because the bipolar transistor is a three-terminal device, the voltage between two terminals controls the current through the third terminal. The three terminals are called the *base*, the *emitter*, and the *collector*. [Figure 9-7](#) shows the circuit symbols for two types of bipolar transistors: NPN and PNP. (For detailed information on

working with transistors, check out *Electronics All-in-One For Dummies*.)



*Illustration by Wiley, Composition Services Graphics*

**Figure 9-7:** Circuit symbols of bipolar transistors.

Because of the versatility of the transistors, there are three basic patterns of design to perform circuit functions (see [Figure 9-8](#) for the visual):

- ✓ **Common emitter:** *Common emitter* means that the emitter terminal is common to both the input and output parts of a circuit. The same holds true for the other two configurations described in this list.
- ✓ **Common base:** When the base terminal is common to both the input and output parts of the circuit, you have a common base circuit.
- ✓ **Common collector:** For the common collector arrangement, the collector terminal comes into play for both input and output circuit pieces.

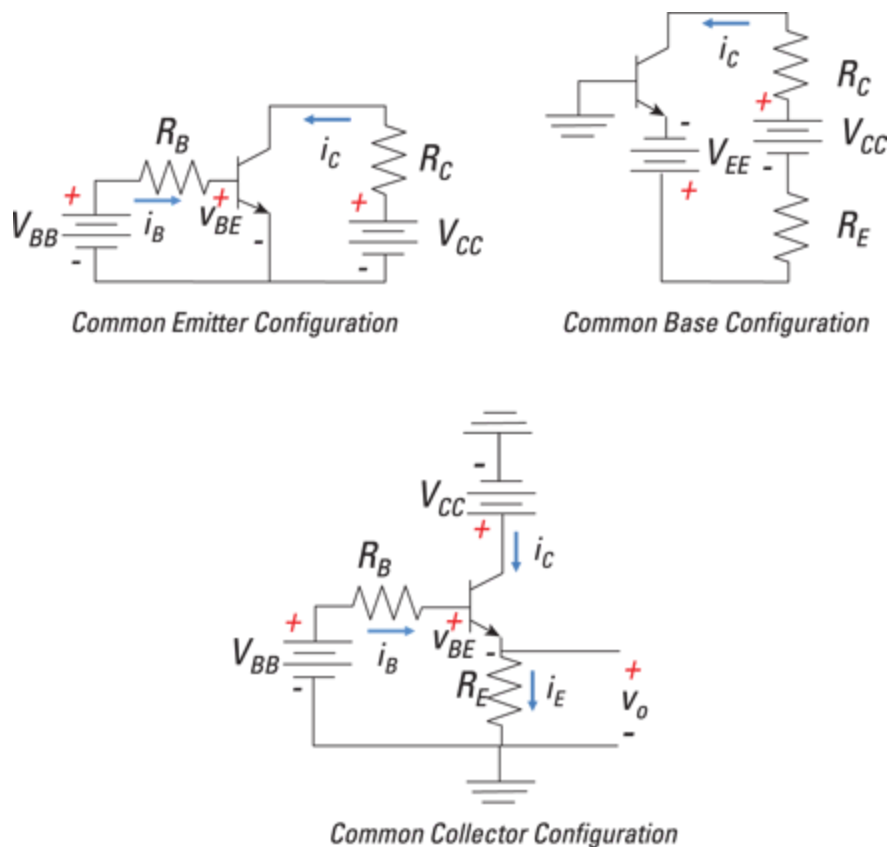


Illustration by Wiley, Composition Services Graphics

**Figure 9-8:** Three common circuit configurations of bipolar transistors.

You can use these circuits in stages or in combination to perform useful functions. That's what you do with an operational amplifier (op amp), which I cover in the next chapter. I don't go through all the benefits of the transistor configurations in the following sections, but I do give you a glimpse of the device's worth.

## ***Making signals louder with the common emitter circuit***

A current-controlled current source (CCCS) is a typical model when you're analyzing a circuit that has a bipolar transistor. [Figure 9-9](#) shows a CCCS as a very simplified DC model of a bipolar transistor. Note that  $\gamma$  denotes some threshold on the voltage.

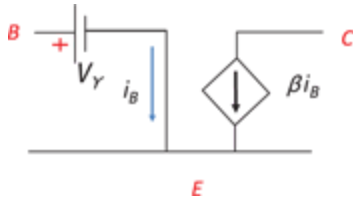


Illustration by Wiley, Composition Services Graphics

**Figure 9-9:** Modified current-controlled current source for a bipolar transistor.

[Figure 9-10](#) shows a common emitter circuit modeled with a dependent source. Note the labeling of the three terminals: base (B), collector (C), and emitter (E). Here, you use mesh-current analysis to find the transistor base current  $i_B$ .

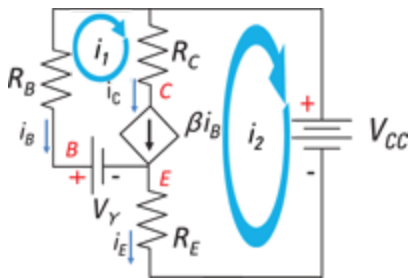


Illustration by Wiley, Composition Services Graphics

**Figure 9-10:** Simple dependent CCCS model of a bipolar transistor.

You see two mesh currents,  $i_1$  and  $i_2$ , in [Figure 9-10](#). By excluding the current source, you can combine Meshes 1 and 2 to create a supermesh. Going through the dependent current source isn't helpful for this analysis, and the supermesh is useful because it lets you avoid the series combination of dependent source  $\beta i_B$  and collector resistance  $R_C$ .

Starting at the bottom of the circuit, you can write the following KVL mesh expression:

$$i_2 R_E - V_Y + i_1 R_B + V_{CC} = 0 \quad \text{KVL equation}$$

This equation has two unknown mesh currents,  $i_1$  and  $i_2$ , through the dependent source  $\beta i_B$ . Use these concepts to write the KCL equation:

$$i_1 - i_2 = \beta i_B \quad \text{KCL equation}$$

You use the KVL and KCL equations to solve for the unknown mesh currents. [Figure 9-10](#) shows that the base current is in the opposite direction of the mesh current  $i_1$ :

$$i_B = -i_1$$

Substitute the value of  $i_B$  into the KCL equation to get a relationship between  $i_1$  and  $i_2$ :

$$i_1 - i_2 = \beta i_B = -\beta i_1$$

$$i_2 = (\beta + 1)i_1$$

You now have current gain to amplify signals:

$$\frac{i_2}{i_1} = \beta + 1$$

The analysis shows the bipolar transistor as a current amplifier. Any change in  $i_1$  (or the base current  $i_B$ ) creates an even larger change in the collector or emitter current related by  $i_2$ . You need current amplifiers for current-hungry devices, such as speakers and magnetic locks to keep unsavory characters out of your house.

To find the input resistance at DC, solve the preceding equation for  $i_2$  and substitute it into the KVL equation. Solve the KVL equation for  $i_1$ , which equals  $-i_B$ :

$$\underbrace{(\beta + 1)i_1}_{=i_2} R_E - V_\gamma + i_1 R_B + V_{CC} = 0$$

$$-i_1 = i_B = \frac{V_{CC} - V_\gamma}{R_B + (\beta + 1)R_E}$$

Here's the input resistance:

$$R_{in} = \frac{V_{CC} - V_{\gamma}}{-i_1}$$

$$R_{in} = \frac{(R_B + (\beta + 1)R_E)i_1}{i_1}$$

$$R_{in} = R_B + (\beta + 1)R_E$$

The dependent source increased the input resistance by adding an emitter resistor  $R_E$  and making  $R_E$  larger by about  $\beta$ . High-input resistance isolates the input and output parts of the circuit from each other. Neato! Why neato? Because there aren't any loading effects. You design input circuits independently from output circuits. When you tie together these two circuits, they perform as expected. Each circuit is unaffected, but they work together to create the desired outcome — redesign isn't needed.

## ***Amplifying signals with a common base circuit***

The common base circuit looks like an ideal current source and is often called a *current buffer*. [Figure 9-11](#) shows a hybrid- $\pi$  model for AC signal analysis with infinite output resistance for an NPN transistor, which I use to analyze the common base configuration. The model has an input resistance  $r_{\pi}$  and a current gain  $\beta$ .



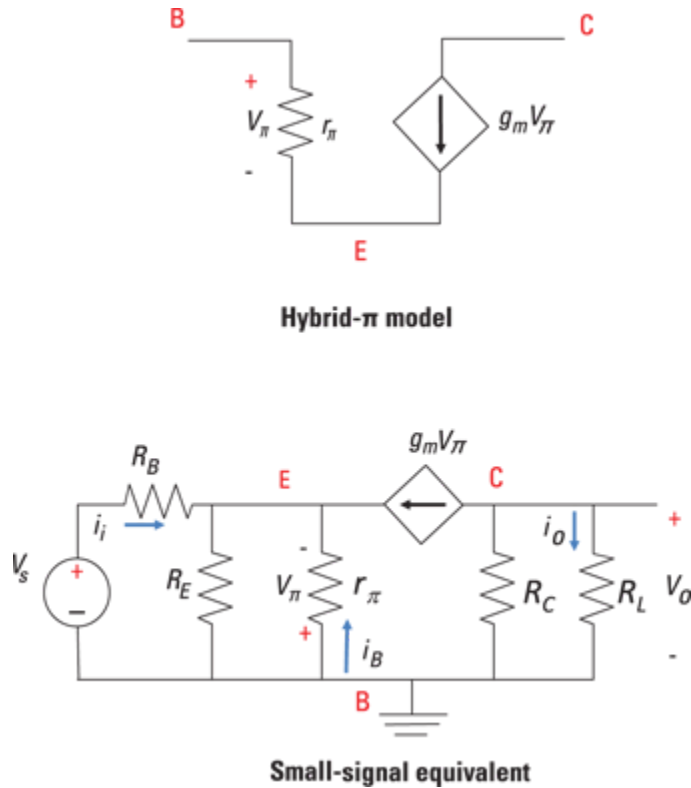


Illustration by Wiley, Composition Services Graphics

**Figure 9-11:** Analysis of a common base circuit.

The circuit shows the parallel connection between the collector resistor  $R_C$  and load resistor  $R_L$  (symbolized by  $R_C || R_L$ ). Applying Ohm's law leads you to the small signal output voltage  $V_o$ :

$$V_o = -(g_m V_\pi)(R_C || R_L)$$

At the emitter node, you apply KCL to get

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{(V_s - (-V_\pi))}{R_B} = 0 \quad (\text{KCL})$$

For the hybrid- $\pi$  model, you have the transistor parameters related by the following equation, with  $\beta$  given earlier as the transistor current gain:

$$\beta = g_m r_\pi$$

Substituting  $\beta$  into the KCL equation, you wind up with

$$V_{\pi} \left( \frac{1+\beta}{r_{\pi}} + \frac{1}{R_E} + \frac{1}{R_B} \right) = -\frac{V_s}{R_B}$$

The terms in brackets form a parallel connection of the given resistors. Solving for  $V_{\pi}$ , you have the following expression:

$$V_{\pi} = -\frac{V_s}{R_B} \left( \left( \frac{r_{\pi}}{1+\beta} \right) \parallel R_E \parallel R_B \right)$$

Substituting  $V_{\pi}$  into the  $V_o$  equation gives you the small-signal gain:

$$A_v = \frac{V_o}{V_s} = g_m \left( \frac{R_C \parallel R_L}{R_B} \right) \left( \left( \frac{r_{\pi}}{1+\beta} \right) \parallel R_E \parallel R_B \right)$$

$$A_v \approx g_m (R_C \parallel R_L) \quad \text{where } R_B \rightarrow 0, \text{ then } \left( \frac{r_{\pi}}{1+\beta} \right) \parallel R_E \parallel R_B \approx R_B$$

To find the current gain, you let the emitter resistance  $R_E$  approach infinity. This is reasonable because  $R_E$  is large compared to  $R_B$  and  $r_{\pi}/(\beta + 1)$ . This means little or no current will flow through  $R_E$ . Using  $\beta = g_m r_{\pi}$ , the KCL at the emitter node is

$$i_i + i_B = -g_m V_{\pi} = -\beta i_B$$

$$i_i = -(\beta + 1)i_B$$

The KCL equation at the collector node is

$$i_o = -g_m V_{\pi} = -\underbrace{g_m r_{\pi}}_{=\beta} i_B = -\beta i_B$$

Using the current divider equation at the output side, the current gain is defined as the ratio of  $i_o$  to  $i_i$ :

$$\frac{i_o}{i_i} = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{R_C}{R_C + R_L} \right)$$

The preceding equation shows the current gain is less than 1. There's some power gain because the voltage gain is greater than 1.

## Isolating circuits with the common collector circuit

The common collector circuit is also known as an *emitter follower*. This means that any variation in the base terminal causes the same variation in the emitter terminal. However, the following analysis deals with signals that are constant, also known as *DC analysis*.

The following analysis of the circuit in [Figure 9-12](#) shows that the emitter follower has a voltage gain that's approximately equal to 1 but provides a high input impedance, isolating the source circuit from the load circuit.

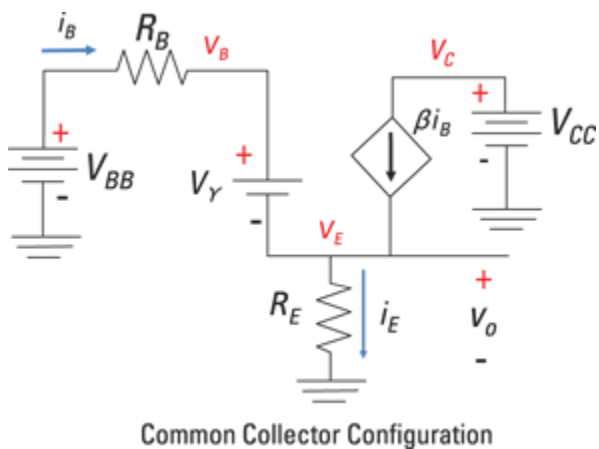


Illustration by Wiley, Composition Services Graphics

**Figure 9-12:** A common collector circuit.

Using [Figure 9-12](#), you get the output voltage with Ohm's law:

$$v_o = i_E R_E$$

Apply KVL to get the base voltage  $v_B$ :

$$v_B = V_Y + v_o$$

Because  $V_Y$  is a constant (about 0.7 volts for a silicon-based transistor and 0.2 volts for a germanium-based

transistor), the output voltage  $v_o$  follows the input voltage  $v_B$ . So the common collector doesn't produce a voltage gain, but it does provide circuit isolation to reduce circuit loading due to its high input impedance.

Find the input resistance as seen by the base terminal as follows:

$$R_{in} = \frac{v_B}{i_B}$$

The output voltage  $v_o$  follows from Ohm's law:

$$v_o = R_E i_E$$

But the emitter current  $i_E$  is related to the input base current  $i_B$ :

$$i_E = (\beta + 1)i_B$$

You now have an output voltage:

$$v_o = R_E i_E = R_E (\beta + 1)i_B$$

If the output voltage  $v_o$  is much bigger than  $V_\gamma$ , you can make the following approximation for the base voltage  $v_B$ :

$$v_B = V_\gamma + v_o \approx v_o$$

$$v_B = R_E (\beta + 1)i_B$$

Solving for the ratio of  $v_B$  to  $i_B$  gives you the input resistance:

$$R_{in} = \frac{v_B}{i_B} = R_E (\beta + 1)$$

The input resistance as seen by the base multiplies the emitter resistance by  $\beta + 1$ . Typical values of current gain  $\beta$  vary from 50 to 150. High-input resistance provides isolation between the input and output parts of the circuit. Also, when little current is drawn from the source, you have longer battery life for portable

applications, letting you play games on your smartphone for longer stretches.

# Chapter 10

## Letting Operational Amplifiers Do the Tough Math Fast

---

### ***In This Chapter***

- ▶ Performing hand calculations electronically with operational amplifiers
  - ▶ Modeling secrets of operational amplifiers with dependent sources
  - ▶ Taking a look at op-amp circuits
  - ▶ Putting together some systems
- 

The operational amplifier (op amp) is a powerful tool when you're working with active devices in modern-day circuit applications. Because op amps can do calculations electronically, they perform mathematical operations (like addition, subtraction, multiplication, division, integration, and derivatives) fast. You can put together basic op-amp circuits to build accurate mathematical models that predict complex and real-world behavior — like when the breakfast pastry in your toaster will turn into a flaming torch.

This chapter introduces op-amp circuits, demonstrates how to use them to perform certain mathematical operations, and gives you a peek at the more complex processing actions that op amps serve as the building blocks for.

# *The Ins and Outs of Op-Amp Circuits*

Commercial op amps first entered the market as integrated circuits in the mid-1960s, and by the early 1970s, they dominated the active device market in analog circuits. The op amp itself consists of a complex arrangement of transistors, diodes, resistors, and capacitors put together and built on a tiny silicon chip called an *integrated circuit*.

You can model the op amp with simple equations with little concern for what's going on inside the chip. You just need some basic knowledge of the constraints on the voltages and currents at the external terminals of the device.

In the following sections, you discover typical diagrams of op-amp circuits, the characteristics of ideal op amps and op amps with dependent sources, and the two equations necessary for analyzing these special circuits.

## *Discovering how to draw op amps*

Unlike capacitors, inductors, and resistors, op amps require power to work. Op amps have the following five key terminals (see their symbols in [Figure 10-1](#)):

- ✓ The positive terminal, called the noninverting input  $v_P$
- ✓ The negative terminal, called the inverting input  $v_N$
- ✓ The output terminal, resulting from the voltage applied between noninverting and inverting inputs:  $v_O = A(v_P - v_N)$
- ✓ Positive and negative power supply terminals, usually labeled as  $+V_{CC}$  and  $-V_{CC}$  and required for the op amp

to operate correctly

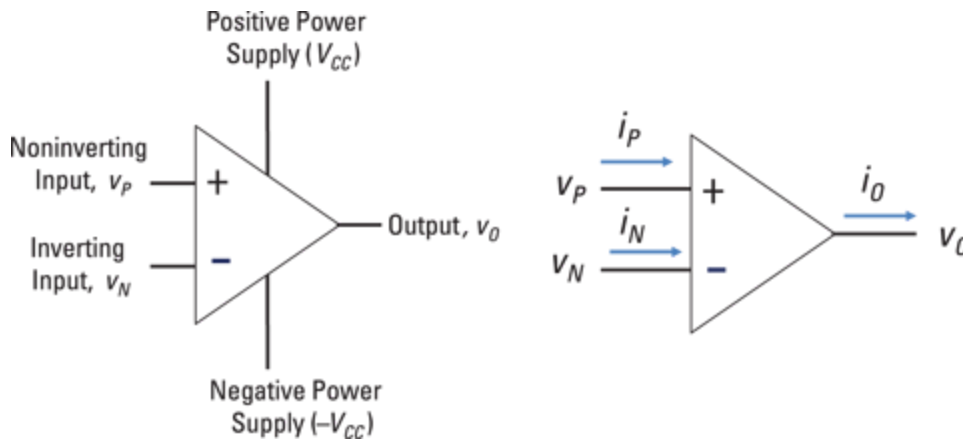


Illustration by Wiley, Composition Services Graphics

**Figure 10-1:** The circuit symbol of the op amp and its five terminals.

Although many op amps have more than five terminals, those terminals aren't normally shown symbolically. Also, to reduce the clutter when you're investigating an op-amp circuit, the power supplies aren't usually shown, either.



When the power supplies aren't shown in a diagram of an op-amp circuit, don't forget that the power supplies provide upper and lower limits of the output voltage, restricting its voltage range. Barring otherworldly powers, you can't get more power output than you supply.

## ***Looking at the ideal op amp and its transfer characteristics***

You can model the op amp with a dependent source if you need accurate results, but the ideal op amp is good enough for most applications.



The op amp amplifies the difference between the two inputs,  $v_P$  and  $v_N$ , by a gain  $A$  to give you a voltage output  $v_O$ :

$$v_O = A(v_P - v_N)$$

The voltage gain  $A$  for an op amp is very large — greater than  $10^5$ .

When the output voltage exceeds the supplied power, the op-amp *saturates*. This means that the output is clipped or maxed out at the supplied voltages and can increase no further. When this happens, the op-amp behavior is no longer linear but operates in the nonlinear region.

You can see this idea in [Figure 10-2](#). The left diagram shows the transfer characteristic, whereas the right diagram shows the ideal transfer characteristic of an op amp with an infinite gain. The graph shows three modes of operation for the op amp. You have positive and negative saturated regions, showing the nonlinear and linear regions. If you want to make signals bigger, you need to operate in the linear region. You can describe the three regions mathematically as follows

|                            |                        |                                      |
|----------------------------|------------------------|--------------------------------------|
| Negative saturated region: | $v_O = -V_{CC}$        | $A_v(v_P - v_N) < -V_{CC}$           |
| Linear active region:      | $v_O = A_v(v_P - v_N)$ | $-V_{CC} < A_v(v_P - v_N) < +V_{CC}$ |
| Positive saturated region: | $v_O = +V_{CC}$        | $A_v(v_P - v_N) > +V_{CC}$           |



To perform math functions (such as addition and subtraction), the op amp must work in linear mode. All op-amp circuits given in this chapter operate in the linear active region.

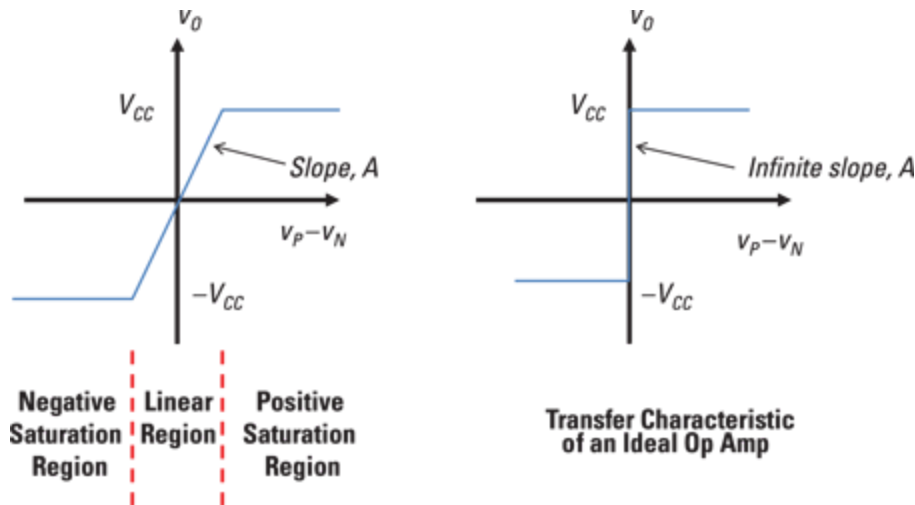


Illustration by Wiley, Composition Services Graphics

**Figure 10-2:** Practical and ideal op-amp properties and a linear dependent source model.

## Modeling an op amp with a dependent source

If you need accurate results, you can model the op amp with a voltage-controlled dependent source, like the one in [Figure 10-3](#). This model consists of a large gain  $A$ , a large input resistance  $R_I$ , and a small output resistance  $R_O$ . The table in [Figure 10-3](#) shows ideal and typical values of these op-amp properties.



High amplification (or gain) makes the analysis simpler, allowing you not to worry about what's going on inside the op amp. As long as the op amp has high gain, the op-amp math circuits will work. High-input resistance draws little current from the input source circuit, increasing battery life for portable applications. Low- or no-output resistance delivers maximum voltage to the output load.

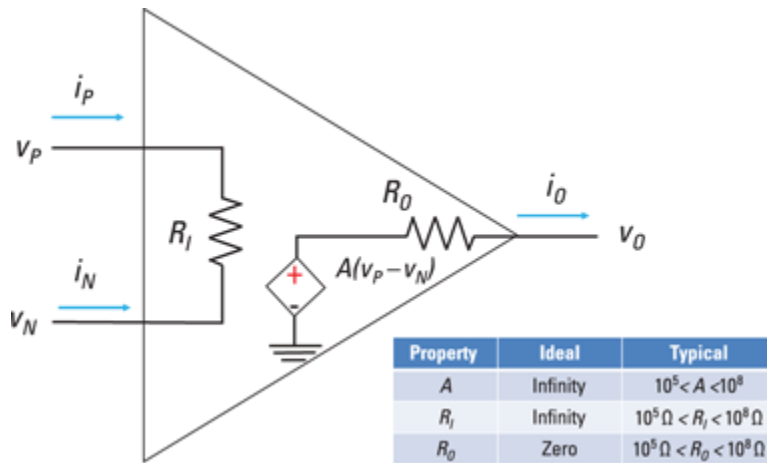


Illustration by Wiley, Composition Services Graphics

**Figure 10-3:** Dependent source model of an op amp.

The dependent voltage-controlled current source is shown in [Figure 10-3](#). The output is restricted between the positive and negative voltages when the op amp is operating in the linear region.

## Examining the essential equations for analyzing ideal op-amp circuits

The ideal properties of an op amp produce two important equations:

$$v_P = v_N$$

$$i_P = i_N = 0$$

These equations make analyzing op amps a snap and provide you with valuable insight into circuit behavior. Why? Because feedback from the output terminals to one or both inputs ensures that  $v_P$  and  $v_N$  are equal.

To get the first constraint, consider that the linear region of an op amp is governed by when the output is restricted by the supply voltages as follows:

$$-V_{CC} \leq \underbrace{v_O}_{v_O = A(v_P - v_N)} \leq +V_{CC}$$

You can rearrange the equation to limit the input to  $v_P - v_N$ :

$$\frac{-V_{CC}}{A} \leq v_P - v_N \leq \frac{+V_{CC}}{A}$$

For an ideal op amp, the gain  $A$  is infinity, so the inequality becomes

$$0 \leq v_P - v_N \leq 0$$

Therefore, the ideal op amp (with infinite gain) must have this constraint:

$$v_P = v_N$$



An op amp with infinite gain will always have the noninverting and inverting voltages equal. This equation becomes useful when you analyze a number of op-amp circuits, such as the op-amp noninverter, inverter, summer, and subtractor.



The other important op amp equation takes a look at the input resistance  $R_I$ . An ideal op amp has infinite resistance. This implies that no input currents can enter the op amp:

$$i_P = i_N = 0$$

The equation says that the op-amp input terminals act as open circuits.



You need to connect the output terminal to the inverting terminal to provide negative feedback in order to make the op amp work. If you connect the output to the positive side, you're providing positive feedback, which isn't good for linear operation. With positive feedback, the op amp would either saturate or cause its output to undergo oscillations.

## *Looking at Op-Amp Circuits*

The mathematical uses for signal processing include noninverting and inverting amplification, addition, and subtraction. Doing these math operations simply requires resistors and op amps. (As a gentle reminder to some students who are troubleshooting their circuits, you also need power.) In the following sections, you analyze several op-amp configurations using the op-amp equations:  $v_P = v_N$  and  $i_P = i_N = 0$ .

### *Analyzing a noninverting op amp*

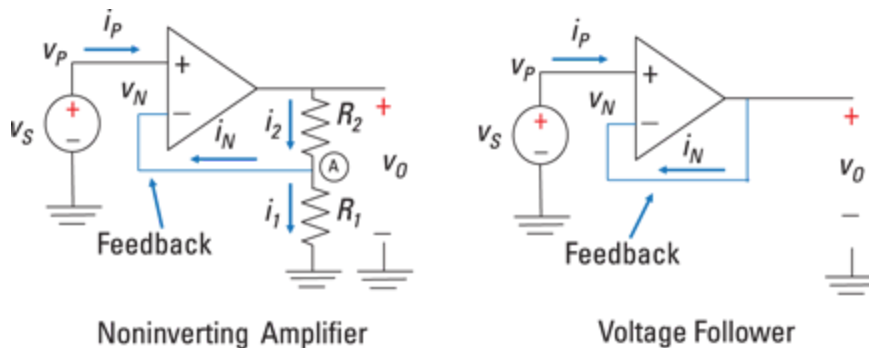
One of the most important signal-processing applications of op amps is to make weak signals louder and bigger. The following example shows how the feedback affects the input-output behavior of an op-amp circuit. Consider [Figure 10-4](#), which first shows the input connected to the noninverting input. You have a feedback path from the output circuit leading to the inverting input.

The voltage source  $v_S$  connects to the noninverting input  $v_P$ :

$$v_P = v_S$$

You gotta first find the voltage at the inverting input so you can figure out how the input and output voltages are related. Apply Kirchhoff's current law (KCL) at Node A between resistors  $R_1$  and  $R_2$ . (Remember that KCL says the sum of incoming currents is equal to the outgoing currents.) Applying KCL gives you

$$i_2 = i_N + i_1$$



*Illustration by Wiley, Composition Services Graphics*

**Figure 10-4:** Noninvert-ing amplifier op amp and voltage follower.

At the output side of the op amp, the inverting current  $i_N$  is equal to zero because you have infinite resistance at the inverting input. This means that all the current going through resistor  $R_2$  must go through resistor  $R_1$ . If the current is the same,  $R_1$  and  $R_2$  must be connected in series, giving you

$$i_2 = i_1$$

Because resistors  $R_1$  and  $R_2$  are connected in series, you can use voltage division (see [Chapter 4](#) for details). Voltage division gives you the voltage relationship between the inverting input  $v_N$  and output  $v_O$ :

$$v_N = \left( \frac{R_1}{R_1 + R_2} \right) v_O$$

The inverting input  $v_N$  and noninverting input  $v_P$  are equal for ideal op amps. So here's the link between the input source voltage  $v_S$  and output voltage  $v_O$ :

$$v_P = v_N \rightarrow v_S = \left( \frac{R_1}{R_1 + R_2} \right) v_O$$

You now have the ratio of the voltage output to the input source:

$$\frac{v_O}{v_S} = \frac{R_1 + R_2}{R_1}$$

$$\frac{v_O}{v_S} = 1 + \frac{R_2}{R_1}$$

You just made the input voltage  $v_S$  larger by making sure the ratio of the two resistors is greater than 1. You read right: To make the input signal louder, the feedback resistor  $R_2$  should have a larger value than input resistor  $R_1$ . Piece of cake! For example, if  $R_2 = 9 \text{ k}\Omega$  and  $R_1 = 1 \text{ k}\Omega$ , then you have the following output voltage:

$$v_O = \left( 1 + \frac{9 \text{ k}\Omega}{1 \text{ k}\Omega} \right) v_S$$
$$v_O = 10v_S$$

You've amplified the input voltage by ten. Awesome!

## ***Following the leader with the voltage follower***

A special case of the noninverting amplifier is the *voltage follower*, in which the output voltage follows in lock step with whatever the input signal is. In the voltage follower (pictured in [Figure 10-4](#)),  $v_S$  is connected to the noninverting terminal. You can express this idea as

$$v_P = v_S$$

You also see that the output  $v_O$  is connected to the inverting terminal, so

$$v_N = v_O$$

An ideal op amp has equal noninverting and inverting voltage. This means that the preceding two equations are equal. In other words

$$v_O = v_S$$

You can also view the voltage follower as a special case of the noninverting amplifier with a gain of 1, because the feedback resistor  $R_2$  is zero (a short circuit) and resistor  $R_1$  is infinite (open circuit):

$$\frac{v_O}{v_S} = 1 + \frac{\overbrace{R_2}^0}{\underbrace{R_1}_{\infty}} = 1$$

$$v_O = v_S$$

The output voltage  $v_O$  is equal to the input source voltage  $v_S$ . The voltage gain is 1 where the output voltage follows the input voltage. But a piece of wire gives a gain of 1, too, so what good is this circuit? Well, the voltage follower provides a way to put together two separate circuits without having them affect each other. When they do affect each other in a bad way, that's called *loading*. A voltage follower solves the loading problem.

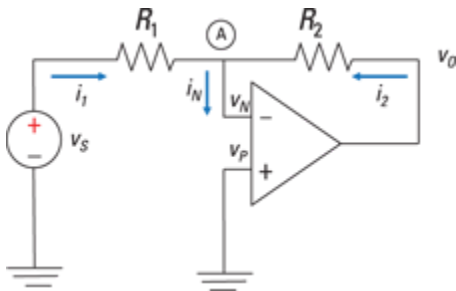
## ***Turning things around with the inverting amplifier***

An inverting amplifier takes an input signal and turns it upside down at the op-amp output. When the value of the input signal is positive, the output of the inverting amplifier is negative, and vice versa.

[Figure 10-5](#) shows an inverting op amp. The op amp has a feedback resistor  $R_2$  and an input resistor  $R_1$  with one end connected to the voltage source. The other end of



the input resistor is connected to the inverting terminal, and the noninverting terminal is grounded at 0 volts. The amount of amplification depends on the ratio between the feedback and input resistor values.



*Illustration by Wiley, Composition Services Graphics*

**Figure 10-5:** A standard inverting amplifier.

Because the noninverting input is grounded to 0 volts, you have

$$v_P = v_N = 0$$

For ideal op amps, the voltages at the inverting and noninverting terminals are equal and set to zero. The inverting terminal is connected to a virtual ground because it's indirectly connected to ground by  $v_P$ .

Apply Kirchhoff's current law (KCL) to Node A to get the following:

$$\frac{v_S - v_N}{R_1} + \frac{v_O - v_N}{R_2} - i_N = 0 \quad (\text{KCL})$$

Simplify the equation with the following constraints for an ideal op amp:

$$v_P = v_N = 0$$

$$i_P = i_N = 0$$

The constraints make the KCL equation simpler:

$$\frac{v_S}{R_1} + \frac{v_O}{R_2} = 0$$

You wind up with the following relationship between the input and output voltages:

$$\frac{v_O}{v_S} = -\frac{R_2}{R_1}$$

Again, the amplification of the signal depends on the ratio of the feedback resistor  $R_2$  and input resistor  $R_1$ . You need only external components of the op amp to make the signal way bigger. The negative sign means the output voltage is an amplified but inverted (or upside down) version of the input signal.

For a numerical example, let  $R_2 = 10 \text{ k}\Omega$  and  $R_1 = 1 \text{ k}\Omega$ . In that case, the inverted output voltage  $v_O$  is ten times as big as the input voltage  $v_S$ . In nothing flat, you just made a weak signal stronger. Nice work — you deserve a raise!



Resistors should be around the  $1 \text{ k}\Omega$  to  $100 \text{ k}\Omega$  range to minimize the effects of variation in the op-amp characteristics and voltage sources.

## ***Adding it all up with the summer***

You can extend the inverting amplifier to more than one input to form a *summer*, or *summing amplifier*. [Figure 10-6](#) shows an inverting op amp with two inputs. The two inputs connected at Node A (called a *summing point*) are connected to an inverting terminal.

Because the noninverting input is grounded, Node A is also connected as a virtual ground. Applying the KCL equation at Node A, you wind up with

$$i_1 + i_2 + i_F - i_N = 0$$

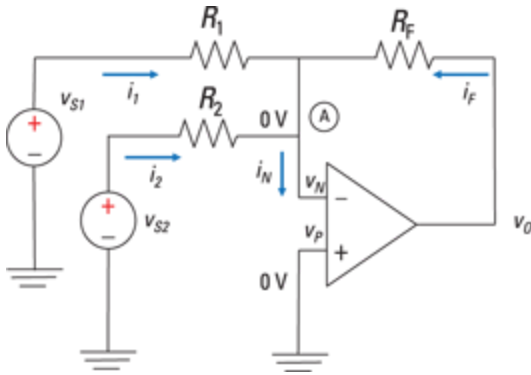


Illustration by Wiley, Composition Services Graphics

**Figure 10-6:** An inverting op-amp summer.

Replace the input currents in the KCL equation with node voltages and Ohm's law ( $i = v/R$ ):

$$\frac{v_{S1}-0}{R_1} + \frac{v_{S2}-0}{R_2} + \frac{v_O-0}{R_F} - \underbrace{i_N}_0 = 0$$

Because  $i_N = 0$  for an ideal op amp, you can solve for the output voltage in terms of the input source voltages:

$$v_O = \underbrace{\left(-\frac{R_F}{R_1}\right)}_{G_1} v_{S1} + \underbrace{\left(-\frac{R_F}{R_2}\right)}_{G_2} v_{S2}$$

$$v_O = G_1 v_{S1} + G_2 v_{S2}$$

The output voltage is a weighted sum of the two input voltages. The ratios of the feedback resistance to the input resistances determine the gains,  $G_1$  and  $G_2$ , for this op-amp configuration.

To form a summing amplifier (or inverting summer), you need to set the input resistors equal with the following constraint:

$$R_1 = R_2 = R$$

Applying this constraint gives you the output voltage:

$$v_O = \left(-\frac{R_F}{R}\right)(v_{S1} + v_{S2})$$

This shows that the output is proportional to the sum of the two inputs. You can easily extend the summer to more than two inputs.

Plug in the following values for [Figure 10-6](#) to test this mumbo jumbo:  $v_{S1} = 0.7$  volts,  $v_2 = 0.3$  volts,  $R_1 = 7 \text{ k}\Omega$ ,  $R_2 = 3 \text{ k}\Omega$ , and  $R_F = 21 \text{ k}\Omega$ . Then calculate the output voltage  $v_o$ :

$$v_o = \left(-\frac{R_F}{R_1}\right)v_{S1} + \left(-\frac{R_F}{R_2}\right)v_{S2}$$

$$v_o = \left(-\frac{21 \text{ k}\Omega}{7 \text{ k}\Omega}\right)(0.7 \text{ V}) + \left(-\frac{21 \text{ k}\Omega}{3 \text{ k}\Omega}\right)(0.3 \text{ V})$$

$$v_o = -4.2 \text{ V}$$

The signals are bigger — mission accomplished. If signals are changing in time, the summer adds these signals instantly with no problem.

## ***What's the difference? Using the op-amp subtractor***

You can view the next op-amp circuit — which is a *differential amplifier*, or *subtractor* — as a combination of a noninverting amplifier and inverting amplifier (see the earlier related sections for the scoop on these amplifiers). [Figure 10-7](#) shows an op-amp subtractor.

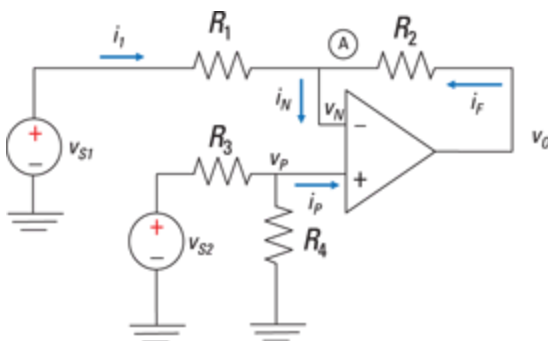


Illustration by Wiley, Composition Services Graphics

**Figure 10-7:** An op-amp subtractor.

You use superposition to determine the input and the output relationship. As I explain in [Chapter 7](#), the superposition technique involves the following steps:

- 1. Turn on one source and turn off the others.**
- 2. Determine the output of the source that's on.**
- 3. Repeat for each input, taking the sources one at a time.**
- 4. Algebraically add up all the output contributions for each input to get the total output.**

For [Figure 10-7](#), first turn off voltage source  $v_{S2}$  so that there's no input at the noninverting terminal ( $v_P = 0$ ). With the noninverting input grounded, the circuit acts like an inverting amplifier. You wind up with output contribution  $v_{O1}$  due to  $v_{S1}$ :

$$v_{O1} = \left( -\frac{R_2}{R_1} \right) v_{S1}$$

You next turn off voltage source  $v_{S1}$  ( $v_N = 0$ ) and turn  $v_{S2}$  back on. The circuit now acts like a noninverting amplifier. Because this is an ideal op amp, no current ( $i_P = 0$ ) is drawn from the series connection of resistors  $R_3$  and  $R_4$ , so you can use the voltage divider equation to determine  $v_P$ . The voltage at the noninverting input is given by

$$v_P = \left( \frac{R_4}{R_3 + R_4} \right) v_{S2}$$

The noninverting input  $v_P$  is amplified to give you an output  $v_{O2}$ :

$$v_{O2} = \left( 1 + \frac{R_2}{R_1} \right) v_P$$

$$v_{O2} = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{R_4}{R_3 + R_4} \right) v_{S2}$$

You then add up the outputs  $v_{O1}$  and  $v_{O2}$  to get the total output voltage:

$$\begin{aligned} v_O &= v_{O1} + v_{O2} \\ &= -\underbrace{\left(\frac{R_2}{R_1}\right)}_{G_1} v_{S1} + \underbrace{\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)}_{G_2} v_{S2} \\ &= -G_1 v_{S1} + G_2 v_{S2} \end{aligned}$$

Here,  $-G_1$  is the inverting gain and  $G_2$  is the noninverting gain. You need the following constraint to form a subtractor:

$$\frac{R_3}{R_1} = \frac{R_4}{R_2}$$

Applying the constraint simplifies the output voltage, giving you

$$v_O = \frac{R_2}{R_1} (v_{S2} - v_{S1})$$

There you have it! You now have the output proportional to the difference between the two inputs.

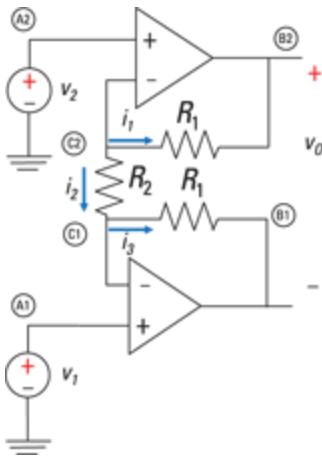
## ***Increasing the Complexity of What You Can Do with Op Amps***

If you've already read the earlier section "[Looking at Op-Amp Circuits](#)," then you have the basic building blocks of op-amp circuits and are ready to tackle the complex processing actions I describe next.

### ***Analyzing the instrumentation amplifier***

The instrumentation amplifier is a differential amplifier suited for measurement and test equipment. [Figure 10-8](#)

shows the input stage of an instrumentation amplifier. Your goal is to find the voltage output  $v_O$  proportional to the difference of the two inputs,  $v_1$  and  $v_2$ . Getting the desired output requires some algebraic gymnastics, but you can handle it.



*Illustration by Wiley, Composition Services Graphics*

**Figure 10-8:** Input stage of an instrumentation amplifier.

At Node C2, you apply KCL ( $i_1 + i_2 = 0$ ) and Ohm's law ( $i = v/R$ ) and wind up with

$$\text{Node C2: } \frac{v_{C2} - v_{B2}}{R_1} + \frac{v_{C2} - v_{C1}}{R_2} = 0$$

At Node C1, the KCL equation ( $-i_2 + i_3 = 0$ ) with Ohm's law leads you to

$$\text{Node C1: } \frac{v_{C1} - v_{C2}}{R_2} + \frac{v_{C1} - v_{B1}}{R_1} = 0$$

[Figure 10-8](#) shows the noninverting input connected to independent voltages  $v_1$  and  $v_2$ . Use the op-amp voltage constraint  $v_P = v_N$  to get the following:

$$v_{C2} = v_2 \text{ and } v_{C1} = v_1$$

Substitute  $v_1$  and  $v_2$  into KCL equations, which gives you

$$\text{Node C2: } \frac{v_2 - v_{B2}}{R_1} + \frac{v_2 - v_1}{R_2} = 0$$

$$\text{Node C1: } \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_{B1}}{R_1} = 0$$

Now solve for  $v_{B2}$  and  $v_{B1}$ , because the output voltage  $v_O$  depends on these two values:

$$v_{B2} = v_2 + \frac{R_1}{R_2}(v_2 - v_1)$$

$$v_{B1} = v_1 + \frac{R_1}{R_2}(v_1 - v_2)$$

The output voltage  $v_O$  is the difference between the  $v_{B1}$  and  $v_{B2}$ :

$$\begin{aligned} v_O &= v_{B2} - v_{B1} \\ &= \left[ v_2 + \frac{R_1}{R_2}(v_2 - v_1) \right] - \left[ v_1 + \frac{R_1}{R_2}(v_1 - v_2) \right] \\ &= \left( 1 + \frac{2R_1}{R_2} \right) (v_2 - v_1) \end{aligned}$$

Cool! Resistor  $R_2$  can be used to amplify the difference  $v_2 - v_1$ . After all, it's easier to change the value of one resistor  $R_2$  than of two resistors  $R_1$ .

## ***Implementing mathematical equations electronically***

As an example of how op amps can solve equations, consider a single output and three voltage input signals:

$$v_O = 10v_1 + 5v_2 - 4v_3$$

You can rewrite the equation in many ways to determine which op-amp circuits you need to perform the math. Here's one way:

$$v_O = -10(-v_1) - 5(-v_2) - 4(v_3)$$

The equation suggests that you have an inverting summer with three inputs:  $-v_1$ ,  $-v_2$ , and  $v_3$ . You need an inverting amplifier with a gain of  $-1$  for  $v_1$  and  $v_2$ . Input



$v_1$  has a summing gain of  $-10$ , input  $v_2$  has a summing gain of  $-5$ , and input  $v_3$  has a summing gain of  $-4$ . You can see one of many possible op-amp circuits in the top diagram of [Figure 10-9](#). The dashed boxes indicate the two inverting amplifiers and the inverting summer.

The outputs of the two inverting amplifiers are  $-v_1$  and  $-v_2$ , and they're inputs to the inverting summer. The third input to the summer is  $v_3$ . Adding up the three inputs with required gains entails an inverting summer, which you see in [Figure 10-9](#).

For input  $v_1$ , the ratio of the inverting summer's feedback resistor of  $200\text{ k}\Omega$  to its input resistor of  $20\text{ k}\Omega$  provides a gain of  $-10$ . Similarly, for input  $v_2$ , the ratio of the feedback resistor of  $200\text{ k}\Omega$  to its input resistor of  $40\text{ k}\Omega$  gives you a gain of  $-5$ . Finally, for input  $v_3$ , the ratio of the feedback resistor of  $200\text{ k}\Omega$  to its input resistor of  $50\text{ k}\Omega$  provides a gain of  $-4$ . You can use other possible resistor values as long as the ratio of resistors provides the correct gains for each input.

Reducing the number of op amps during the design process helps lower costs. And with some creativity, you can reduce the number of op amps in the circuit by rewriting the math equation of the input-output relationship:

$$v_o = 10v_1 + 5v_2 - 4v_3$$

$$v_o = -2[-5v_1 - 2.5v_2] - 4v_3$$

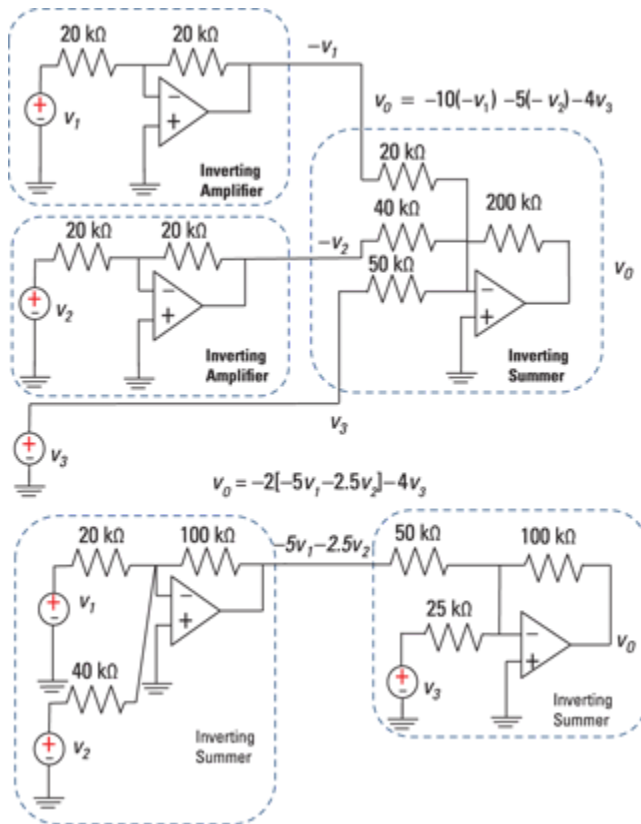


Illustration by Wiley, Composition Services Graphics

**Figure 10-9:** Doing calculations with op amps.

This suggests you need two op amps. One input is a combination of inputs  $v_1$  and  $v_2$  formed by an inverting summer. When you take the output of the first summer and feed it and another input to a second inverting summer, the result is proportional to  $v_3$  with gain  $-4$ . The bottom diagram of [Figure 10-9](#) shows one way to implement this equation.

For  $v_1$ , the ratio of the feedback resistor of  $100\text{ k}\Omega$  to the input resistor of  $20\text{ k}\Omega$  produces a gain of  $-5$ . For  $v_2$ , the input resistor of  $40\text{ k}\Omega$  gives you a gain of  $-2.5$ . The output of the first summer is then multiplied by  $-2$  because of the ratio of the second inverting summer's feedback resistor of  $100\text{ k}\Omega$  to the input resistor of  $50\text{ k}\Omega$ . The input  $v_3$  to the second summer is multiplied by  $-$

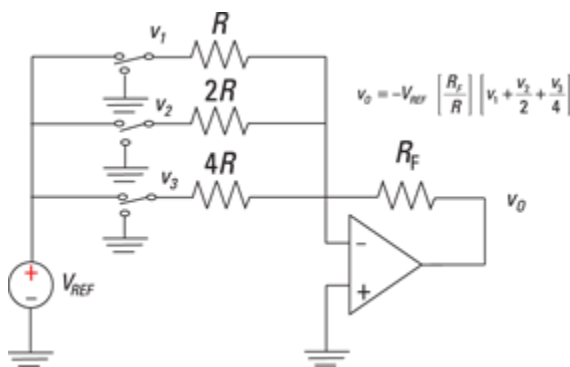
4 because of the ratio between the 100-k $\Omega$  feedback resistor to the 25-k $\Omega$  resistor.

## ***Creating systems with op amps***

Op-amp circuits are basic building blocks for many applications in signal processing, instrumentation, process control, filtering, digital-to-analog conversion, and analog-to-digital conversion.

For example, you can do a digital-to-analog conversion (DAC) using the inverting summer. The primary purpose of this common device is to convert a digital signal consisting of binary 1s and 0s (perhaps coming from your personal computer) to an analog and continuous signal (to run your DC motor in your remote-control toy). The device has extensive applications in robotics, high-definition televisions, and cellphones.

To simplify the example, I focus on 3-bit devices (even though most applications use 8- to 24-bit DACs). DACs have one output voltage  $v_o$  with a number of digital inputs ( $b_0, b_1, b_2$ ), along with a reference voltage  $V_{REF}$ . You see a block diagram of a 3-bit input in [Figure 10-10](#).



*Illustration by Wiley, Composition Services Graphics*

**Figure 10-10:** A digital-to-analog converter.

The following equation gives you the relationship between the digital input and analog output:

$$v_O = V_{REF} \left( b_2 + \frac{b_1}{2} + \frac{b_0}{4} \right)$$

Bit  $b_2$  is the most significant bit (MSB) because it's weighted with the largest weight in the sum; bit  $b_0$  is the least significant bit (LSB) because it has the smallest weight.

To implement a DAC, you can use an inverting summer, as in [Figure 10-10](#). Also shown are the digital inputs that can have only one of two voltage values: A digital 1 is equal to  $V_{REF}$ , and a digital 0 is equal to 0 volts. The inputs  $v_1$ ,  $v_2$ , and  $v_3$  to the summer are weighted appropriately to give you the voltage output  $v_O$  based on the three inputs. Input  $v_1$  has the most weight, and input  $v_3$  has the least. That's my two bits on making DACs.