Part I

Getting Started with Circuit Analysis

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In this part . . .

- Discover what circuit analysis is all about.
- Get the scoop on current and voltage behaviors in common circuit components and find out how to read circuit diagrams.
- Familiarize yourself with Kirchhoff's voltage law and Kirchhoff's current law — two laws essential for creating connection equations.
- Use source transformation and current and voltage divider techniques to simplify circuit analysis.

Chapter 1
Introducing Circuit Analysis

In This Chapter

- Understanding current and voltage
- Applying laws when you connect circuit devices
- Analyzing circuits with algebra and calculus
- Taking some mathematical shortcuts

Circuit analysis is like the psychoanalysis of the electrical engineering world because it's all about studying the behavior of circuits. With any circuit, you have an input signal, such as a battery source or an audio signal. What you want to figure out is the circuit's *output* — how the circuit responds to a given input.

A circuit's output is either a voltage or a current. You have to analyze the voltages and currents traveling through each element or component in the circuit in order to determine the output, although many times you don't have to find *every* voltage and *every* current within the circuit.

Circuit analysis is challenging because it integrates a variety of topics from your math and physics courses in addition to introducing techniques specific to determining circuit behavior. This chapter gives you an overview of circuit analysis and some of the key concepts you need to know before you can begin understanding circuits.

Getting Started with Current and Voltage

Being able to analyze circuits requires having a solid understanding of how voltage and current interact within a circuit. Chapter 2 gives you insight into how voltage and current behave in the types of devices normally found in circuits, such as resistors and batteries. That chapter also presents the basic features of circuit diagrams, or *schematics.*

The following sections introduce you to current and voltage as well as a direction-based convention that’s guaranteed to come in handy in circuit analysis.

Going with the flow with current

Current is a way of measuring the amount of electric charge passing through a given point within a certain amount of time. Current is a flow rate. The mathematical definition of a current is as follows:

$$i = \frac{dq}{dt}$$

The variable *i* stands for the current, *q* stands for the electrical charge, and *t* stands for time.

TECHNICAL STUFF The charge of one electron is 1.609×10^{-19} coulombs (C).

Current measures the flow of charges with dimensions of coulombs per second (C/s), or *amperes* (A). In engineering, the current direction describes the net flow of positive charges. Think of current as a *through variable,* because the flow of electrical charge passes through a point in the circuit. The arrow in Figure 1-1 shows the current direction.

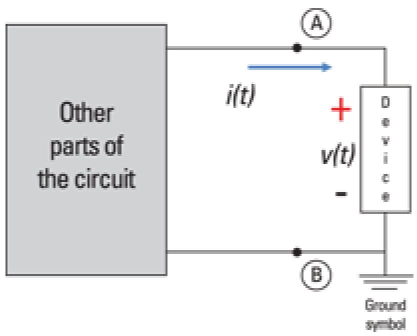

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Figure 1-1: Current direction, voltage polarities, and the passive sign convention.

Measuring current through a device requires just one point of measurement. As an analogy, say you're asked to count the number of cars flowing through your long stretch of residential street for 10 minutes. You can count the number of cars from your home or your friend's home next door or the house across the street. You need just one location point to measure the flow of cars.

REMEMBER Two types of current exist: alternating current (AC) and direct current (DC). With AC, the charges flow in both directions. With DC, the charges flow in just one direction.

TIP If you have trouble keeping AC and DC straight, try this mnemonic device: AC means "always changing," and DC means "doesn't change."

Recognizing potential differences with voltage

From physics, you know that plus and minus charges attract each other and that like charges repel each other. You need energy to separate the opposite charges. As long as the charges are separated, they have electric potential energy.

Voltage measures the amount of energy *w* required to move a given amount of charge *q* as it passes through the circuit. You can think of voltage as electric potential difference. Mathematically, voltage is defined as

$$v = \frac{dw}{dq}$$

Voltage has units of *volts* (V), which is the same as joules per coulomb (J/C). In a 12-volt car battery, the opposite charges on the battery terminals have a separation of 12 units of energy per unit of charge (or 12 V = 12 J/C). When terminals are separated, there's no current flow. If you provide a conducting path between the opposite charges, you now have charges flowing, resulting in an electric current.

> It takes two points to measure voltage across a device, just like it takes two points to measure height or distance. That's why you can think of voltage as an *across variable.*

Refer to Figure 1-1 to see the positive and negative voltage signs (called *polarities*) of a device, labeled at Terminals A and B.

Staying grounded with zero voltage

Because measuring voltage requires two points, you need a common reference point called a *ground*. You assign ground as 0 volts, where all other points in a circuit are measured with respect to ground. This is analogous to defining sea level as a reference point of 0 feet so you can measure the height of mountains. When the sea canyon is below sea level, you assign a negative algebraic sign. In circuits, the negative sign means the answer is less than the ground potential of 0 volts. Refer to Figure 1-1 to see an example of a typical ground symbol.

From algebra to calculus and back to algebra

When you're looking at simple circuits, such as a direct current (DC) circuit that involves only resistors and a constant battery source, you can get by with just algebra in your analysis. Because you don't have to worry about any fancy math, you can focus on analytical approaches such as node-voltage analysis (Chapter 5) and superposition (Chapter 7).

But when you start looking at more complex circuits, such as an alternating current (AC) circuit, the math becomes more complex. AC circuits have time-varying sources, capacitors, and inductors, so you need calculus to deal with the changes of electrical variables over time. Applying connection constraints (Kirchhoff's laws) to AC circuits gives you differential equations, but never fear! The chapters in Part IV show you how to solve differential equations of first-order and second-order circuits when you have capacitors and inductors connected to resistors.

Of course, sometimes you can use advanced techniques to skip over calculus entirely. You can convert differential equations into simpler algebraic problems using the Laplace transform method, which I introduce in Chapter 16.

You may have seen a streetcar with one trolley pole and an electric bus with two trolley poles. Why the difference? The streetcar uses its wheels as its ground point, so current coming from the generator flows back to the generator through the earth via the wheels grounded at 0 volts. For the electric bus, you need two wires because current can’t flow through the rubber tires.

Getting some direction with the passive sign convention

Along with the algebraic signs of your calculated answers, the *passive sign convention* orients you to what’s happening in a circuit. Specifically, it tells you that current enters a passive device at its positive voltage terminal.

Here’s how passive sign convention works: You assign plus and minus signs to each device to serve as reference marks. After you arbitrarily assign the polarities of a device, you define the current direction so that it enters the positive side. (You can see what I mean by referring to Figure 1-1.) If your answers for voltage or current are positive, then the polarities line up with your assigned polarities or current direction. If your answers come up negative, they’re opposite to your assigned polarities or current direction. A negative answer isn’t wrong; it’s just reverse to your assigned reference marks.

The way you assign your polarities and current direction doesn’t control the circuit behavior. Rather, the algebraic signs of your answers tell you the actual directions of voltages and current in the circuit.

Beginning with the Basic Laws

A circuit is basically a collection of electrical devices, such as resistors, batteries, capacitors, and inductors, arranged to perform a certain function. Each component of a circuit has its own constraints. When you connect devices in any circuit, the devices follow certain laws:

- **Ohm's law:** This law describes a linear relationship between the voltage and current for a resistor. You can find details about resistors and Ohm's law in Chapter 2.
- **Kirchhoff's voltage law (KVL):** KVL says the algebraic sum of the voltage drops and rises around a loop of a circuit is equal to zero. You can find an explanation of voltage drops, voltage rises, circuit loops, and KVL in Chapter 3.
- **Kirchhoff's current law (KCL):** KCL says the algebraic sum of incoming and outgoing currents at a node is equal to zero. Chapter 3 provides info on applying KCL and defines nodes in a circuit.

With these three laws, you can solve for the current or voltage in any device.

Applying Kirchhoff's laws can become tedious, but you can take some shortcuts. Source transformation allows you to convert circuits to either parallel or series circuits. Then, with all the devices connected in series or in parallel, you can use the voltage divider and current divider techniques to find the voltage or current for any device. I cover these techniques in Chapter 4.

Surveying the Analytical Methods for More-Complex Circuits

When you have many simultaneous equations to solve or too many inputs, you can use the following techniques to reduce the number of simultaneous equations and simplify the analysis:

- **Node-voltage analysis:** A node is a point in the circuit. This technique has you apply Kirchhoff's current law (KCL), producing a set of equations that you use to find unknown node voltages. When you know all the node voltages in a circuit, you can find the voltage across each device. I cover node-voltage analysis in Chapter 5.
- **Mesh-current analysis:** Mesh-current analysis deals with circuits that have many devices connected in many loops. You use Kirchhoff's voltage law (KVL) to develop a set of equations with unknown mesh currents. Because you can describe the device currents in terms of the mesh currents, finding the mesh currents lets you calculate the current through each device in the circuit. See Chapter 6 for info on mesh-current analysis.
- **Superposition:** When you have multiple independent power sources in a linear circuit, superposition comes to your rescue. Analyzing linear circuits involves using

only devices (such as resistors, capacitors, and inductors) and independent sources. By applying superposition, you can take a complex circuit that has multiple independent sources and break it into simpler circuits, each with only one independent source. The circuit's total output then is the algebraic sum of output contributions due to the input from each independent source. Turn to Chapter 7 for details on superposition.

- **Thévenin's and Norton's theorems:** Thévenin and Norton equivalent circuits are valuable tools when you're connecting and analyzing two parts of a circuit. The interaction between the source circuit (which processes and delivers a signal) and load circuits (which consume the delivered signal) offers a major challenge in circuit analysis. Thévenin's theorem simplifies the analysis by replacing the source circuit's complicated arrangement of independent sources and resistors with a single voltage source connected in series with a single resistor. Norton's theorem replaces the source circuit with a single current source connected in parallel with a single resistor. You can find out more about both theorems, including how to apply them, in Chapter 8.

Introducing Transistors and Operational Amplifiers

Although transistors and operational amplifiers (op amps) are modeled with dependent sources, they're referred to as *active devices* because they require power to work. Transistors, which are made of semiconductor material, are used primarily as current amplifiers (see Chapter 9). Op amps are linear devices consisting of many transistors, resistors, and capacitors. They're used to perform many mathematical and processing operations, including voltage amplification (see Chapter 10). You can think of op amps as very high-gain DC amplifiers.

The op amp is one of the leading linear active devices in modern circuit applications. This device does mathematical operations (addition, subtraction, multiplication, division, integration, derivatives, and so on) quickly because it does them electronically. You put together basic op-amp circuits to build mathematical models.

Dealing with Time-Varying Signals, Capacitors, and Inductors

Circuits deal with signals that carry energy and information. *Signals* are time-varying electrical quantities processed by the circuit. Throughout the book, you deal with linear circuits, where the output signal is proportional to the input signal.

Chapter 11 introduces you to signal sources that change with time (unlike batteries, whose signals don't change with time). Signals that change in time can carry information about the real world, like temperature, pressure, and sound. You can combine basic functions such as sine and exponential functions to create even more interesting signals.

When you add passive, energy-storing elements (such as capacitors and inductors) to a circuit, the analysis gets a little tougher because now you need differential equations to analyze the circuit’s behavior. In fact, the circuits created with capacitors and inductors get their names from the differential equations that result when you apply Kirchhoff’s laws in the course of analysis:

- *First-order circuits,* which have a resistor and capacitor or a resistor and inductor, are described with first-order differential equations. The capacitor’s current is related to the first derivative of the voltage across the capacitor, and the inductor’s voltage is related to the first derivative of the current through the inductor. See Chapter 13 for help analyzing first-order circuits.
- *Second-order circuits* consist of capacitors, inductors, and resistors and are described by second-order differential equations. Flip to Chapter 14 for pointers on analyzing these circuits.

Avoiding Calculus with Advanced Techniques

I don’t know about you, but I hate using calculus when I don’t have to, which is why I’m a fan of the advanced circuit analysis techniques that allow you to convert calculus-based problems into problems requiring only algebra.

Phasors make your life simple when you’re dealing with circuits that have capacitors and inductors, because you don’t need differential equations to analyze circuits in the phasor domain. Phasor analysis investigates circuits that have capacitors and inductors in the same way you analyze circuits that have only resistors. This technique applies when your input is a sine wave (or a sinusoidal signal). See Chapter 15 for details on phasors.

Analyzing circuits with software

When circuits get too complex to analyze by hand, today's software offers many capabilities. Here are some commonly used software tools:

- **SPICE:** This software was originally developed at the Electronics Research Laboratory of the University of California, Berkeley. SPICE stands for Simulation Program for Integrated Circuit Emphasis. PSpice is a PC version of SPICE, and several companies, such as Cadence (`www.cadence.com`) and Linear Technology (`www.linear.com`), produce various versions of SPICE. Both companies offer demos and free versions of their software.
- **National Instrument's Multisim:** This is one of the granddaddies of circuit analysis software as well as a great tool for beginners. It also has a cool feature that shows changing voltages in real time of the circuit. The trial version pretty much lets you do anything you want. A student version is available as well. Visit `www.ni.com/multisim/` for more information.
- **Ngspice:** This tool is a widely used open-source circuit simulator from Sourceforge. The free Ngspice software (available at `ngspice.sourceforge.net`) is developed by many users, and its code is based on several major open-source software packages.

Chapter 16 describes a more general technique that's handy when your input isn't a sinusoidal signal: the Laplace transform technique. You use the Laplace transform to change a tough differential equation into a simpler problem involving algebra in the Laplace domain (or *s*-domain). You can then study the circuit's behavior using only algebra. The *s*-domain method I cover in Chapter 17 gives you the same results you'd get from calculus methods to solve differential equations, which you find in Chapters 13 and 14. The algebraic approach in the *s*-domain follows along the same lines as the approach you use for resistor-only circuits, only in place of resistors, you have *s*-domain impedances.

A major component found in older entertainment systems is an electronic filter that shapes the frequency content of signals. In Chapter 18, I present low-pass, high-pass, band-pass, and band-stop (or band-reject) filters based on simple circuits. This serves as a foundation for more-complex filters to meet more stringent requirements.

Chapter 18 also covers Bode diagrams to describe the frequency response of circuits. The Bode diagrams help you visualize how poles and zeros affect the frequency response of a circuit. The frequency response is described by a transfer (or network) function, which is the ratio of the output signal to the input signal in the *s*-domain. The *poles* are the roots of the polynomial in the transfer function’s denominator, and the *zeros* are the roots of the polynomial in the numerator.

Chapter 2

Clarifying Basic Circuit Concepts and Diagrams

In This Chapter

- Sorting out current-voltage relationships
- Mapping out circuits with schematics
- Understanding a circuit's loops and nodes

Before you can begin working with circuits, you need to have a basic understanding of how current and voltage behave in some of the devices most commonly found in circuits. You also need to be able to read basic circuit diagrams, or *schematics*. This chapter is all about helping you get comfortable with these basics so you can dive confidently into the world of circuit analysis.

Looking at Current-Voltage Relationships

Given that power is a rate of energy transfer, electrical power $p(t)$ is defined as the product of the voltage $v(t)$ and current $i(t)$ as a function of time:

$$p(t) = i(t) \cdot v(t)$$

TIP

To remember the formula $p = iv$, I tell students to remember the phrase *poison ivy.* It may be corny, but it works.

An electrical device absorbs power when $p(t)$ is positive, implying that the current and voltage have the same algebraic sign. The device delivers power when $p(t)$ is negative, implying current and voltage have opposite algebraic signs. See Chapter 1 for details on the passive sign convention and what negative current and negative voltage mean.

Power has units of *watts* (W), or joules per second. The units of current (coulombs per second) and voltage (joules per coulomb) should cancel out to give you the desired units for power. Here's the dimensional analysis:

$$p(t) = i(t) \cdot v(t) \quad \rightarrow \quad \left(\frac{\text{joules}}{\text{sec}}\right) = \left(\frac{\cancel{\text{coulombs}}}{\text{sec}}\right) \cdot \left(\frac{\text{joules}}{\cancel{\text{coulombs}}}\right)$$

So the power relationship works out as far as units are concerned!

Because power involves current and voltage, understanding the current-voltage (i-v) characteristics of various devices, such as resistors and batteries, is important. Resistors have a very straightforward relationship with voltage and current. In fact, for circuits that contain only resistors and independent power sources, the relationship between current and voltage simply depends on a device's resistance, which is a constant *R*. In the following sections, I introduce you to some devices and circuit configurations that provide a certain amount of resistance, no resistance, or infinite resistance.

Absorbing energy with resistors

Resistors are simple electrical devices that appear in almost every circuit. They suck up energy and give it off as heat. An everyday object like a toaster or an incandescent light bulb can be modeled as a resistor.

You may think resistors don't do much because they waste energy, but they actually have a few important purposes:

- **Reducing voltage:** A resistor can use up some voltage so that not all of the supplied voltage falls on another device. You're basically dividing the supplied voltage into smaller voltages by adding resistors to a circuit.
- **Limiting current:** If you don't want current to burn up a device, you can limit current by connecting a resistor to the device.
- **Timing and filtering:** You can use resistors, along with capacitors, to create timing circuits or filters. I discuss timing and filtering in Chapters 12 and 13 and filtering specifically in Chapter 18.

The following sections introduce current-voltage relationships and graph the *i*-*v* curves for resistors. They also show you how to calculate the power dissipated as heat.

Applying Ohm's law to resistors

Ohm's law says that the current through a linear resistor is proportional to the voltage across the resistor. Mathematically, you have Ohm's law described as

$$v = Ri$$

$$i = Gv \qquad \rightarrow \qquad R = \frac{1}{G}$$

where v is voltage, i is current, R is resistance, and G is conductance. The resistance R or conductance G is a proportionality constant relating the resistor voltage and its current. For example, if the voltage is doubled, then the current is doubled.

Resistance provides a measure of difficulty in pushing electricity through a circuit. The unit of resistance is *ohms* (Ω), and the unit of conductance is *siemens* (S). For fun (if you call algebra fun), you can rearrange Ohm's law:

$$i = \frac{v}{R}$$

$$R = \frac{v}{i}$$

When you don't know the current, use the top equation, and when you don't know the resistance, use the bottom equation.

Figure 2-1 shows the symbol and *i*-*v* characteristic for a linear resistor. The slope of the line gives you conductance *G,* and the reciprocal of the slope produces the resistance value *R*.

You can have large current flow for a small applied voltage if the resistance is small enough. Some materials cooled to very low temperatures are superconductors, having near-zero resistance. As soon as current flows in a superconducting circuit, current flows forever unless you disconnect the voltage source.

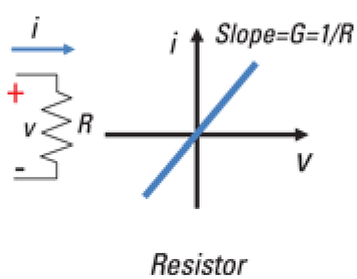

Illustration by Wiley, Composition Services Graphics

Figure 2-1: The *i*-*v* characteristic for resistors.

Calculating the power dissipated by resistors

Because power is $p = iv$, you can use Ohm's law, $v = iR$, to figure the amount of heat a resistor gives off when current flows or voltage is applied across the resistor. Here are two versions of the power-dissipation formula, which you get by plugging in the voltage or current value from Ohm's law into $p = iv$:

$$p = iv = i(iR) = i^2R$$

$$p = \left(\frac{v}{R}\right)v = \frac{v^2}{R}$$

So by knowing either the voltage or current for a given resistor R, you can find the amount of power dissipated. If you calculate the power dissipated as 0.1 watts, then a $^1/_4$-watt (0.25-watt) resistor can handle this amount of power. A $^1/_8$-watt (0.125-watt) resistor should be able to handle that amount as well, but when it comes to power ratings, err on the larger side.

Offering no resistance: Batteries and short circuits

Batteries and short circuits have different *i*-*v* characteristics but the same slope (or equivalently, zero resistance), as Figure 2-2 shows. In certain situations, you can remove a battery from a circuit by replacing it with a short circuit, 0 volts (I explain how in Chapter 7). Read on for the details on batteries, short circuits, and their *i*-*v* characteristics.

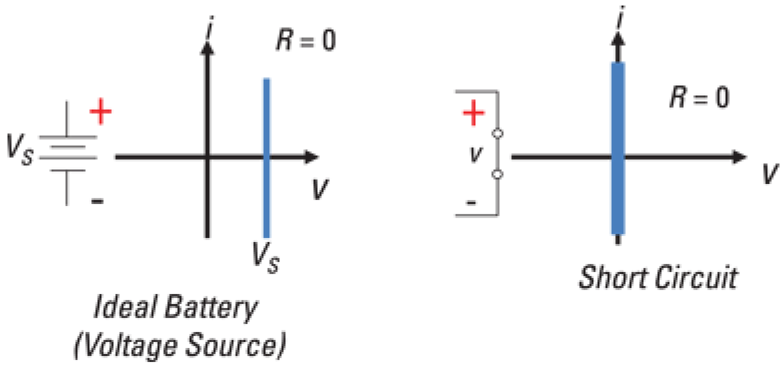

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Figure 2-2: You get zero resistance and constant voltage from an ideal voltage source or short circuit.

Short stuff: Why birds on a wire don’t get zapped

Have you ever wondered how birds sit on a bare high-voltage line of 25,000 volts without getting shocked? The entire length of wire is at 25,000 volts, so the entire bird on the wire is also at 25,000 volts. Because there’s no voltage difference, current doesn’t flow through the bird. Now, if the bird decides to stretch its wing and touches an adjacent wire at 0 volts, well, bad things happen, and it’s bye-bye birdie! Fortunately for the birds, the power lines are strung apart so they aren’t short circuited.

Batteries: Providing power independently

In circuit analysis, batteries are referred to as *independent sources.* Specifically, batteries are *independent sources of voltage,* supplying the circuit with a constant voltage that’s independent of the current. So no matter how much current is drawn from the battery, you still have the same voltage. Figure 2-2 shows the electrical symbol and the *i*-*v* characteristic of a battery. Because the slope is infinite, an ideal battery has zero resistance.

You can convert a battery into an independent current source through source transformation, as I explain in Chapter 4. I cover independent current sources later in “Facing infinite resistance: Ideal current sources and open circuits.”

Short circuits: No voltage, no power

Figure 2-2 shows that, like a battery, a short circuit has an infinite slope (and therefore infinite resistance) in its *i*-*v* characteristic. And just like a battery, the voltage is also constant: In a short circuit, there’s zero voltage across a wire, no matter how much current flows through it. Because there’s no voltage across a short circuit, there’s zero absorbed power (p = 0 watts).

When you connect two points in a circuit that have different voltages, you get a *short circuit*. When this happens, you bypass the other parts of a circuit (called the *load*) and establish a path of low resistance, causing most of the current to flow around or away from some other parts in the circuit. Accidental short circuits, especially between the high and low voltages of a power supply, can cause strong current to flow, possibly damaging or overheating the power supply and the circuit if the circuit isn’t protected by a fuse.

Facing infinite resistance: Ideal current sources and open circuits

Figure 2-3 shows that an ideal current source and an open circuit both have zero slope in their *i*-*v* characteristics, meaning that they have infinite resistance. And in both cases, the current is constant.

The infinite resistance makes sense because if current entered the ideal current source, the current would no longer be constant. In circuit analysis, you can remove a current source by replacing it with an infinite resistor or open circuit. (You can read more about this change in Chapter 7.)

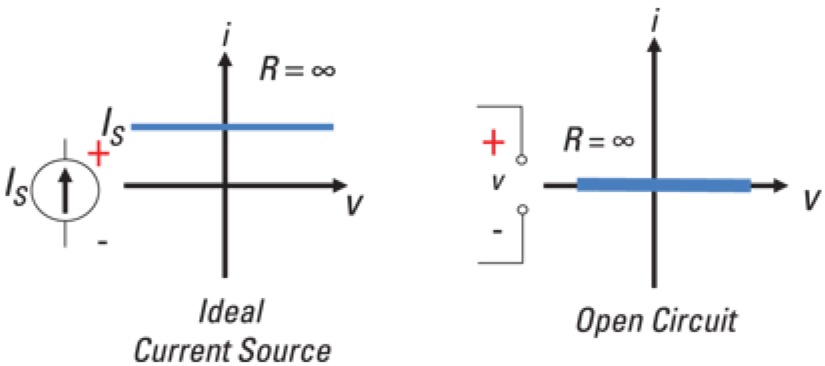

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Figure 2-3: You get infinite resistance and constant current from an ideal current source or open circuit.

An *open circuit* occurs when there's no current flow for any applied voltage, like when you blow a fuse. Because there's no current flow, there's no power absorbed (p = 0 watts) in an open-circuit device.

All or nothing: Combining open and short circuits with ideal switches

Think of ideal switches as a combination of an open circuit and a short circuit. When a switch is on, you have a short circuit, providing current flow in the circuit. When a switch is off, you have an open circuit, leaving zero current flow. Figure 2-4 illustrates an ideal switch's *i*-*v* characteristic along with its symbol, which shows the switch in the *off* state.

Because the switch has zero voltage in its *on* state and zero current in its *off* state, no power (p = 0 watts) is dissipated in the switch.

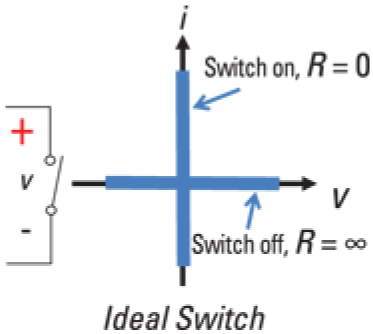

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Figure 2-4: An ideal switch has infinite or zero resistance, depending on whether the switch is on or off.

Mapping It All Out with Schematics

Schematics, which are drawings that symbolize a circuit, help you see the connections between electronic components. They also help you troubleshoot your circuit design during construction. You usually arrange electronic schematics from top to bottom and left to right, following the path to place the components.

Schematics use symbols to represent the different components of circuits. Here are some basic symbols to help you get started:

- **Wires:** Simple conductors, or wires, appear as plain lines in schematics. When two wires cross each other, you know the following:
  - If a dot appears at their intersection, the wires are connected (see the top-left diagram in Figure 2-5).
  - If the dot is absent or you see a curved bridge over one of the wires, the wires are unconnected (see the top-right diagram in Figure 2-5).

Wires that cross over are found in more-complicated circuits, which appear much later in the book.

Lines don’t necessarily depict actual wires, like the rat’s nest of wires you’d see inside an old radio; the lines simply represent a pathway of conductors. Today you have the more common metallic pathway called *traces* on a board. If you’ve ever opened up a desktop computer, you’ve seen traces on a big motherboard and wires connecting various devices like power supplies, sound cards, and hard drives.

- **Gates:** Control lines at the gate terminals of switches are represented by dashed lines (see the bottom-left diagram in Figure 2-5). By applying a voltage to the gate terminal, you can control the *on* and *off* states of the switch.

- **Power supplies:** Power supplies in schematics incorporate the device symbols I show you in Figures 2-2 and 2-3. You see power supply connections at the bottom right of Figure 2-5. The left diagram shows a way to reduce the clutter found in schematics by not drawing the symbol for the power supply. The schematic on the right shows the ground symbol, which marks a reference point of 0 volts.

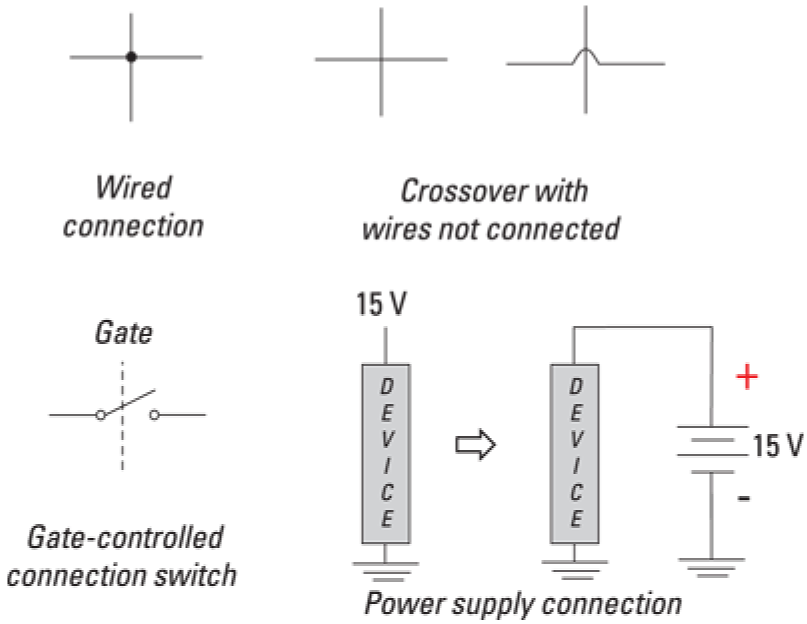

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Figure 2-5: Connection circuit symbols.

Additionally, circuit schematics often depict circular arrangements of electronic devices and junction points. The circular arrangements of electrical devices are called *loops,* and the junction or connection points are called *nodes*. I discuss these features next.

Going in circles with loops

When looking at a circuit schematic such as the one in Figure 2-6, you often see a collection of resistors and a battery connected together in some configuration. The loops form circular connections of devices. By definition, a *loop* occurs when you trace a closed path through the circuit in an orderly way, passing through each device only once.

This method of generating a closed path allows you to get consistent results when analyzing circuits. To form a loop or closed path, you must start at one point in the circuit and end up at the same place, much like going around the block in your neighborhood.

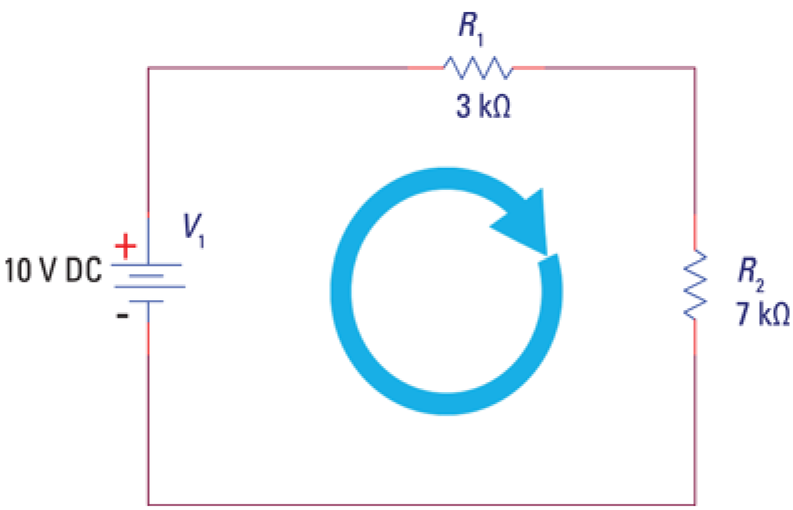

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Figure 2-6: Schematic with a closed path loop.

As more devices are connected to the circuit, there's an increased likelihood that more loops will occur. Figure 2-7 shows a circuit with two inner loops (Loops 1 and 2) and one big outer loop (Loop 3).

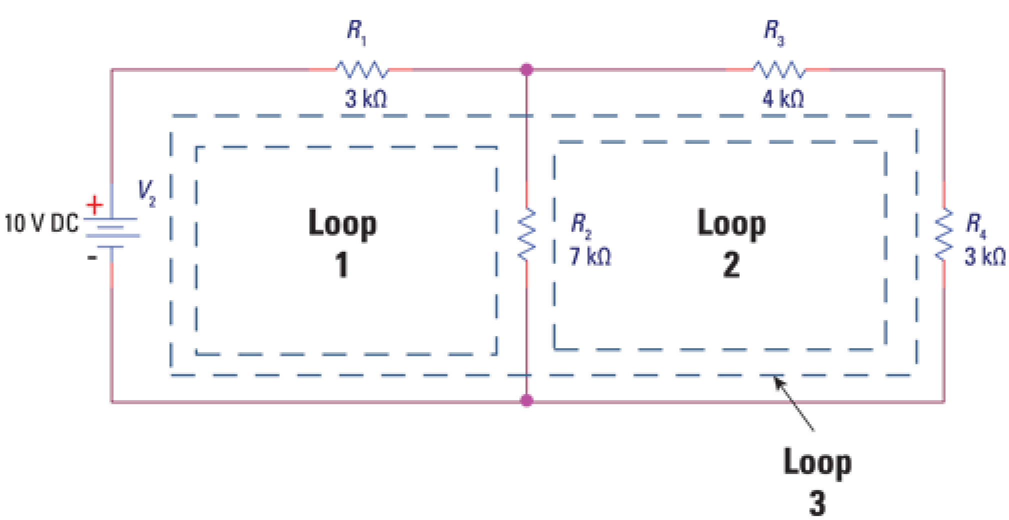

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Figure 2-7: Schematic with three loops.

Getting straight to the point with nodes

A *node* is simply a junction or point where two or more devices are connected. Be sure to add the following important points about nodes to your memory bank:

- A node isn't confined to a point; it includes the wire between devices.
- Wires connected to a node have zero resistance.

Figure 2-8, which depicts three nodes (or junctions), emphasizes the preceding points about nodes. The connected devices, which can be either resistors or independent sources (like batteries), are represented as boxes. The dashed lines outline the node points.

Look at Node A, which consists of points 1, 2, and 3. These points are really the same node or point connected by a zero-resistance wire. Similarly, four devices are connected at Node C.

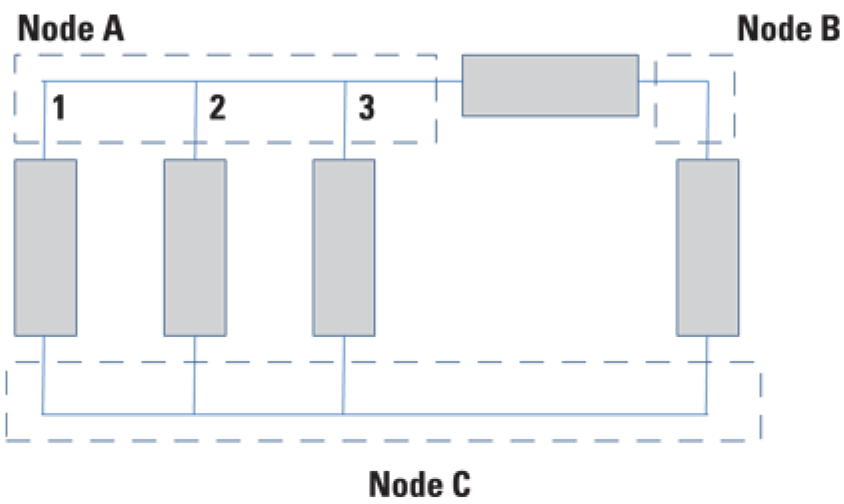

Illustration by Wiley, Composition Services Graphics

Figure 2-8: Circuit schematic with three nodes and five devices.

Chapter 3

Exploring Simple Circuits with Kirchhoff's Laws

In This Chapter

- Discovering Kirchhoff's voltage law and current law
- Analyzing simple circuits with the help of Kirchhoff and Ohm
- Finding the equivalent resistance of series or parallel resistors and their combinations

Just like you follow the law of gravity after jumping out of a perfectly good airplane to go skydiving, the devices (or elements) in any circuit have to follow certain laws. Whereas you follow the laws of nature, circuit elements such as resistors must follow Kirchhoff's laws.

This chapter introduces you to Gustav Kirchhoff's two circuit laws — Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL) — and reveals how to use them in conjunction with Ohm's law, which I introduce in Chapter 2. With these three laws, you can solve for the current and voltage in any device in a circuit. The circuits in this chapter focus primarily on resistors driven by independent sources such as batteries.

Presenting Kirchhoff's Famous Circuit Laws

Gustav Kirchhoff’s two laws — Kirchhoff’s voltage law (KVL) and Kirchhoff’s current law (KCL) — are essential for creating connection equations. *Connection equations* follow the energy and charge conservation laws when you connect devices (such as batteries or resistors) to form a circuit. Conservation laws tell you that electrons must behave in certain ways when you’re connecting devices to form a circuit. In turn, the electron actions govern the behavior of the voltage and current around the loops and at the nodes.

More importantly, these connection equations don’t depend on specific elements in the circuit. In other words, Kirchhoff’s laws work no matter which connected devices you use in forming the circuit. The following sections get you acquainted with KVL and KCL and show you how to apply them to circuits. Rest assured that KVL and KCL are some of your best friends in the world of circuit analysis.

Kirchhoff’s voltage law (KVL): Conservation of energy

Kirchhoff’s voltage law (KVL) states that the algebraic sum of the voltages around a closed loop is zero at every instant. You can write this law as follows:

> Voltage supplied (or delivered) = Voltage drops absorbed (or used up)

Electrical energy relates to *voltage,* the amount of energy required to move a given amount of charge (for more on voltage, see Chapter 2). So you can think of KVL as a mathematical representation of the law of conservation of energy. In a circuit, for example, a battery supplies power (a rate of energy) and a resistor dissipates the delivered power as heat (as when an incandescent light bulb emits heat). Another way of saying this is that the amount of supplied energy is equal to the amount of energy used up or absorbed.

To visualize KVL, suppose you're going for a walk. You start at your house and walk through your neighborhood, going up and down a number of hills. You head home using a different path, walking up and down another set of hills before ending your walk at home. Walking up and down the hills is analogous to voltage rises and drops. After starting and completing your walk, you form a closed path or loop ending at the starting point, and your net elevation (potential energy) doesn't change.

KVL and loops in a circuit go hand in hand. As you go around a loop, you enter and exit circuit devices. Two points are required to measure voltage, so if you enter and exit a device, you have enough points to find the device's voltage using KVL. After that, you can find the device's corresponding current by using relationships such as Ohm's law.

Formulating KVL expressions requires understanding the concept of voltage rises and drops. I get you acquainted with voltage rises and drops and show you how to calculate KVL equations in the next sections.

Identifying voltage rises and drops

Voltage rises occur when you go from a negative terminal to a positive terminal (– to +). They're usually associated with batteries or, more generally, *sources.* Not surprisingly, *voltage drops* occur when you go from a positive terminal to a negative terminal (+ to –), along the direction of electric current flow. They're commonly associated with passive devices, called *loads,* such as resistors.

Figure 3-1 shows you what voltage rises and drops look like in a circuit schematic. In Device 1, going along the direction of the current from the negative terminal to the positive terminal (left to right) results in a voltage rise. For Device 2, going from the positive terminal to the negative terminal (again, left to right) constitutes a voltage drop. As for the direction of the current, the *passive sign convention* tells you that the current flows from the positive terminal to the negative terminal. (For the scoop on the passive sign convention, turn to Chapter 2.)

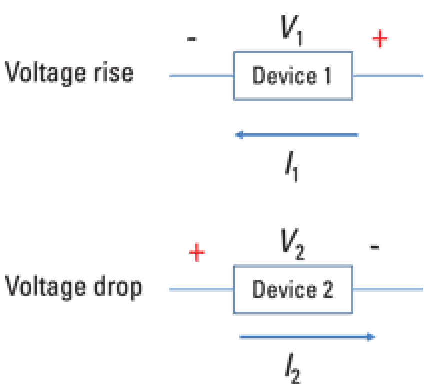

Illustration by Wiley, Composition Services Graphics

Figure 3-1: A voltage rise and voltage drop.

Always label your schematics appropriately so you know where the voltage terminals are and the direction of the current.

Forming a KVL equation

After labeling the voltage polarities (+ and –) in a circuit schematic, you can form the KVL equation. Simply choose your starting point and travel the path of the current through each device, noting the voltage as you enter each device. When you’re back to where you started, add up all the rises and drops in voltage to get your KVL equation. Labeling the circuit appropriately helps you write your KVL equation correctly.

Consider the circuit in Figure 3-2. To account for the voltage rises and drops when going around the loop, you need to keep track of the voltage polarities and pick a *node* (a junction point where two or more devices are connected) to serve as both the starting and ending point. Then go around the loop in either a clockwise or counterclockwise direction. Also, make sure you're not passing through any device more than once for a single loop.

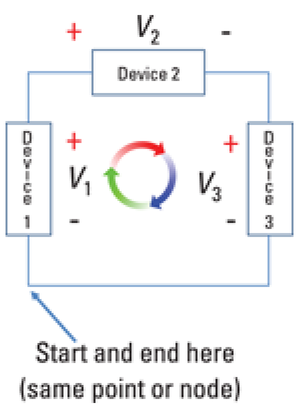

Illustration by Wiley, Composition Services Graphics

Figure 3-2: Circuit diagram illustrating KVL.

Try to build the KVL equation for the circuit in Figure 3-2, starting at the lower-left corner of the diagram and going around the loop in a clockwise direction. The circuit consists of three loads. Keep account of voltage drops in the direction of the arrows. Because you enter Device 1 at its negative terminal and exit at its positive terminal, the resulting voltage rise as you go across Device 1 is $+V_1$. Then you enter Device 2 at its positive terminal and exit at its negative terminal. Consequently, the voltage drop across Device 2 can be expressed as $-V_2$. Next, you enter Device 3 at its positive terminal and exit at its negative terminal, so you wind up with a voltage of drop of $-V_3$. Finally, leaving Device 3 at the negative terminal takes you back to where you started. Now you have the information you need to create your KVL equation. When you add up the voltage rises and drops for all the devices, you get the following KVL equation:

$$V_1 - V_2 - V_3 = 0$$

You can get the same KVL equation for this example if you go counterclockwise and start at the upper-right corner of the circuit in Figure 3-2. In that case, the KVL equation would be

$$V_2 - V_1 + V_3 = 0$$

Algebraically, the preceding two equations are equivalent. Whether you go in a clockwise or counterclockwise direction, or whether you get voltage drops or rises for each device as you go around the loop, you obtain the same KVL equation.

If you want to show that the sum of the voltage rises is equal to the sum of the voltage drops (to reinforce that KVL is really a conservation-of-energy equation), write the KVL equation as follows:

Sum of voltage rises = Sum of voltage drops

Similarly, by using some algebra to isolate V_1 on one side of the equation, you get the following:

$$V_1 = V_2 + V_3$$

This form reinforces the fact that KVL is really like a conservation of energy equation.

Kirchhoff's current law (KCL): Conservation of charge

Kirchhoff's current law (KCL) tells you that the following is true at a node:

Sum of incoming currents = Sum of outgoing currents

So for all intents and purposes, KCL is really a mathematical representation of the conservation of charge. Think about it this way: When you apply voltage pressure using a battery, it supplies a current flow at one end, and the same amount of current is delivered to the rest of the circuit. Charge can't accumulate in the wire, which means the current flow is related to conservation of charge.

> You can envision current — the flow of charges — as the flow of water. When you open a water faucet that has a hose connected to it, the amount of water flowing from the faucet is the same as the amount of water exiting at the other end of the hose. The water pressure from an external energy source creates the water flow, which means the water can't accumulate in the hose. Granted, this example illustrates conservation of mass rather than conservation of charge, but you get the idea.

KCL means that the current entering a node must equal the current going out of a node. Formally, KCL states that the algebraic sum of all the currents at a node is zero, but the in = out version is simpler because you don't have to figure out what's positive or negative for incoming or outgoing currents.

Tracking incoming and outgoing current

To create a KCL equation, you need to keep track of incoming currents and outgoing currents at each node. You're measuring a net current of zero for each node.

For practice following the ins and outs of currents, check out Figure 3-3, which shows three nodes. To minimize the clutter in the figure and show the consistency in the passive sign convention, I limit the voltage polarity notations (+ and –) to Devices 1 and 4. The current I_4 is flowing through Device 4. I_4 is an outgoing current at Node A but an incoming current at Node B. This labeling convention holds true for the other devices and currents as well.

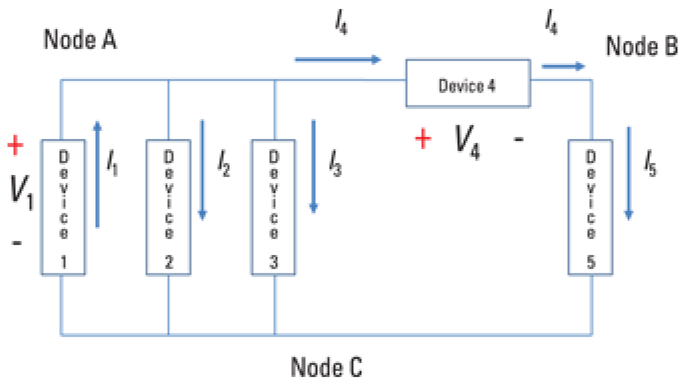

Illustration by Wiley, Composition Services Graphics

Figure 3-3: A three-node circuit diagram illustrating Kirchhoff's current law.

Calculating KCL

When you have one incoming current and one outgoing current at a node, applying KCL is relatively straightforward. Consider Node B in Figure 3-3. The current entering Node B is I_4, and the current exiting Node B is I_5. Algebraically, the KCL equation for Node B is

$$\text{in} = \text{out} \quad \rightarrow \quad I_4 = I_5$$

Now consider Node A. The incoming current is I_1, and the outgoing currents are I_2, I_3, and I_4. The KCL equation for Node A is therefore

$$\text{in} = \text{out} \quad \rightarrow \quad I_1 = I_2 + I_3 + I_4$$

Finally, consider Node C. The incoming currents are I_2, I_3, and I_5, and the outgoing current is I_1. So the KCL equation for Node C is

$$\text{in} = \text{out} \quad \rightarrow \quad I_2 + I_3 + I_5 = I_1$$

The number of independent KCL equations you actually need is one fewer than the number of nodes for any circuit. If you want to save yourself a little time in the case of Figure 3-3, you can find the KCL equation for Node C simply by substituting the KCL equation for Node B into the KCL equation for Node A.

If you have a lot of devices connected at one node, label that node as your circuit reference point and count it as having 0 volts (I discuss the ground symbol and reference nodes in Chapter 2). Doing so will make your calculations cleaner when you start getting into more advanced circuit analysis. In Figure 3-3, Node C has the most devices connected to it, so that's the one you want to set equal to 0 volts as your circuit reference point.

Tackling Circuits with KVL, KCL, and Ohm's Law

Mathematically, you need two basic types of equations to analyze circuits:

- Device equations that describe the behavior between voltage and current for the component in question

- Connection equations derived from Kirchhoff's voltage and current laws (KVL and KCL) for any circuit

In the following sections, I show you how to apply these laws to the following types of circuits: a circuit with a battery and resistors, a series circuit, and a parallel circuit. In a series circuit, the same current flows through all the connected devices. In a parallel circuit, all the connected devices have the same voltage. Although the circuits differ, the general procedure is the same:

1. **Label the device terminals with the proper voltage polarities (+ and –) and voltage variables.**
2. **Assign the directions of the currents for the given circuit.**

   Apply the passive sign convention, with current flowing from the + sign to the – sign.
3. **Formulate KVL or KCL connection equations.**
4. **Apply device equations (such as Ohm's law for resistors) and then substitute the device equations into the connection equations.**
5. **Solve for the voltage and current for any device.**

Getting batteries and resistors to work together

The circuit in Figure 3-4 is made up of a battery (an ideal voltage source) and two resistors. Because the power supply is a source, the current direction is away from the positive terminal. As for the resistors, they follow the passive sign convention: The current flows from + to –.

To find the voltage and current for each device in Figure 3-4, you first need to take stock of the circuit. Because the circuit contains three devices, you have a total of six unknown voltages and currents.

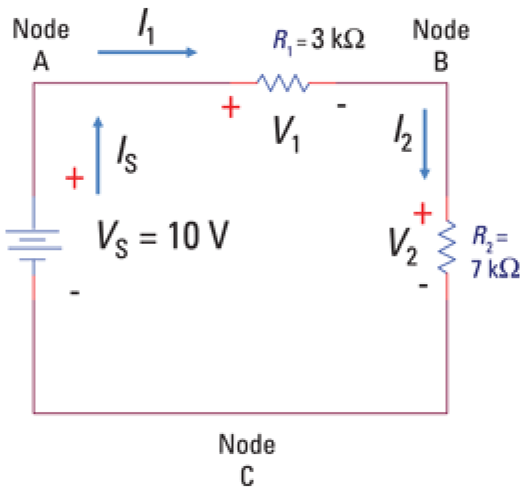

Illustration by Wiley, Composition Services Graphics

Figure 3-4: Circuit diagram of a battery and two resistors.

Starting with voltage

You can begin your analysis with either of Kirchhoff's laws. In this example, go ahead and start with KVL.

You ultimately use both KVL and KCL because you need both of Kirchhoff's laws, along with device equations such as Ohm's law, to help you generate enough equations to solve for unknown voltages and currents. When you have a given number of voltages or currents to solve for — say, six in total — you need six independent equations. *Independent* means each equation can't be derived from the other equations. (If you can only get one equation by using the other equations, then that equation is *dependent.*) For each device, Ohm's law gives you one independent equation; the circuit in Figure 3-4 contains three devices, so you get three equations from Ohm's law. For each node, except for the reference or starting node, you get a KCL equation; this circuit has three nodes, so you get two KCL equations (not counting the ground Node C). You can get the dependent KCL equation for Node C from the other two KCL equations. Finally, this circuit gives you one independent KVL equation.

When formulating KVL, make sure you start and end at the same node. You wind up with the same KVL equation no matter where you start in the circuit, as long as you end at your starting point. For Figure 3-4, I start at the lower right-hand corner and move clockwise, producing the following KVL equation:

$$V_S - V_1 - V_2 = 0$$

Bringing in current

You have many unknowns with KVL and Ohm's law, so you need to establish more relationships between what's known and unknown. Use KCL next to get some more equations. Starting with Node A, the incoming current at Node A is I_s, and the outgoing current at Node A is I_1. So the KCL equation for Node A is

$$\text{in} = \text{out} \quad \rightarrow \quad I_s = I_1$$

For Node B, the incoming current is I_1, and the outgoing current is I_2. The KCL equation for Node B is

$$\text{in} = \text{out} \quad \rightarrow \quad I_1 = I_2$$

Combining the KCL equations for Nodes A and B, you have

$$I_s = I_1 = I_2$$

This is a neat equation because it says if you can figure out one of these currents, you've found the other two currents as well. For example, if you can find I_1, then you automatically know I_s and I_2.

Combining device equations with KVL

The voltage source V_s in Figure 3-4 is 10 volts. Plug this value into the previous KVL equation from the section "Starting with voltage":

$$V_s = 10\text{ V} \rightarrow \qquad V_s = 10\text{ V} = V_1 + V_2$$

Now you need device equations to figure out the unknown currents and voltages. To calculate the voltages for each device in Figure 3-4, use Ohm's law for resistors. You get two more equations relating voltage and current:

$$V_1 = I_1 R_1 = I_1(3\text{ k}\Omega) = I_1(3{,}000\ \Omega)$$

$$V_2 = I_2 R_2 = I_2(7\text{ k}\Omega) = I_1(7{,}000\ \Omega)$$

Note that you can write both equations in terms of I_1 because $I_1 = I_2$.

Now substitute the Ohm's law values of V_1 and V_2 into $V_s = 10\text{ V} = V_1 + V_2$:

$$10\text{ V} = (3{,}000\ \Omega)I_1 + (7{,}000\ \Omega)I_1 = (10{,}000\ \Omega)I_1$$

Solving for I_1 gives you

$$I_1 = \frac{10\text{ V}}{10{,}000\ \Omega} = 0.001\text{ A} = 1\text{ mA}$$

Because $I_2 = I_s = I_1 = 0.001$ A, you know the current for all three devices.

Finally, you can plug in the values to figure out the voltages for the three devices using Ohm's law:

$$V_1 = I_1(3{,}000\ \Omega) = (0.001\text{ A})(3{,}000\ \Omega) = 3\text{ V}$$

$$V_2 = I_1(7{,}000\ \Omega) = (0.001\text{ A})(7{,}000\ \Omega) = 7\text{ V}$$

Verify the KVL equation by substituting in the voltages to show that the total sum of the voltages is equal to zero. It’s good practice to check your results.

Note that the 10-volt power supply is divided proportionally between the two resistors of 3 volts and 7 volts.

Summarizing the results

To give you some insight into the calculations in this section, the following table lists the voltage and current for each device in the circuit. It also shows the power supplied by the voltage source and the power dissipated by the resistor using $P = IV$, where P is the power supplied by a source or absorbed by a load device, V is the voltage across the device, and I is the current through the device. ***Note:*** You don’t need to develop a similar table for each circuit you analyze; this table just serves to illustrate some points.

Device	*Current (I)*	*Voltage (V)*	*Power (W)*
V_s	–1 mA	10 V	–100 mW
R_1	1 mA	3 V	30 mW
R_2	1 mA	7 V	70 mW

As you can see from the table, you have the same amount of current flowing through each device, which tells you that the circuit in question is a series circuit. As for the power, remember that negative power is supplied power and that positive power is dissipated or absorbed power; therefore, the power supplied by the battery V_S is equal to the sum of the power dissipated or absorbed by the two resistors, which illustrates the conservation of energy.

Sharing the same current in series circuits

Two devices or elements are connected *in series* when they have one common node where no other devices have currents flowing through them. In other words, the same current flows through each device, and the current can only flow forward. It basically has a one-way ticket to ride with no alternative routes.

> If you're having trouble picturing series circuits, imagine that you've just bought a brand-new blanket that does a good job of trapping body heat. You also have a thin blanket, but it's a poor heat insulator. You know it's going to be a cold night, so you're trying to figure out how best to keep warm. Should the new blanket go on top or bottom of the old blanket? If your gut feeling is that it shouldn't matter, you're correct. Why? Because the blankets are connected in series. Heat must go through both blankets before it escapes. Just like heat flows from hot to cold places, current flows from high (+) to low (–) voltage. The blankets are heat insulators in series behaving like resistors connected in series. If the order of the resistors changes, you still get the same amount of current going through each of them.

Figure 3-5 shows three resistors connected in series and three resistors that aren't connected in series. You can see that the same current I flows through each of the three resistors in the series circuit.

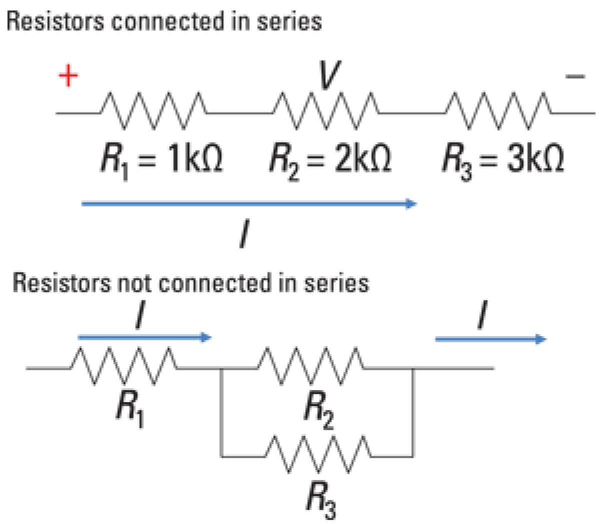

Illustration by Wiley, Composition Services Graphics

Figure 3-5: Resistors connected in series (top) and not connected in series (bottom).

In the bottom circuit, the current I through R_1 splits proportionally between R_2 and R_3. Intuitively, if R_2 and R_3 have the same resistance value, then the current I splits in half: $I/2$. If R_2 has a bigger resistance value than R_3, then the current through R_2 will have a smaller value than the current through R_3. Either way, the currents flowing out between R_2 and R_3 will add up to the same current I.

The voltage across the three resistors connected in series is given as V, and the voltage for each of the resistors R_1, R_2, and R_3 is V_1, V_2, and V_3, respectively. Now apply Ohm's law, $V = IR$, for the voltages. Because the current is the same for each resistor in a series connection, this implies that

$$
\begin{aligned}
V &= V_1 + V_2 + V_3 \\
&= IR_1 + IR_2 + IR_3 \\
&= I\left(R_1 + R_2 + R_3\right) \\
&= IR_T
\end{aligned}
$$

where R_T is the total resistance given as $R_T = R_1 + R_2 + R_3$. The preceding equation is a form of Ohm's law. So in Figure 3-5, the total resistance is R_T = 1 kΩ + 2 kΩ + 3 kΩ = 6 kΩ.

For resistors connected in series, the total resistance is simply the sum of the resistances. Whenever you see two or more resistors connected to a series circuit, you can replace the individual resistances with the total equivalent resistance for the series circuit — a tactic that comes in handy when you want to simplify a circuit for analysis. Similarly, you can break up a single resistor into smaller resistors that add up to the value of the single resistor. And don't forget that the same current flows through each series resistor.

When you check your answer, remember that the total resistance for resistors connected in series is always greater than the value of any one resistor.

Climbing the ladder with parallel circuits

Connected devices have the same voltage when they're connected in parallel. The lights on a string of Christmas lights are connected in parallel, as are all major appliances in a house. Figure 3-6 illustrates a parallel circuit that consists of three devices.

You can tell when you’re looking at a parallel circuit because the circuit diagram looks like a ladder lying on its side. You can also say that devices are connected in parallel when they form a loop that doesn’t encircle any other elements. Devices are connected in parallel when they have two nodes in common.

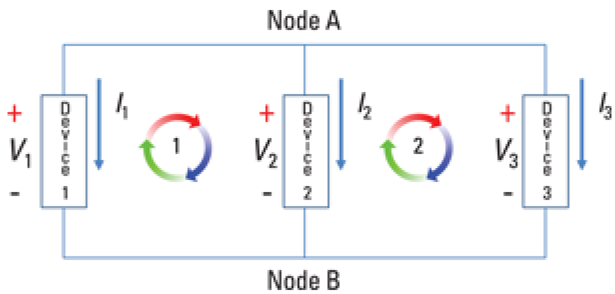

Illustration by Wiley, Composition Services Graphics

Figure 3-6: Circuit diagram of parallel connections.

To start analyzing this circuit, first formulate a KVL equation for Loop 1. You can start anywhere in the circuit, but I’ve started at the lower-left corner, which gives me the following equation:

$$V_1 - V_2 = 0 \quad \rightarrow \quad V_1 = V_2$$

For Loop 2, start from the lower-left corner of Loop 2, giving you another a KVL equation:

$$V_2 - V_3 = 0 \quad \rightarrow \quad V_3 = V_2 \quad \rightarrow \quad V_1 = V_2 = V_3$$

You now know that with the circuit configuration in Figure 3-6, the KVL analysis yields the same voltage for each device given in the circuit.

Next, apply KCL for Node A, where there are no incoming currents and where outgoing currents consist of I_1, I_2, and I_3. The result?

$$\text{in} = \text{out} \qquad \rightarrow \qquad 0 = I_1 + I_2 + I_3$$

If you suppose that the devices are resistors — R_1, R_2, and R_3 — then you can use Ohm's law to find the following device equations:

$$I_1 = \frac{V}{R_1} \;, \qquad I_2 = \frac{V}{R_2} \;, \qquad I_3 = \frac{V}{R_3}$$

When you substitute Ohm's law into the KCL equation, 0 = I_1 + I_2 + I_3, you get the following (where R_T is the total equivalent resistance for these three resistors connected in parallel):

$$0 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$0 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_T}$$

I simplified that last equation by dividing both sides of the equation by V. As you can see, finding the total resistance for resistors connected in parallel is a bit more complicated than finding the total resistance for a series circuit, which I explain how to do in the preceding section.

Describing total resistance using conductance

For more than two resistors connected in parallel, you can get an alternate description of the total resistance by using the definition of *conductance, G = 1/R,* which I introduce in Chapter 2. Using G = 1/R for Figure 3-6 gives you the following equation to find the total conductance (which is simply the sum of the conductances for each device that's connected in parallel):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \rightarrow \qquad G_T = G_1 + G_2 + G_3$$

Try your hand at using the definition of conductance to find the total resistance for the circuit in Figure 3-7.

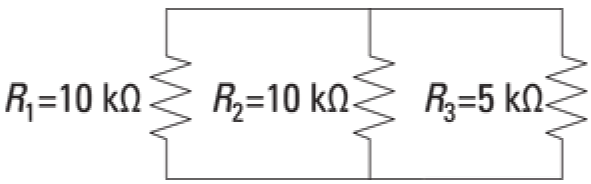

Illustration by Wiley, Composition Services Graphics

Figure 3-7: Parallel connections of resistors.

Here’s the calculation of the total resistance given the three resistors:

$$\frac{1}{R_T} = \frac{1}{10\ \text{k}\Omega} + \frac{1}{10\ \text{k}\Omega} + \frac{1}{5\ \text{k}\Omega}$$

$$G_T = 0.1\ \text{mS} + 0.1\ \text{mS} + 0.2\ \text{mS} = 0.4\ \text{mS}$$

$$R_T = \frac{1}{0.4\ \text{mS}} = 2.5\ \text{k}\Omega$$

Using a shortcut for two resistors in parallel

Two resistors connected in parallel commonly appear in many circuits, so it’s convenient to have a simple formula for this case. In words, the total resistance for two parallel resistors is the product of the two resistors divided by the sum of the two resistors. Algebraically, that looks like the following:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \quad \rightarrow \quad R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Finding equivalent resistor combinations

To better understand equivalent resistances, try calculating the total equivalent resistance of the three parallel resistors in Figure 3-7. You can find the equivalent resistance in pairs, as in Circuit A of Figure 3-8. Using the preceding equation for pairs of resistors yields

$$\frac{1}{R_{eq1}} = \frac{1}{10\text{ k}\Omega} + \frac{1}{10\text{ k}\Omega} \quad \rightarrow \quad R_{eq1} = \frac{10\text{ k}\Omega \cdot 10\text{ k}\Omega}{10\text{ k}\Omega + 10\text{ k}\Omega} = 5\text{ k}\Omega$$

The two 10-kΩ resistors in parallel in Circuit A are replaced with a 5-kΩ resistor in Circuit B, where you now have two 5-kΩ resistors connected in parallel. Transform the circuit again, replacing these two resistors in parallel with their equivalent:

$$\frac{1}{R_{eq_2}} = \frac{1}{5\text{ k}\Omega} + \frac{1}{5\text{ k}\Omega} \quad \rightarrow \quad R_{eq_2} = \frac{5\text{ k}\Omega \cdot 5\text{ k}\Omega}{5\text{ k}\Omega + 5\text{ k}\Omega} = 2.5\text{ k}\Omega$$

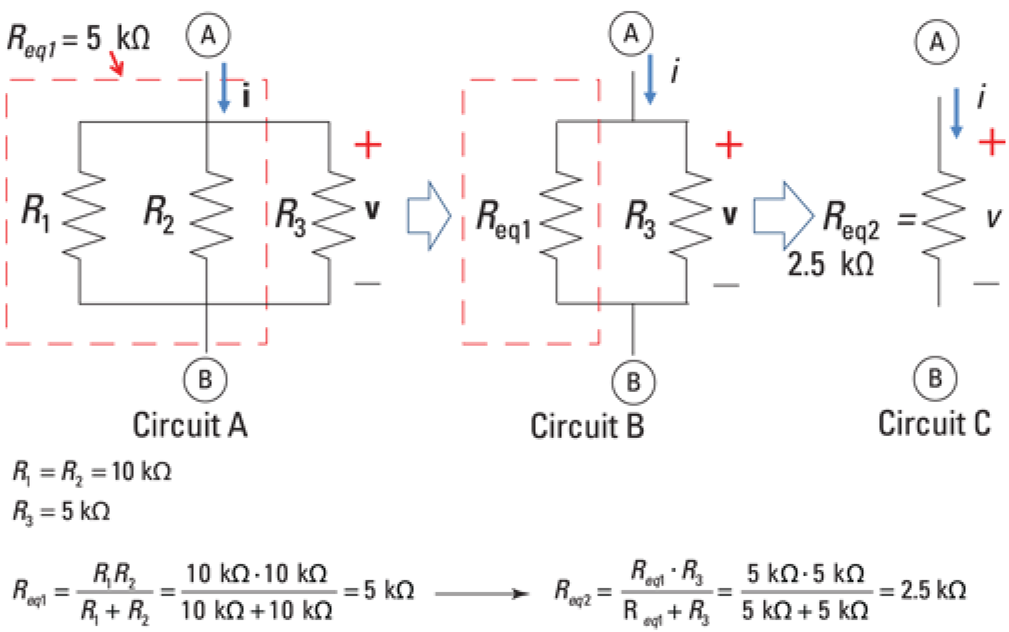

Illustration by Wiley, Composition Services Graphics

Figure 3-8: Circuit combining series and parallel resistors.

Circuit B is transformed to Circuit C, which has an equivalent resistance of 2.5 kΩ.

This example shows that if you want to cut the value of a particular resistor in half, you can connect two equal resistors in parallel. Circuits A, B, and C are all equivalent circuits because they have the same voltage *v* across Terminals A and B and the same *net* current *i* going through the resistor network.

Finding the faulty bulb in a string of Christmas lights

Early Christmas lights were frustrating for reasons beyond the tangled mess they created if you weren’t careful. The individual light bulbs were connected in series, so when one bulb went out, none of the bulbs would light. It was difficult to determine which bulb had burnt out because there wasn’t a current going through each of the light bulbs. You had to check every single bulb to find the culprit.

Fortunately, manufacturers saw the (multicolored, twinkling) light. Most of today’s Christmas lights are connected in parallel. So when one light bulb is bad, the other lights stay on because they all share the same voltage — making it easy to find the one faulty bulb you need to replace. As for the tangled mess, even electrical engineers haven’t solved that one!

Combining series and parallel resistors

You can use the concepts of series and parallel resistors to transform a complex circuit into a simpler circuit. Replacing part of a complicated-looking circuit with a simpler but equivalent circuit simplifies the math.

Figure 3-9 shows a complex circuit with resistors connected in series and in parallel. Your job is to find the total resistance so you can replace all those resistors with a single resistor.

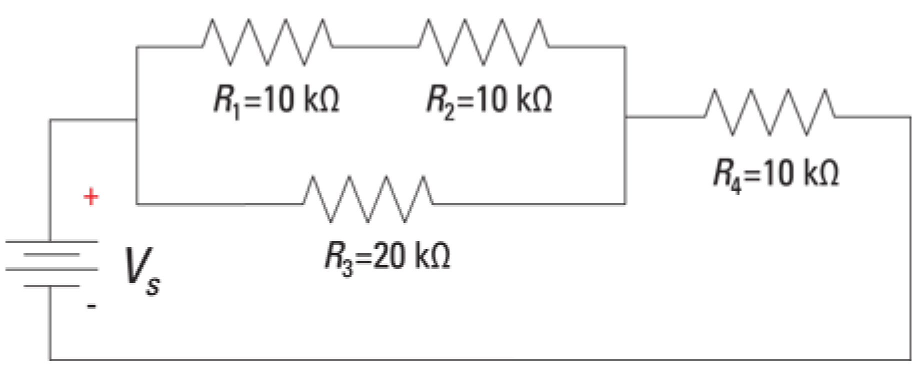

Illustration by Wiley, Composition Services Graphics

Figure 3-9: A combination of parallel and series resistors.

In Figure 3-9, resistors R_1 and R_2 are connected in series. This series combination is equivalent to

$$R_{eq1} = R_1 + R_2 = 10\ \text{k}\Omega + 10\ \text{k}\Omega = 20\ \text{k}\Omega$$

R_{eq1} is connected in parallel with R_3. You calculate the equivalent resistance value for this parallel combination as

$$R_{eq2} = \frac{R_{eq1}R_3}{R_{eq1} + R_3} = \frac{(20\ \text{k}\Omega)(20\ \text{k}\Omega)}{20\ \text{k}\Omega + 20\ \text{k}\Omega} = 10\ \text{k}\Omega$$

R_{eq2} is in series with R_4. This series combination yields

$$R_{eq} = R_{eq2} + R_4 = 10\ \text{k}\Omega + 10\ \text{k}\Omega = 20\ \text{k}\Omega$$

Chapter 4

Simplifying Circuit Analysis with Source Transformation and Division Techniques

In This Chapter

- Recognizing equivalent circuits
- Transforming circuits into equivalent series and parallel circuits
- Analyzing circuits with voltage and current divider techniques

Using Kirchhoff's laws and Ohm's laws (see Chapter 3) can get pretty laborious when you're analyzing complex circuits. Fortunately, you can make analyzing circuits easier by replacing part of the circuit with a simpler but equivalent circuit.

Through a *makeover* or *transformation technique,* you modify a complex circuit so that in the transformed circuit, the devices are all connected in series or in parallel. After the transformation, you no longer need to systematically apply Kirchhoff's laws, because you can use shortcuts: the current divider technique and the voltage divider technique.

In this chapter, I explain how to make the transformation and apply both types of divider techniques. Rest assured that the info in this chapter can make your life a little easier when you start analyzing more-complex circuits.

Equivalent Circuits: Preparing for the Transformation

When you're analyzing a complex circuit, you can simplify the math by replacing part of the circuit with a simpler, equivalent circuit. Two circuits are said to be *equivalent* if they have the same *i-v* characteristics at a pair of terminal connections. (You can find information about the *i-v* characteristics of various electrical devices in Chapter 2.)

You find the *i-v* characteristic for each circuit by using Kirchhoff's laws and Ohm's law, which give you the equations that relate the current *i* and voltage *v* across two terminals (see Chapter 3 for details). Then you compare the *i-v* relationships associated with the pair of terminals to find out in which conditions the circuits are equivalent. Even better, after you understand how to do source transformations, you no longer need to rely completely on Kirchhoff's and Ohm's laws to complete your analysis.

Take a look at the practical models of independent voltage and current sources in Figure 4-1. Circuit A depicts an ideal voltage source connected in series with a resistor, and Circuit B depicts an ideal current source connected in parallel with a resistor. In the following example, I show you that these two circuits are considered equivalent because they have the same *i-v* characteristics at the terminal pair A and B.

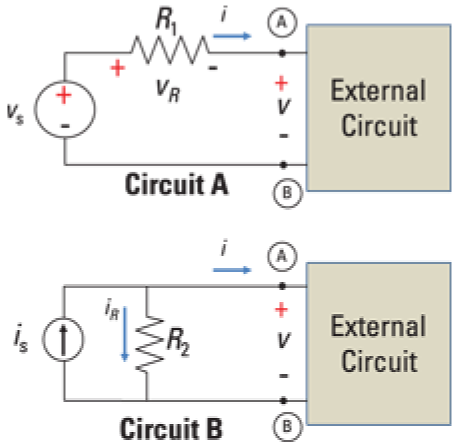

Illustration by Wiley, Composition Services Graphics

Figure 4-1: Models of equivalent circuits with voltage and current sources.

To find the *i*-*v* characteristic of Circuit A, you have to develop the relationship between the current *i* and voltage *v* for Terminals A and B. You do this by using Kirchhoff's and Ohm's laws.

Kirchhoff's voltage law (KVL) says that the sum of the voltage drops and rises around a loop is zero. In other words, the voltage source has to equal the voltage drops across the resistors. Therefore, using KVL for Circuit A produces

$$v_s = v_R + v$$

Using Ohm's law for resistor R_1 gives you the following voltage:

$$v_R = iR_1$$

Substituting the value of v_R into $v_s = v_R + v$ yields

$$v_s = R_1 i + v$$

One way to get the *i*-*v* characteristic for Circuit A is to solve for *v*, which yields the following:

$$v = v_s - R_1 i$$

The resulting equation relates the voltage *v* and the current *i* at Terminals A and B in Circuit A of Figure 4-1. If you know the current and voltage from the input voltage source, you can find the voltage output.

An alternate form of the *i*-*v* characteristic requires manipulating $v = v_s - R_1 i$

by solving for current *i* to obtain $i = \frac{v_s}{R_1} - \frac{v}{R_1}$. When you do this, knowing the

voltage input provides you with the current output at Terminals A and B.

Look at Circuit B in Figure 4-1 to find a similar *i*-*v* relationship at Terminals A and B. You use Kirchhoff's current law (KCL), which states that the sum of the incoming currents is equal to the sum of the outgoing currents at any node or terminal — here, at Terminal A or Terminal B. KCL yields

$$i_s = i_R + i$$

Using Ohm's law for R_2 gives you the following:

$$i_R = \frac{v}{R_2}$$

When you substitute the value of i_R into $i_s = i_R + i$, you get

$$i_s = \frac{v}{R_2} + i$$

Solving for *v* gives you the following *i*-*v* characteristic:

$$v = R_2 i_s - R_2 i$$

This equation relates the voltage *v* and the current *i* at Terminals A and B for Circuit B.

Now you can compare the result for Circuit B, $v = R_2 i_s - R_2 i$, with the result for Circuit A, $v = v_s - R_1 i$, to find the conditions for equivalent circuits. One way of doing so is to equate *v* between the two circuits. For this example, this approach gives you:

$$R_2 i_s - R_2 i = v_s - R_1 i$$

If you rearrange these equations to group the independent sources and collect like terms for the current *i,* you wind up with the following conditions:

$$\underbrace{\left(R_2 i_s - v_s\right)}_{=0} + \underbrace{\left(R_1 i - R_2 i\right)}_{=0} = 0$$

The first expression in parentheses deals with the independent sources, and the second collects like terms with current *i*. For this equation to be equal to zero, you set the terms in parentheses equal to zero, which gives you the following two equations:

$$R_1 = R_2 = R$$
$$v_s = R_2 i_s = i_s R$$

Because R_1 and R_2 are equal to each other, removing their subscripts yields a general resistor value of *R* for the two circuits. These are the conditions in which the two circuits are said to be equivalent.

Transforming Sources in Circuits

Each device in a series circuit has the same current, and each device in a parallel circuit has the same voltage. Therefore, finding the current in each device in a circuit is easier when the devices are all connected in parallel, and finding the voltage is easier when they're all connected in series. Through a circuit *transformation,* or *makeover,* you can treat a complex circuit as though all its devices were arranged the same way — in parallel or in series — by appropriately changing the independent source to either a current or voltage source.

Changing the practical voltage source to an equivalent current source (or vice versa) requires the following conditions (see the preceding section to find out why these conditions characterize equivalent circuits):

- The resistors must be equal in both circuits.
- The source transformation must be constrained by $v_s = i_s R$.

The constraining equation, $v_s = i_s R$, looks like Ohm's law, which should help you remember what to do when transforming between independent voltage and current sources.

In this section, I show you how to transform a circuit.

Converting to a parallel circuit with a current source

Transformation techniques let you convert a practical voltage source with a resistor connected in series to a current source with a resistor connected in parallel. Therefore, you can convert a relatively complex circuit to an equivalent circuit if all the devices in the external circuit are connected in parallel. You can then find the current of individual devices by applying the current divider techniques that I discuss later in "Cutting to the Chase Using the Current Divider Technique."

When switching from a voltage source to a current source, the resistors have to be equal in both circuits, and the source transformation must be constrained by $v_s = i_s R$. Solving the constraint equation for i_s allows you to algebraically convert the practical voltage source into a current source:

$$i_s = \frac{v_s}{R}$$

Figure 4-2 illustrates the conversion of a voltage source, in Circuit A, into an equivalent current source, in Circuit B. The resistors, *R,* are equal, and the constraint equation was applied to change the voltage source into a current source.

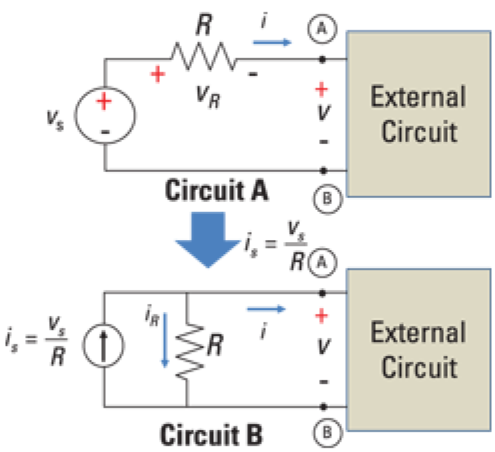

Illustration by Wiley, Composition Services Graphics

Figure 4-2: Transforming a voltage source into a current source.

Figure 4-3 shows the conversion with some numbers plugged in. Both circuits contain the same 3-kΩ resistor, and the source voltage in Circuit A is 15 volts. With this information, you can find the source current, i_s, for the transformed Circuit B.

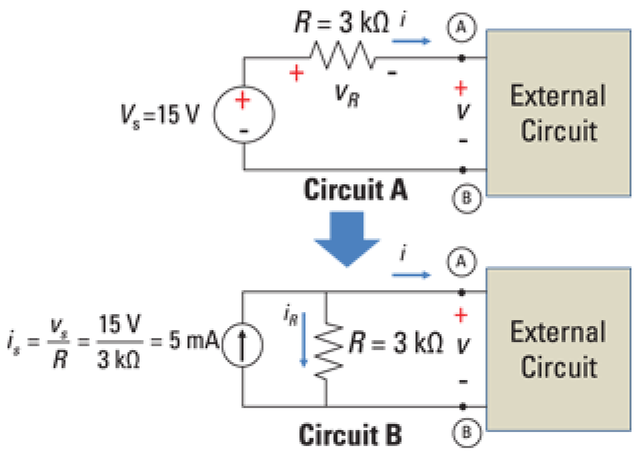

Illustration by Wiley, Composition Services Graphics

Figure 4-3: A numerical example of transforming a voltage source into a current source.

Use the constraint equation to find the source current in Circuit B. Here's what you get when you plug in the numbers:

$$i_s = \frac{v_s}{R} = \frac{15\text{ V}}{3\text{ k}\Omega} = 5\text{ mA}$$

Changing to a series circuit with a voltage source

You can convert a current source connected in parallel with a resistor to a voltage source connected in series with a resistor. You use this technique to form an equivalent circuit when the external circuit has devices connected in series.

Converting a practical current source connected with a resistor in parallel to a voltage source connected with a resistor in series follows the conditions for equivalent circuits:

- The resistors must be equal in both circuits.

- The source transformation must be constrained by $v_s = i_s R$.

Figure 4-4 illustrates how to convert a current source into a voltage source.

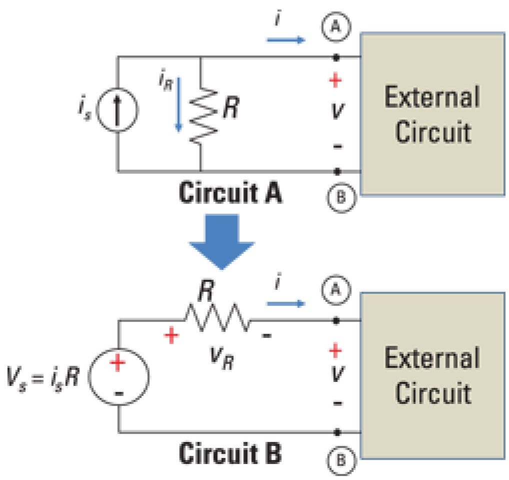

Illustration by Wiley, Composition Services Graphics

Figure 4-4: Transforming a current source into a voltage source.

Figure 4-5 depicts the same transformation of a current source to a voltage source with some numbers plugged in. Both circuits contain the same 3-kΩ resistor, and the current source in Circuit A is 5 mA.

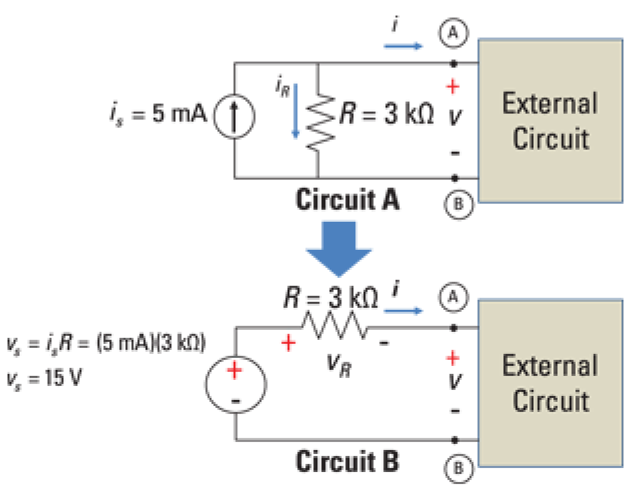

Illustration by Wiley, Composition Services Graphics

Figure 4-5: Numerical example of transforming a current source into a voltage source.

You can use the constraint equation to find the source voltage for Circuit B. Plugging in the numbers produces the following:

$$v_s = i_s R = (5\ \text{mA})(3\ \text{k}\Omega) = 15\ \text{V}$$

Suppose you have a complex circuit that has a current source, a resistor connected in parallel, and an external circuit with multiple resistors connected in series. You can transform the circuit so that it has a voltage source connected with all the resistors in series.

Consider Circuit A in Figure 4-6, where the right side of Terminals A and B consists of two resistors connected in series. On the left side of Terminals A and B is a practical current source modeled as an ideal current source in parallel with a resistor.

You want all the devices to be connected in series, so you need to move *R* when you transform the circuit. To transform the circuit, change the current source to a voltage source and move *R* so that it's connected in series rather than in parallel. When you use the constraint equation $v_s = i_s R$ to find the source voltage, remember that *R* is the resistor you moved.

Circuit B is a series circuit where all the devices share the same current. You can find the voltage through R, R_1, and R_2 using voltage divider techniques, which I discuss in the next section.

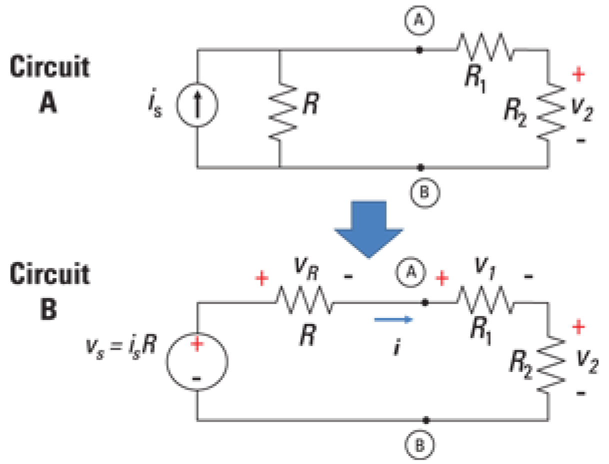

Illustration by Wiley, Composition Services Graphics

Figure 4-6: Transforming a complex circuit into a series circuit.

Divvying It Up with the Voltage Divider

The *voltage divider* technique allows you to calculate the voltage for each device connected in series with an input voltage source. In the preceding section, I show you how to transform a circuit to a series circuit with a voltage source. This section shows you how to formulate the voltage divider equation. Then you see the voltage divider equation at work.

If a circuit problem with a current source asks you to find the voltage for a particular device, you may find it easier to convert the circuit to a series circuit first. Then you can find voltage using the voltage divider technique.

Getting a voltage divider equation for a series circuit

You use the voltage divider when the device in the circuit is connected in series and is driven by a voltage source. The input voltage source is divided proportionally according to the resistor values.

To get the voltage divider equation, you start with the fact that for a series circuit, the same current flows through each resistor. With this current, you use Kirchhoff's voltage law (KVL) and Ohm's law to obtain the voltage across a particular resistor. You can solve for (or eliminate) the current in the expression. In the resulting equation, the desired voltage across the resistor is proportional to the input source voltage. Because the voltage source is multiplied by a ratio of resistors having a value of less than 1, the desired output and device voltage are always less than the input source voltage.

Look at Circuit B of Figure 4-6, which has a voltage source and three resistors connected in series. You want to calculate the voltage across the resistors.

KVL says that the sum of the voltage rises and drops around a loop is equal to zero. So applying KVL to Circuit B produces the following:

$$v_s = v_R + v_1 + v_2$$

Because the circuit is connected in series, the same current *i* flows through each of the resistors. Using Ohm's law for each resistor yields

$$v_R = iR$$
$$v_1 = iR_1$$
$$v_2 = iR_2$$

Substituting v_R, v_1, and v_2 into $v_s = v_R + v_1 + v_2$ and factoring out the current *i* gives you

$$v_s = i\left(R + R_1 + R_2\right)$$

When you divide the voltage across Resistor 1, which is $v_1 = iR_1$, by $v_s = i(R + R_1 + R_2)$, you get one form of the desired voltage divider equation:

$$\frac{v_1}{v_s} = \frac{iR_1}{i(R + R_1 + R_2)}$$

$$\frac{v_1}{v_s} = \frac{R_1}{R + R_1 + R_2}$$

This form of the voltage divider is often referred as a *voltage transfer function,* which relates the output voltage (voltage v_1 for this example) to the input voltage source (which is v_s in this case). You can find the output voltage if you know the input voltage source.

Solving for v_1 yields the following voltage divider equation:

$$v_1 = v_s\left(\frac{R_1}{R + R_1 + R_2}\right)$$

You can find similar voltage divider relationships for v_2 and v_R:

$$v_2 = v_s\left(\frac{R_2}{R + R_1 + R_2}\right)$$

$$v_R = v_s\left(\frac{R}{R + R_1 + R_2}\right)$$

These divider equations show that to find the voltage across a particular resistor, you simply multiply the input source voltage by the desired resistor and divide by the total resistance of the series circuit. That is, the voltage of each device of Figure 4-6 depends on the ratio of resistors multiplied by the source voltage.

The voltage across each resistor is always less than and proportional to the supplied independent source voltage because the ratio of resistors is always less than 1. This idea offers a handy check of your calculations. The largest voltage goes across the largest resistor, and the lowest voltage goes across the smallest resistor.

Figuring out voltages for a series circuit with two or more resistors

Voltage divider techniques work well for a series circuit that has two or more resistors. You calculate the output voltage by multiplying the input source voltage by the desired resistor and dividing by the total resistance in the circuit.

I use Figure 4-7 to illustrate the voltage divider technique numerically. The given Circuit A has a source current of 5 milliamps as well as a 4-kΩ resistor arranged in parallel with a series combination of 6-kΩ and 10-kΩ resistors. To find the voltage across the resistors, you first transform Circuit A so that it has a voltage source and all three resistors in series.

Start by finding the source voltage in the transformed circuit, Circuit B. The transformation must be constrained by $v_s = i_s R$, so here's the source voltage:

$$v_s = i_s R = (5\ \text{mA})(4\ \text{k}\Omega) = 20\ \text{V}$$

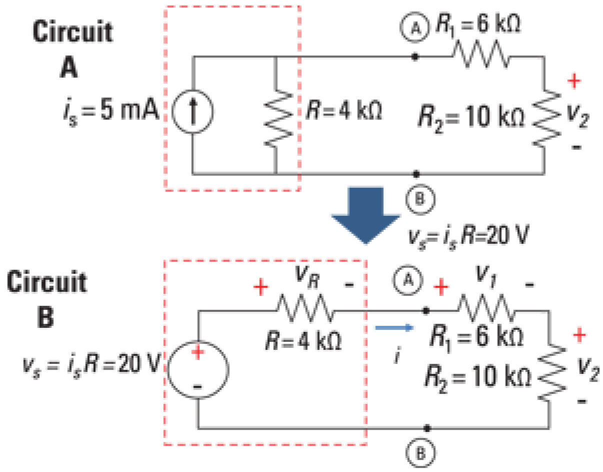

Illustration by Wiley, Composition Services Graphics

Figure 4-7: Numerical example of the voltage divider method.

According to the voltage divider equation, you find the voltage across a resistor by multiplying the source voltage by the desired resistor and then dividing by the total resistance of the series circuit (see the preceding section for details). Try calculating the voltage for each resistor shown. Use the voltage divider shortcut and plug in the numbers:

$$v_1 = 20\ \text{V} \cdot \left(\frac{6\ \text{k}\Omega}{4\ \text{k}\Omega + 6\ \text{k}\Omega + 10\ \text{k}\Omega} \right) = 6\ \text{V}$$

$$v_2 = 20\ \text{V} \cdot \left(\frac{10\ \text{k}\Omega}{4\ \text{k}\Omega + 6\ \text{k}\Omega + 10\text{k}\Omega} \right) = 10\ \text{V}$$

$$v_R = 20\ \text{V} \cdot \left(\frac{4\ \text{k}\Omega}{4\ \text{k}\Omega + 6\ \text{k}\Omega + 10\ \text{k}\Omega} \right) = 4\ \text{V}$$

Finding voltages when you have multiple current sources

Analyzing a circuit that has multiple current sources and parallel resistors would be tedious if you could only use Kirchhoff’s laws and Ohm’s law. However, thanks to the power of source transformation and the voltage divider technique, the analysis is relatively straightforward.

Circuit A in Figure 4-8 has two current sources and two parallel resistors. What's the voltage, v_1, through resistor R_1?

You transform this circuit in two stages. First transform the circuit so that it has two voltage sources and all the resistors arranged in series. Then combine the voltage sources to get one equivalent voltage source. After that, you can find v_1, the voltage across R_1, using the voltage divider technique.

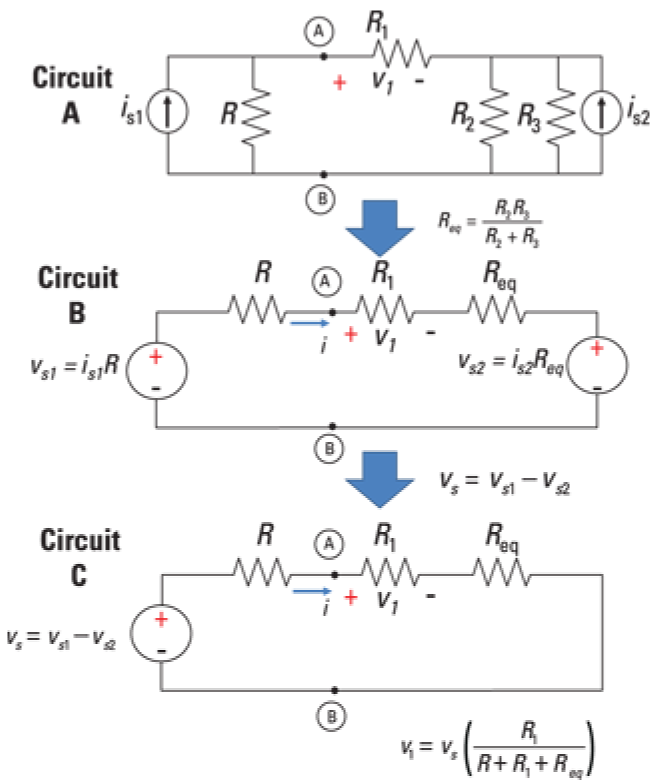

Illustration by Wiley, Composition Services Graphics

Figure 4-8: Circuit analysis with multiple current sources.

Circuit B of Figure 4-8 is the transformation of Circuit A using several operations. On the right side of Circuit A, you want to move both R_2 and R_3 so that they're connected in series. These resistors are connected in parallel, so find their equivalent resistance, R_{eq}:

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

Next, transform the two current sources by converting them to voltage sources:

$$v_{s1} = i_{s1}R$$
$$v_{s2} = i_{s2}R_{eq}$$

Circuit C of Figure 4-8 completes the transformation. The two voltage sources connected in series are combined, forming one equivalent voltage source. Observing the voltage polarities results in one voltage source, as follows:

$$v_s = v_{s1} - v_{s2}$$

For a numerical example, check out the circuit in Figure 4-9. Your goal is to find v_1, the voltage of the 11-kΩ resistor. Start the circuit transformation by putting the resistors in series and switching to voltage sources.

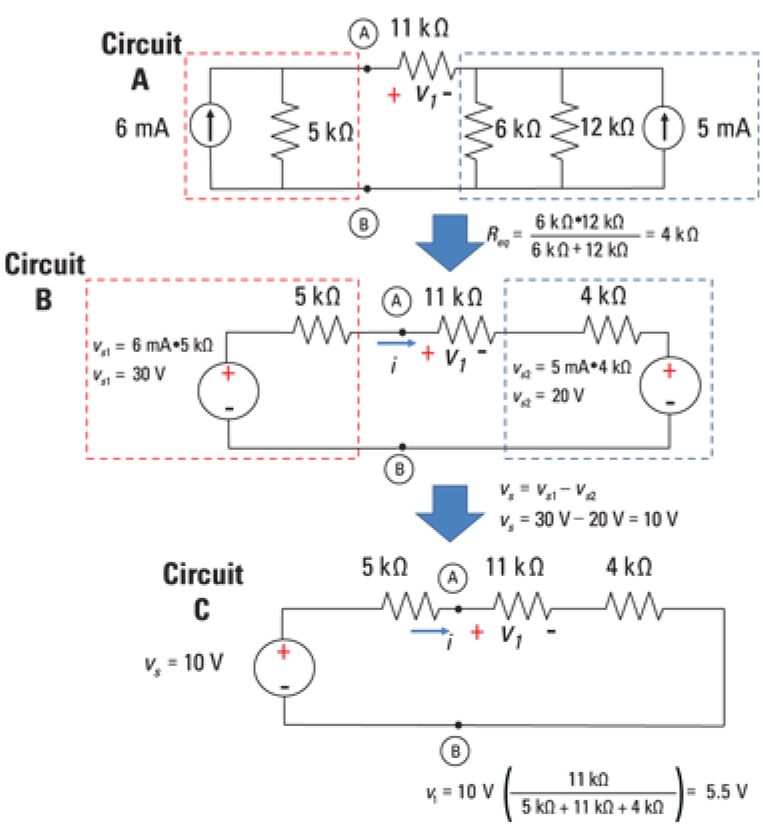

Illustration by Wiley, Composition Services Graphics

Figure 4-9: Numerical example with multiple current sources.

Here’s the equivalent total resistance for two parallel resistors of 6 kΩ and 12 kΩ:

$$R_{eq} = \frac{6\ \text{k}\Omega \cdot 12\ \text{k}\Omega}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = 4\ \text{k}\Omega$$

Use the transformation equations $v_{s1} = i_{s1}R$ and $v_{s2} = i_{s2}R_{eq}$ to convert the two current sources to two voltage sources:

$$v_{s1} = 6\ \text{mA} \cdot 5\ \text{k}\Omega = 30\ \text{V}$$

$$v_{s2} = 5\ \text{mA} \cdot 4\ \text{k}\Omega = 20\ \text{V}$$

You can see the source transformations and equivalent resistance in Circuit B of Figure 4-9.

The voltage sources are connected in series, so combine them, noting their polarities:

$$v_s = v_{s1} - v_{s2} = 30\ \text{V} - 20\ \text{V} = 10\ \text{V}$$

Circuit C of Figure 4-9 shows the completed and simplified transformation. Now you can use the voltage divider equation to find v_1:

$$v_1 = 10\ \text{V} \cdot \frac{11\ \text{k}\Omega}{4\ \text{k}\Omega + 11\ \text{k}\Omega + 5\ \text{k}\Omega} = 5.5\ \text{V}$$

Even though the voltage divider shortcut for series circuits lets you find an unknown voltage without using Kirchhoff’s and Ohm’s laws, this technique was developed from the foundational equations of Kirchhoff’s and Ohm’s laws.

Using the voltage divider technique repeatedly

When a part of a circuit has a combination of series and parallel resistors, you can use the voltage divider technique repeatedly. For example, you may use this approach when resistors are connected in parallel and one of the parallel branches has a series combination.

Consider Circuit A in Figure 4-10. You could find all the components' voltages and currents using Kirchhoff's voltage and current laws. But if all you want is the voltage across a specific device, you can take a shortcut with source transformation and the voltage divider technique. You can find the voltage v_x across the 3-kΩ resistor using the voltage divider technique repeatedly.

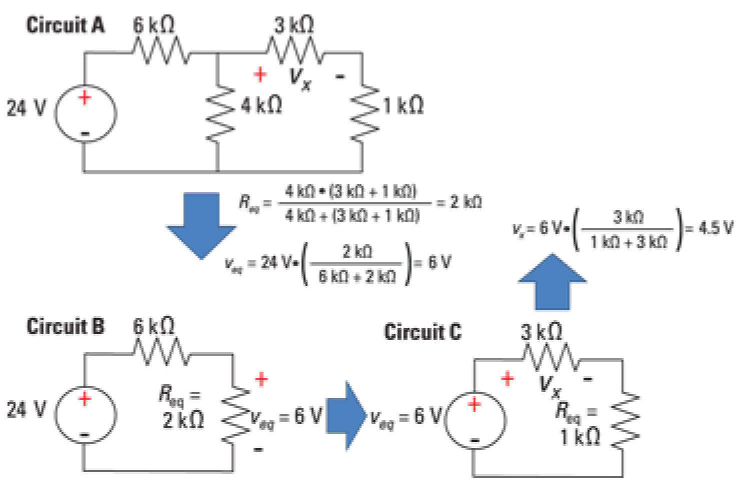

Illustration by Wiley, Composition Services Graphics

Figure 4-10: Circuit analysis using voltage divider methods repeatedly.

On the right side of Circuit A, you have a resistor series combination of 3 kΩ and 1 kΩ connected in parallel with the 4-kΩ resistor. The total resistance for this combination is

$$R_{eq} = \frac{4\ \text{k}\Omega \cdot (3\ \text{k}\Omega + 1\ \text{k}\Omega)}{4\ \text{k}\Omega + (3\ \text{k}\Omega + 1\ \text{k}\Omega)} = 2\ \text{k}\Omega$$

I tell you how to find equivalent resistance in Chapter 3.

Circuit B shows the circuit after you combine these resistors. Calculate the voltage across R_{eq} using the voltage divider method:

$$v_{eq} = 24\text{ V}\cdot\frac{2\text{ k}\Omega}{6\text{ k}\Omega+2\text{ k}\Omega} = 6\text{ V}$$

However, the 6 volts also go across the resistor series combination of 3 kΩ and 1 kΩ, as depicted in Circuit C of Figure 4-10. Use the voltage divider method once again to find v_x:

$$v_x = 6\text{ V}\cdot\frac{3\text{ k}\Omega}{3\text{ k}\Omega+1\text{ k}\Omega} = 4.5\text{ V}$$

Cutting to the Chase Using the Current Divider Technique

The *current divider* technique lets you easily calculate the current for each device connected in parallel when the devices are driven by an input current source. Earlier in "Converting to a parallel circuit with a current source," I show you how to transform your circuit. This section shows you where the current divider equation comes from and how to apply it.

Getting a current divider equation for a parallel circuit

For devices connected in parallel with a current source, the current divider technique allows you to find the current through each device. Basically, you're looking at how the current source distributes its supplied current to each device, depending on the ratio of conductances (or resistances) in the circuit.

The current divider shortcut replaces using Kirchhoff's current law and Ohm's law in finding the current through each device. Of course, the shortcut works only because it's based on these fundamental laws. To see where the current divider equation comes from, look at Figure 4-11. Circuit A is a complex circuit with a voltage source. You want to find an equation to calculate the current through each resistor.

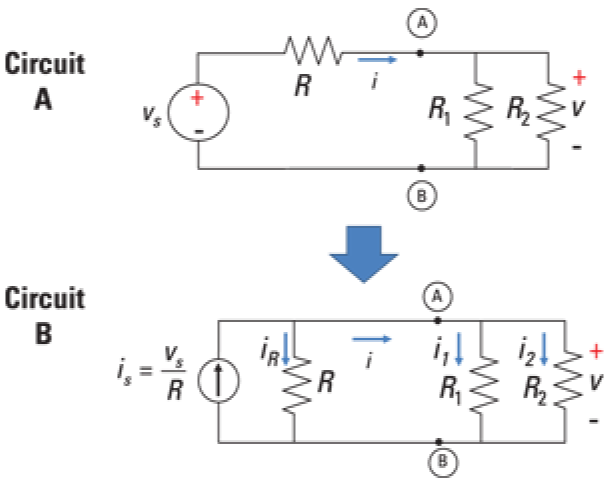

Illustration by Wiley, Composition Services Graphics

Figure 4-11: Source transformation of a complex circuit into a parallel circuit.

Start by transforming the circuit. Circuit B is Circuit A transformed into a parallel circuit with a current source. Kirchhoff's current law (KCL) says that the sum of the incoming currents is equal to the sum of the outgoing currents. Applying KCL to Circuit B of Figure 4-11 gives you the following:

$$i_s = i_R + i_1 + i_2$$

Because Circuit B shows devices connected in parallel, the voltage v is the same across each resistor. Using Ohm's law for each resistor and using the definition of conductance G (found in Chapter 3) yields the following expressions for the currents:

$$i_R = \frac{v}{R} = Gv \qquad i_1 = \frac{v}{R_1} = G_1 v \qquad i_2 = \frac{v}{R_2} = G_2 v$$

where $G = 1/R$, $G_1 = 1/R_1$, and $G_2 = 1/R_2$.

Substituting the values of i_R, i_1, and i_2 into $i_s = i_R + i_1 + i_2$ and factoring out the voltage v gives you

$$i_s = v(G + G_1 + G_2)$$

When you divide $i_1 = vG_1$ by $i_s = v(G + G_1 + G_2)$, you wind up with the following form of the desired current divider equation:

$$\frac{i_1}{i_s} = \frac{vG_1}{v(G + G_1 + G_2)}$$

$$\frac{i_1}{i_s} = \frac{G_1}{G + G_1 + G_2}$$

This form of the current divider equation is often referred to as a *current transfer function,* and it relates the ratio output current (current i_1 for this example) to the input source current (i_s in this example). With this equation, you can find the output current going through any device for a given input source current.

Algebraically solving for i_1 yields the following form of the current divider equation:

$$i_1 = i_s\left(\frac{G_1}{G + G_1 + G_2}\right)$$

You can find similar relationships for i_2 and i_R:

$$i_2 = i_s\left(\frac{G_2}{G + G_1 + G_2}\right)$$

$$i_R = i_s\left(\frac{G}{G + G_1 + G_2}\right)$$

These equations show that to find the current through a desired conductance, you simply multiply the input source current by the desired conductance divided by the total conductance of the parallel combination in the circuit. Thus, in Figure 4-11, the current through each resistor depends on the ratio of resistors multiplied by the input source current.

> The current through each resistor is always less than and proportional to the supplied independent source current because the ratio of conductance is always less than 1. That idea offers a neat way to check your answer: For a parallel combination of resistors, the largest current goes through the smallest resistor because it has the least resistance, and the smallest current goes through the largest resistor.

Figuring out currents for parallel circuits

The current divider method provides a shortcut in finding the current through each device when all the devices are connected in parallel.

Try using current divider techniques to calculate the current through each resistor in Circuit A of Figure 4-12. Circuit A is a complex circuit with a voltage source, so first transform it into an equivalent circuit that has a current source and all the resistors connected in parallel, as in Circuit B.

Start by finding the source current in Circuit B. The transformation must be constrained by $v_s = i_s R$, where V = 24 volts and v_s = 6 Ω, so the source current for Circuit B, i_s, is 4 amps.

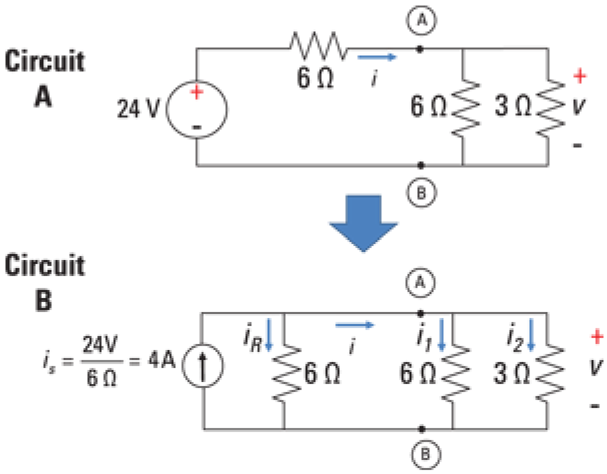

Illustration by Wiley, Composition Services Graphics

Figure 4-12: Numerical example to convert a complex circuit to a parallel circuit.

After you know the source current, i_S, you can use the current divider equation for each resistor. Using the current divider method yields the following (see the preceding section for the derivation of this equation):

$$i_1 = 4\text{ A} \cdot \left(\frac{\frac{1}{6\,\Omega}}{\frac{1}{6\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}} \right) = 1\text{ A}$$

$$i_2 = 4\text{ A} \cdot \left(\frac{\frac{1}{3\,\Omega}}{\frac{1}{6\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}} \right) = 2\text{ A}$$

$$i_R = 4\text{ A} \cdot \left(\frac{\frac{1}{6\,\Omega}}{\frac{1}{6\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}} \right) = 1\text{ A}$$

These results show how easy it is to use the current divider method after you’ve transformed the circuit into a parallel circuit driven by a practical current source.

Finding currents when you have multiple voltage sources

Circuit A of Figure 4-13 has multiple voltage sources. What is i_1, the current through R_1? You can find the answer by transforming the circuit and using the current divider technique.

Transforming this circuit involves two stages. The first is converting the voltage sources to current sources and connecting all resistors in parallel. The second is combining the two current sources. You can then apply the current divider technique to find i_1.

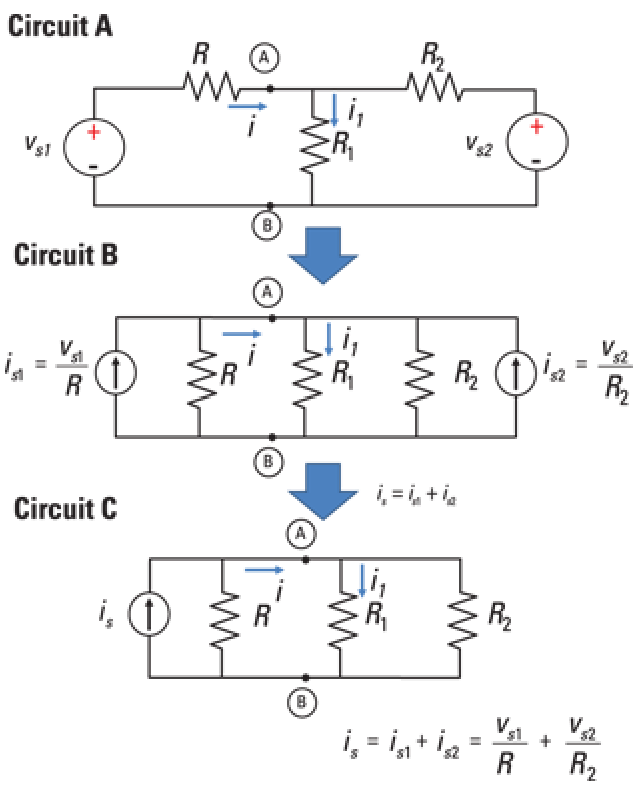

Illustration by Wiley, Composition Services Graphics

Figure 4-13: Circuit analysis with multiple voltage sources.

Circuit B shows the first part of the transformation of Circuit A: switching from voltage to current sources. For the transformed circuit to be equivalent to the original, the following equations have to hold true (see the earlier section "Converting to a parallel circuit with a current source" for these constraint equations):

$$i_{s1} = \frac{v_{s1}}{R}$$

$$i_{s2} = \frac{v_{s2}}{R_2}$$

Circuit C of Figure 4-13 completes the transformation. The two current sources are connected in parallel and combined to form one equivalent current source. The current sources point in the same direction, so you can add them up to get the following:

$$i_s = i_{s1} + i_{s2}$$

Figure 4-14 provides a numerical example for the circuit shown in Figure 4-13. Start by switching to current sources.

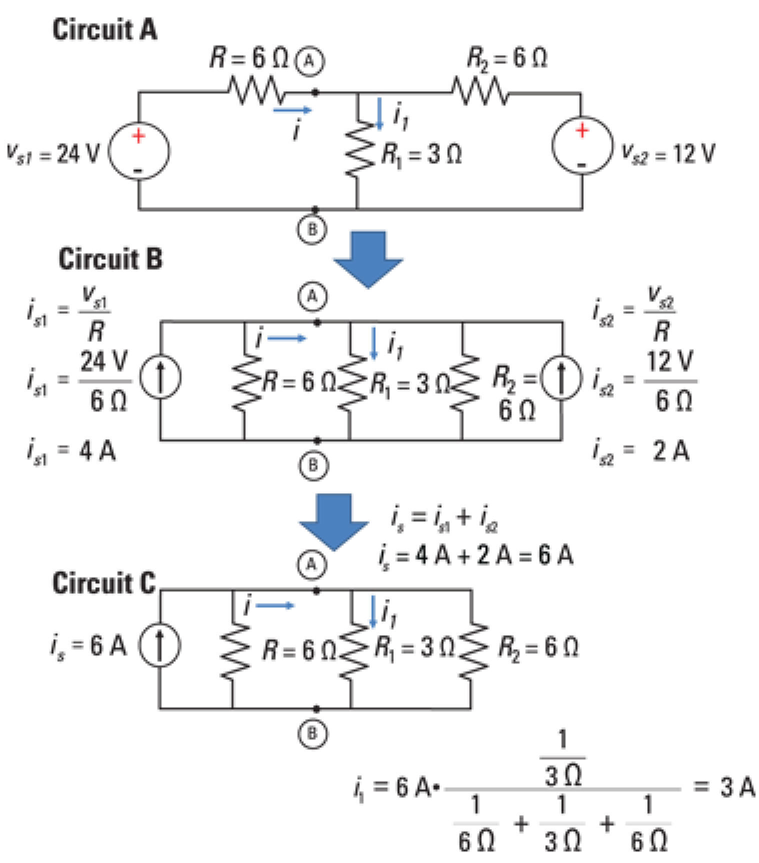

Illustration by Wiley, Composition Services Graphics

Figure 4-14: Numerical example of a circuit with multiple voltage sources.

The transformation of the two voltage sources to two current sources yields

$$i_{s1} = \frac{v_{s1}}{R} = \frac{24\ \text{V}}{6\ \Omega} = 4\ \text{A}$$

$$i_{s2} = \frac{v_{s2}}{R_2} = \frac{12\ \text{V}}{6\ \Omega} = 2\ \text{A}$$

You can see the results of the source transformations and equivalent resistance in Circuit B of Figure 4-14.

The current sources are connected in parallel and point in the same direction, so add them together:

$$i_s = i_{s1} + i_{s2} = 4\ \text{A} + 2\ \text{A} = 6\ \text{A}$$

Circuit C of Figure 4-14 shows the completed and simplified transformation that you use to calculate i_1. Use the current divider technique to find the current through the 3-Ω resistor:

$$i_1 = 6\ \text{A} \cdot \frac{\frac{1}{3\ \Omega}}{\frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega}} = 3\ \text{A}$$

Using the current divider technique repeatedly

When you see parts of a circuit with resistor combinations connected in parallel within other combinations of devices connected in parallel, you can use current divider techniques repeatedly. To see how this works, consider Circuit A in Figure 4-15. You can use current divider shortcuts repeatedly to find the current i_x through the 8-kΩ resistor.

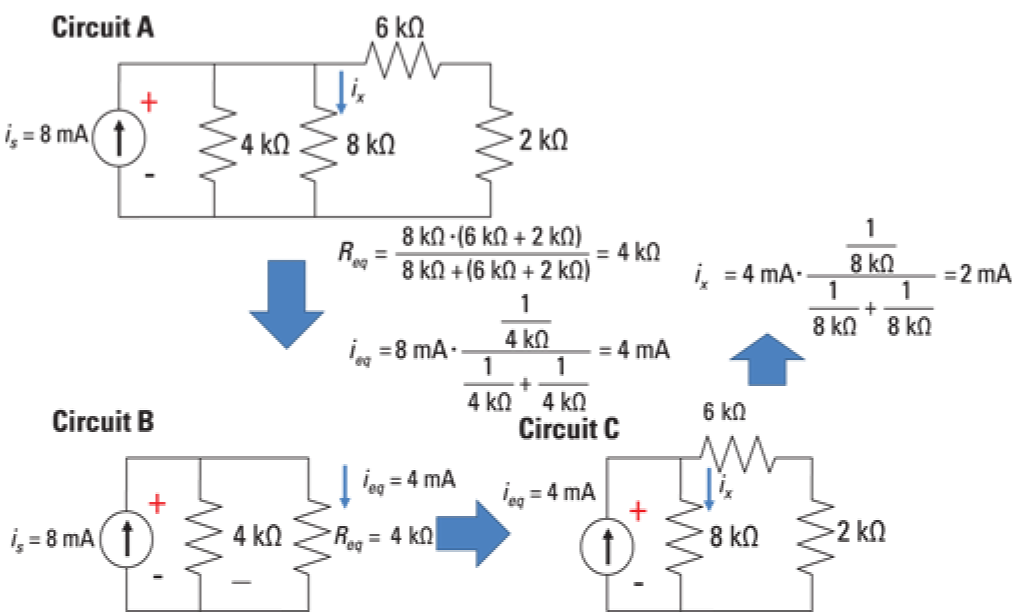

Illustration by Wiley, Composition Services Graphics

Figure 4-15: Finding current in a complex circuit using current divider techniques repeatedly.

The circuit includes a resistor series combination of 6 kΩ and 2 kΩ connected in parallel with the 8-kΩ resistor. For this resistor combination, total resistance yields

$$R_{eq} = \frac{8\text{ k}\Omega\cdot(6\text{ k}\Omega+2\text{ k}\Omega)}{8\text{ k}\Omega+(6\text{ k}\Omega+2\text{ k}\Omega)} = 4\text{ k}\Omega$$

Circuit B shows the equivalent resistance. Calculate the current through R_{eq} using the current divider equation:

$$i_{eq} = 8\text{ mA}\cdot\frac{\frac{1}{4\text{ k}\Omega}}{\frac{1}{4\text{ k}\Omega}+\frac{1}{4\text{ k}\Omega}} = 4\text{ mA}$$

However, the 4-milliamp current is split between the 4-kΩ resistor and the series resistor combination of 6 kΩ and 2 kΩ, as in Circuit C of Figure 4-15. Use the current divider method once again to find the current through the 8-kΩ resistor:

$$i_x = 4\text{ mA}\cdot\frac{\frac{1}{8\text{ k}\Omega}}{\frac{1}{8\text{ k}\Omega}+\frac{1}{6\text{ k}\Omega+2\text{ k}\Omega}} = 2\text{ mA}$$

Even though the current divider shortcut for parallel circuits makes it so you don’t have to use Kirchhoff’s laws and Ohm’s laws to find an unknown current, this technique was developed from the foundational equations of Kirchhoff’s laws and Ohm’s law.