Astrophysical Shocks and Particle Acceleration

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Intended Learning Objectives

- Shocks in astrophysics
- Shock jump conditions
- Diffusive shock acceleration
- Energy distribution of high energy particles

What is a shock?

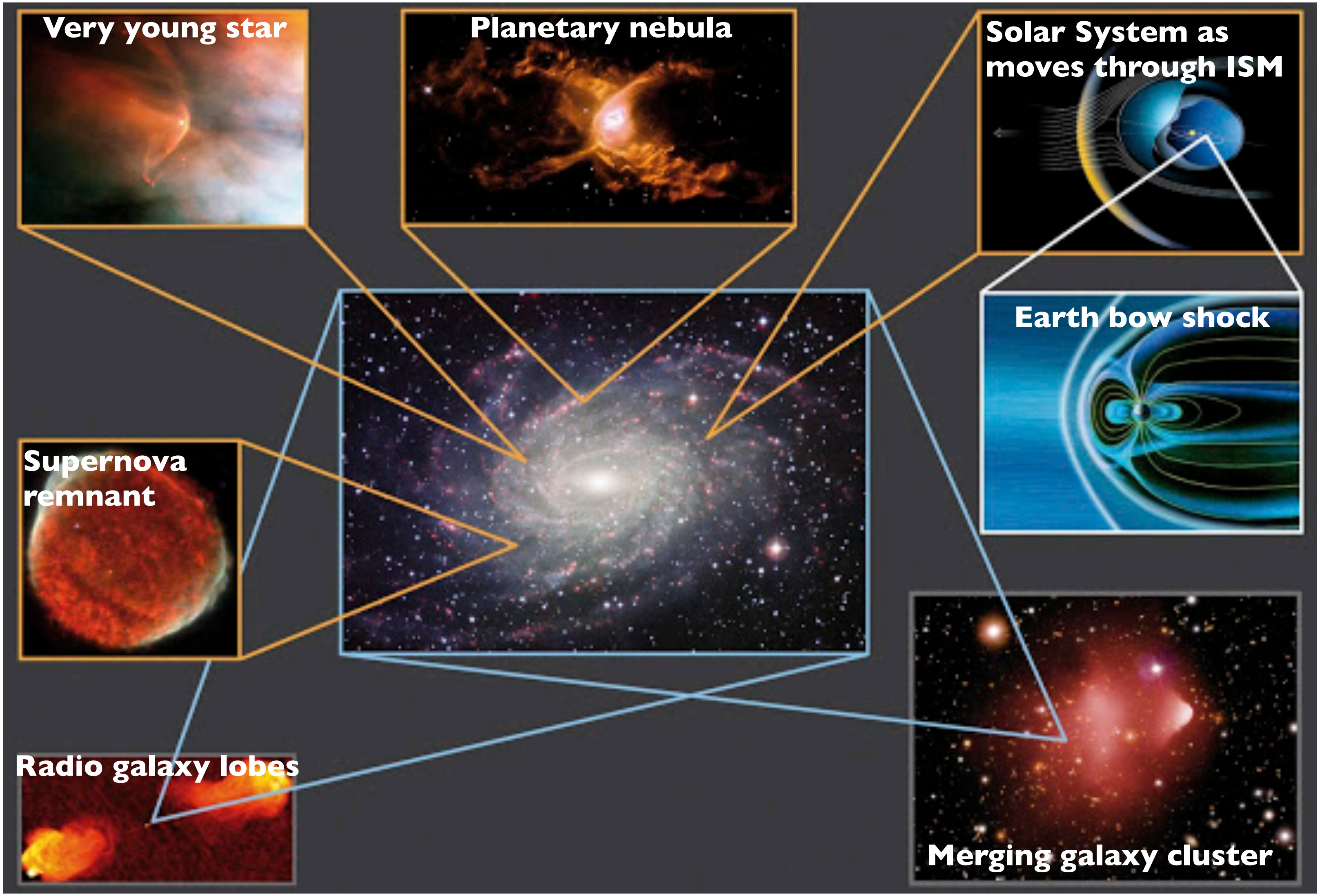
Shocks

- A shock occurs when a disturbance moves through a medium faster than the medium sound speed.
- The shock wave carries energy and propagates through the medium.
- A sudden change in bulk velocity, density, medium temperature occurs across the shock. No forewarning for the material hit by the shock.

Example of shock: sonic boom

Image credit: Navy News Photo/wiki commons

Shocks in Astrophysics



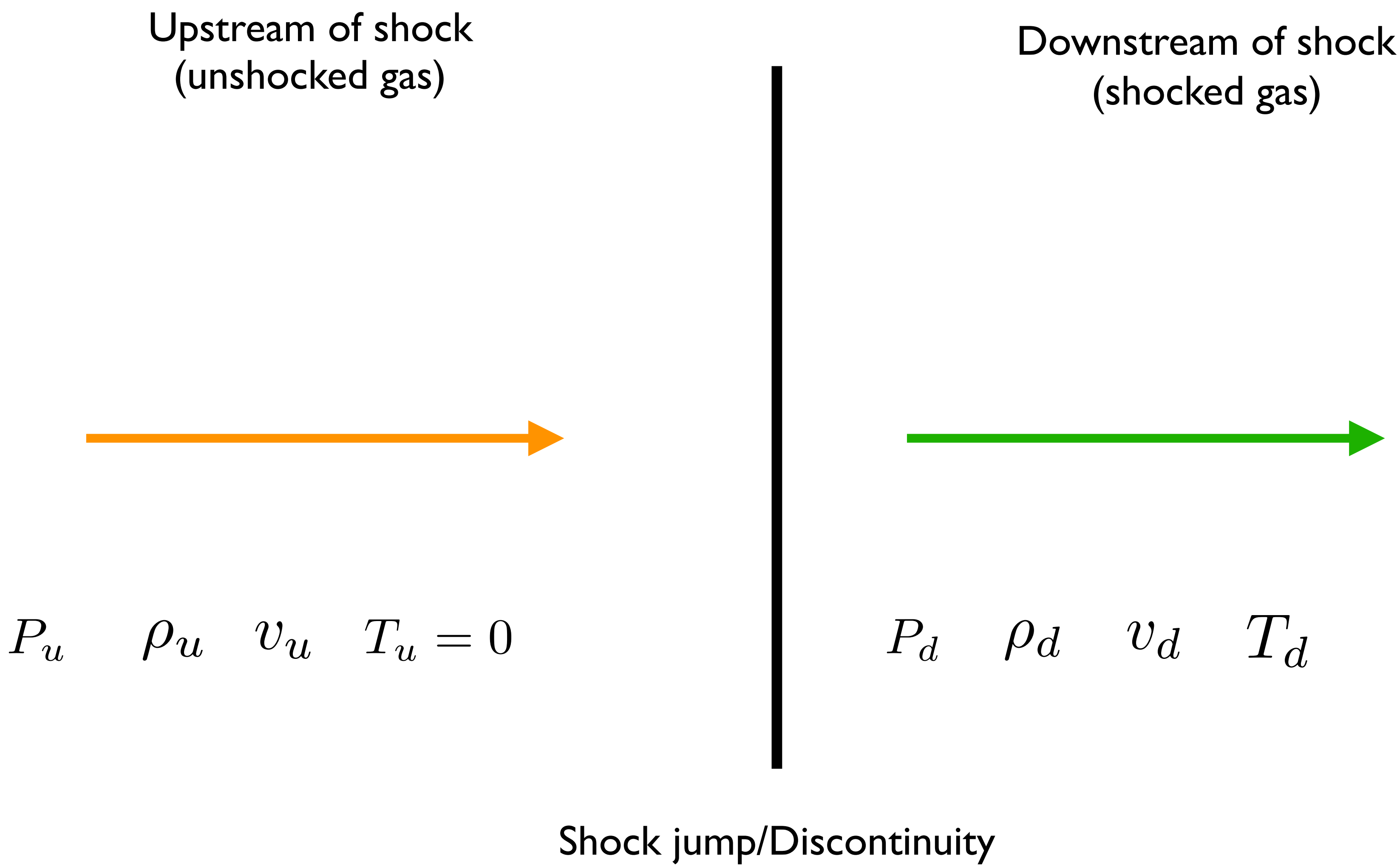
- Astrophysical shocks occur on all scales from planets to clusters of galaxies.
- Astrophysical shocks are collisionless (the shock front is thinner than the particle mean free path).

Image credit: http://minsex.blogspot.com/2011/11/astrophysical-shock-waves.html

Shock Physics

To understand the shock conditions, let's go into the instantaneous rest frame of the shock.

In restframe of shock



Shock Physics

Assumptions

- Monoatomic gas (e.g. fully-ionized hydrogen plasma).
- Thermal (internal) energy of unshocked gas is negligible.
- Pressure and temperature of unshocked gas are approximately zero.

Conservation laws

Conservation of mass (the mass per unit area flowing across the shock is conserved)

$$v_d \rho_d = v_u \rho_u$$

Conservation of momentum (shock does not accelerate in its rest frame; the difference between upstream and downstream pressures is provided by the gas pressure downstream)

$$v_d^2 \rho_d = v_u^2 \rho_u + P_d$$

Conservation of energy (PdV is the work being done on the gas at the shock; the rate per unit area at which this work is done is $v_d P_d$)

$$v_u(\frac{1}{2}\rho_u v_u^2) - v_d(\frac{1}{2}\rho_d v_d^2 + \frac{3}{2}P_d) = v_d P_d$$

Shock Physics

If we use the fact that $v_d \rho_d = v_u \rho_u$ and remove P_d, we obtain

$$\frac{\rho_d^2}{\rho_u^2} v_u^3 - 5 \frac{\rho_d}{\rho_u} v_u^3 + 4 v_u^3 = 0$$

This can be re-written as

$$\left(\frac{\rho_d}{\rho_u} - 4 \right) \left(\frac{\rho_d}{\rho_u} - 1 \right) = 0$$

The equation above has the trivial solution $v_u = v_d$, and otherwise leads to the **strong shock jump conditions**

$$\boxed{\frac{\rho_d}{\rho_u} = 4 \qquad\qquad \frac{v_u}{v_d} = 4}$$

This is a highly simplified case; however, it is a reasonable approximation in many astrophysical environments. For a full derivation, see Longair or Blundell & Blundell.

Characteristics of Shocks

In order to characterize the shock, the Mach number is adopted

$$M = \frac{u_u}{c_u}$$

i.e., the ratio between the speed of the shock and the speed of sound in the unperturbed medium. For the sake of simplicity, we just derived the jump conditions for $M \gg 1$.

The temperature of the interstellar medium (ISM) is $T \simeq 10^4\ \mathrm{K}$ and its sound speed is

$$c_s = \left(\frac{\gamma k T}{m}\right)^{1/2} \simeq 12\ \mathrm{km/s}$$

- Stars have velocities $15\ \mathrm{km/s}$; therefore, stellar bow shocks have Mach number $M \simeq 1.2$ (weak shocks).

- Some stars have fast stellar winds $(100 - 3000\ \mathrm{km/s})$; the wind meets the ISM in bow shocks with Mach number $M = 8 - 250$ (strong shocks).

- Supernova ejecta have velocities $10^4\ \mathrm{km/s}$; therefore, the ejecta meet the ISM in shocks with Mach number $M = 10^3$ (very strong shocks).

Astrophysical Shocks

Image credits: https://apod.nasa.gov/apod/ap130529.html

What is the shape of the energy distribution of particles?

Power Law Energy Distribution

We will now explore the mechanisms behind the power-law energy distributions of particles in astrophysical environments

$$N(E) \propto E^{\alpha}$$

with α being a constant. This energy distribution is a non-thermal one because it differs from the Maxwellian distribution.

Energy Distribution

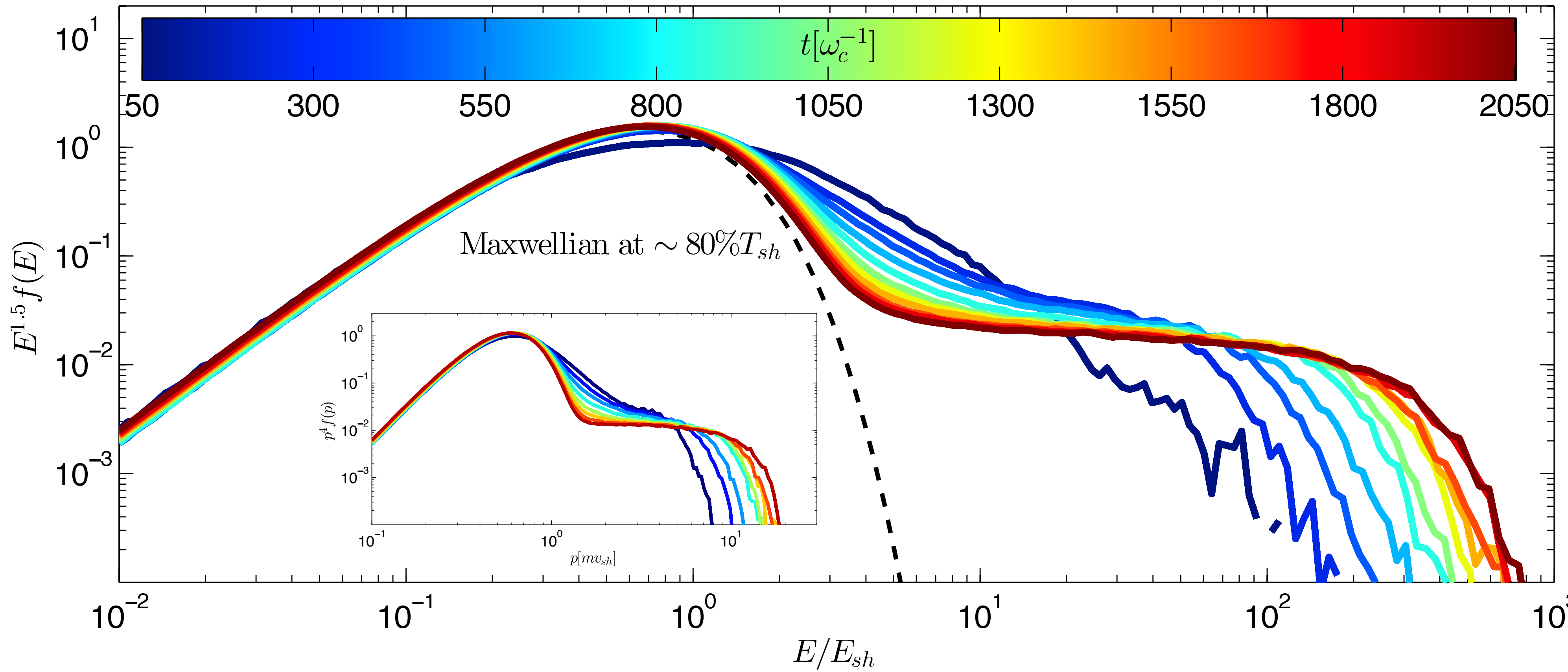
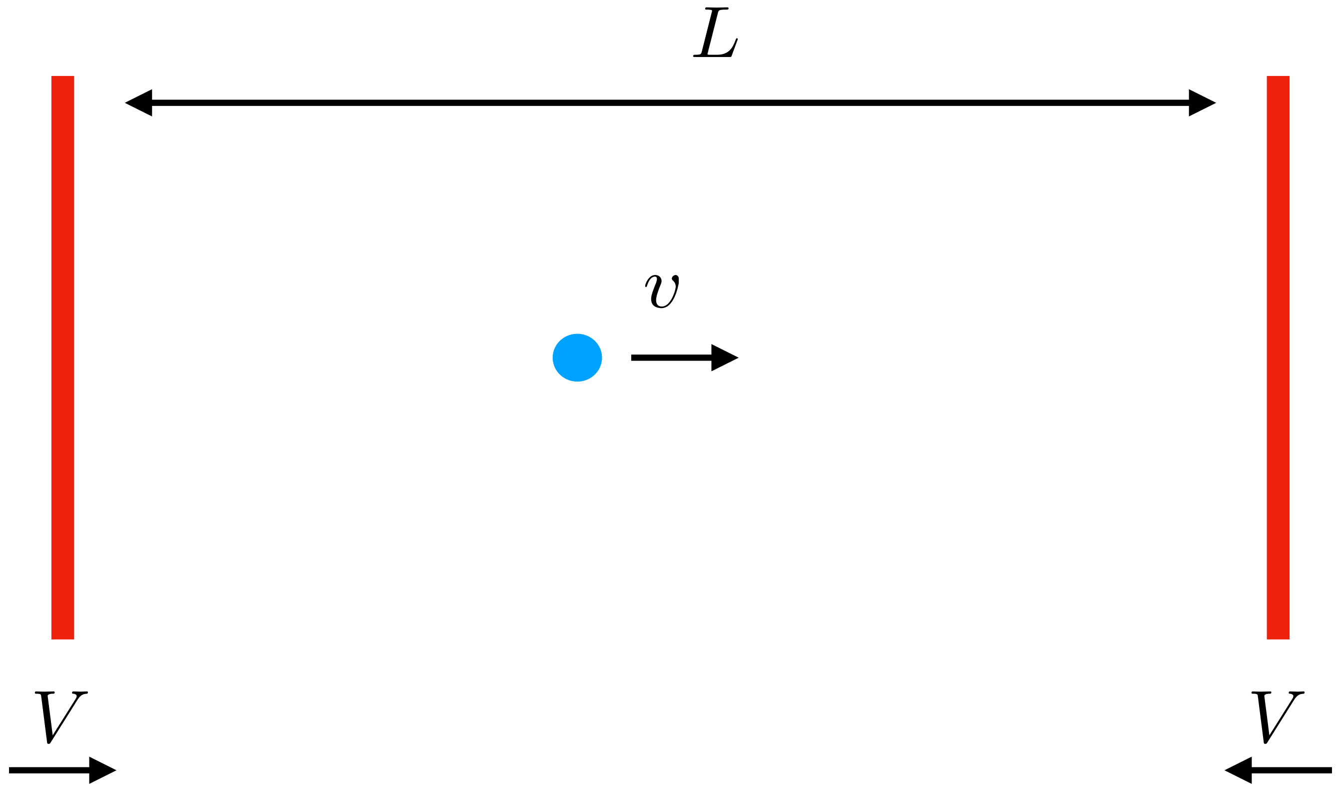


Figure 1. Downstream ion energy spectrum at different times, as in the legend. It is possible to note the thermal distribution, well-fitted by a Maxwellian with temperature about 80% the one expected for Mach number 20 shock that does not accelerate particles (dashed line), and a non-thermal power-law tail extending to larger energies at later times. The spectrum is plotted multiplied by $E^{1.5}$, to emphasize the agreement with the energy scaling predicted by DSA, which reads p^{-4} in momentum (see inset and eq. 3).

Caprioli & Spitkovsky, ApJ (2014), arXiv: 1310.2943.

How can we obtain a power law?

Intuitive Picture



A particle of mass m is traveling at mildly relativistic speed v between two scattering surfaces at a distance L from each other. The two surfaces are approaching each other with speed $V \ll c$.

Intuitive Picture

If the particle collides alternately head-on with each scatterer, it will gain energy at a rate

$$\frac{dE}{dt} = \text{rate of collisions} \times \text{energy change per collision}$$

If the particle is relativistic, the momentum increases of $\gamma m V$ after each collision. The total energy increase is $E = pc$. Therefore, taking $v \simeq c$, we have

$$\frac{dE}{dt} \simeq \frac{v}{L} \times \gamma m V c \simeq \frac{\gamma m c^2 V}{L} \simeq \frac{EV}{L}$$

This implies that the particle gains energy as $\boxed{\frac{dE}{dt} = \frac{E}{\tau}}$

where τ is the timescale defining the crossing time between scatterers. The particle energy will increase exponentially.

Intuitive Picture

This example tells us that we can look for a mechanism allowing multiple scatterings.

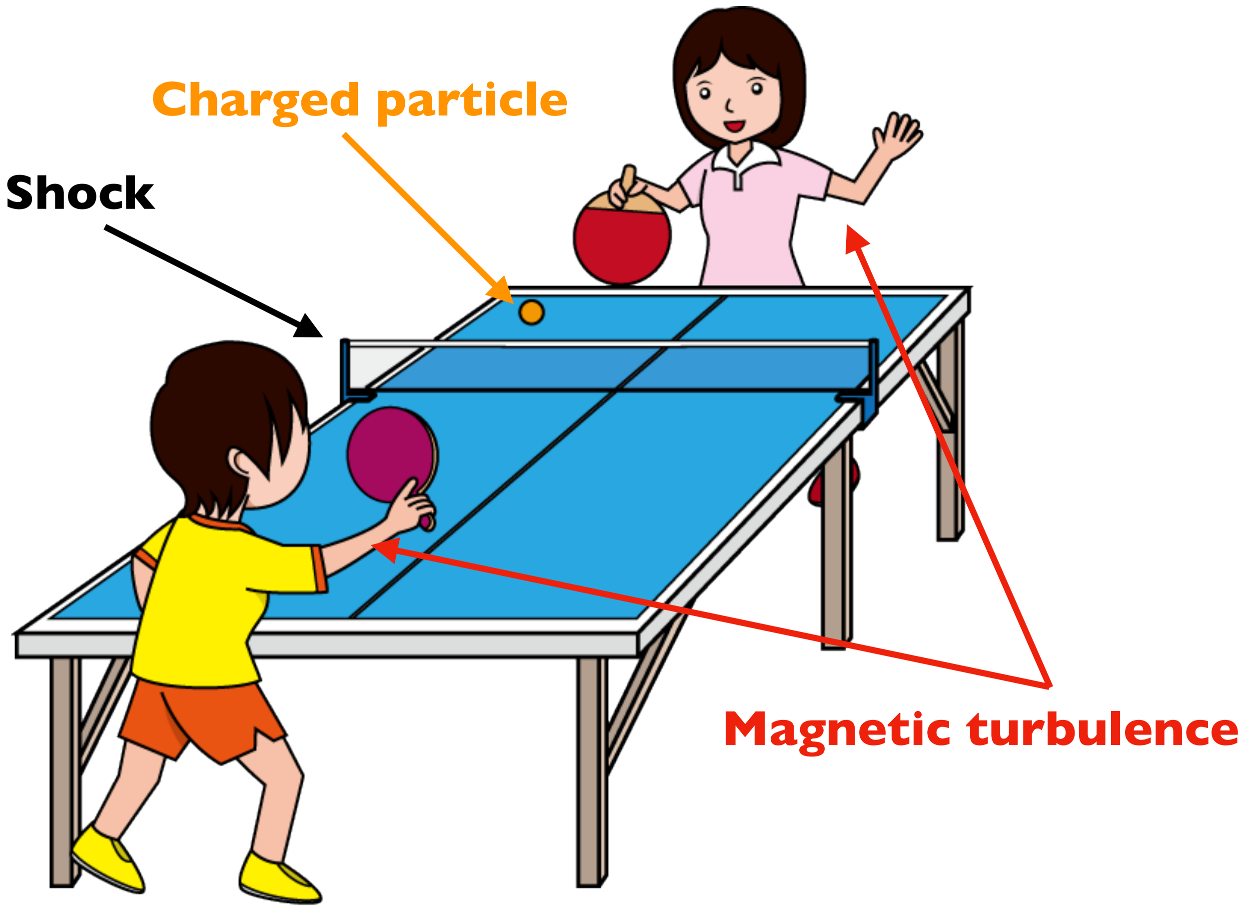
We can use shocks!

Diffusive Shock Acceleration

Diffusive shock acceleration (first order Fermi acceleration) leads to a power-law energy distribution.

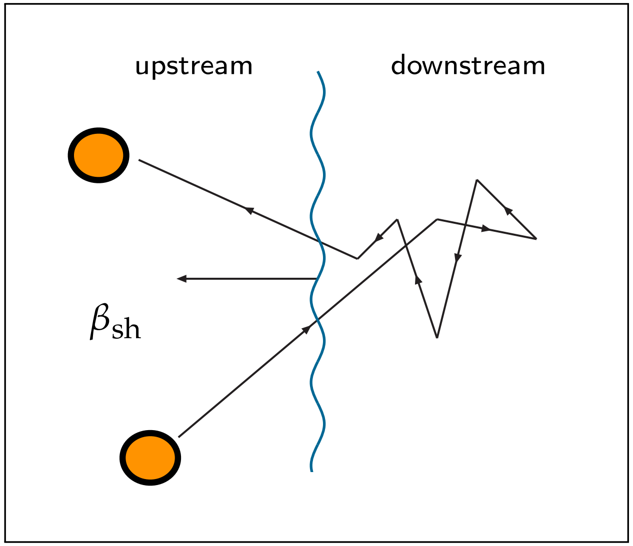
Let us assume to have plasma on both sides of the shock and an isotropic distribution of charged particles (obtained via diffusion in the magnetic field).

Charged particles (or cosmic rays) cross the shock back and forth multiple times, each time gaining energy.



Diffusive Shock Acceleration

A particle crosses the shock from upstream to downstream and back.



The energy in the downstream region in the shock frame ($\beta = V/c$)

$$E'_d = \gamma E_d (1 - \beta \cos\theta_d)$$

The elastic scattering in the downstream region conserves energy ($E'_d = E'_u$).The direction is isotropized.

The emitted energy in the lab frame will then be

$$E_u = \gamma E'_u (1 + \beta \cos\theta'_u)$$

Therefore the energy gain per cycle is

$$\frac{\Delta E}{E_d} = \frac{E_u - E_d}{E_d} = \gamma^2 (1 + \beta \cos\theta'_u)(1 - \beta \cos\theta_d) - 1$$

The energy gain is always positive since $\cos\theta_d < 0$ and $\cos\theta'_u > 0$.

Diffusive Shock Acceleration

The distributions of θ_d and θ'_u(relative to $v_{\rm sh}$) follow projection onto the shock

$$\frac{dn}{d\cos\theta_d} = -2\cos\theta_d(\cos\theta_d < 0) \quad \text{and} \quad \frac{dn}{d\cos\theta'_u} = -2\cos\theta'_u(\cos\theta'_u > 0)$$

Averaging each angle on its own distribution, we obtain

$$\langle\cos\theta'_u\rangle = \frac{2}{3} \quad \text{and} \quad \langle\cos\theta_d\rangle = -\frac{2}{3}$$

Hence the average energy gain is

$$\frac{\langle\Delta E\rangle}{E_d} = \gamma^2(1+\beta\langle\cos\theta'_u\rangle)(1-\beta\langle\cos\theta_d\rangle) - 1 \simeq \frac{4}{3}\beta$$

On average then $\boxed{\frac{\langle\Delta E\rangle}{E_d} \propto \beta}$ Acceleration at a planar shock is first order in shock velocity.

Each "encounter" (i.e., a pair of in and out crossings) results in an energy gain.

Exercise

Let us now proof that the diffusive shock acceleration leads to a power-law in the particle energy distribution.

Hint 1: Remember that the fractional change in kinetic energy at each crossing is β and assume that the particle crossing the shock has an initial energy E_0.

Hint 2: Assume an initial ensemble of N_0 particles. Assume that the particles will not continue to cross the shock indefinitely. The net momentum flux of the shocked gas downstream will carry them away after a certain time. Hence, there will be a probability P of remaining in the shock-crossing region after each crossing.

After n crossings, the particle will have energy

$$E = E_0 \beta^n$$

After n crossings, the particle number will be

$$N = N_0 P^n$$

$$\frac{\log(N/N_0)}{\log(E/E_0)} = \frac{\log(P)}{\log(\beta)} \quad \Rightarrow \quad N(E)dE \propto E^{-1+\frac{\log P}{\log \beta}}$$

Diffusive Shock Acceleration

From the exercise above, we have recovered a power law energy distribution

$$\boxed{N(E)dE \propto E^{-k}dE}$$

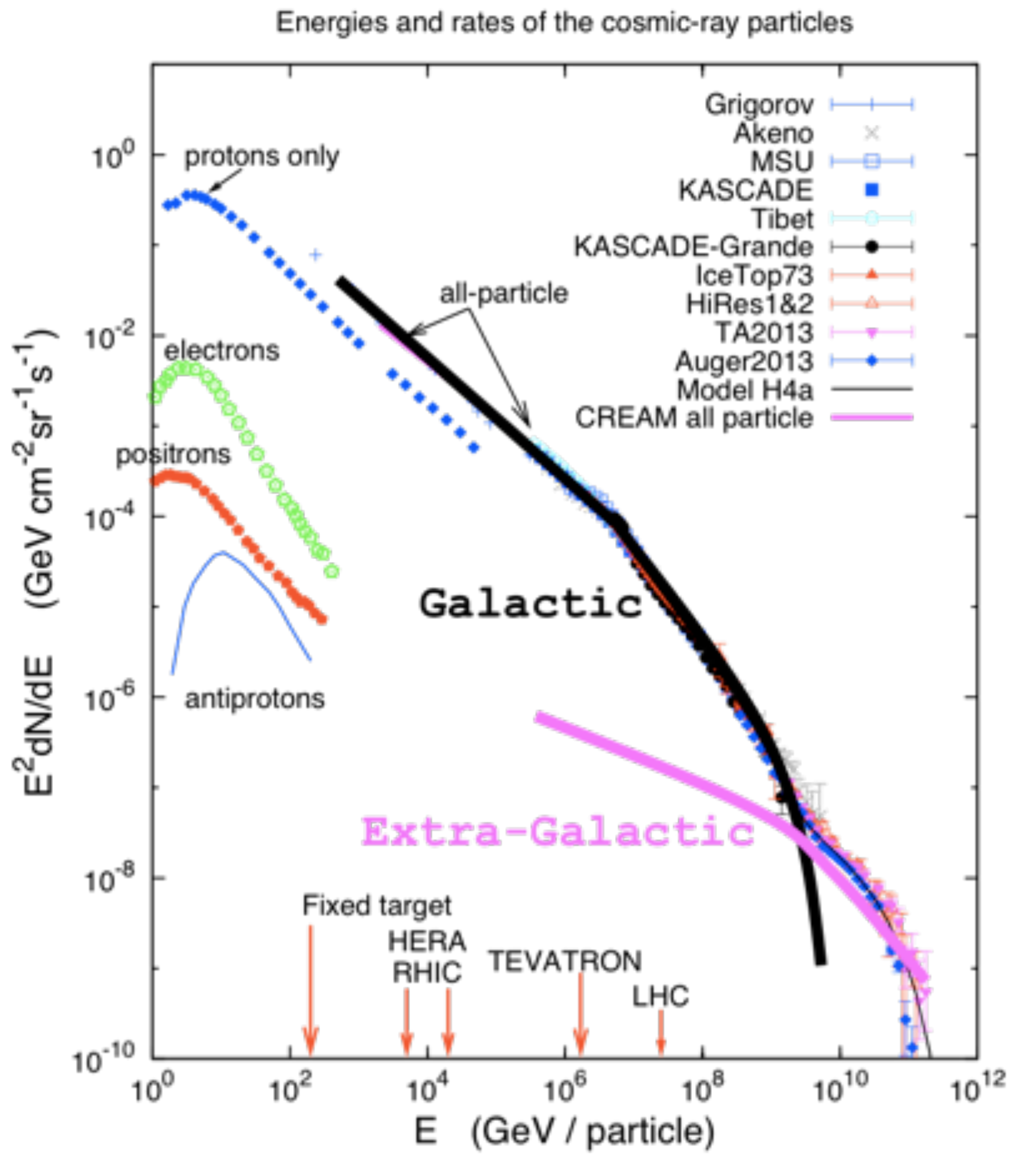
with $k = 1 - (\ln P / \ln \beta)$. It can be proved that

$$\frac{\ln P}{\ln \beta} = -1$$

therefore $N(E)dE \propto E^{-2}dE$.

This is close to the spectral index observed in cosmic rays and other cosmic accelerators such as AGNs. In reality the index is slightly steeper than 2 and this requires a more refined treatment.

Cosmic Ray Energy Spectrum



Cosmic rays include particles such as electrons, protons, and heavier nuclei. Their sources are not yet known, but we expect them to be both galactic and extragalactic.

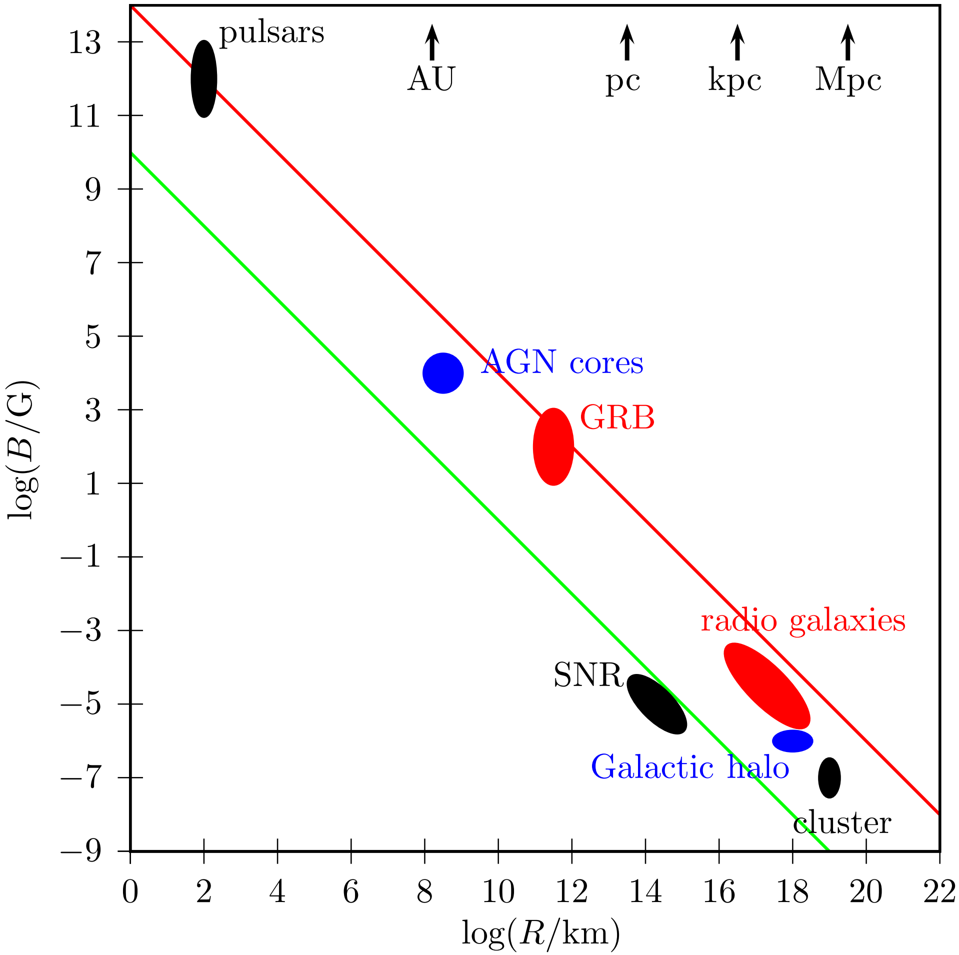
Image credit: https://masterclass.icecube.wisc.edu/en/analyses/cosmic-ray-energy-spectrum

Hillas Criterion

If a particle escapes the acceleration region, it will be unable to gain more energy. This imposes a limit on its maximum energy

$$E_{\mathrm{max}} = qBR$$

We need to demand that the Larmor radius of the particle does not exceed the acceleration region.



Hillas Plot

Sources above the red line accelerate protons up to 10^21 eV, sources above the green line accelerate iron up to 10^20 eV.

Image credit: Kachelriess, arXiv:astro-ph/0801.4376

Take Home Messages

- A shock occurs when a disturbance moves through a medium faster than the medium sound speed.
- The jump conditions describe the variation of the quantities characterizing the shock in its proximity.
- Multiple scatterings of particles across the shock lead to a power law in the particle energy distribution.
- The maximum energy can be estimated through the Hillas criterion.

References

- Longair, High Energy Astrophysics, Cambridge University Press.
- Blundell and Blundell, Concepts in thermal physics, Oxford University Press.
- Blandford and Eichler, Physics Reports, Volume 154, Issue 1, p. 1-7 (1987).
- Gaisser, Engel, Resconi, Cosmic Rays and Particle Physics, 2nd edition, Cambridge University Press.
- High Energy Radiation from Black Holes, Dermer and Menon, Princeton Series in Astrophysics.

An example is: energy injected into the ambient medium suddenly by a supersonically-moving solid object: the sonic boom. The air ahed of a supersonic aircraft does not have time to move out of the way before the shot hits it; temperature and density are suddenly (almost discontinuously) increased as the shock passes

This is similar to the blast wave of a supernova. The huge energy injection heats cold interstellar gas suddenly as the blastwave moves outward.

To have a better understanding of the change of conditions at the shock, we transform into the instantaneous rest frame of the shock itself.

Depicted in the composition are: a bow shock around the very young star, LL Ori, in the Great Orion Nebula (upper row, left image); shock waves around the Red Spider Nebula, a warm planetary nebula (upper row, central image); very thin shocks on the edge of the expanding supernova remnant SN 1006 (central row, left image); artist's impressions of the bow shock created by the Solar System as it moves through the interstellar medium of the Milky Way (upper row, right image) and of Earth's bow shock, formed by the solar wind as it encounters our planet's magnetic field (central row, right image); shock-heated shells of hot gas on the edge of the lobes of the radio galaxy Cygnus A (lower row, left image); a bow shock in the hot gas in the merging galaxy cluster 1E 0657-56, also known as the 'Bullet Cluster'.

Shock Physics

In order to characterize the shock, the Mach number is adopted

$$M = \frac{u_u}{c_u}$$

the Mach number is the ratio between the speed of the shock and the speed of sound in the unperturbed medium. The jump conditions above have been derived for $M \gg 1$.

Fermi Acceleration (2nd order)????

Deriv exponent?

A mechanism by which particles obtain a power-law energy distribution was originally investigated by Fermi to explain highest energy cosmic rays. In the 70s it was demonstrated to be an efficient mechanism for the production of synchrotron-emitting electrons in AGN

Characteristics of Shocks

If we use the ideal gas law $P = nk_B T$ to characterize our shocked gas, then for a gas with mean particle mass m, the ideal gas law becomes

$$P = \frac{\rho}{m} k_B T$$

From here, we obtain $T_d = \frac{3}{16} \frac{m v_u^2}{k_B}$

Can we use shocks to accelerate particles to ultra-relativistic energies?

Fermi's 'ping-pong' acceleration process

Fermi acceleration: simple first-order

where alpha is some constant. Such distributions are called non-thermal because they vary significantly from Maxwellian distribution

suppose we inject a particle of mass m travelling at a mildly relativistic speed v between two scattering surfaces separated by distance L and approaching each other with speed V <<c. If the particle collides alternately head-on with each scatterer, it will gain energy at a rate

dE/dt = rate of collisions x energy change per collisions

If we rely on the fact that the particle is relativistic, the momentum increase in each collision is (gamma m V) so the energy increase is E=p c.

Fermi's 'ping-pong' acceleration process

Fermi acceleration: simple first-order

dE/dt = EV/L where in the last step we're approximating v=c.

dE/dt = E/tau where tau is some timescale, in this case the crossing time between scatterers. the particle's energy will increase exponentially.

If a population of particles with slightly different initial energies enter a region where this sort of scattering occurs, the slightly higher energy particles will be accelerated to much greater energies than the slightly lower energy ones.

This situation is an idealization but it shows how to proceed. If we can find a mechanism for multiple back-and-forth accelerations, we can get a power-law distribution. The most significant caveat is that we don't know how to make particles mildly relativistic to start with, in order to make use of E=p c.

Real Fermi accelerationat shocks

The way to do this in practice is to make use of shocks: in the jets and at the hotspots of radiosource and at the blast-wave shock in a supernova.

Let's take the case of a shock propagating into 'cold' gas at speed v_u. In the frame of the shock, we have our familiar results: we see unshocked gas ahed of us approaching at speed v_u, and shocked gas streaming away behind us at v_d = 1/4 v_u.

Now consider electrons initially at rest in the unshocked gas frame. They see the shock approaching at v_u but they also see the hot shocked gas approaching at 3/4 v_u. As they cross the shock they are accelerated to a mean speed of 3/4 v_u, as viewed from the frame of the unshocked gas, and are also thermalized to a high temperature.

Consider what would happen if, say as a result of its thermal motion or tangled magnetic field, an electron is carried back over the shock front. With respect to the frame it has just come from-the shocked gas frame-it is once again accelerated by 3/4 v_u.

So we have a system of symmetric head-on collisions that we can use to generate a power-law energy distribution.

Real Fermi accelerationat shocks: calculation

Now we want to demonstrate explictly that this gives a power-lay in electron energies

Let us say that the fractional change in kinetic energy at each crossing is beta. After n crossings, a particle with initial energy E_0 will have energy E=E_0 beta^n.

The particles will not continue crosing the shock indefinitely. the net momentum flux of the shock gas downstream will carry them away in due course. So let us call P the probability of remaning in the shock-crossing region after each crossing. Then after n crossings there will be N=N_0 P^n of the original N_0 electrons left.

we can eliminate n from these expressions to find

log(N/N_0)/log(E/E_0) = log P/log beta

which gives
N/N_0 = (E/E_0)^(log P/log beta)

If we get the differential form of this we get

N(E)dE = const E^(-1+ log P/log beta) ---> N(E)dE propto E^-k dE

Real Fermi accelerationat shocks:power law

The detailed derivation of the power-law index is not examinable.

The result which you should remember is that it can be shown that

ln P/ln beta = -1

Hence the power-law index is -2, i.e., N(E)dE propto E^-2 dE

which is close to-but not quite-the spectral index observed in cosmic rays, and as we will see over the course of the next few lectures it closely underlies the spectrum of sync radiation in AGN.

Much work is still done to build models in which the index is slightly steeper than 2 as is observed in the real cosmic ray energy spectrum.