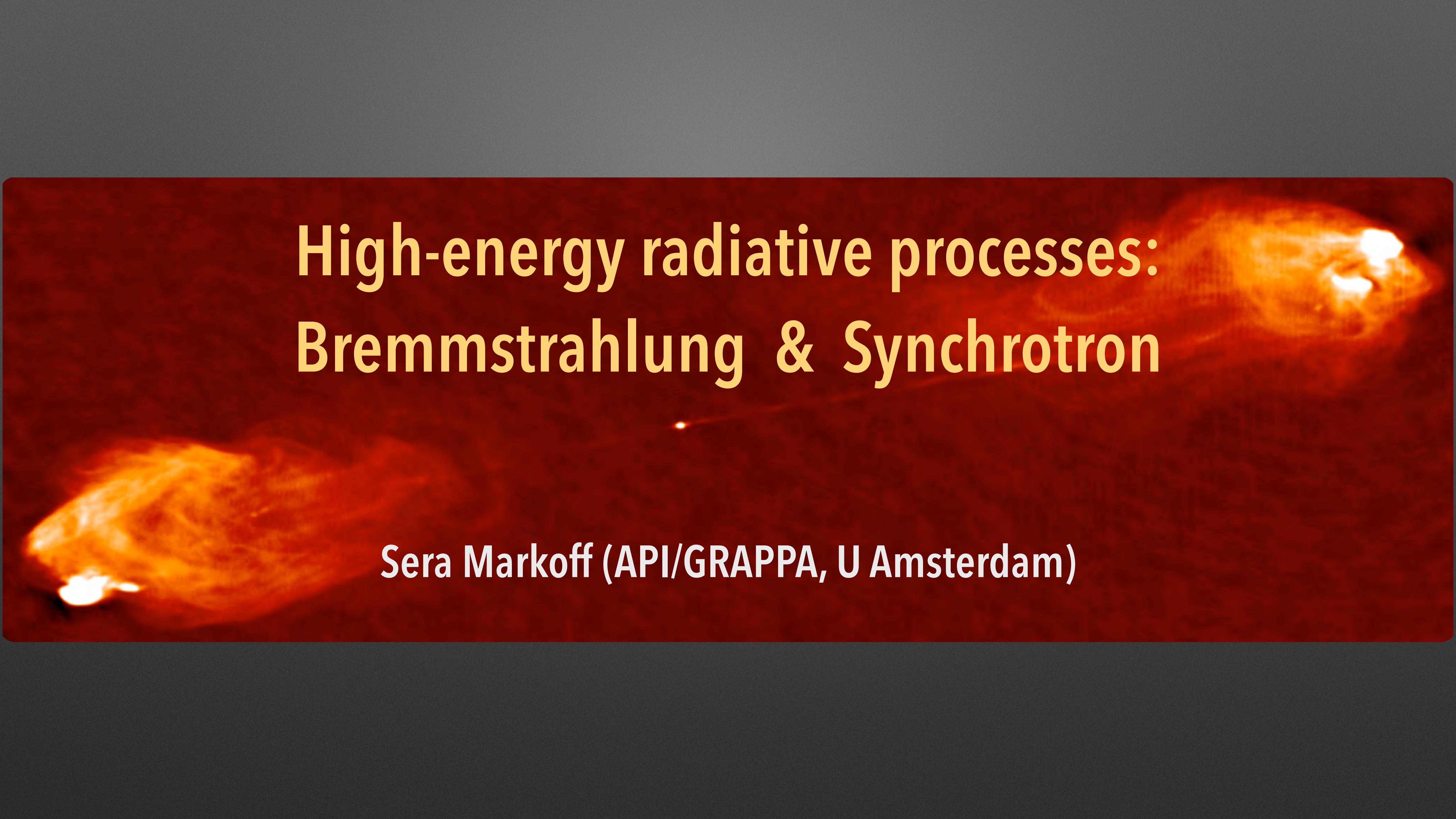


Relativity Processes I





High-energy radiative processes: Bremmstrahlung & Synchrotron

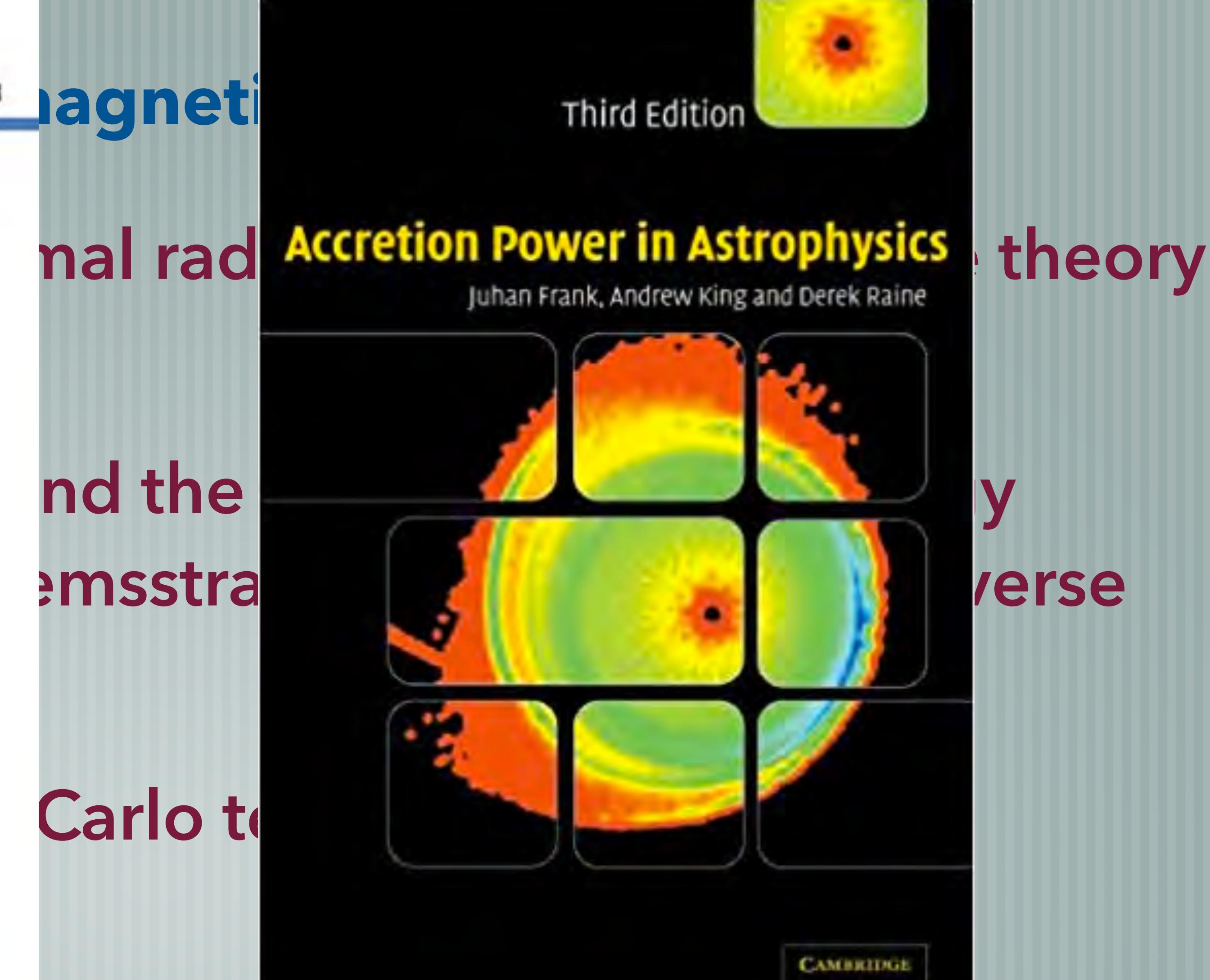
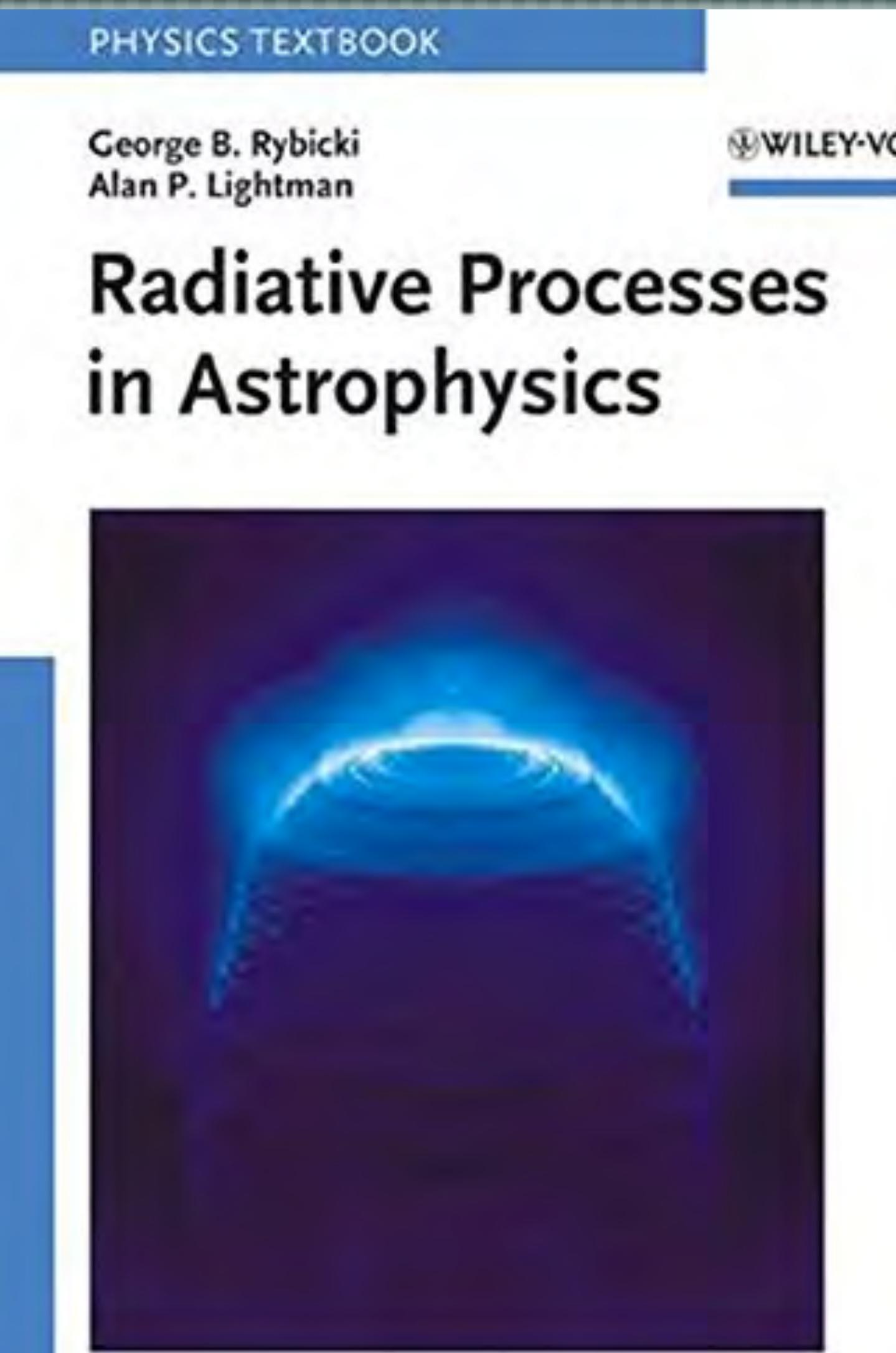
Sera Markoff (API/GRAPPA, U Amsterdam)

What Sebastian and I will try to cover in 4 hours (!?!)

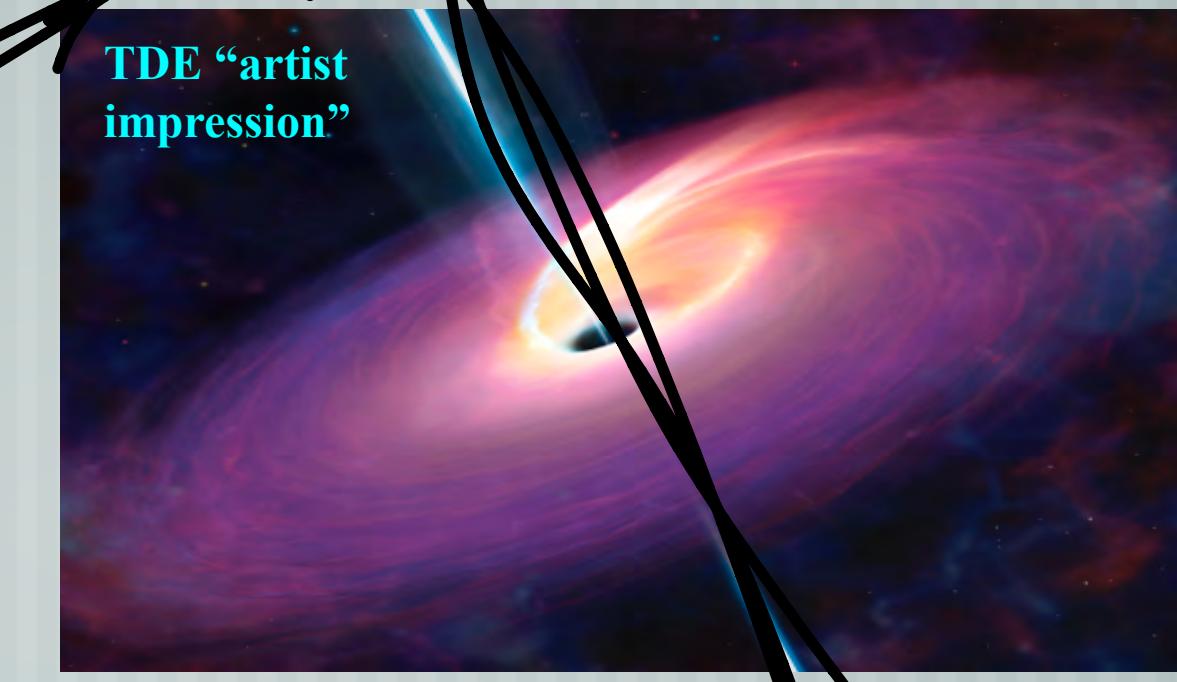
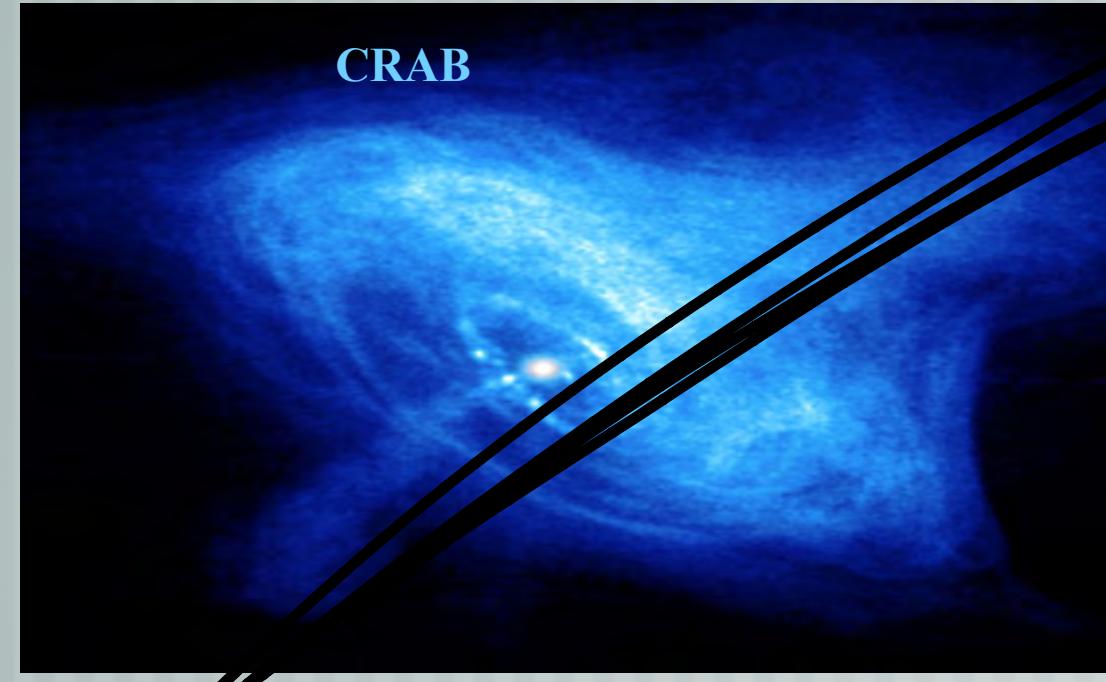
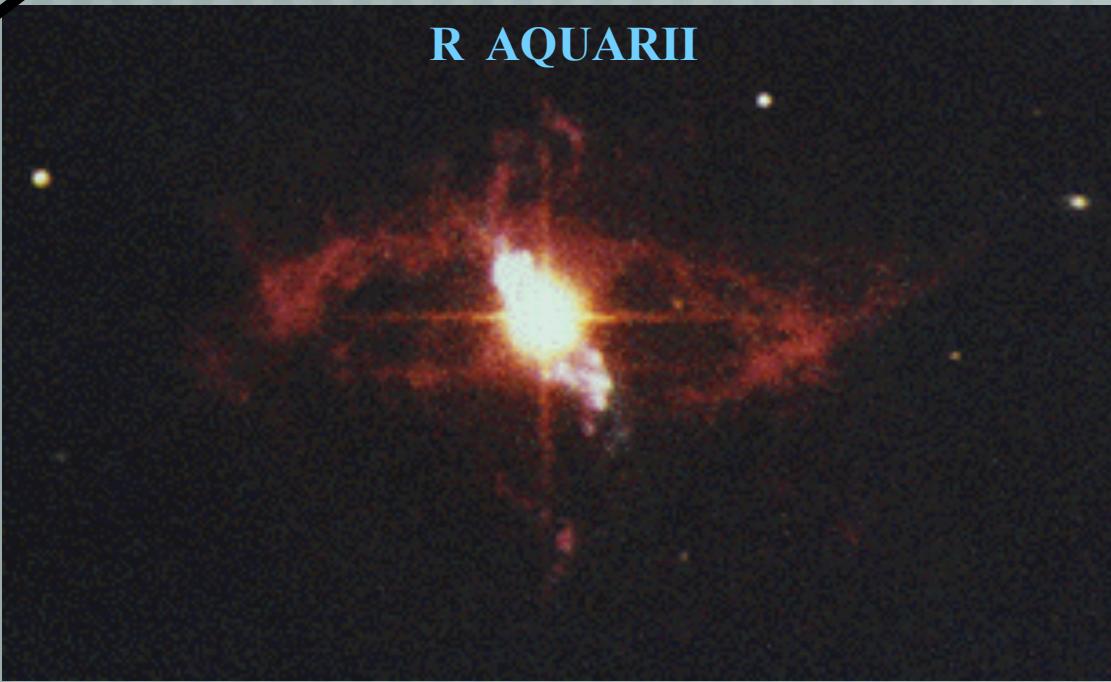
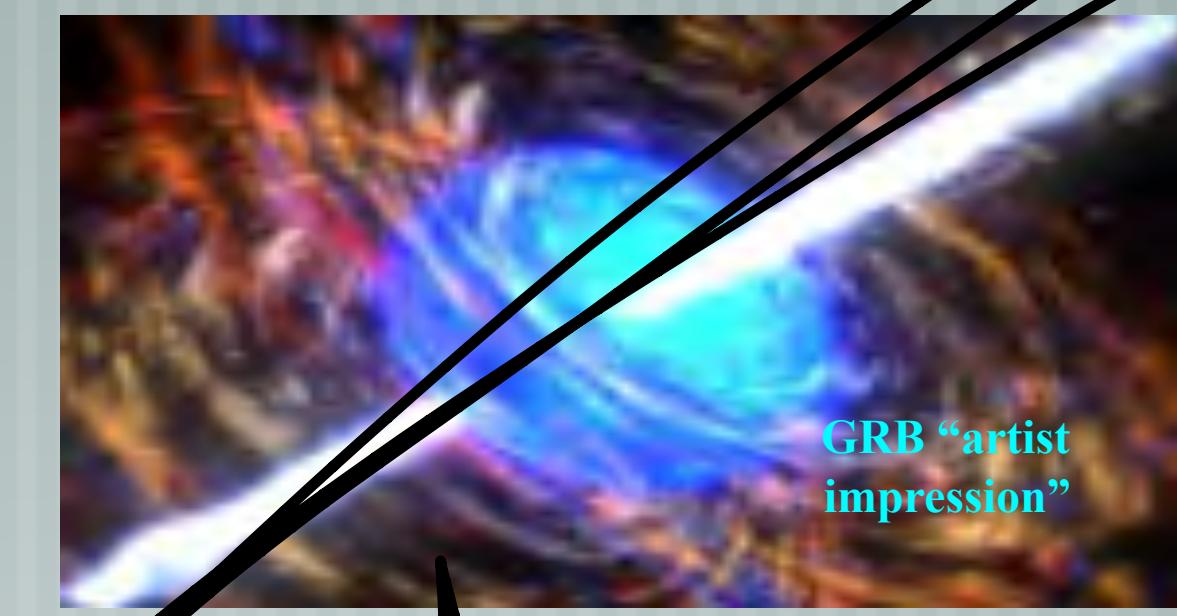
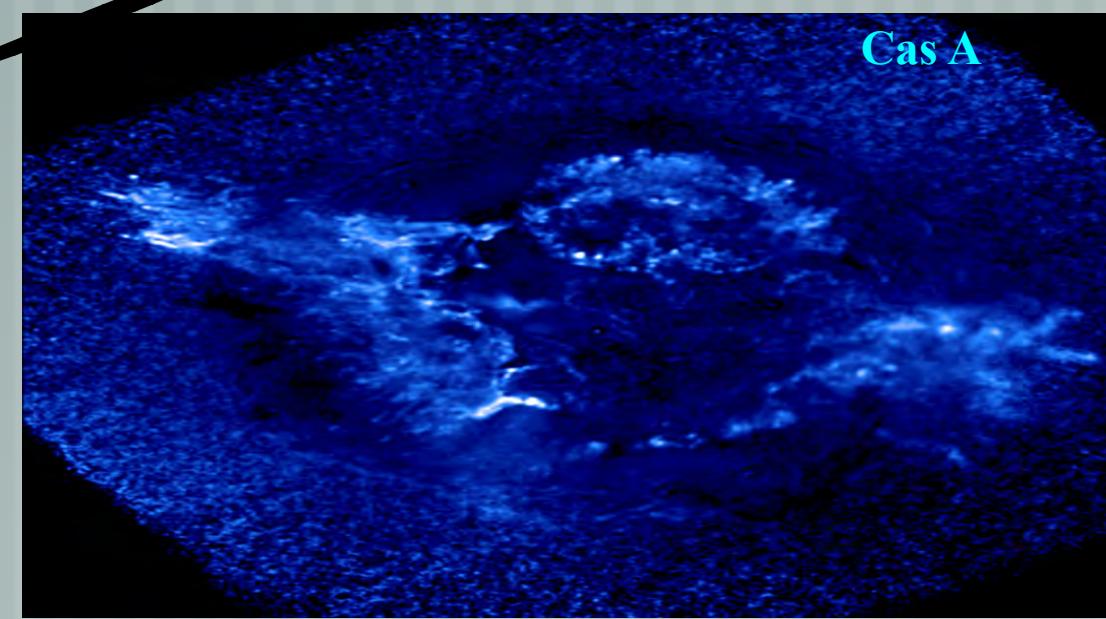
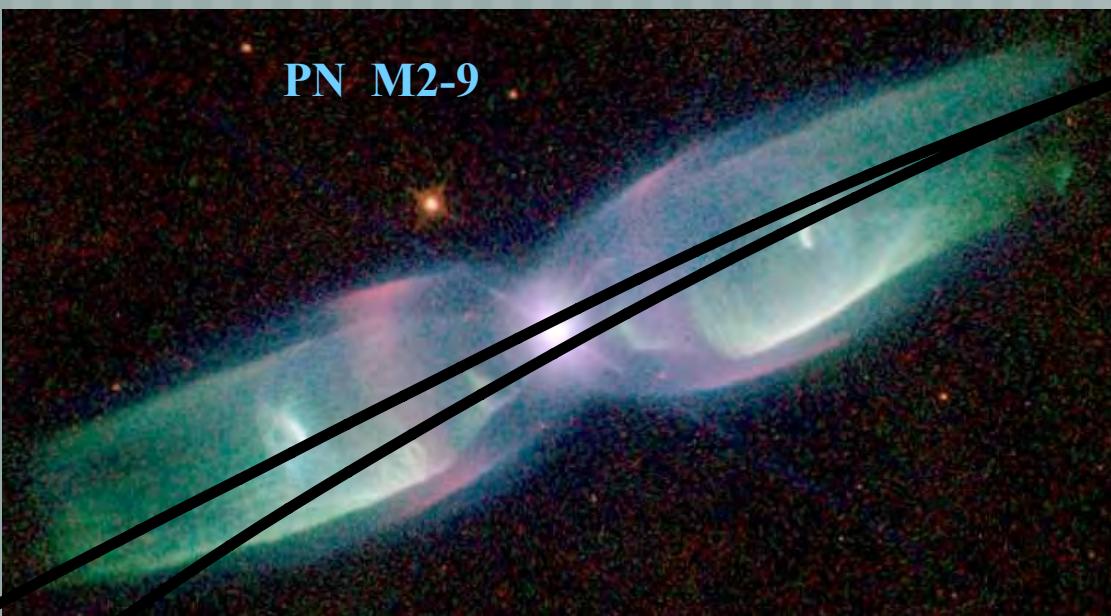
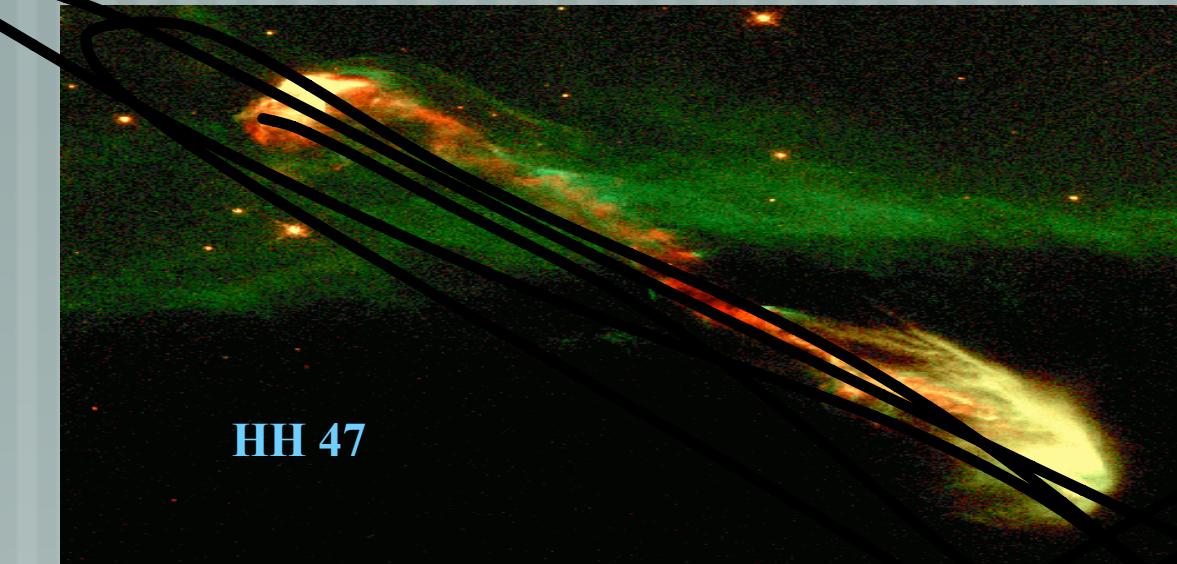
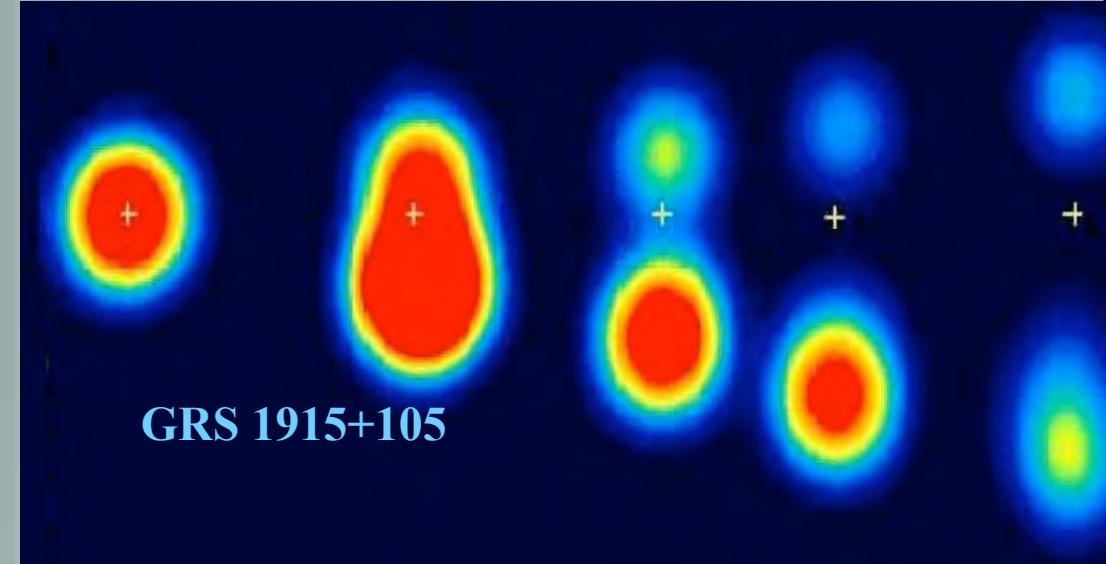
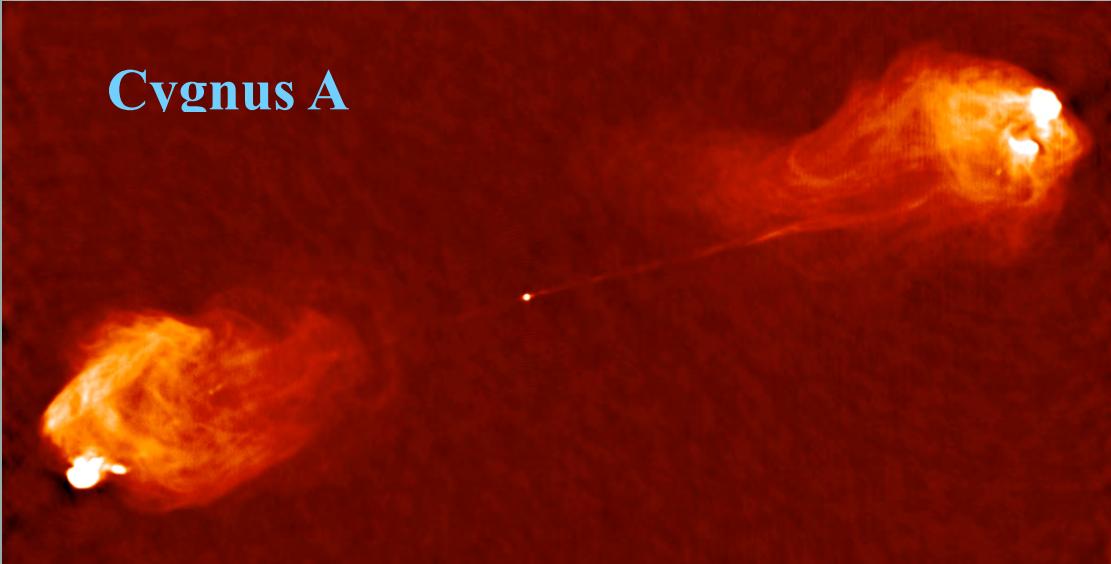
High-energy electromagnetic radiation

- * Introduction to nonthermal radiation and some of the theory behind it
- * The basic concepts behind the three main high-energy radiation processes: bremsstrahlung, synchrotron, inverse Compton
- * Experience with Monte Carlo technique for inverse Compton

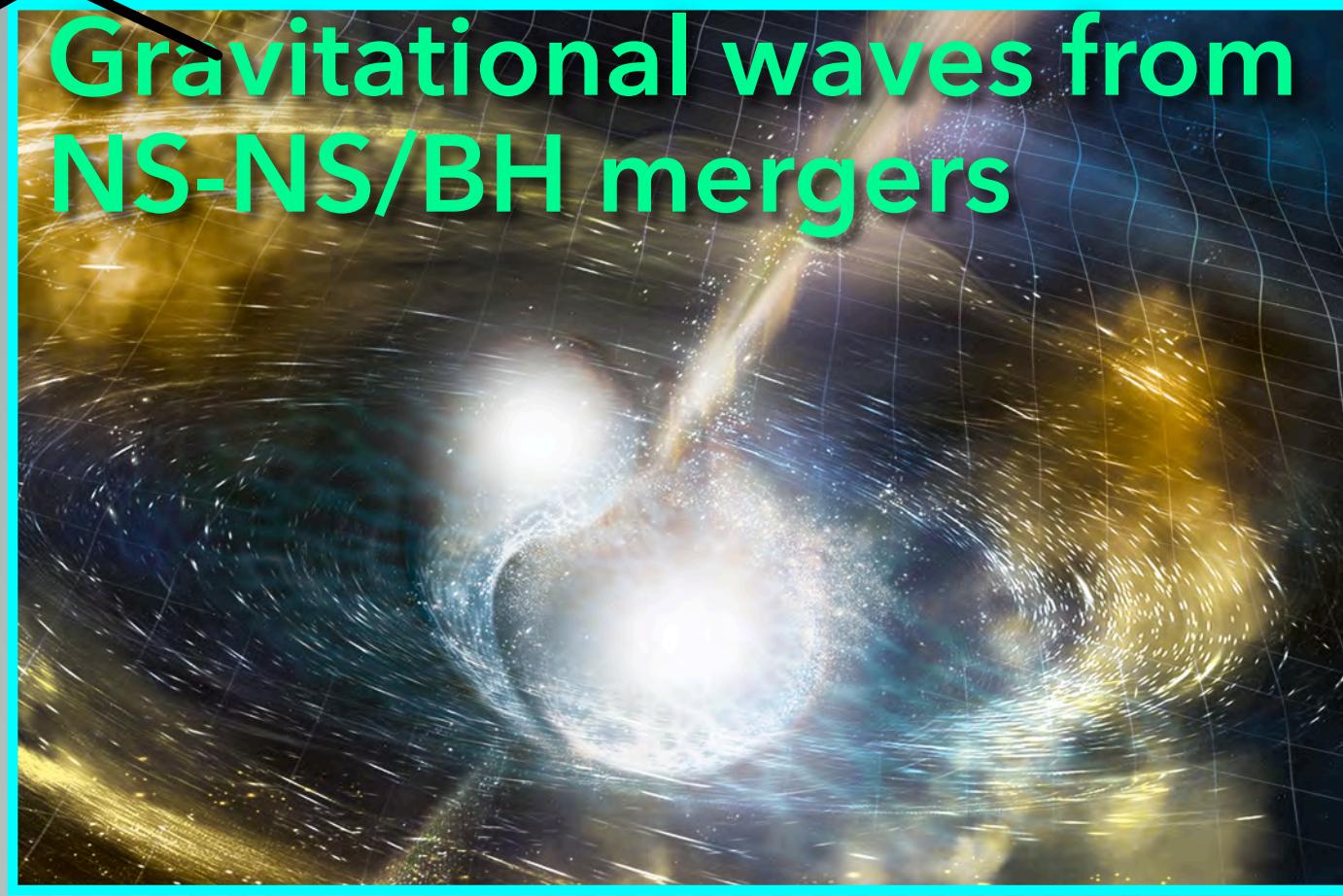
What Sebastian and I will try to cover in 4 hours (!?!)



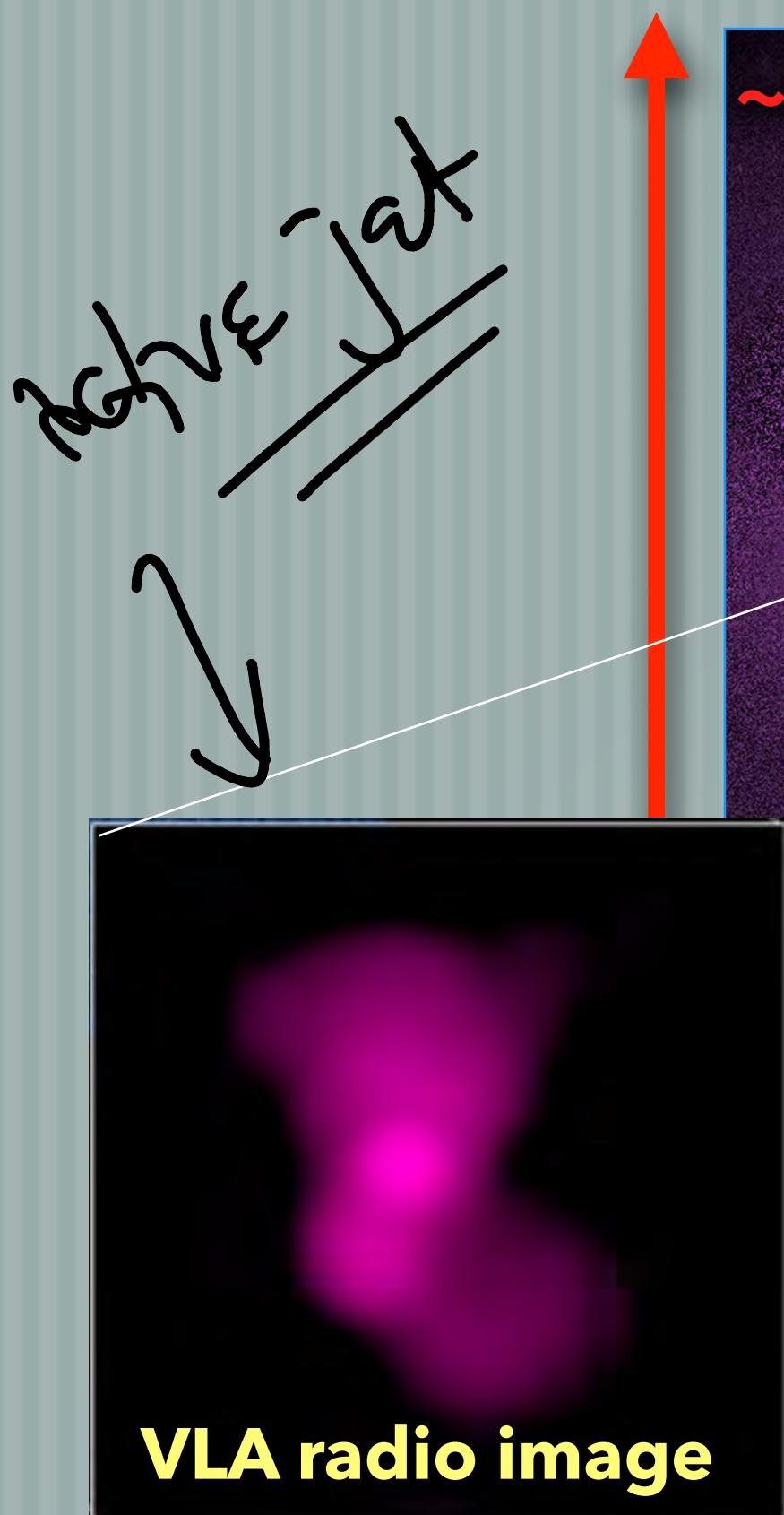
Cast of Characters



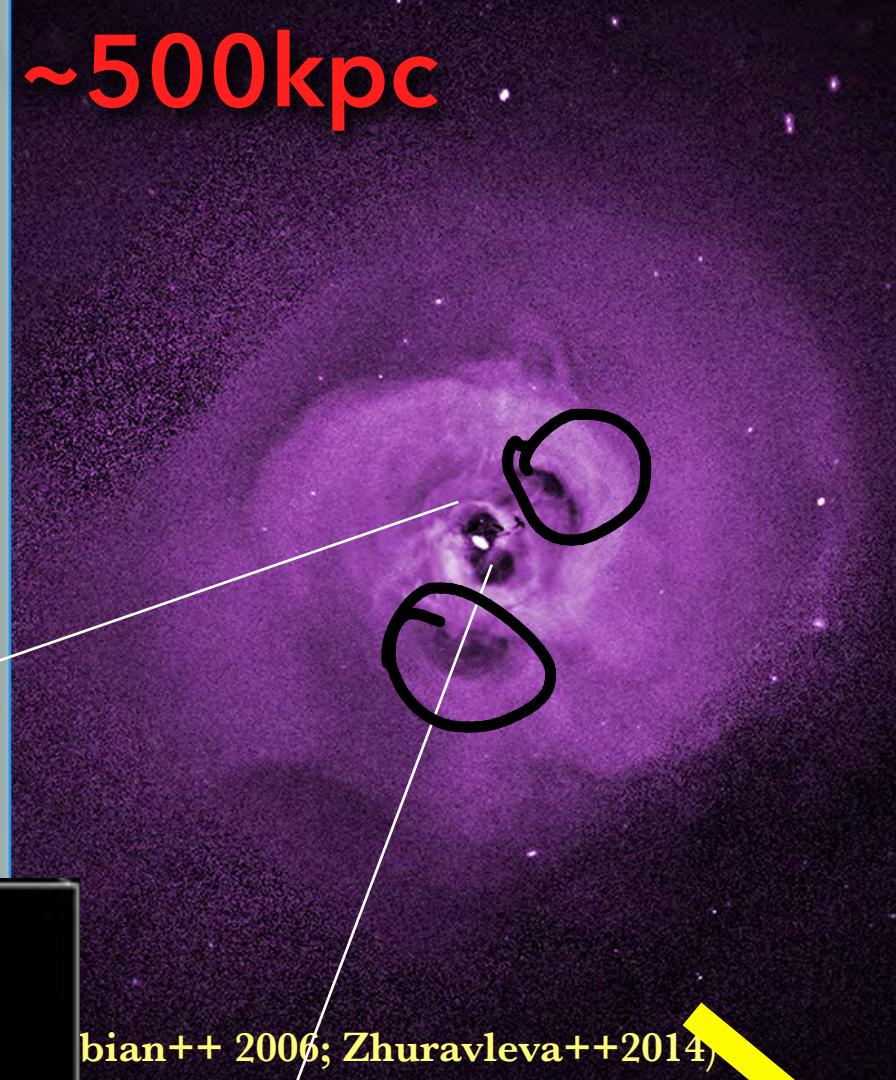
Gravitational waves from
NS-NS/BH mergers



We need to understand how accretion works in order to understand...



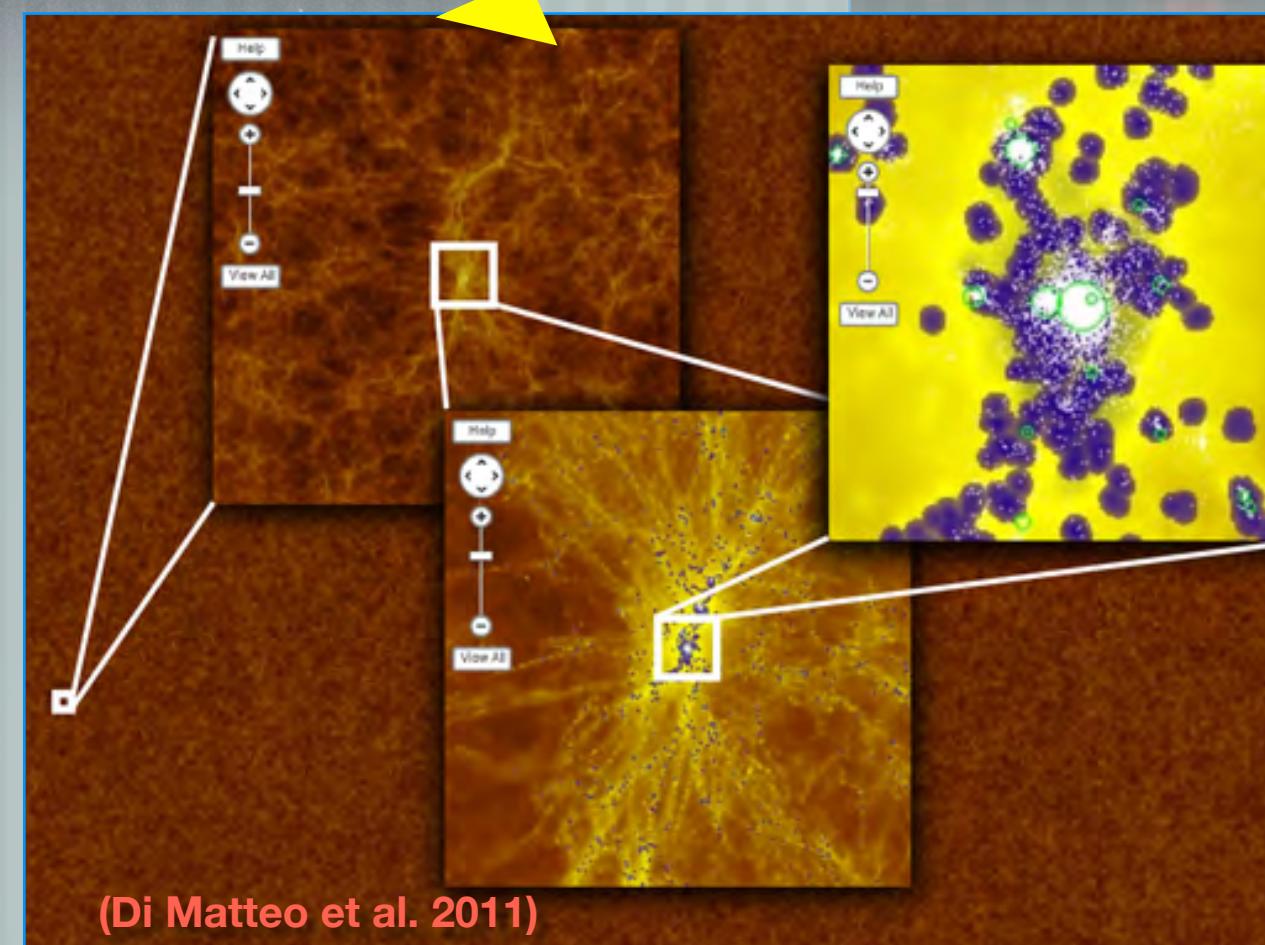
GALAXY EVOLUTION/
AGN FEEDBACK



~500kpc

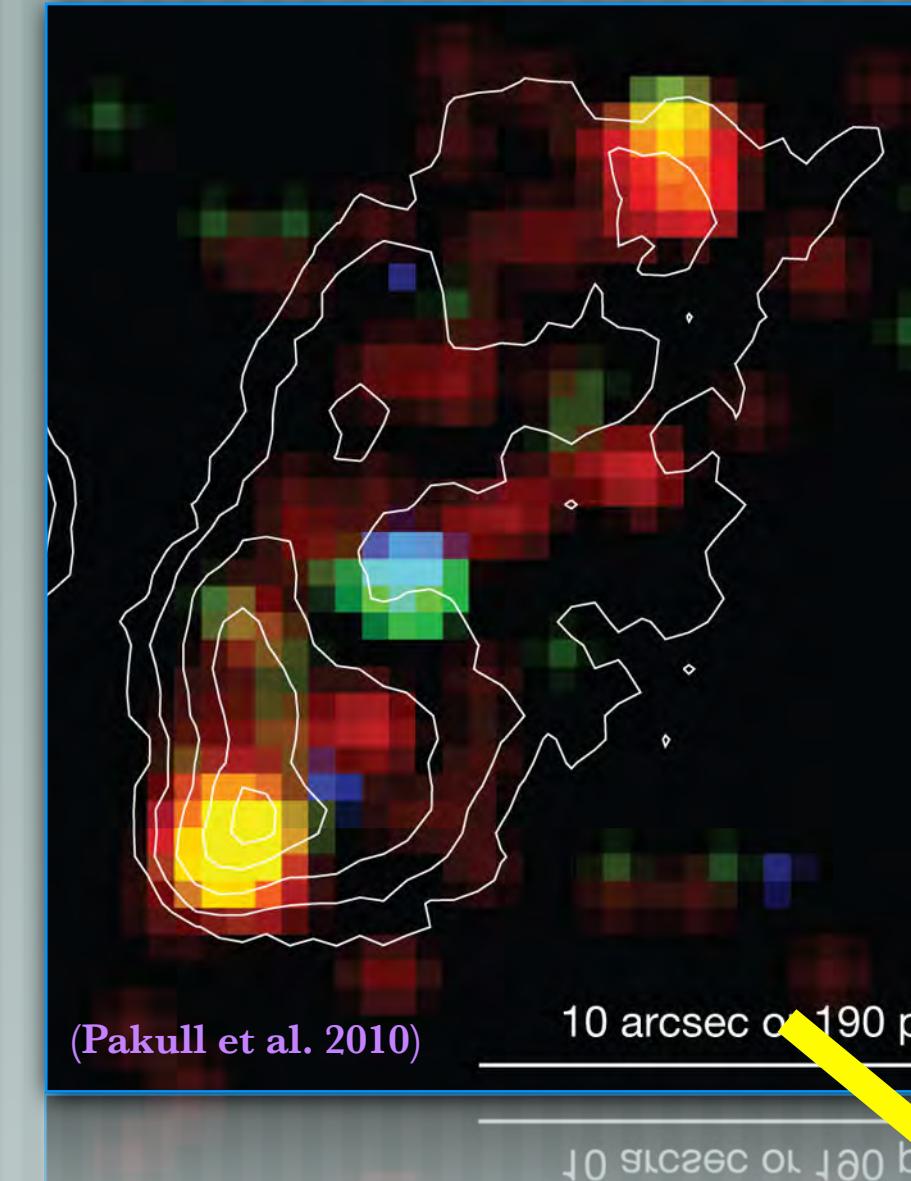
bian++ 2006; Zhuravleva++2014)

Cosmological
Simulations:



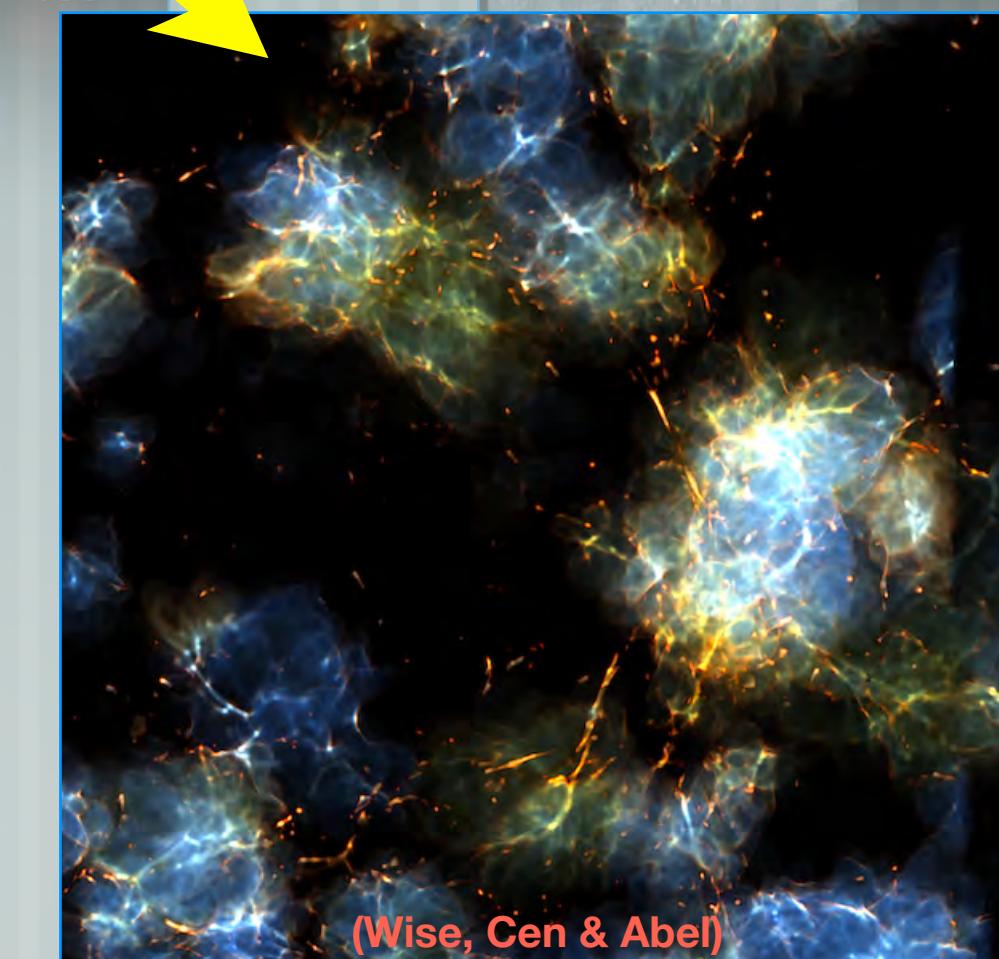
(Di Matteo et al. 2011)

extra-galactic
Shocks
IONIZATION OF BH
SURROUNDING GAS



(Pakull et al. 2010)

10 arcsec or 190 pc

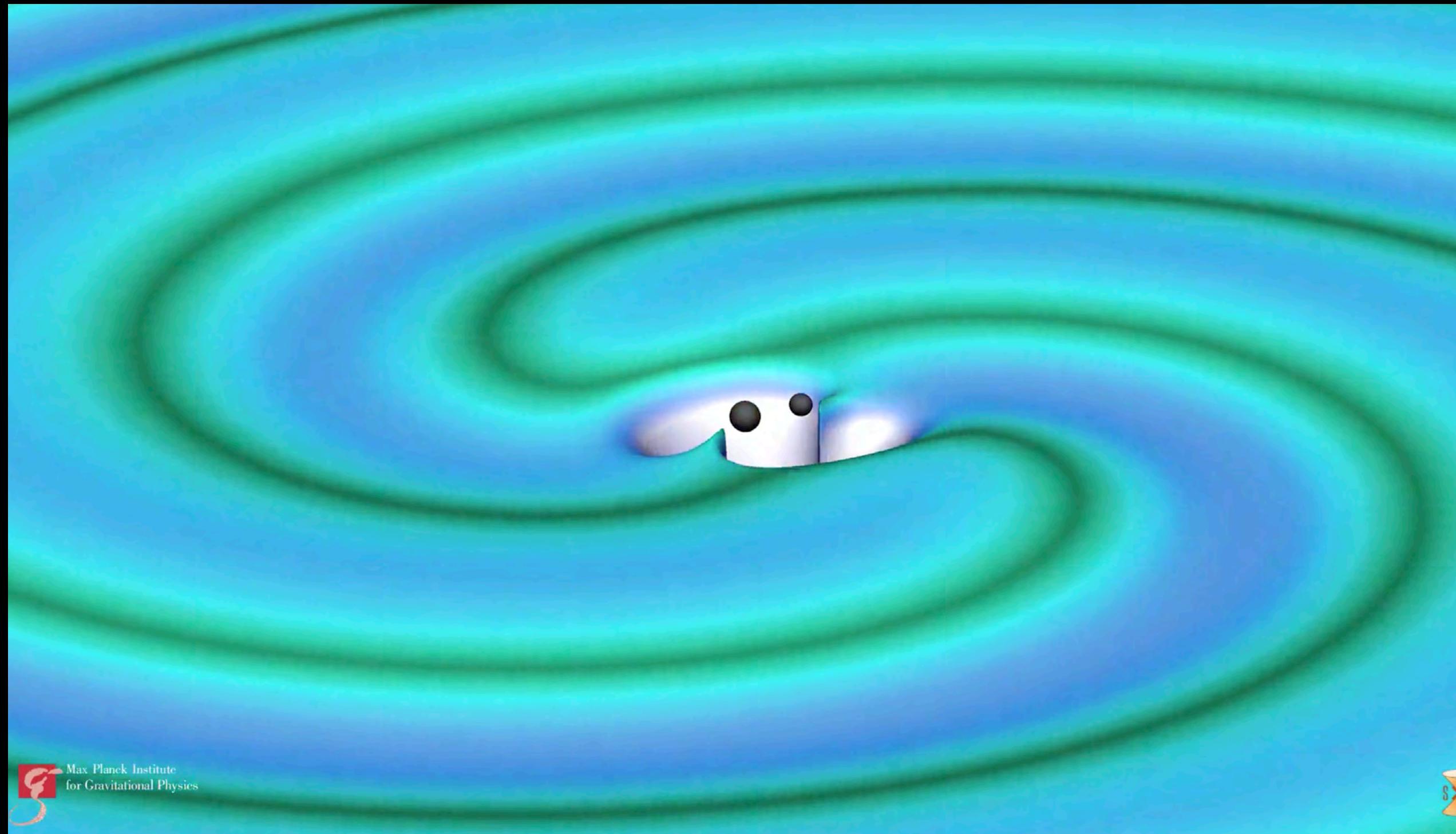


(Wise, Cen & Abel)

HIGH-ENERGY PARTICLE
ACCELERATION

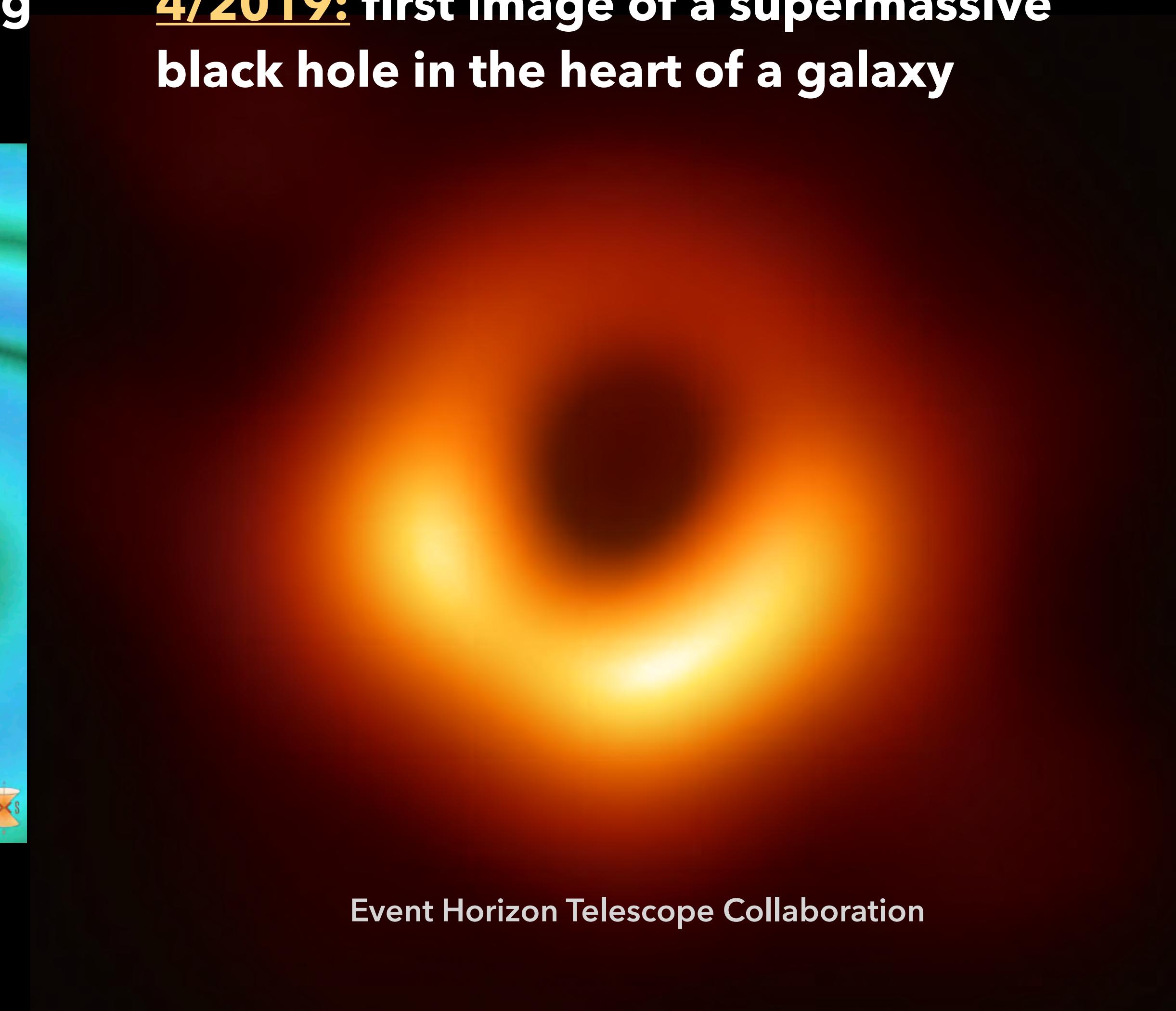
The new revolution in (astro)physics: "seeing" black holes

2015: discovery of gravitational waves from merging black holes (2017 Physics Nobel Prize)



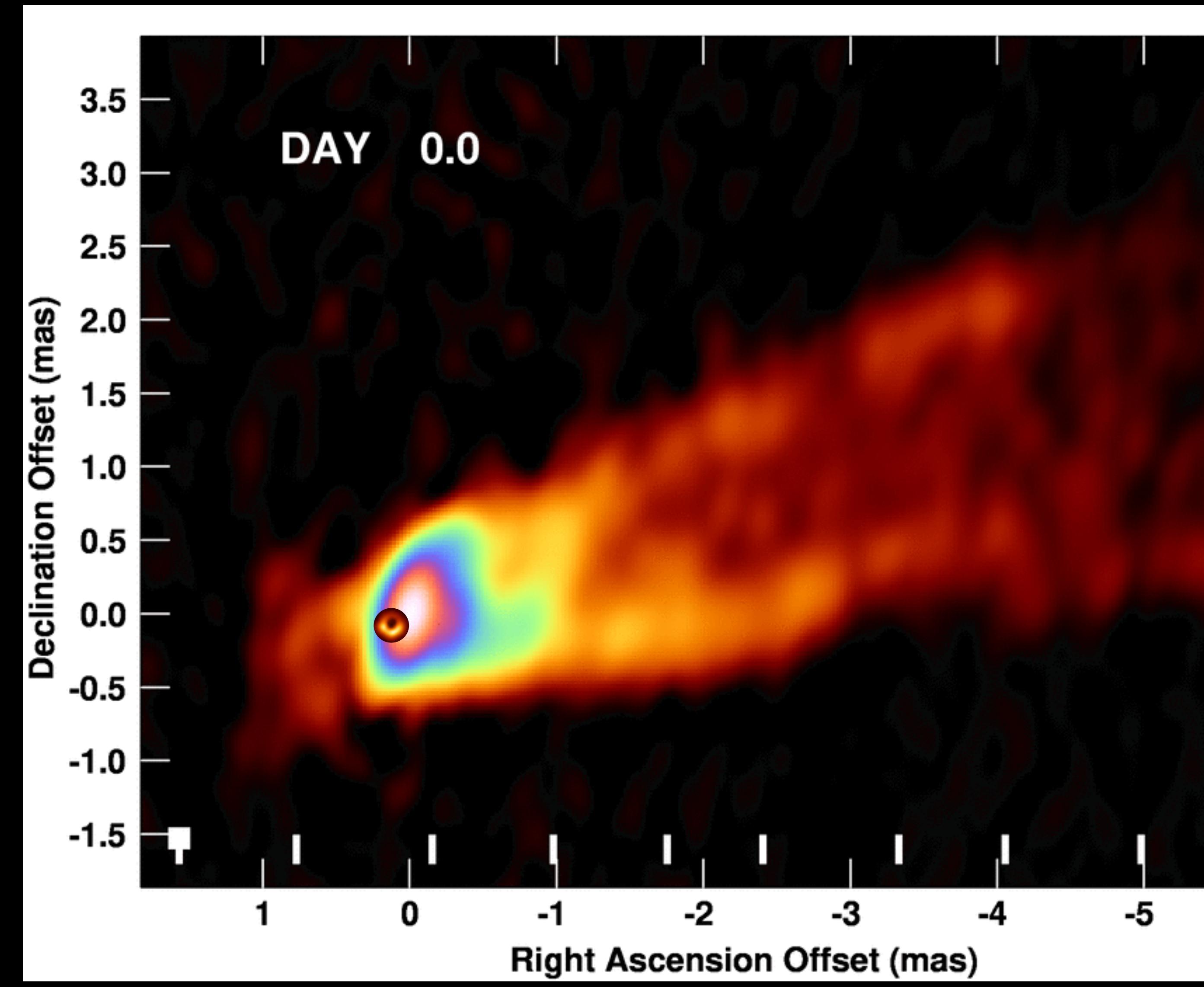
LIGO/VIRGO collaboration

4/2019: first image of a supermassive black hole in the heart of a galaxy

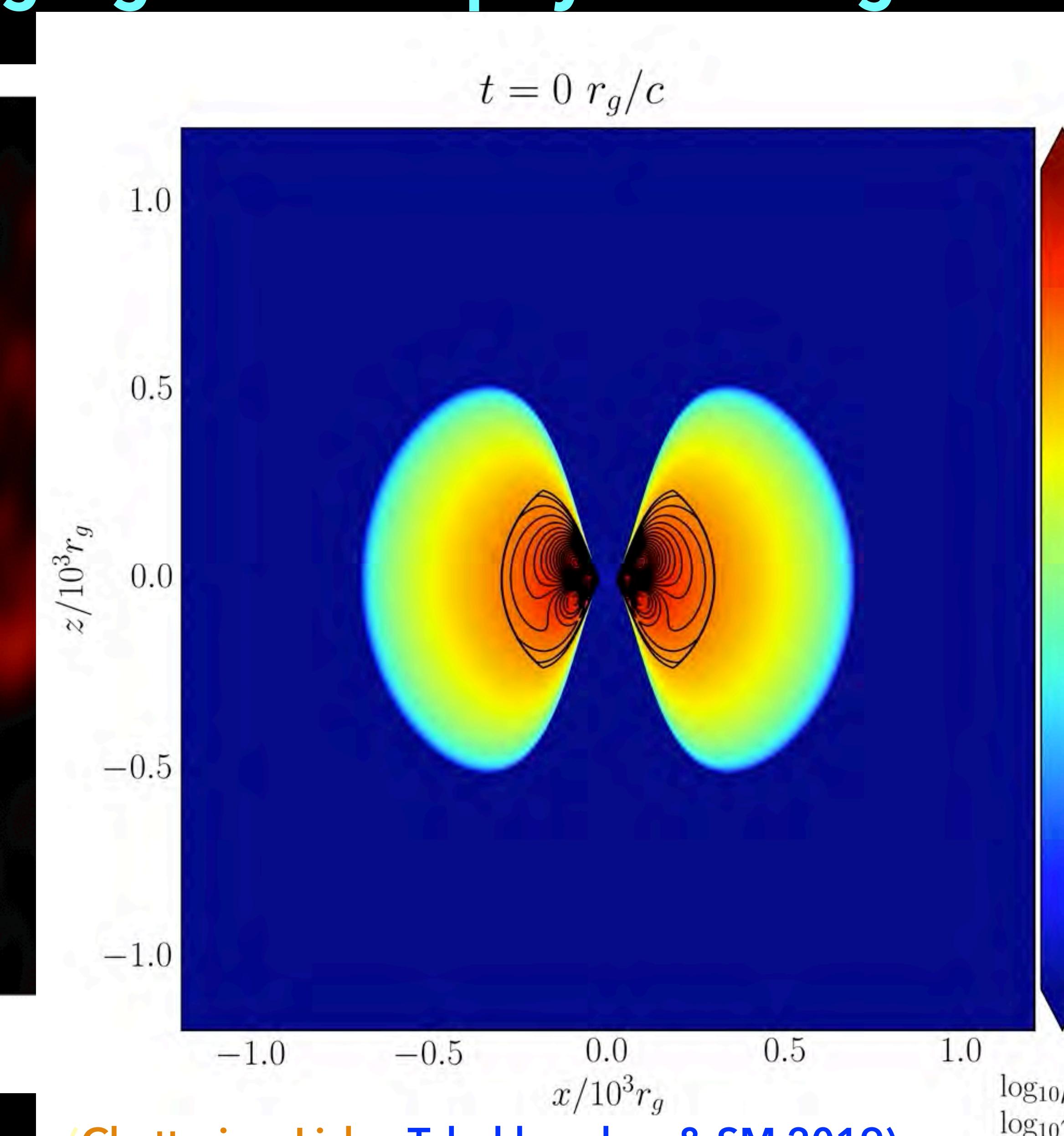


Event Horizon Telescope Collaboration

Closing in on the dynamics, but lagging on microphysics of light

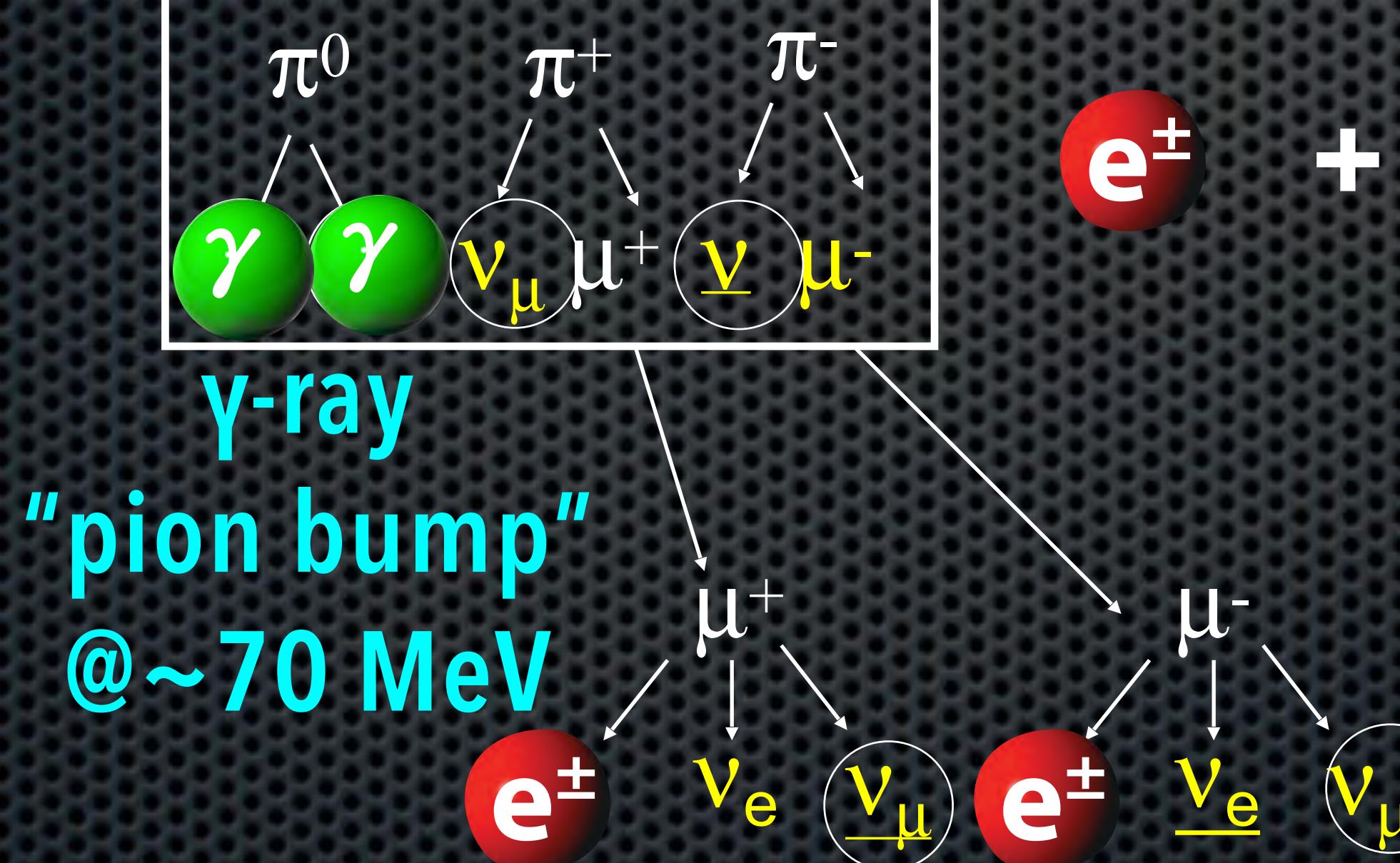
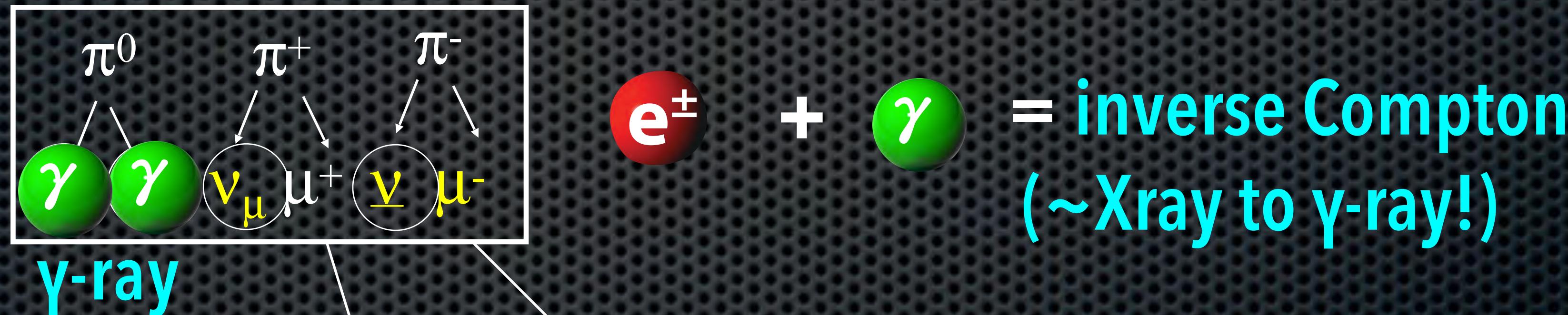
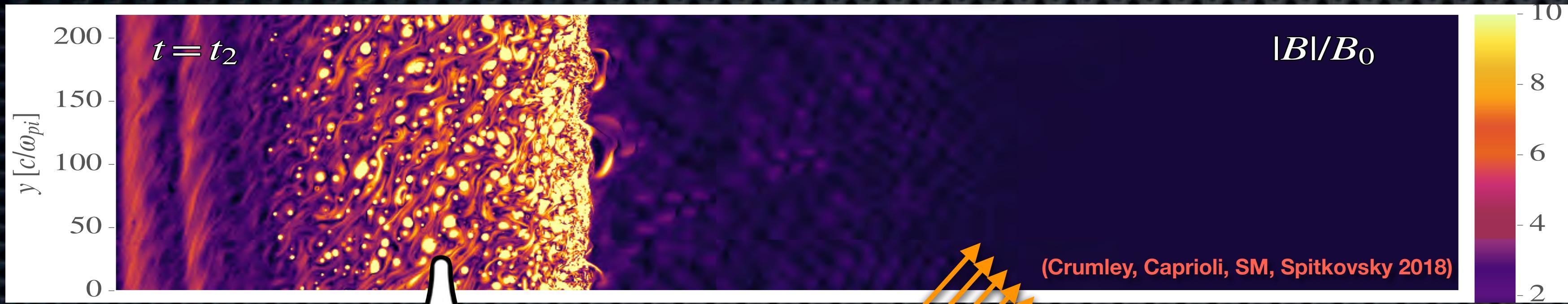


(M87 inner jet, Walker++2007/2008)

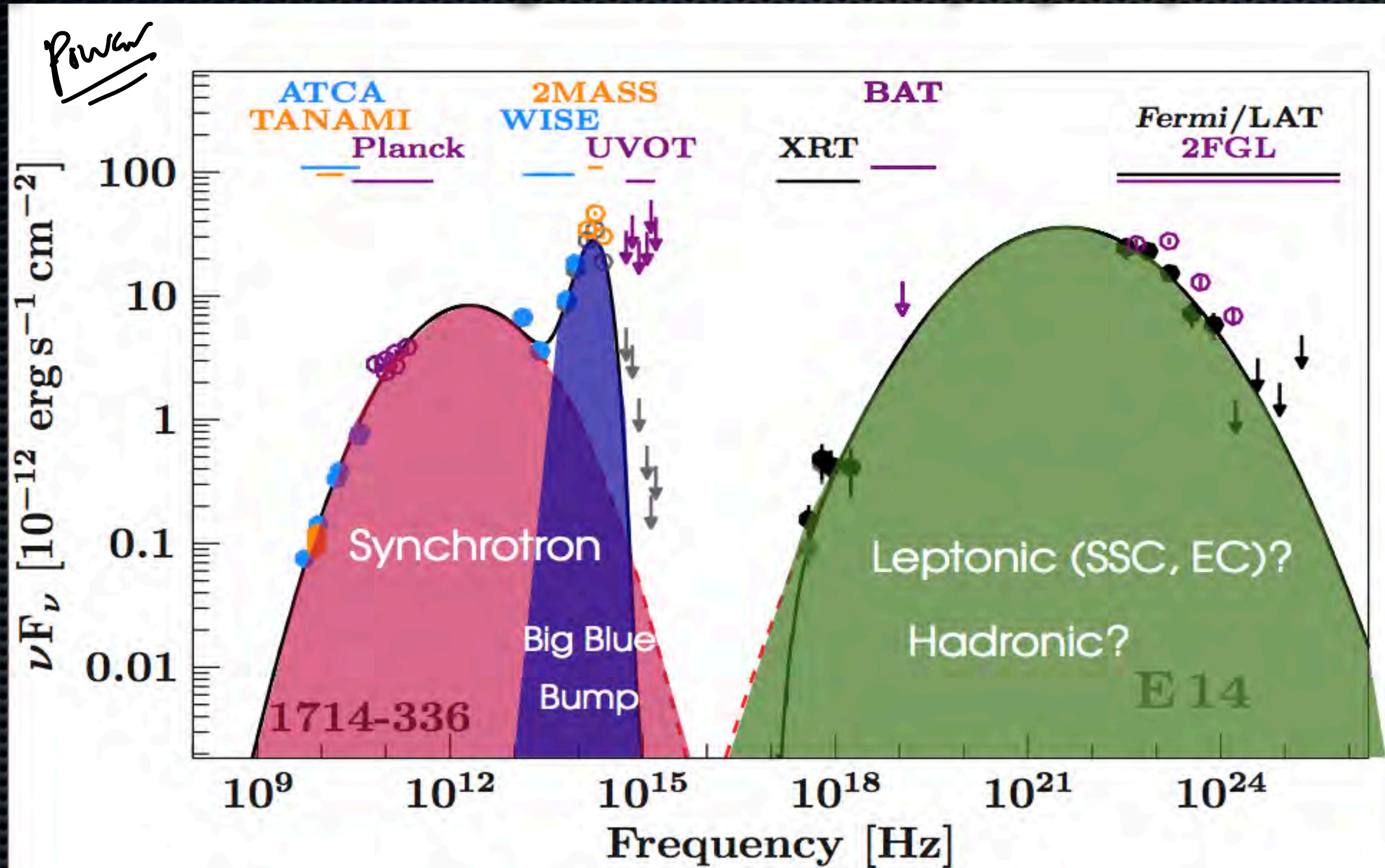


(Chatterjee, Liska, Tchekhovskoy & SM 2019)

'Nonthermal' emission traces particle acceleration



Complex (and thus degenerate) interplay of processes



A quick primer on important properties of light

Radiative transfer: absorption/scattering & optical depth

In target medium of particle density n (cm^{-3}), with cross section (absorption/scattering area) σ_v (cm^2):

"absorption coefficient" $\propto_v \equiv n\sigma_v$ $[\propto_v] = \frac{1}{\text{cm}}$ "Skin depth" =

$\tau =$ "optical depth" $\tau_v(s) = \int_0^s \propto_v(s') ds' = \int n(s') \sigma_v ds'$

if homogeneous, $\tau \equiv n\sigma_v s = \propto_0 s$

In RT $e^{-\tau}$ | "e-folding" $\tau_0 \approx 1 \implies s = \frac{1}{\propto_0} = \frac{1}{n\sigma_v} = \frac{\text{mean free path}}{\text{path}}$!

$\tau \gg 1 \rightarrow$ Thick

$\tau \ll 1 \Rightarrow$ opt. Thin

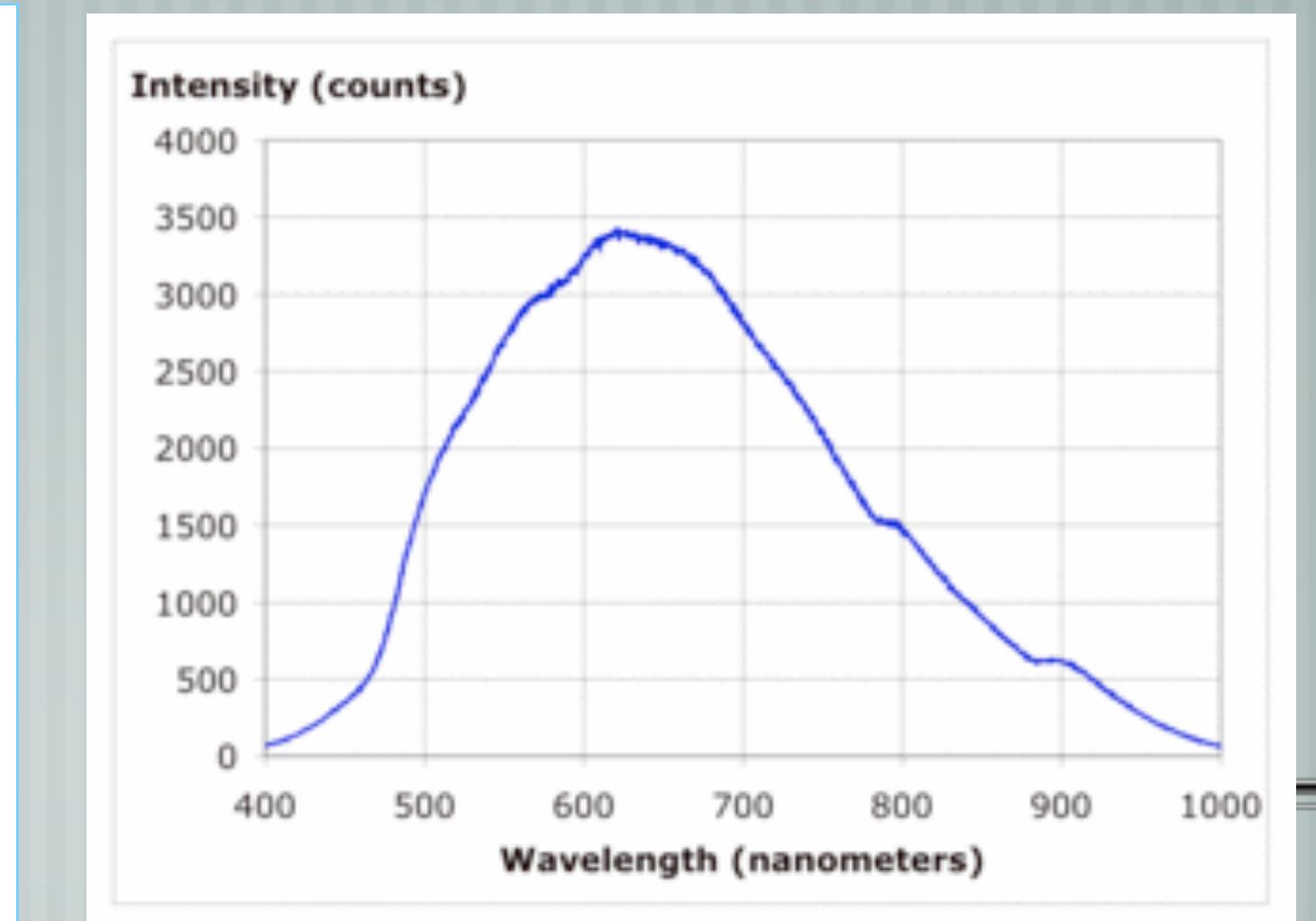
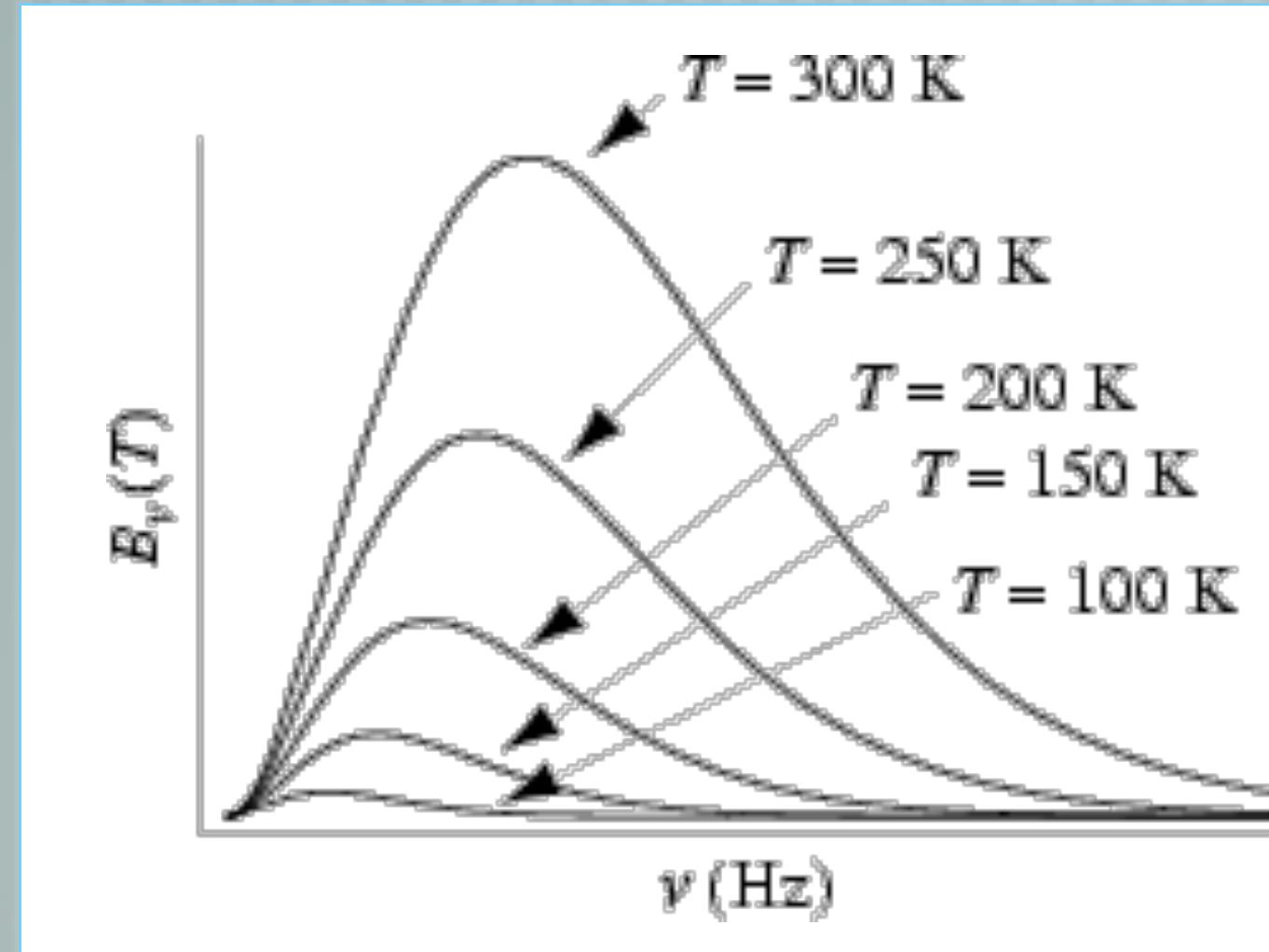
$\tau \sim 1$

Thermal equilibrium + high opacity = Blackbody emission

Plasma in 'causal contact', all global properties of photons depend only on T

➡ Specific intensity given by the "blackbody" (Planck) formula:

$$I_\nu(T) \equiv B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$



Blackbody emission II

[Handy to know the essential characteristics like:

- ➡ Averaging over the total distribution gives a mean photon energy of $E_{\text{mean}} = h\nu_{\text{mean}} \approx 2.7kT$, close to the peak of the spectrum
- ➡ Integrating over frequency gives total energy density $\epsilon_\gamma = aT^4$
- ➡ Total luminosity of a blackbody $L = (4\pi R^2)\sigma_B T^4$

Few E&M definitions (in vacuum)

How particle moves

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}$$

$$\vec{j} \cdot \vec{E} + \frac{1}{8\pi} \frac{\partial}{\partial t} (E^2 + B^2) = -\vec{\nabla} \cdot \vec{S}$$

$$\frac{dU_{\text{field}}}{dT}$$

$\rightarrow +/-$ ch.
m particles

$$U_B = \frac{B^2}{8\pi}$$

\rightarrow change in energy of
field

loss term
transport of energy
in fields $\propto \gamma$
"radiation"

$$\vec{S} = \text{Poynting vector}$$

$$= \frac{c}{4\pi} (\vec{E} \times \vec{B})$$

$$\epsilon_0 \Rightarrow \frac{c}{4\pi} |\vec{E}|^2 = \frac{c}{4\pi} |\vec{B}|^2$$

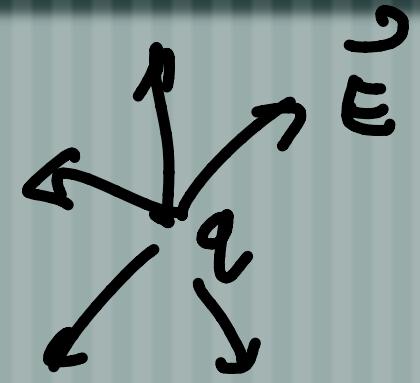
How is it we see things from so far away??
How can EM radiation travel without attenuation?

————— [Need radiation field to somehow not decrease (much) with distance

- > i.e., how do we maintain a signal over extremely long (astrophysical) distances?
- > Typically static radial EM fields go as $1/R^2$
- > What is the scaling for energy transport (ie., radiation)?

$$\sim 1/R^4$$

Radiating charges



$$\vec{E} \approx \frac{r}{R^2} \rightarrow S \propto 1/r^n \rightarrow \text{no way!}$$

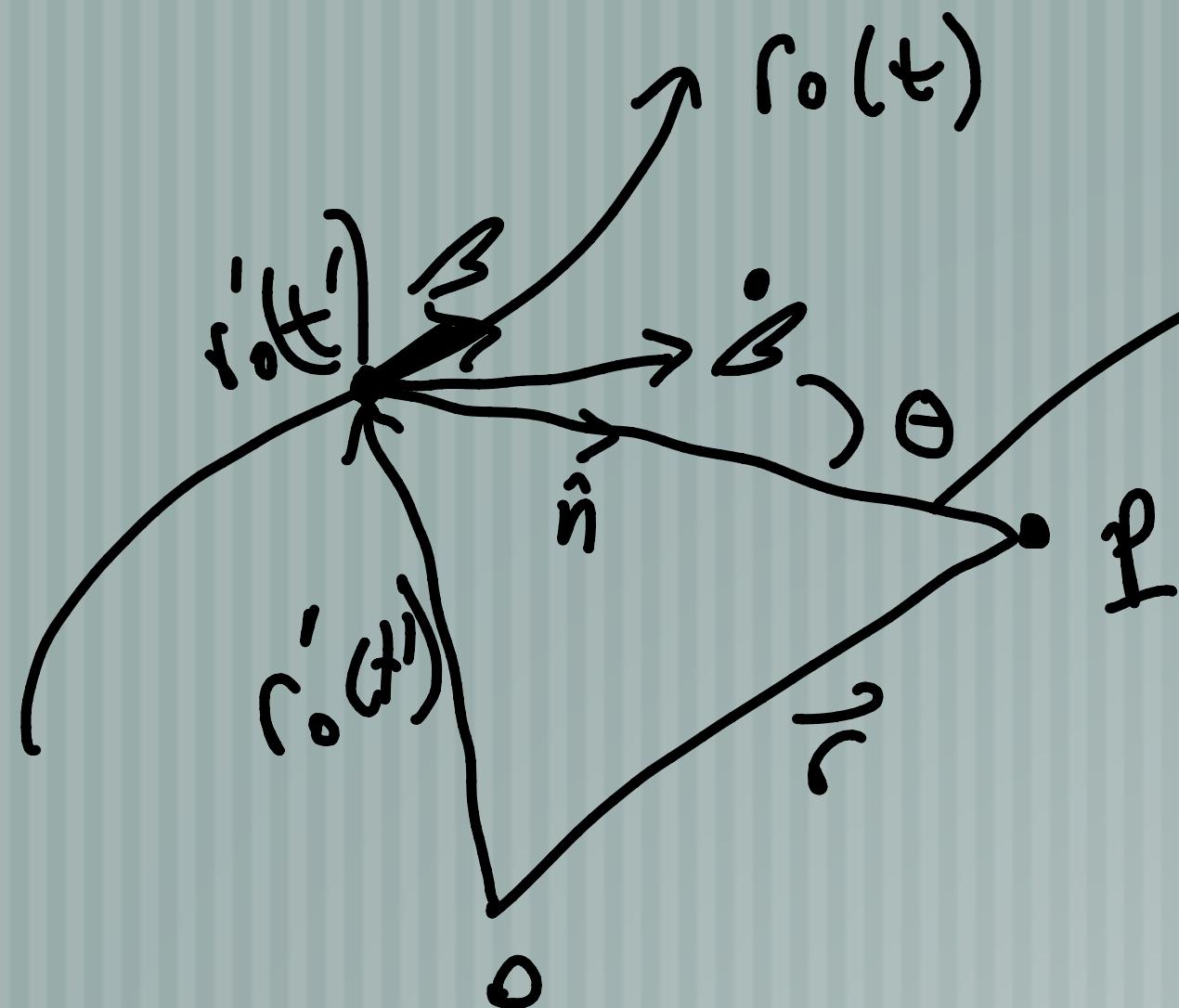


$$\vec{E} \sim \frac{1}{r} \rightarrow S \propto \frac{1}{r^2} \rightarrow \text{better}$$

\rightarrow radio / pulse \Rightarrow acceleration

Geometry with relativistic motion = retarded time

Coordinate system of observing rel. particle emitting light:



$$R = |\vec{r} - r'_0(t')|$$

"P" feels / sees EM radiation from source at

$$\boxed{t' \equiv t - R/c}$$

$$\hat{n} \equiv \frac{\vec{R}}{R} \text{ at } t'$$

$$\frac{dt'}{dt} \equiv \kappa = (1 - \hat{n} \cdot \vec{\beta})$$

Electric field from Liénard-Wiechert Potential

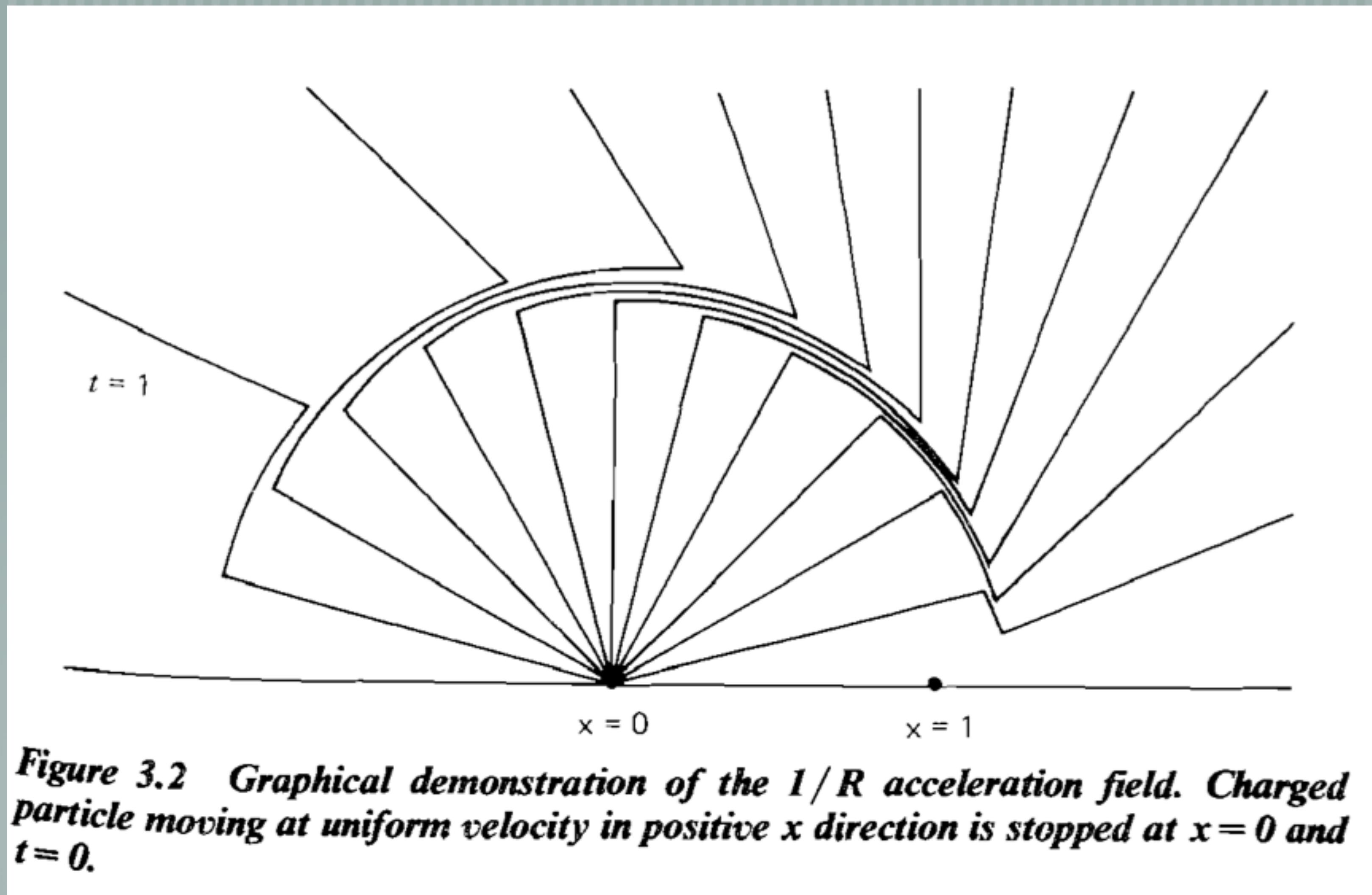
$$\vec{E}_{\text{rel}}(\vec{r}, t) = q \left[\frac{(\hat{n} - \vec{\beta})(1 - \beta^2)}{\kappa^3 R^2} \right] + \frac{q}{c} \left[\frac{\hat{n}}{\kappa^3 R} \times (\dot{\hat{n}} - \dot{\vec{\beta}}) \times \vec{\beta} \right]$$

"velocity field" =
 $\beta \rightarrow 0$, reduces to a/r^2
 $|a|=1$

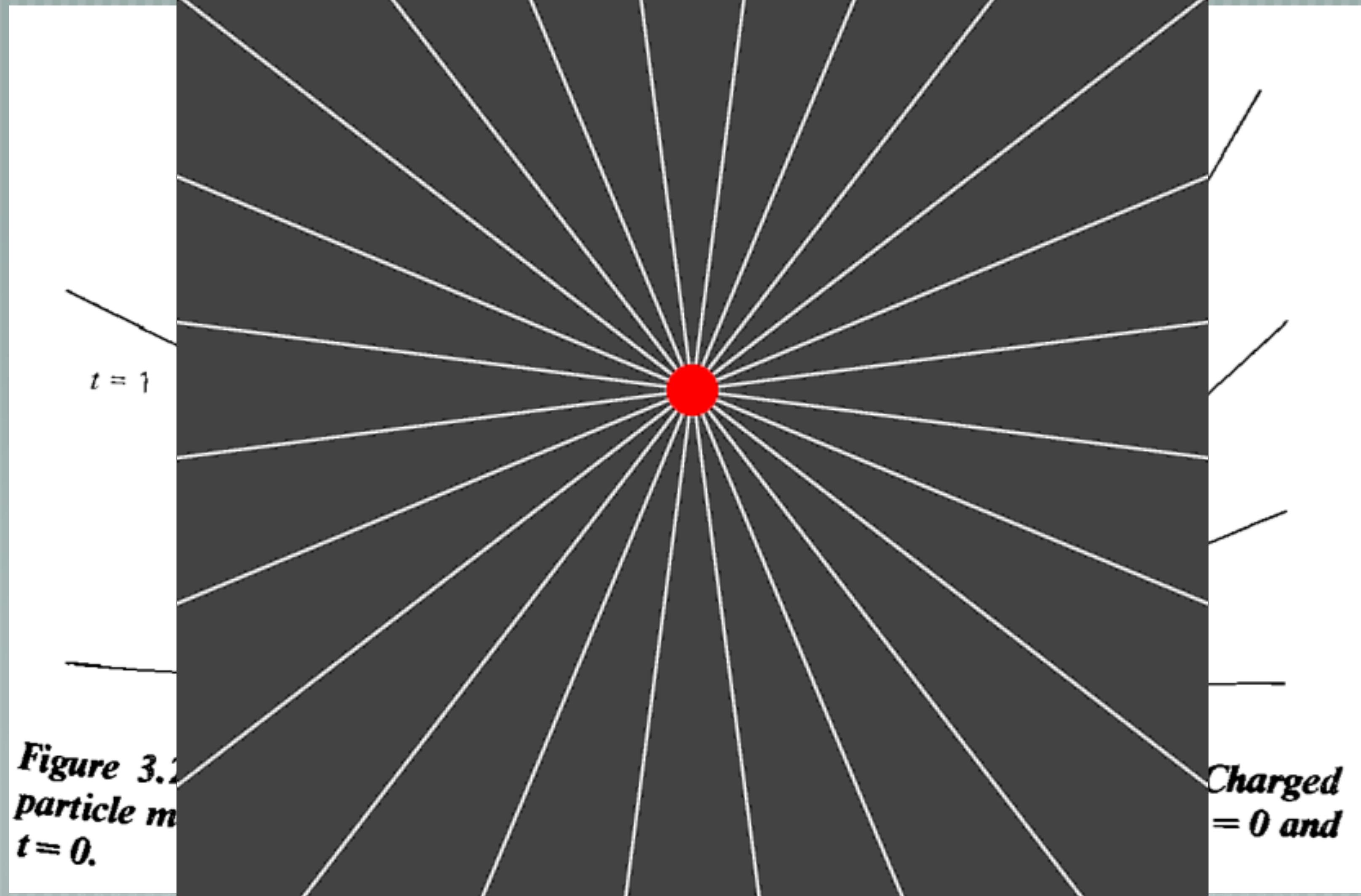
"acceleration field" = "radiation field" =
 $\vec{s} \sim 1/r^2$

most interesting
($\vec{s} \sim \gamma r^n$)

How you get a transverse/radial field from acceleration!

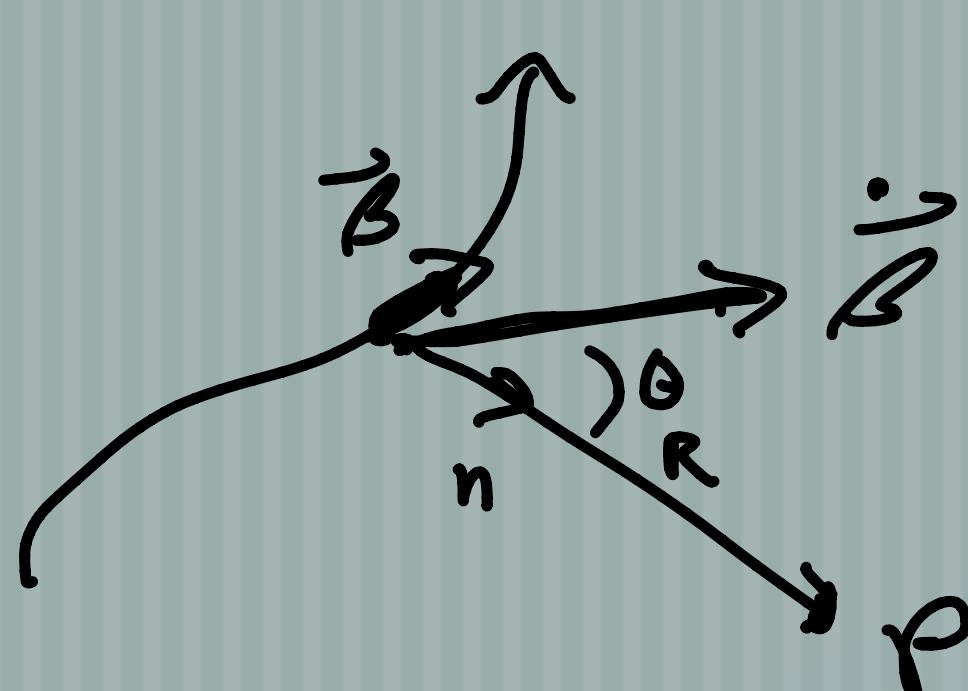


How you get a transverse / radial field from acceleration!



Radiation pattern & power from accelerating charges

Simplify & think non-relativistic $\gamma \rightarrow 0, \kappa = 1$

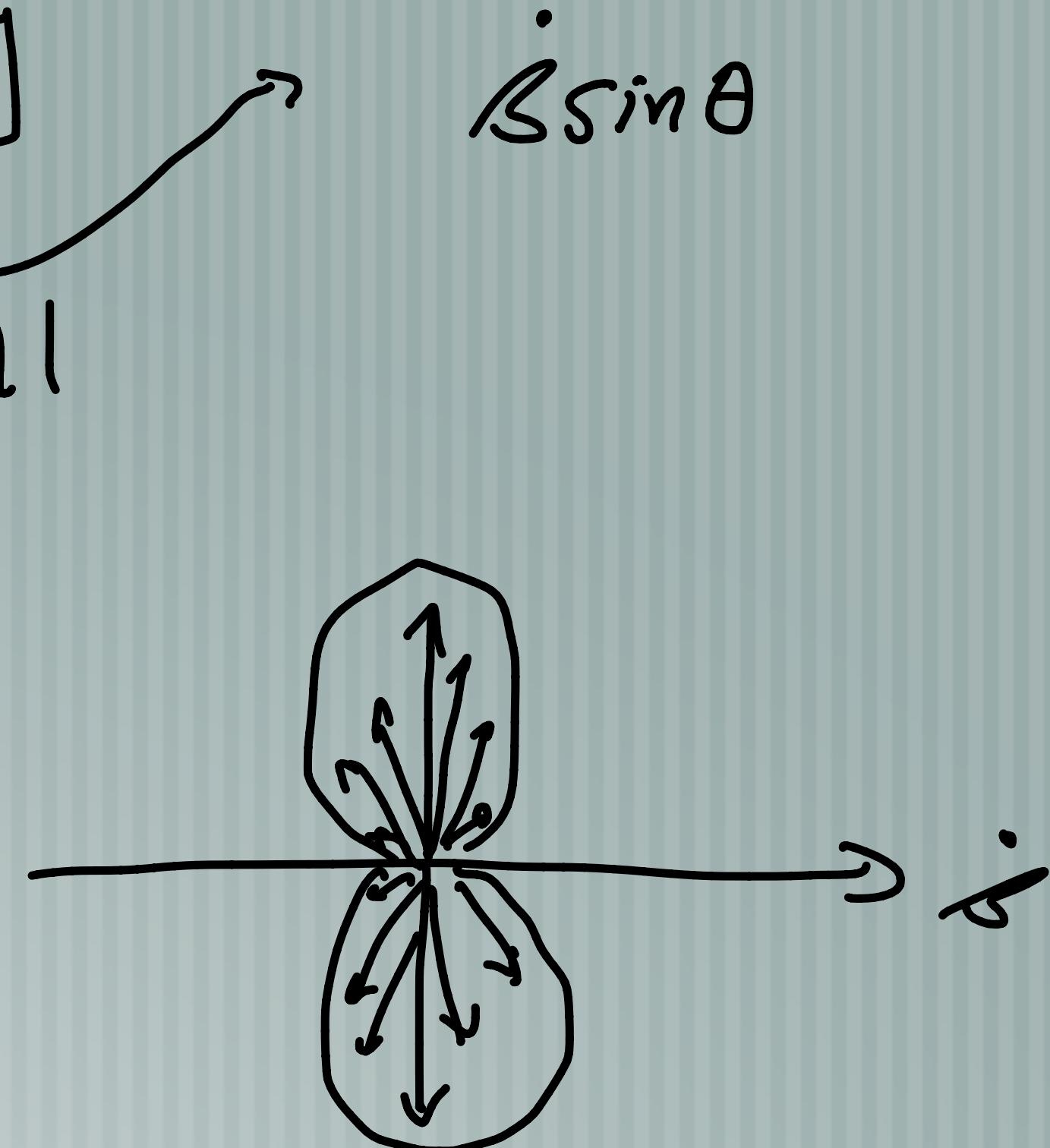


$$\vec{E}_{rad} = \frac{q}{cr} [\hat{n} \times (\hat{n} \times \vec{B})]$$

$$|E_{rad}| = \frac{q}{cr} \dot{\beta} \sin\theta = |\mathbf{B}_{rad}|$$

$$|S| = \frac{c}{4\pi} |E|^2 = \frac{r^2 \dot{\beta}^2 \sin^2\theta}{4\pi c R^2}$$

$$\frac{dS}{dr} = \frac{dw}{dt dr} = R^2 |S| = \frac{q^2 \dot{\beta} s n^2 \theta}{4\pi c}$$



Power from dipole radiation: Larmor's formula

$$\vec{j} = q \vec{r}$$

$$\vec{j} = q \vec{v}$$

$$\ddot{\vec{j}} = q \vec{v} = q \vec{\beta} c$$

$$\frac{dP}{d\Omega} = \frac{\ddot{\vec{j}}^2 \sin^2 \theta}{4\pi c^2}$$

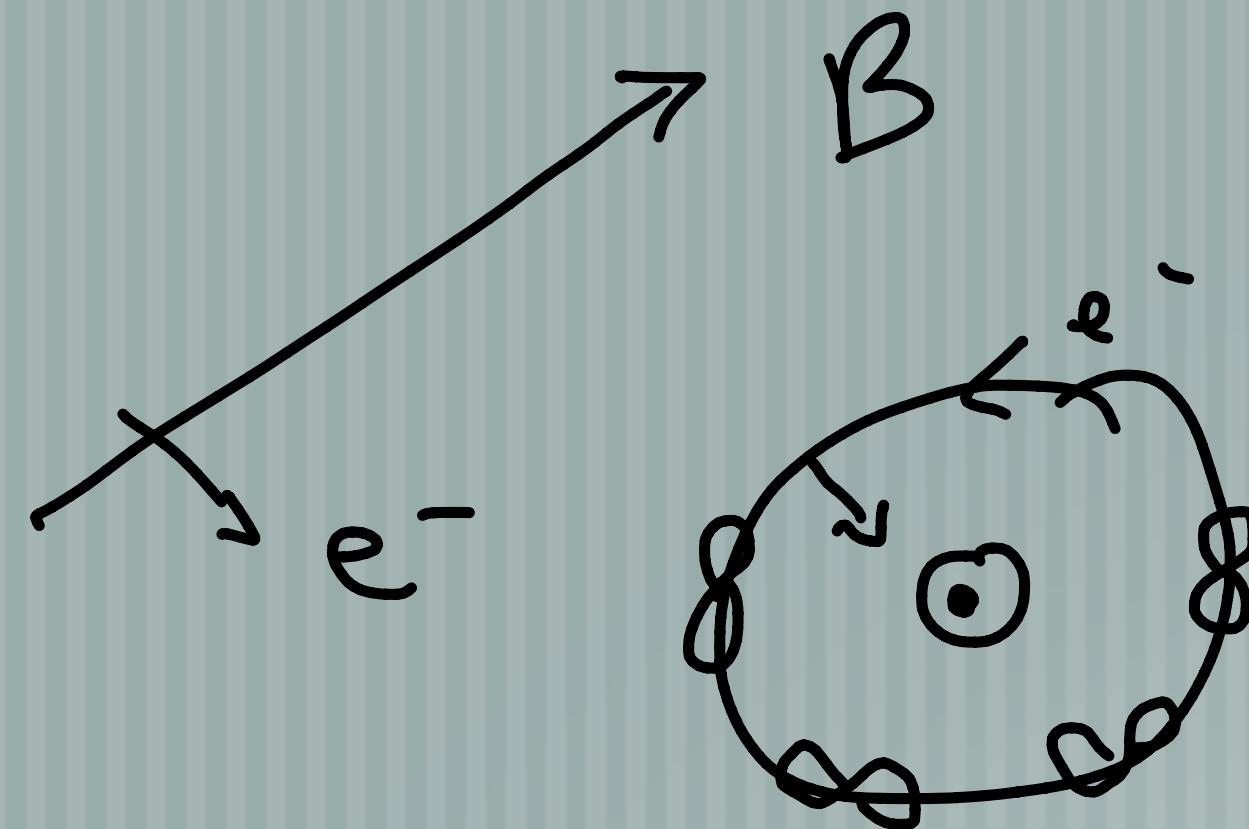
$$d\Omega = \sin \theta d\theta d\phi$$

$$\int \frac{dP}{d\Omega} d\Omega = \left(\frac{8\pi}{3}\right) \frac{\ddot{\vec{j}}^2}{4\pi c^2} \quad \text{"Larmor's =}$$

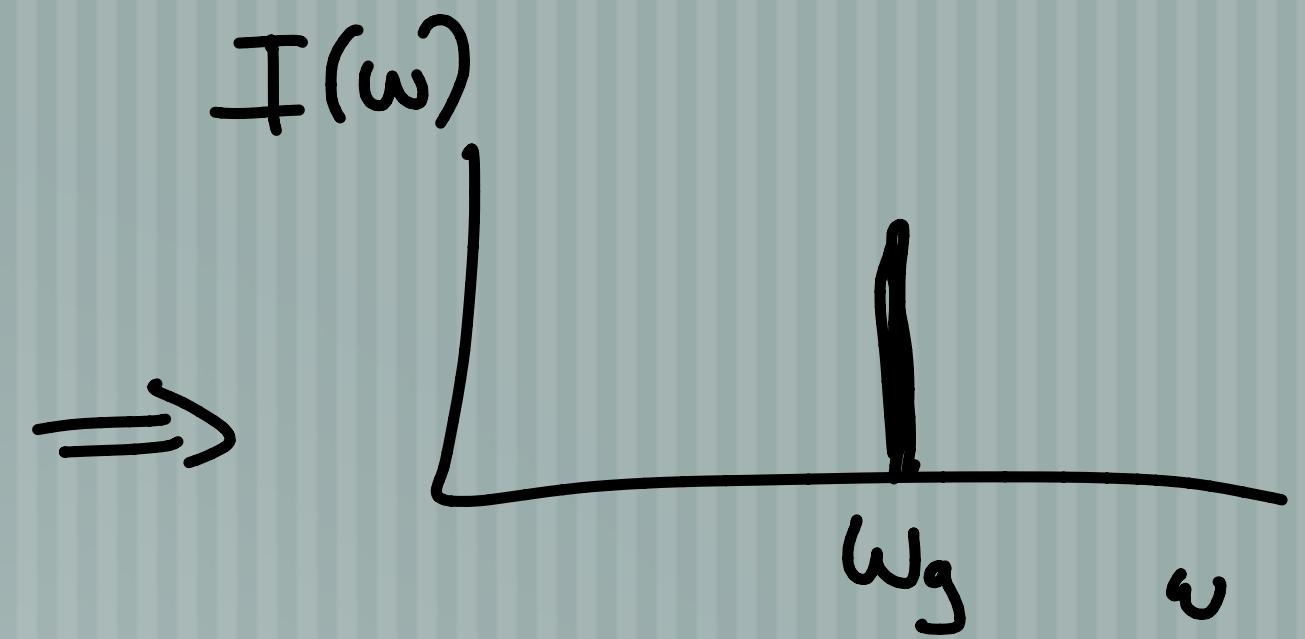
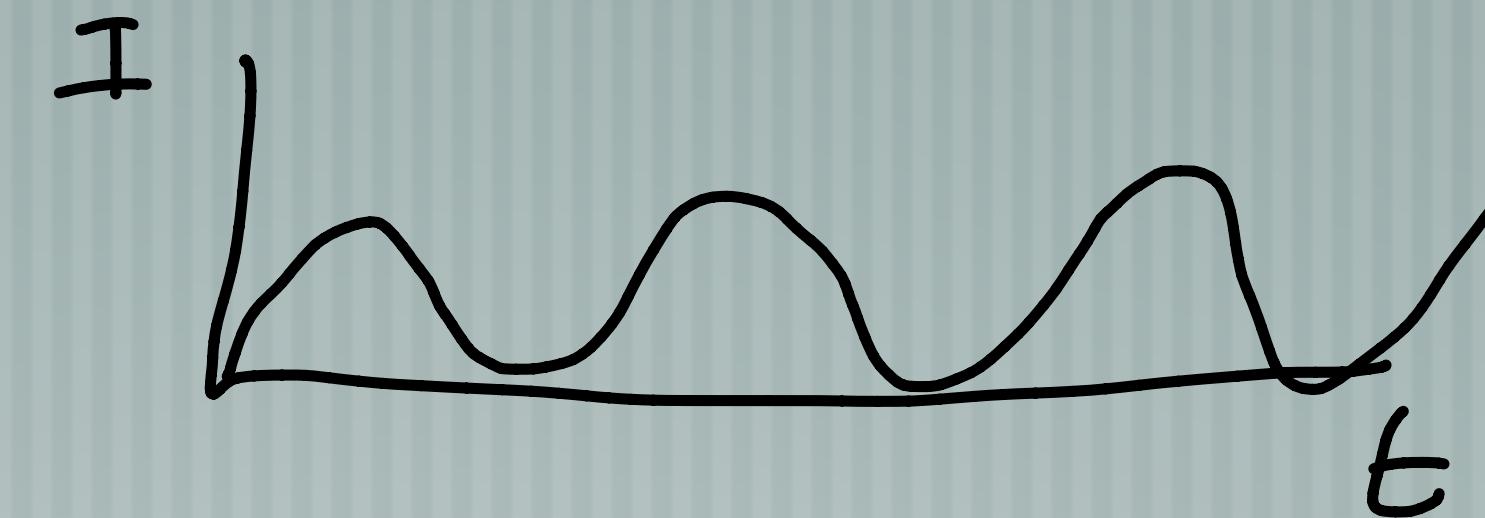
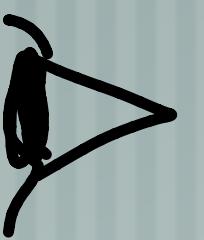
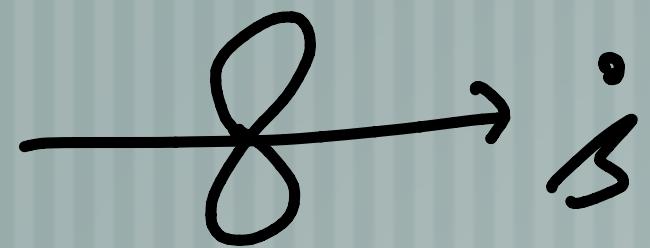
Takeaway : * accelerating charges radiate like dipole
(if non-RL), can always transform to that frame

* main effect of relativity (sp.) is framing +
angle aberration \rightarrow math

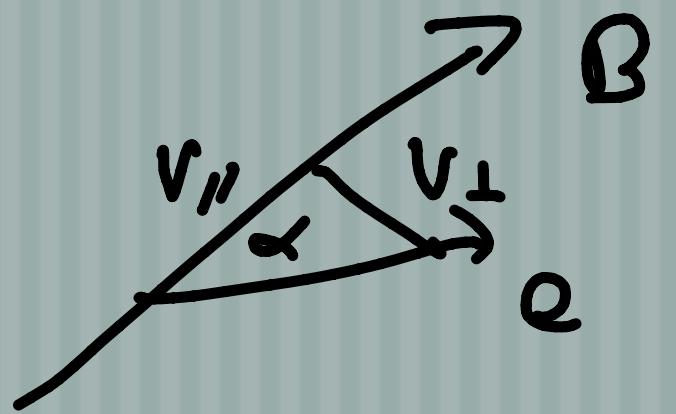
"Prelude" to cyclo-synchrotron



$$F_L = -\frac{e}{c} (\vec{v} \times \vec{B})$$



Power from non-relativistic particle in a magnetic field (Cyclotron)



$v_{||} \Rightarrow$ acceleration? No \rightarrow no light L

$$v_{\perp} \Rightarrow v \sin \alpha$$

$$\omega_g = \frac{v_{\perp}}{r} \Rightarrow \frac{eB}{mc} = \boxed{1.8 \times 10^7 \text{ Gauss}}$$

Want $\ddot{d} = -e \vec{v} = e v \sin \alpha \omega_g$

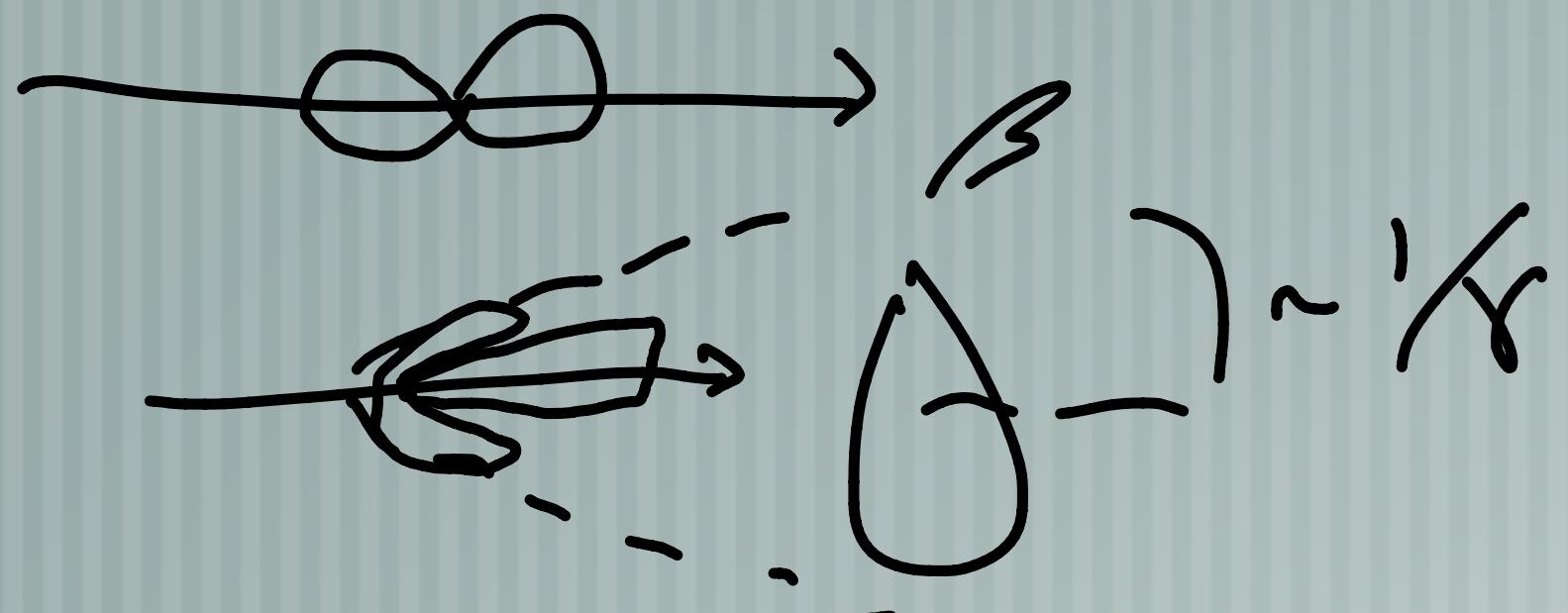
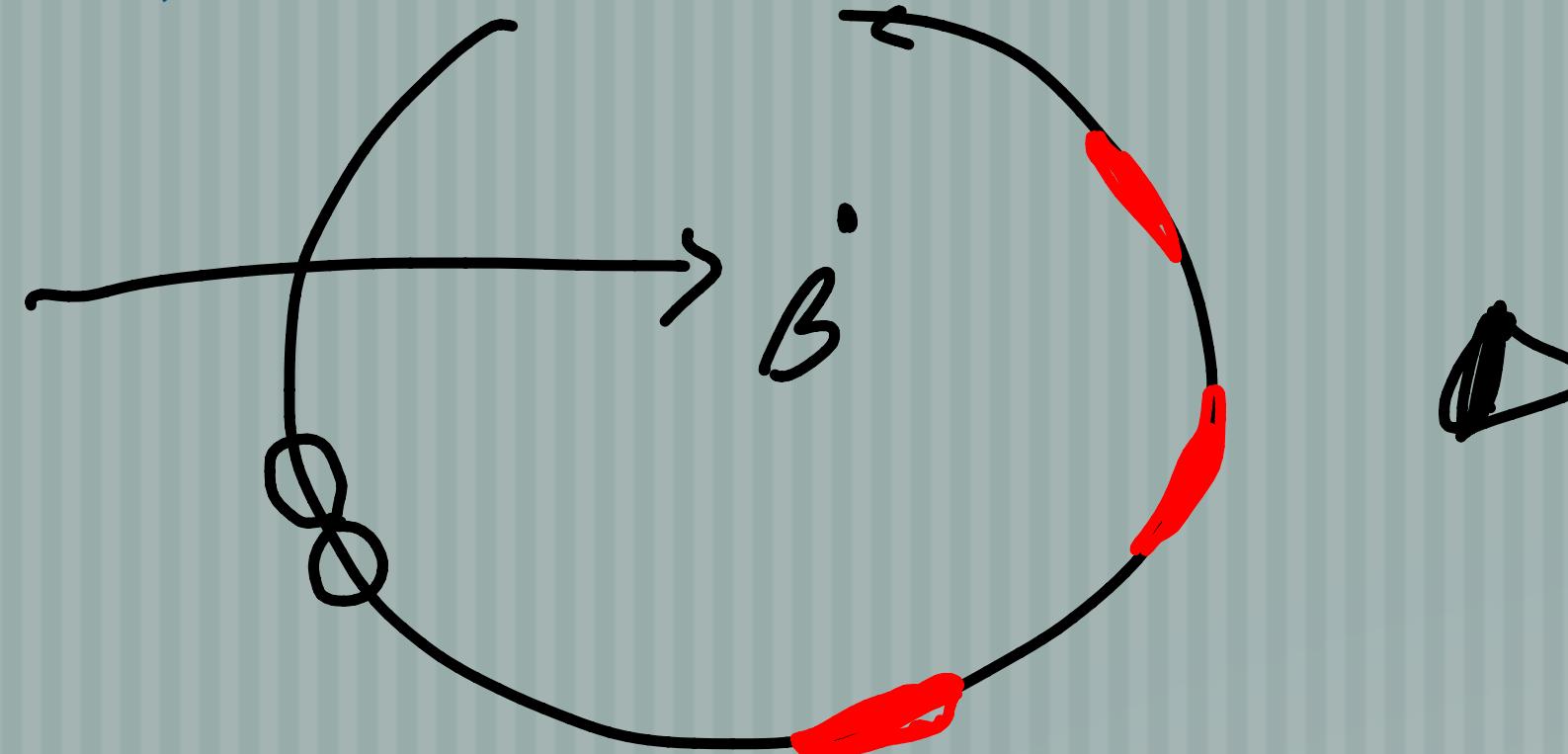
$$F = ma$$

$$\frac{e}{c} v B \sin \alpha = m \ddot{v}$$

$$P_{\text{cycl.}} = \frac{2}{3} \frac{\ddot{d}^2}{c^3} = \frac{2}{3} \frac{e^2 v^2 \sin^2 \alpha \omega_g^2}{c^3}$$

How does P depend on Energy $\sim V^2 = KE$

Qualitative extension to relativistic particle motion (synchrotron)



$$P_{\text{Larmor}} = \frac{2}{3} \frac{e^2 v^2 \sin^2 \alpha}{c^3} \gamma^2$$

$$\frac{\int P d\nu}{\int d\nu} = \frac{1}{4\pi} \left(\frac{8\pi}{3} \right) \frac{2}{3} \frac{e^2}{c^3} \gamma^2 \beta^2 \omega_g^2$$

$$\beta = v/c$$

$$f_0 = \frac{e^2}{m c^2}$$

$$\Rightarrow P_{\text{Synch, pt}} = \frac{4}{3} \left(\frac{8\pi r_0^2}{3} \right) \left(\frac{\beta^2}{8\pi} \right) \gamma^2 \beta^2 c$$

$$= \boxed{\frac{4}{3} \sigma_T U_B \gamma^2 \beta^2 c} \quad \times n$$

$$E^2$$

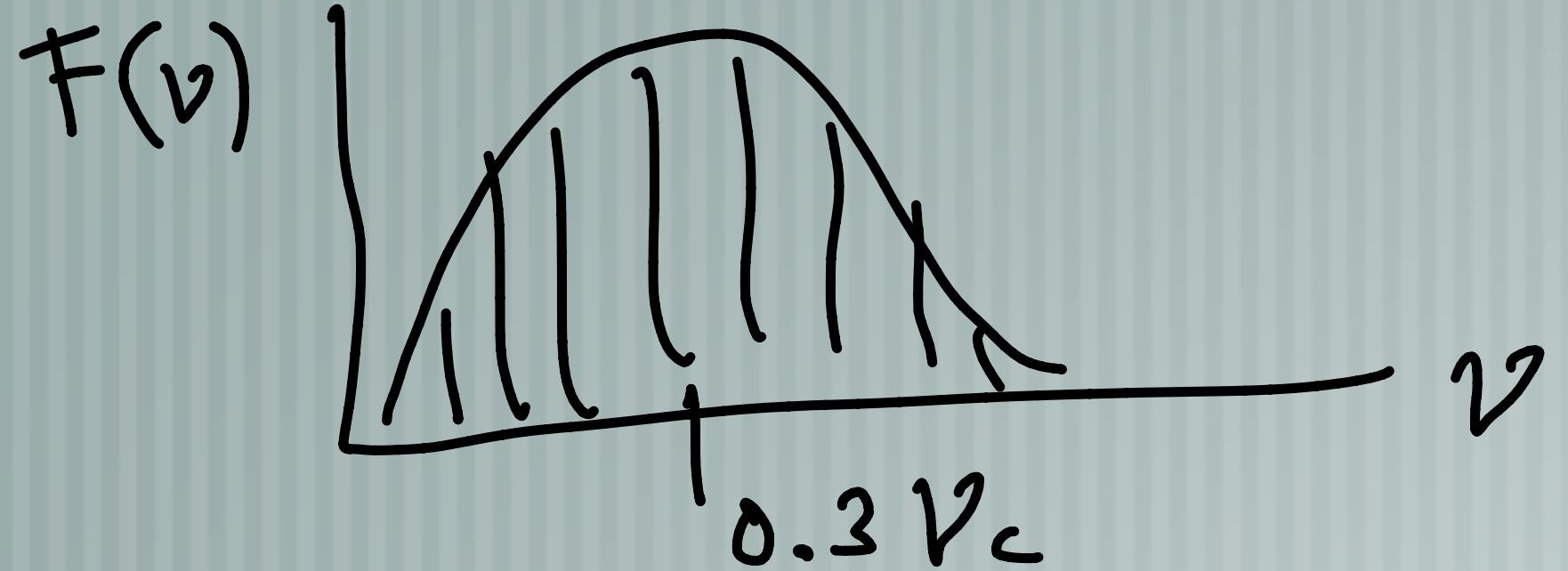
Dependence on what?

Form of frequency dependence $dW/d\omega$

Spectrum : $\frac{dP}{dt d\omega} = \frac{J\omega}{dt d\omega}$

$\frac{dW}{d\omega}$ encodes the Fourier transform

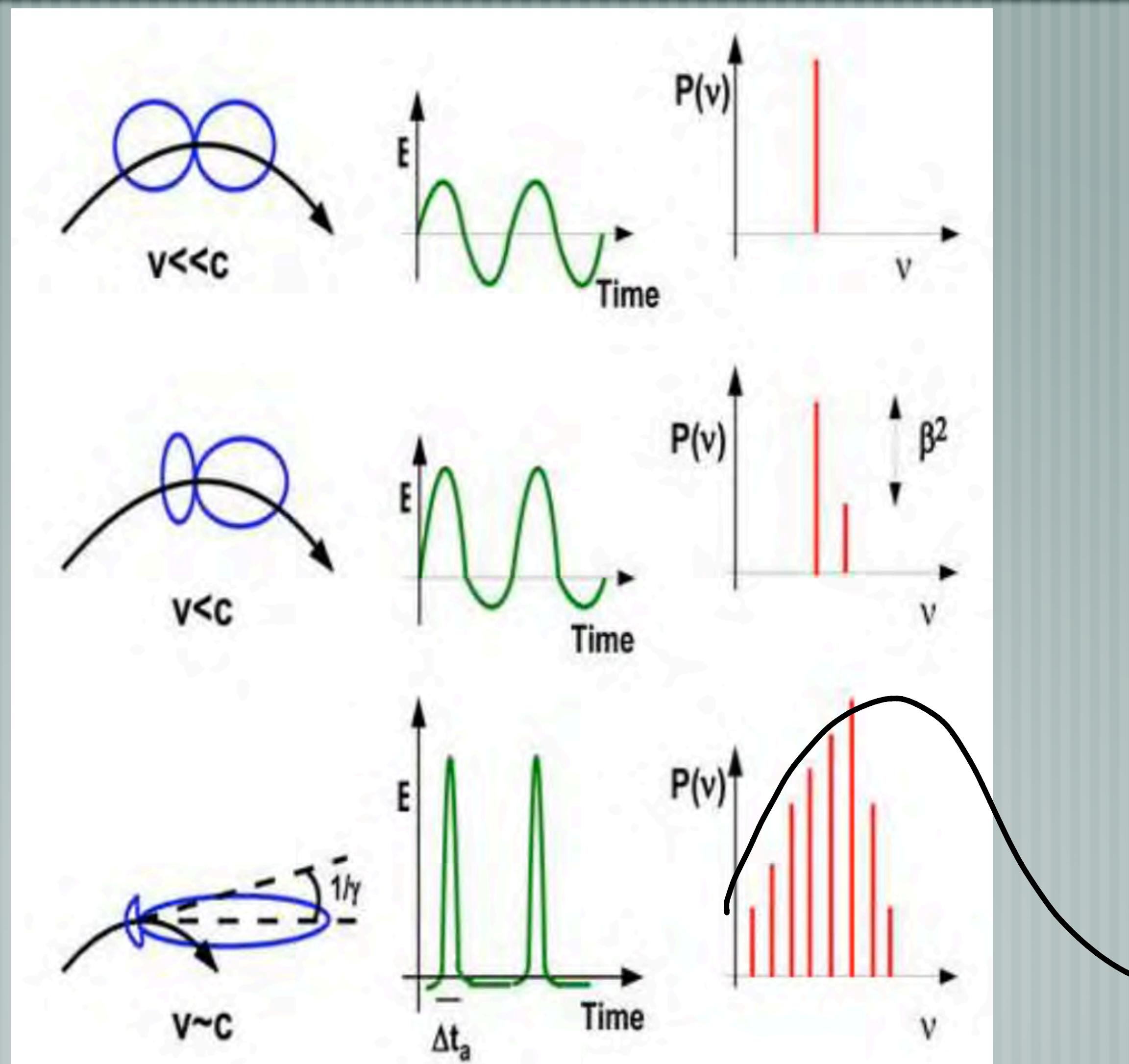
$$\frac{dW}{d\omega} = \frac{8\pi}{3} \frac{\omega^4}{c^3} |\tilde{J}(\omega)|^2$$



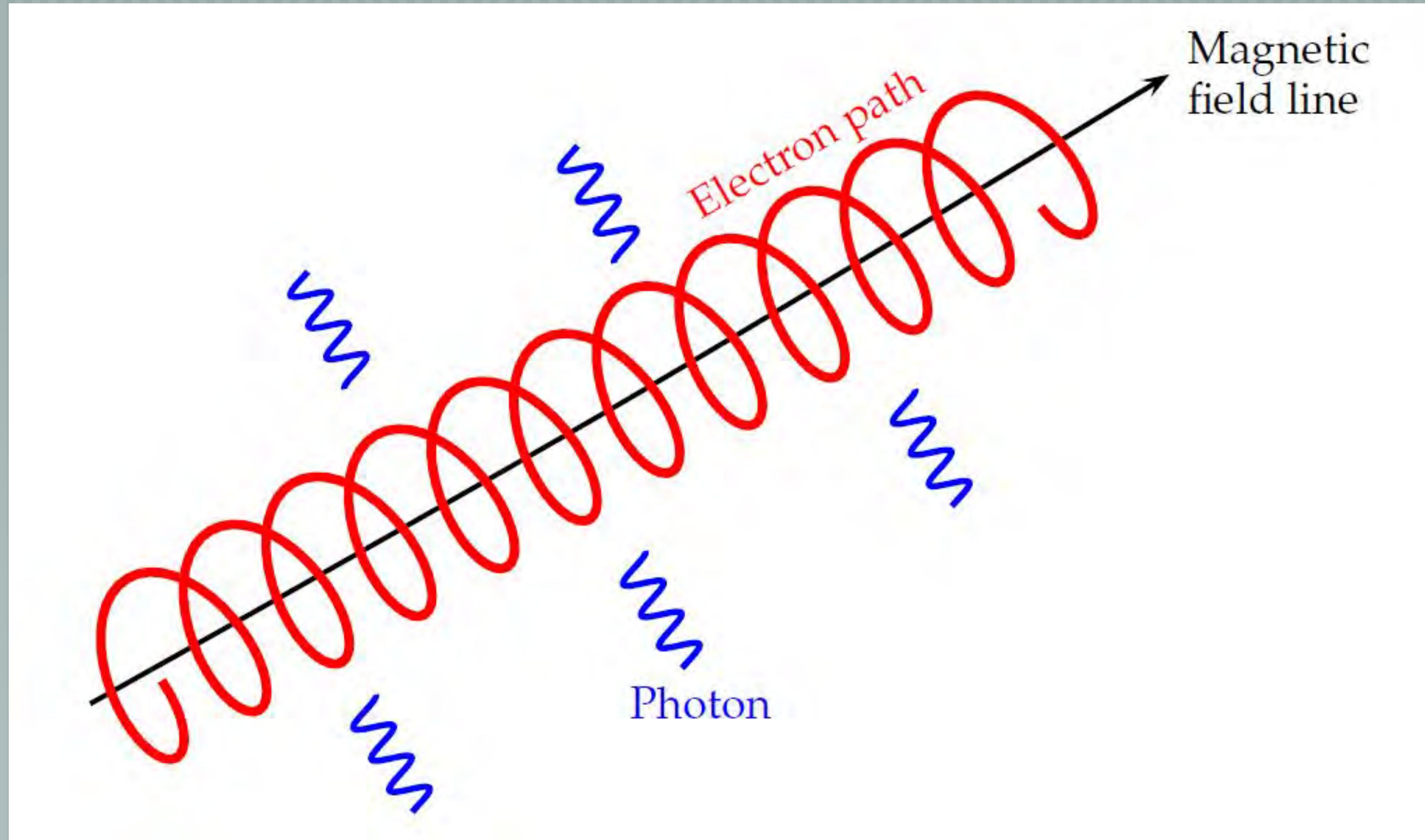
$$V_c \sim \frac{3}{4\pi} \gamma^2 w_g \sin \zeta$$

$$\sim \frac{3}{4\pi} \frac{\gamma^2 e B}{mc}$$

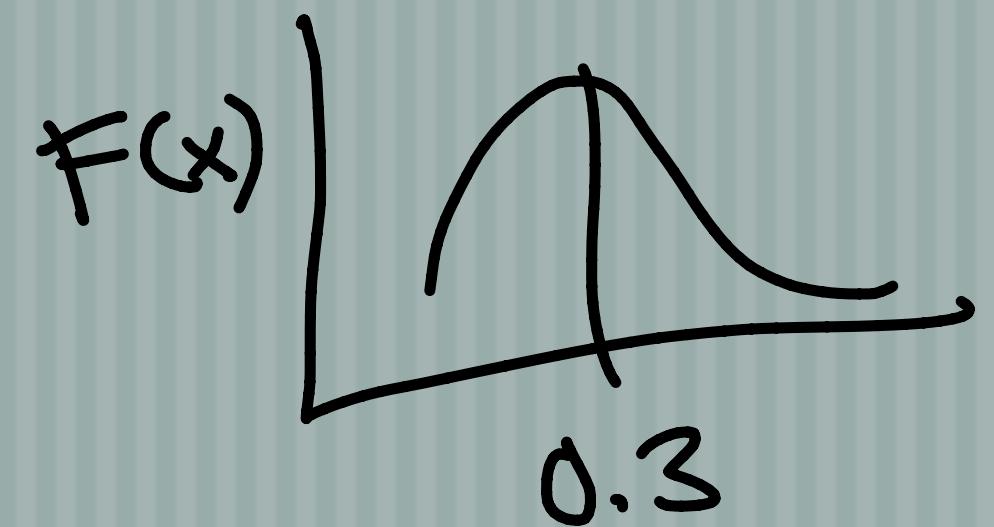
Relativistic motion/aberration effects spectrum



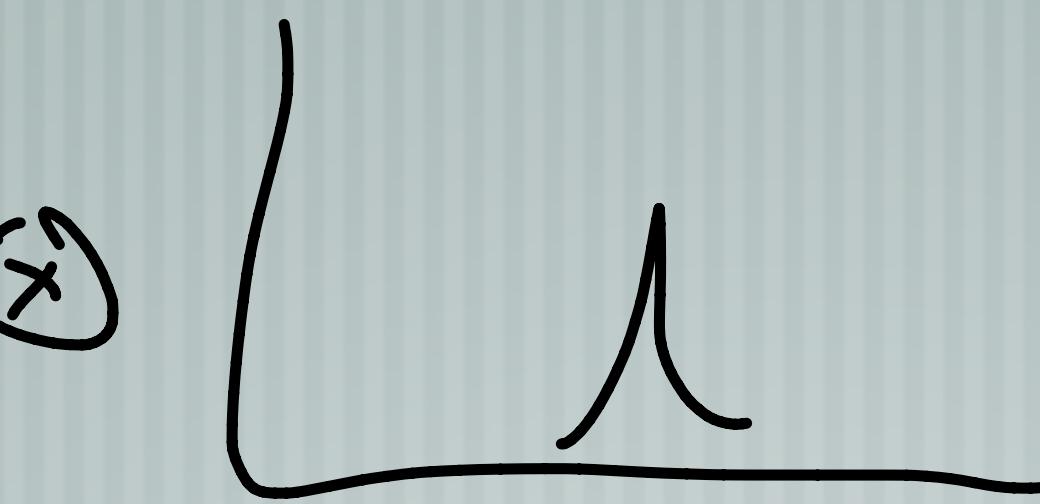
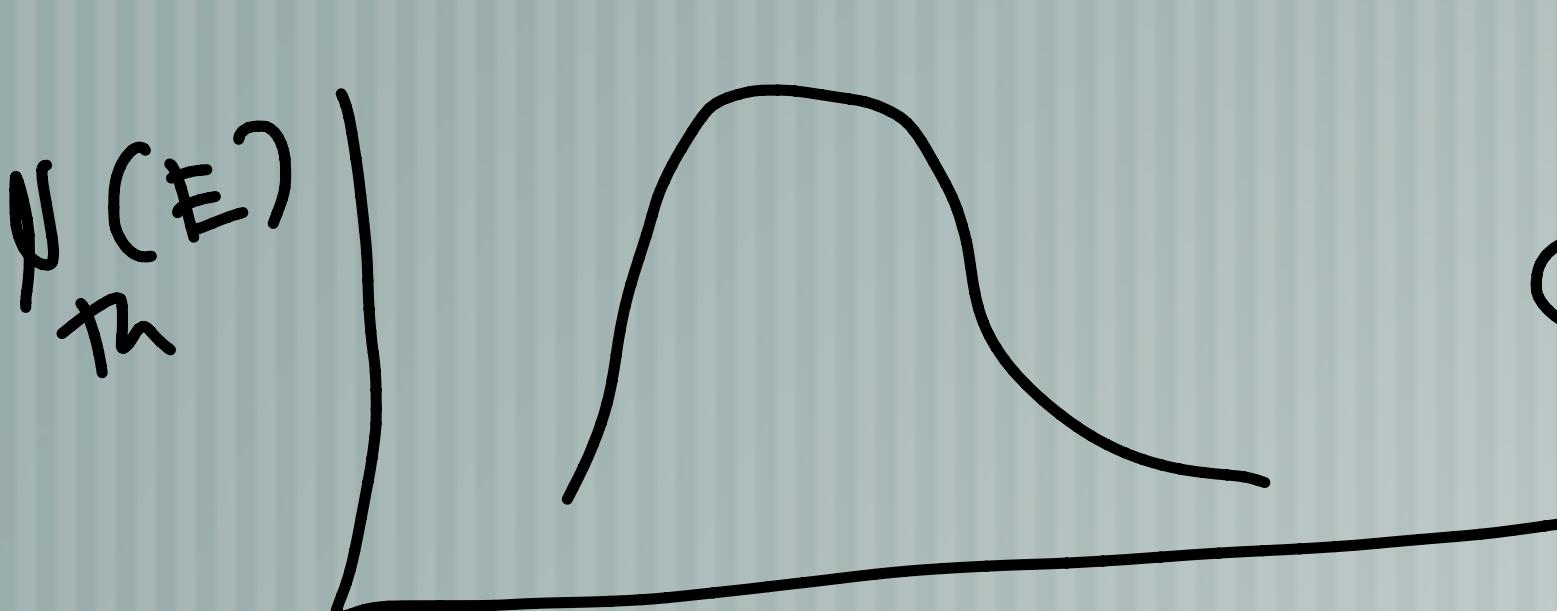
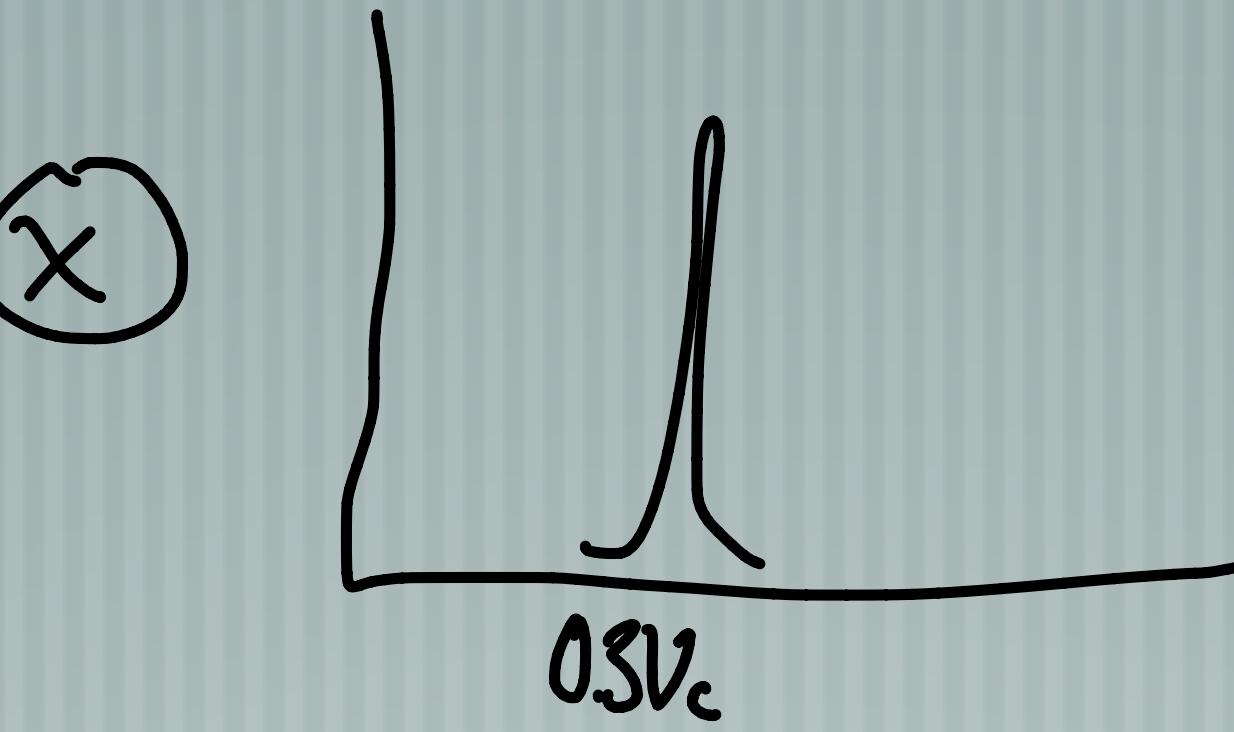
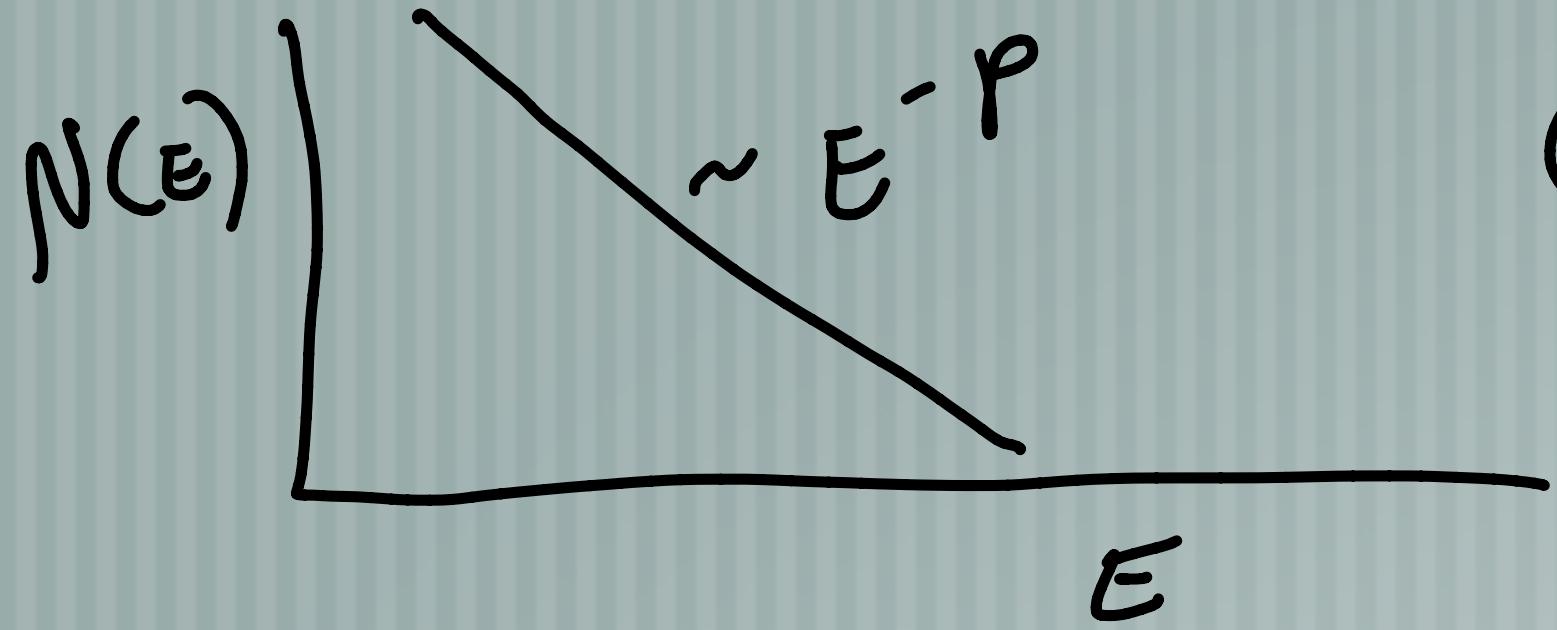
Synchrotron



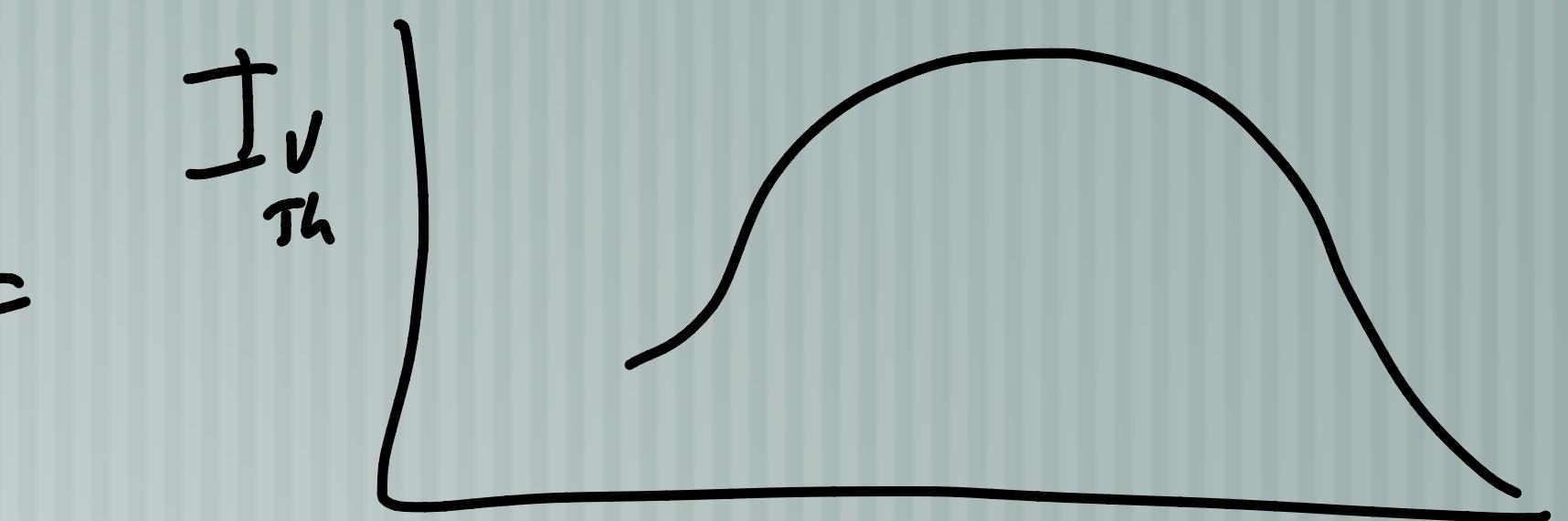
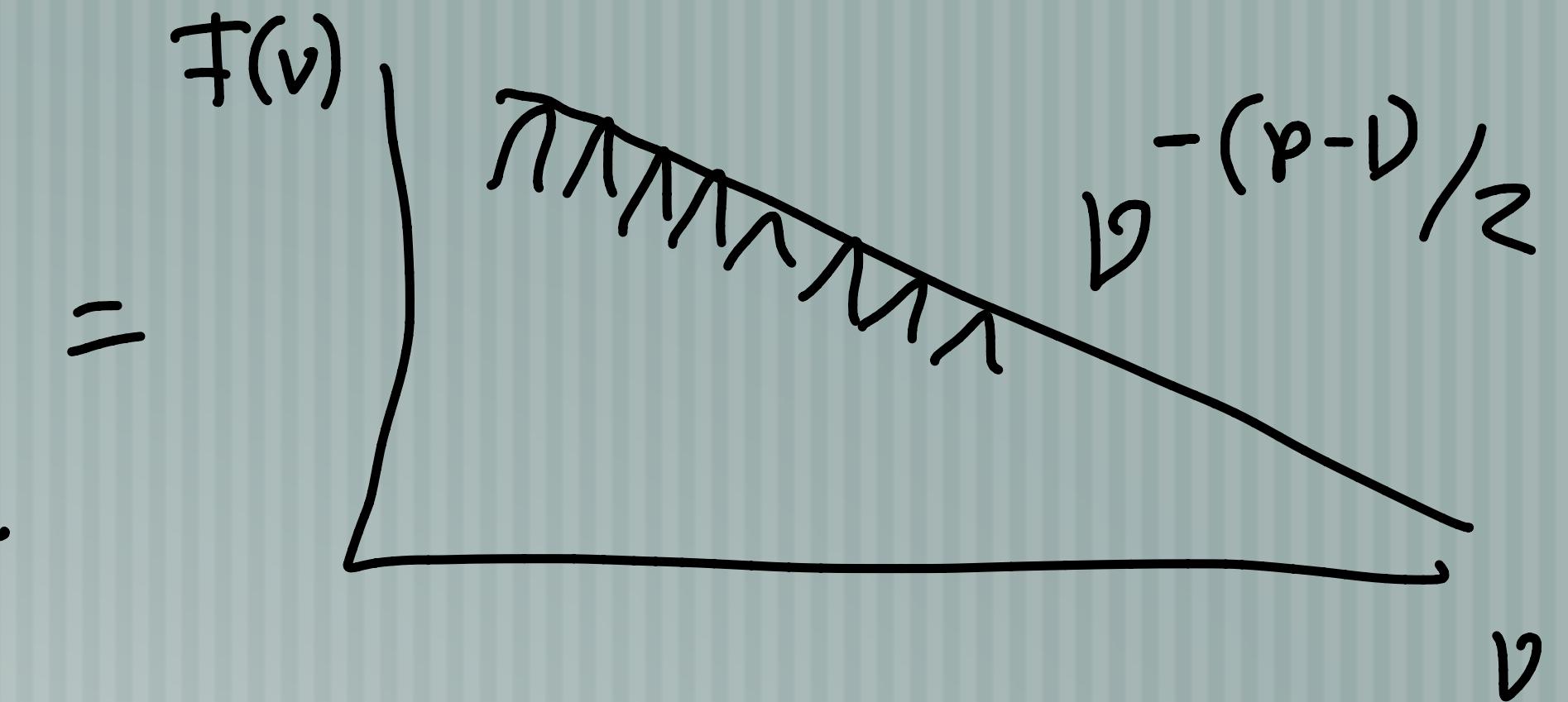
Synchrotron spectral form $P(v, E)$



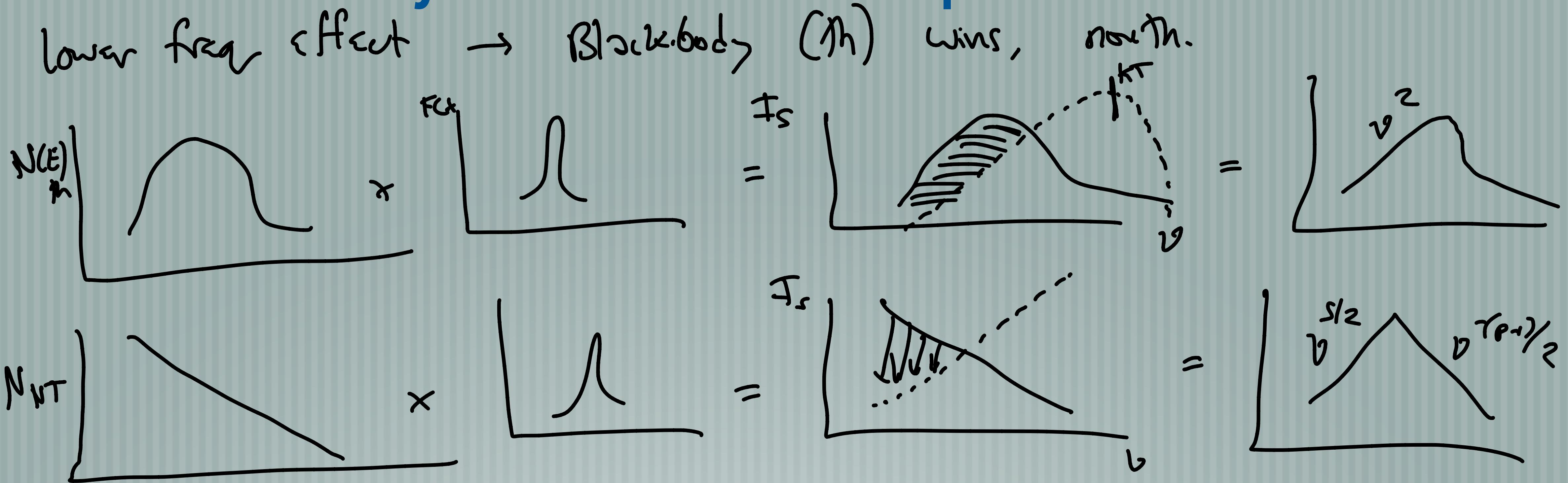
$$x = v/v_c$$



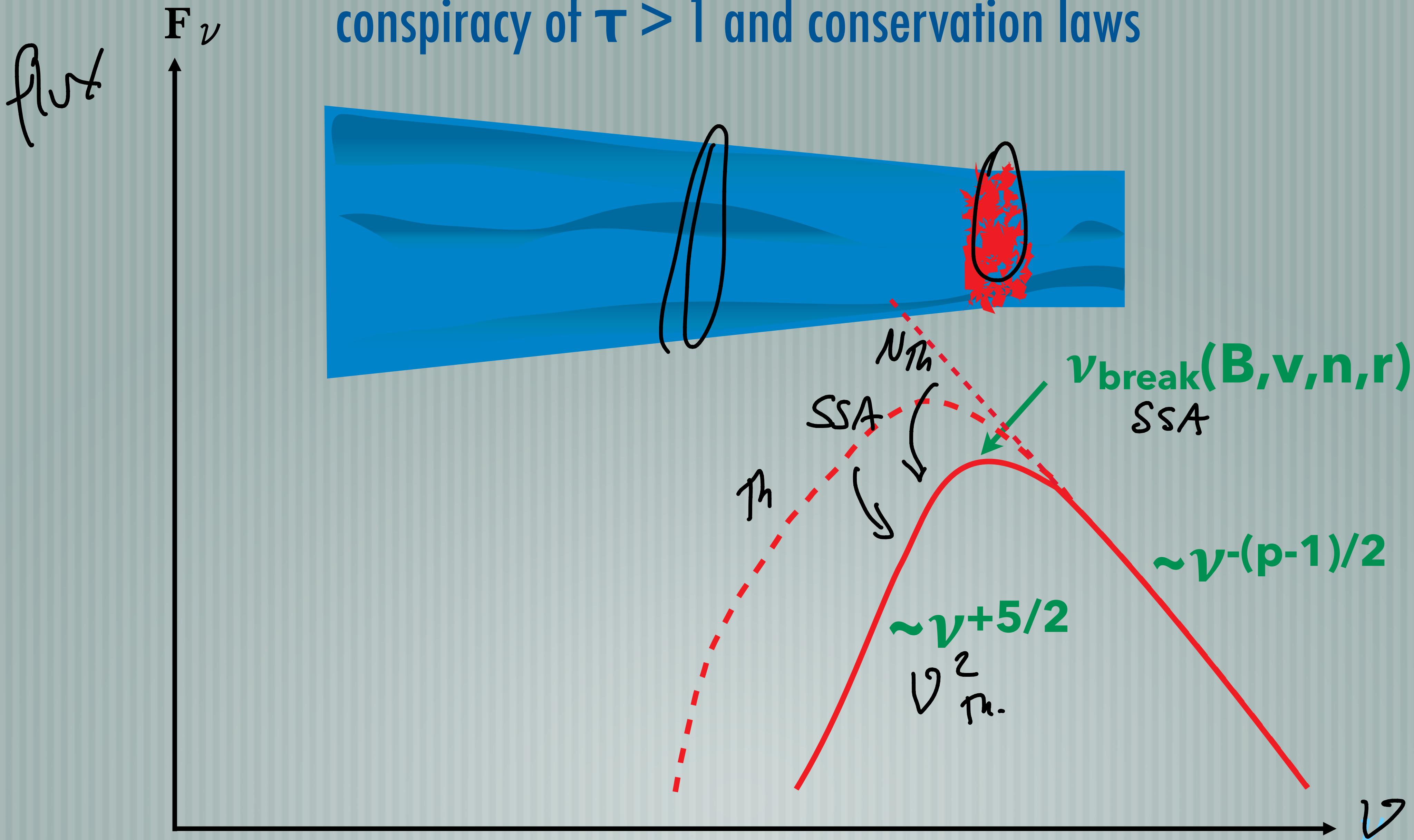
$$P(v, E_0) = \frac{\sqrt{3} e^3 \beta \sin \alpha}{mc^2} F(x) \quad F(x)$$



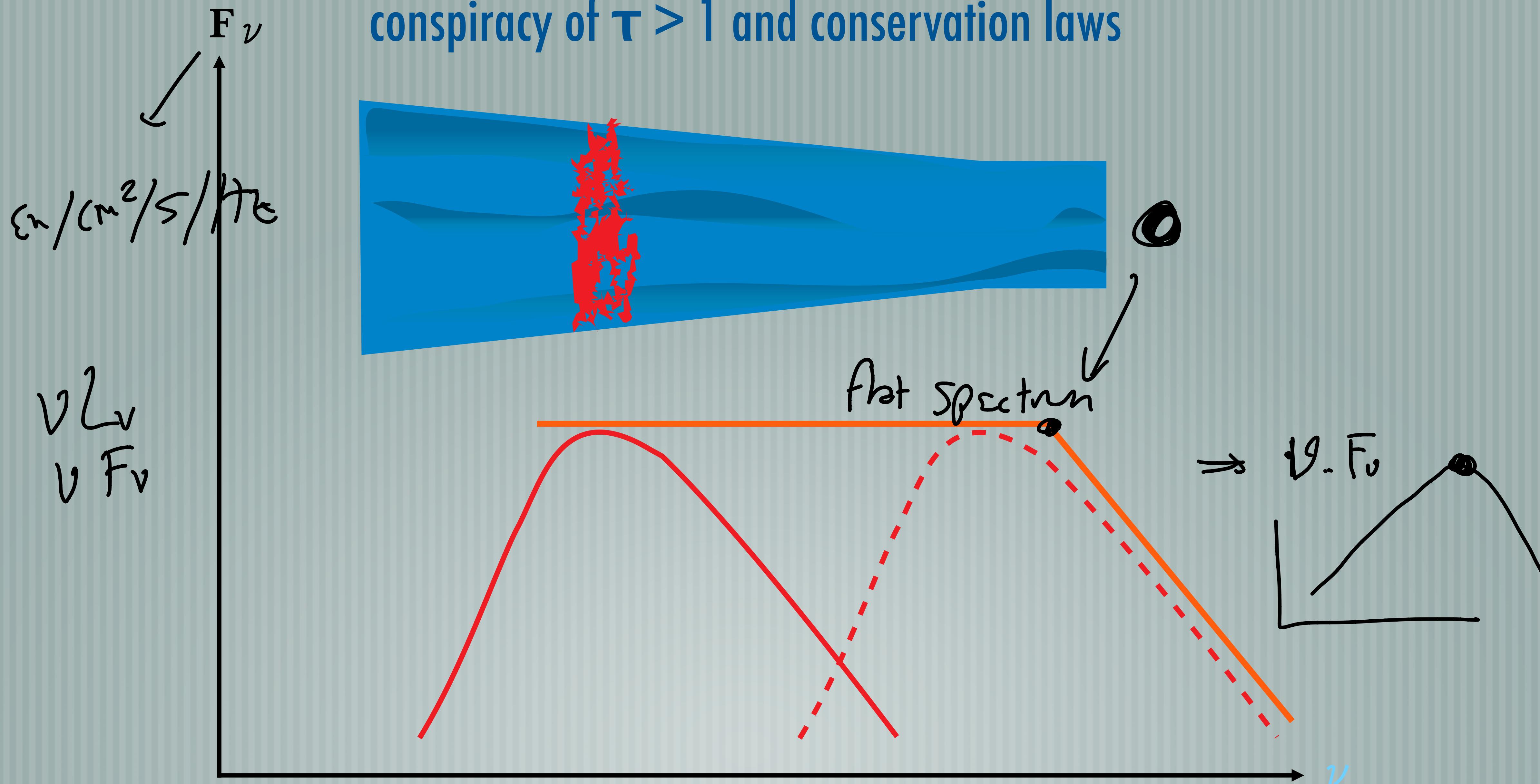
Synchrotron self-absorption



“Signature” flat(ish) emission of compact jets (“cores”) is a conspiracy of $\tau > 1$ and conservation laws



"Signature" flat(ish) emission of compact jets ("cores") is a conspiracy of $\tau > 1$ and conservation laws

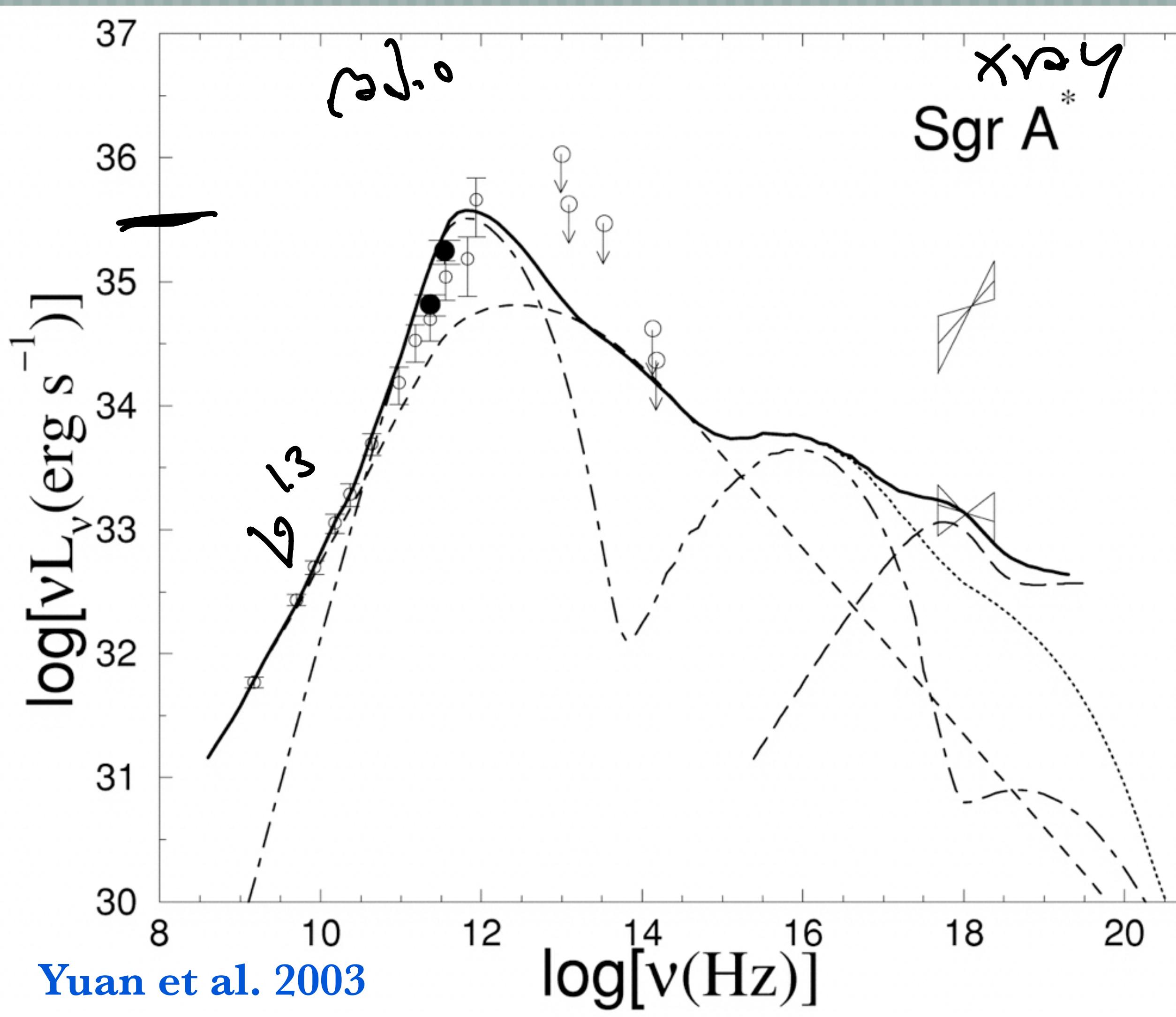


SSA in active galactic nuclei (AGN) jets

$$V_{SSA} = 100 \text{ MHz} \left(\frac{B_{mg}}{R_{kpc}} \right)^{4/3} \left(\frac{R_{kpc}}{100} \right)^{1/3}$$

τ_v

Application to Sgr A* I: my favourite black hole!



$$R = 10r_g$$

$$M = 4 \times 10^6 M_\odot$$

$$= 10^{(6.6)}$$

$$r_g \approx 10^5 \left(\frac{m}{M_\odot}\right) \text{ cm}$$

$$kpc = 3 \times 10^{21} \text{ cm}$$

$$= 10^{21.5}$$

$$v_{ssA} \sim 100 \text{ MHz} \quad \beta_{ma}^{4/3} R_{kpc}$$

$$r_g = 10^{(5+6.6)} = 10^{11.6}, \quad 10r_g = 10^{12.6}$$

$$10^{12} \text{ Hz} \sim 10^3 \text{ Hz} \cdot \beta_{ma}^{4/3} R_{kpc}$$

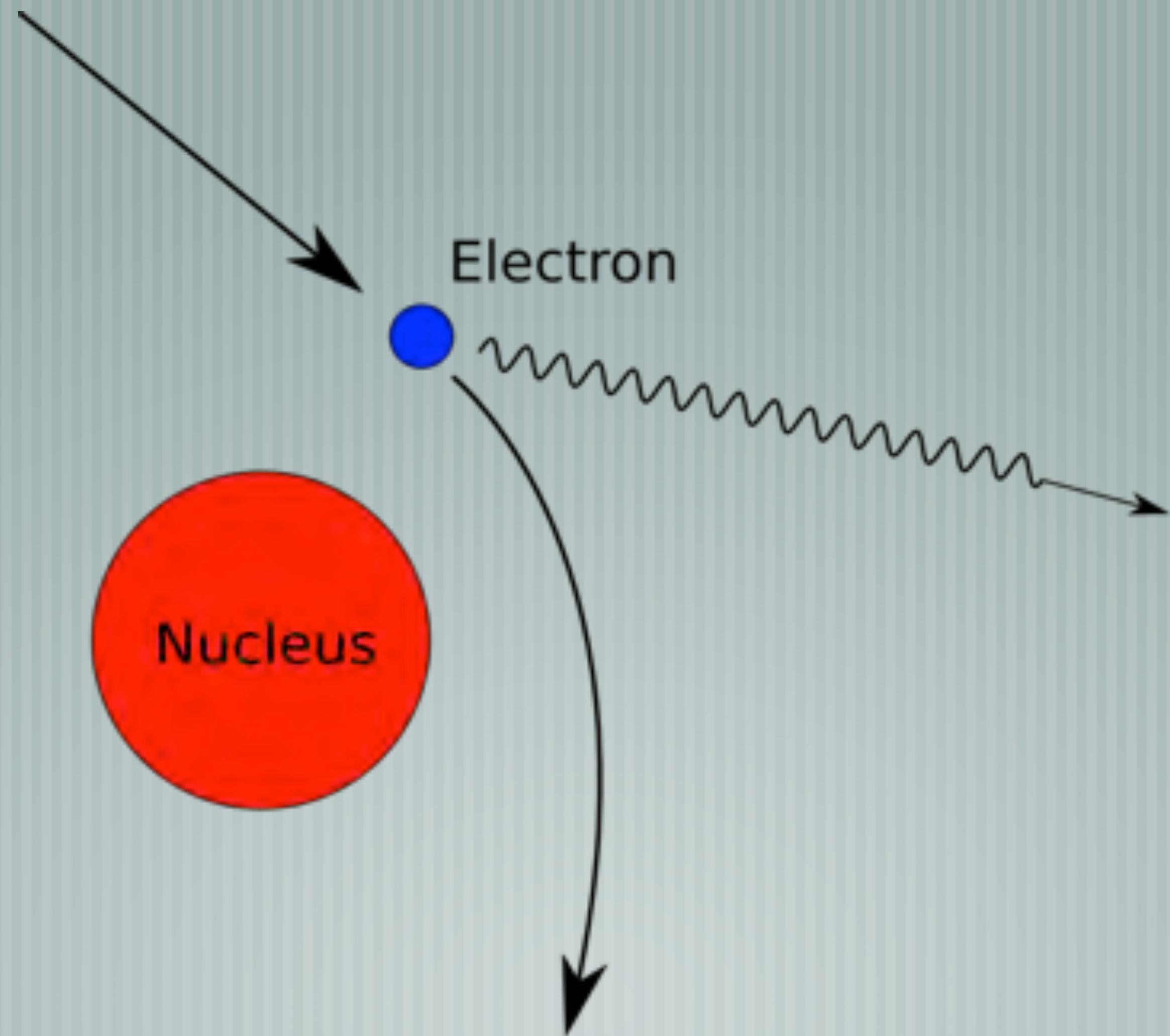
$$\beta_{ma}^{4/3} = 10^{12-8+3}$$

$$= 10^7$$

$$\beta_{ma} = 10^{7.3/4} \approx 10^{21/4} \approx 10^5 \text{ m/s}$$

$$\sqrt{100 \text{ g}}$$

Bremsstrahlung

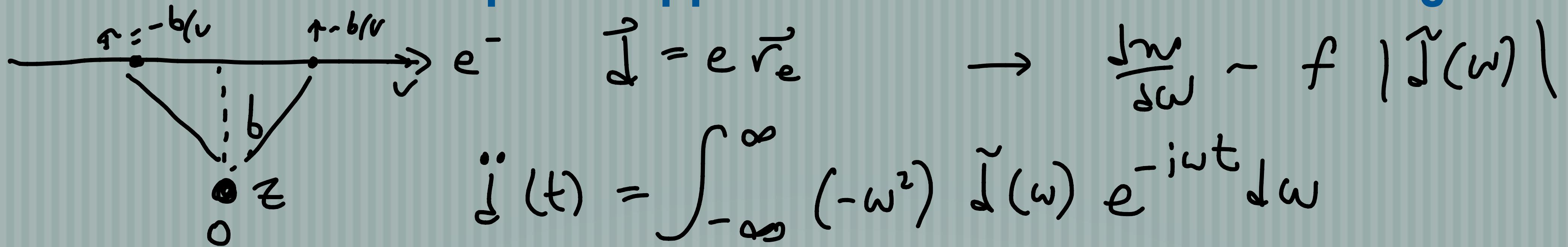


Bremsstrahlung

Radiation emitted as a particle de/accelerates in the Coulomb field of another charge

- * “Braking radiation”, also called “free-free” emission
- * QED process, but we can go pretty far with classical picture using dipole approximation for case of e-ion interactions
- * If interested in seeing the full QM derivations:
 - e-p: Karzas & Latter 1961 ApJ Suppl., 6, 167
 - e-e+: Haug 1987, A&A, 178, 292
 - e-e: Haug 1989, A&A, 218, 330

Semi-classical "dipole" approximation for bremsstrahlung



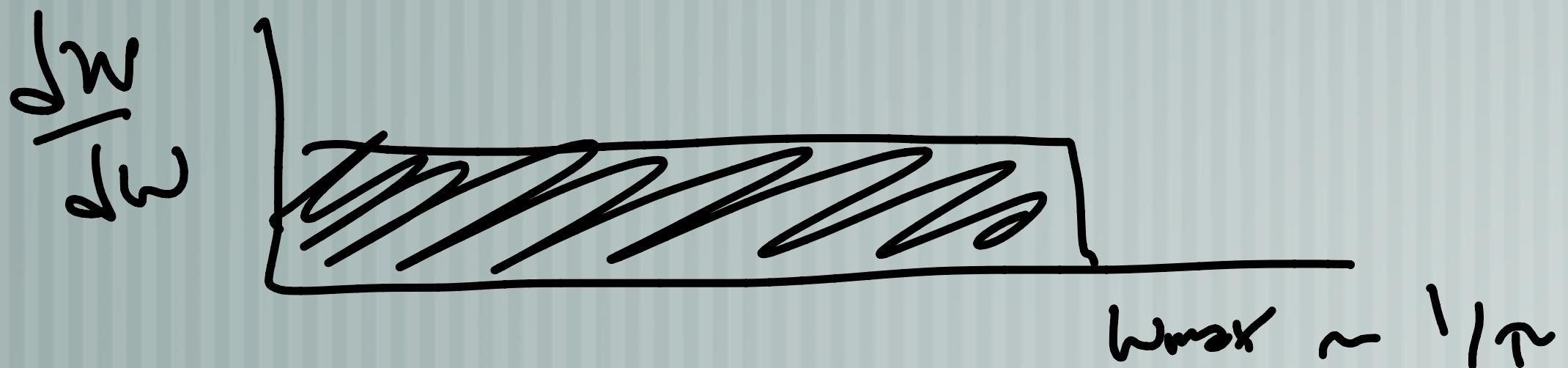
$$\ddot{j}(t) = \int_{-\infty}^{\infty} (-\omega^2) \tilde{J}(\omega) e^{-i\omega t} d\omega$$

$$(-\omega^2) \tilde{J}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \ddot{j}(t) e^{i\omega t} dt$$

assume $\tau \ll 1/\omega \Rightarrow \omega\tau \ll 1 \Rightarrow e^{i\omega\tau} \approx 1$ our decel. per.

$$(-\omega^2) \tilde{J}(\omega) = -\frac{e}{2\pi} \int_{-\tau}^{\tau} \dot{v}_e(t) dt$$

$$F = m \dot{v}_e = -\frac{ze^2}{r^2}$$

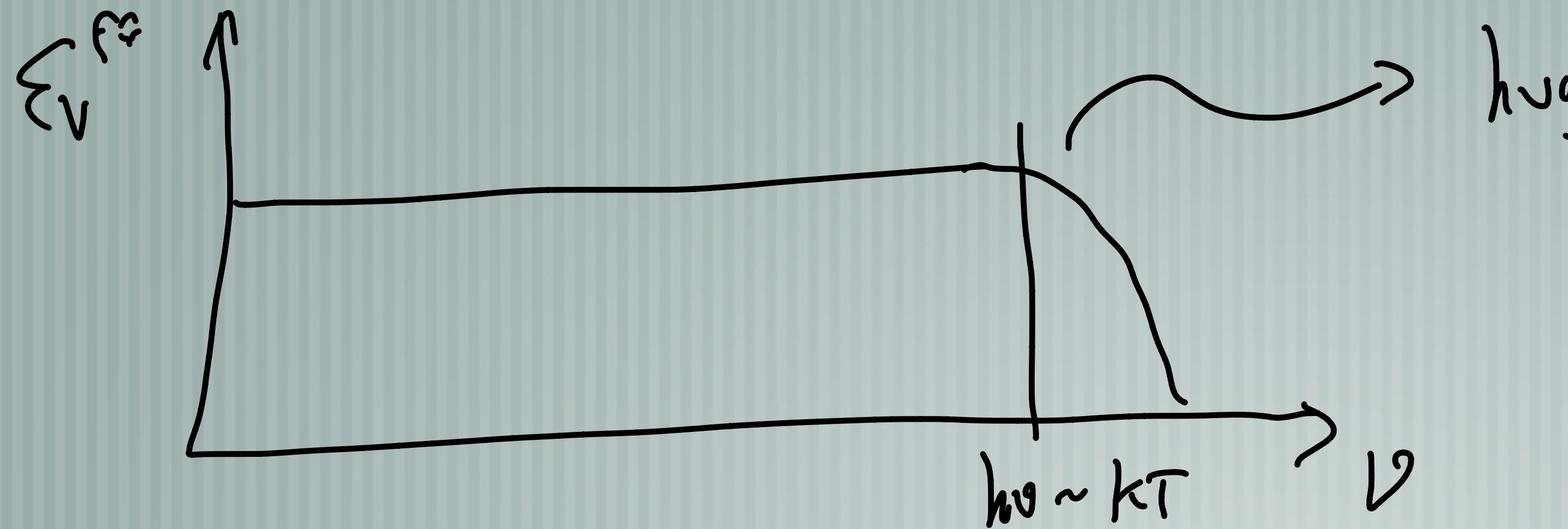


Application: astrophysical thermal plasma (relativistic:)

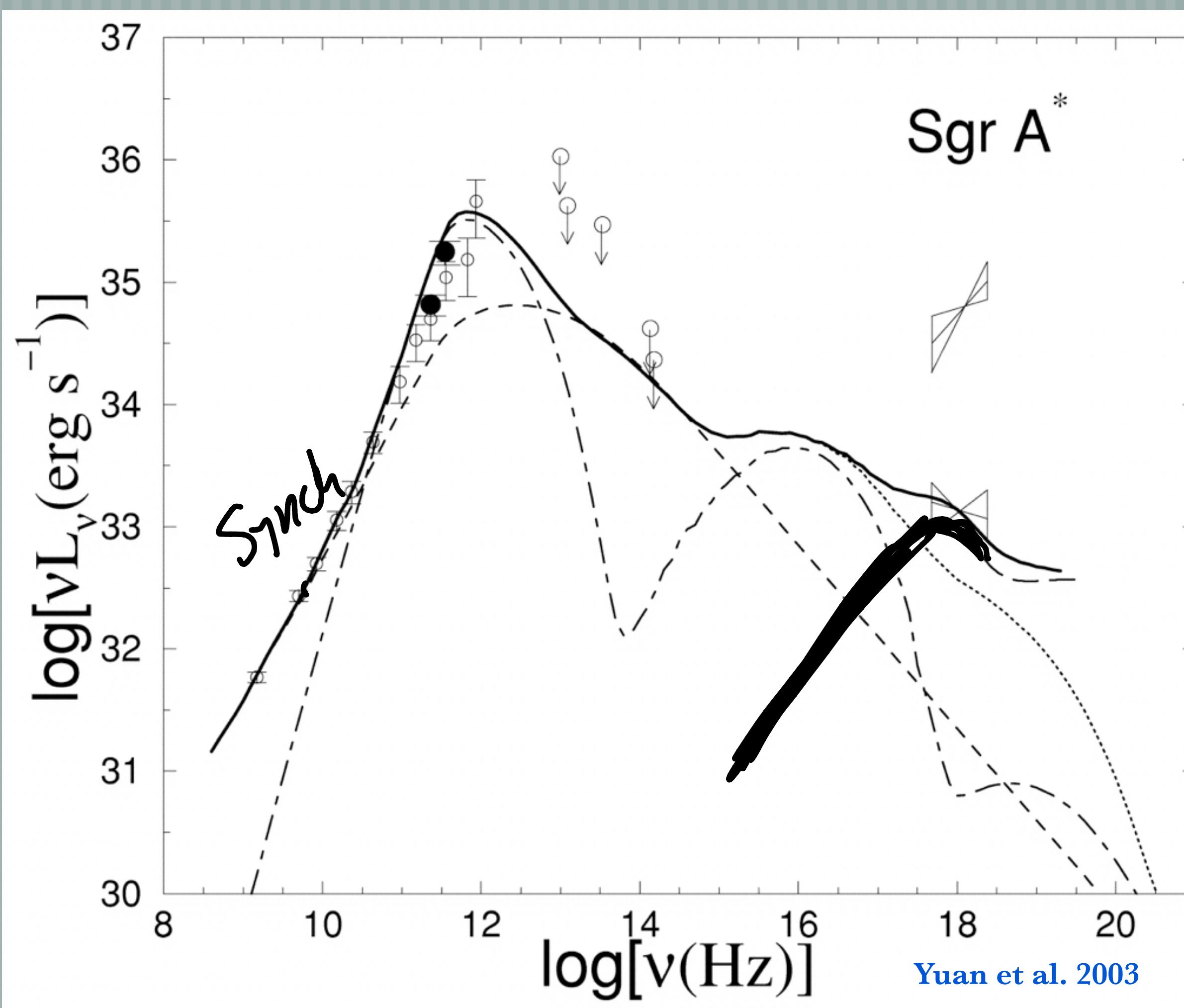
Thermal distribution in plasma, N_e, N_i, T

- average over M-B velocity distribution, incl. Gauntt (QM)

$$\epsilon_v^{ff} = 7 \times 10^{-38} Z^2 N_e N_i T^{-1/2} e^{-hv/kT} \tilde{g}_{ff} \text{ erg/cm}^3/\text{s/Hz}$$



Sgr A*: What's the T?



$$(K)\tau \sim 10^7 K \Rightarrow 10^{17} \text{ Hz}$$

$$\tau = 4 \times 10^7 \text{ K}$$

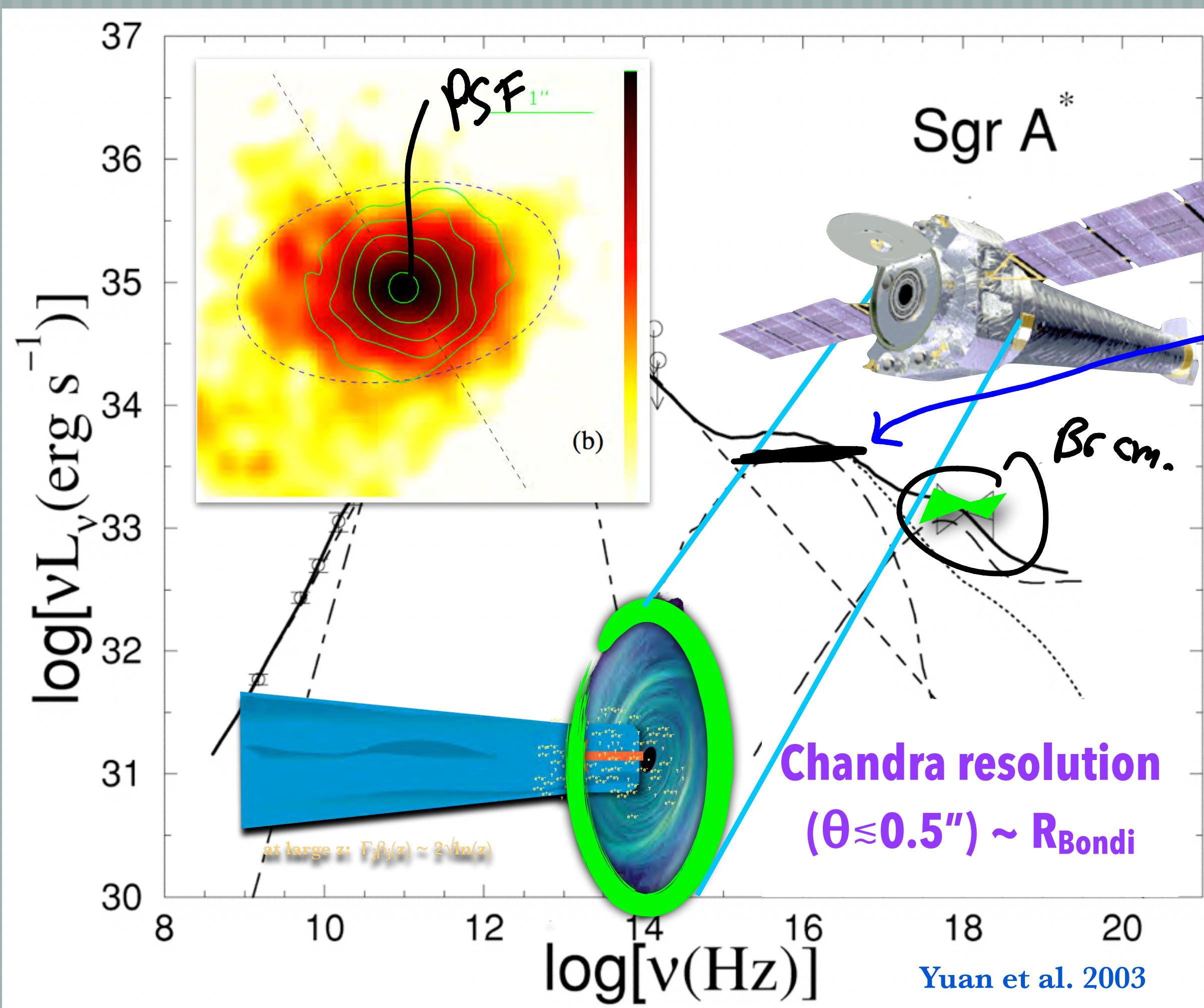
$$At r = 10^5 \text{ cm!}$$

How do we know that size?



Sgr A*: What's \dot{M} feeding the disk?

$$T \sim 4 \times 10^7$$



assume: $n_e = n_i \neq Z = 1$

$$10^{33.5} \text{ c}\gamma/\text{s} = \epsilon_{\nu}^{\text{ff}} \cdot \text{vol} \cdot V$$

$$\frac{4}{3} \pi r^3 \downarrow 10^{18}$$

Solve for n , using the formula:

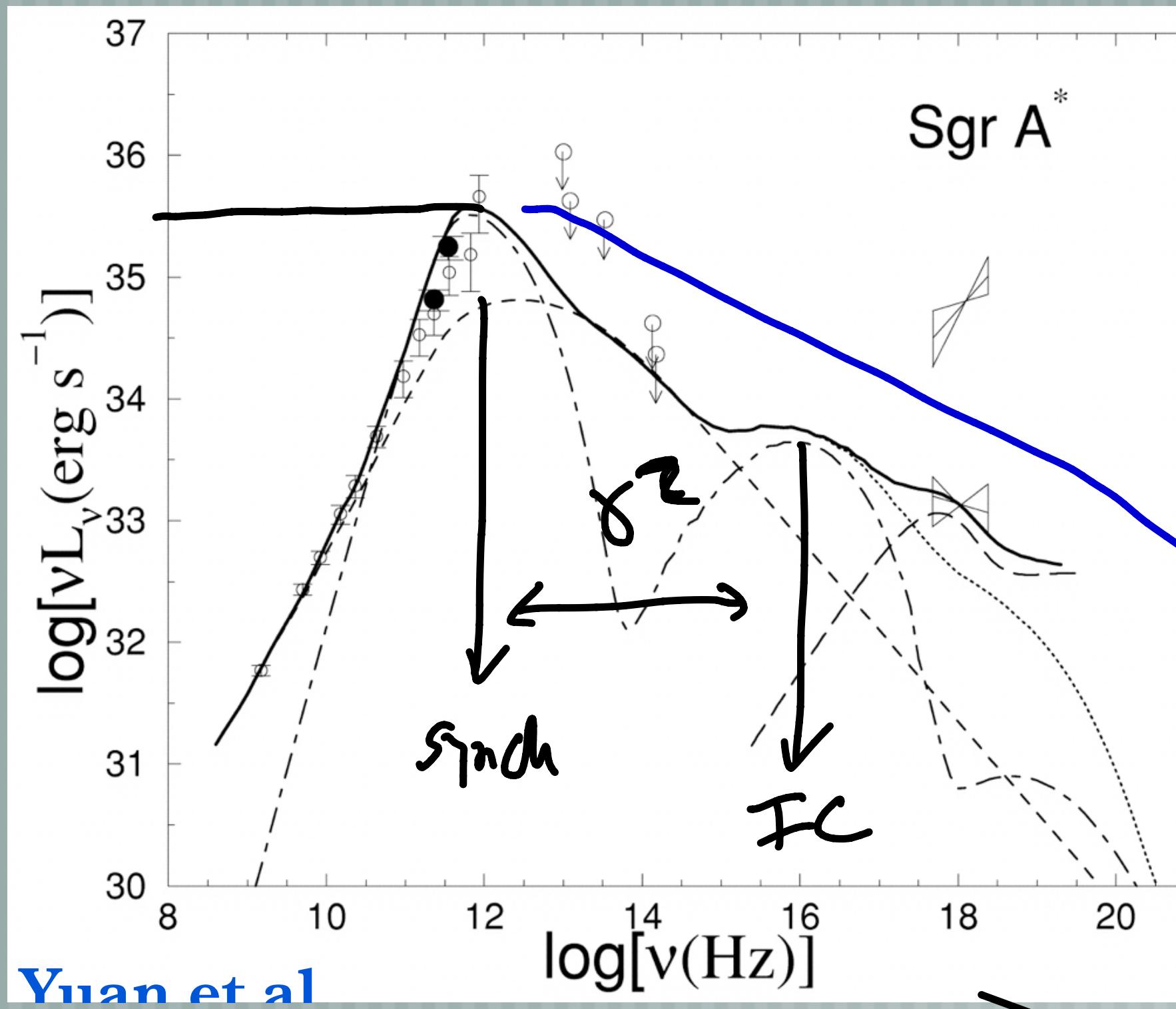
$$L_x \sim 10^{33} = 10^{0.85 - 38} n^2 \cdot 10^{-7.6/2} \cdot 10^{18} \cdot 10^{17.5 - 3} \cdot 10^{-10} \cdot \frac{[33 - 0.85 + 38 + 3.8 - 18 - 52.5 - 0.6]}{2}$$

$$n = 10^{\frac{[33 - 0.85 + 38 + 3.8 - 18 - 52.5 - 0.6]}{2}} \sim 1.5 \text{ cm}^{-3}$$

$$n = \frac{\dot{m}}{4\pi r^2 c_s m_p} \Rightarrow \dot{m} \sim 5 \times 10^{19} \text{ g m/s} \sim 10^{-6} M_\odot/\text{yr}$$

$c_s \sim 10^7 \text{ cm/s}$, local sound speed

Estimate temperature, return to synchrotron!



Sgr A* is in low state, has hot, puffy disk but lots just estimate the T_{in} of inflow using thin disk, $T \propto r^{-3/4}$

$$\frac{T_{in}}{T_{out}} \sim \left(\frac{r_{in}}{r_{out}}\right)^{-3/4} \sim \left(\frac{10^9}{10^5 r_g}\right)^{-3/4} \sim 10^{4 \cdot -3/4} = 10^3$$

So $T_{in} \sim 10^{7.6+3} = 10^{10.6} \text{ K}$, $\gamma_e = 1$ is $\sim 10^{9.6}$

So $\gamma_e \sim 10$ [This is about 10x too low but ok!]

$P_{tot, synch} \approx 10^{35.5} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 \frac{B^2}{8\pi} \cdot n \cdot v_{rel} @ 10^9 g$

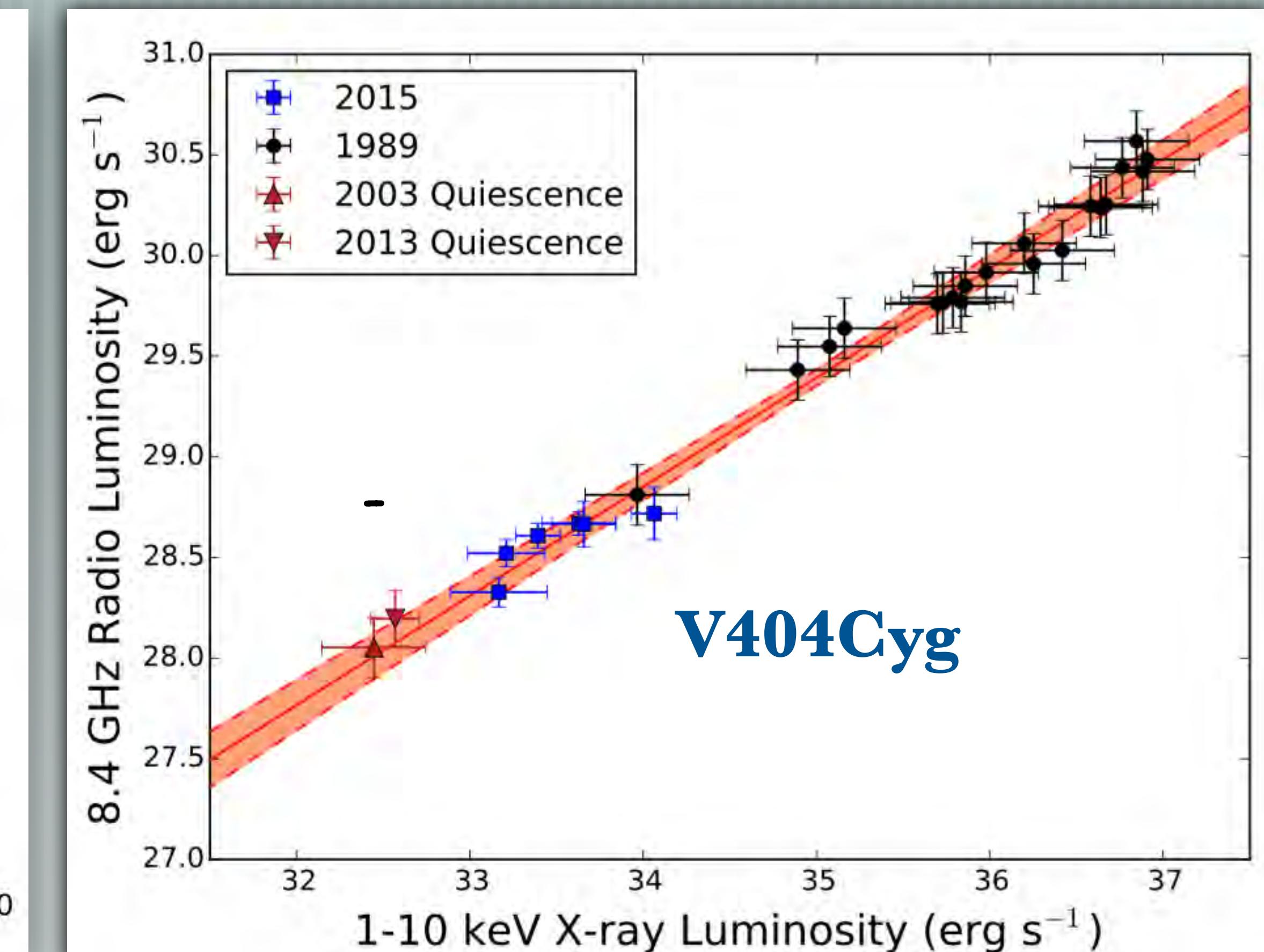
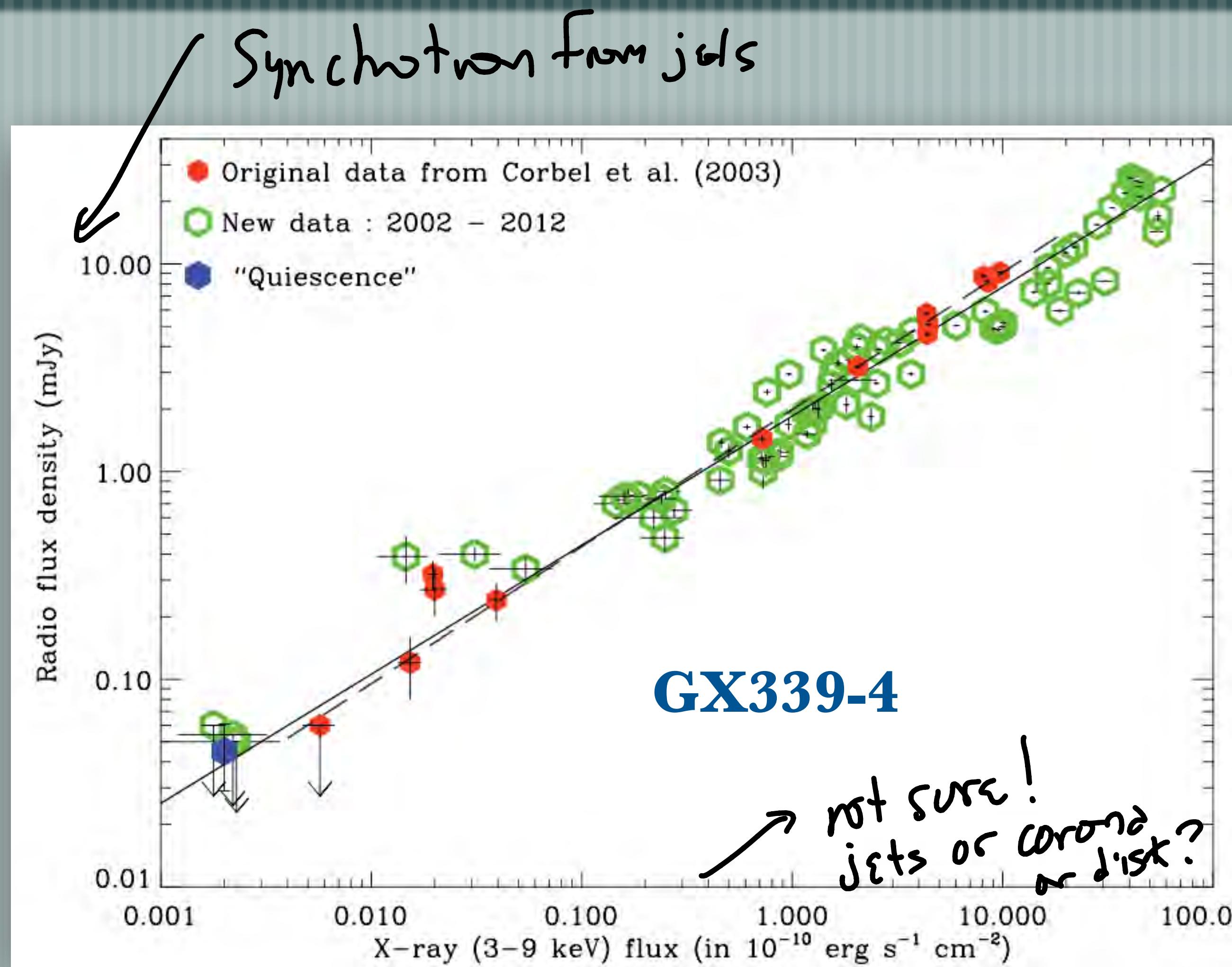
$\beta = 100 \text{ km/s}, \gamma = 10 :$

$$10^{35.5} = 10^{-24 + 10.5 + 2 + 4 - 1.4 + 0.5 + 3 \times 12.6} \cdot n$$

$n \sim 10^4 \text{ cm}^{-3} \rightarrow$ bit low, but given θ magnitude not bad, w/in factor of 10?

point: just from spectrum & simple power of 10 calculations, we got within a small factor of the physical conditions of a real black hole!!

Black hole XRBs have “built in” radio/Xray coupling



Radio/Xray correlation = ratio of radiative efficiencies

$$L_R \sim L_X \xrightarrow[\text{range of observed slopes}]{0.55 - 0.7}$$

$$m^{17/12} \sim m^q \cdot (0.55 - 0.7)$$

$$\Rightarrow q \sim \frac{17/12}{0.55 - 0.7} \sim \frac{1.4}{0.55 - 0.7} \sim 2 - 2.5 \Rightarrow \text{has to be radiatively inefficient process!}$$

Synchrotron: $\propto n \cdot B^2$, both $\propto m$ so $q = 2$

($L \sim m$ is efficient)

SSC: $n \cdot nB^2 \Rightarrow q \leq 3$, since $\tau \leq 1$ will be $2 - 3$

IC: $n \cdot m \Rightarrow q = 2$

Bremss: $n \cdot n$, both $\propto m$ so $q = 2$

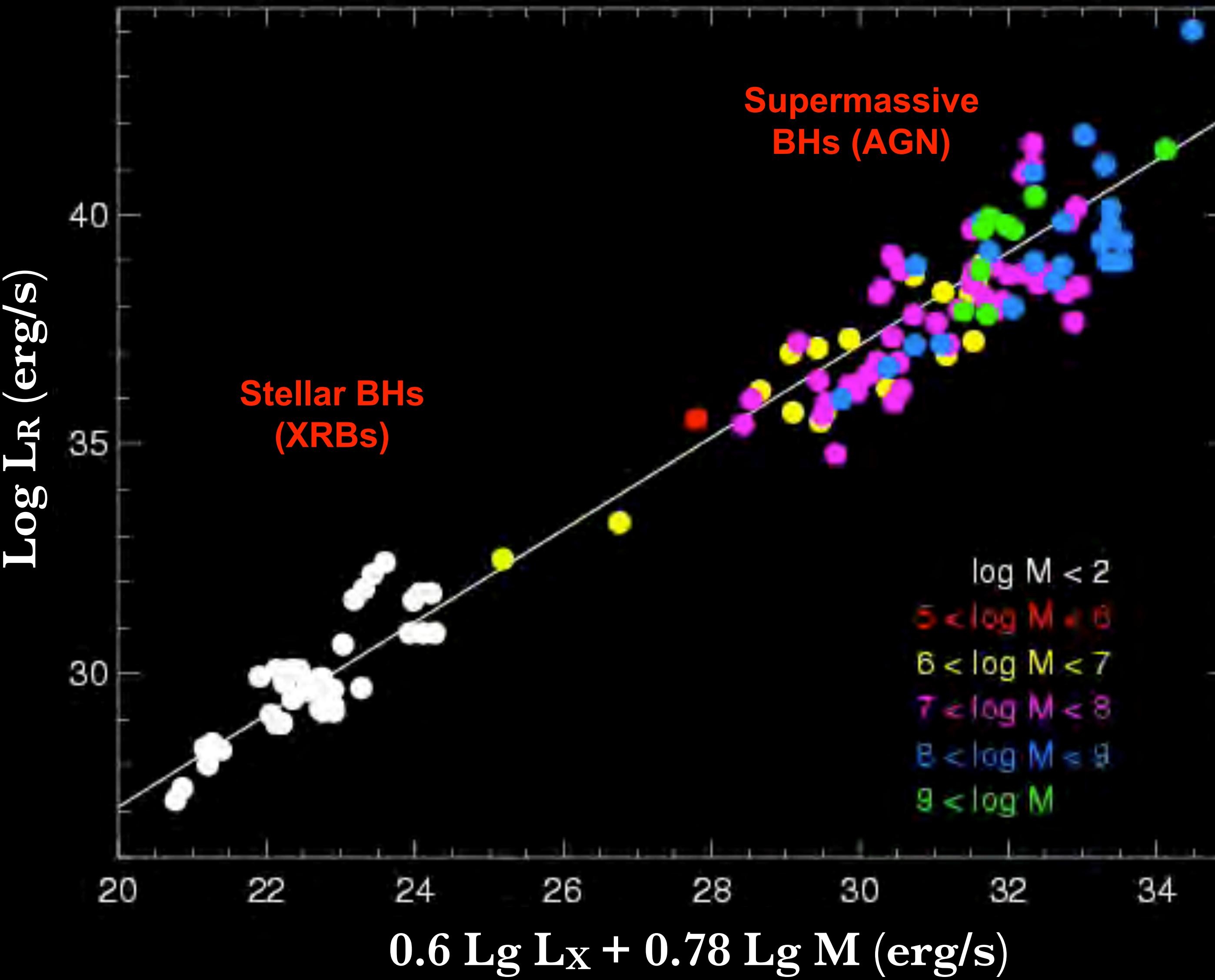
From synchrotron theory plus jet model
conserving particle + magnetic energy:

$$L_R \sim m^{17/12} \text{ in optically thick (flat spectrum) regime}$$

$$L_X \sim m^q \Rightarrow \text{we want } q \text{ to be}$$

But classic Shakura + Sunyaev disk won't work!

Fundamental Plane of Black Hole Accretion: connecting (\dot{M}) black holes of all masses



(SM et al. 2003; Merloni, Heinz & diMatteo 2003; Falcke, Körding & SM 2004; SM 2005; Merloni et al. 2006; Kording et al. 2006; Gültekin et al. 2009; Plotkin, SM et al. 2011)