Orbit Notes

Smadar Naoz

Why Orbits matter?

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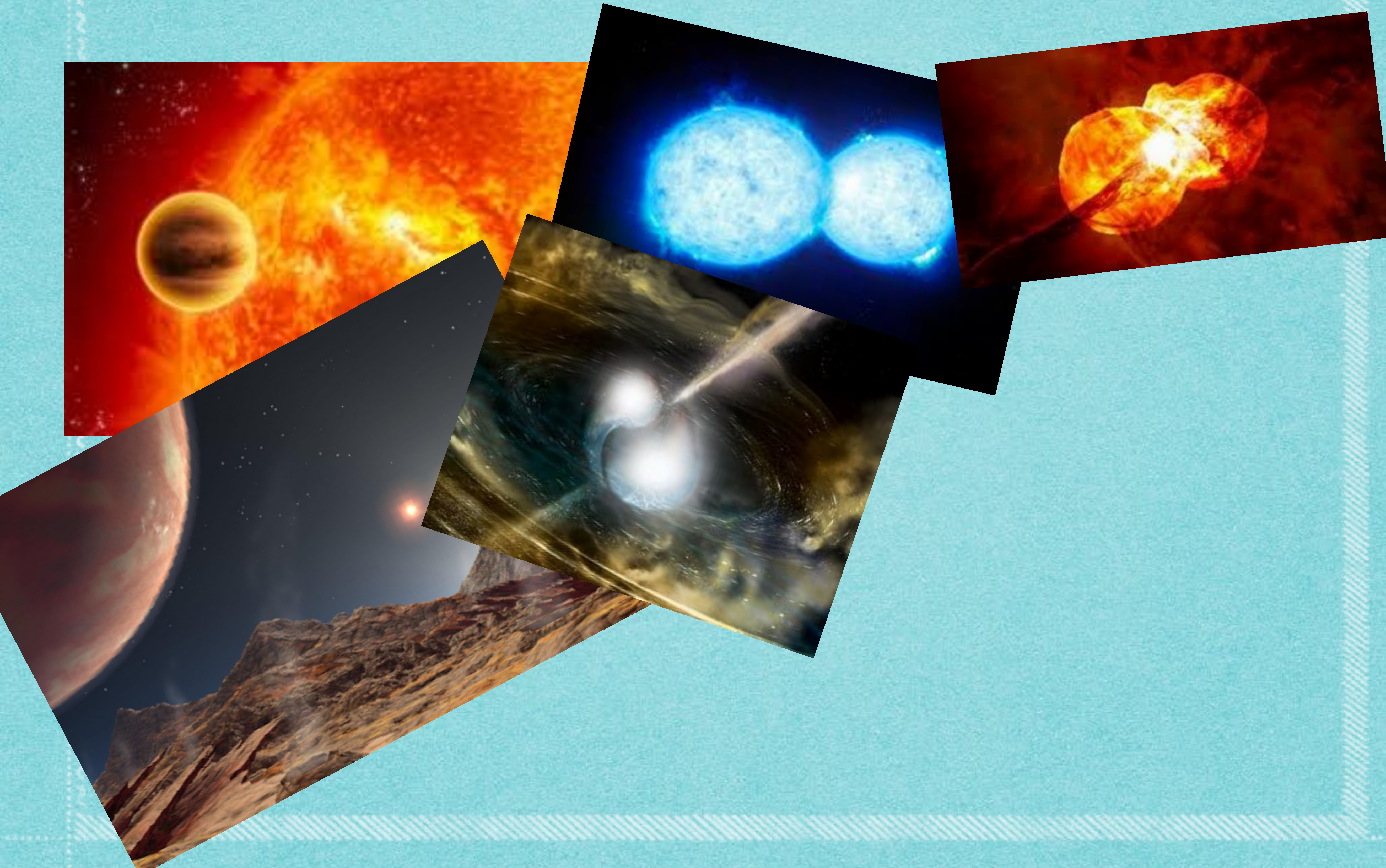

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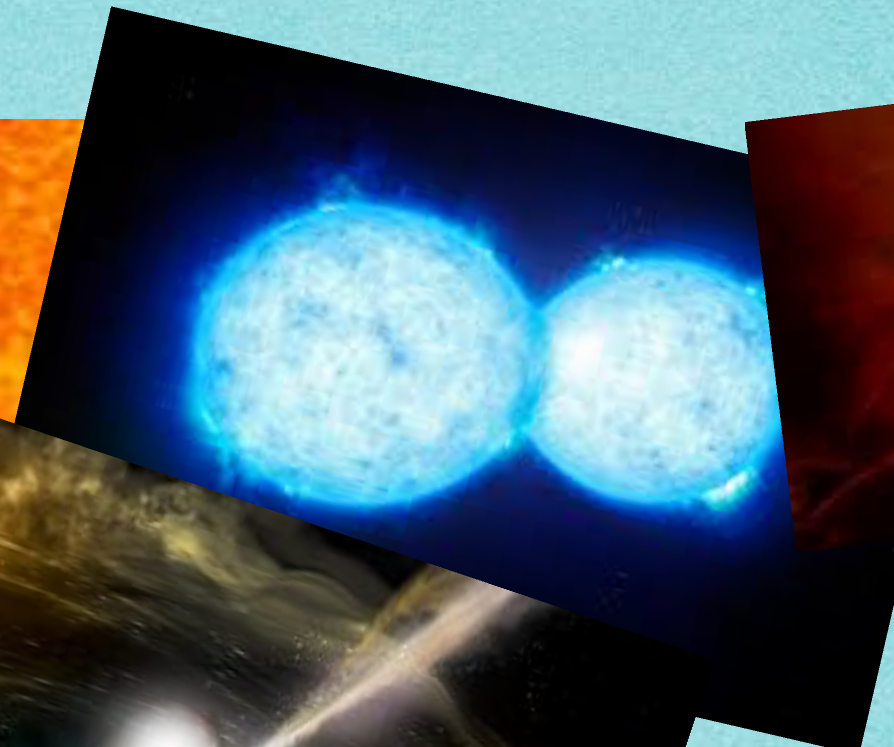

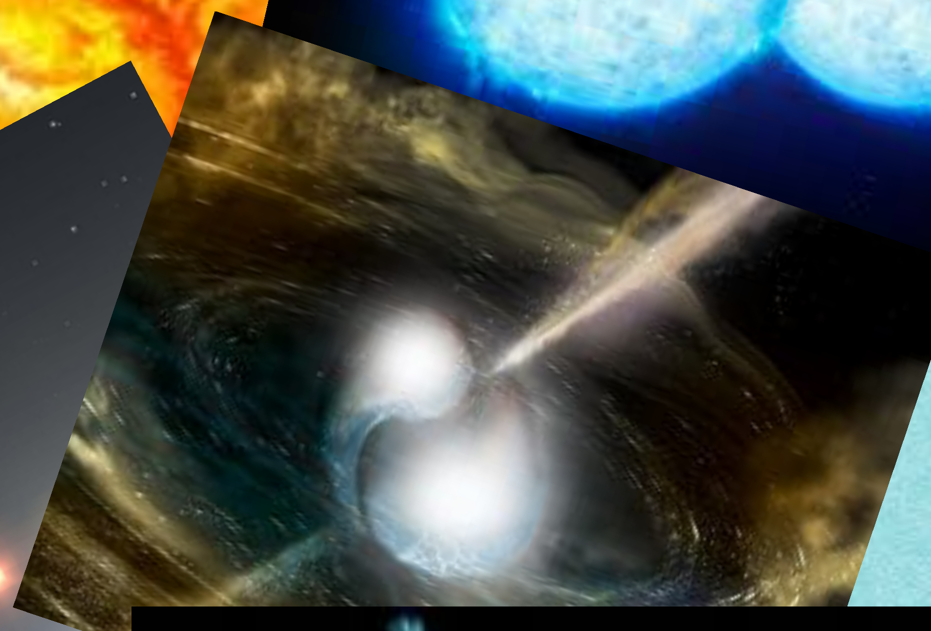

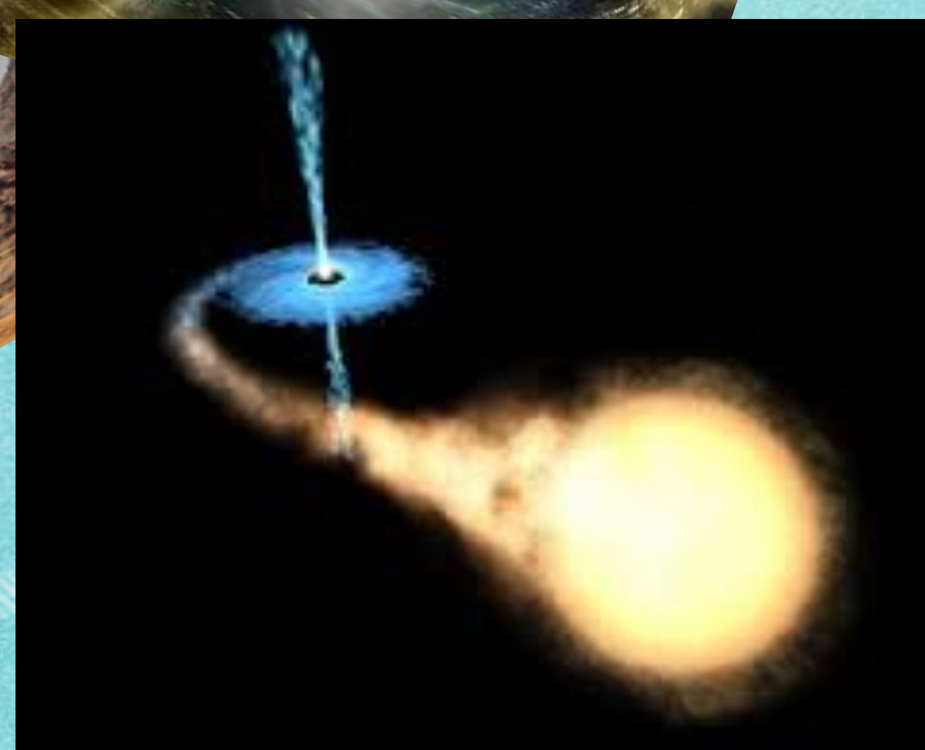

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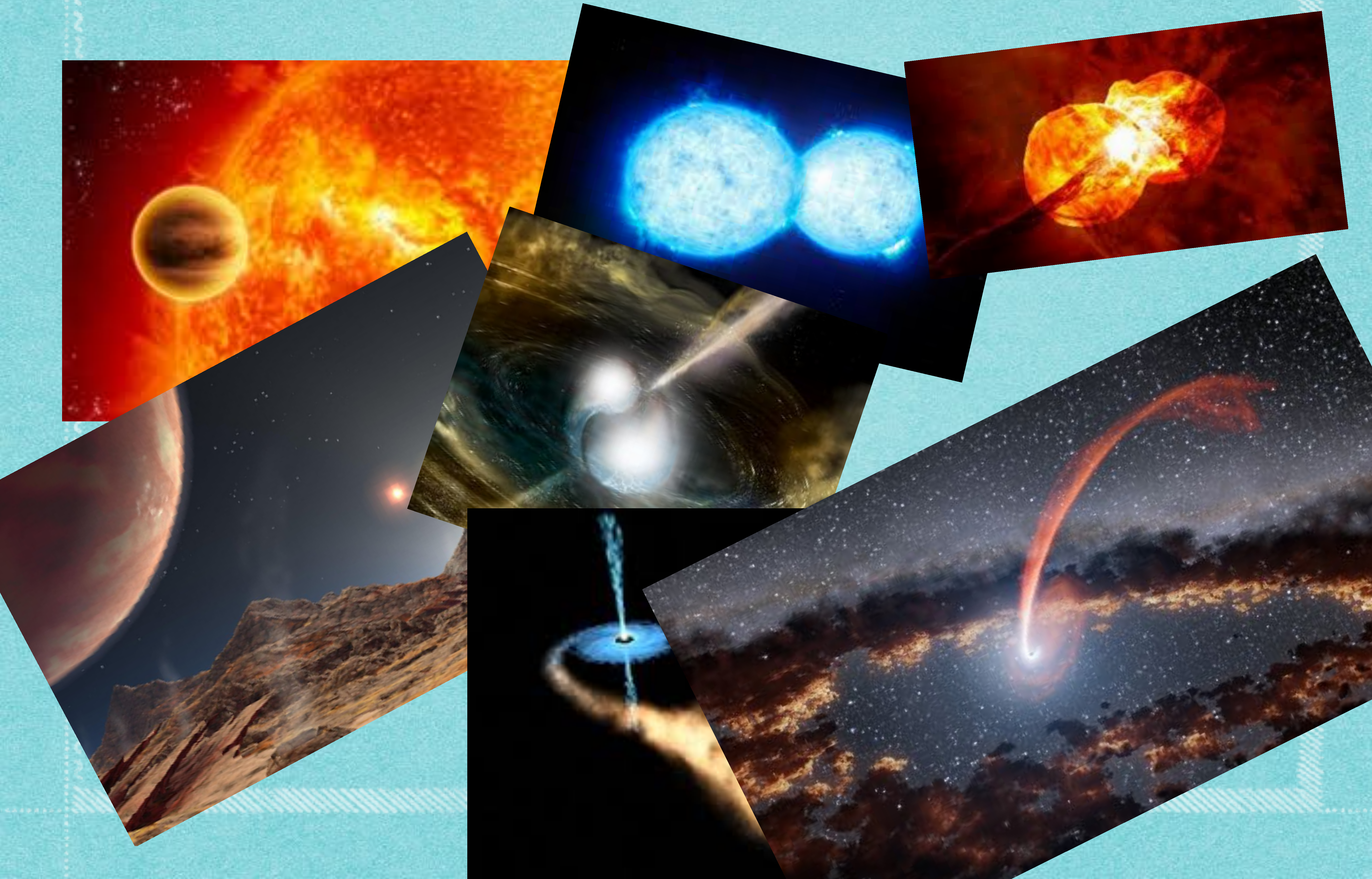

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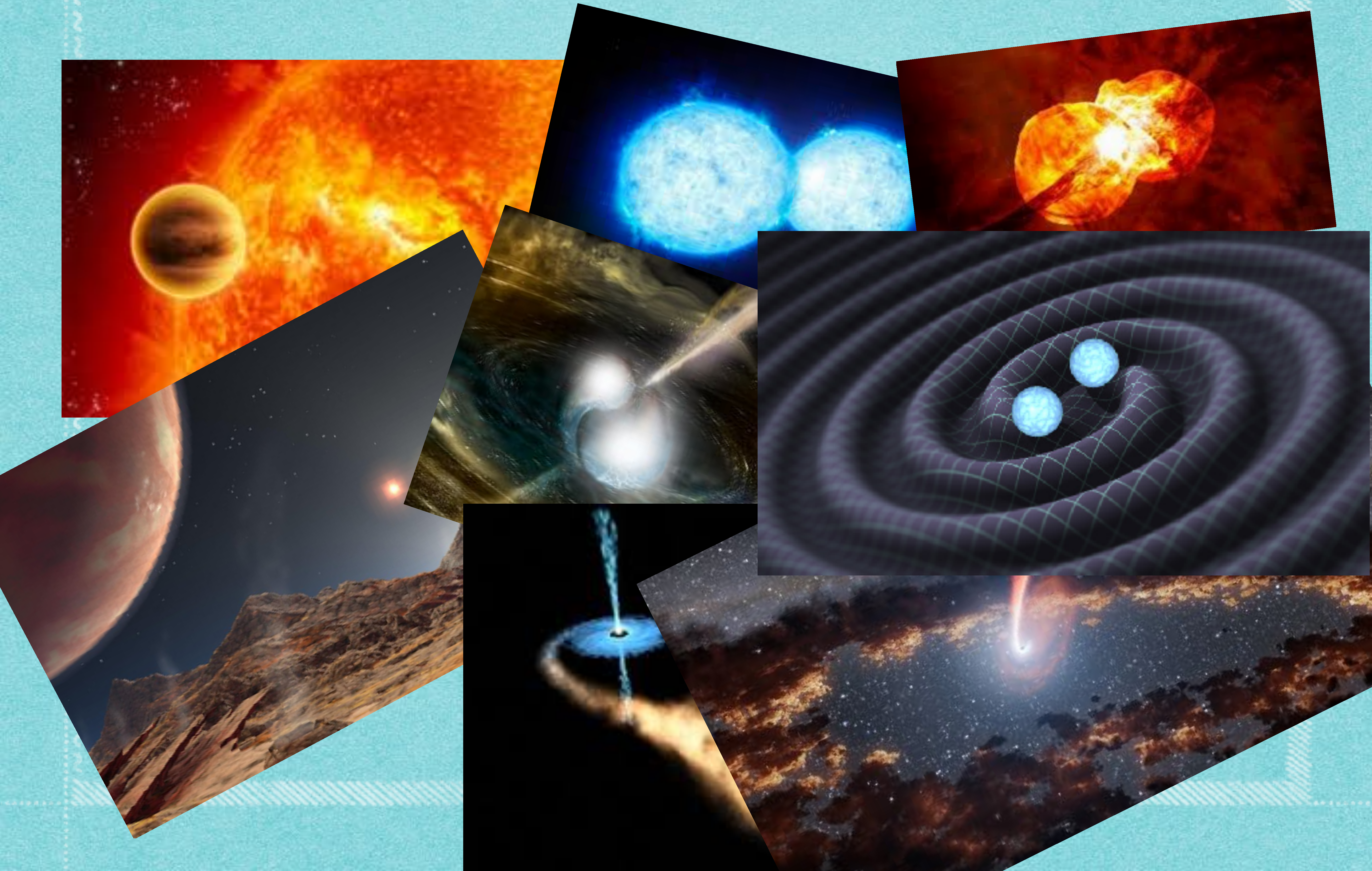
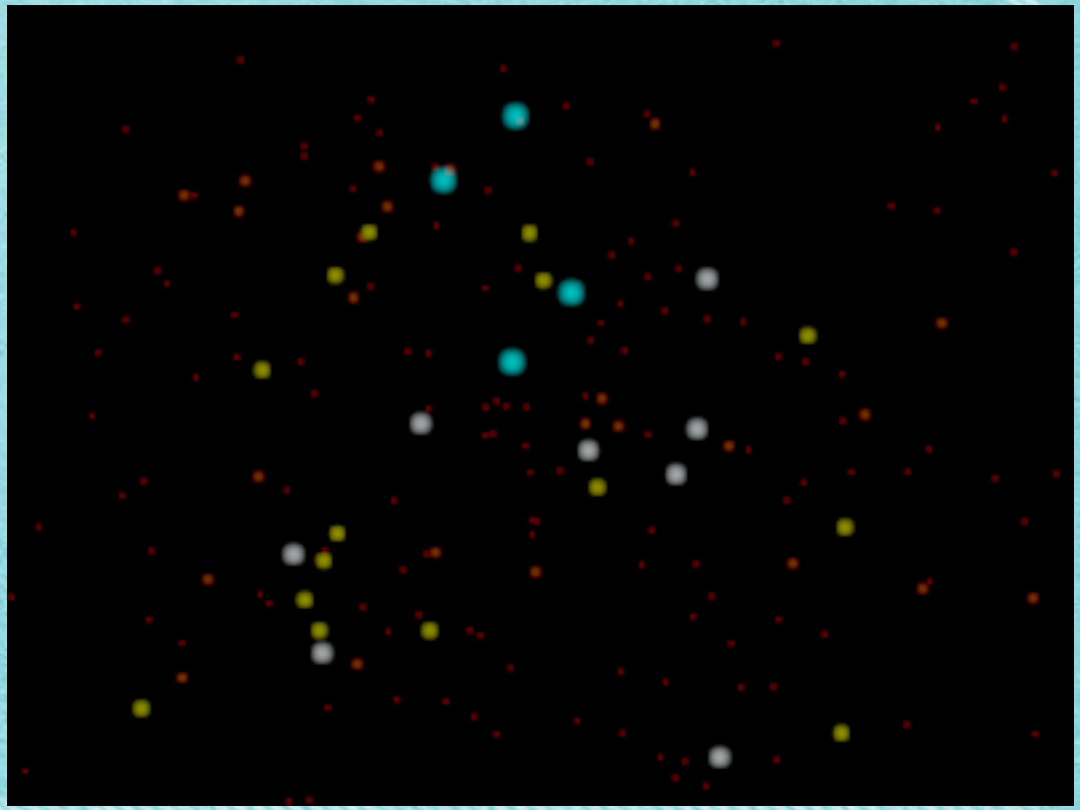


Things move in space



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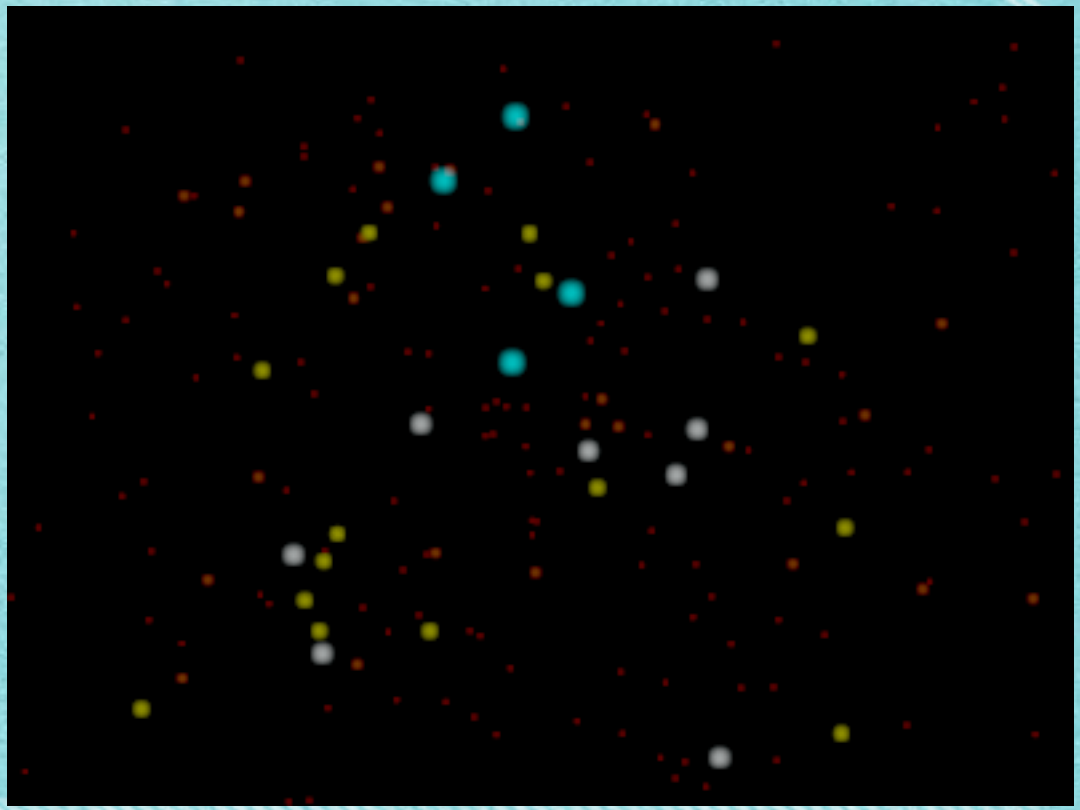

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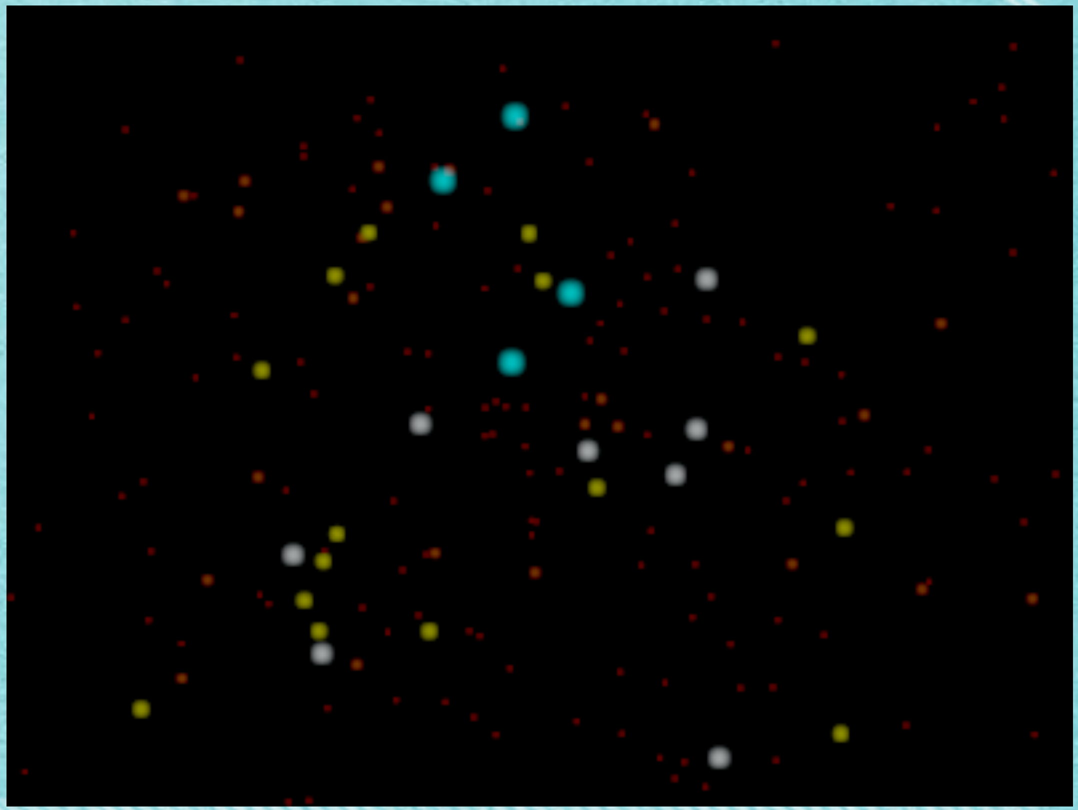
The most general problem to describe a moment of body in galaxy/star cluster/etc

$$\ddot{\vec{r}} = \vec{F} = -\vec{\nabla}\Phi$$

Assume Φ is smooth



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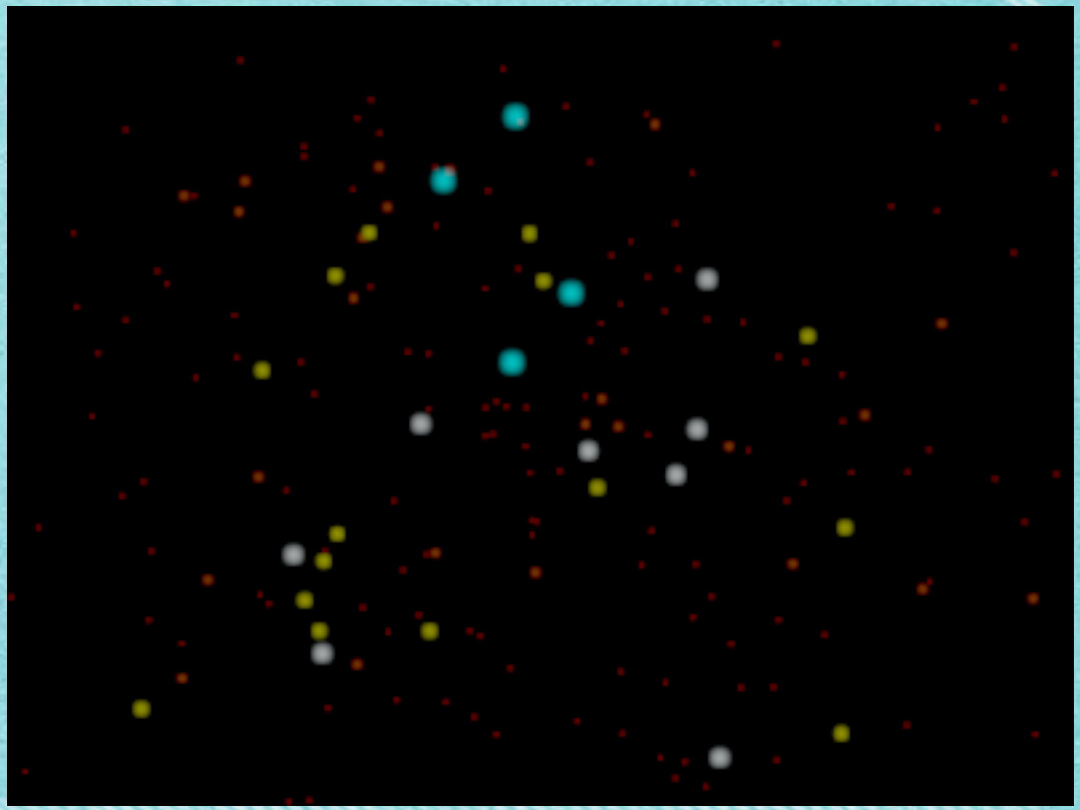

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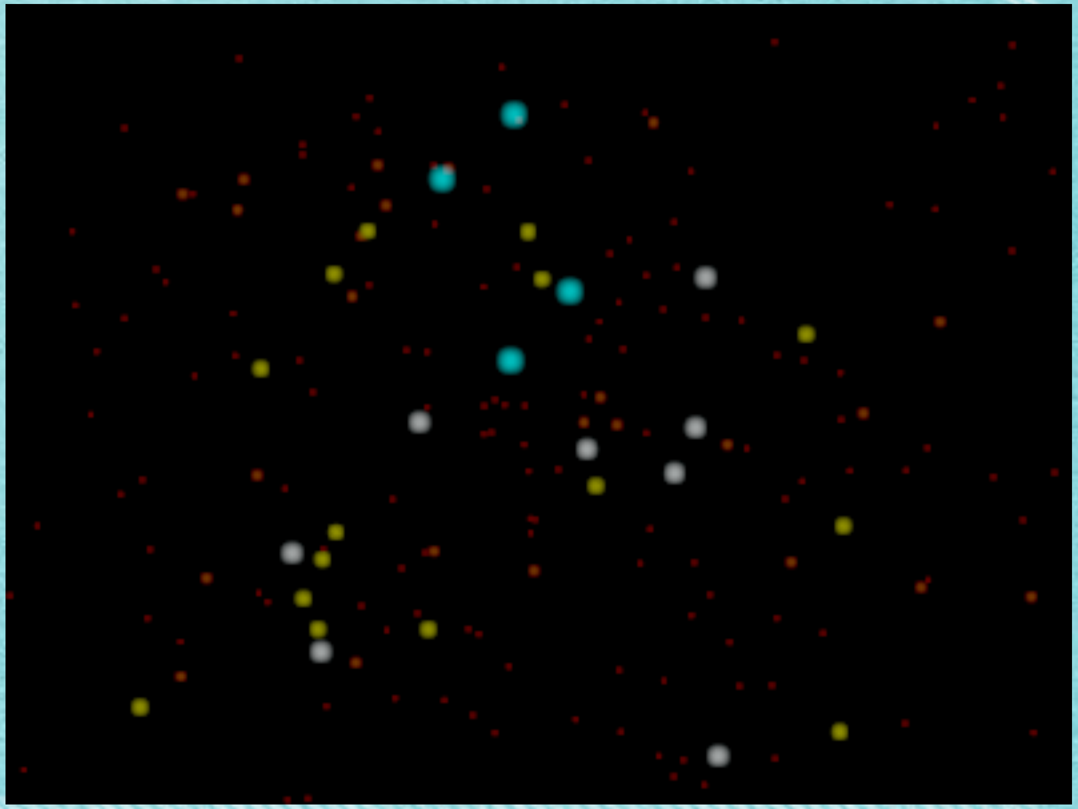
A: Neglecting interactions/encounters with individual particles



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Spherical potential - famous conserved quantity

$$\ddot{\vec{r}} = \vec{F}(r) = -\vec{\nabla}\Phi(r)$$

So:

$$\ddot{\vec{r}} = F(r)\hat{r}$$

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Conservation of angular momentum

$$\frac{d}{dt}\overrightarrow{L} = \frac{d}{dt}(\vec{r}\times\overrightarrow{v}) = \dot{\vec{r}}\times\overrightarrow{v} + \vec{r}\times\ddot{\vec{r}}$$

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Q: What does it means that angular momentum is conserved?

Q: Orbits are confined to a plane

Angular momentum conservation

- symmetry under rotations (isotropic in space)

  ➪ conservation of angular momentum

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Emmy Noether (1882-1935)

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Polar coordinates

Some algebra

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$$\mathbf{L} = \mathbf{r} \times \dot{\mathbf{r}} = \begin{vmatrix} \hat{R} & \hat{\theta} & \hat{z} \\ R & 0 & z \\ \dot{R} & R\dot{\theta} & \dot{z} \end{vmatrix}$$

$$L_z = R^2\dot{\theta}$$

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$$L_z = R^2\dot{\theta} \longrightarrow L_z = \text{const}$$

integral of motion

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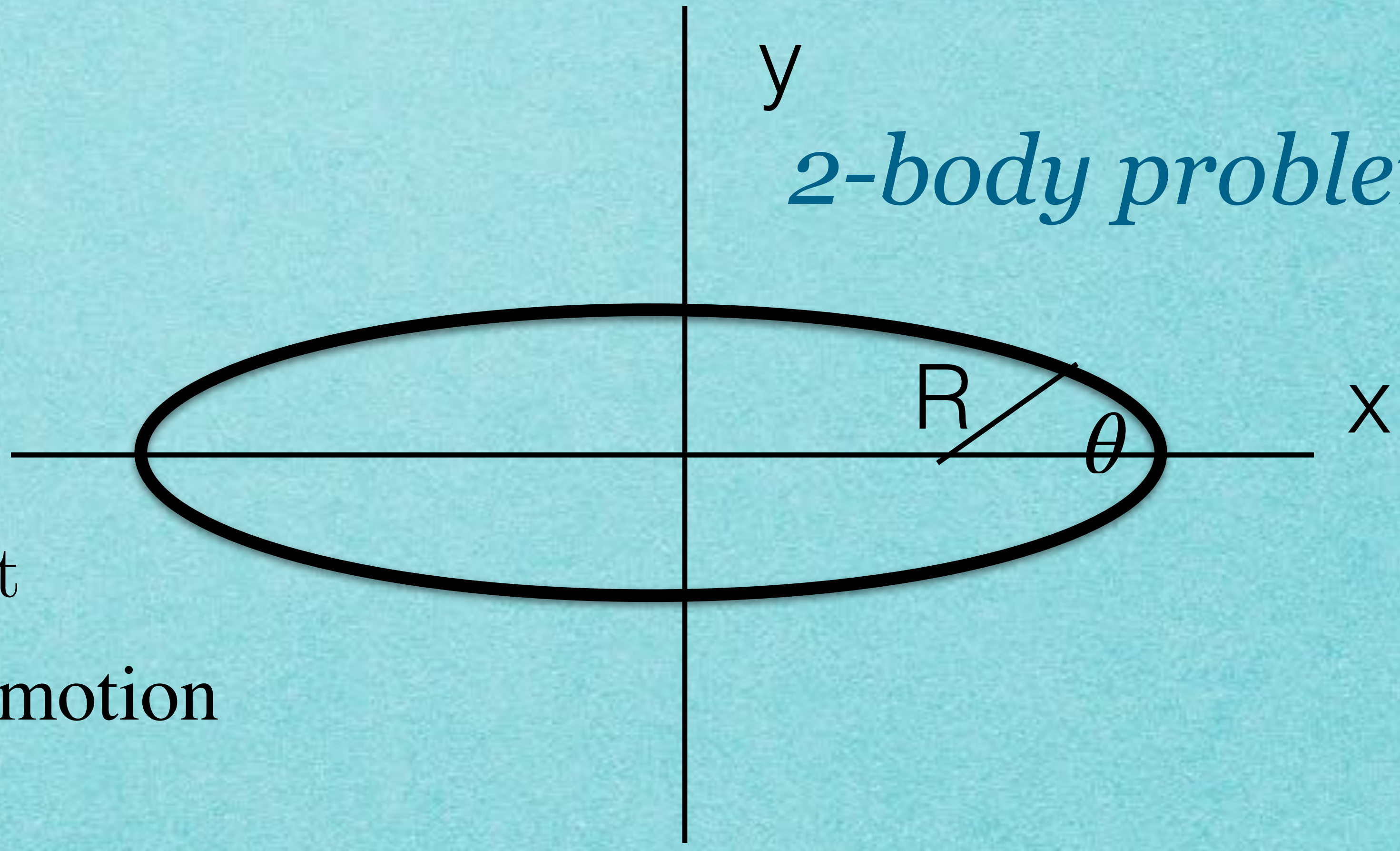
2-body problem



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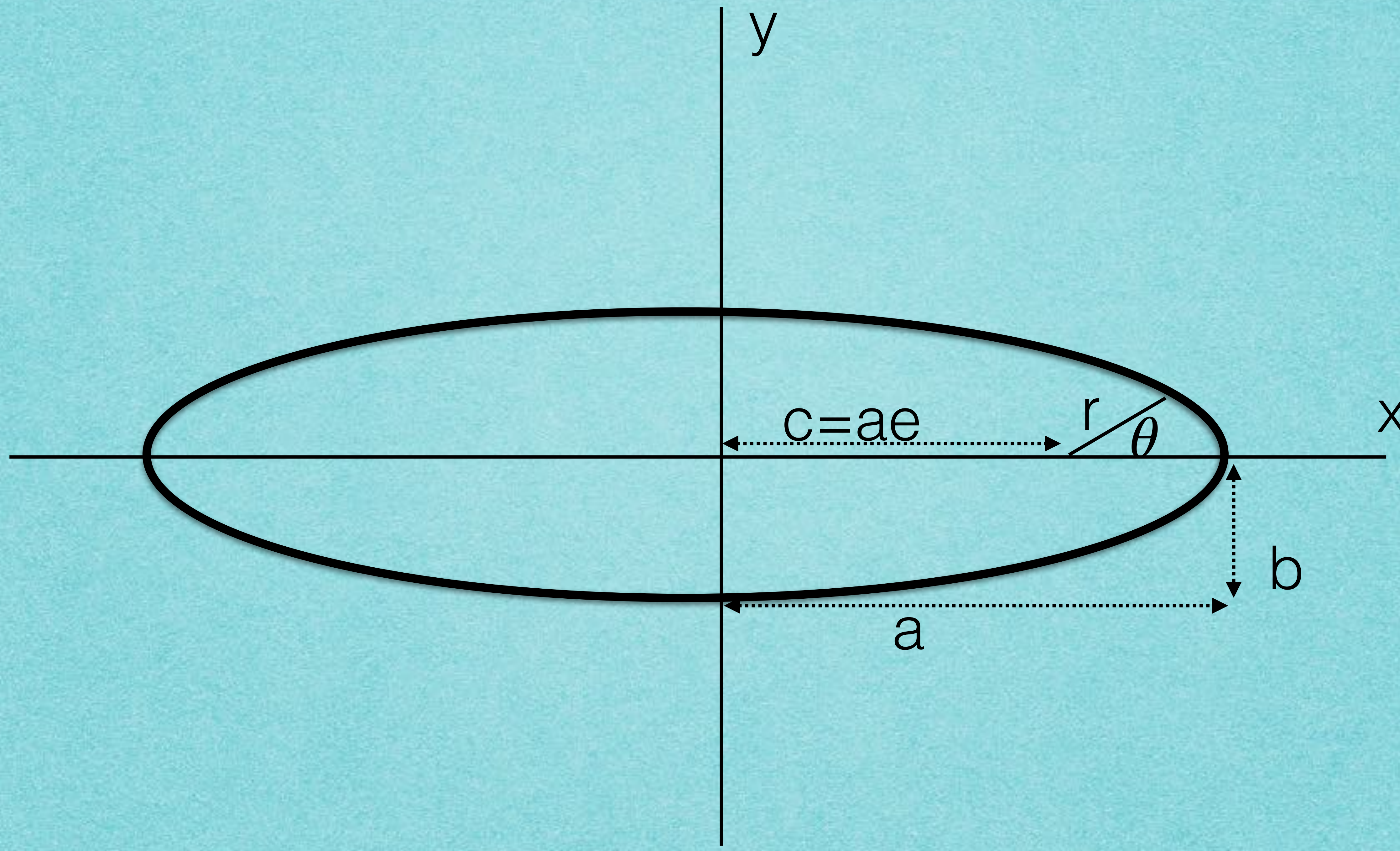
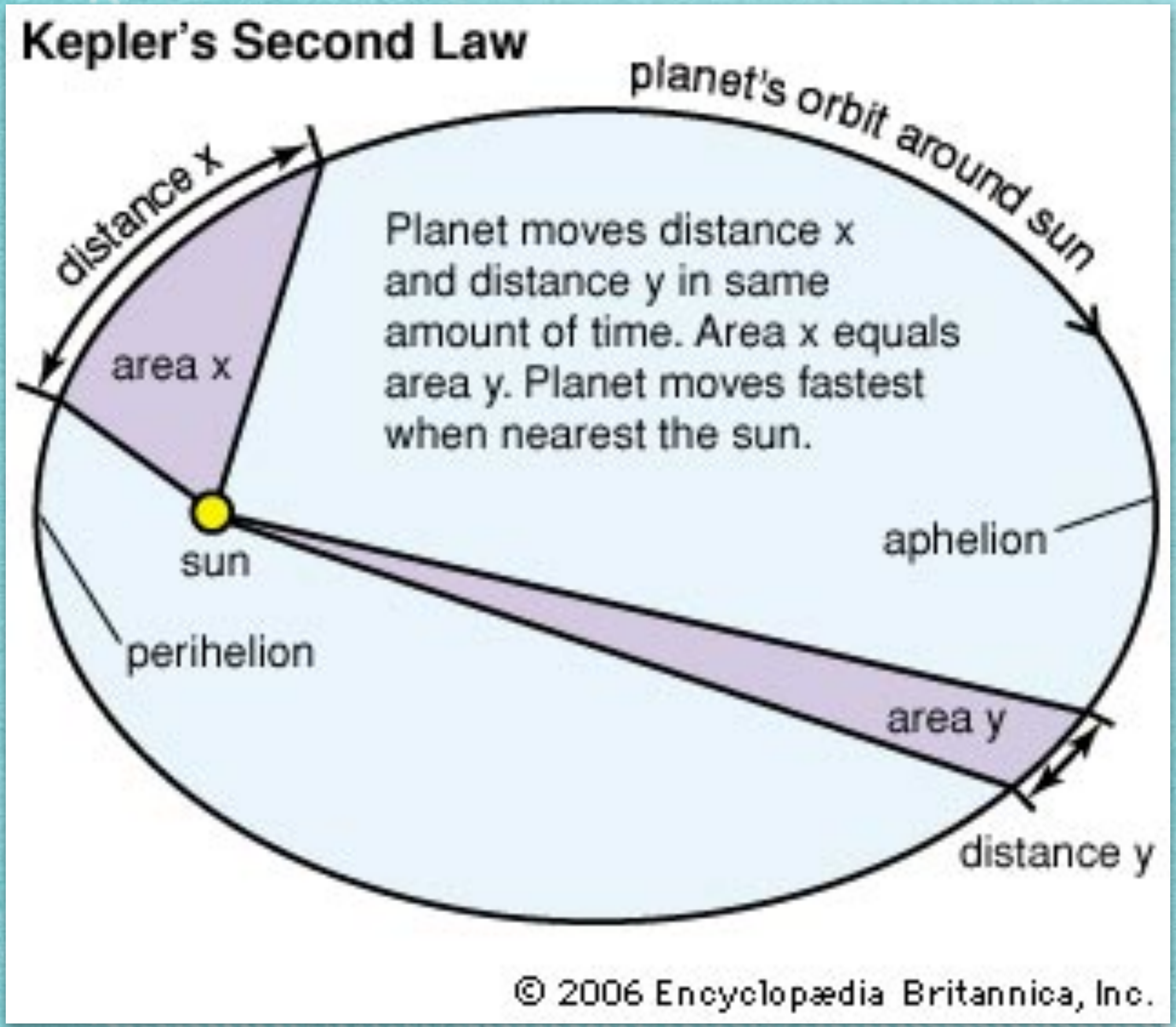
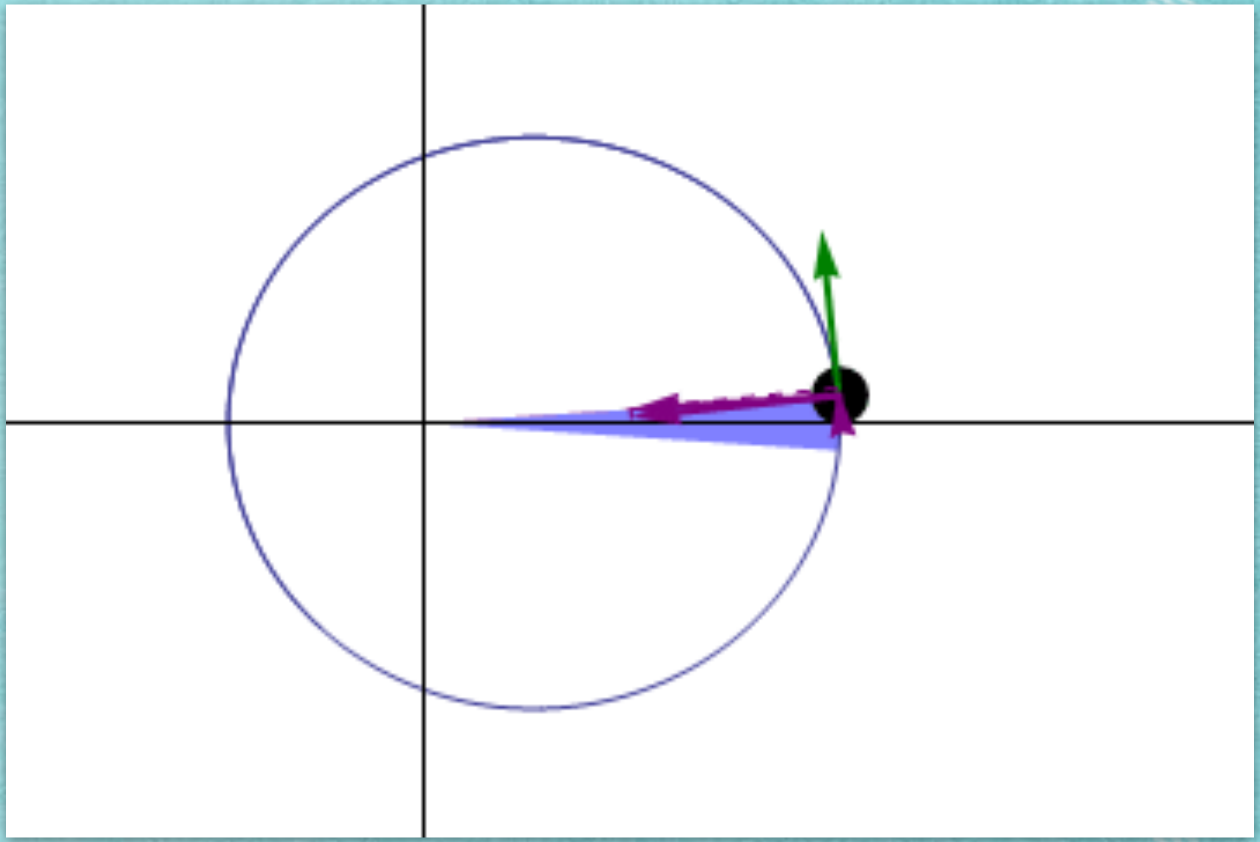


2-body problem

Kepler's second law

equal areas are swept in equal times





The same (blue) area is swept out in a fixed time period. The green arrow is velocity. The purple arrow directed towards the Sun is the acceleration. The other two purple arrows are acceleration components parallel and perpendicular to the velocity.

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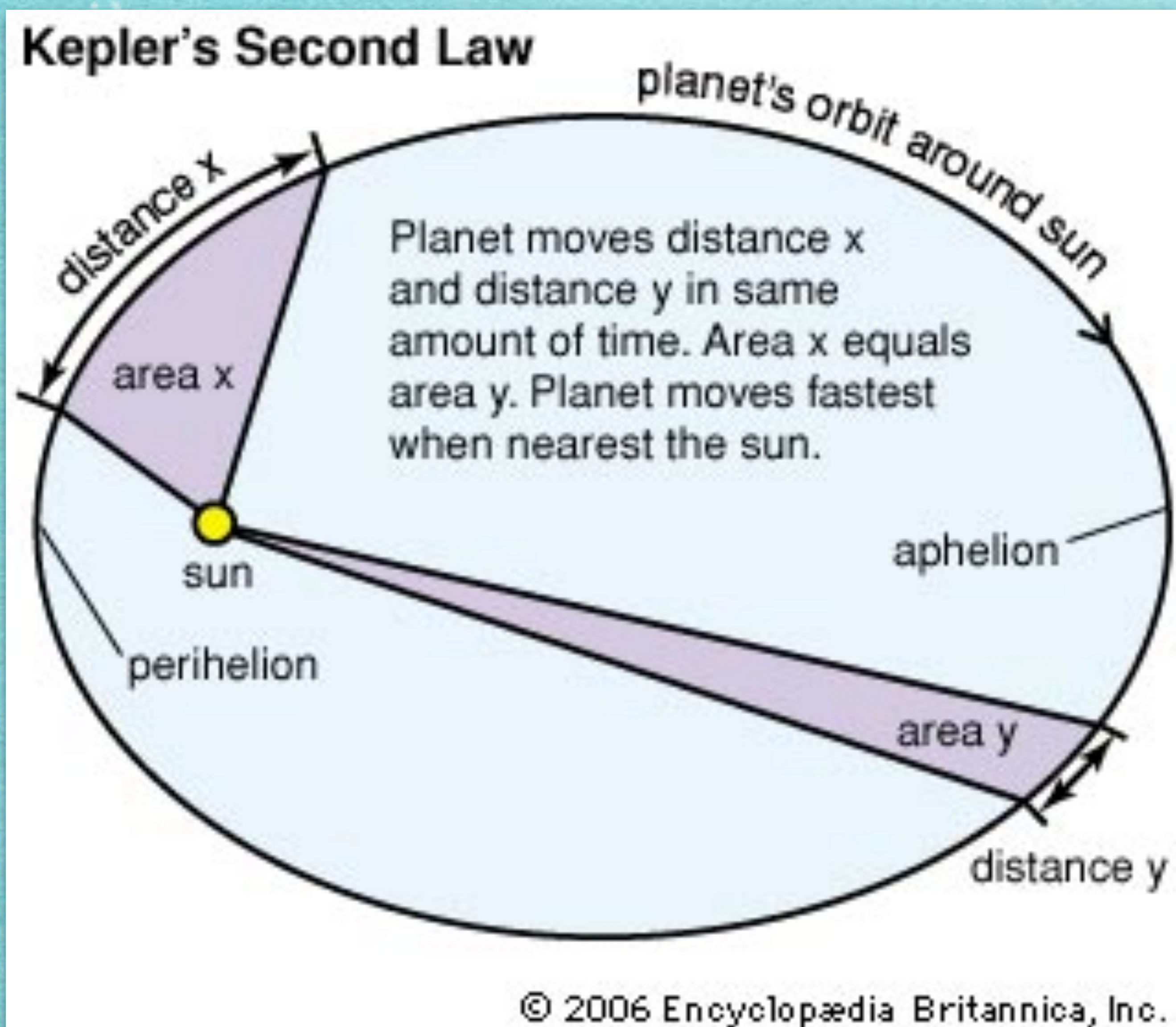

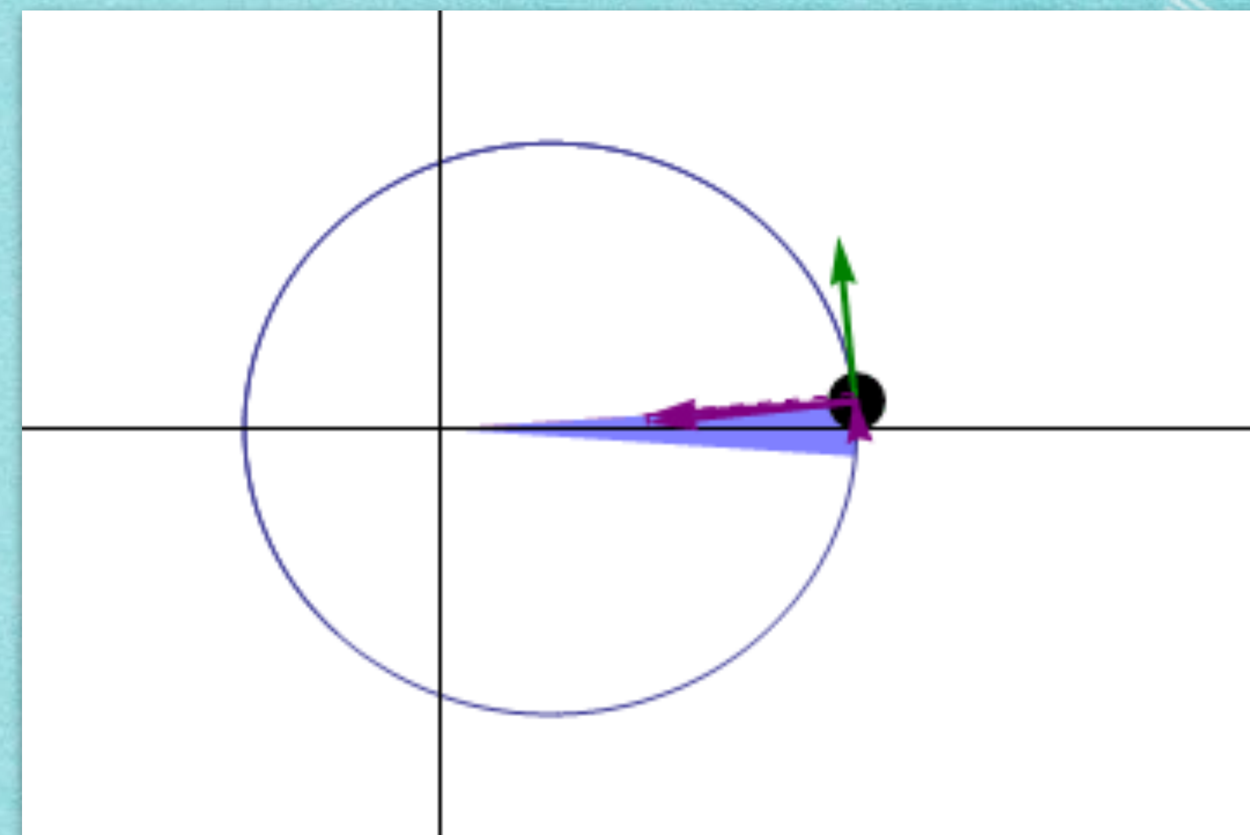

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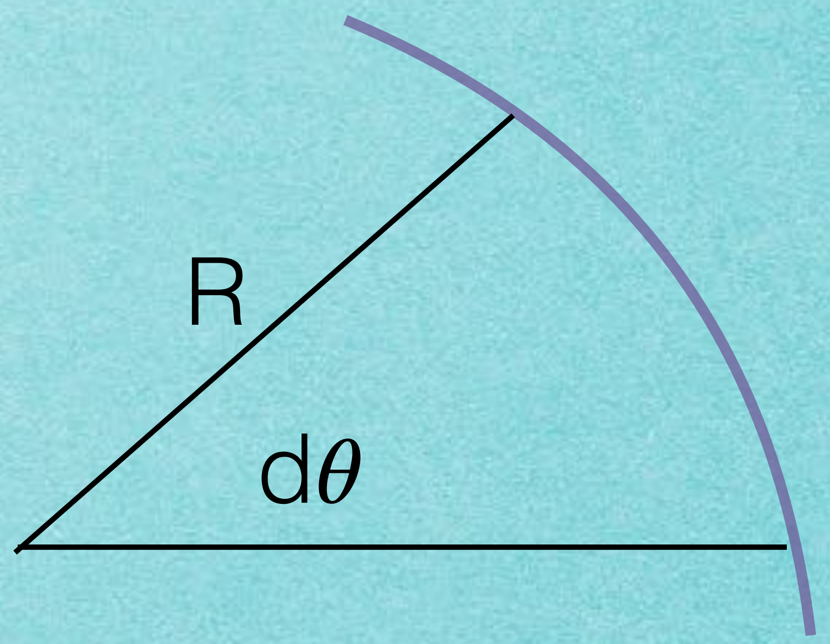

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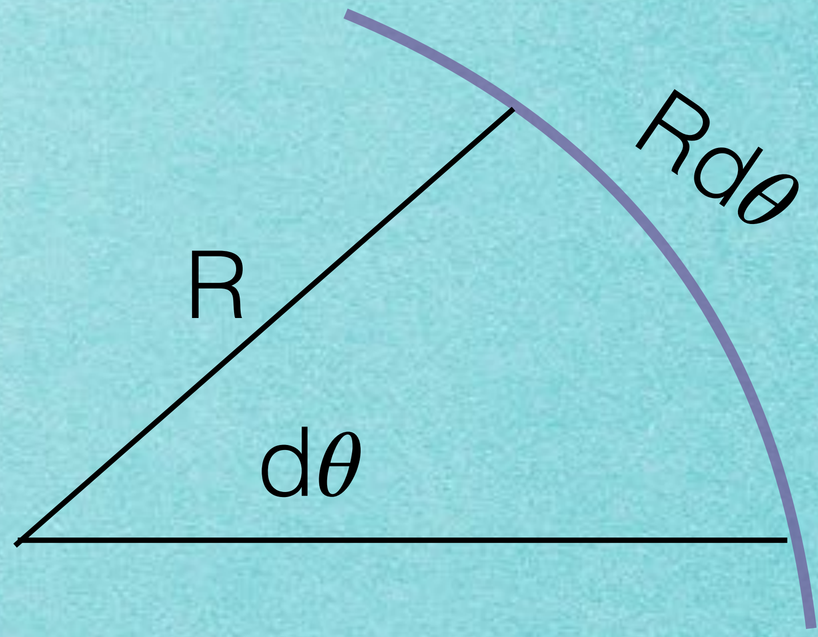

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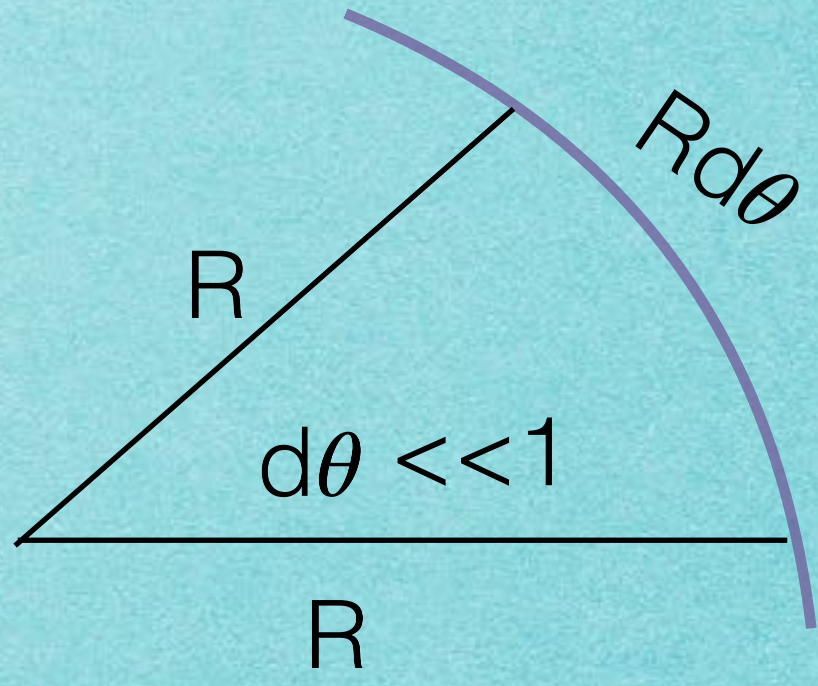

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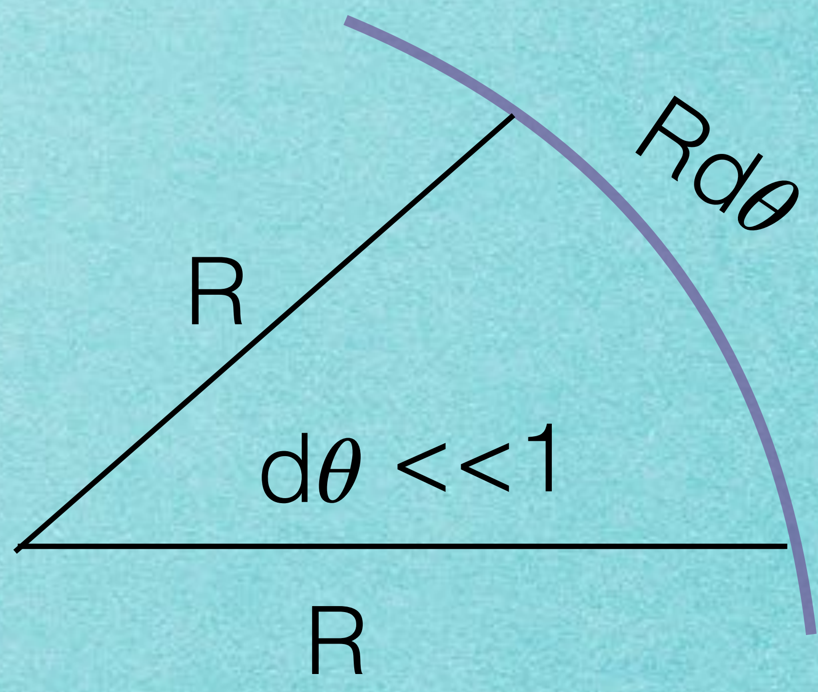
$$\begin{matrix}\text{Triangle}\\ \text{area}\end{matrix} = dA = \frac{1}{2}R^2 d\theta$$



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/dt

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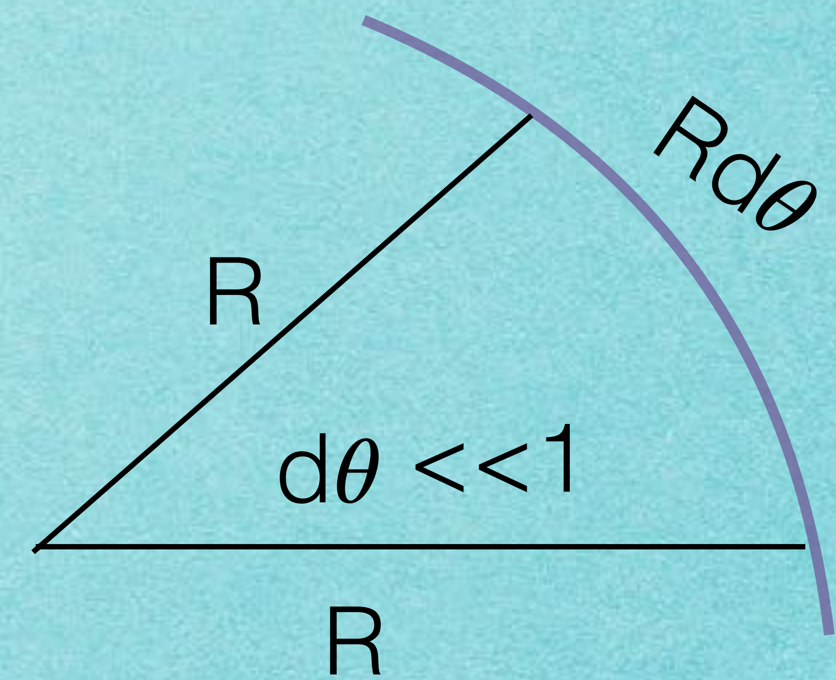

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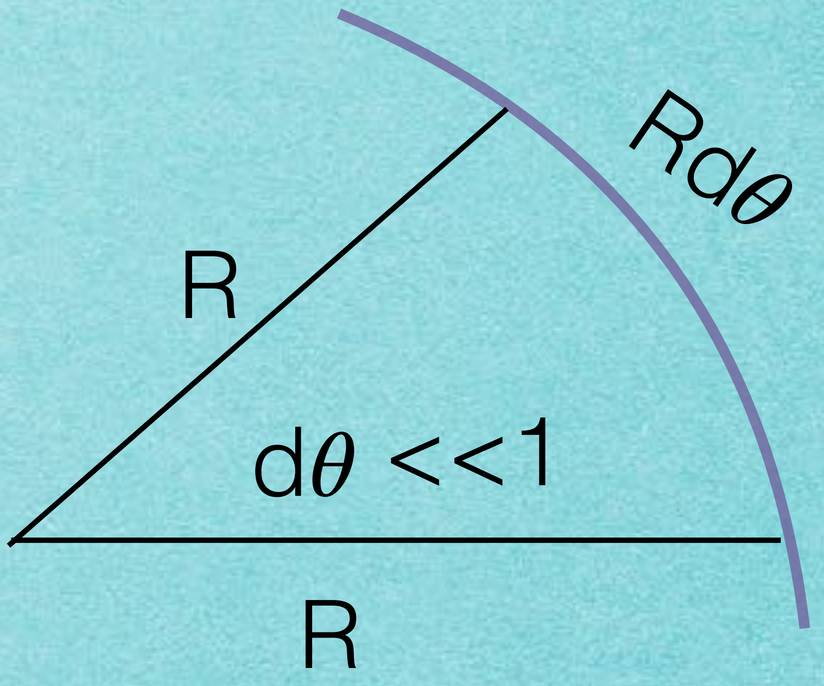

Kepler's third law is also angular momentum conservation

$$P = 2\pi\sqrt{\frac{a^3}{G(m_1+m_2)}}$$

Another conserved parameter

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Energy!

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$$E = \frac{1}{2}\dot{\vec{r}}^2 + \Phi = \frac{1}{2}\dot{r}^2 + \frac{1}{2}(r\dot{\theta})^2 + \Phi$$

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$$L_z = r^2\dot{\theta}$$

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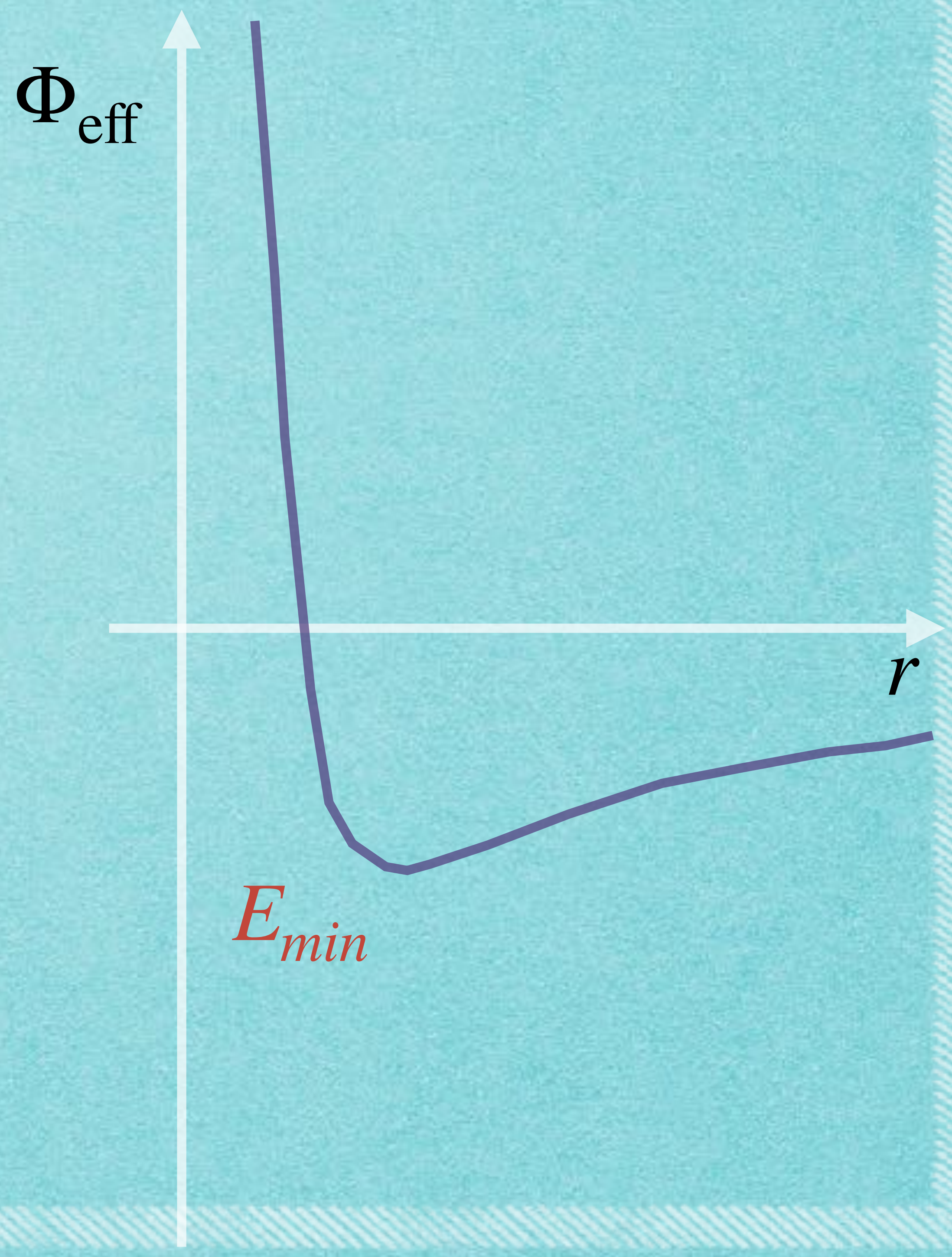

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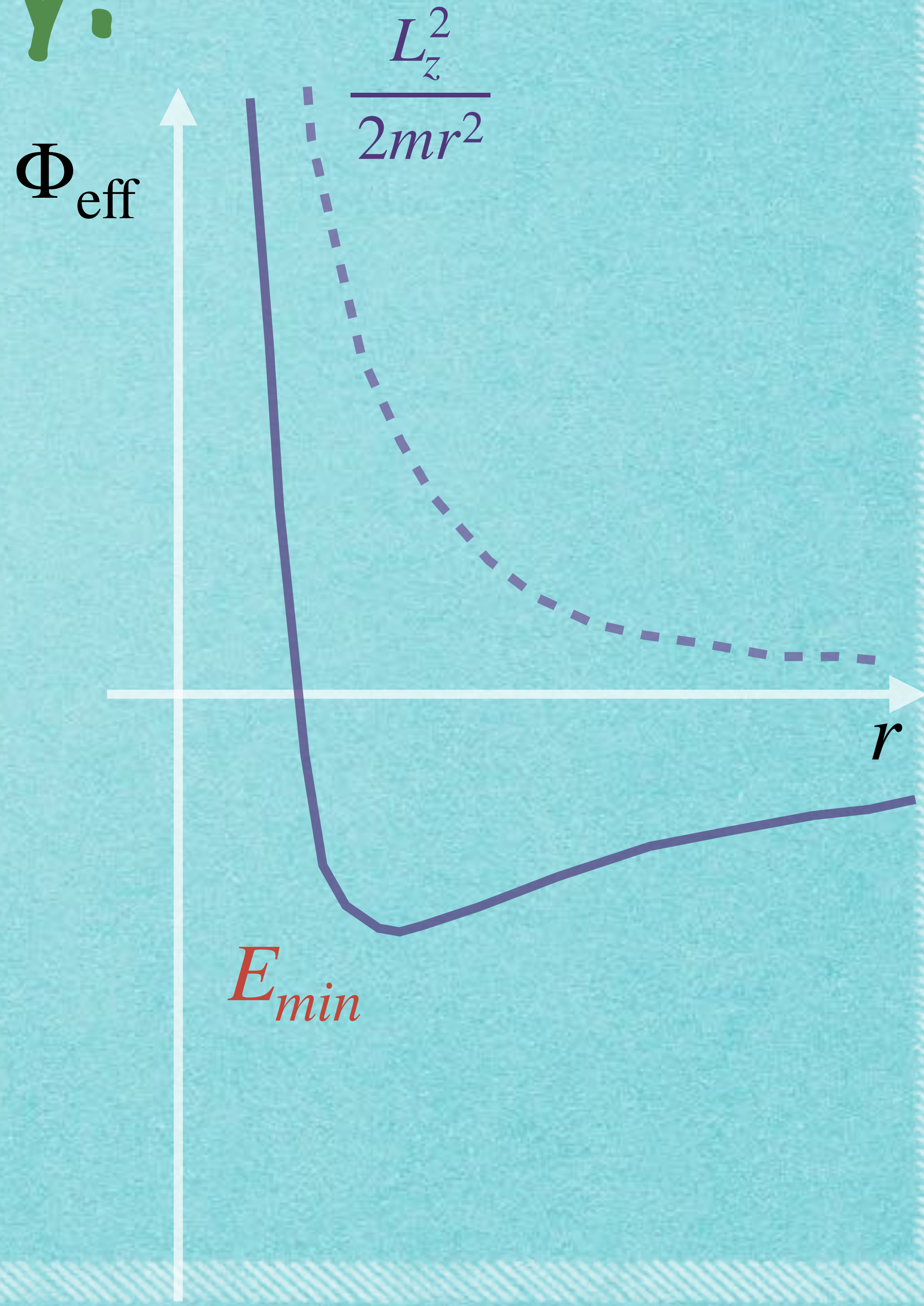

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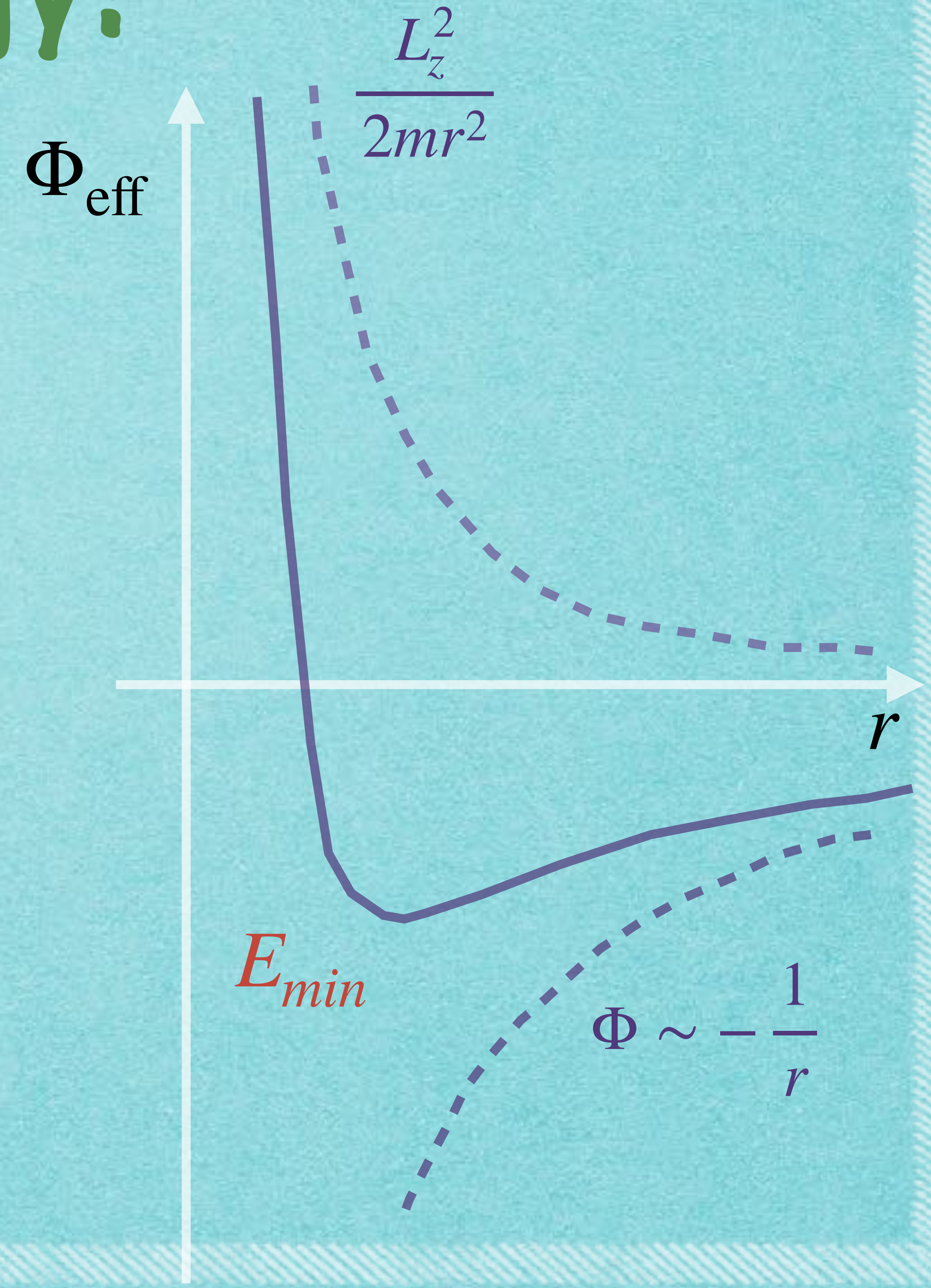

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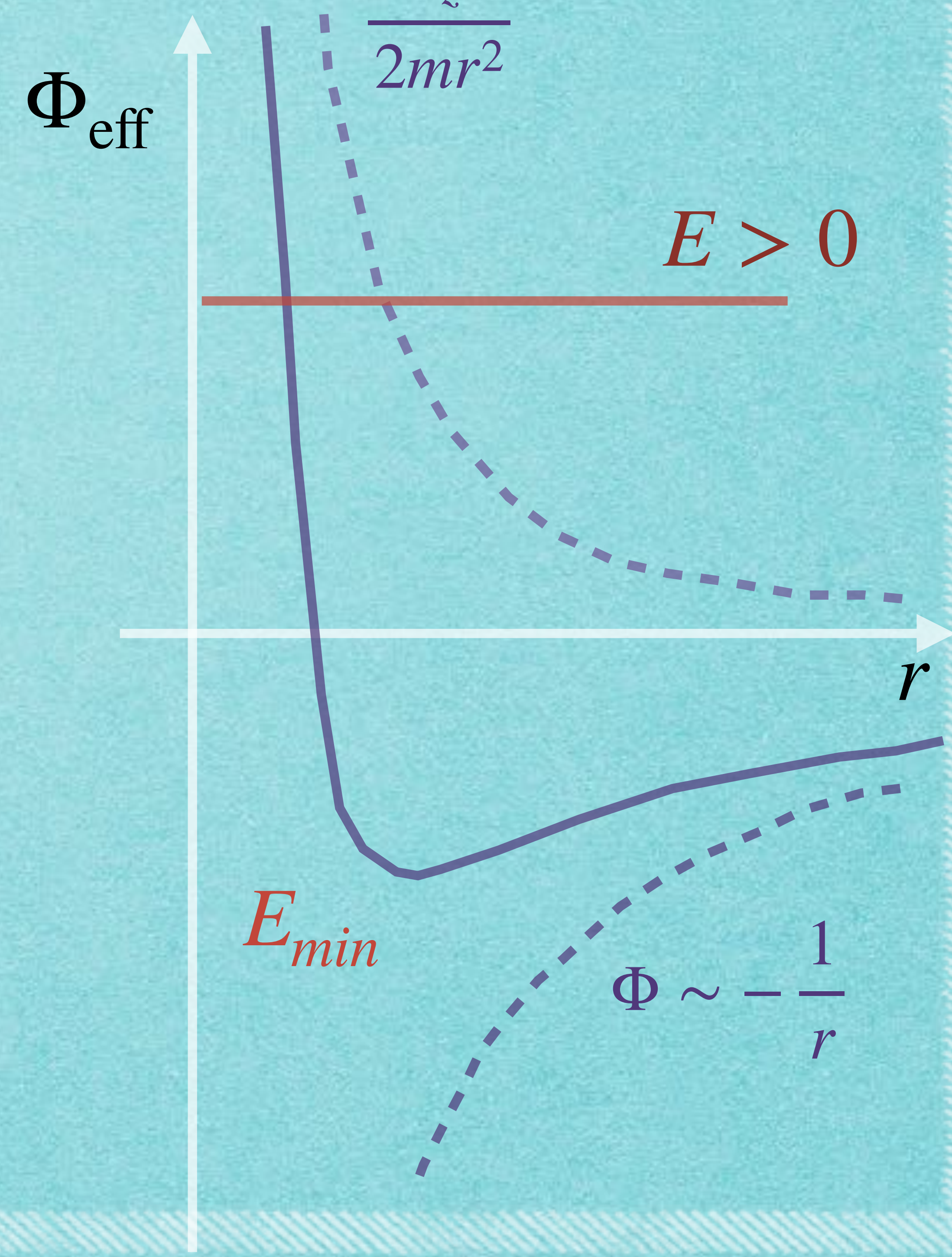

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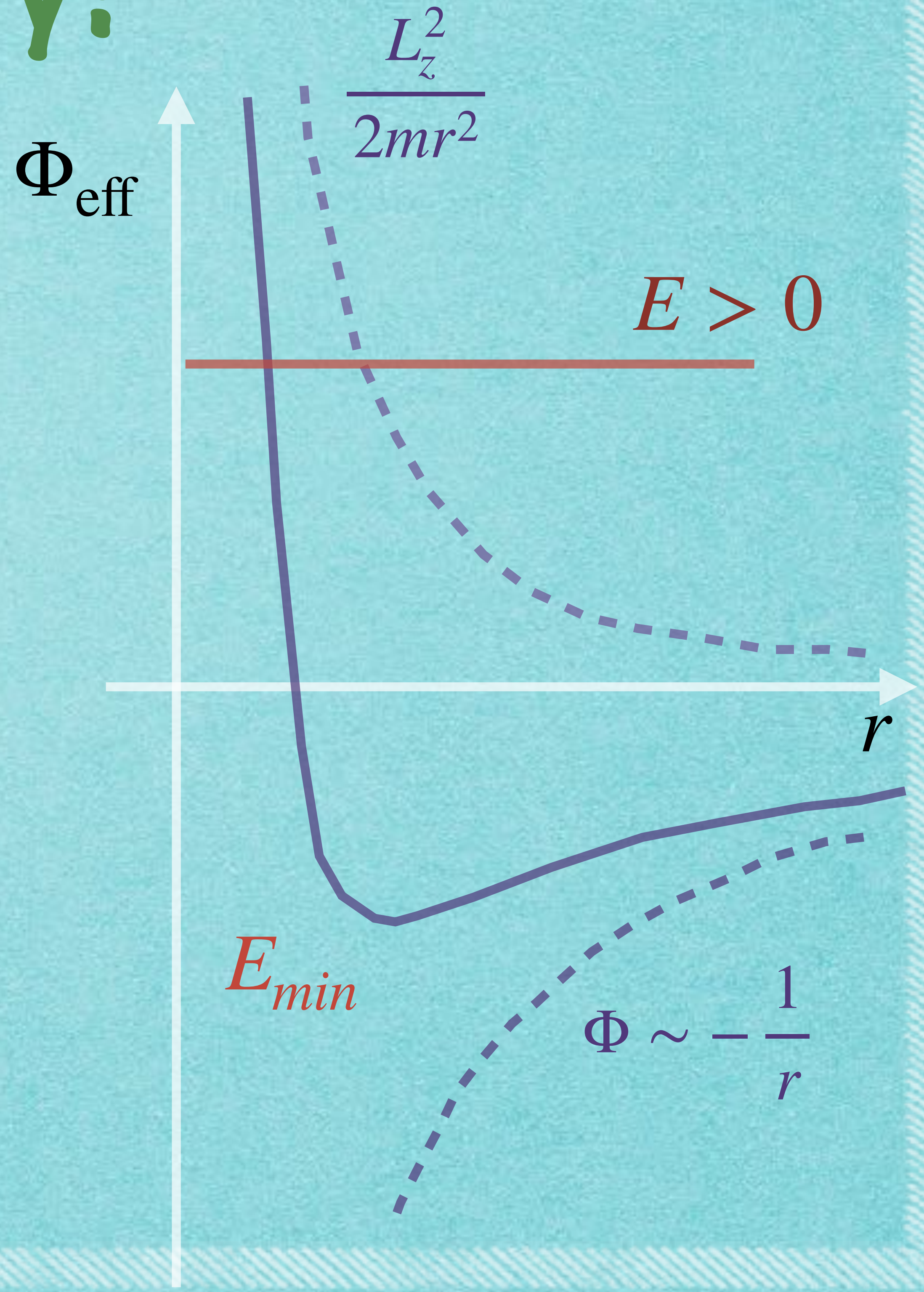

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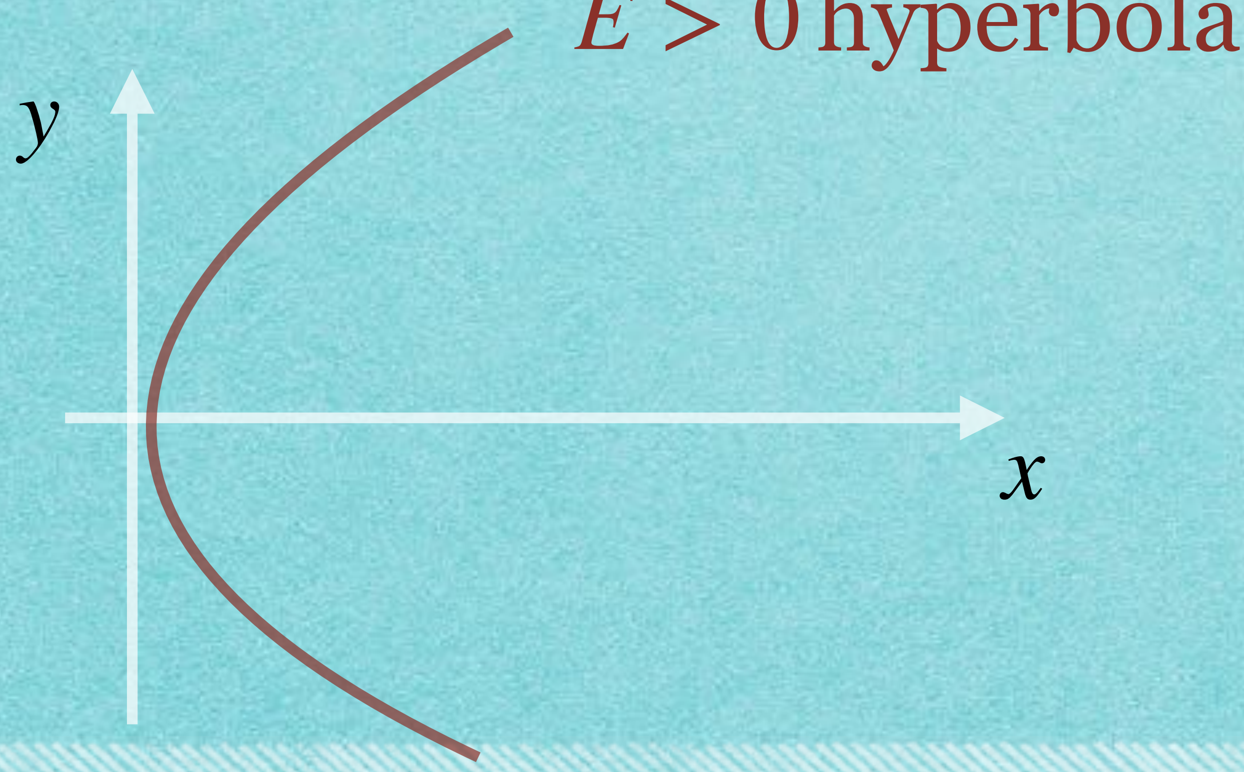

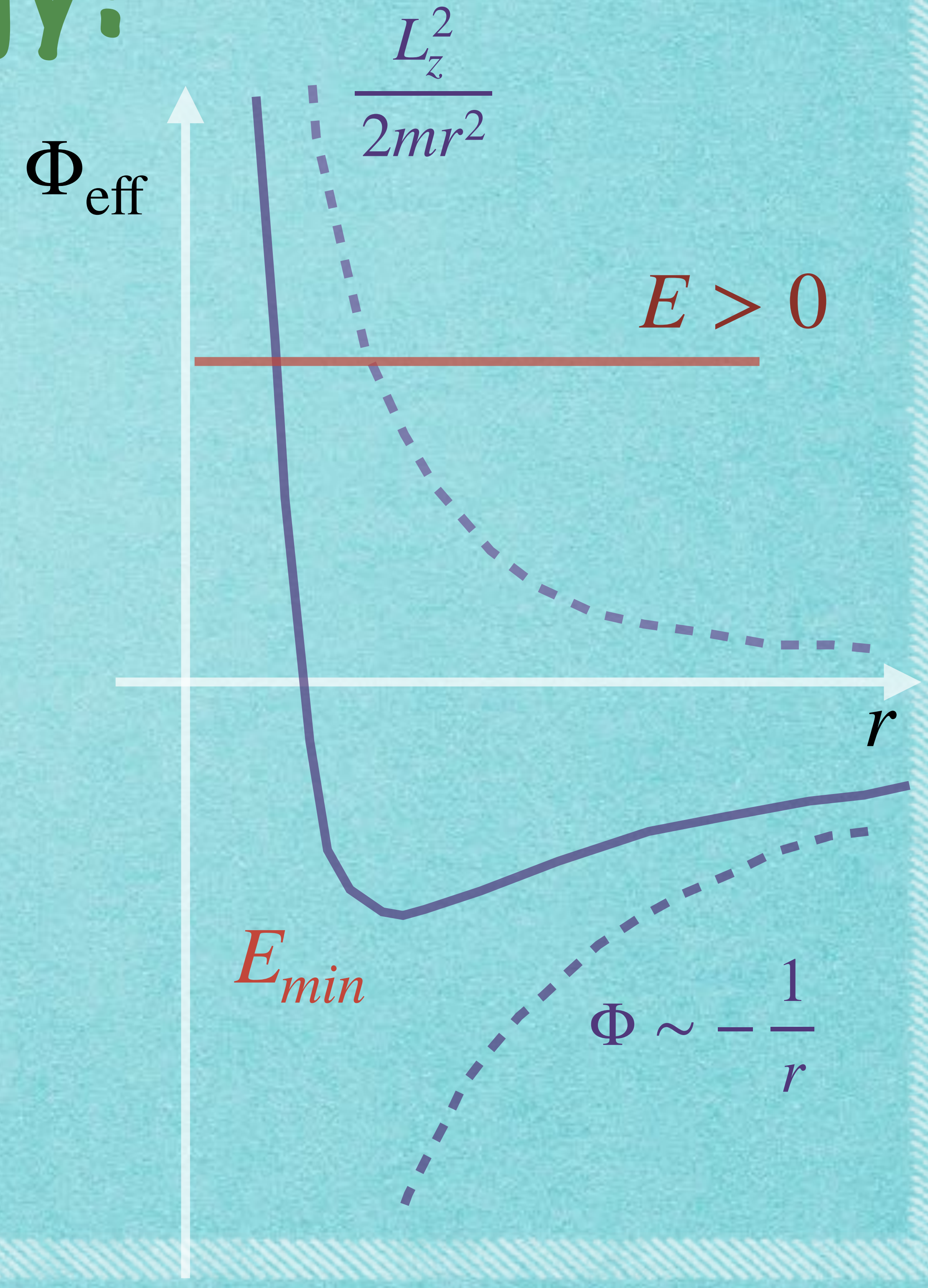

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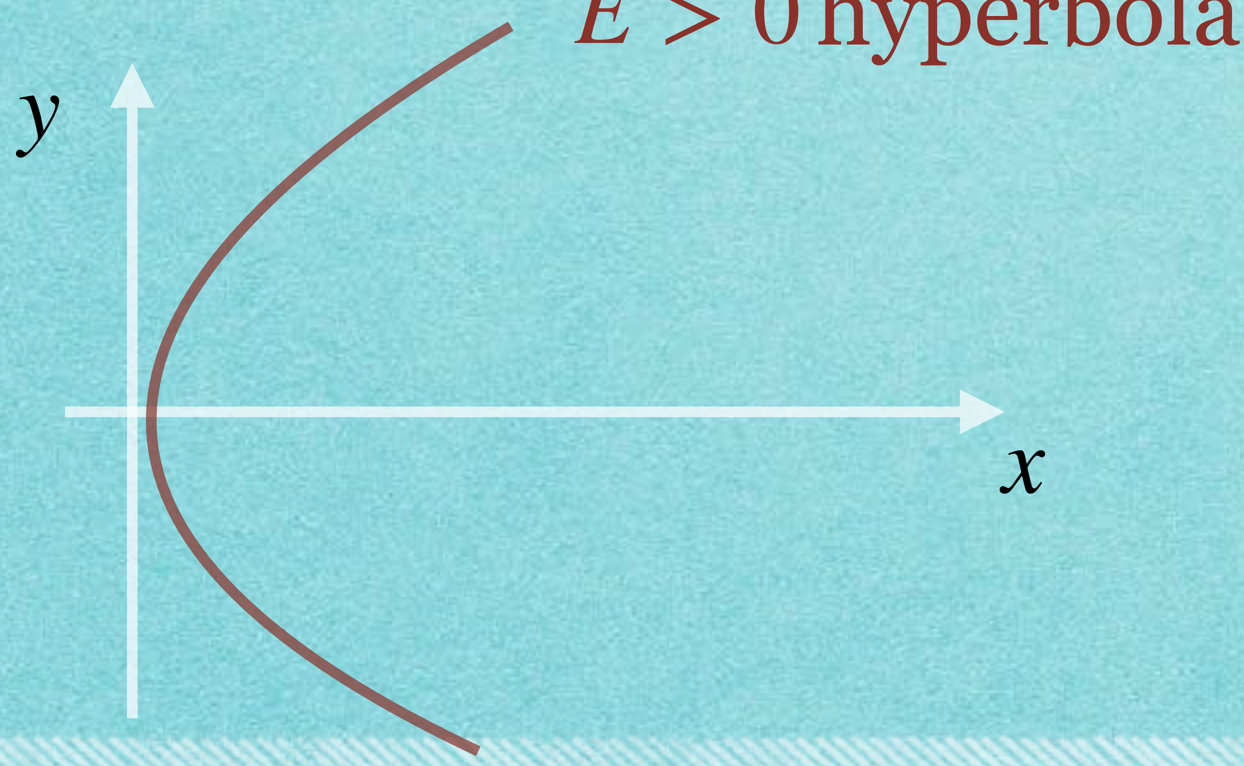

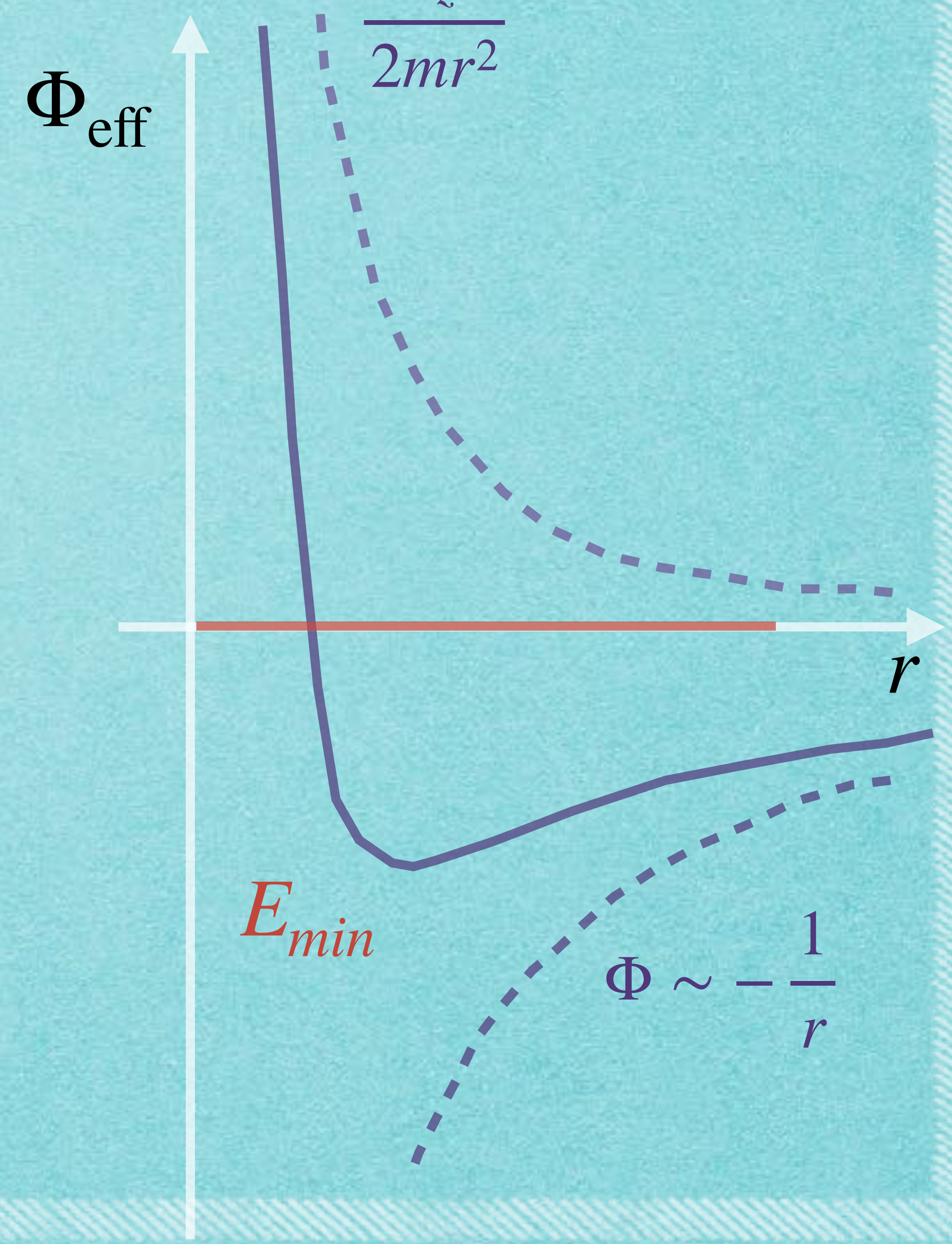

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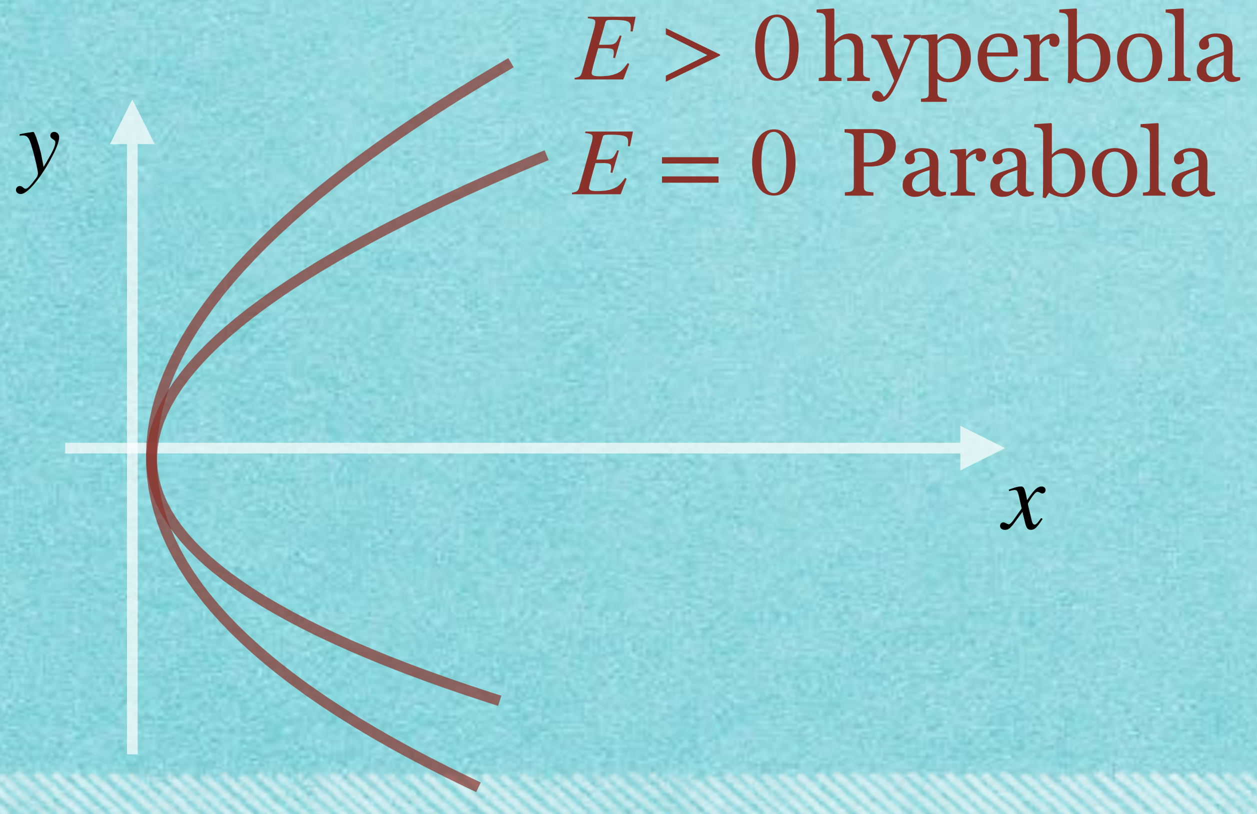

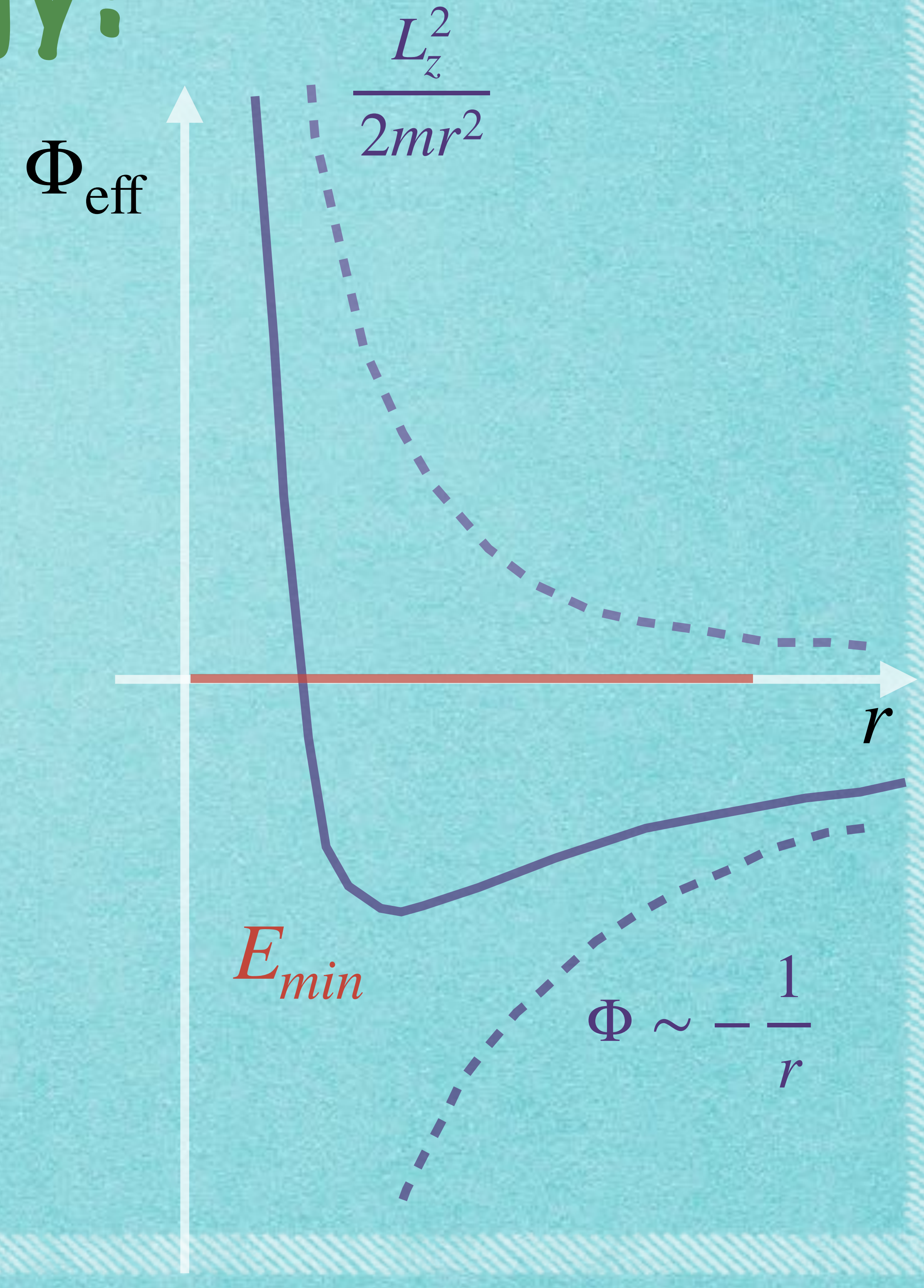

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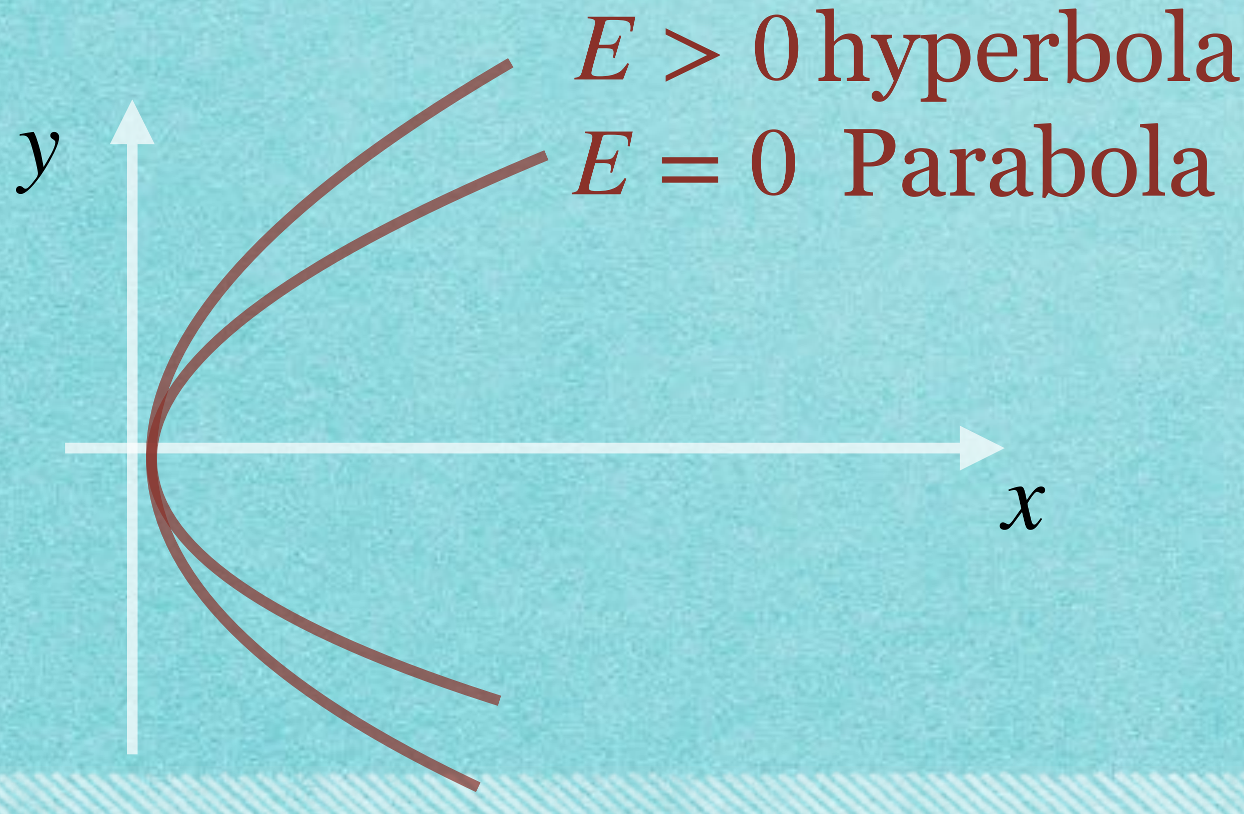

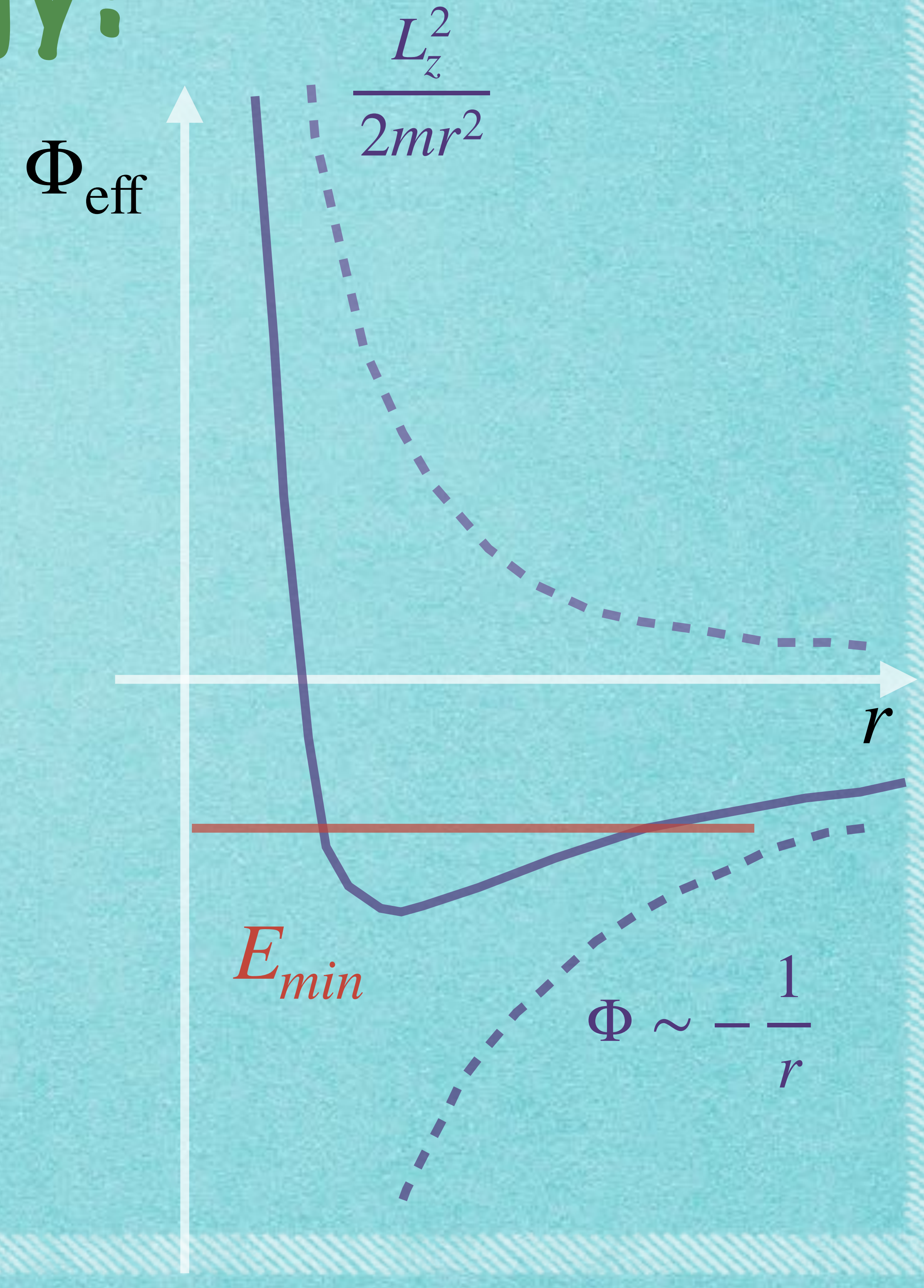

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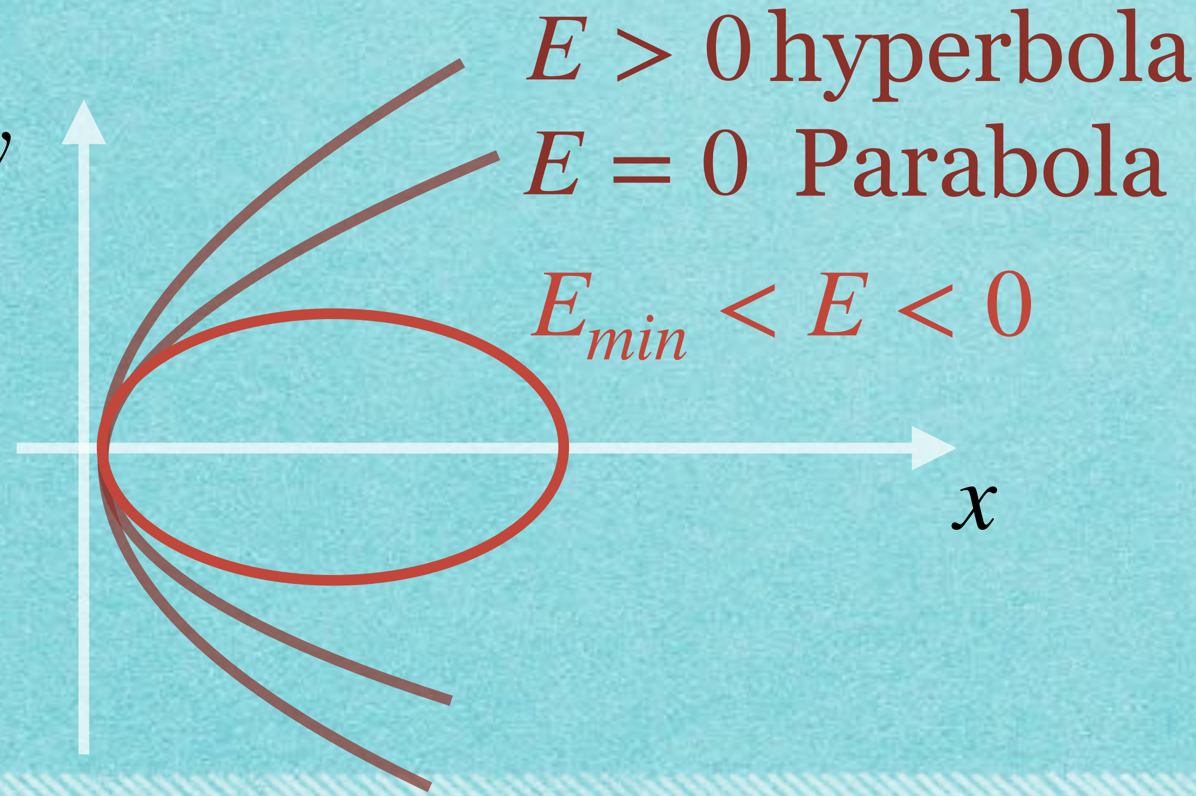

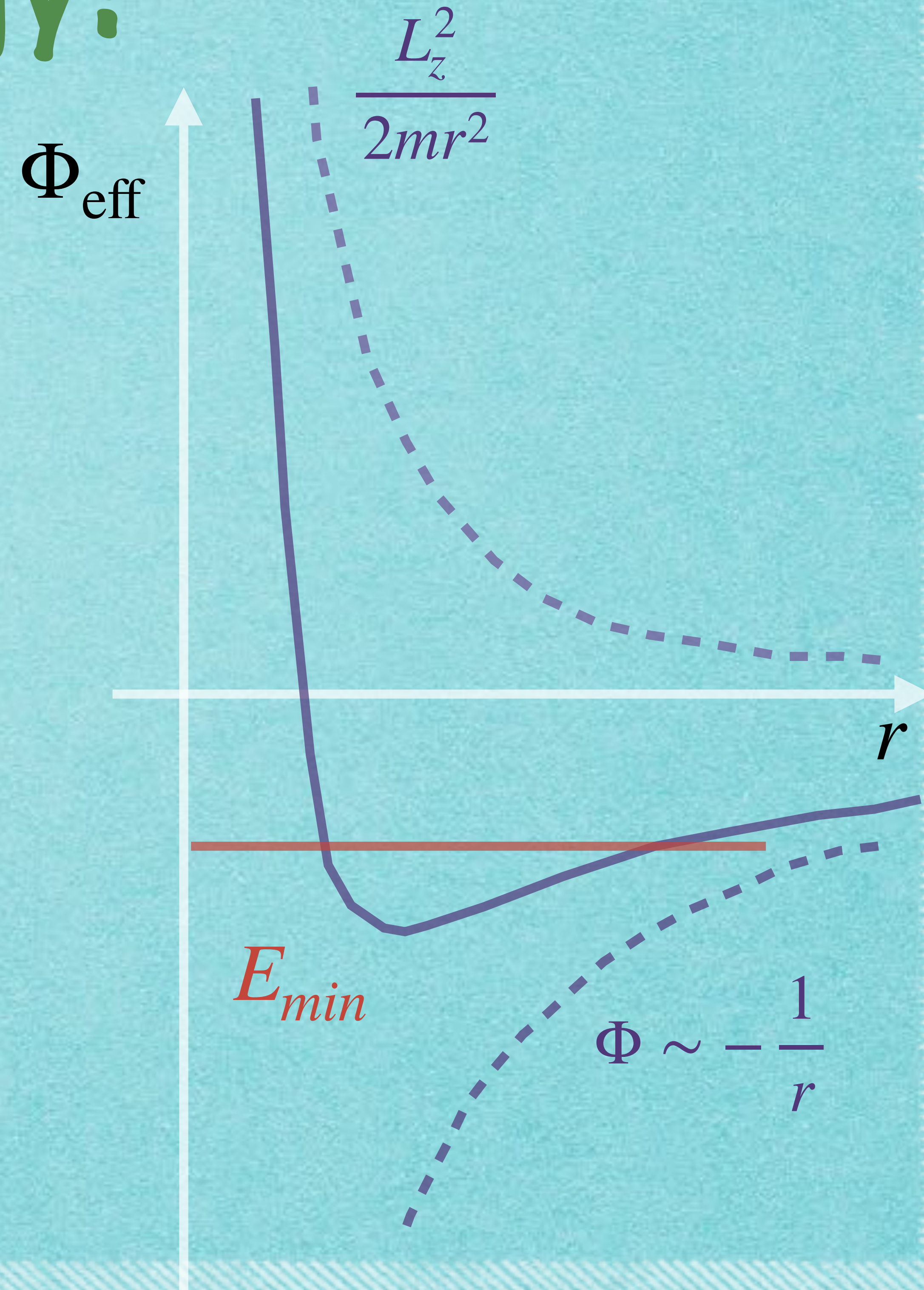

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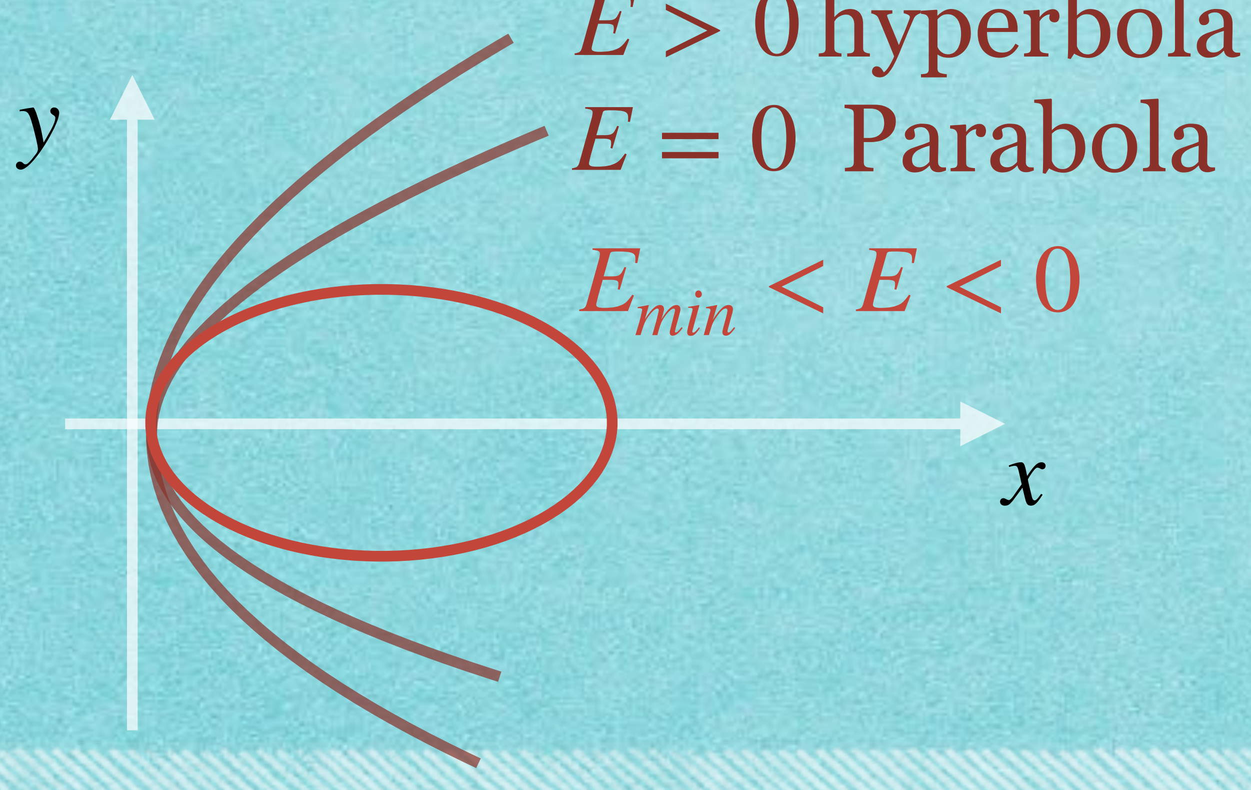
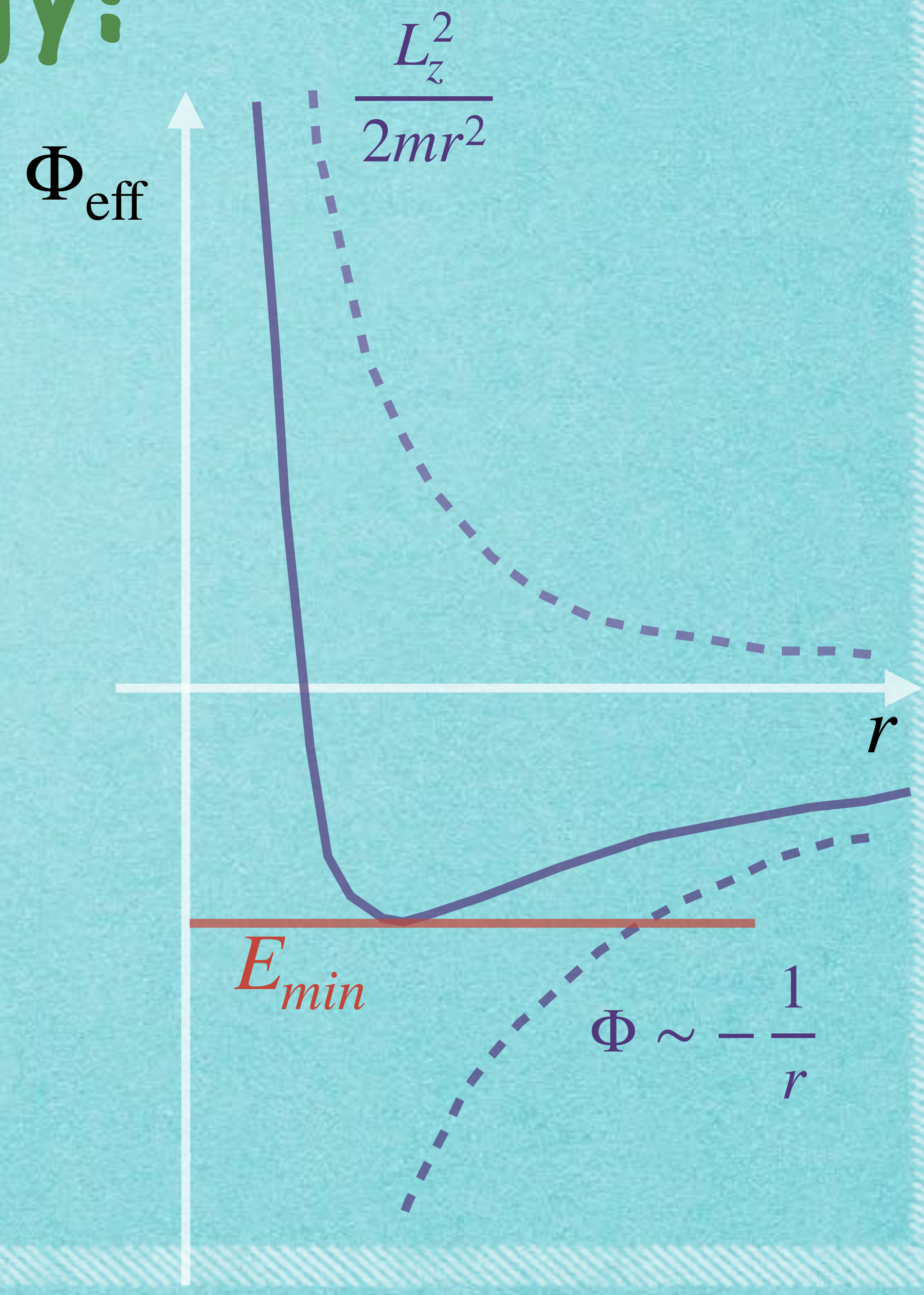
For $\Phi \sim -\dfrac{1}{r}$

$$\Phi_{\text{eff}} = \frac{L_z^2}{2mr^2} + \Phi$$





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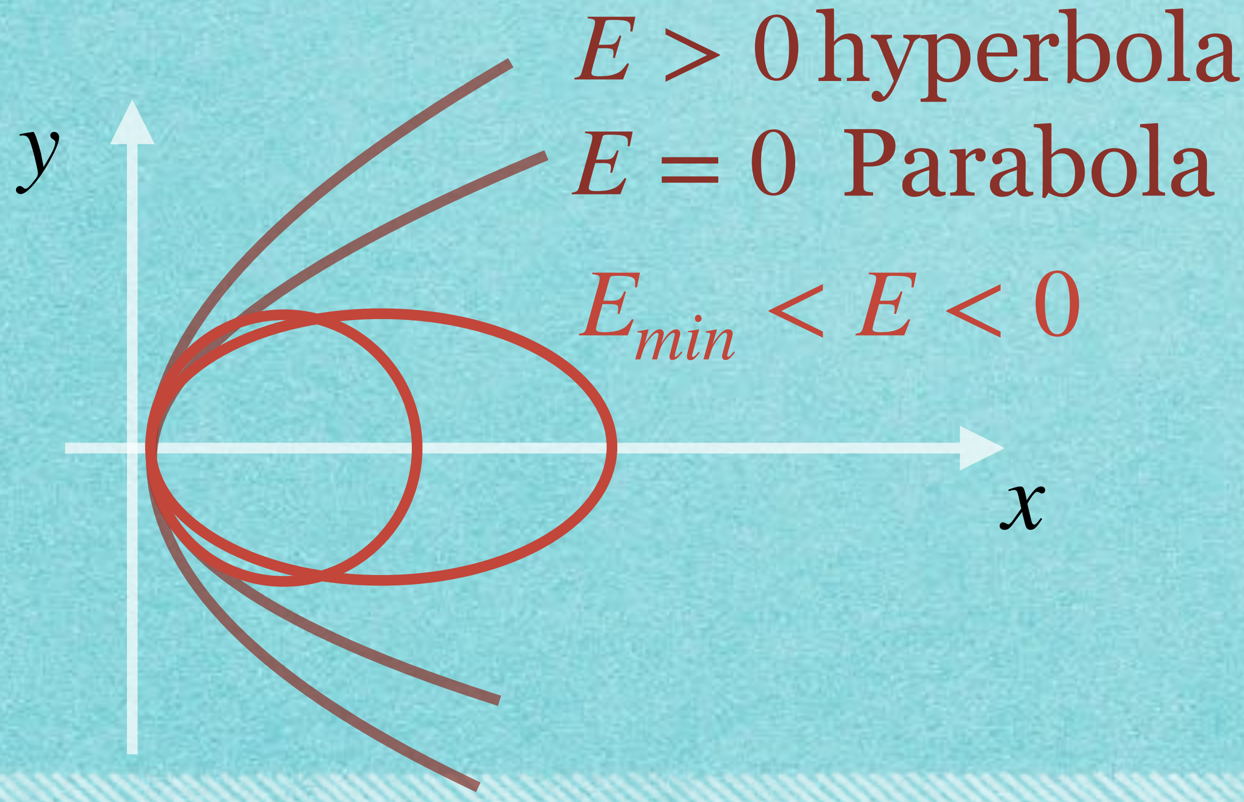
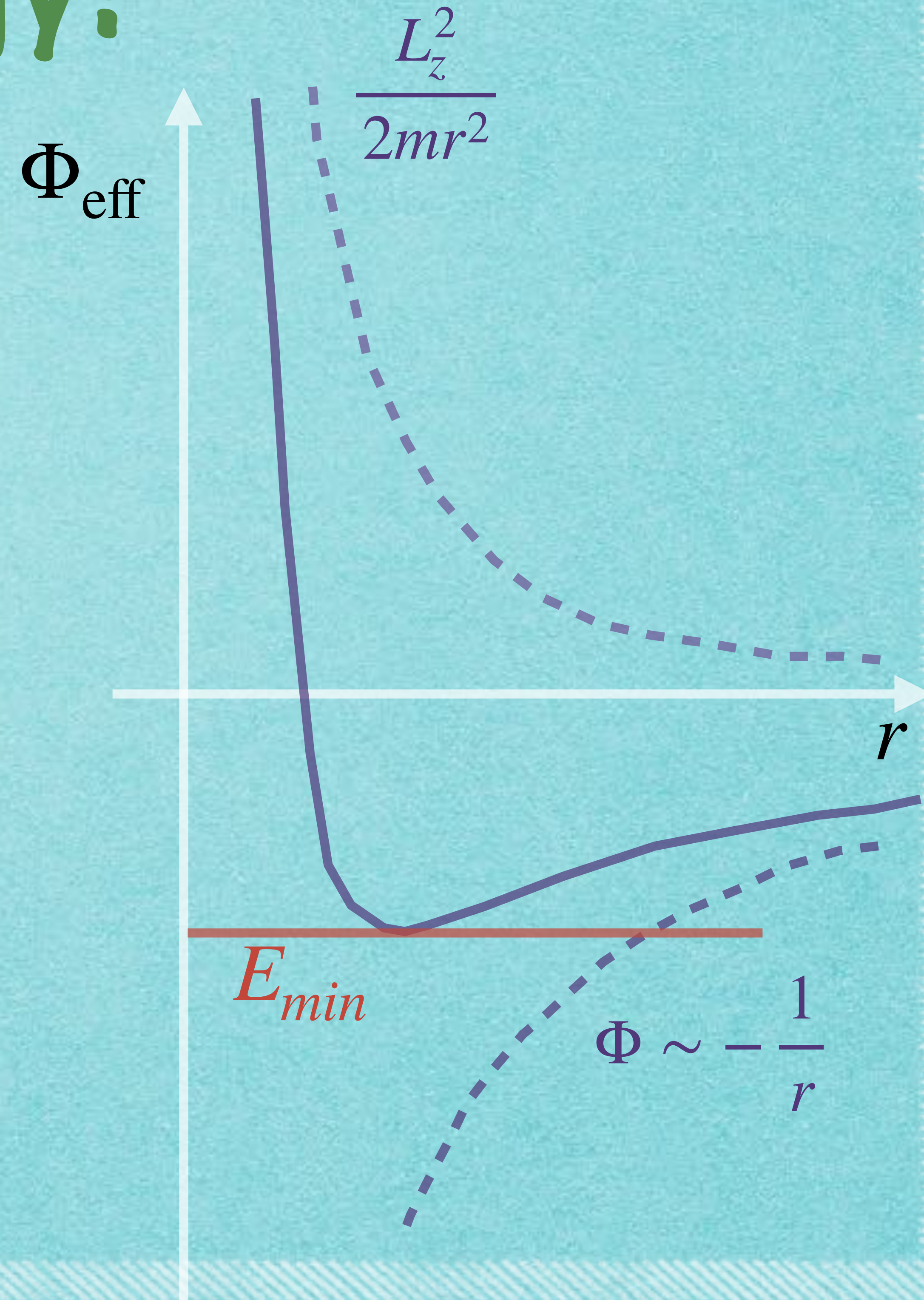




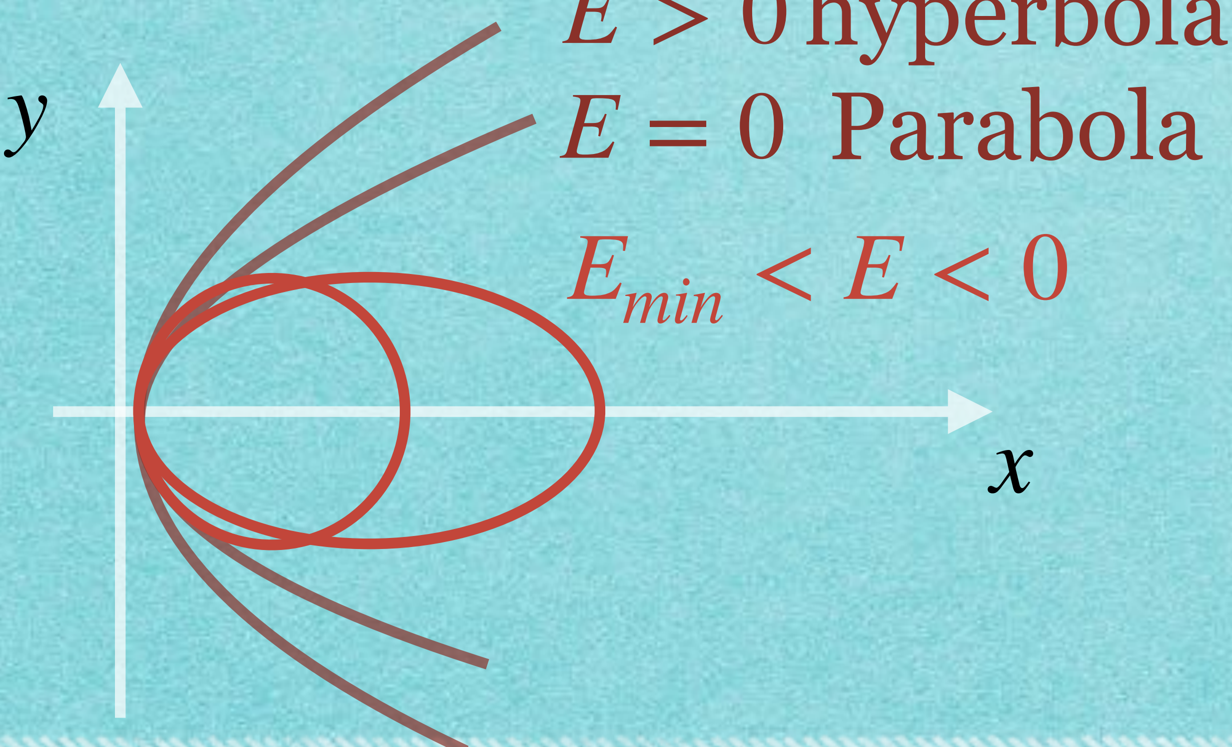
Another conserved parameter

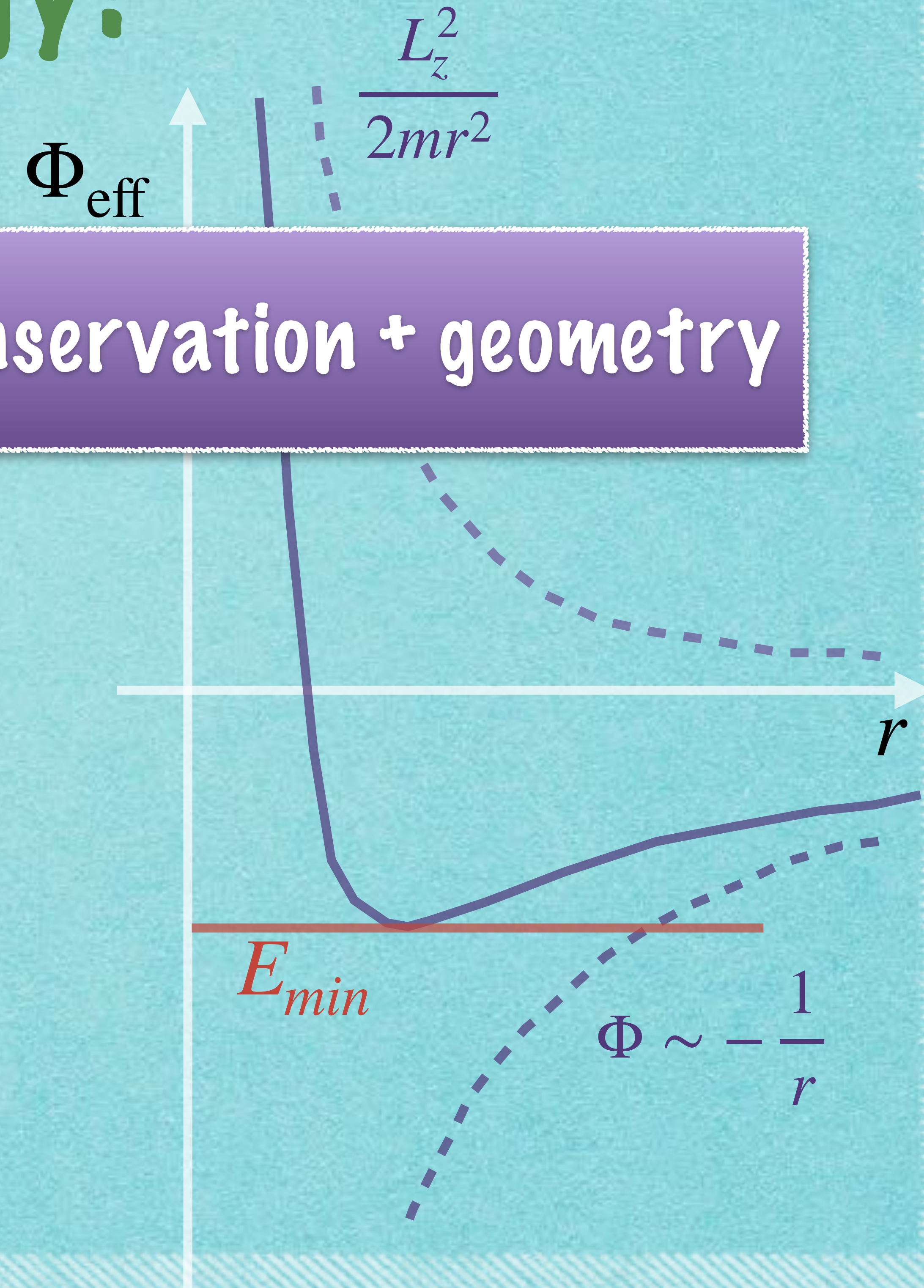
Energy!

$$E = \frac{1}{2}\dot{r}^2 + \frac{L_z^2}{2mr^2} + \Phi$$

Kepler's first law is Energy conservation + geometry

$$\Phi_{\text{eff}} = \frac{L_z^2}{2mr^2} + \Phi$$





3-body problem

3-body problem

Goal - describe the 3-body dynamics

3-body problem

Goal - describe the 3-body dynamics

Some simplification

Result: When mass transfer, and accretion begins

3-body problem

Goal - describe the 3-body dynamics

↓ Some simplification

Result: When mass transfer, and accretion begins

↓ Some generalization

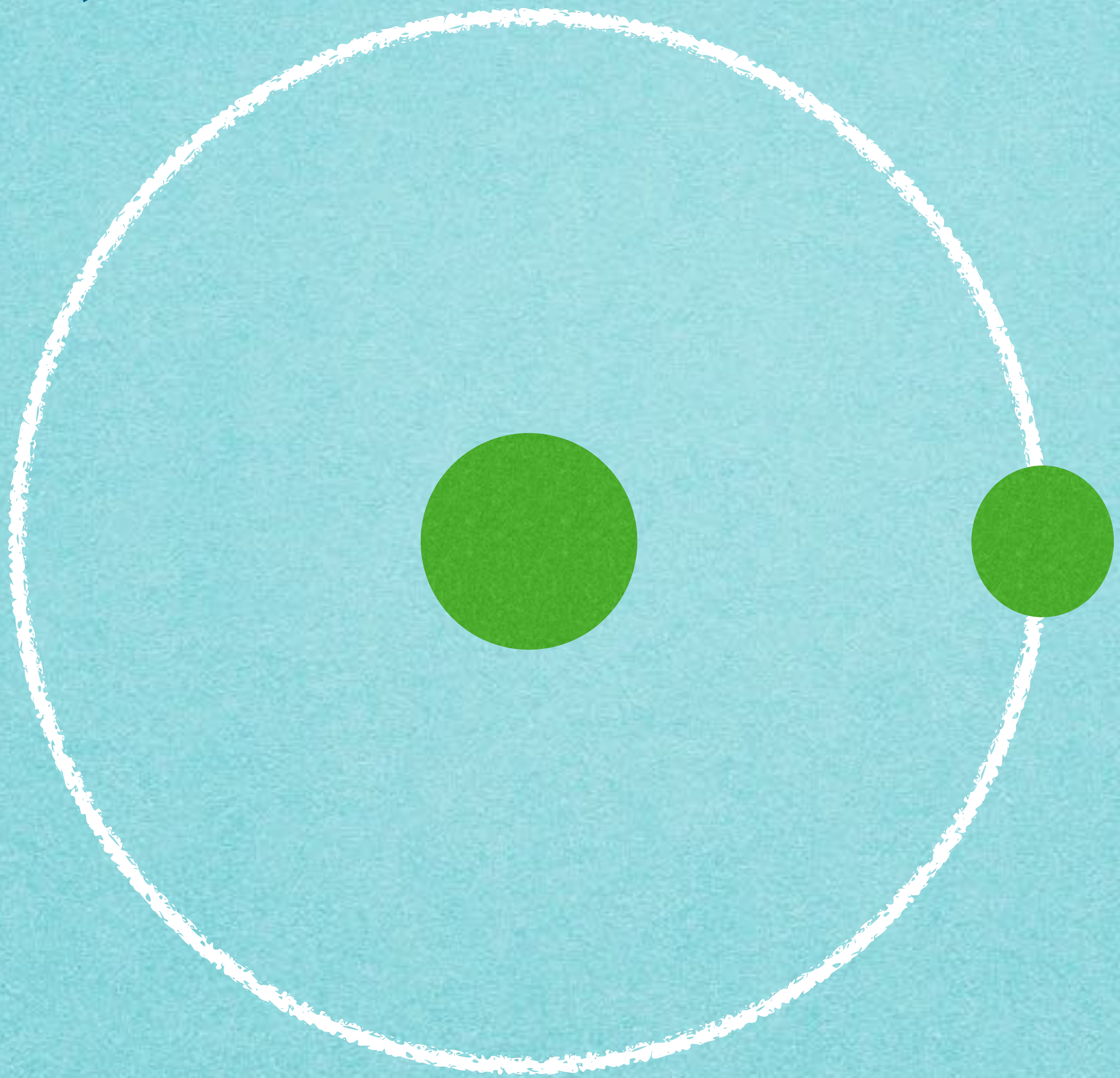
Result: merging stars/black holes/planets and chaos

3-body problem

Circular, restricted

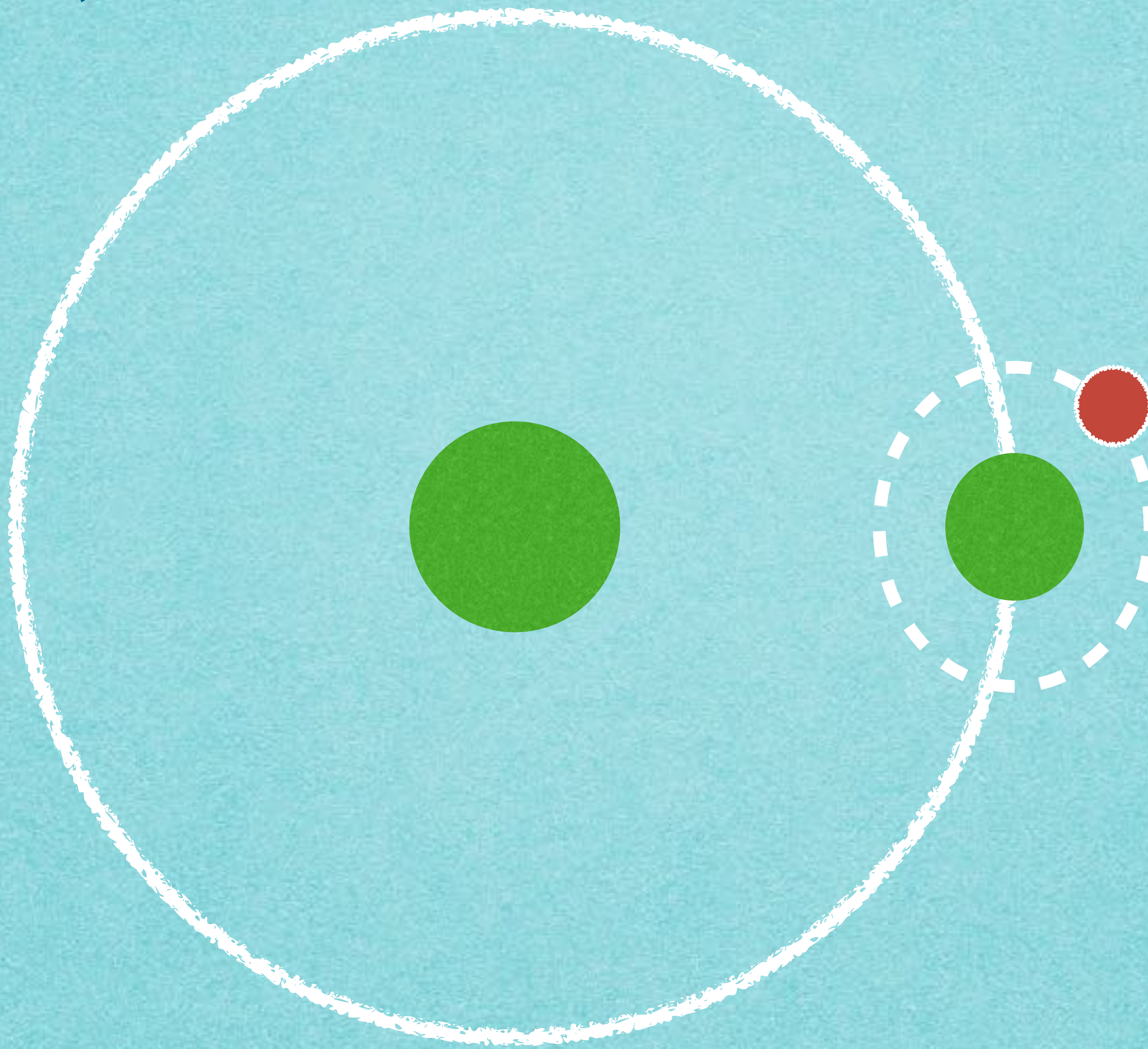
3-body problem

Circular, restricted



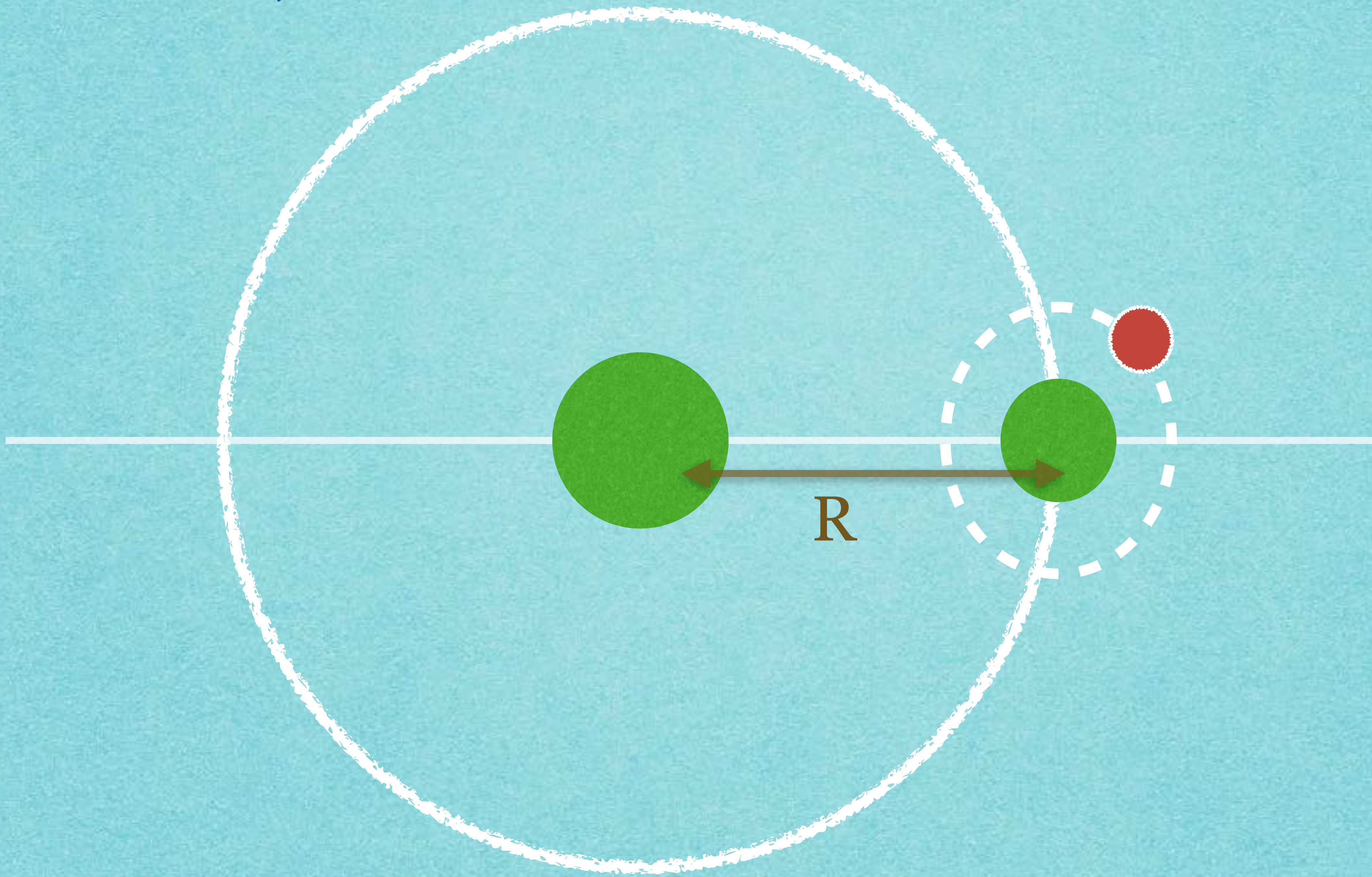
3-body problem

Circular, restricted



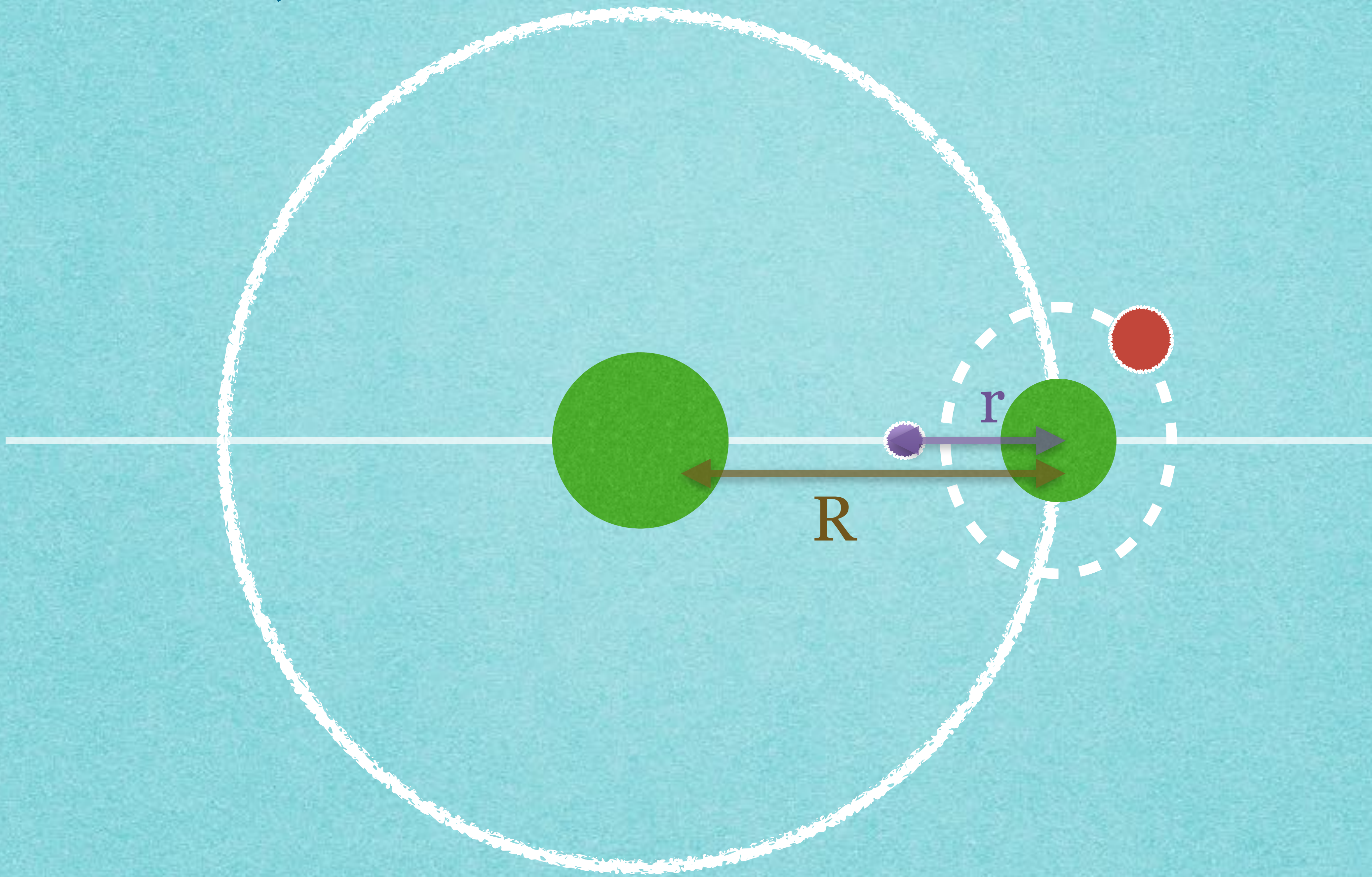
3-body problem

Circular, restricted



3-body problem

Circular, restricted



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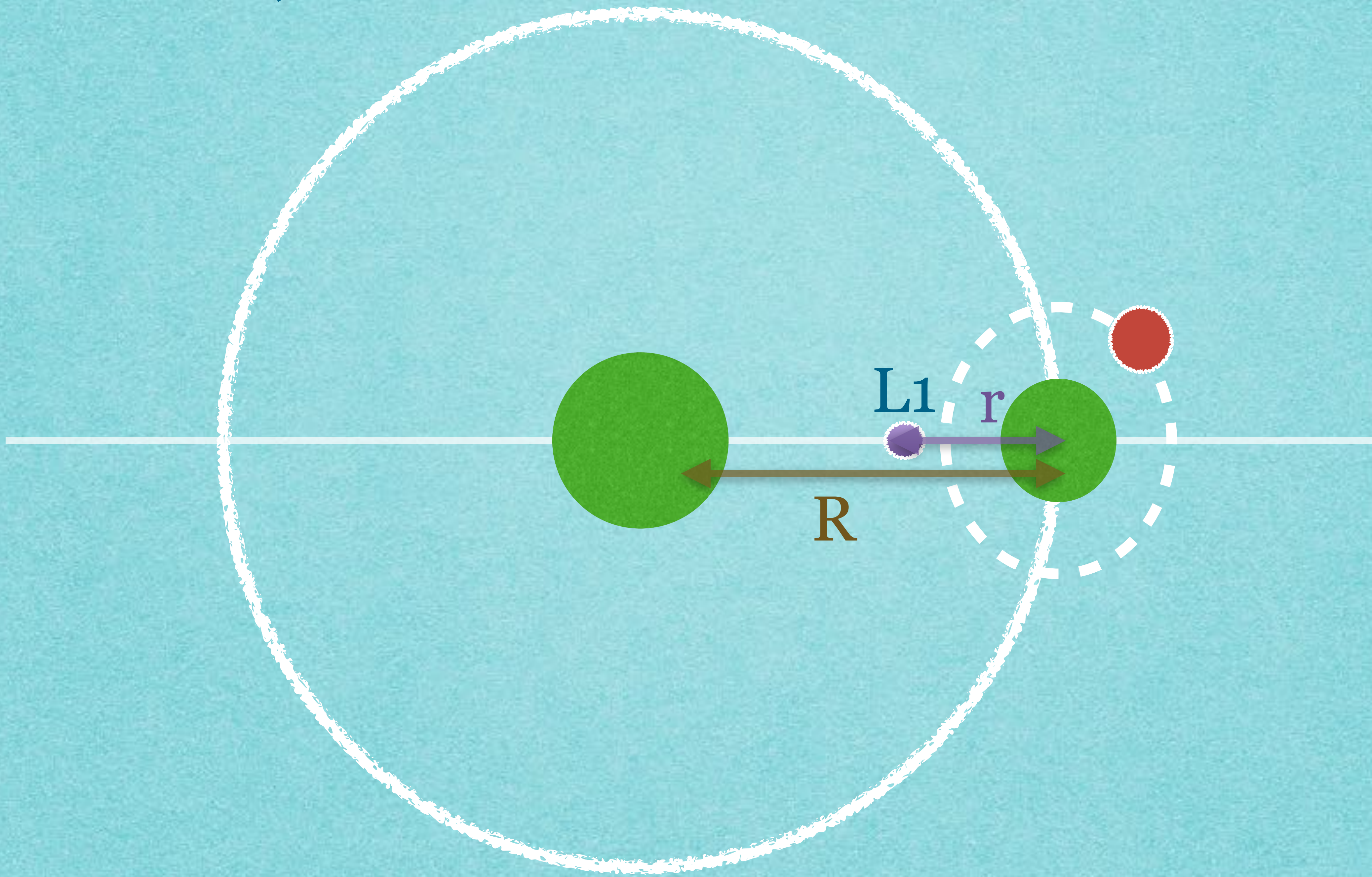
Circular, restricted



3-body problem

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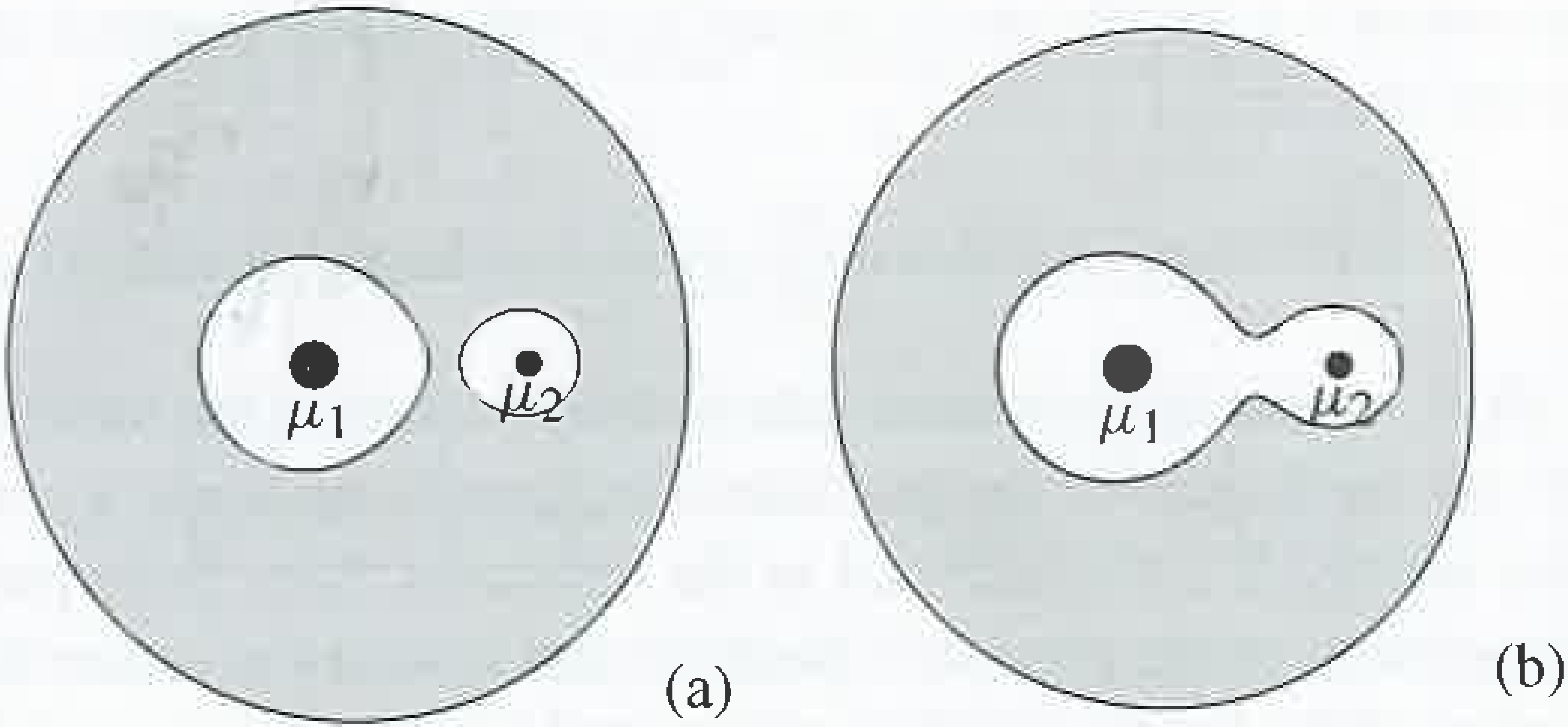
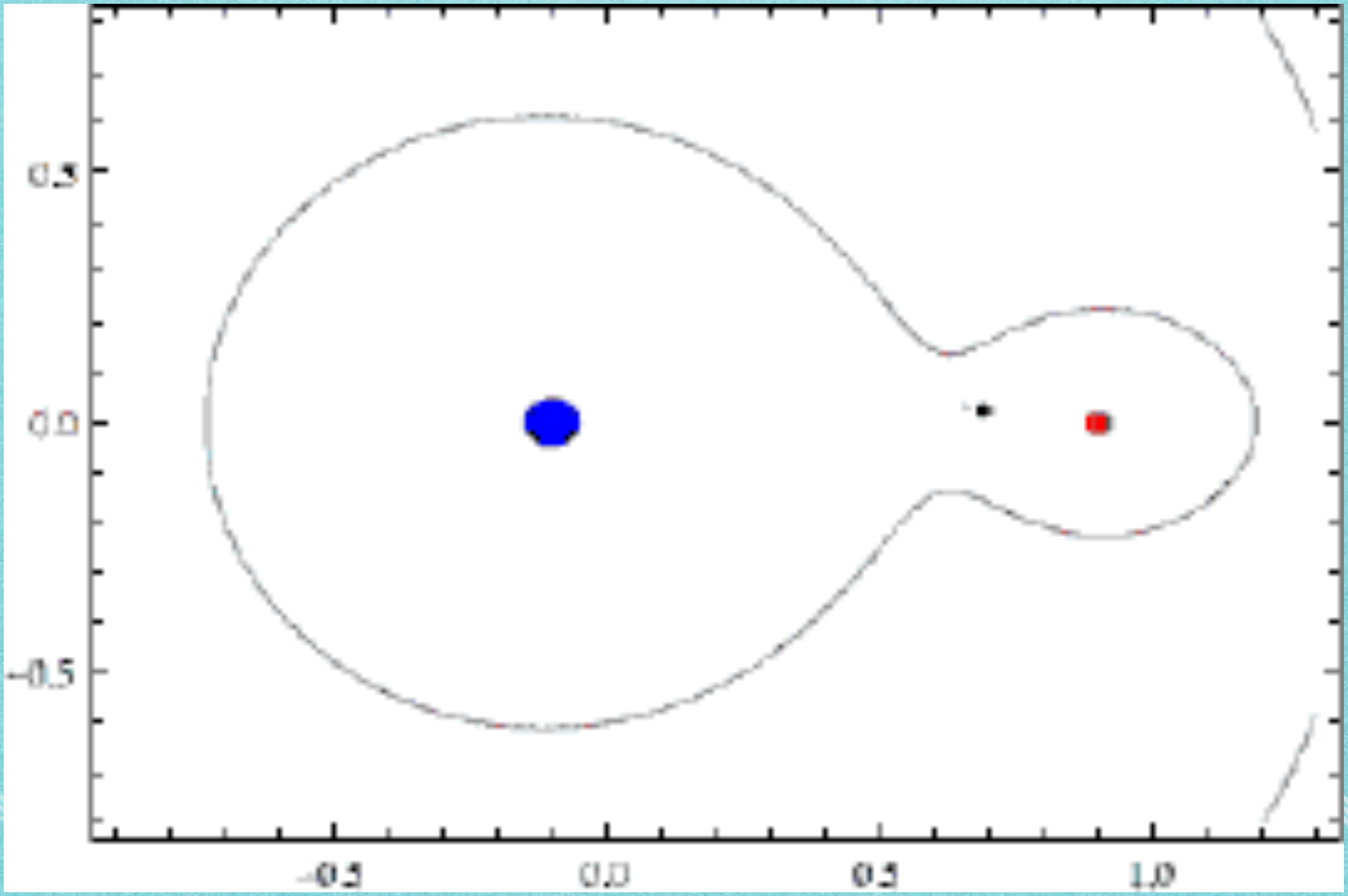


Fig. 3.2. Zero-velocity curves for two values of the Jacobi constant for the case when $\mu_2 = 0.2$. The values of C_J are (a) $C_J = 3.9$, (b) $C_J = 3.7$. The shaded areas denote the excluded regions. (See also Fig. 9.11 for the case $\mu_2 = 10^{-3}$.)

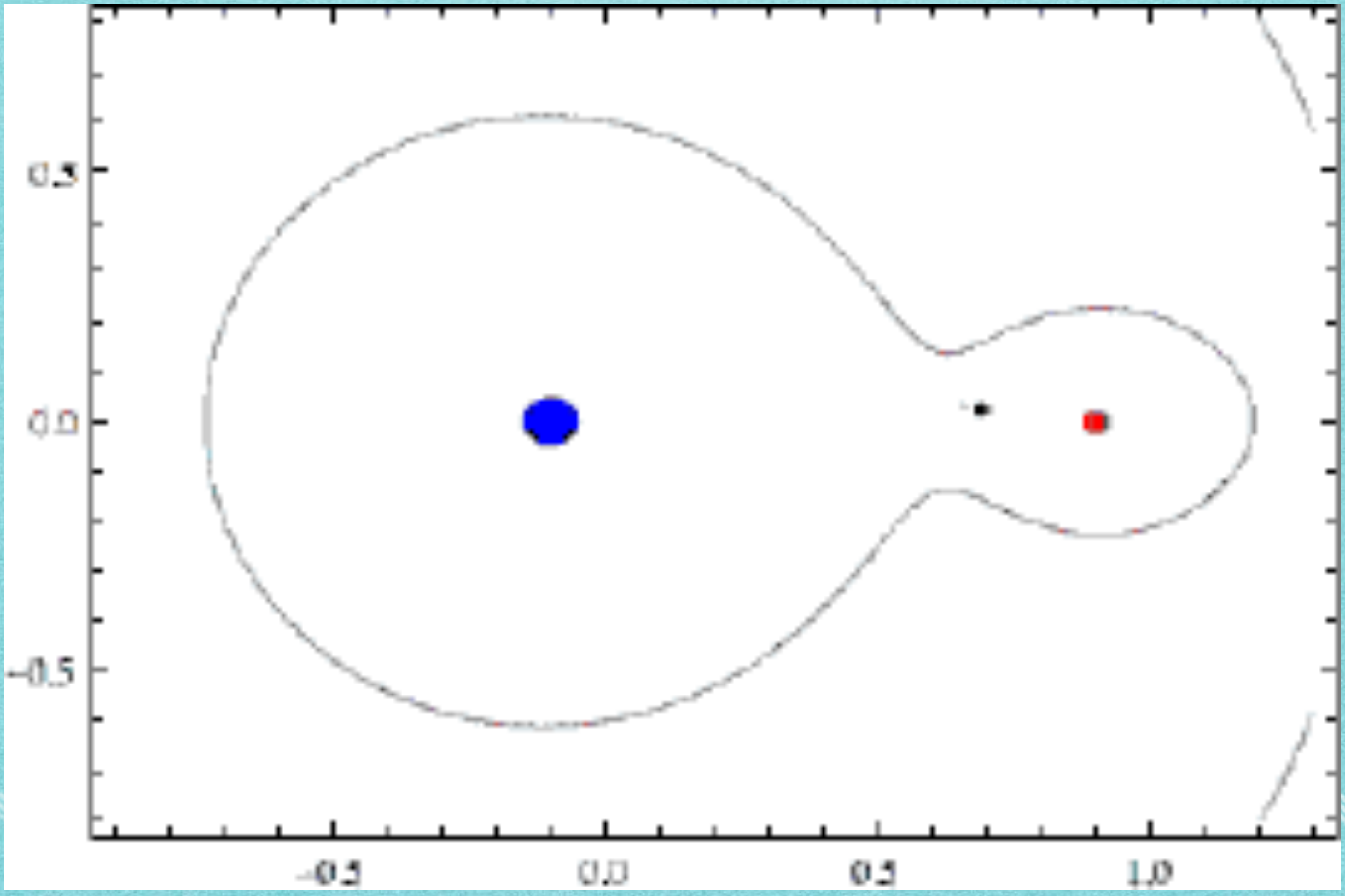
3-body problem

Circular, restricted **Lagrange points/tidal/Hill radius**



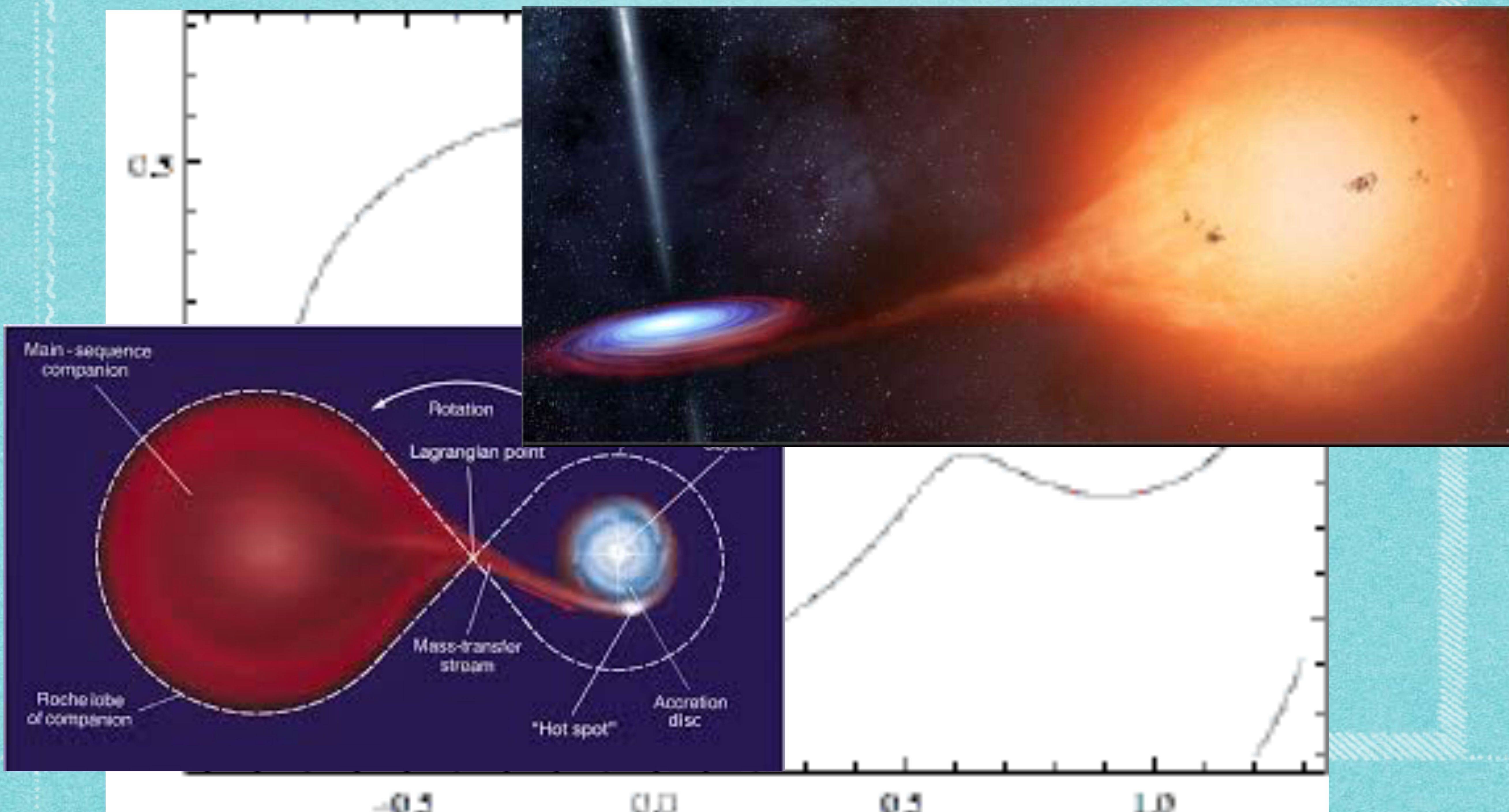
3-body problem

Circular, restricted ***Lagrange points/tidal/Hill radius***



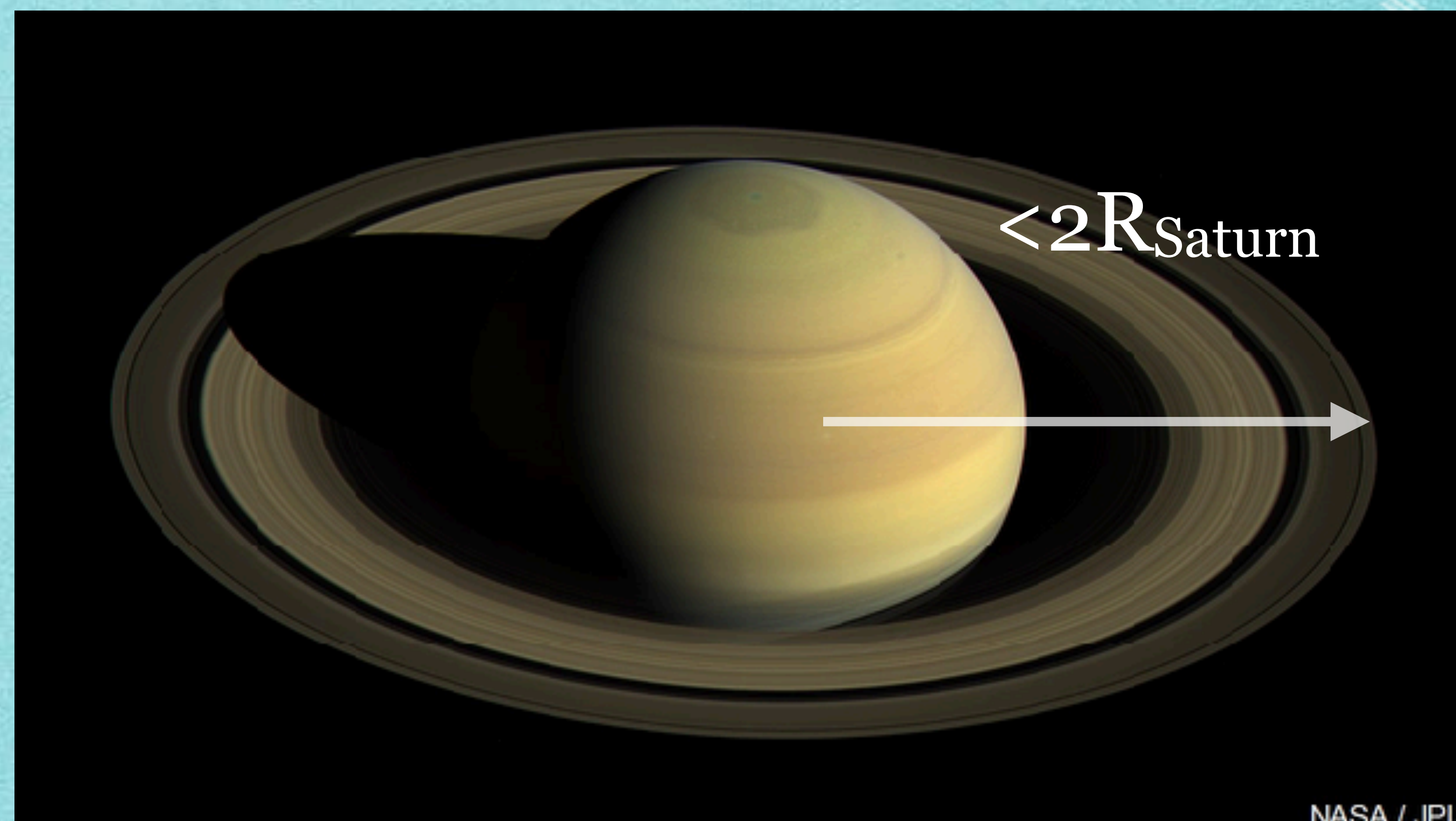
3-body problem

Circular, restricted **Lagrange points/tidal/Hill radius**



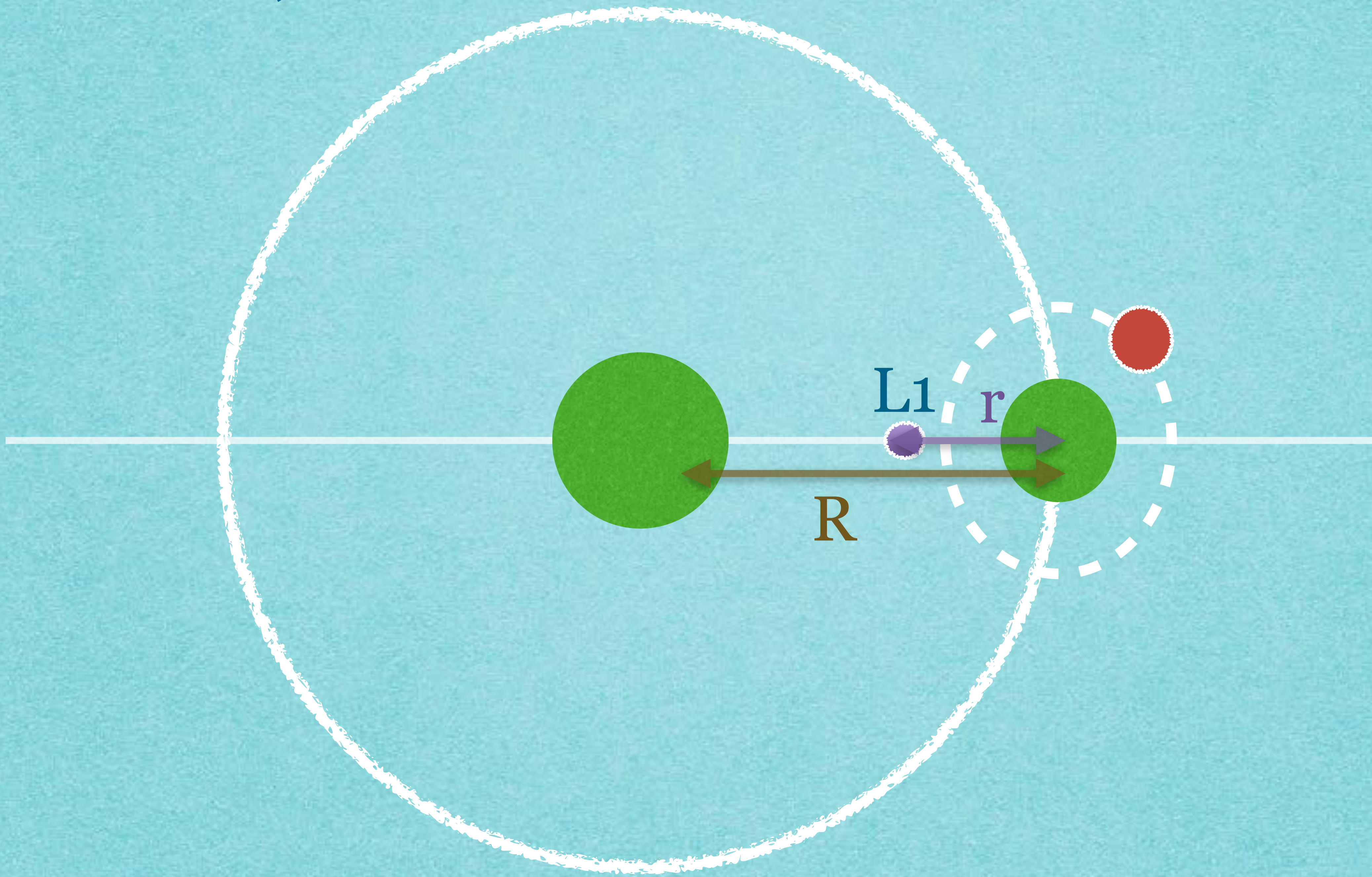
3-body problem

Lagrange points/tidal/Hill radius



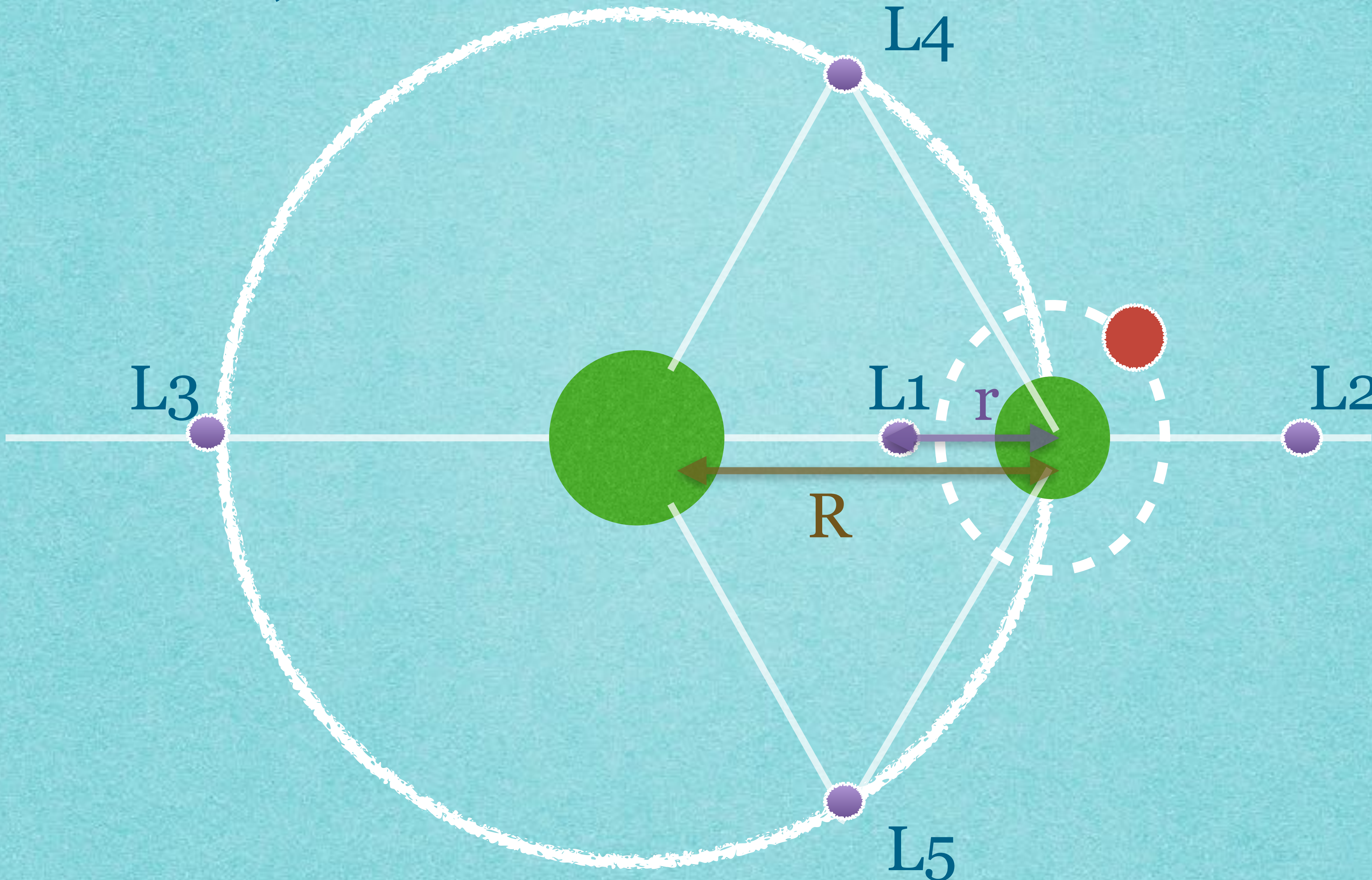
3-body problem

Circular, restricted



3-body problem

Circular, restricted

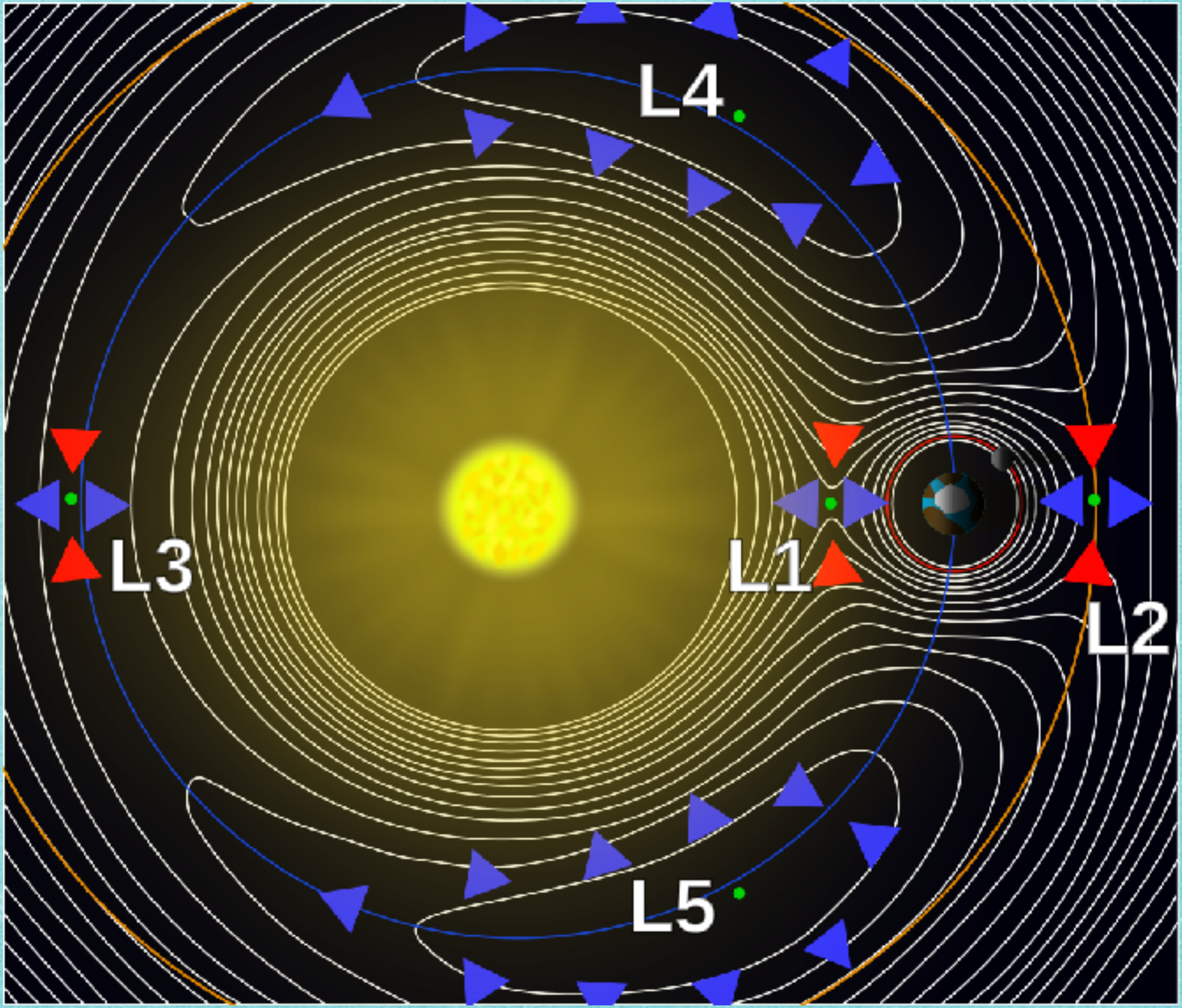


3-body problem

Circular, restricted

Lagrange points/tidal/Hill radius

A contour plot of the effective potential due to gravity and the centrifugal force of a two-body system in a rotating frame of reference. The arrows indicate the gradients of the potential around the five Lagrange points—downhill toward them (red) or away from them (blue). Counterintuitively, the L4 and L5 points are the high points of the potential. At the points themselves these forces are balanced.



3-body problem

Goal - describe the 3-body dynamics

Some simplification

Circular, restricted

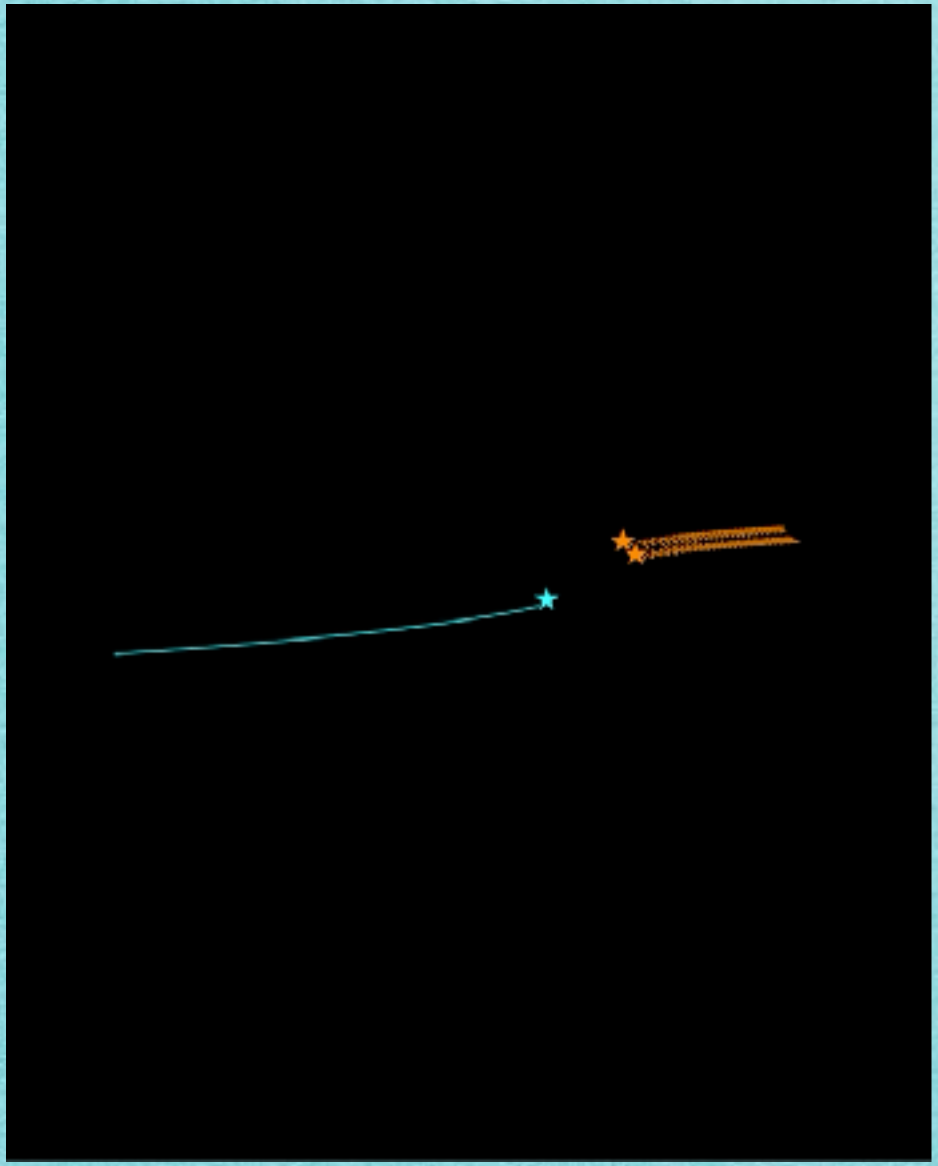
Result: When mass transfer, and accretion begins

Some generalization

Result: merging stars/black holes/planets and chaos

3-body problem

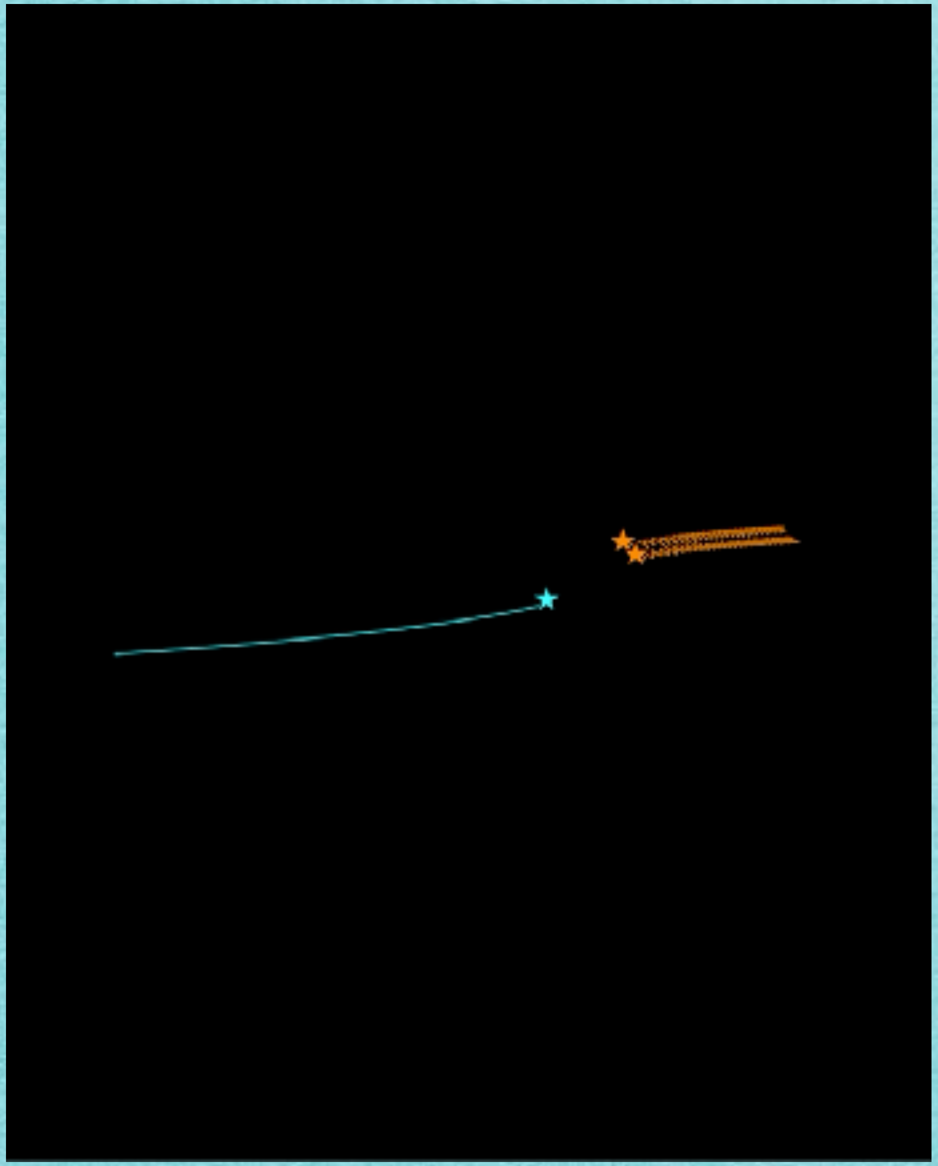
General masses and general orientations



- not bound

3-body problem

General masses and general orientations



Movie credit: Carl Rodriguez

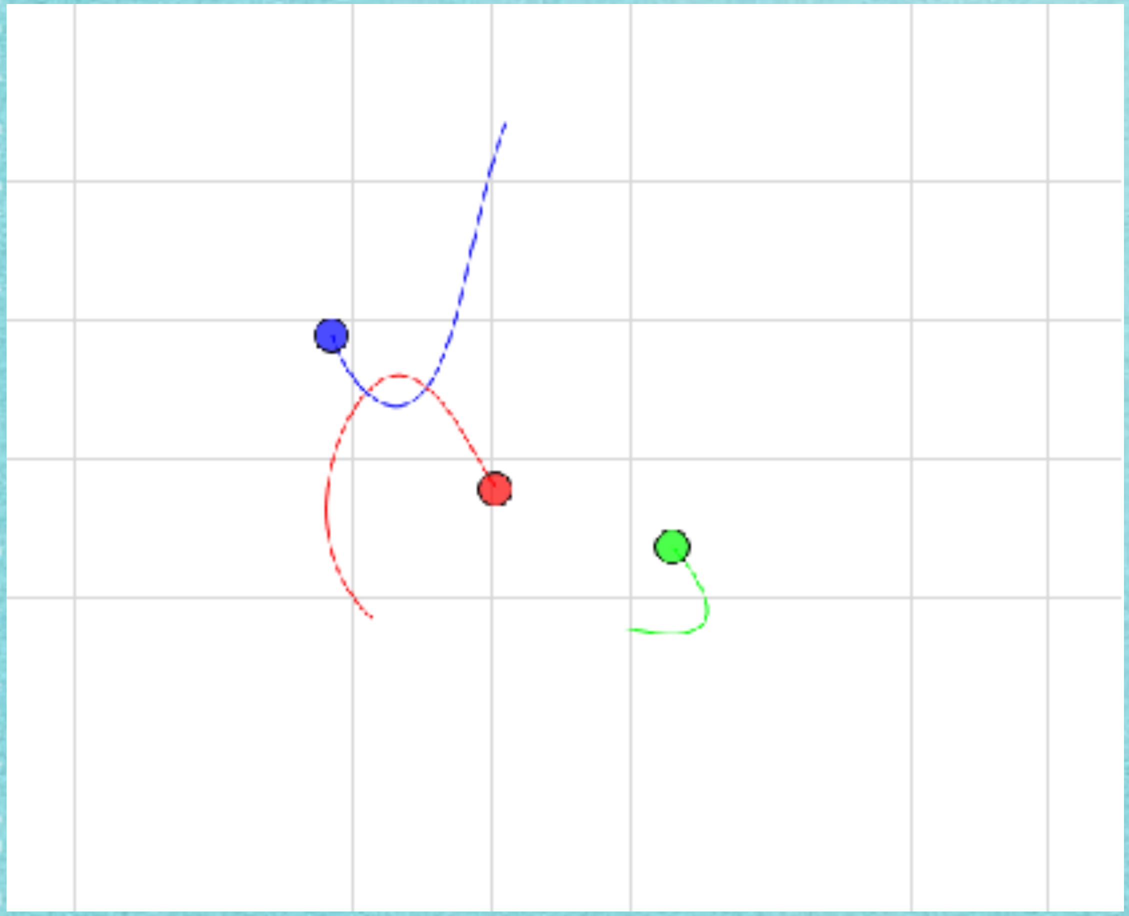
- *not bound*

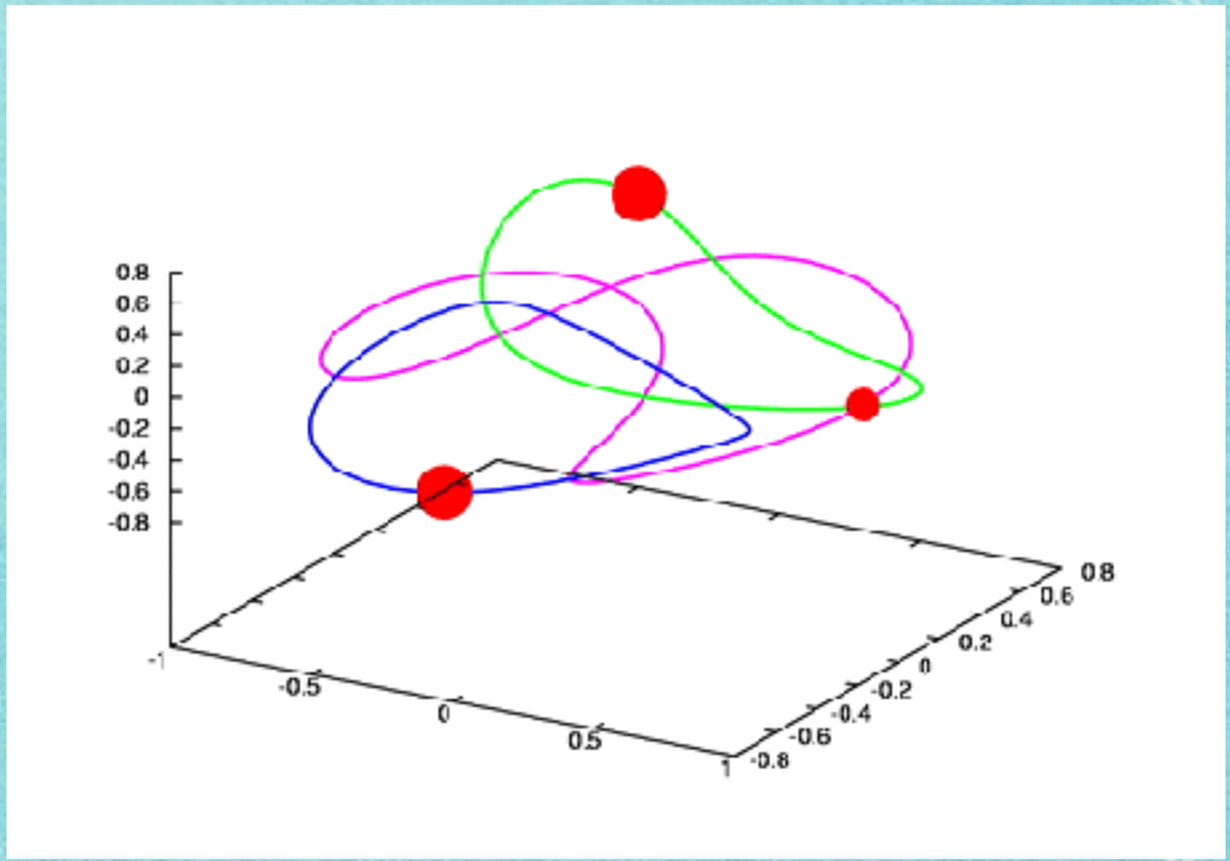
3-body problem

General masses and general orientations

Stability?

Stable examples



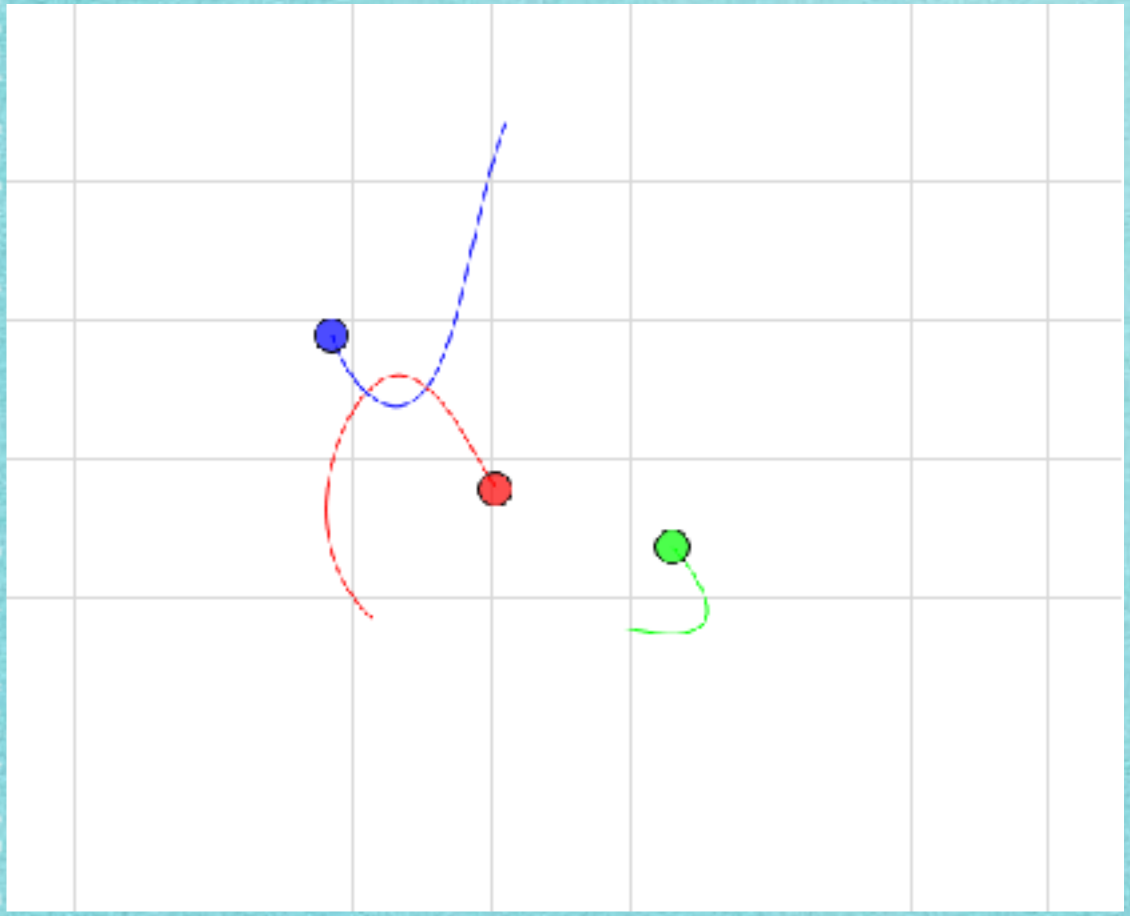


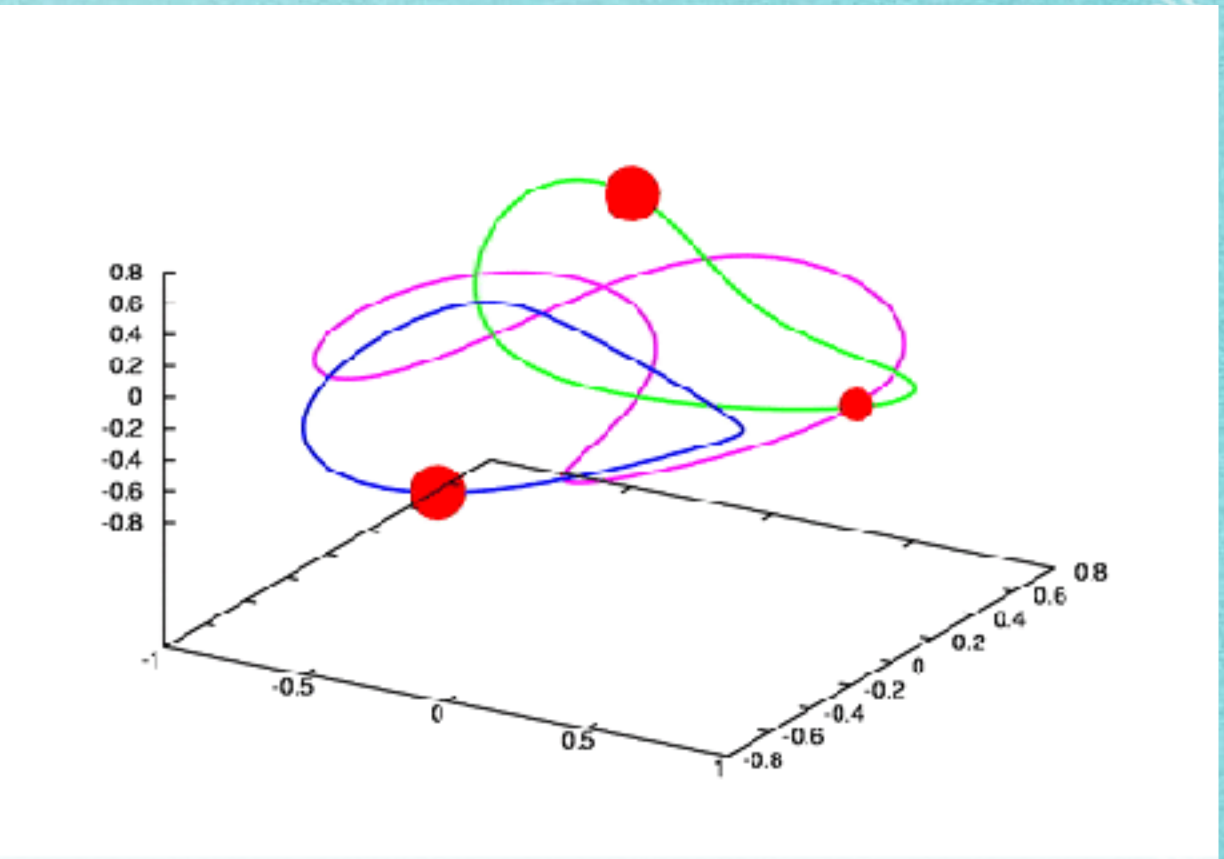
3-body problem

General masses and general orientations

Stability?

Stable examples



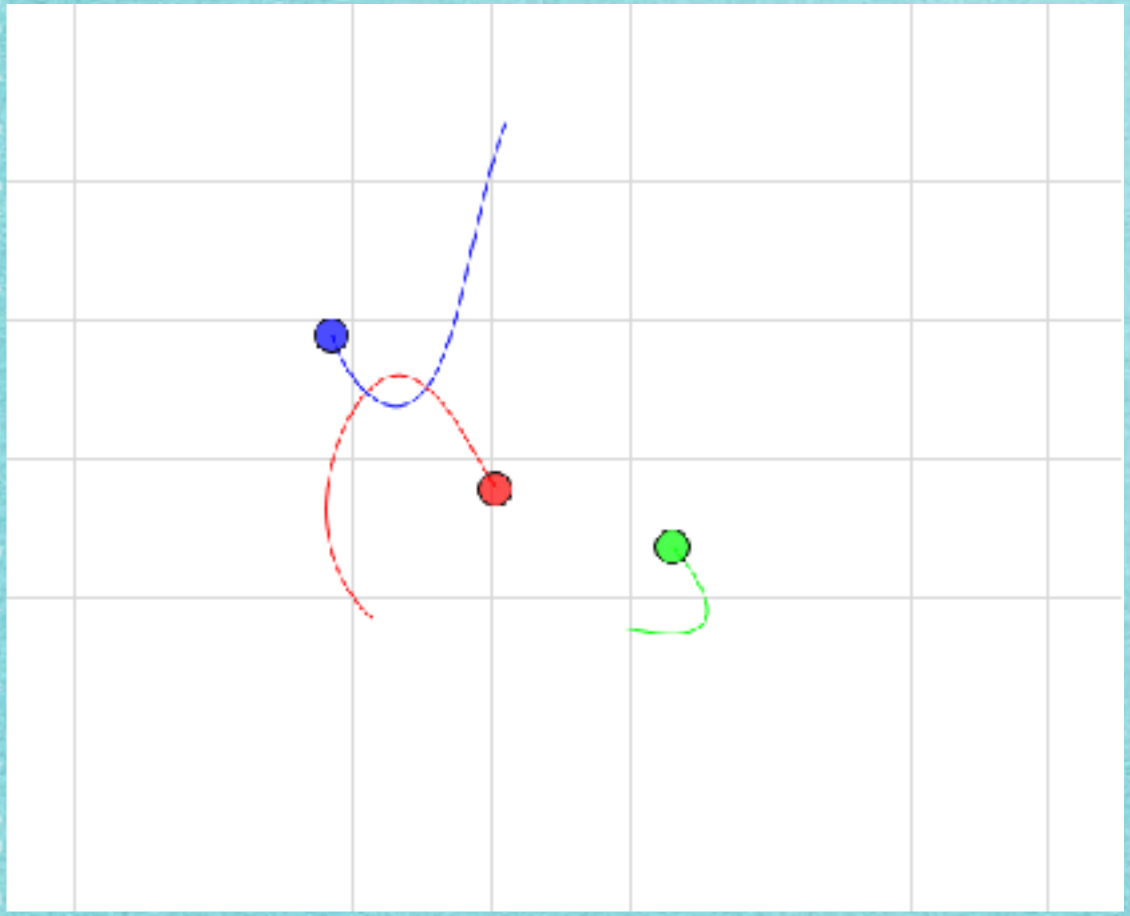


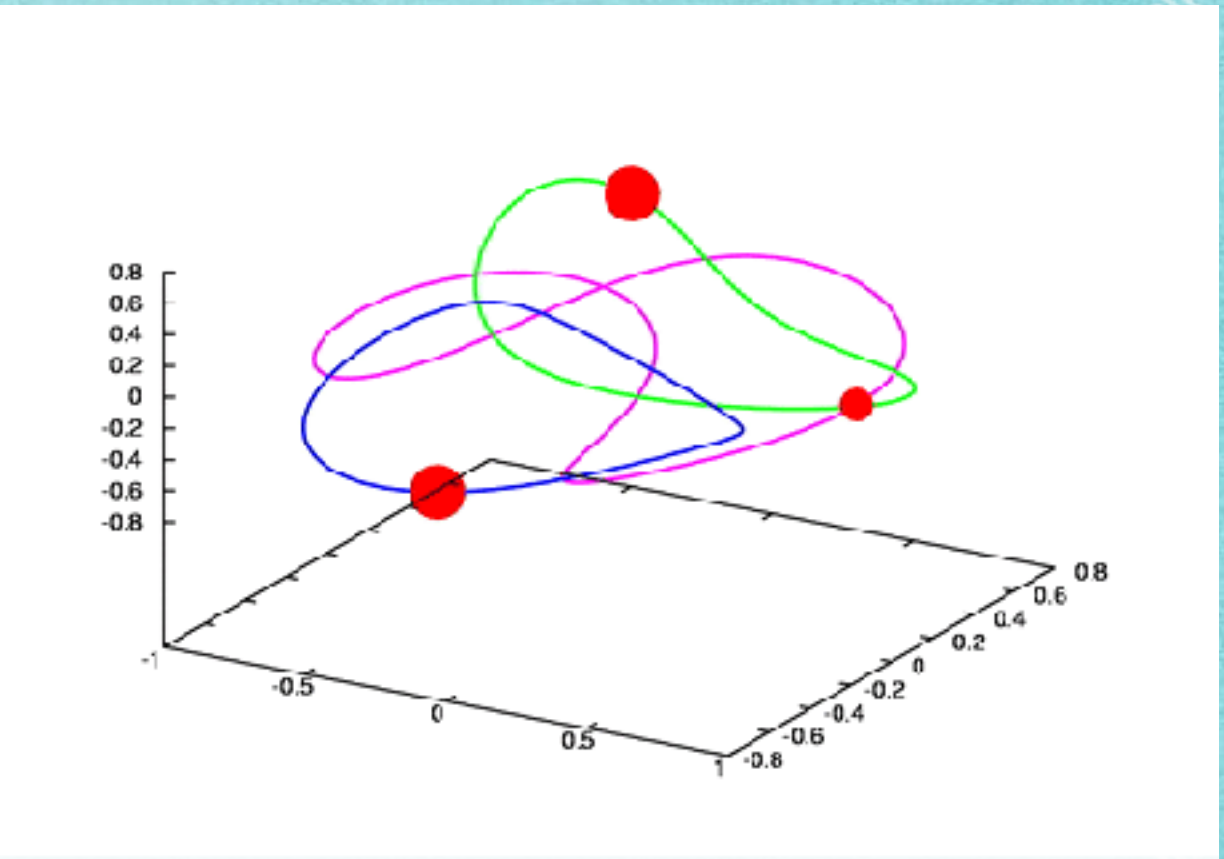
3-body problem

General masses and general orientations

Stability?

Stable examples

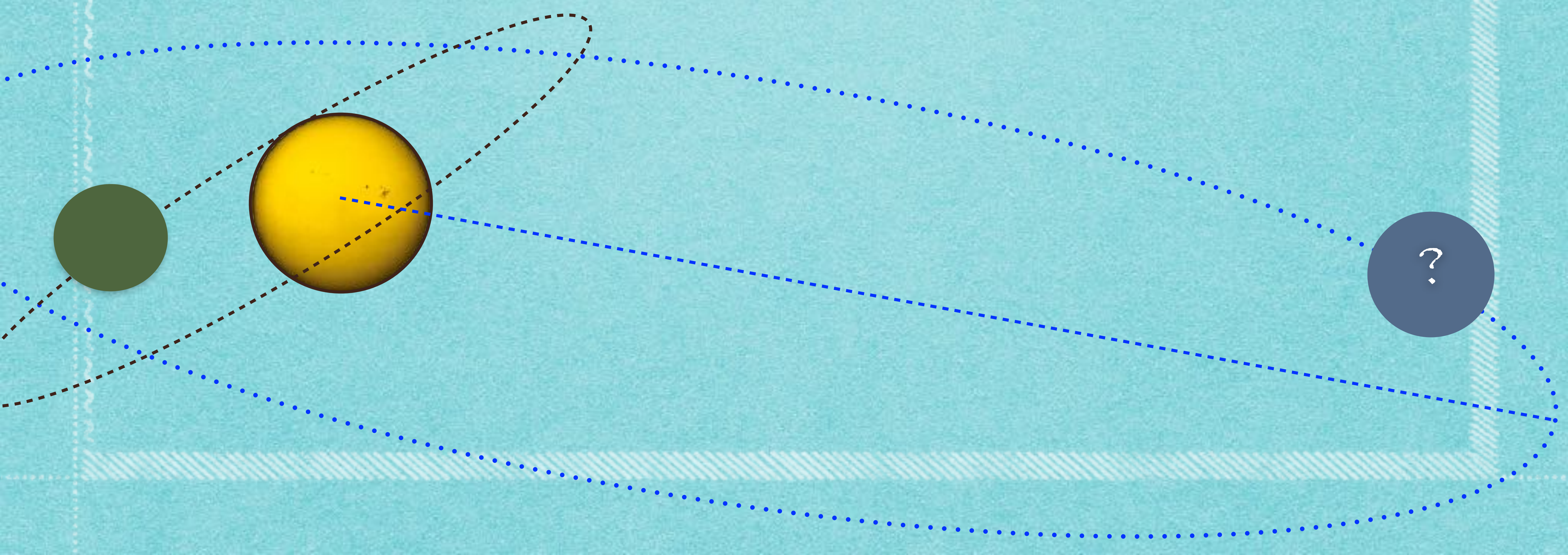




3-body problem

General masses and general orientations

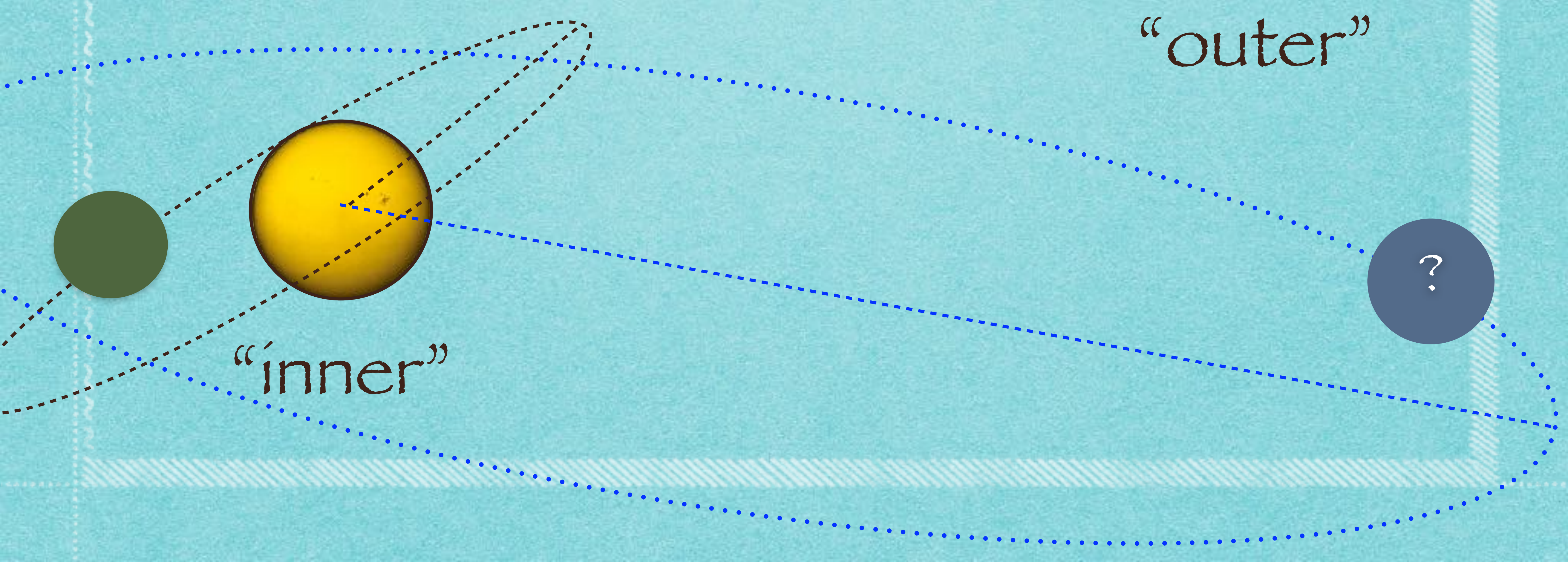
General stability requirement



3-body problem

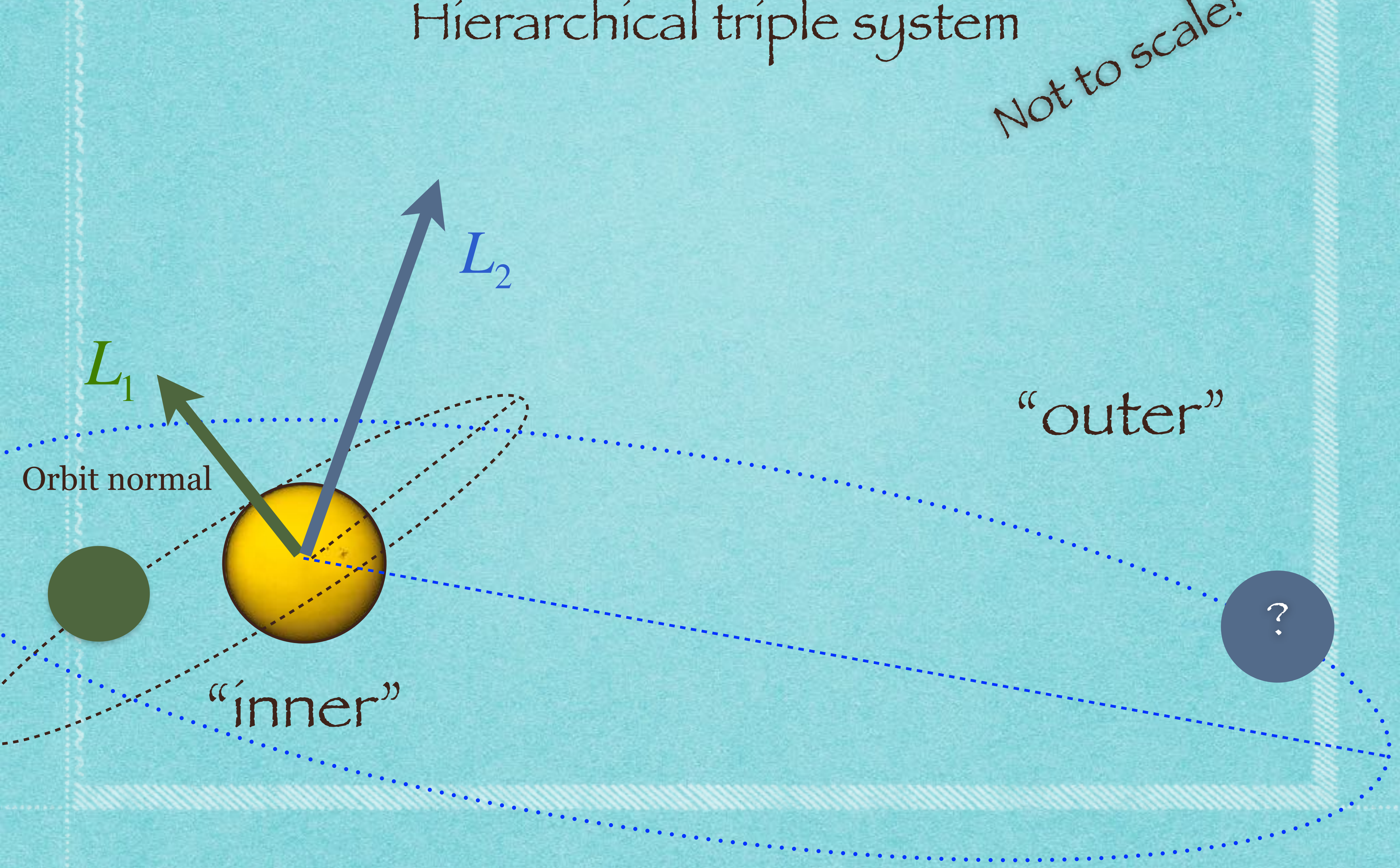
Hierarchical triple system

Not to scale!



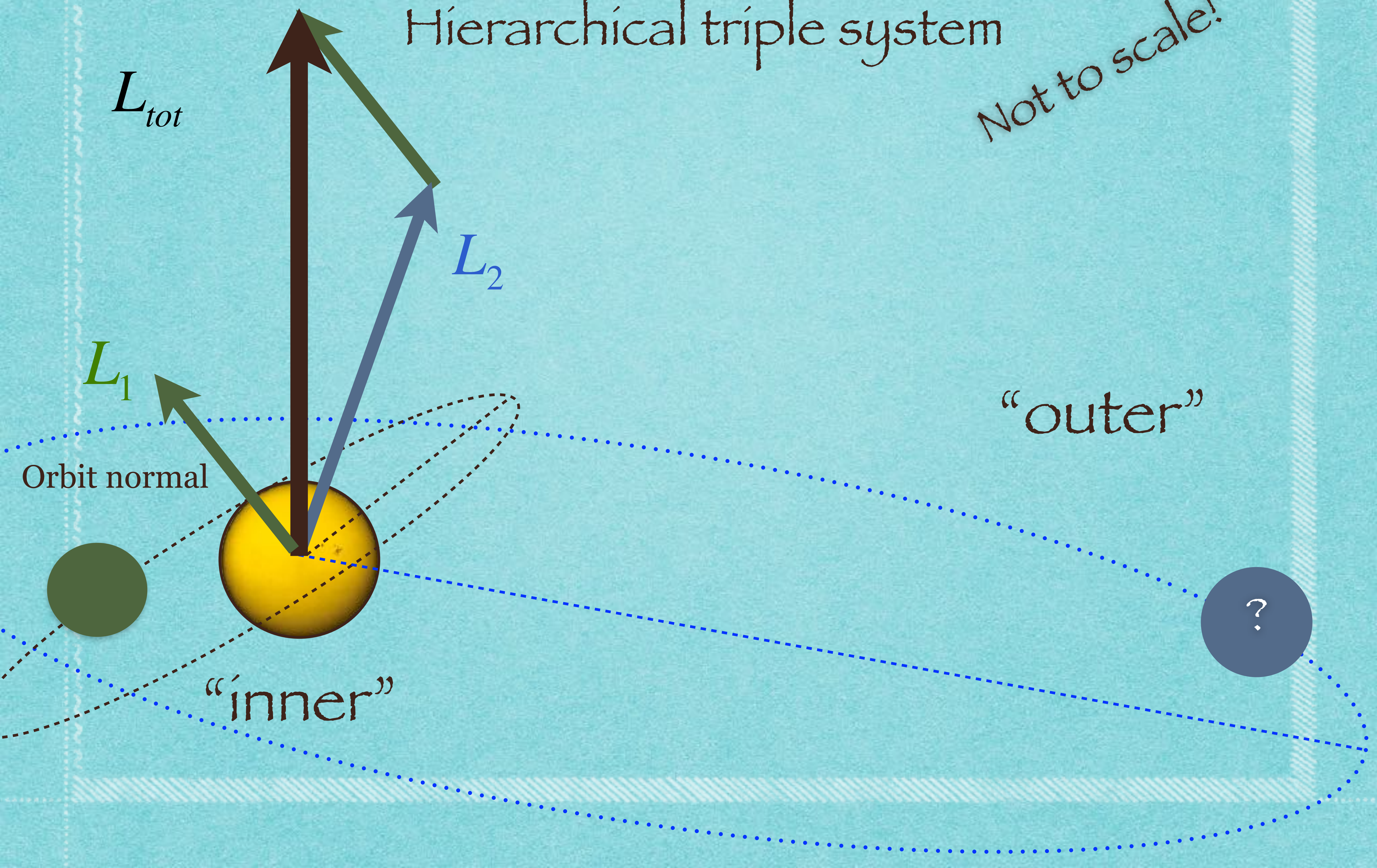
3-body problem

Hierarchical triple system



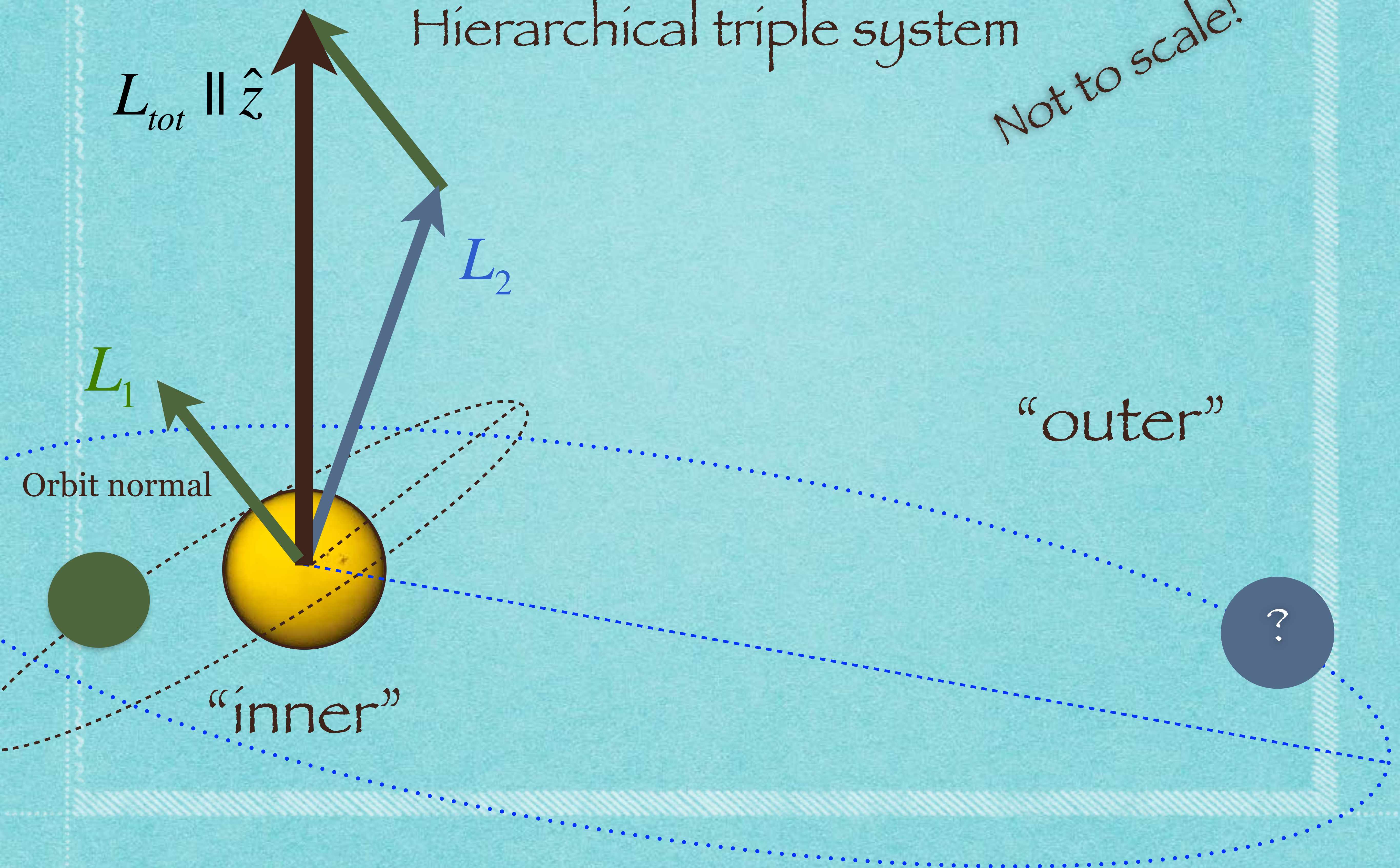
3-body problem

Hierarchical triple system



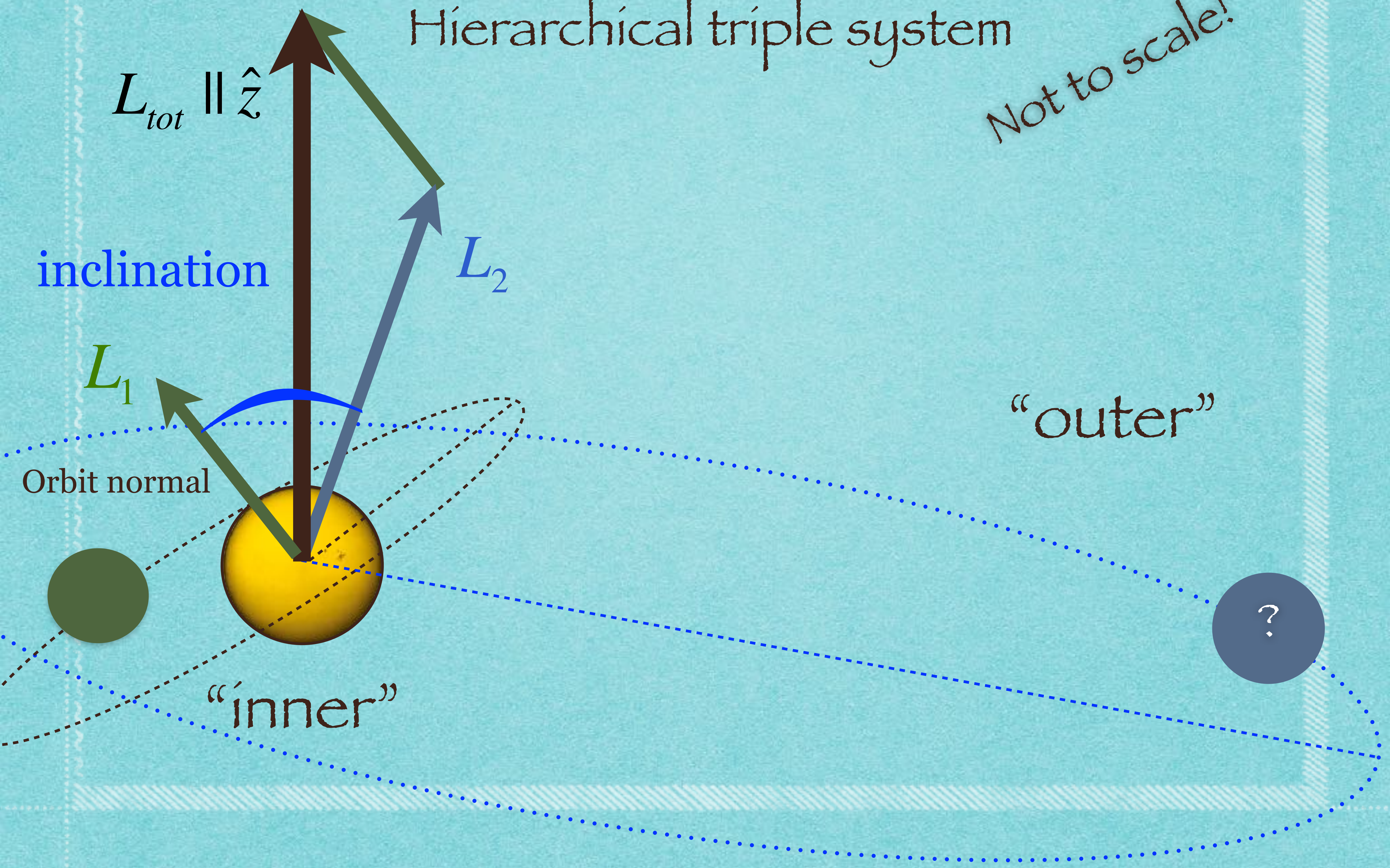
3-body problem

Hierarchical triple system



3-body problem

Hierarchical triple system

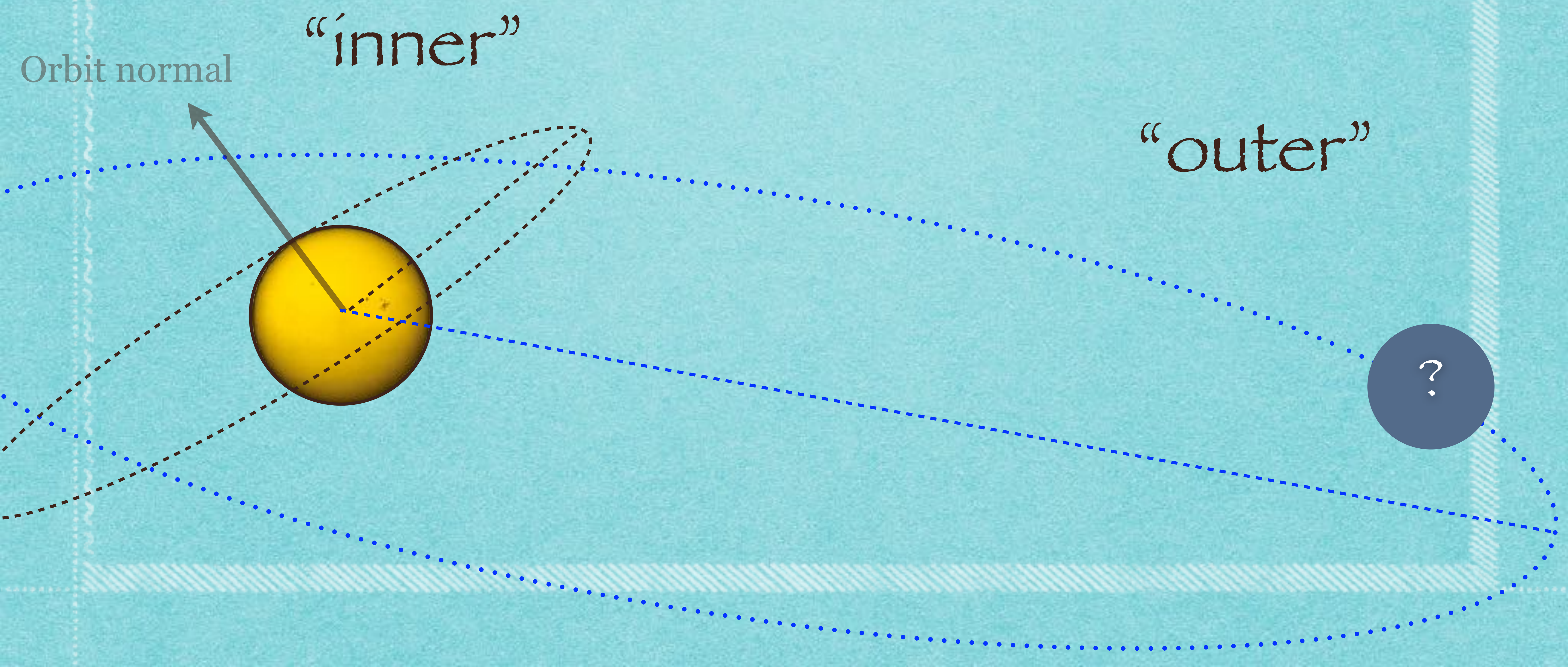


The Eccentric Kozai-Lidov (EKL)

Hierarchical triple system

Not to scale!

Kozai 1962, Lidov 1962

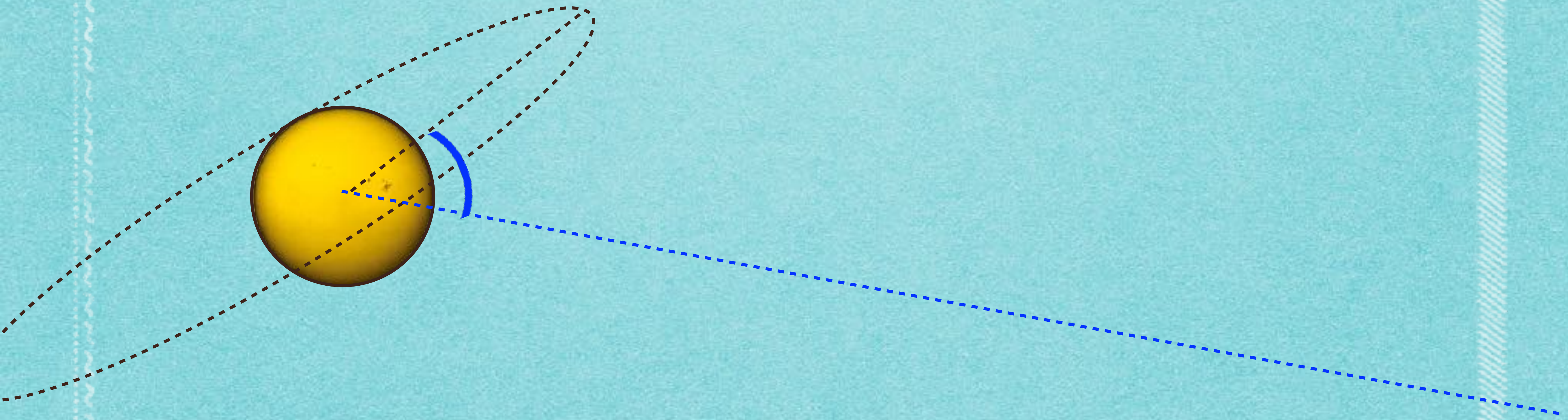


The Eccentric Kozai-Lidov (EKL)

The eccentricity and inclination oscillate

Kozai 1962, Lidov 1962

For initially inclined system ≳ 40°



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Kozai 1962, Lidov 1962

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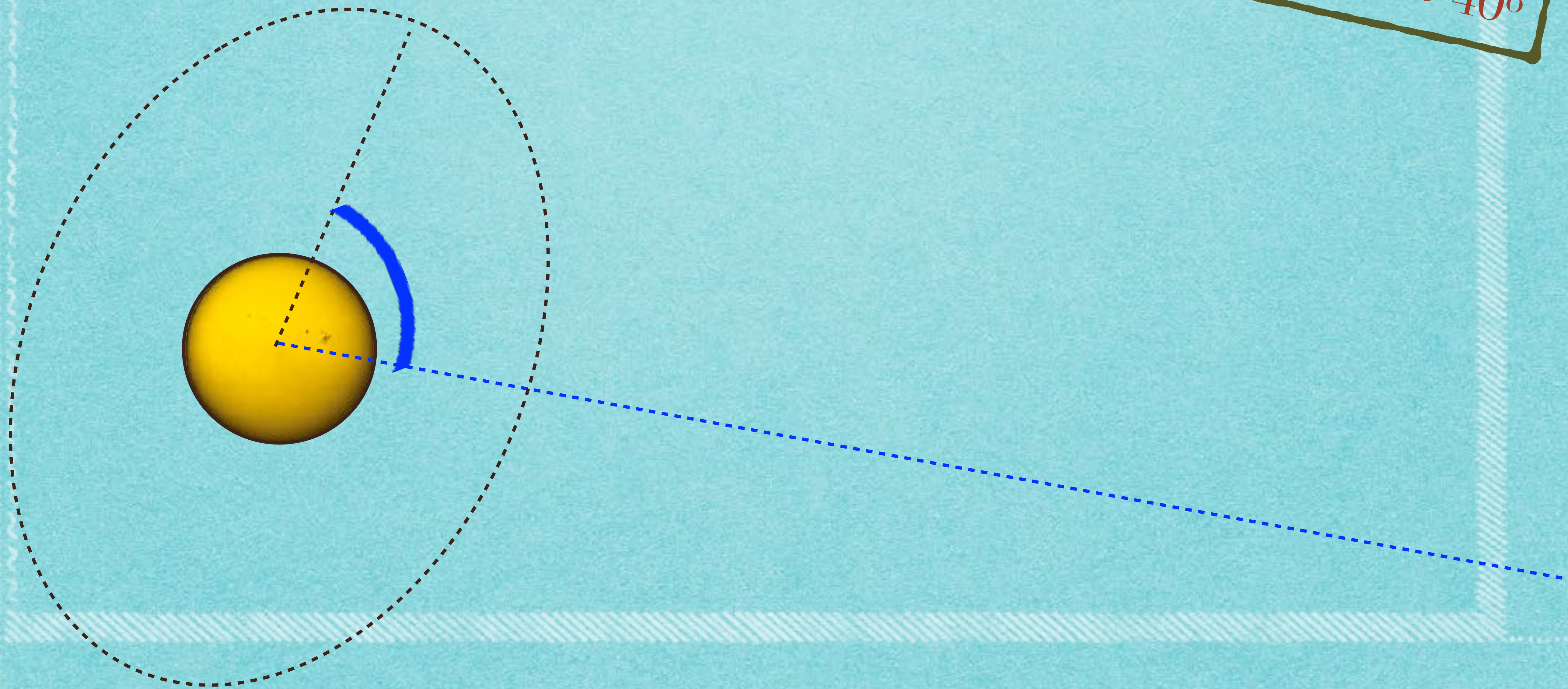
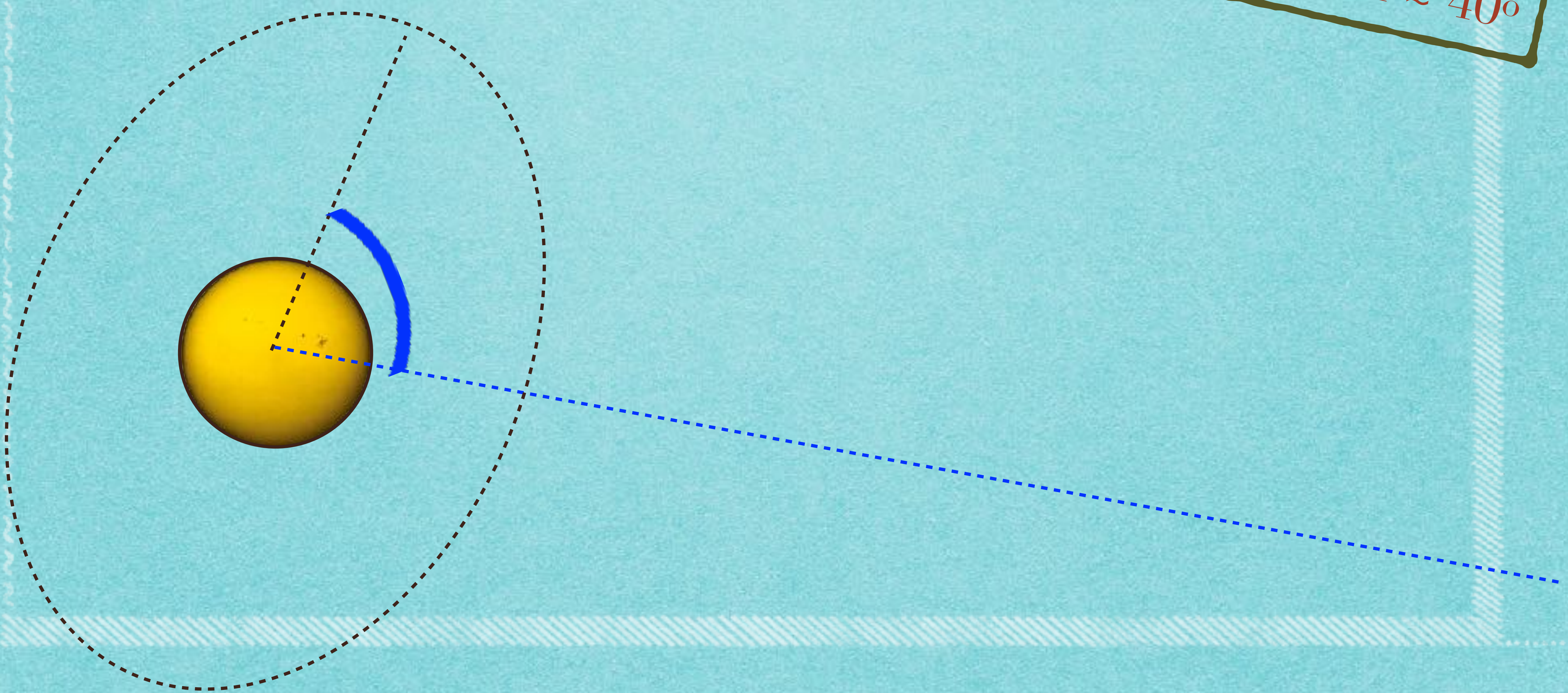


The Eccentric Kozai-Lidov (EKL)

The eccentricity and inclination oscillate

Kozai 1962, Lidov 1962

For initially inclined system $\gtrsim 40^o$

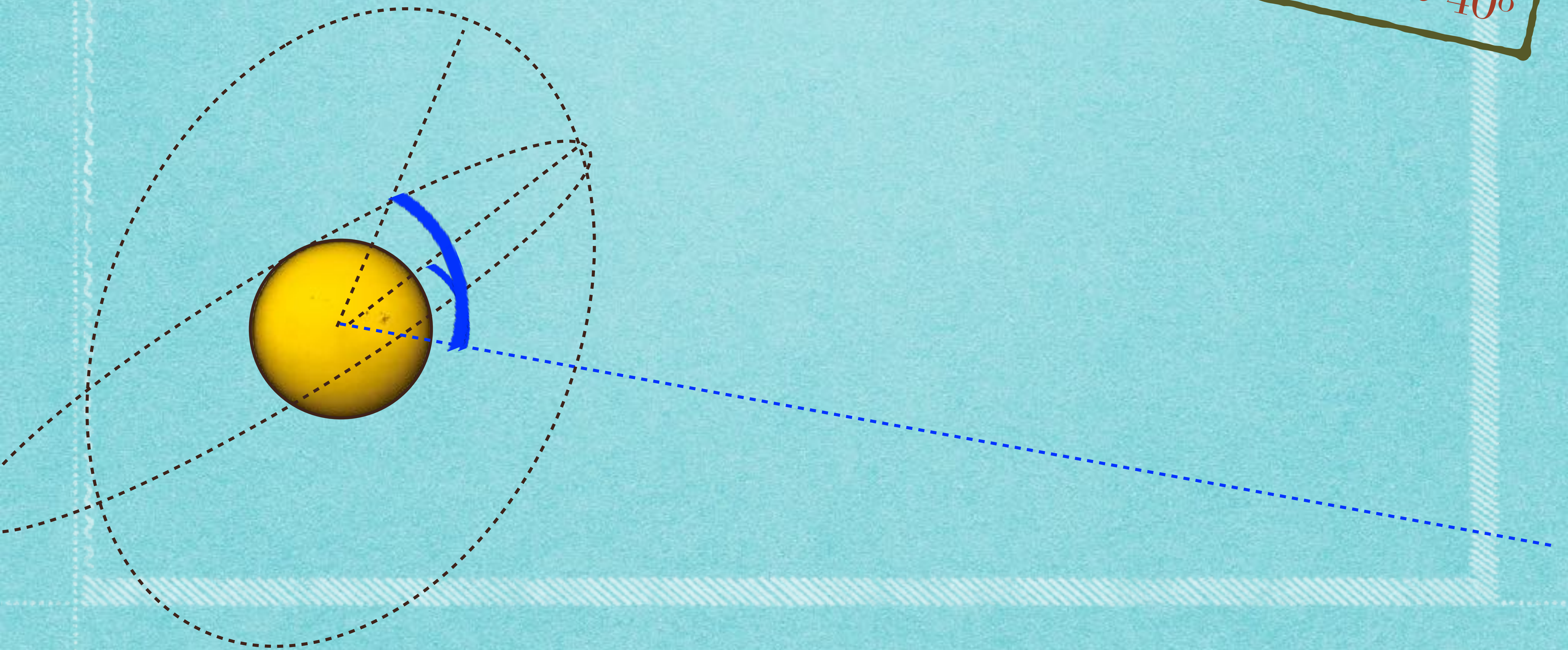


The Eccentric Kozai-Lidov (EKL)

The eccentricity and inclination oscillate

Kozai 1962, Lidov 1962

For initially inclined system ≳ 40°



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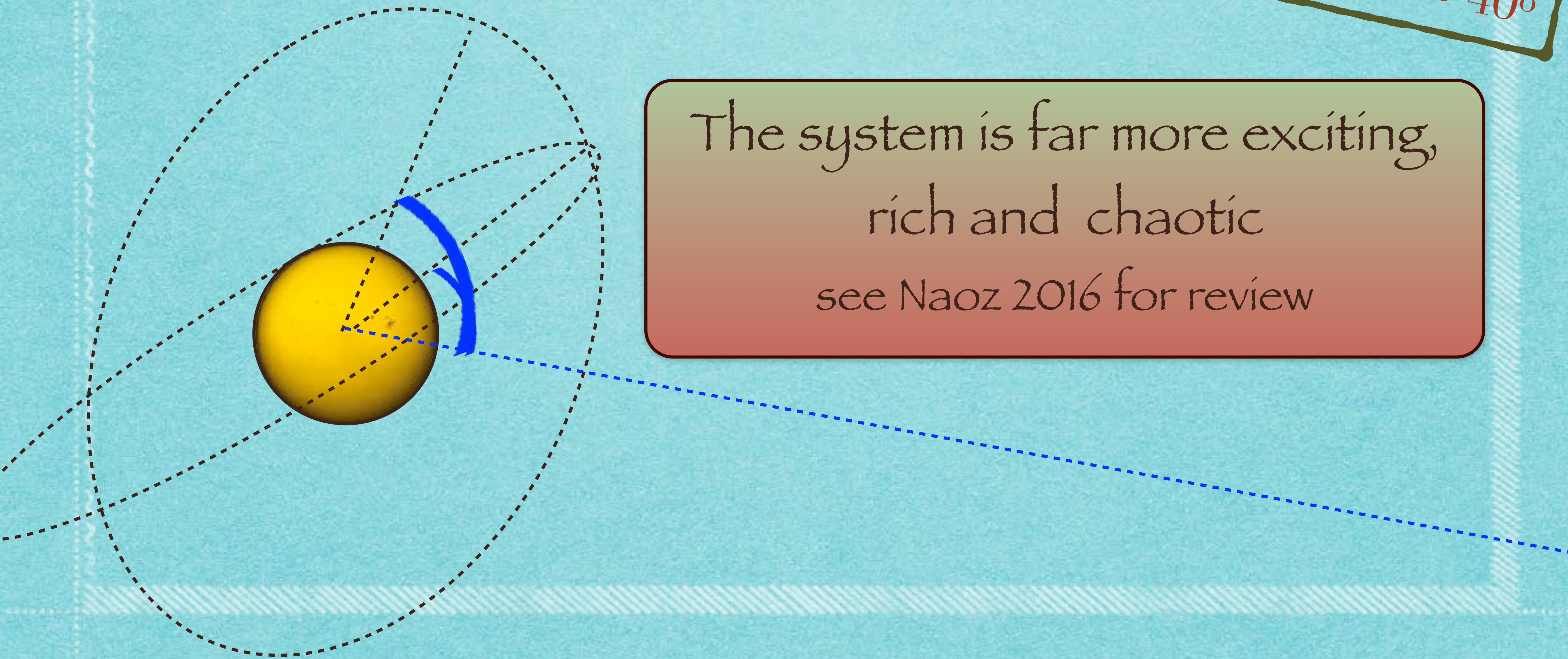
The eccentricity and inclination oscillate

Kozai 1962, Lidov 1962

For initially inclined system ≳ 40º



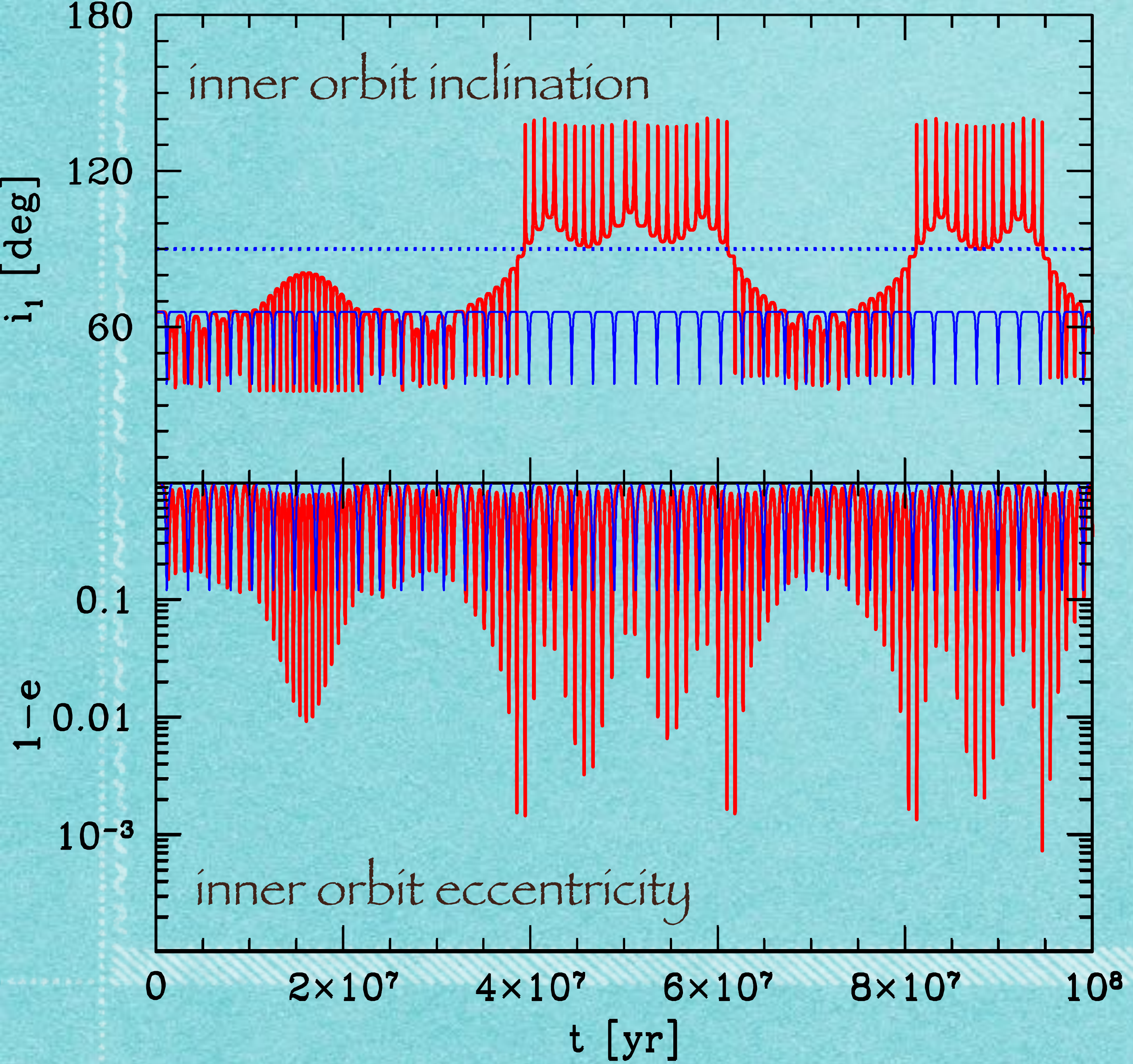
The system is far more exciting, rich and chaotic

see Naoz 2016 for review

The Eccentric Kozai-Lidov (EKL)

The Eccentric Kozai-Lidov (EKL)

Extremely high eccentricities and flips



$M_1 = 1\ \mathrm{M}_\odot$
$M_2 = 1\ \mathrm{M}_J$
$M_3 = 4\ \mathrm{M}_J$

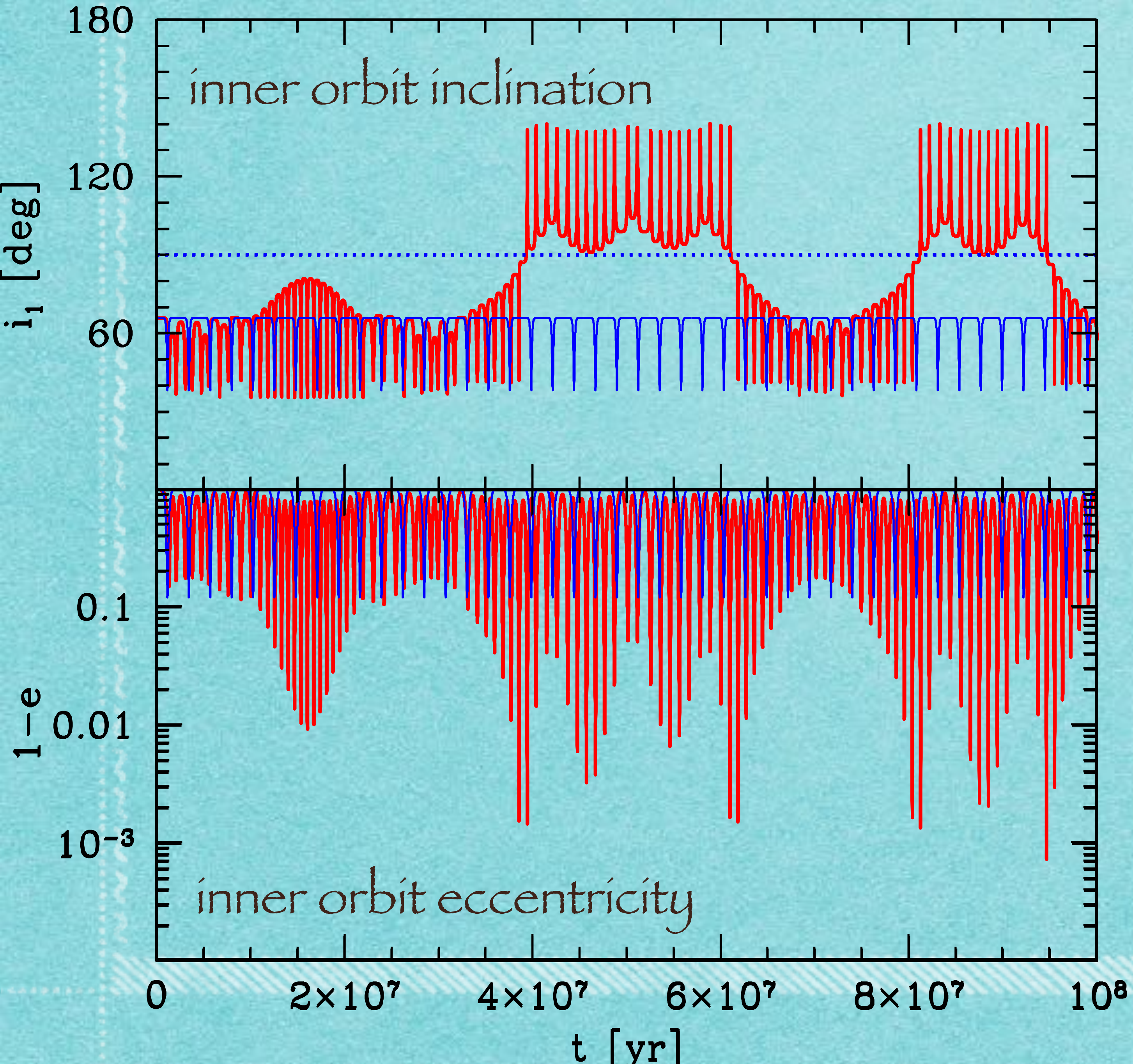
$a_1 = 5$ AU
$a_2 = 51$ AU

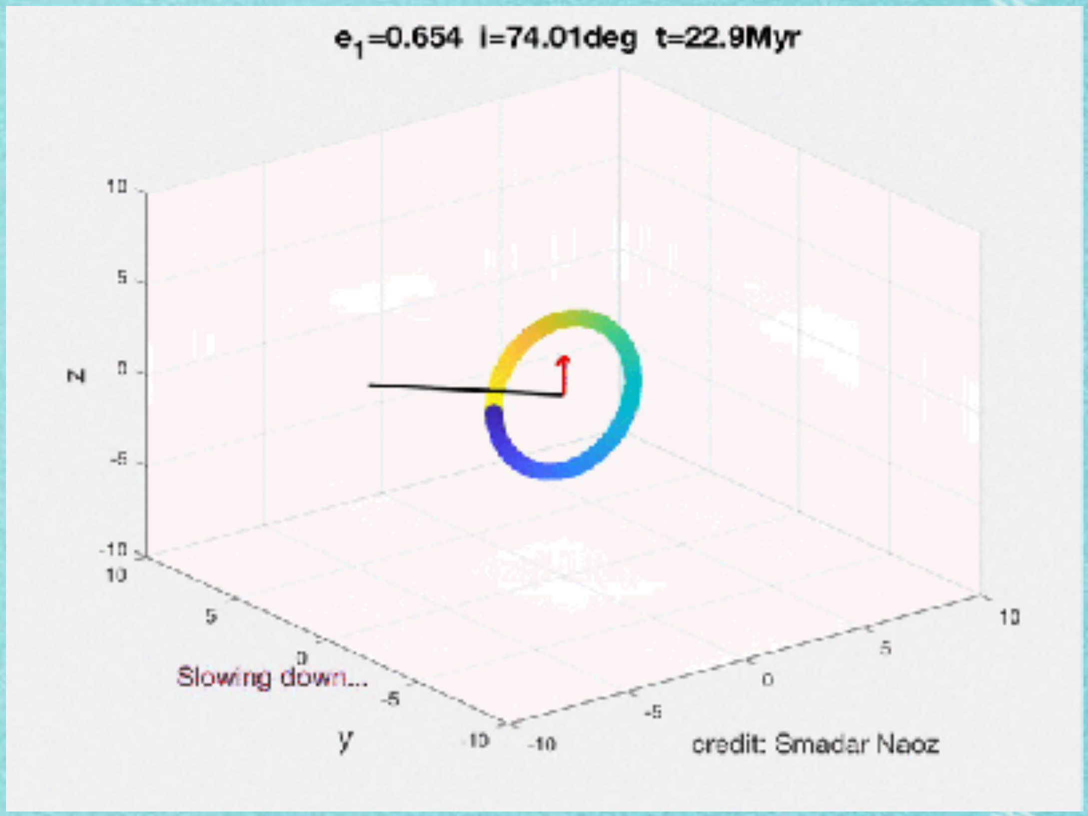
$i = 71°$

Naoz et al (2013),MNRAS, arXiv:1107.2414

The Eccentric Kozai-Lidov (EKL)

Extremely high eccentricities and flips





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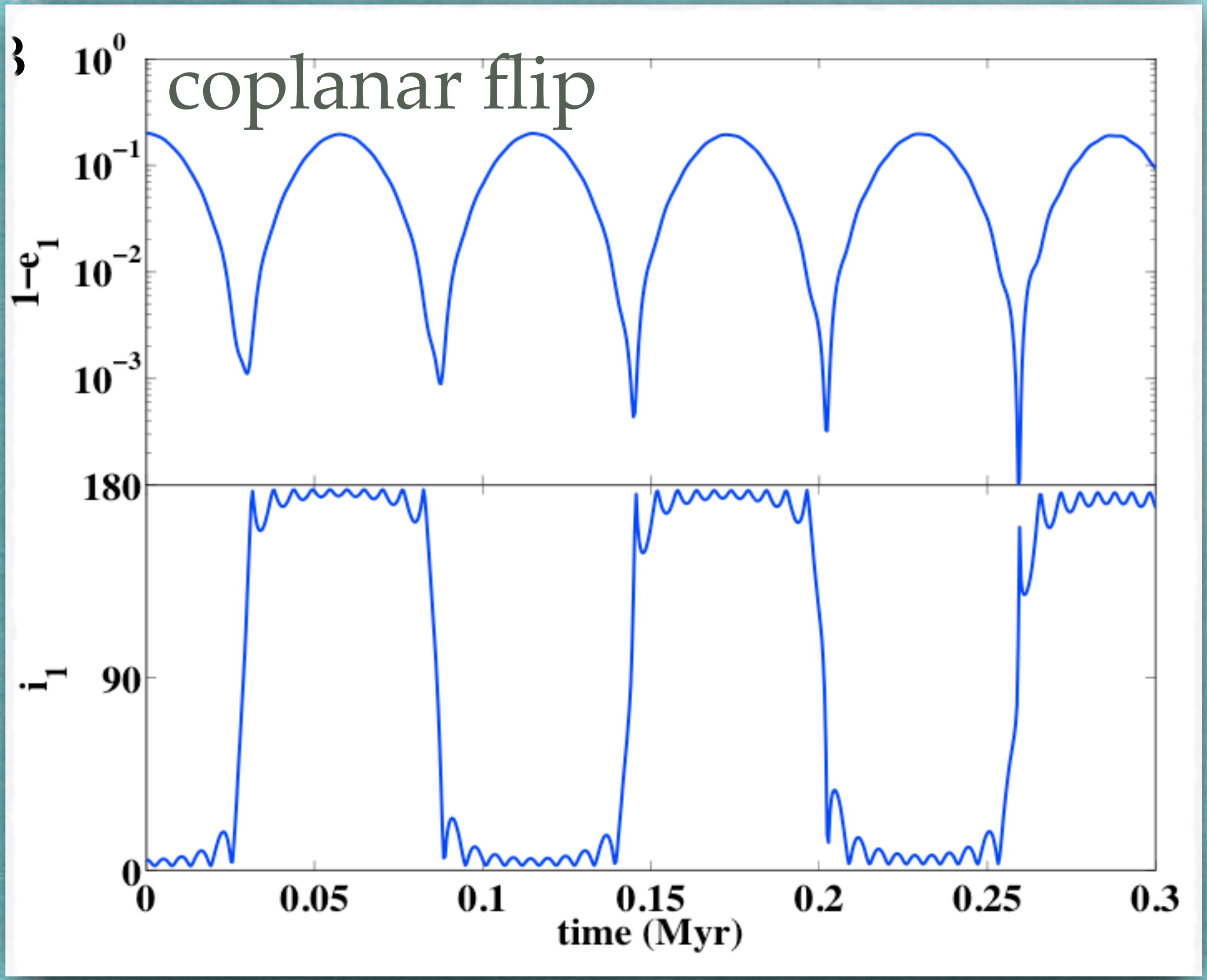
$i = 71^\circ$

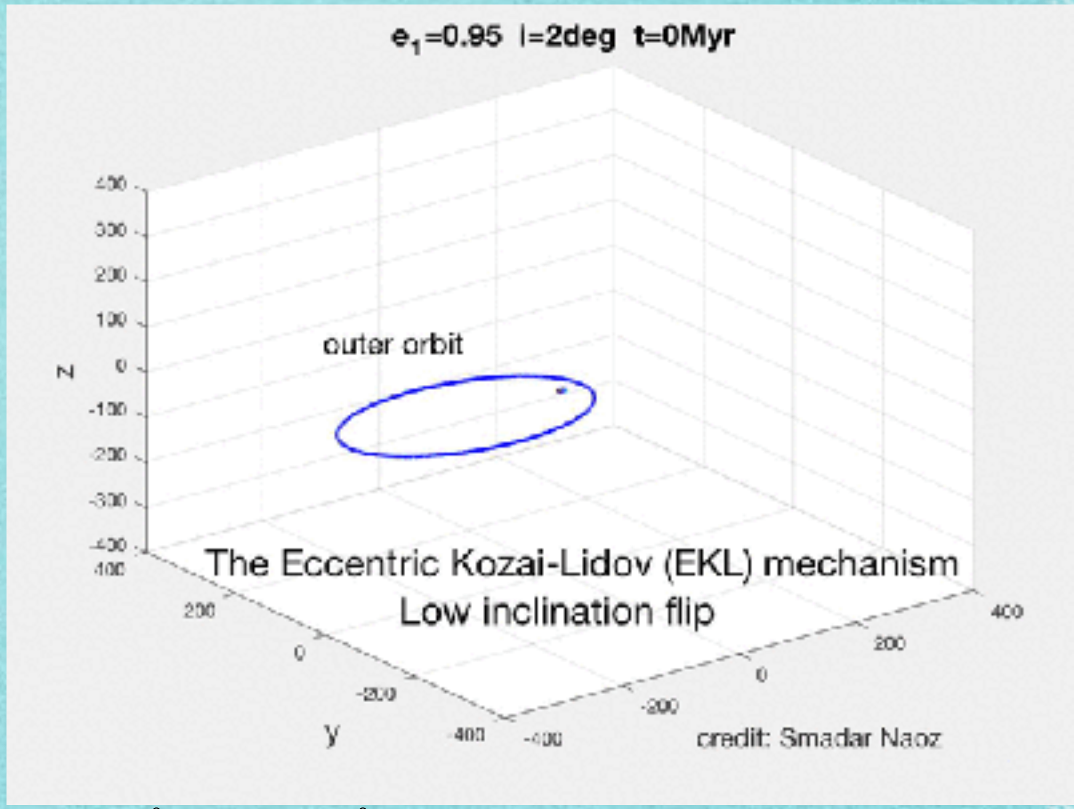
Naoz et al (2013),MNRAS, arXiv:1107.2414

The Eccentric Kozai-Lidov (EKL)

The Eccentric Kozai-Lidov (EKL)

Going beyond Kozai angles





$\omega_1 = 0^\circ, \Omega_1 = 180^\circ,$

$e_2 = 0.6, a_1 = 4AU, a_2 = 50AU$

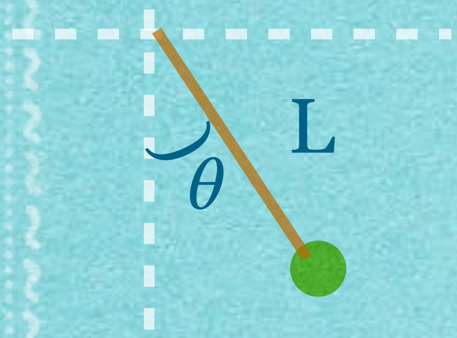
$e_1 = 0.8, i = 5^\circ$

$m_1 = 1M_\odot, m_2 = 1M_J, m_3 = 0.3M_\odot$

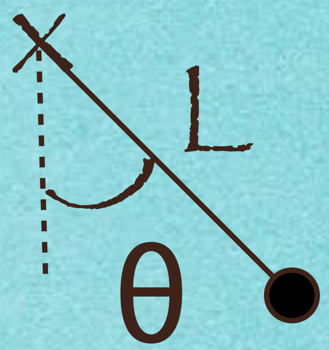
Li, **Naoz,** Kocsis, Loeb 2014a, ApJ arXiv:1310.6044

Li, **Naoz,** Holman, Loeb 2014b, ApJ, arXiv:1405.0494

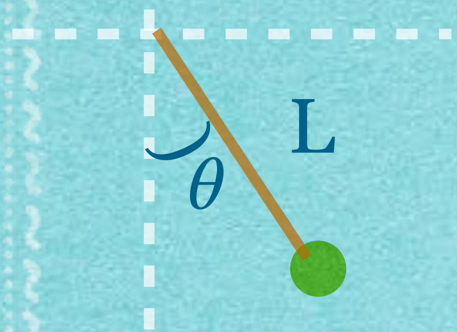
The Eccentric Kozai-Lidov (EKL)



$$H(\theta, p) = \frac{p^2}{2mL^2} - mgL\cos\theta$$



The Eccentric Kozai-Lidov (EKL)



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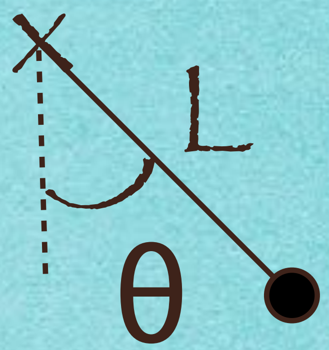
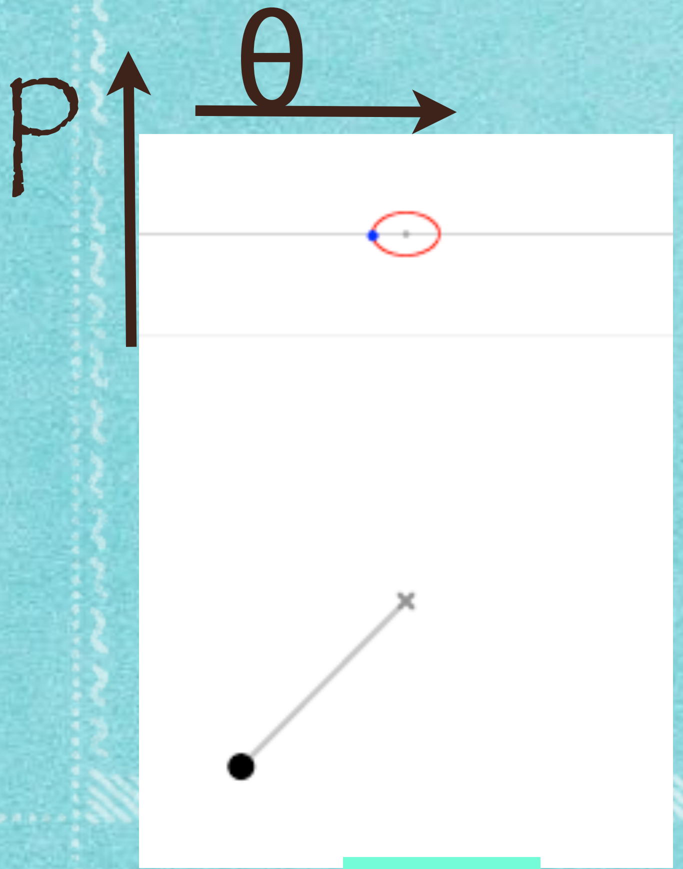
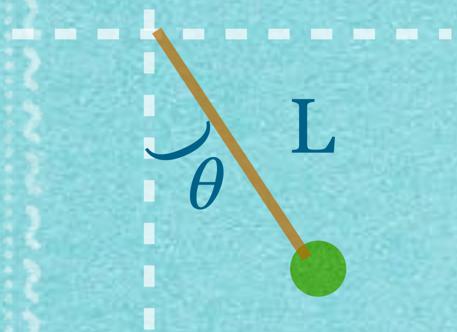




$\theta_0=45^o$

The Eccentric Kozai-Lidov (EKL)



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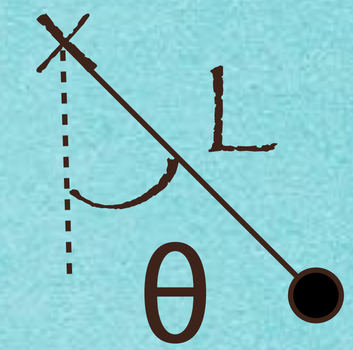
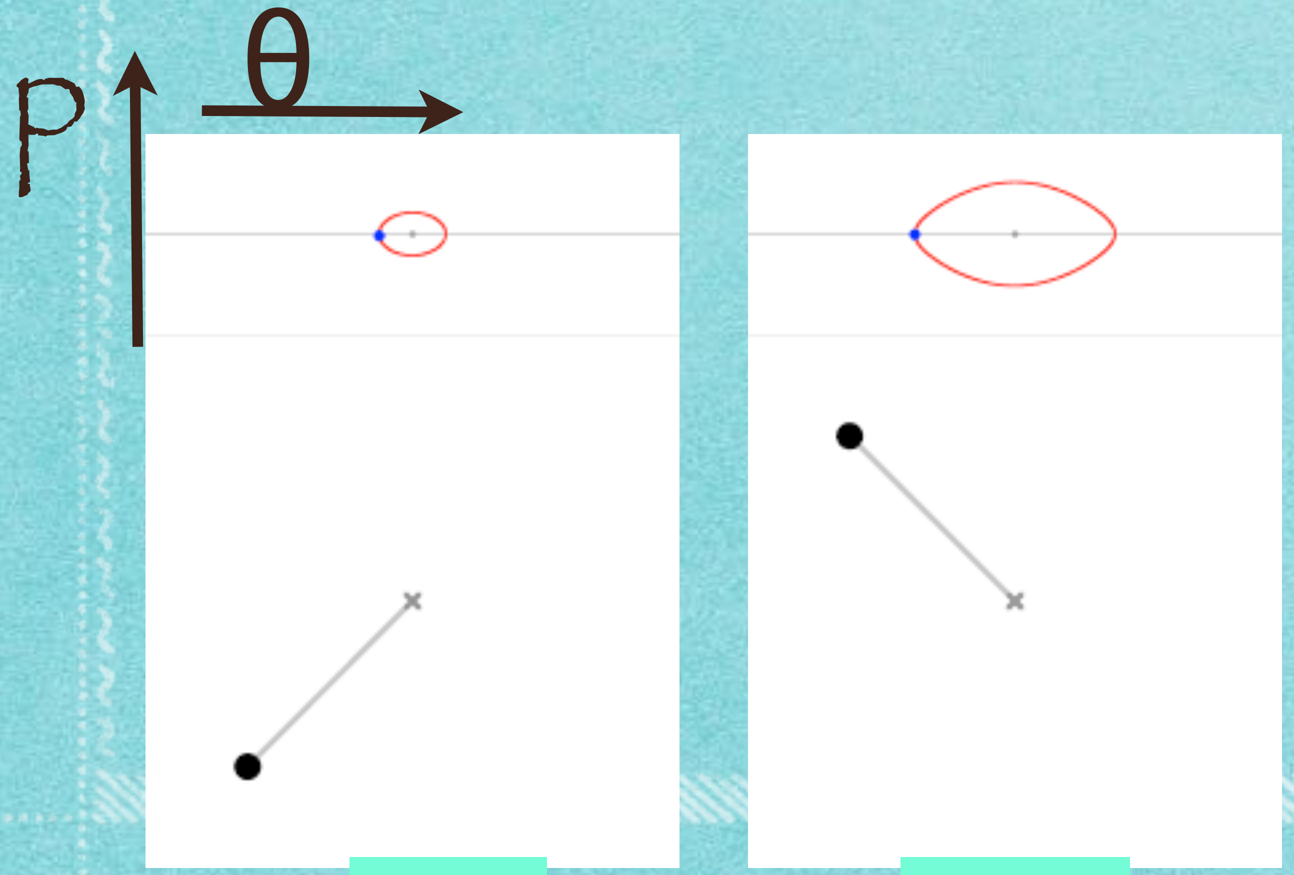
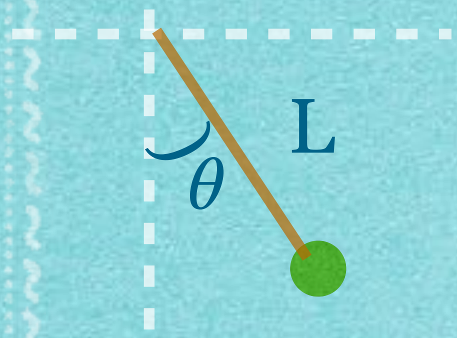




$\theta_0 = 45^o$ $\theta_0 = 135^o$

The Eccentric Kozai-Lidov (EKL)



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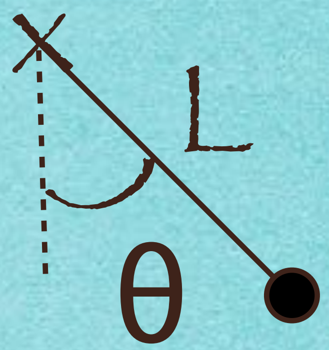
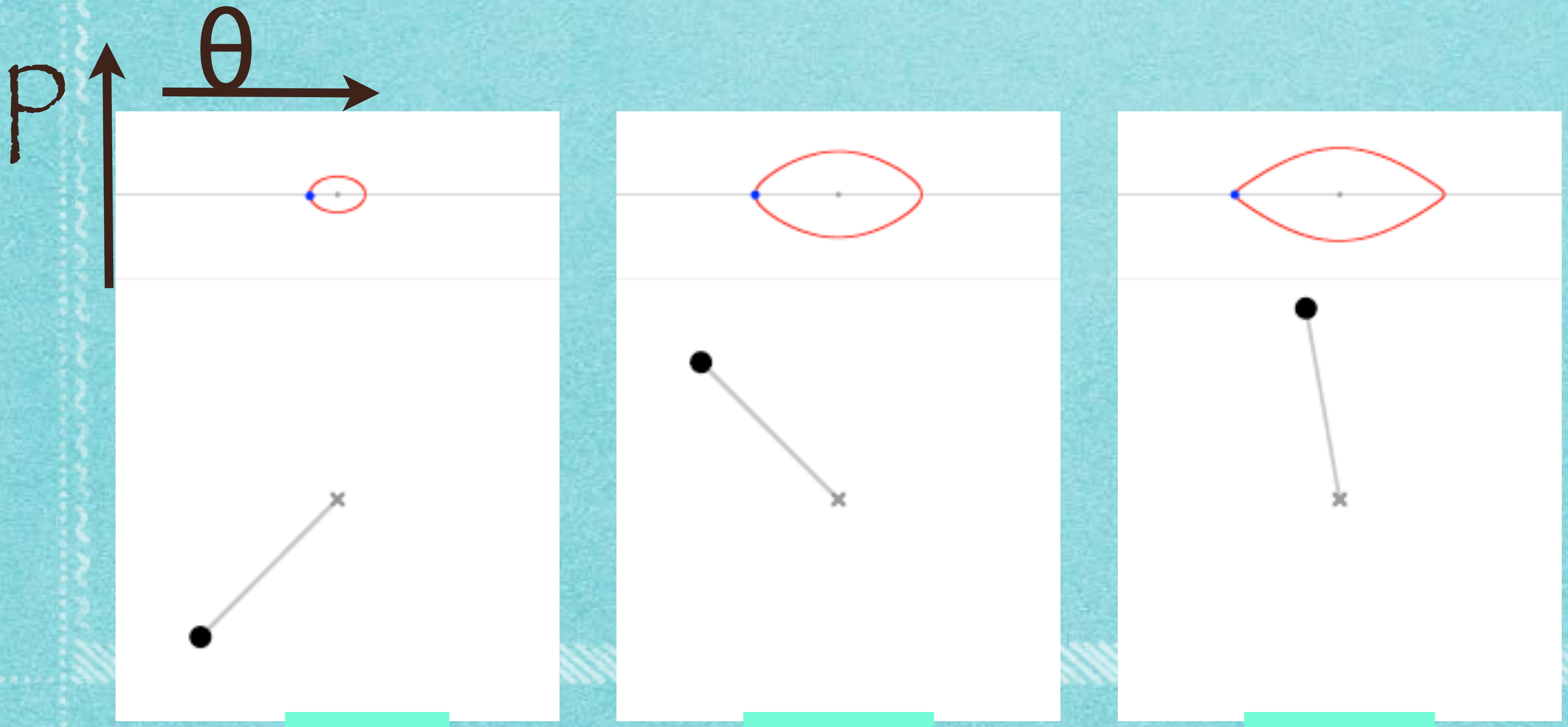


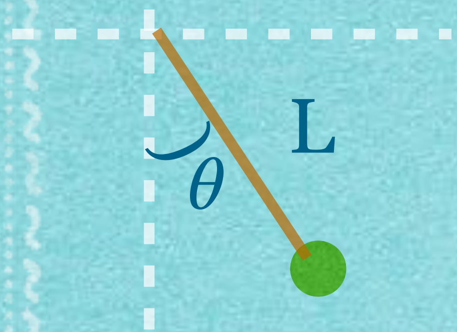


θ_0=45°

θ_0=135°

θ_0=170°

The Eccentric Kozai-Lidov (EKL)



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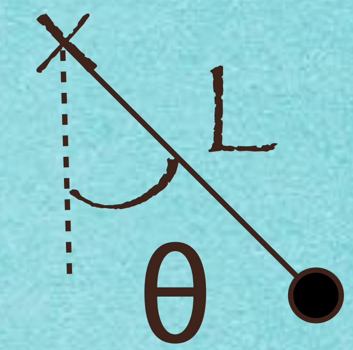
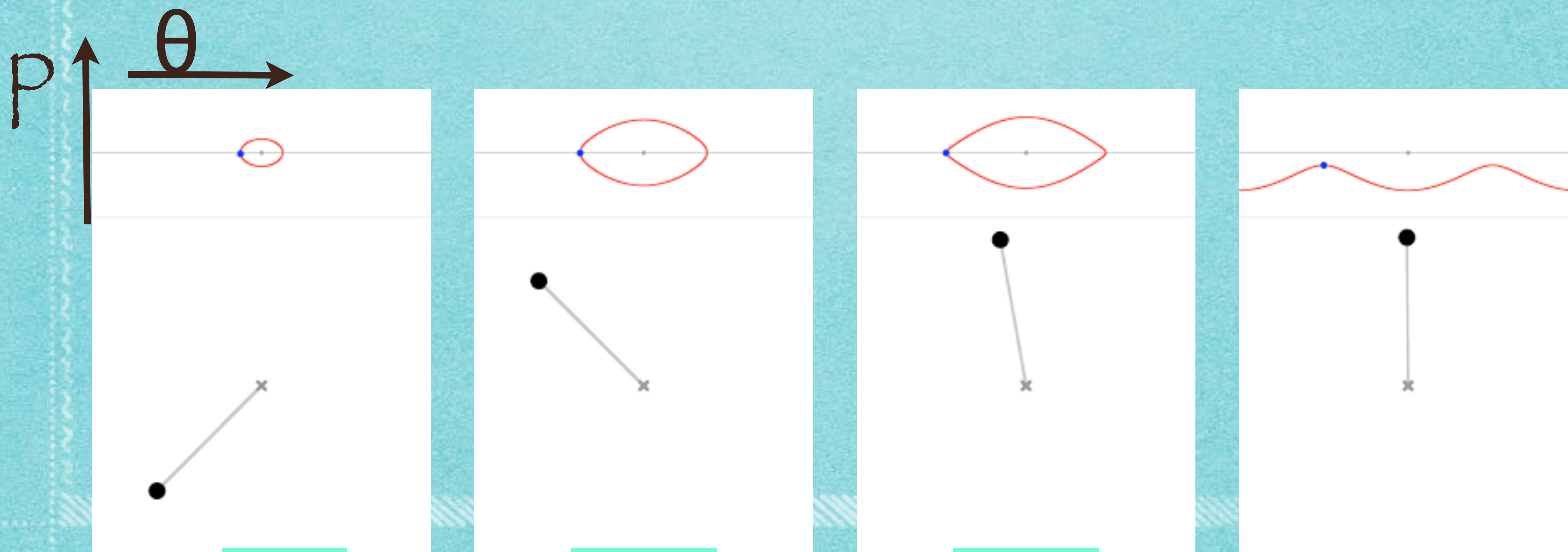
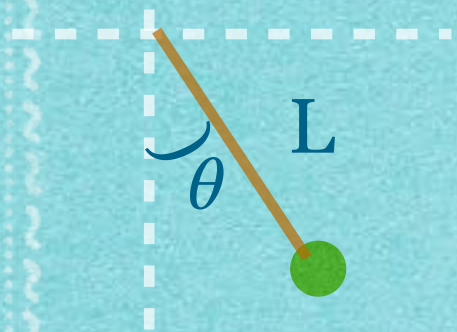


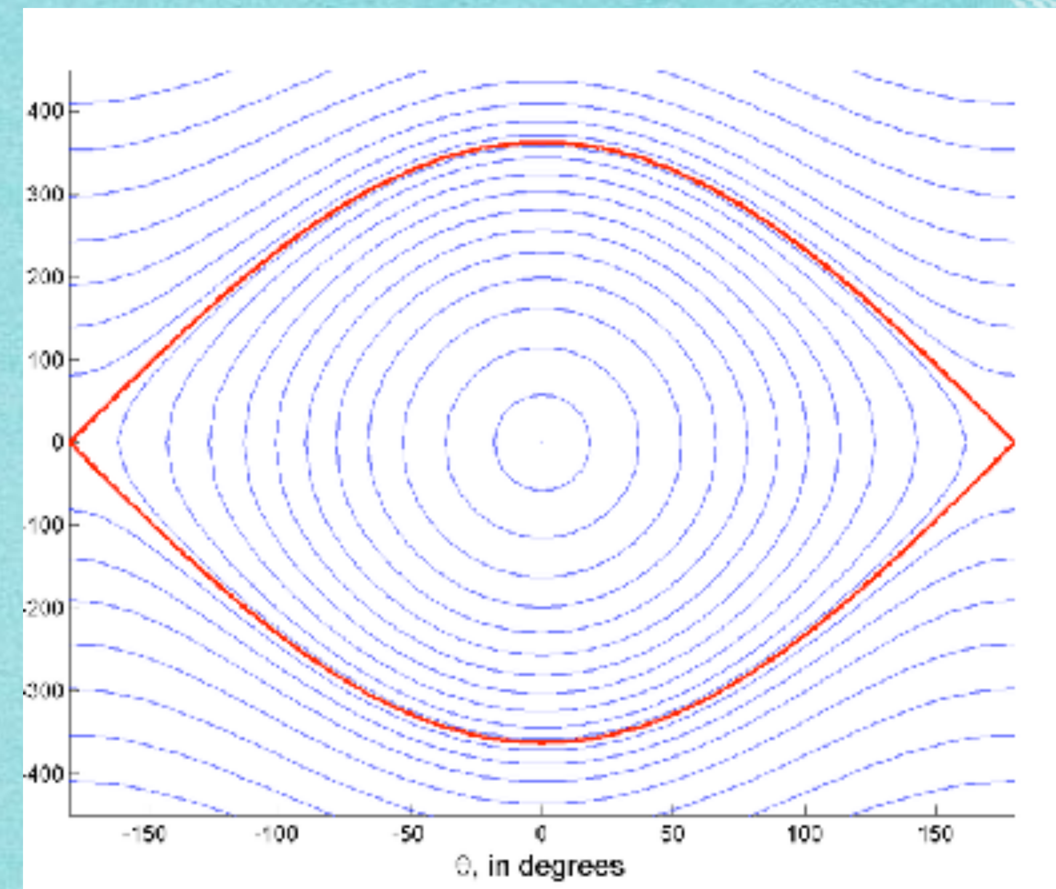


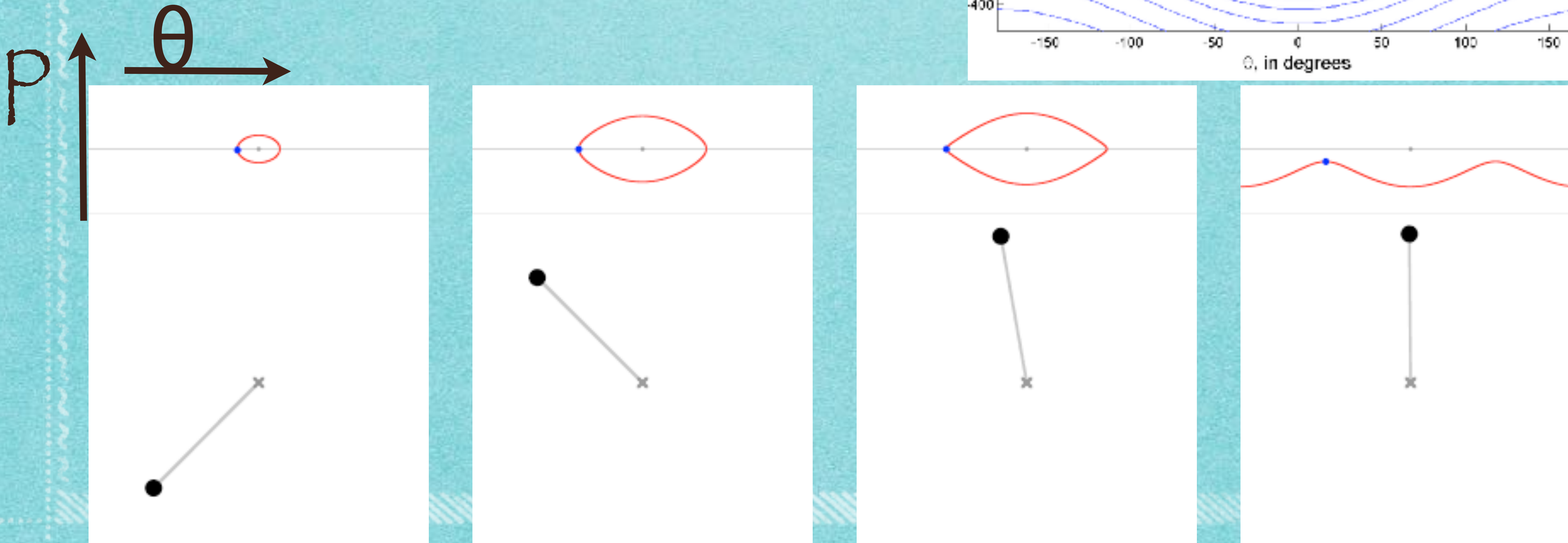
$\theta_0 = 45^o$ $\theta_0 = 135^o$ $\theta_0 = 170^o$

The Eccentric Kozai-Lidov (EKL)



$$H(\theta, p) = \frac{p^2}{2mL^2} - mgL\cos\theta$$





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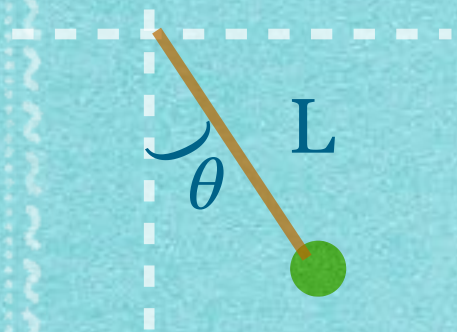

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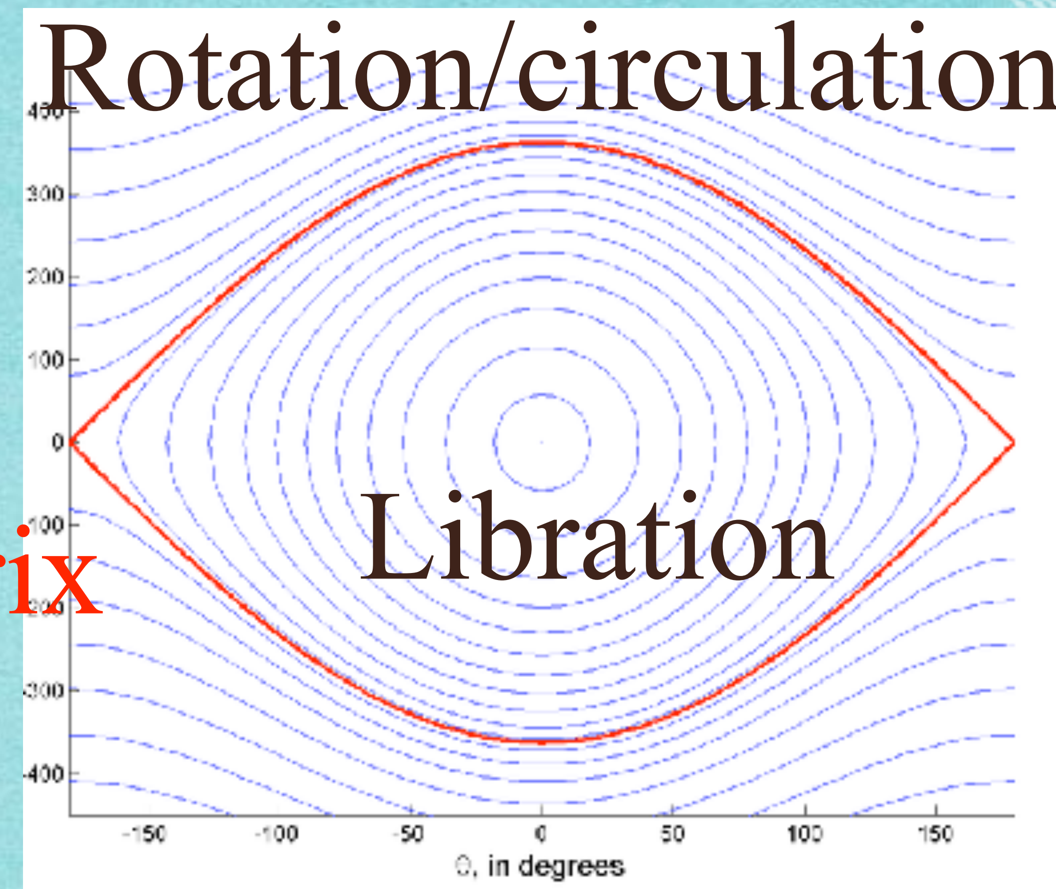
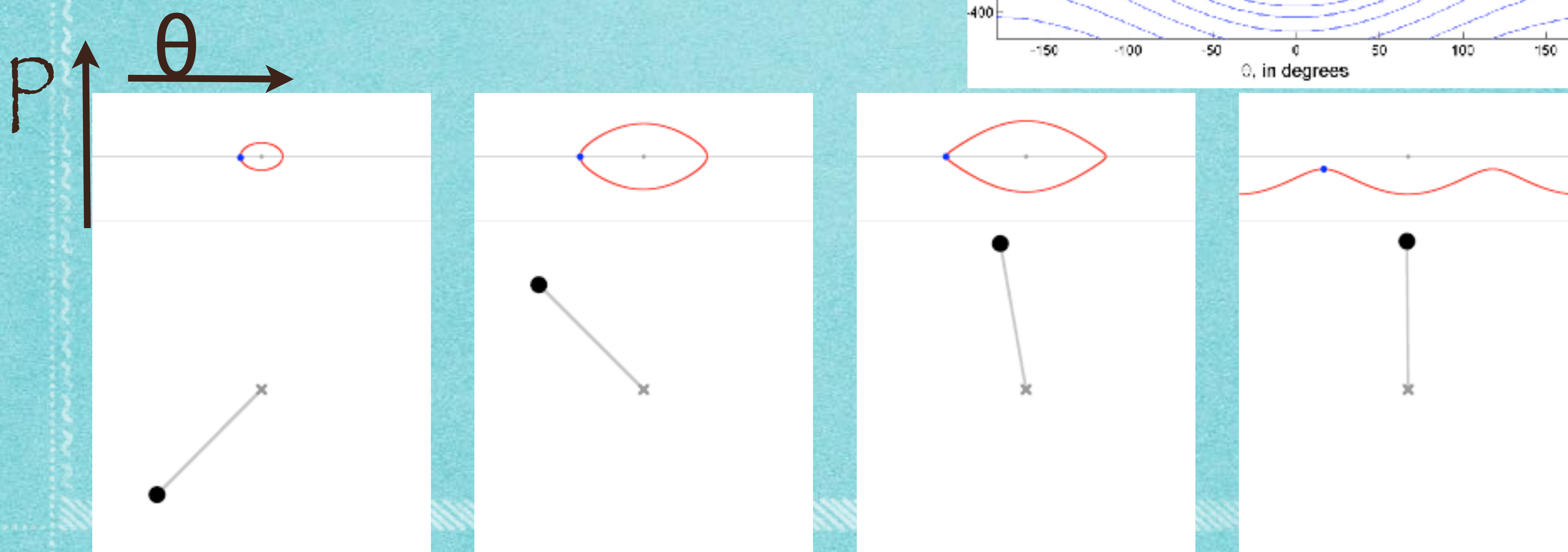


The separatrix



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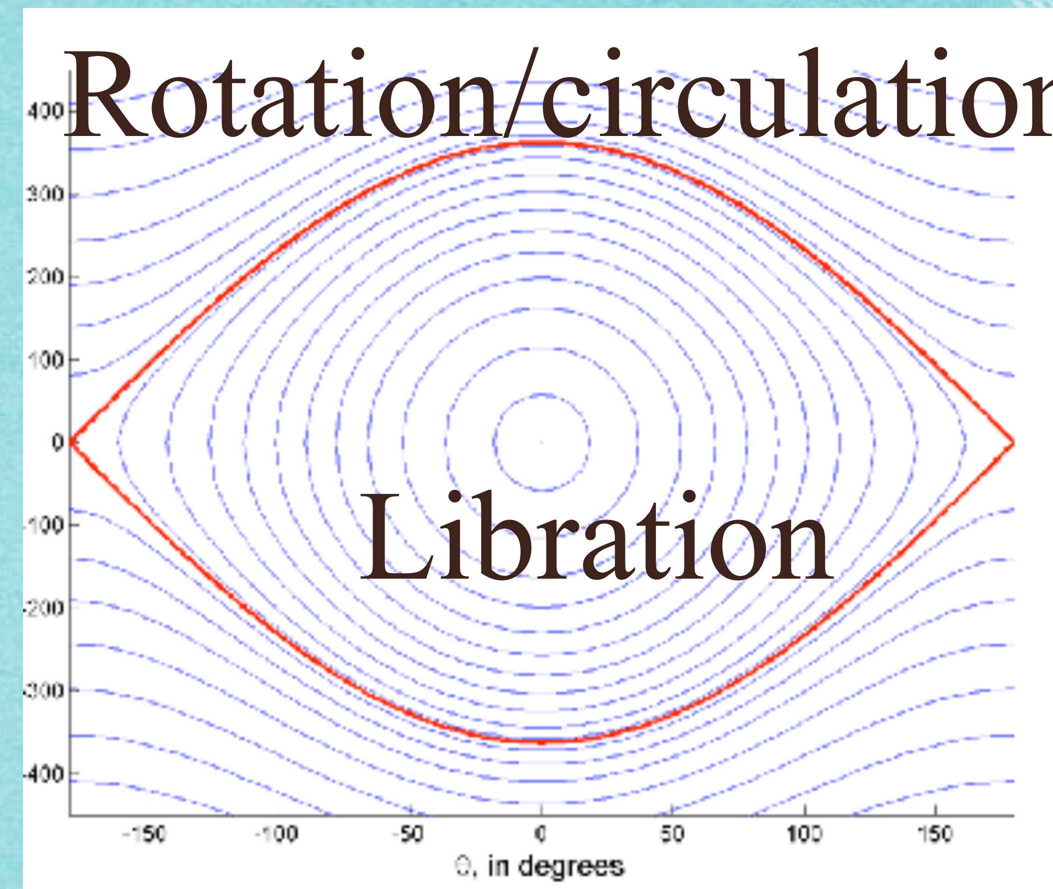
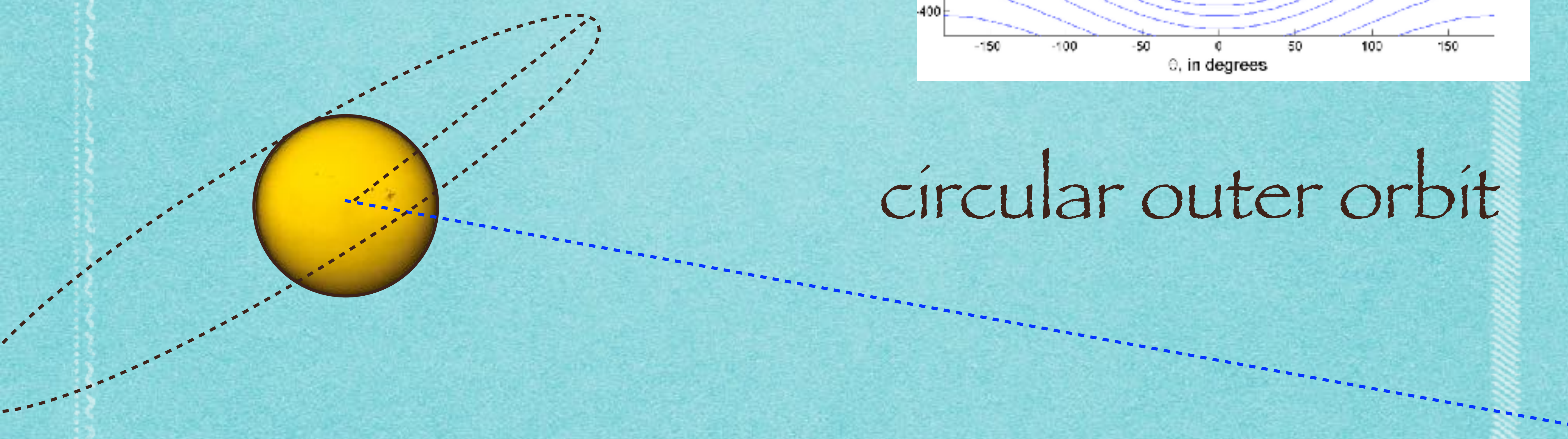
The Eccentric Kozai-Lidov (EKL)

Quadrupole test particle limit:





Lithwick & **Naoz** (2011), ApJ

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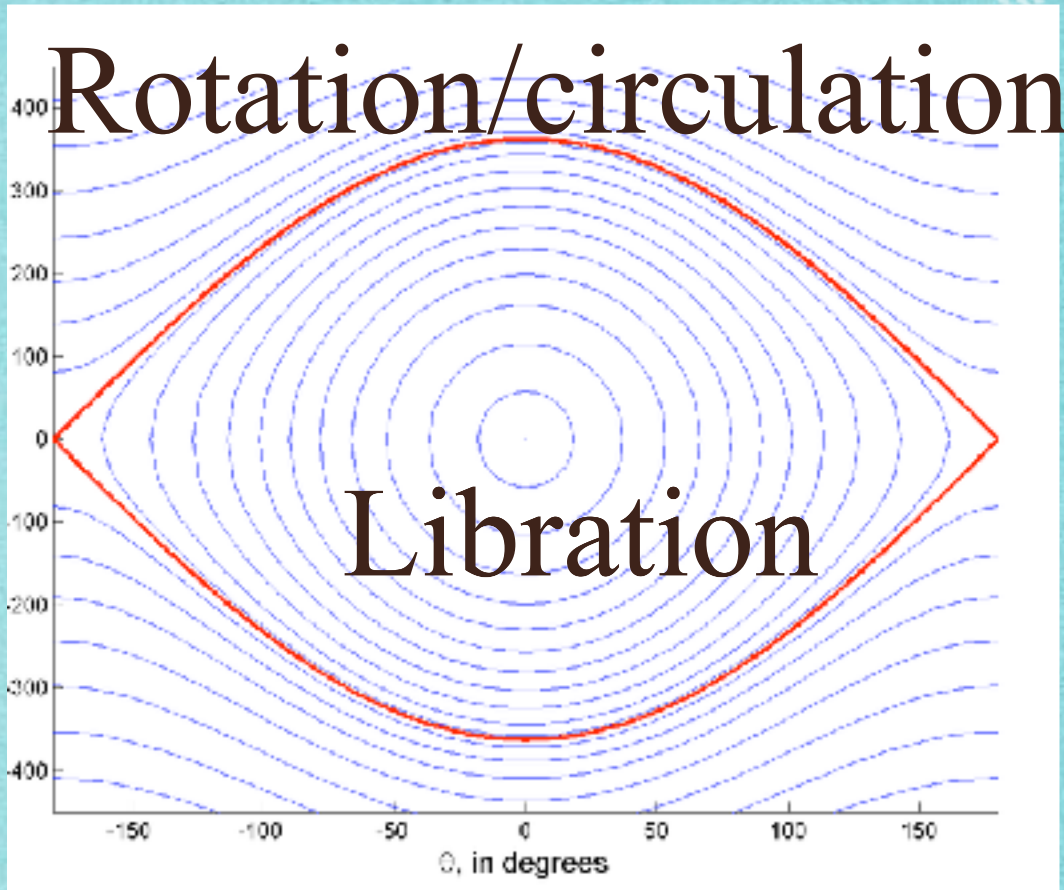
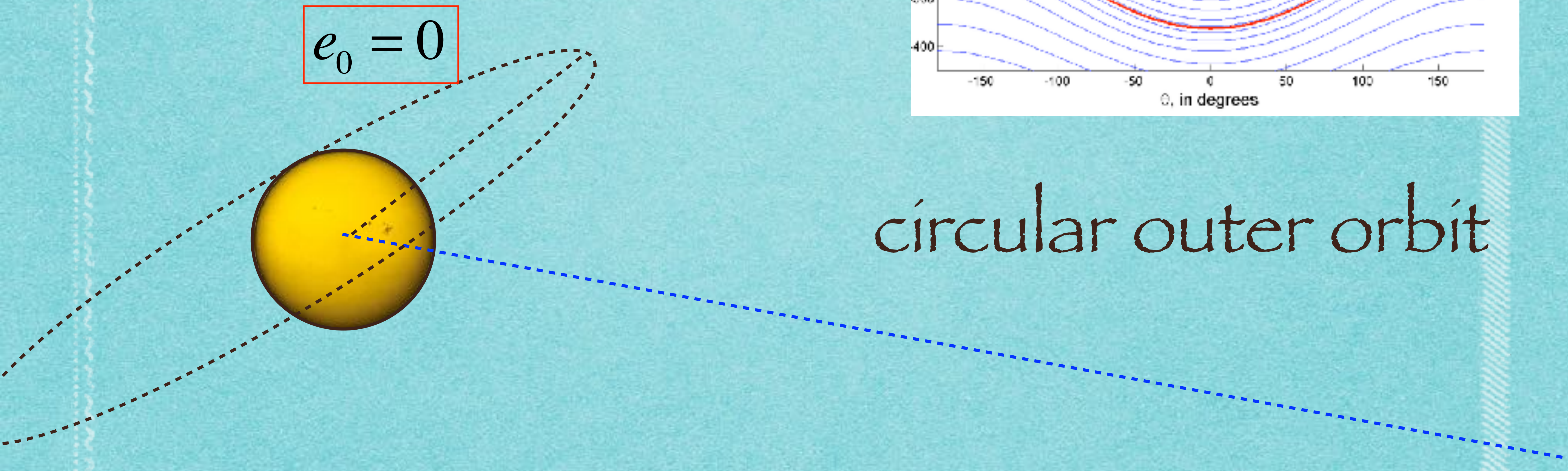
Quadrupole test particle limit:





Lithwick & **Naoz** (2011), ApJ

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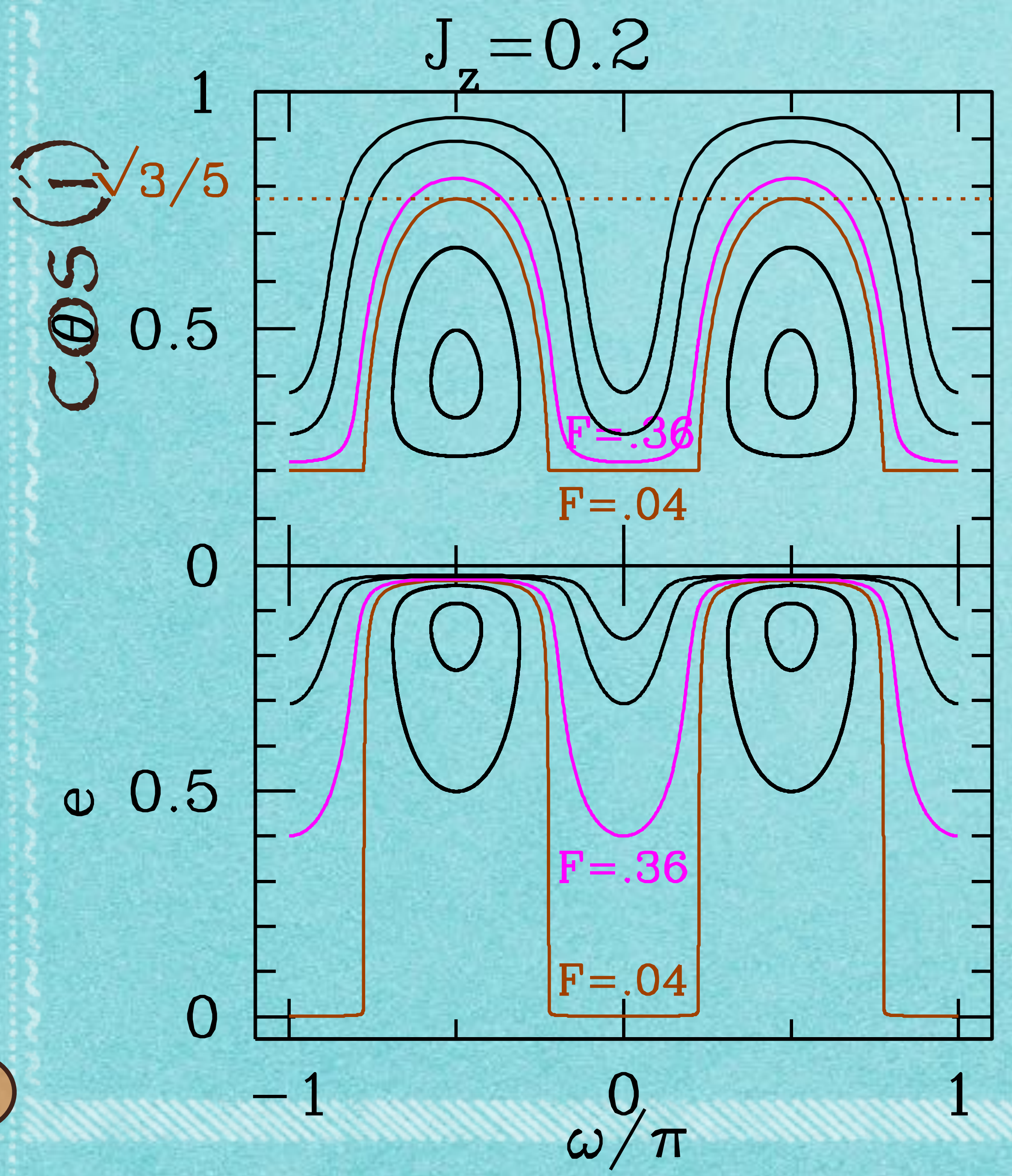

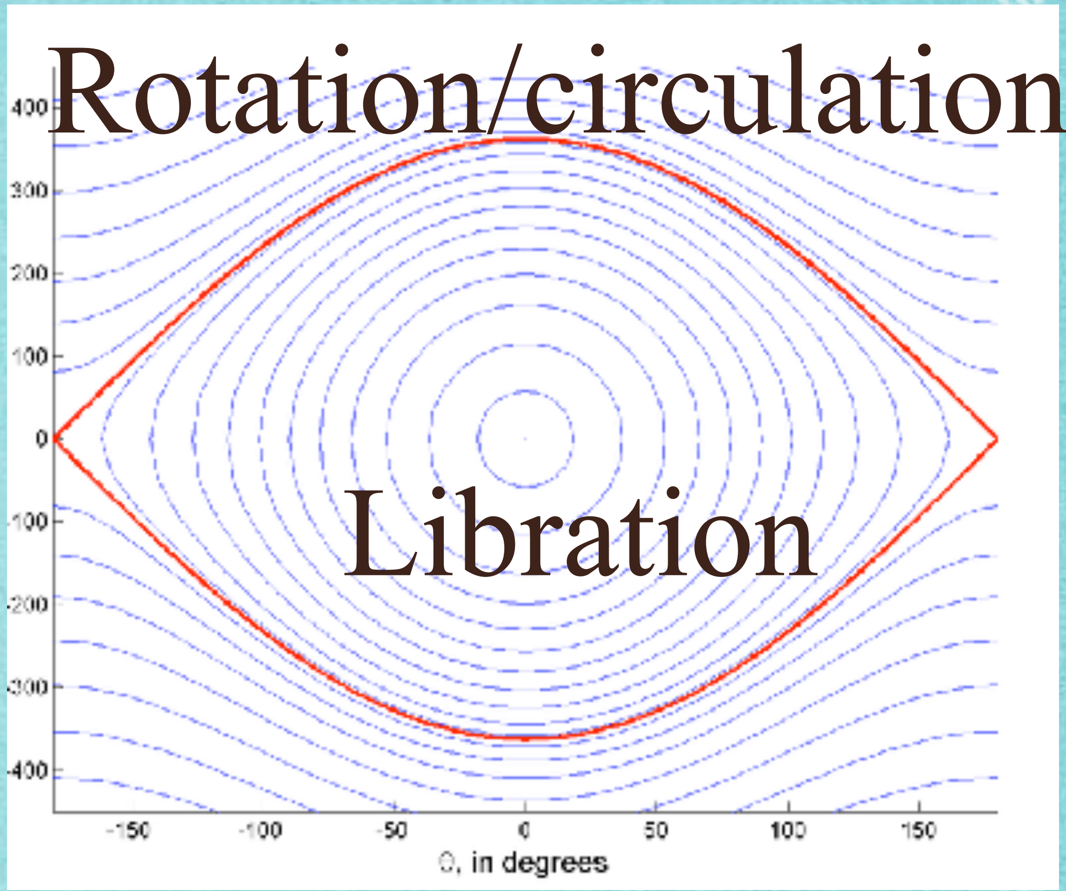

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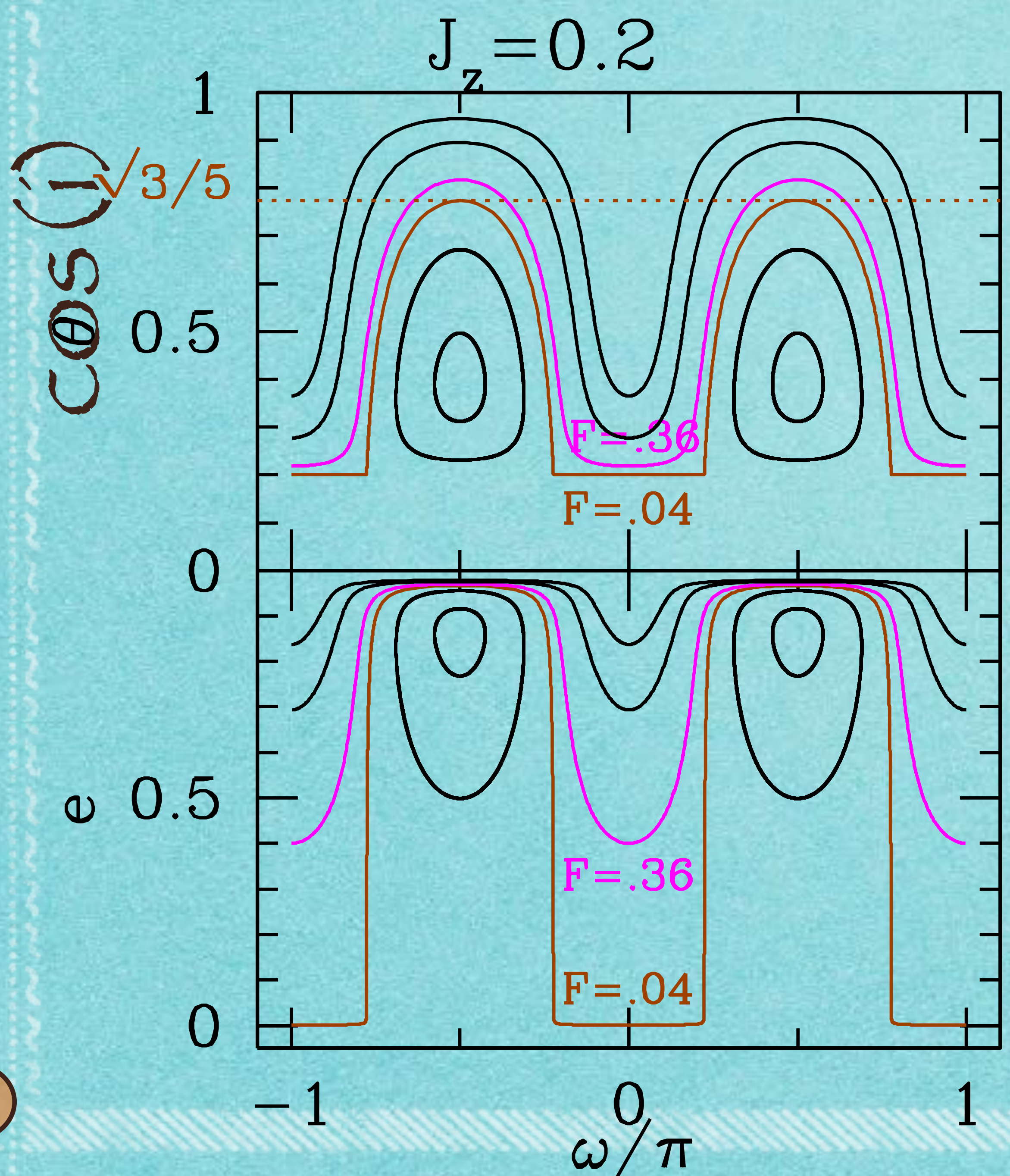
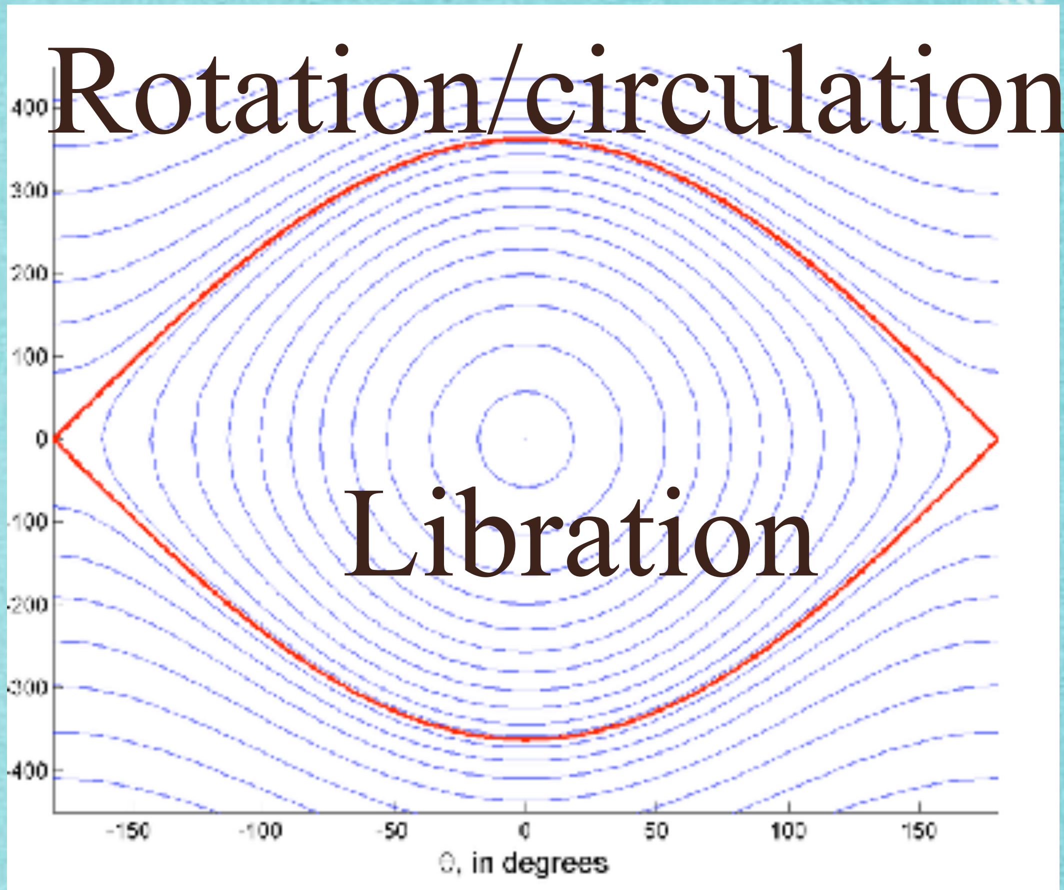


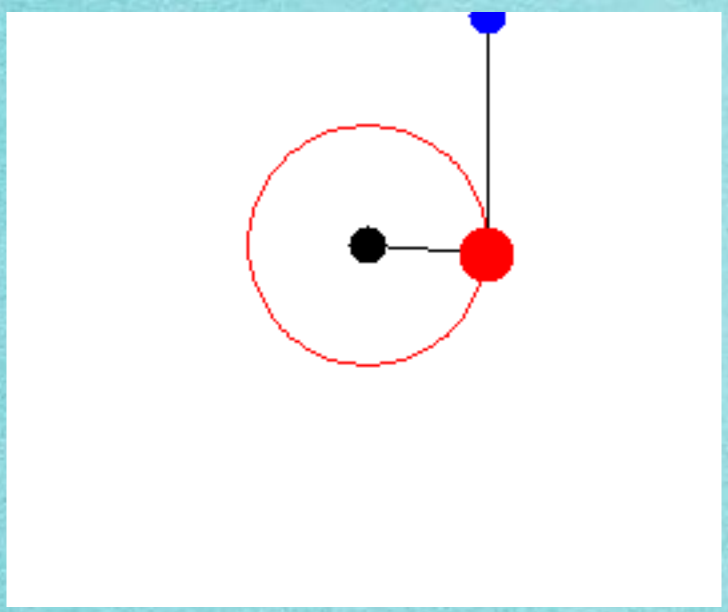


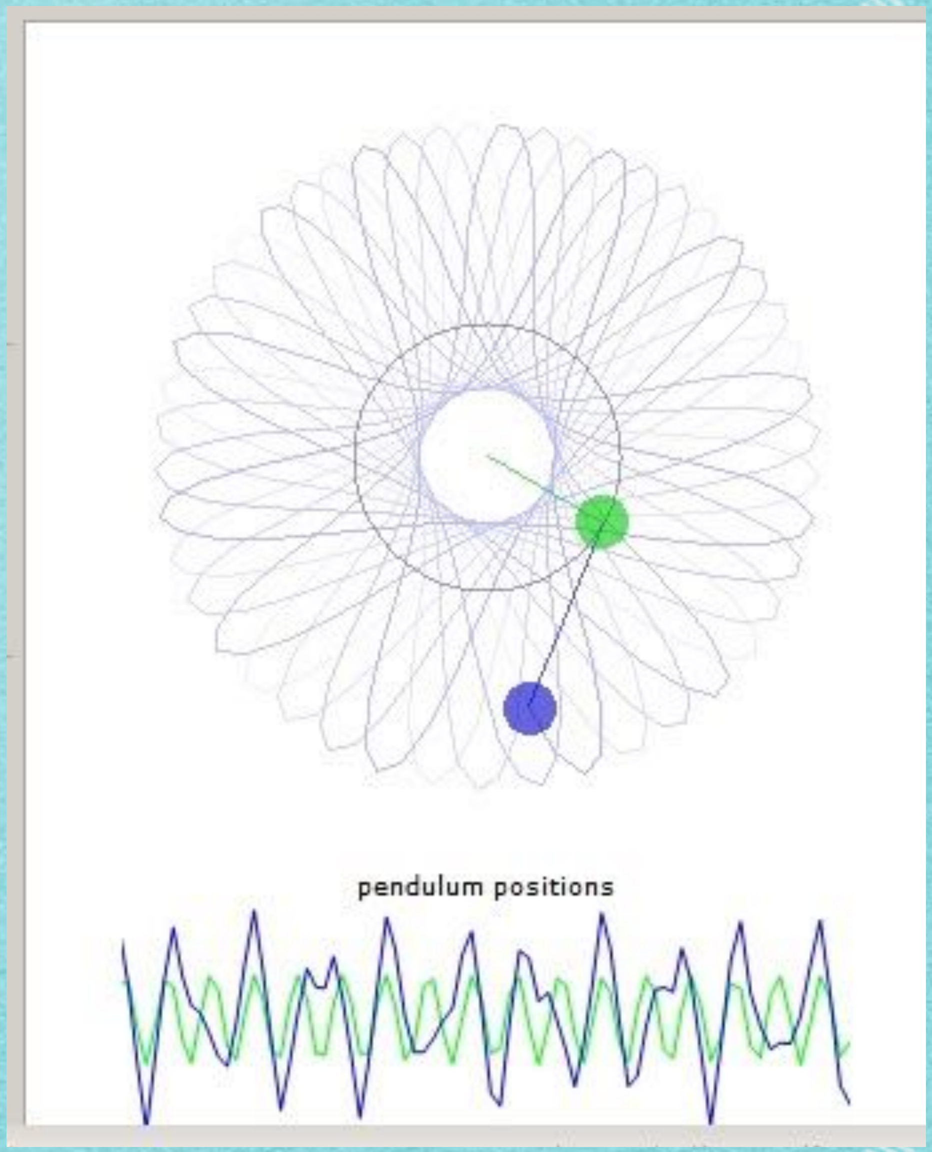
A: The separatrix has:

$$e_0 = 0, \cos i_0 = \sqrt{\frac{3}{5}}$$

Lithwick & **Naoz** (2011), ApJ

The double Pendulum





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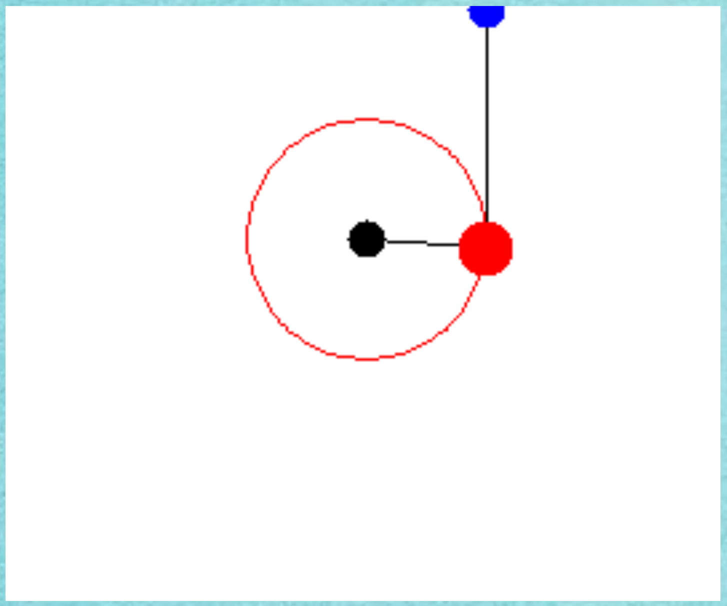
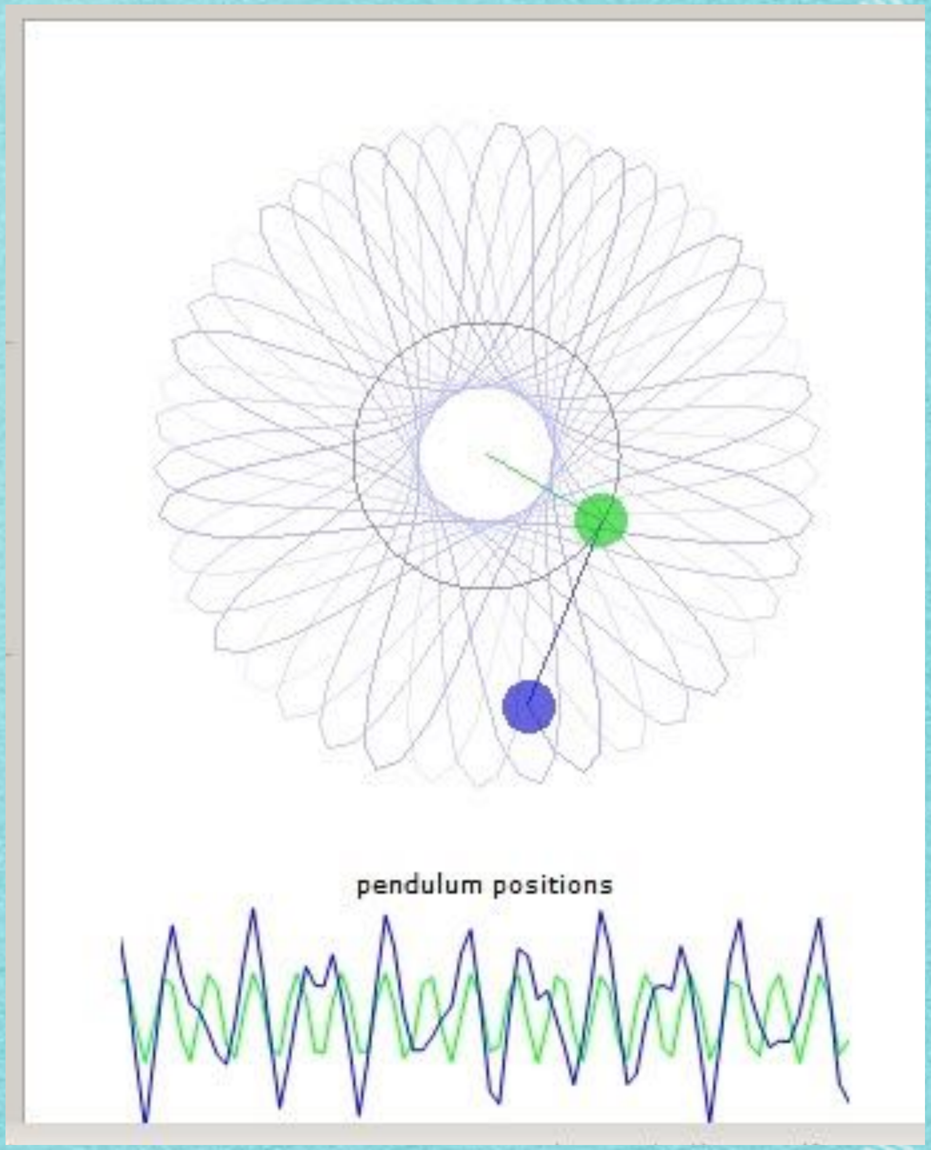
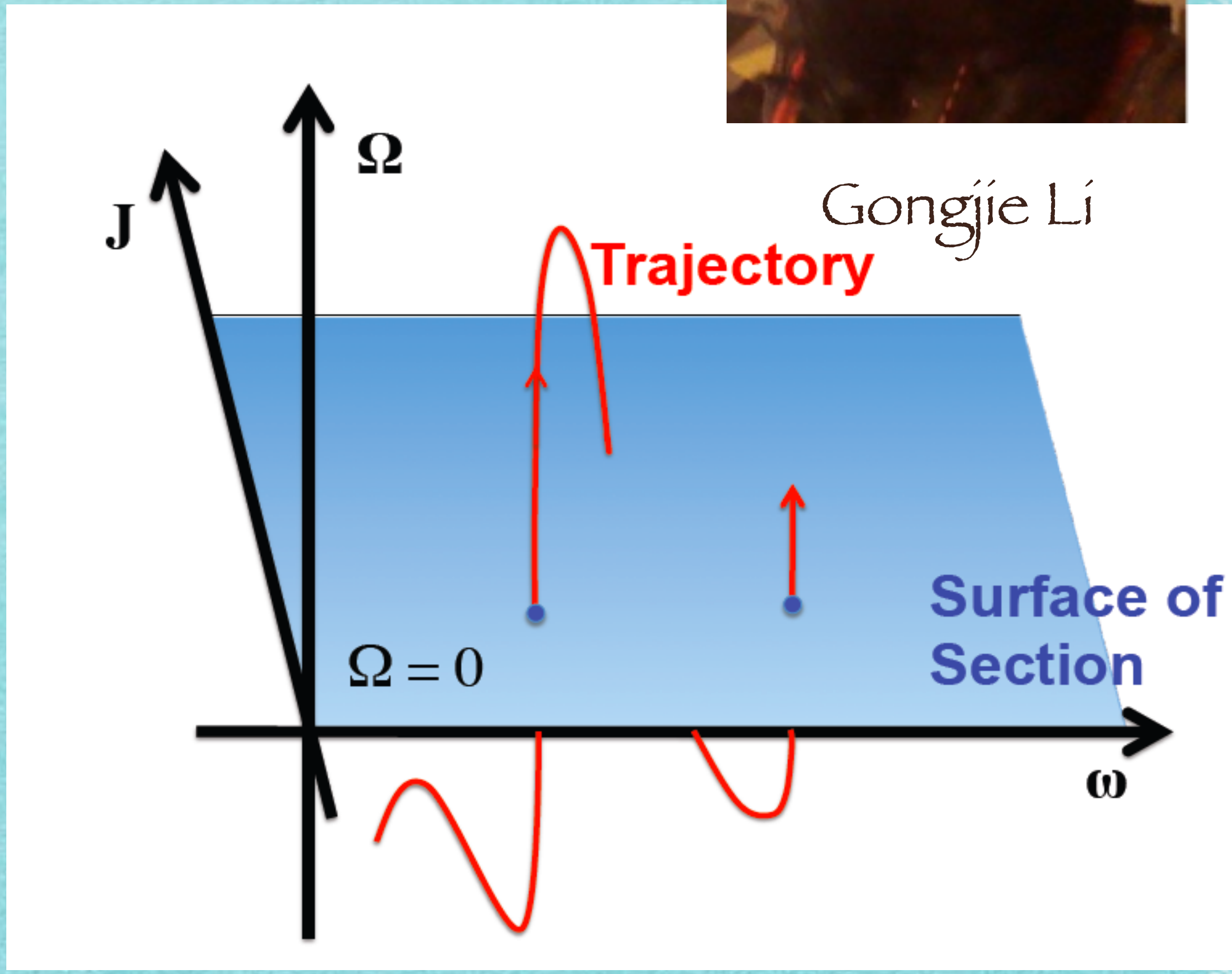




The Eccentric Kozai-Lidov (EKL)



Li, **Naoz,** Kocsis, Loeb 2014a, ApJ arXiv:1310.6044
Li, **Naoz,** Holman, Loeb 2014b, ApJ, arXiv:1405.0494

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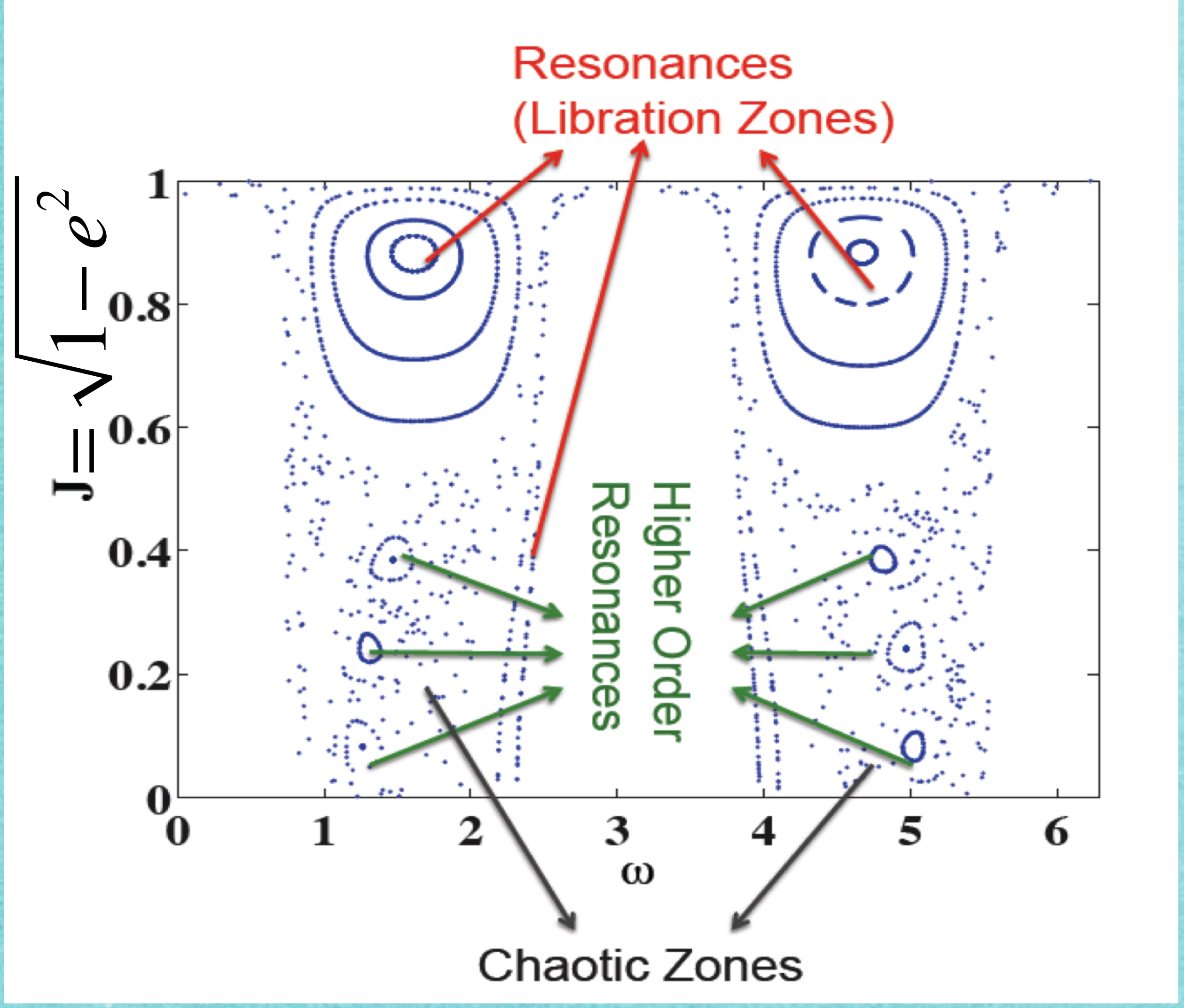
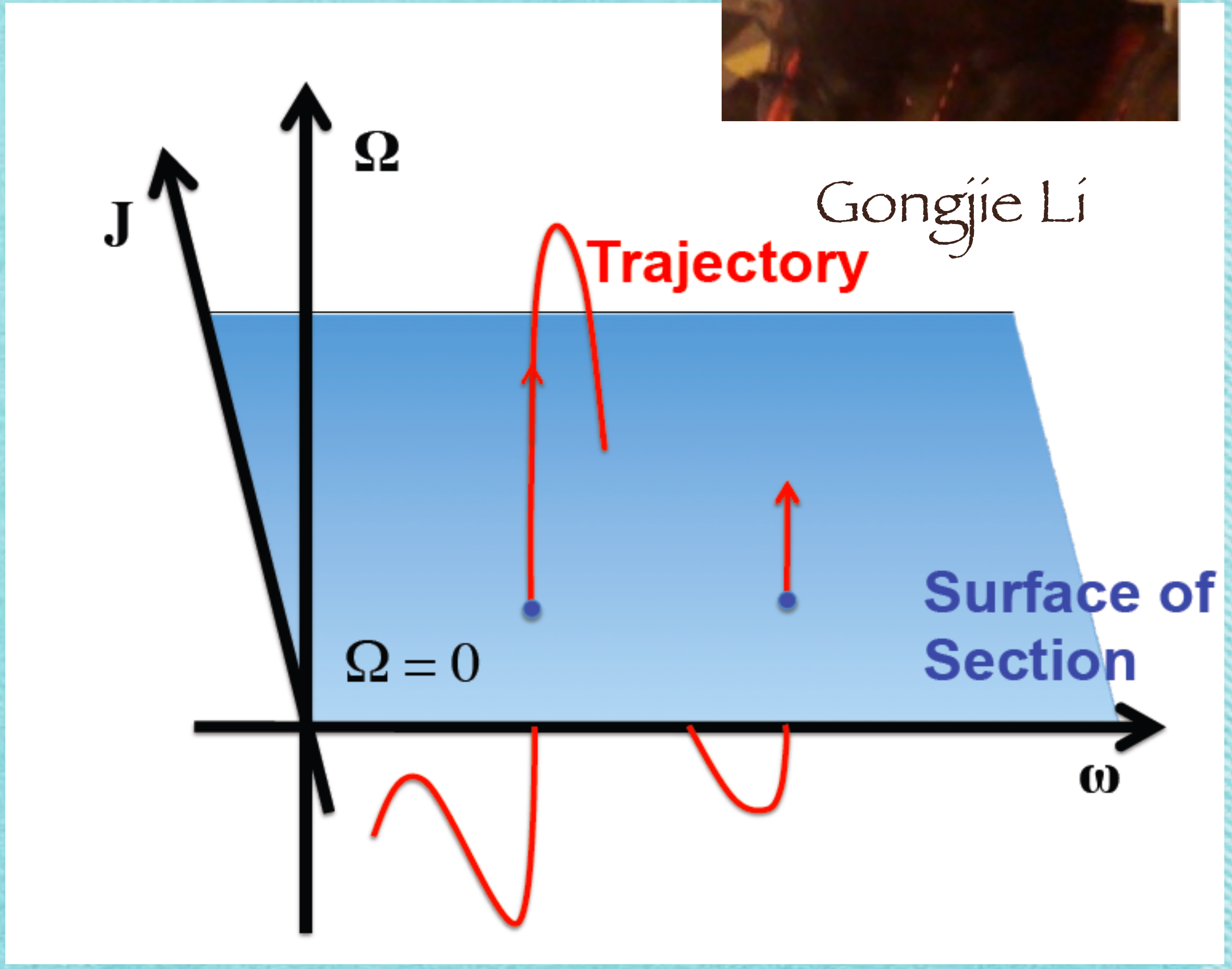
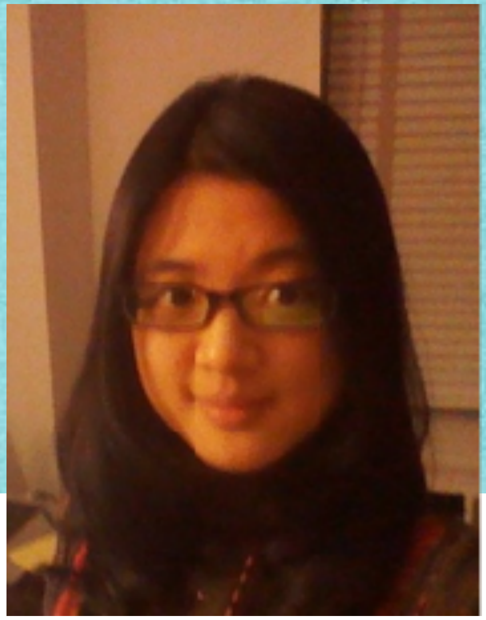






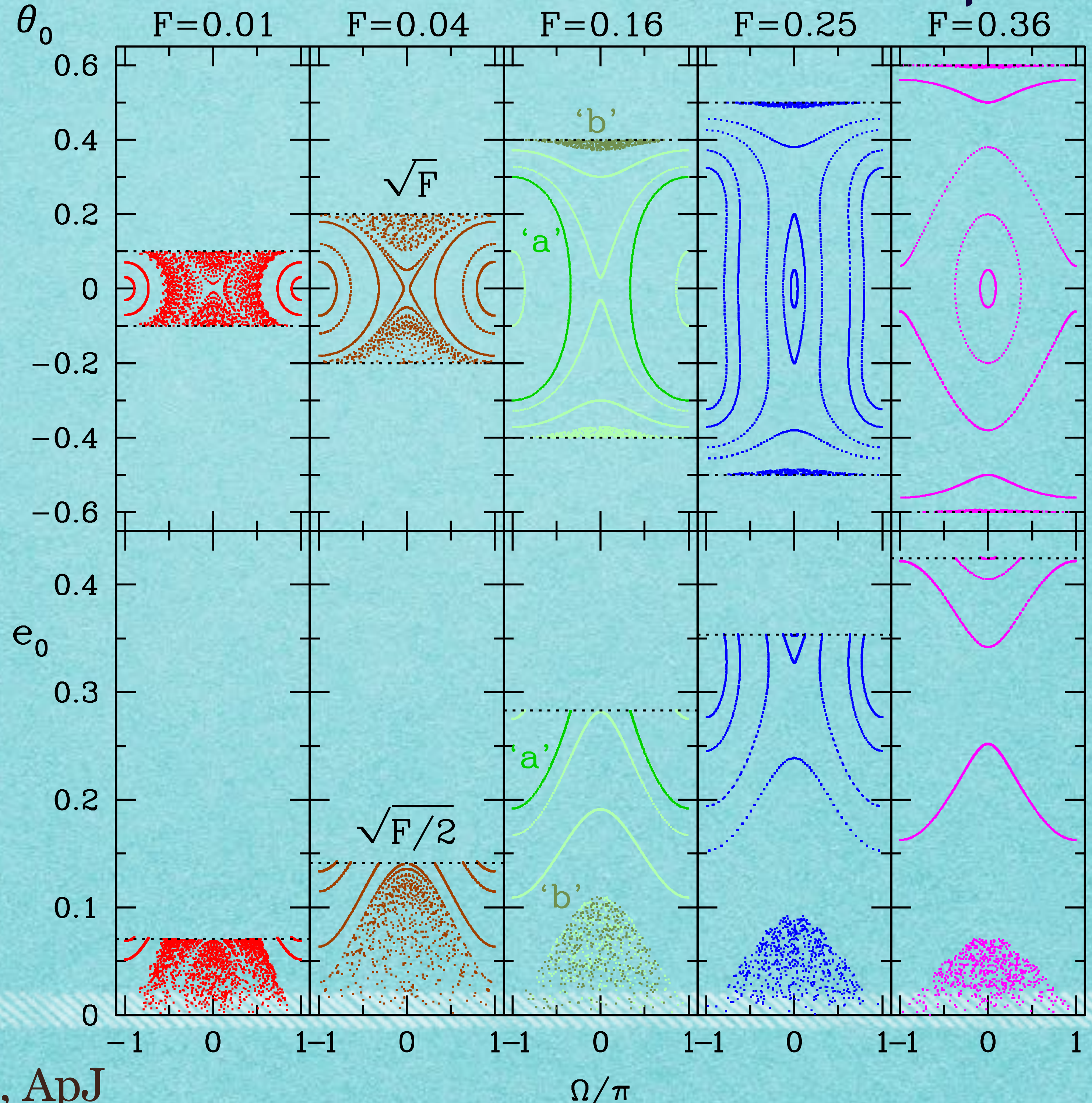
Gongjie Li

Li, **Naoz,** Kocsis, Loeb 2014a, ApJ arXiv:1310.6044
Li, **Naoz,** Holman, Loeb 2014b, ApJ, arXiv:1405.0494

The Eccentric Kozai-Lidov (EKL)

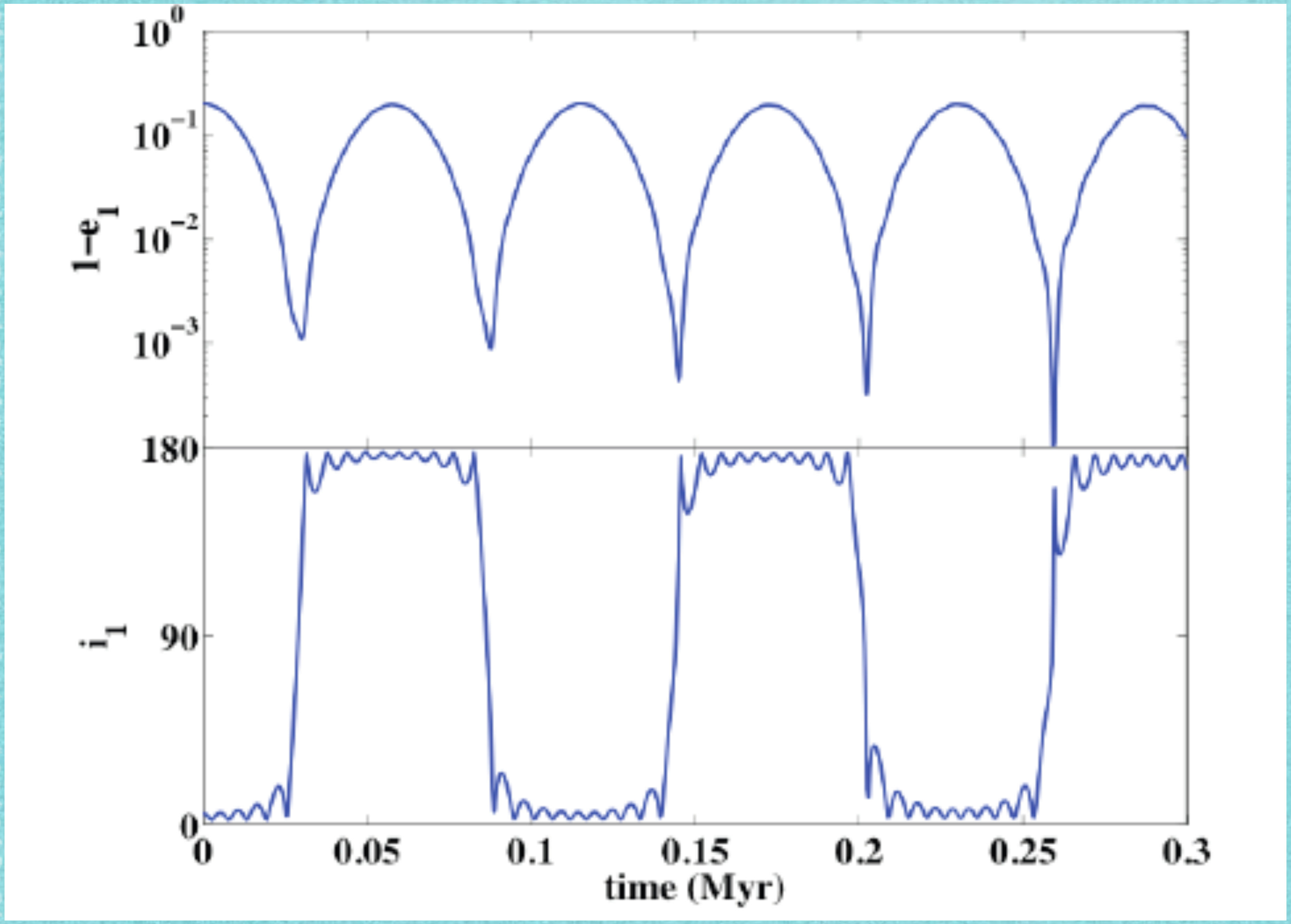
A: Octupole - chaotic behavior crossing the separatrix

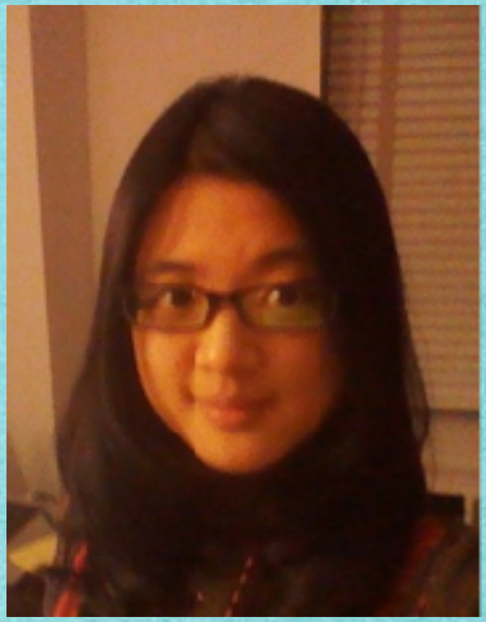
First to show chaos in these systems: Holman, Touma & Tremaine (1997)



Lithwick & **Naoz** (2011), ApJ

The Eccentric Kozai-Lidov (EKL)





Gongjie Li

$\omega_1 = 0^\circ, \Omega_1 = 180^\circ,$

$e_2 = 0.6, a_1 = 4AU, a_2 = 50AU$

$e_1 = 0.8, i = 5^\circ$

$m_1 = 1M_\odot, m_2 = 1M_J, m_3 = 0.3M_\odot$

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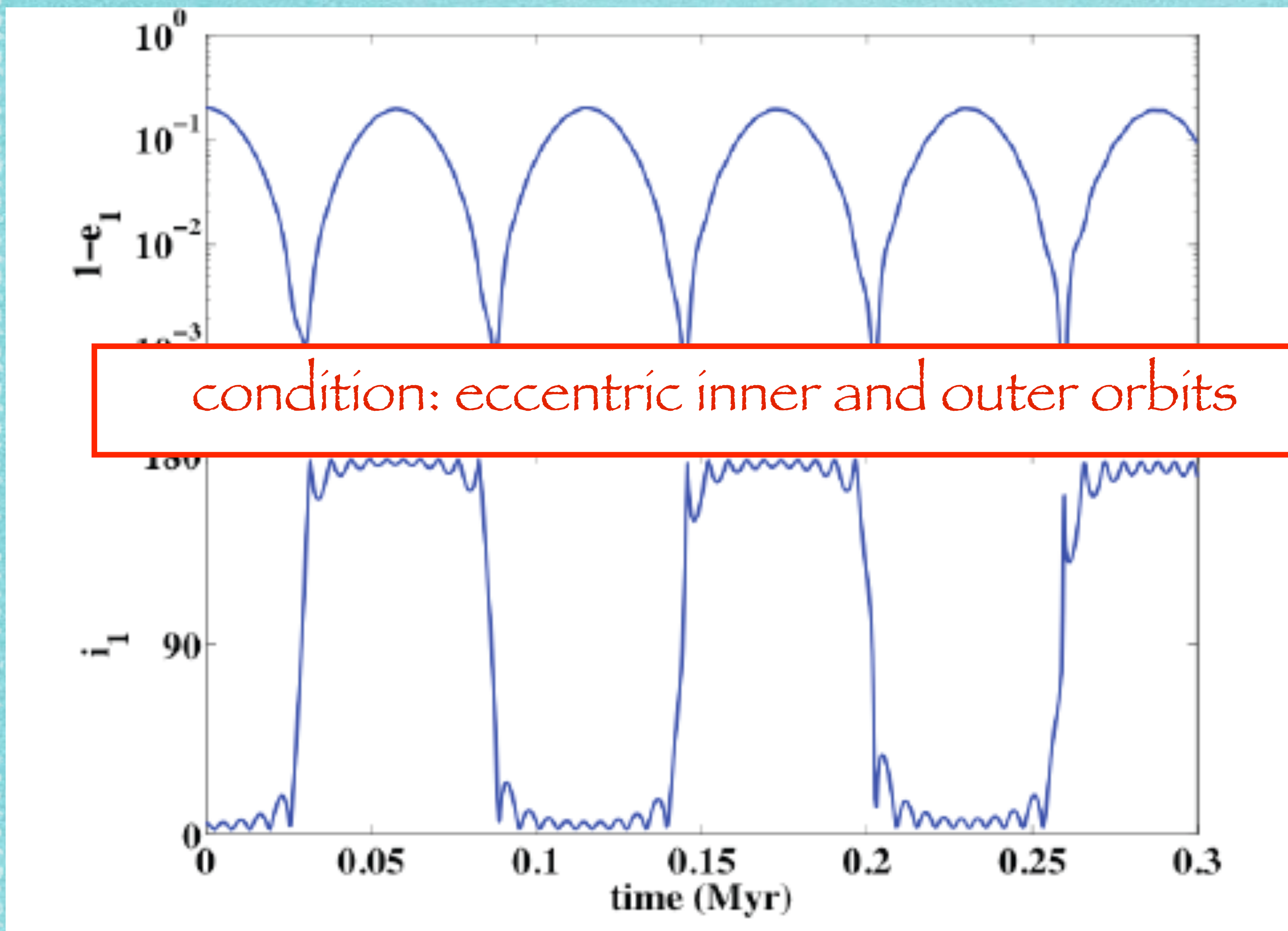
The Eccentric Kozai-Lidov (EKL)



condition: eccentric inner and outer orbits

Gongjie Li

$\omega_1 = 0^\circ, \Omega_1 = 180^\circ,$

$e_2 = 0.6, a_1 = 4AU, a_2 = 50AU$

$e_1 = 0.8, i = 5^\circ$

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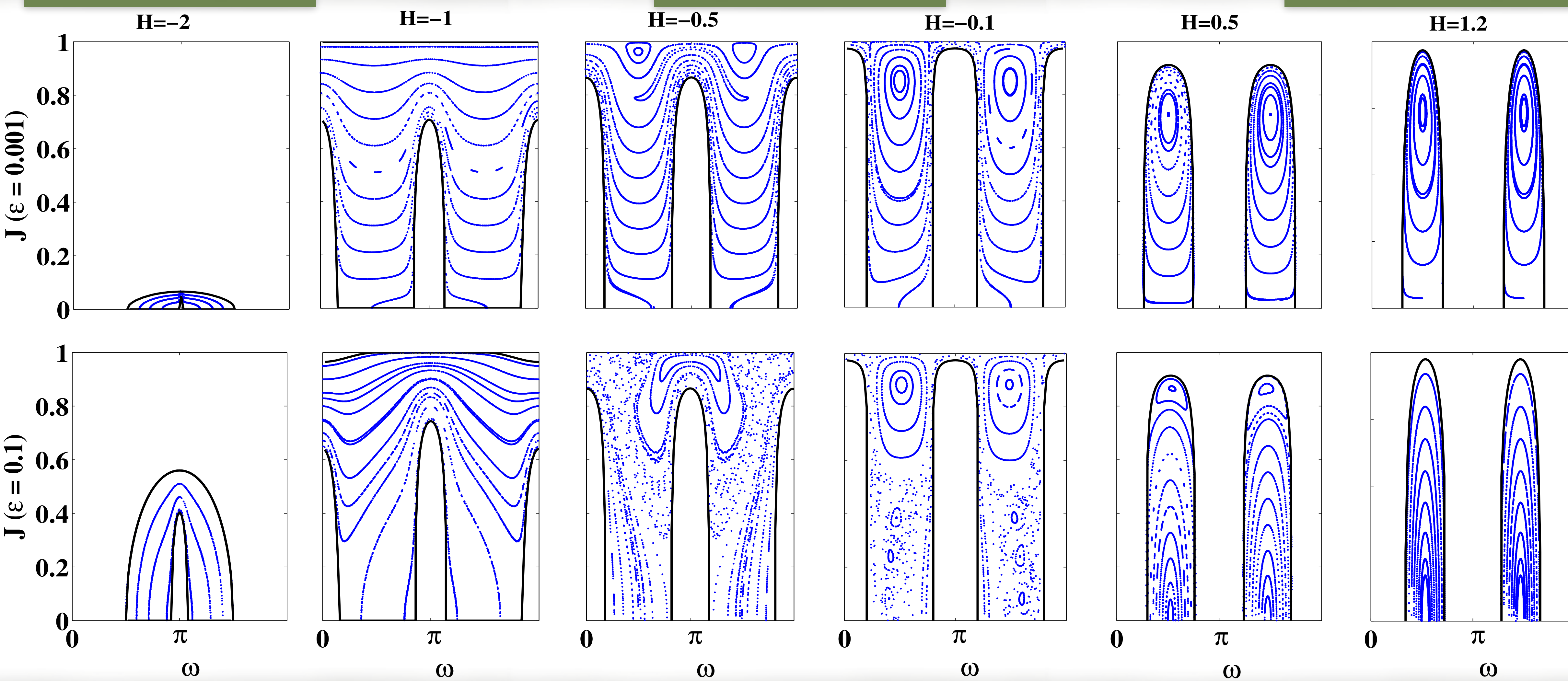
The Eccentric Kozai-Lidov (EKL)

Low i High e

High i Low e

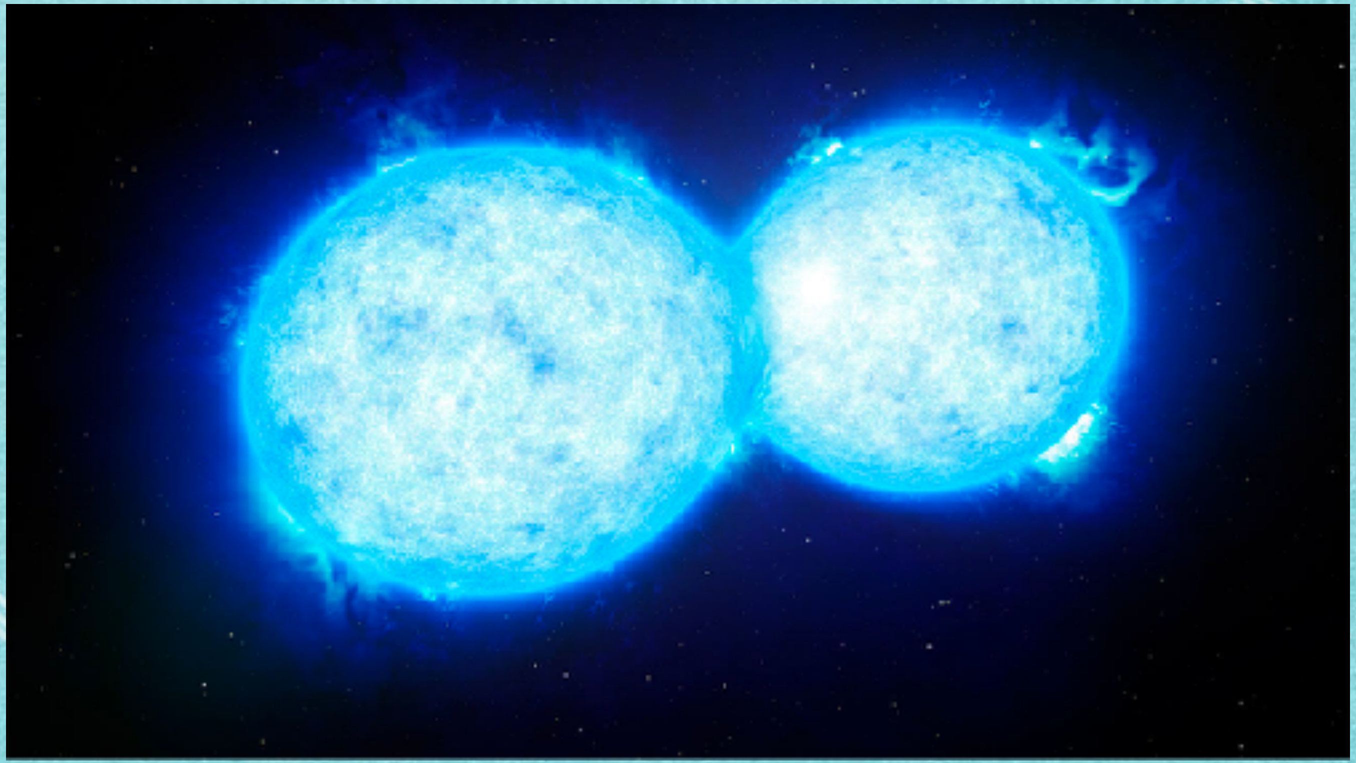
High i High e



octupole plays important role

Li, **Naoz,** Holman, Loeb 2014b, ApJ, arXiv:1405.0494

A Recipe for Merging Binary Black Holes



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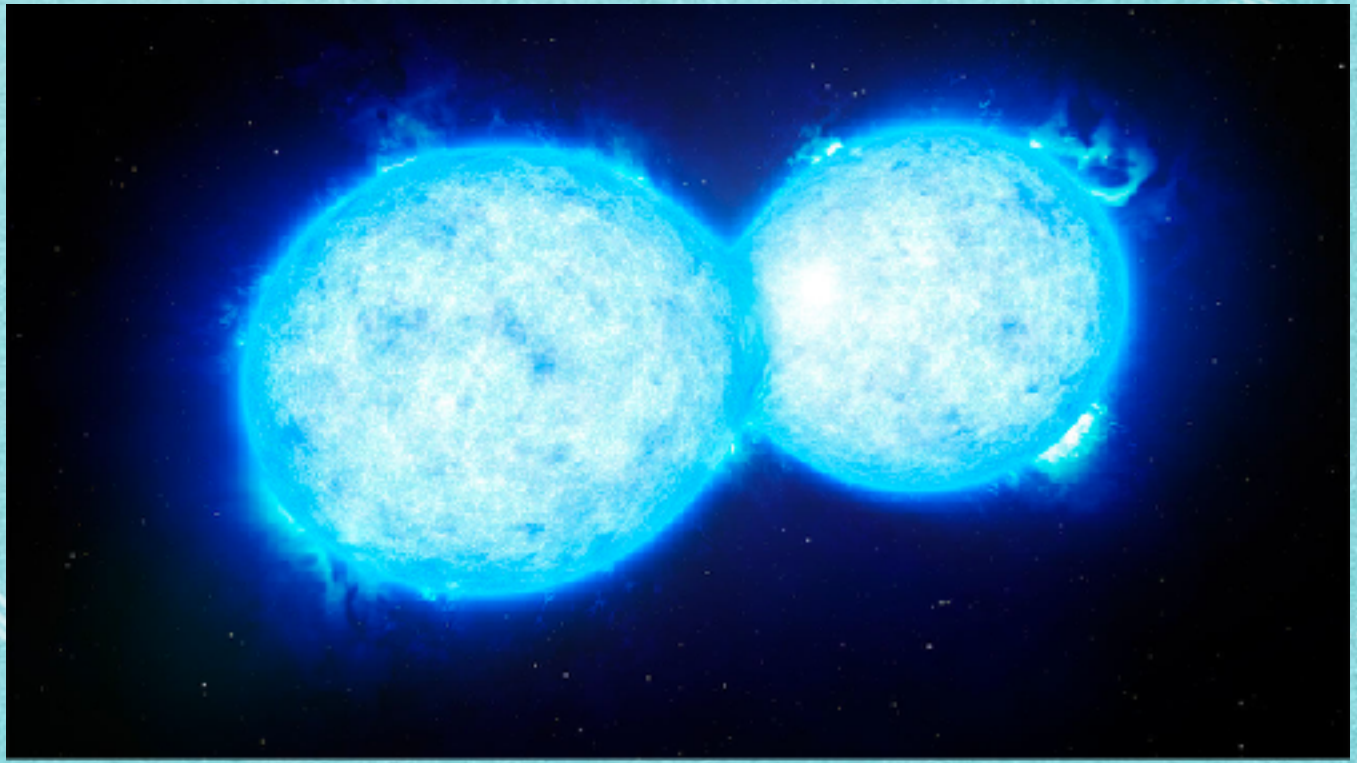

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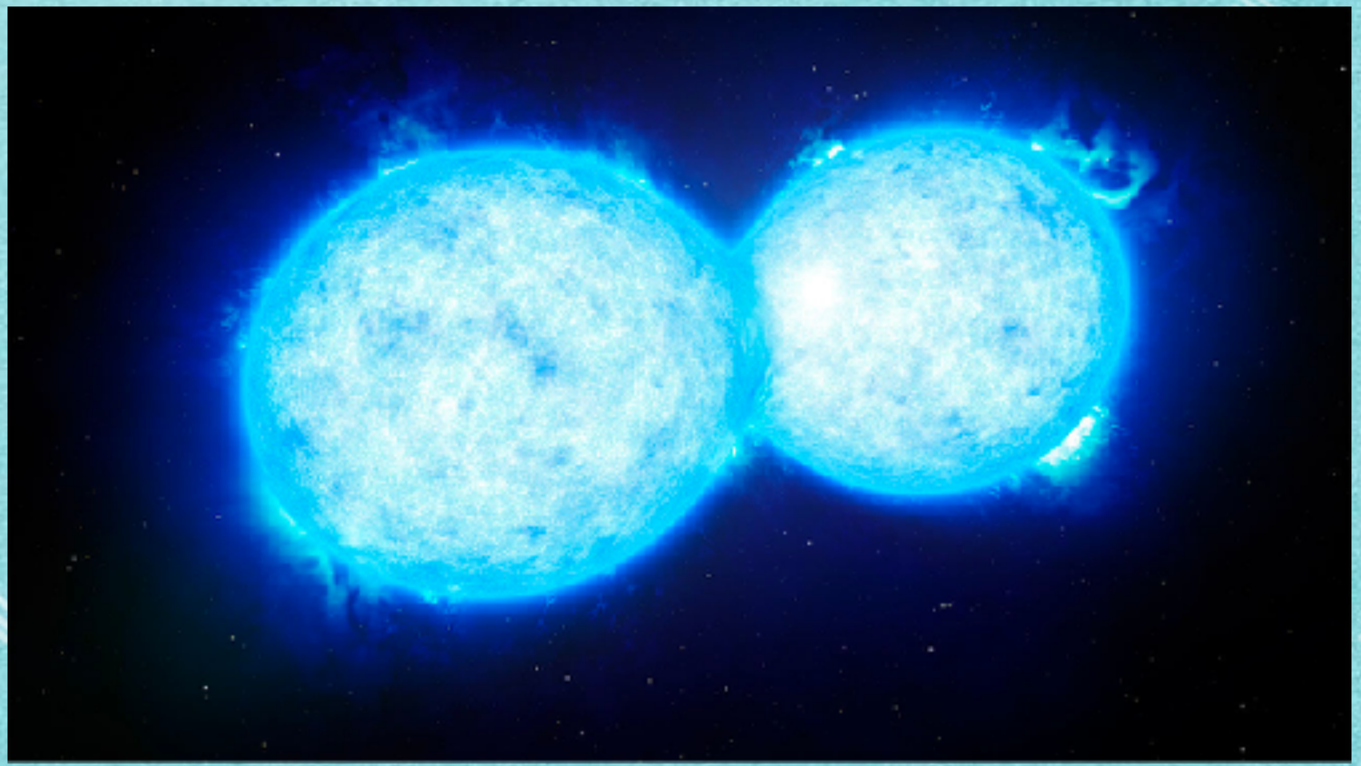

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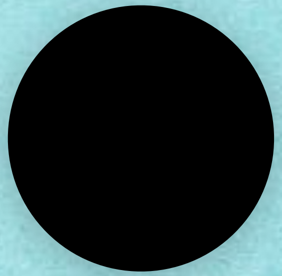
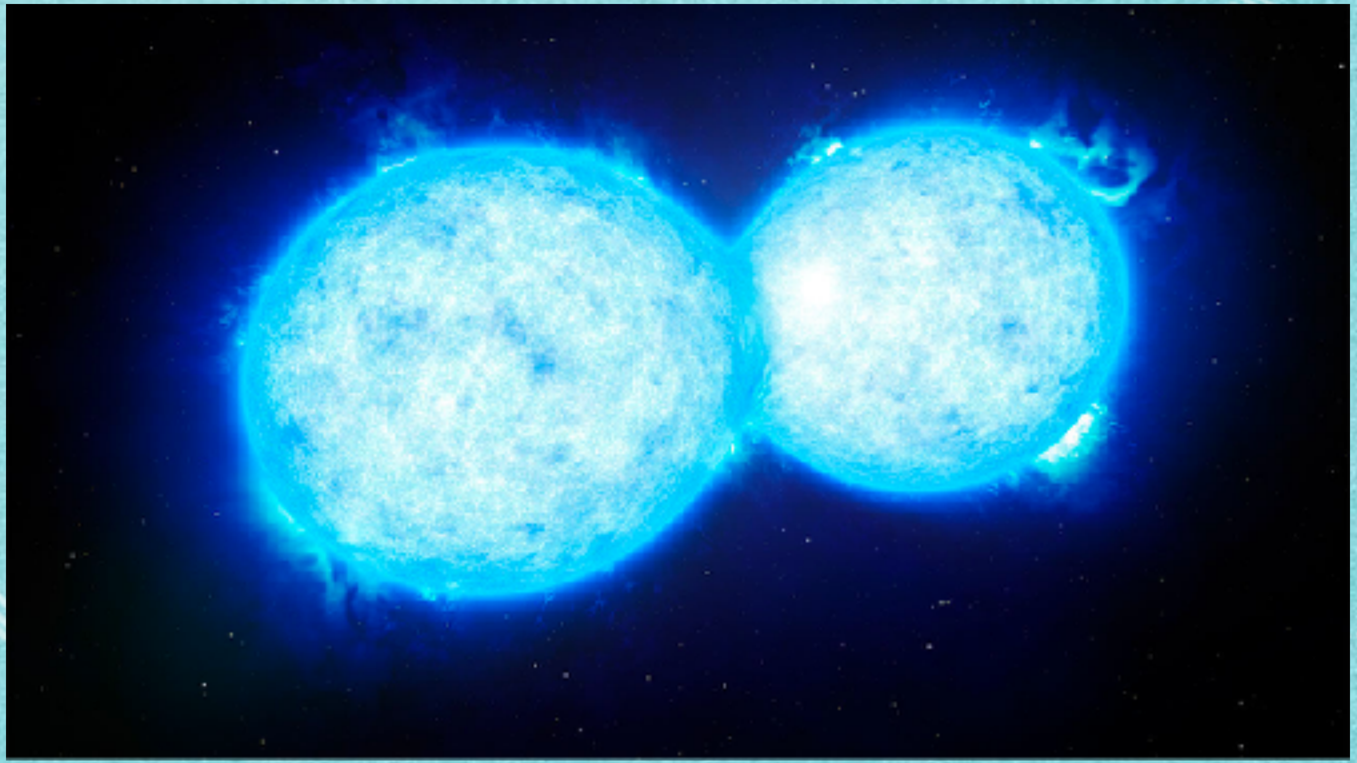




A Recipe for Merging Binary Black Holes

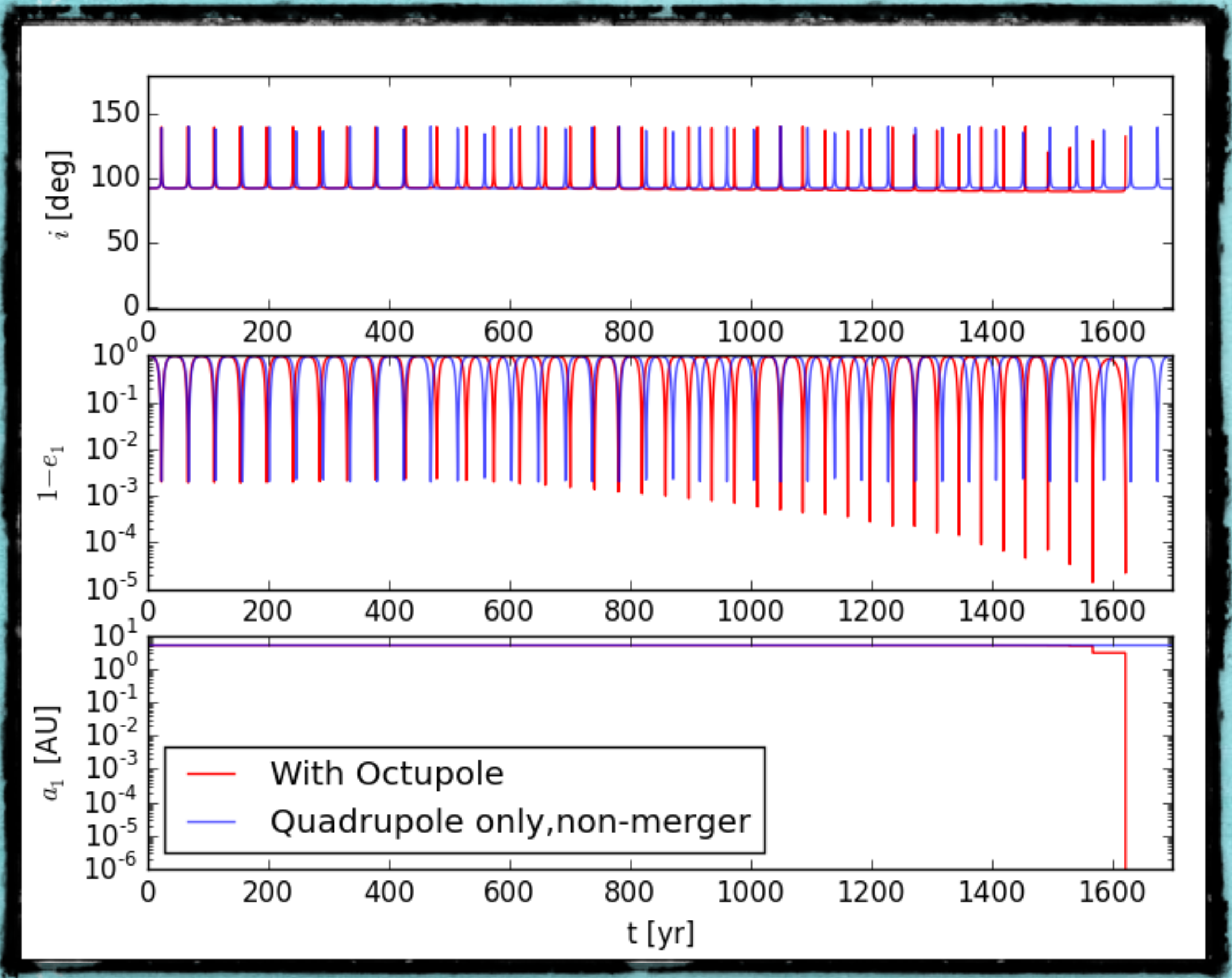
A Recipe for Merging Binary Black Holes

A Recipe for Merging Binary Black Holes

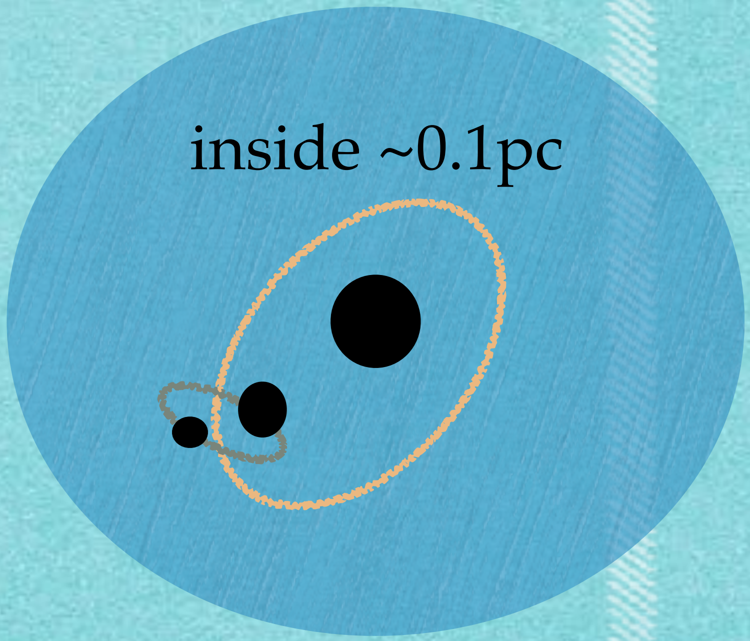
$t_{merge} \gtrsim$ Hubble time

Helping Binary Black Holes Merge

Binaries In Galactic Nuclei



Bao-Minh Hoang



Hoang, **Naoz** et al (2018), ApJ, arXiv:17060.9896

Binaries In Galactic Nuclei

Alexander Stephan

- EKL
- General relativity (1PN)
- Tides
- Post main sequence stellar evolution (single and binary)
- Unbinding the binary
- Disruption due to the SMBH

Stephan, **Naoz** et al (2016,2019)

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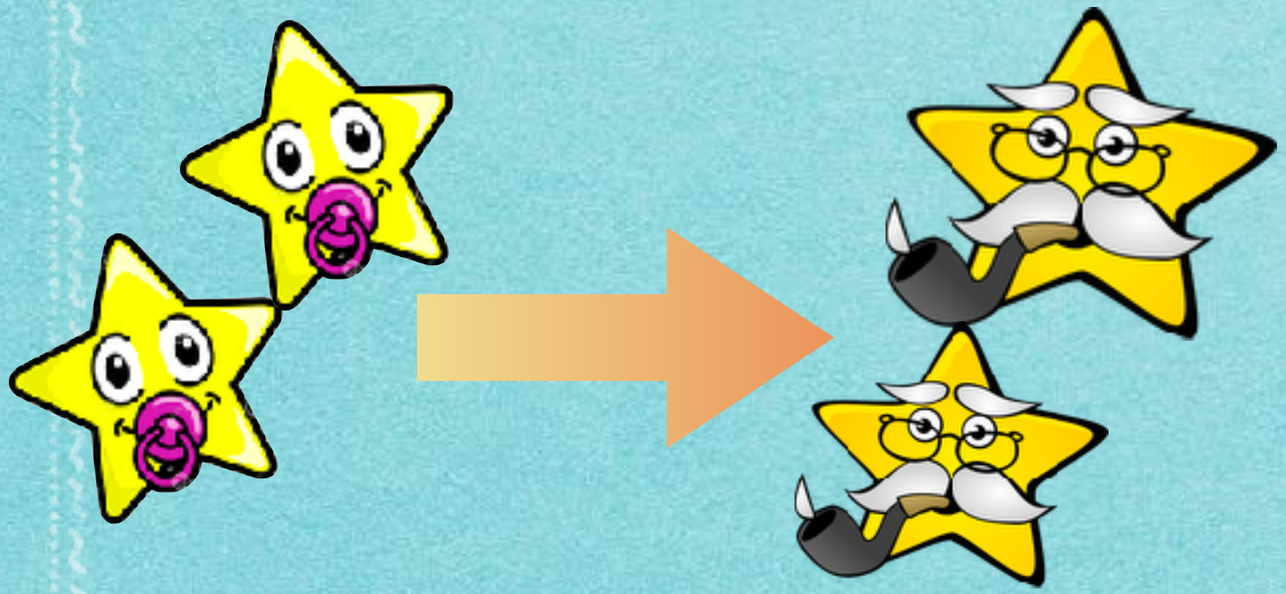

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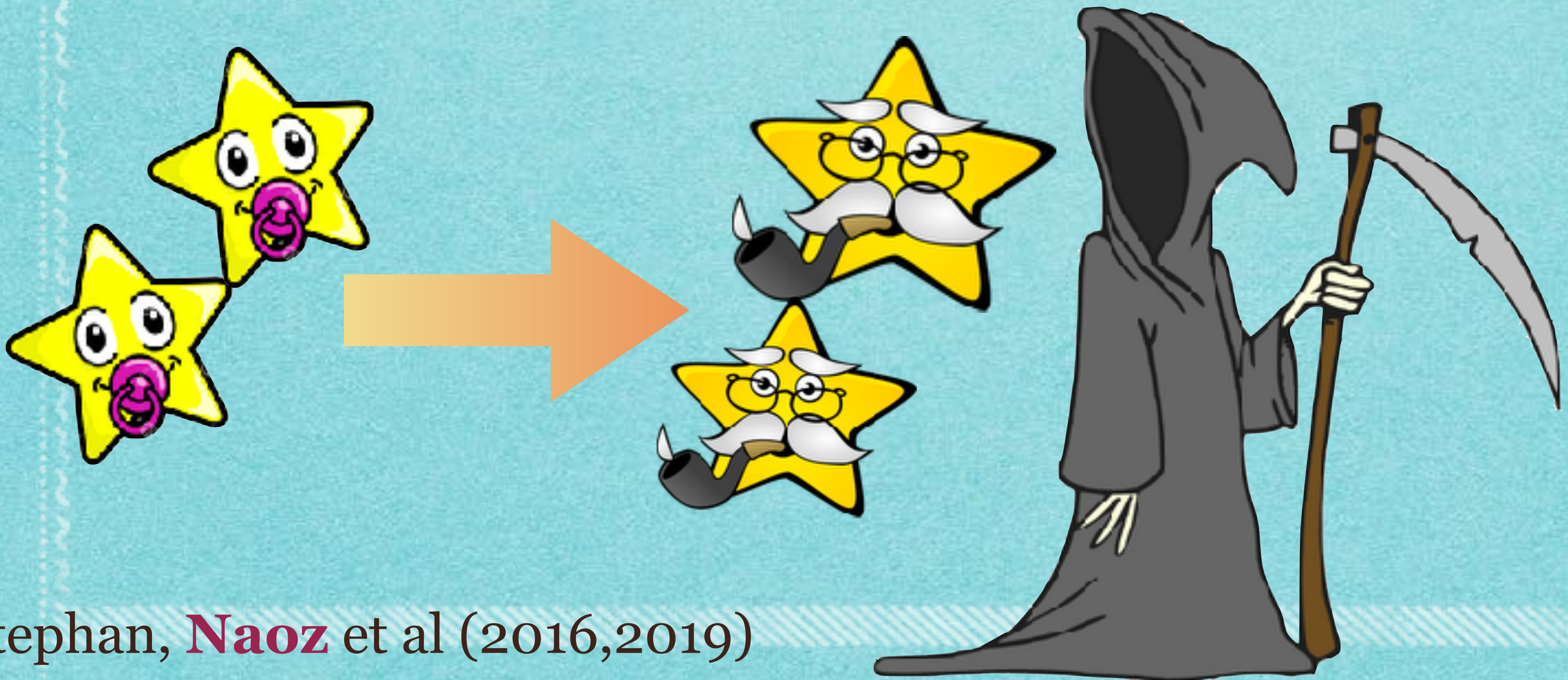

Stephan, **Naoz** et al (2016,2019)

Binaries In Galactic Nuclei

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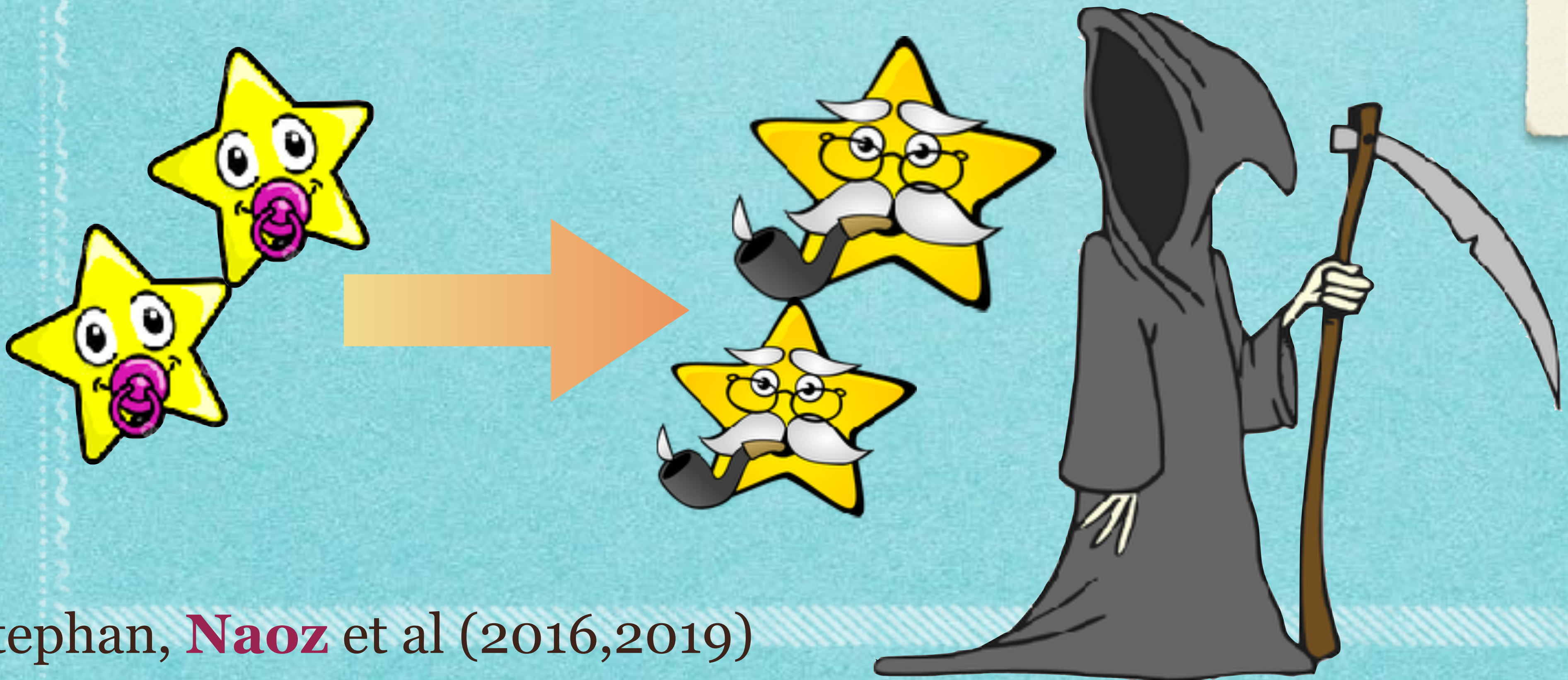
Alexander Stephan

BH-BH Rate
~20 $\mathrm{Gpc^{-3}\ yr^{-1}}$



Stephan, **Naoz** et al (2016,2019)

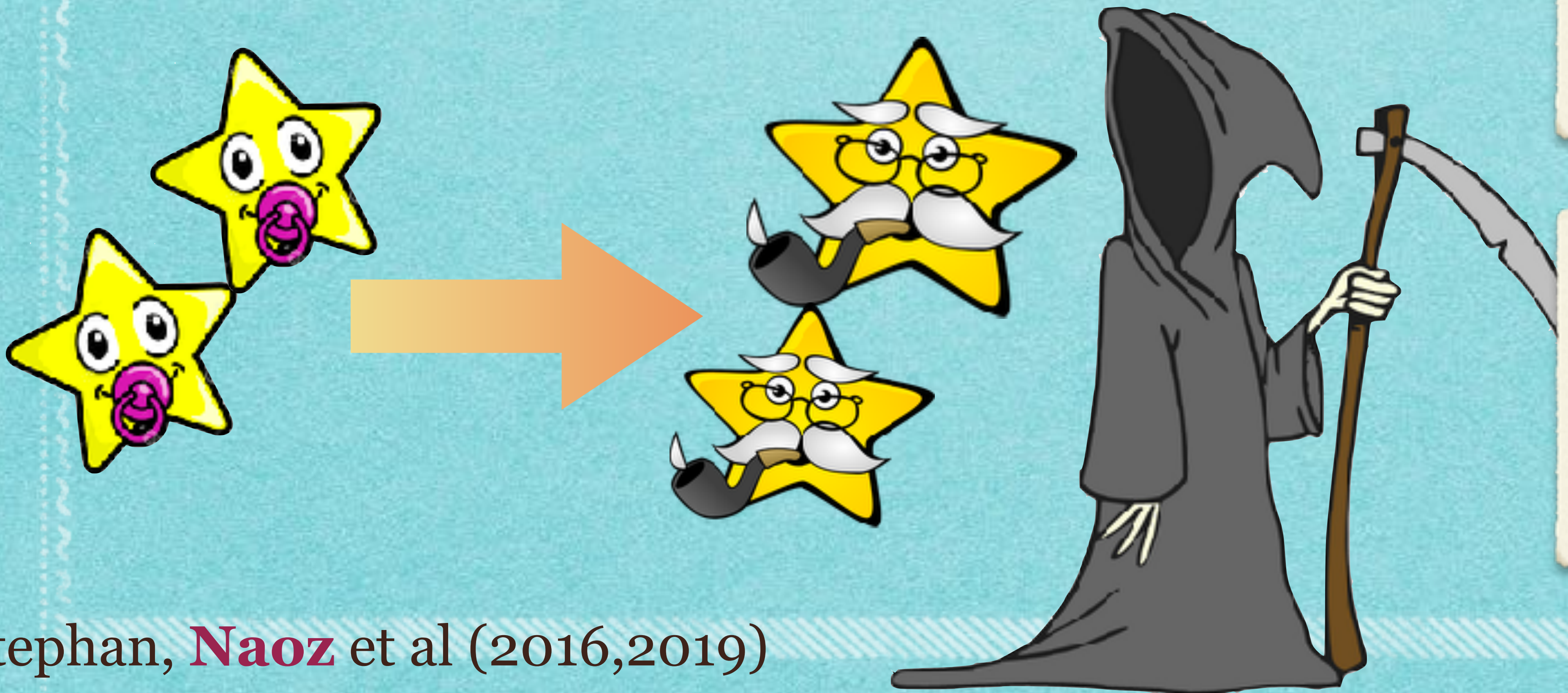
Binaries In Galactic Nuclei

olution (single


Alexander Stephan

BH-BH Rate
~20 Gpc^{-3} yr^{-1}

NS-BH Rate
~7 Gpc^{-3} yr^{-1}



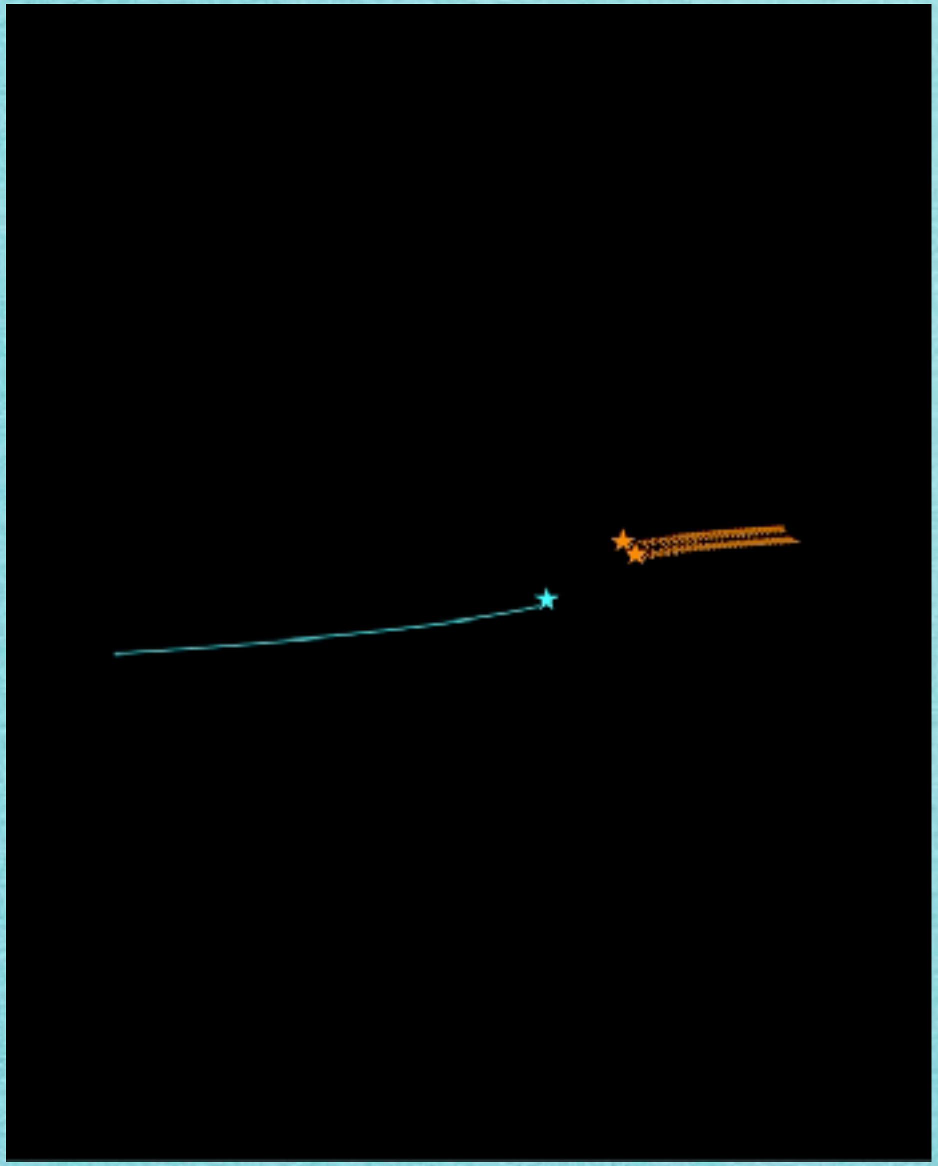
Stephan, **Naoz** et al (2016,2019)

3-body problem

General masses and general orientations
- not bound

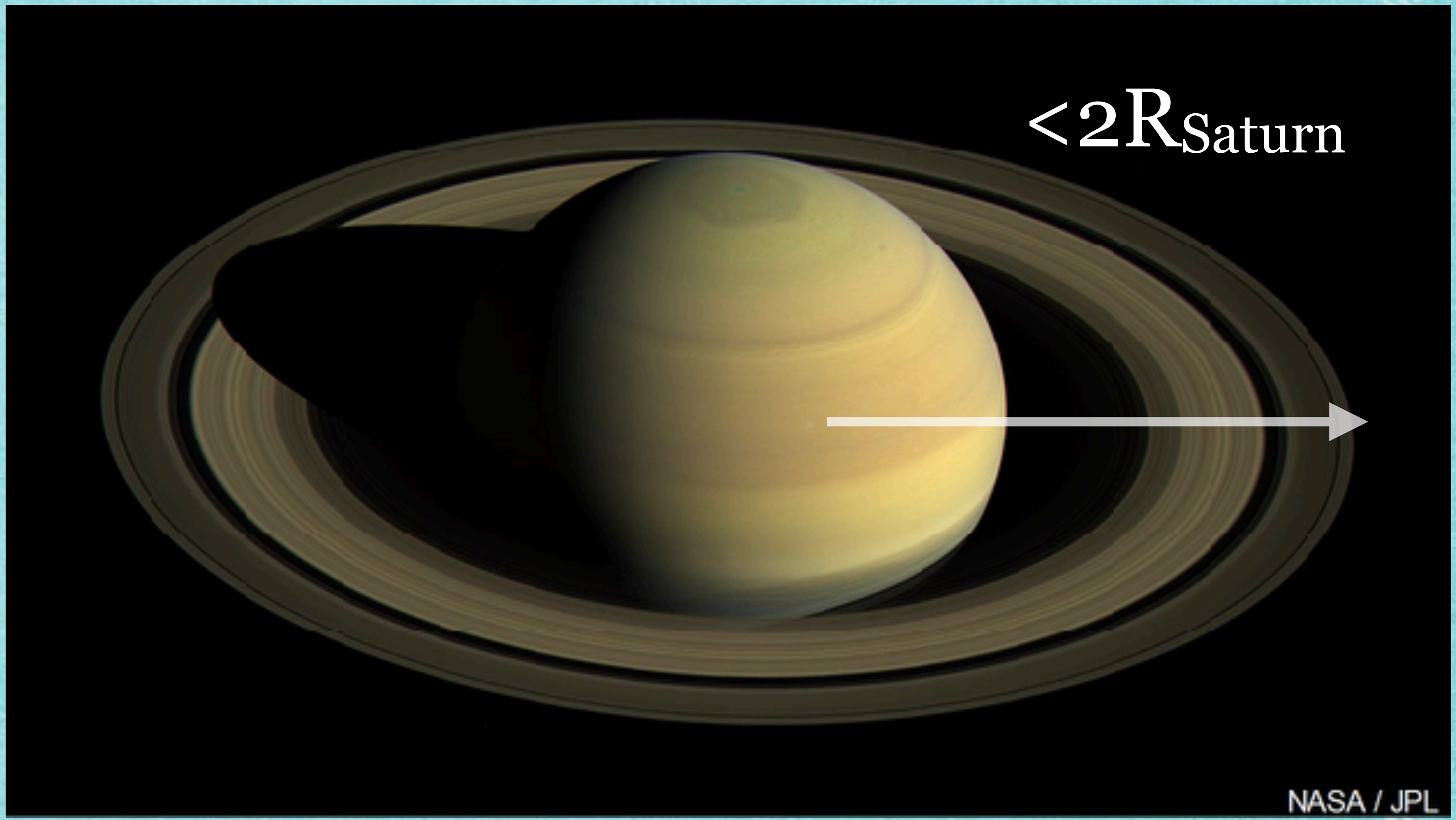
3-body problem

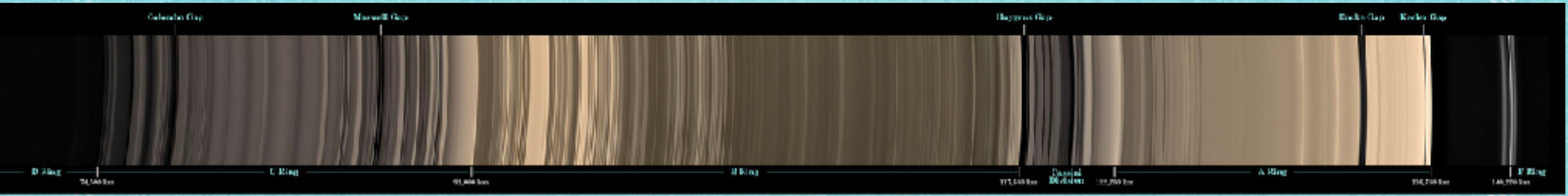
General masses and general orientations - not bound



3-body problem

Lagrange points/tidal/Hill radius





3-body problem

Lagrange points/tidal/Hill radius

Tidal Disruption and the Roche Limit

This problem examines why ring systems about all the giant planets occupy planetocentric distances that are less than ∼2 planetary radii.

Consider a perfectly rigid, spherical satellite of radius R_s, mass m_s, and density ρ_s orbiting a planet of radius R_p, mass m_p, and density ρ_p. Assume the satellite to be in synchronous lock, so that its spin period matches its orbital period. Take the satellite's orbital semi-major axis to be a_s and its orbital eccentricity to be zero.

A marble rests on the surface of this spinning satellite. The spin of the satellite tries to spin it off. The tidal field of the planet also tries to pull it off. The only force trying to keep it glued to the satellite is the satellite's own gravity. For small enough a_s, the marble will fly off. What is this minimum semi-major axis, $a_{s,1}$? Express in terms of ρ_s, ρ_p, and R_p.

3-body problem

Lagrange points/tidal/Hill radius

Answer: The marble is at its most unstable when it lies right in between the planet and the host satellite. Then the tidal acceleration from the planet acting to pull the marble off is $|(d/da)(Gm_p/a^2)R_s| = 2(Gm_p/a^3)R_s$. The centrifugal acceleration of the satellite acting to spin the marble off is $(Gm_p/a^3)R_s$. At $a = a_{s,1}$, these accelerations add to barely balance the satellite's gravitational pull, Gm_s/R_s^2. Then

$$3\frac{Gm_p}{a_{s,1}^3}R_s = \frac{Gm_s}{R_s^2}$$

Solve for $a_{s,1}$ we find:

$$a_{s,1} = \left(\frac{3\rho_p}{\rho_s}\right)^{1/3} R_p = 1.44\left(\frac{\rho_p}{\rho_s}\right)^{1/3} R_p$$

3-body problem

Lagrange points/tidal/Hill radius

How does your answer in (a) relate to the radius of the Hill sphere of the satellite, $r_H = (m_s/3m_p)^{1/3}a_s$?

Answer:

When $a = a_{s,1}$, then

$$r_H = (\rho_s/3\rho_p)^{1/3}(R_s/R_p)(3\rho_p/\rho_s)^{1/3}R_p = R_s$$

the satellite just fills its Hill sphere (Roche lobe). When $a < a_{s,1}$, we say the satellite overfills its Roche lobe. When $a > a_{s,1}$, we say the satellite underfills its Roche lobe.