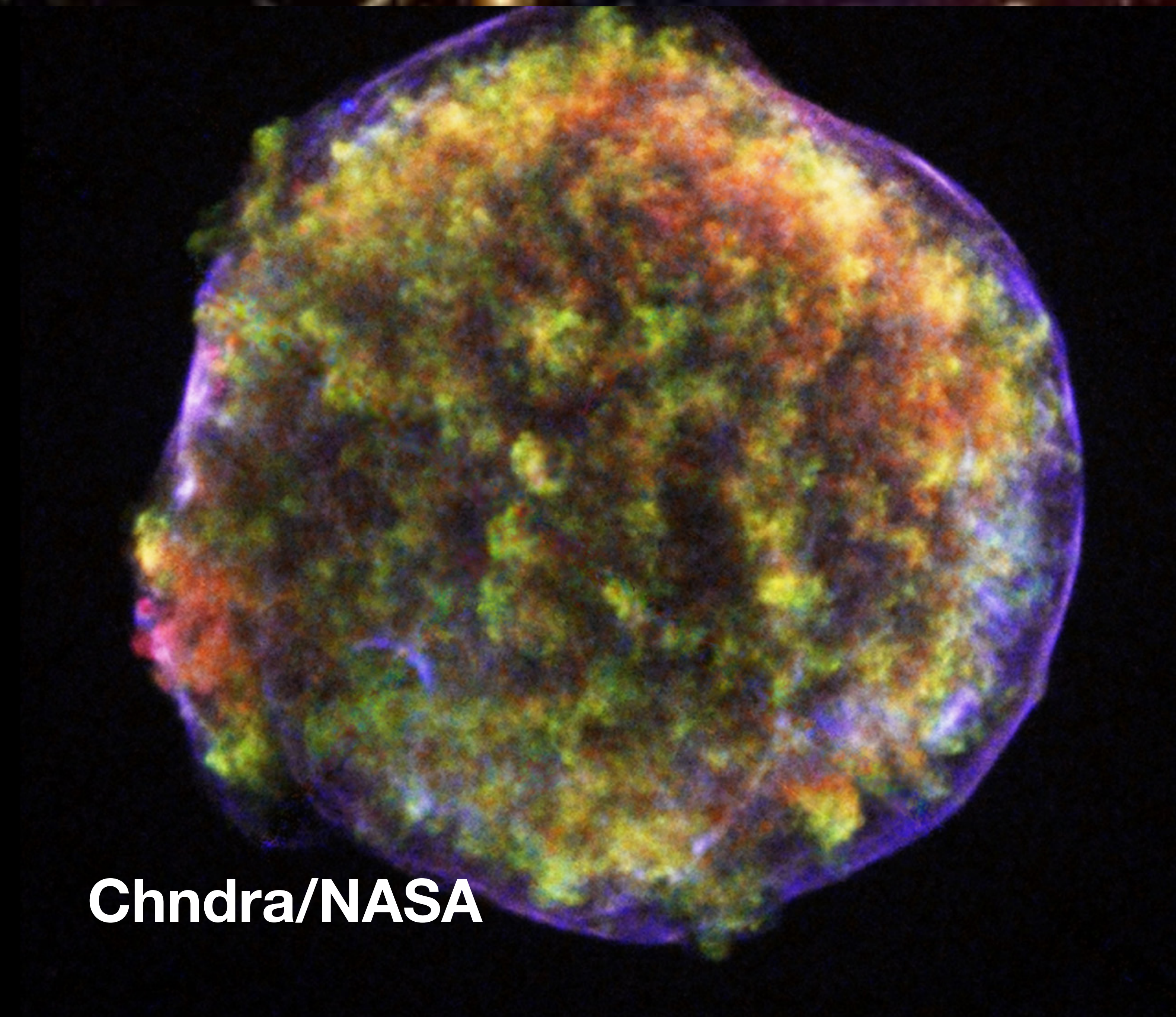
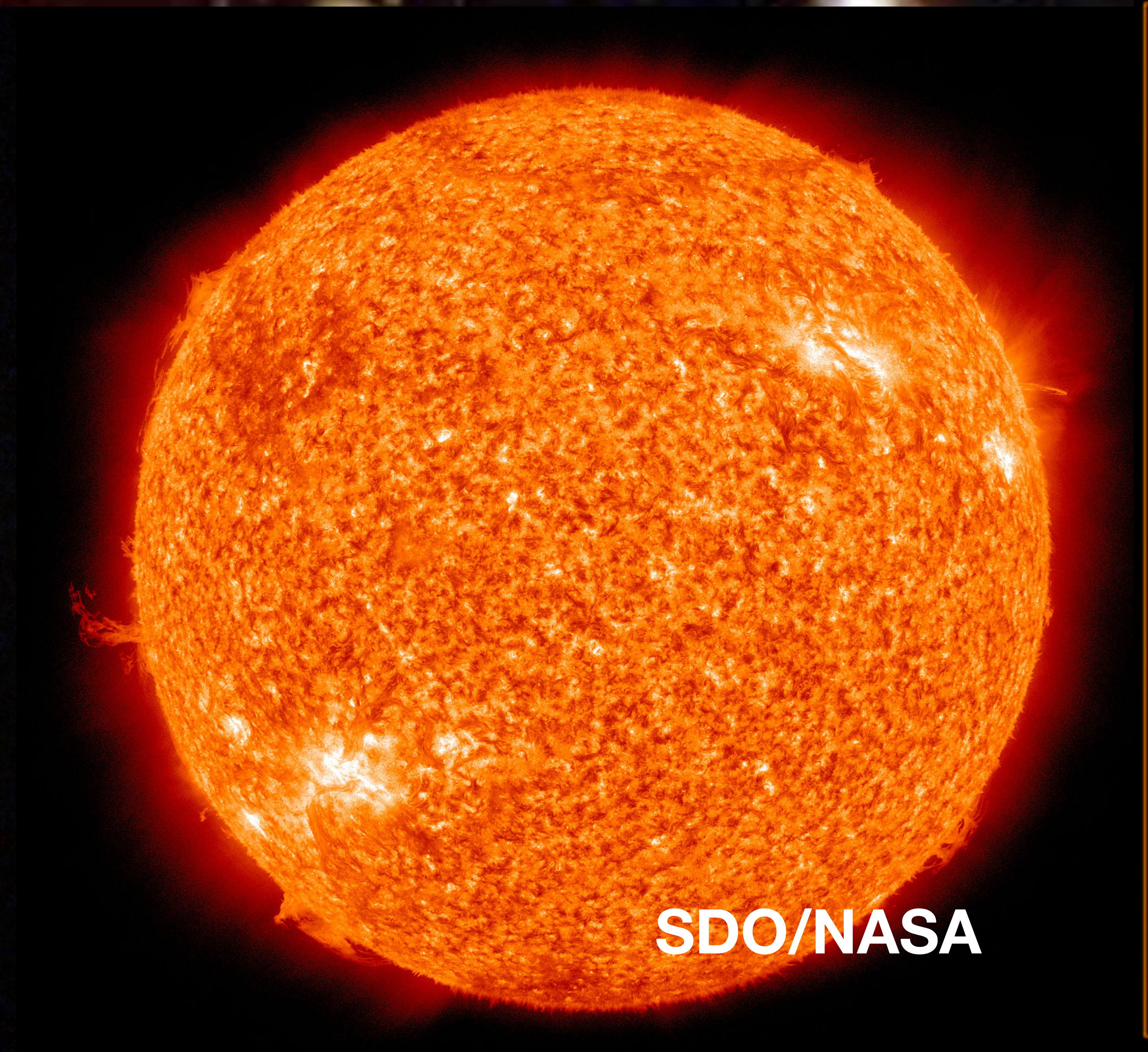
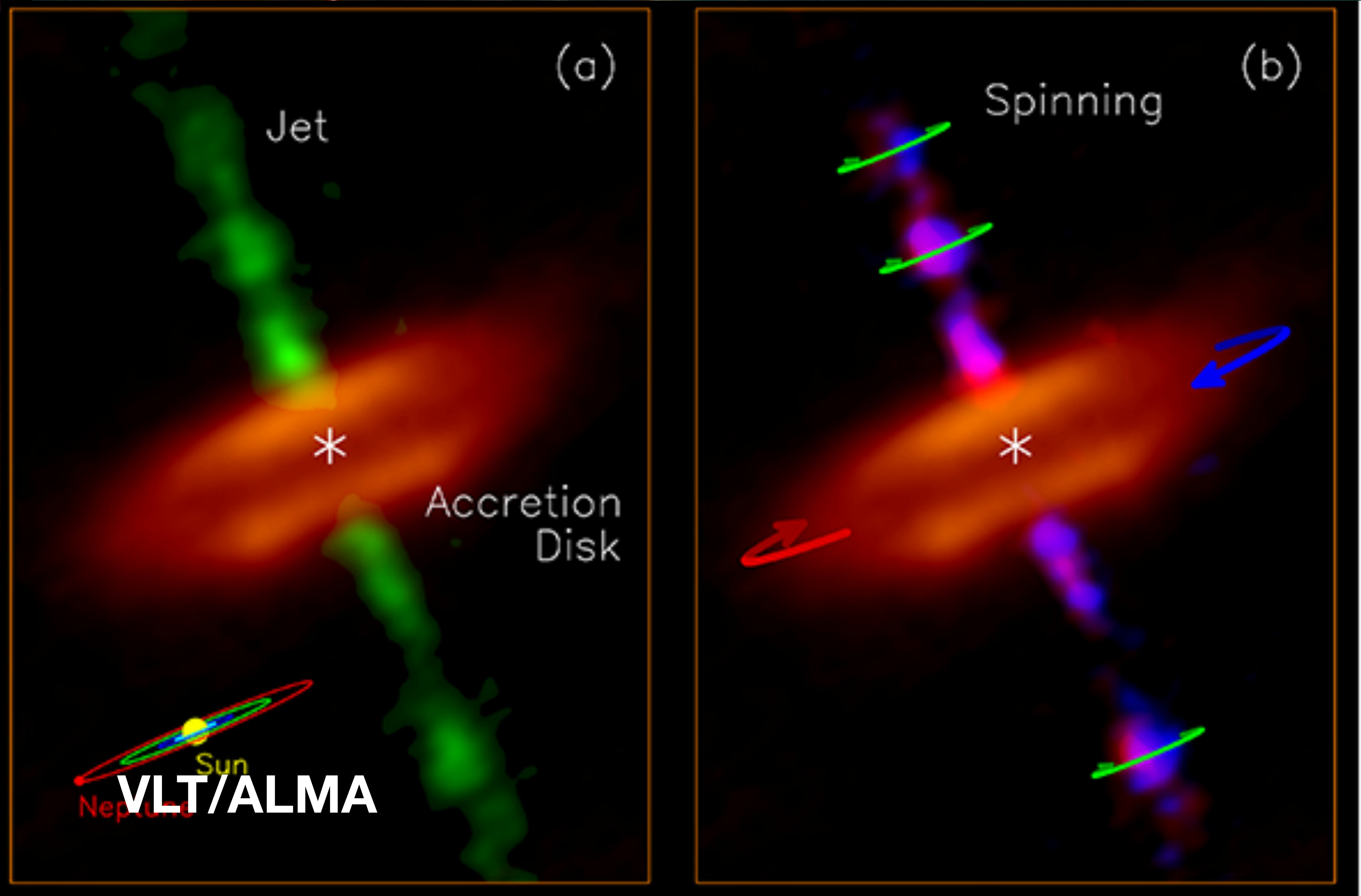
Short Primer on Fluid Dynamics

Smadar Naoz, Sebastian Heinz

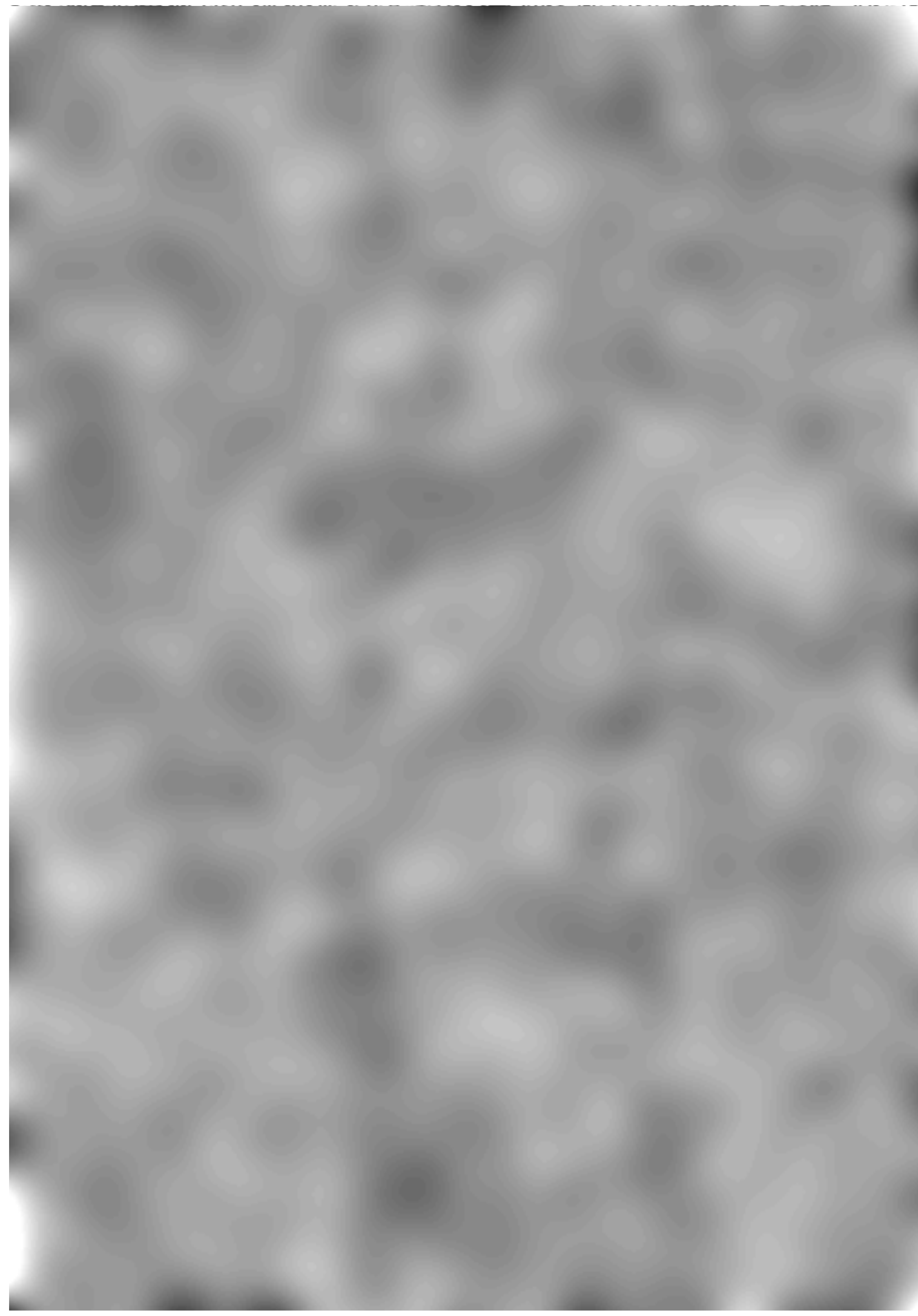
Flickr/Steve Johnson


Chndra/NASA

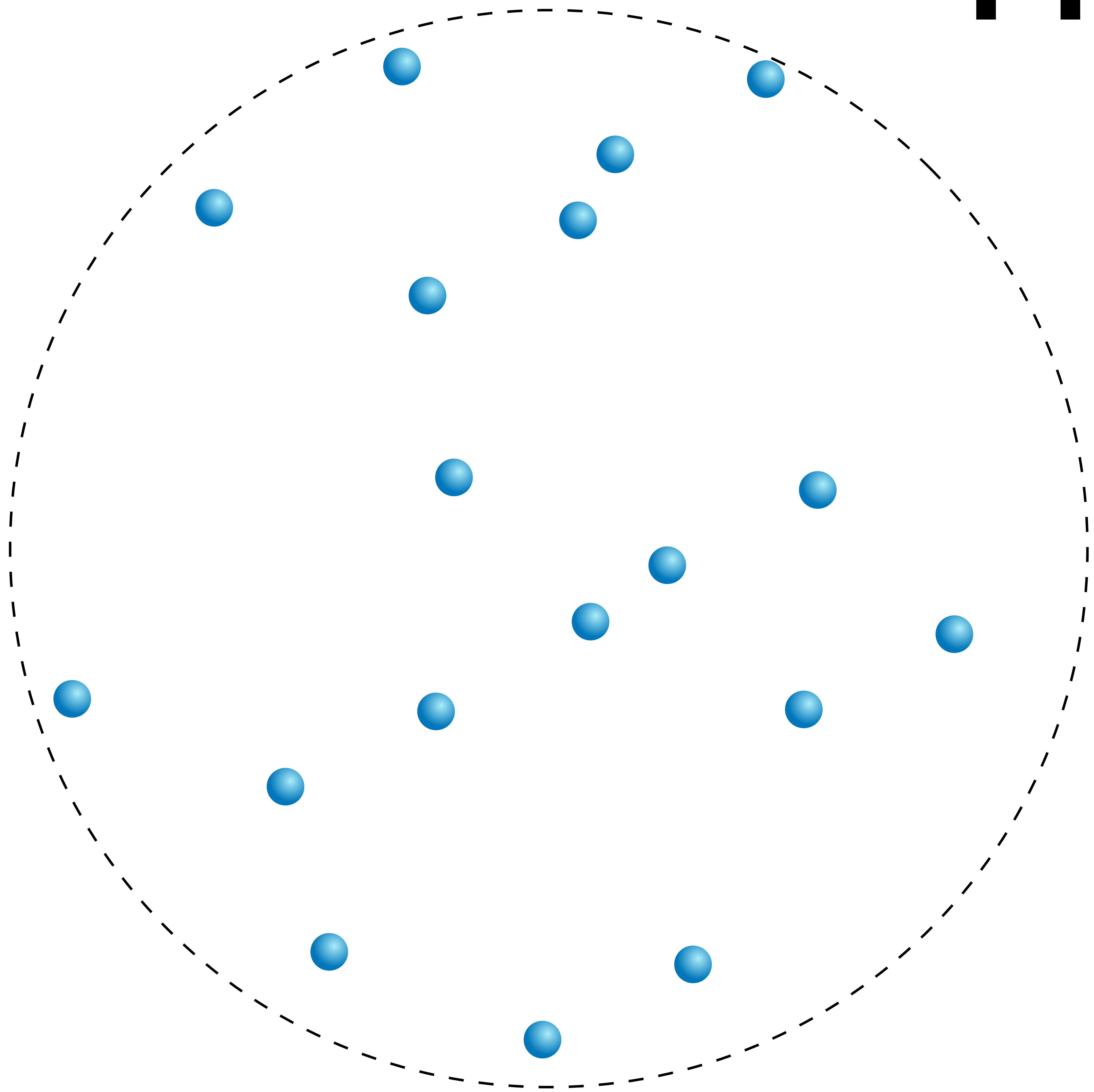
HST/NASA

Chndra/NASA

SDO/NASA

(a)
Jet
Accretion
Disk
Sun
VLT/ALMA
(b)
Spinning

Kinetic vs. Fluid Approach:

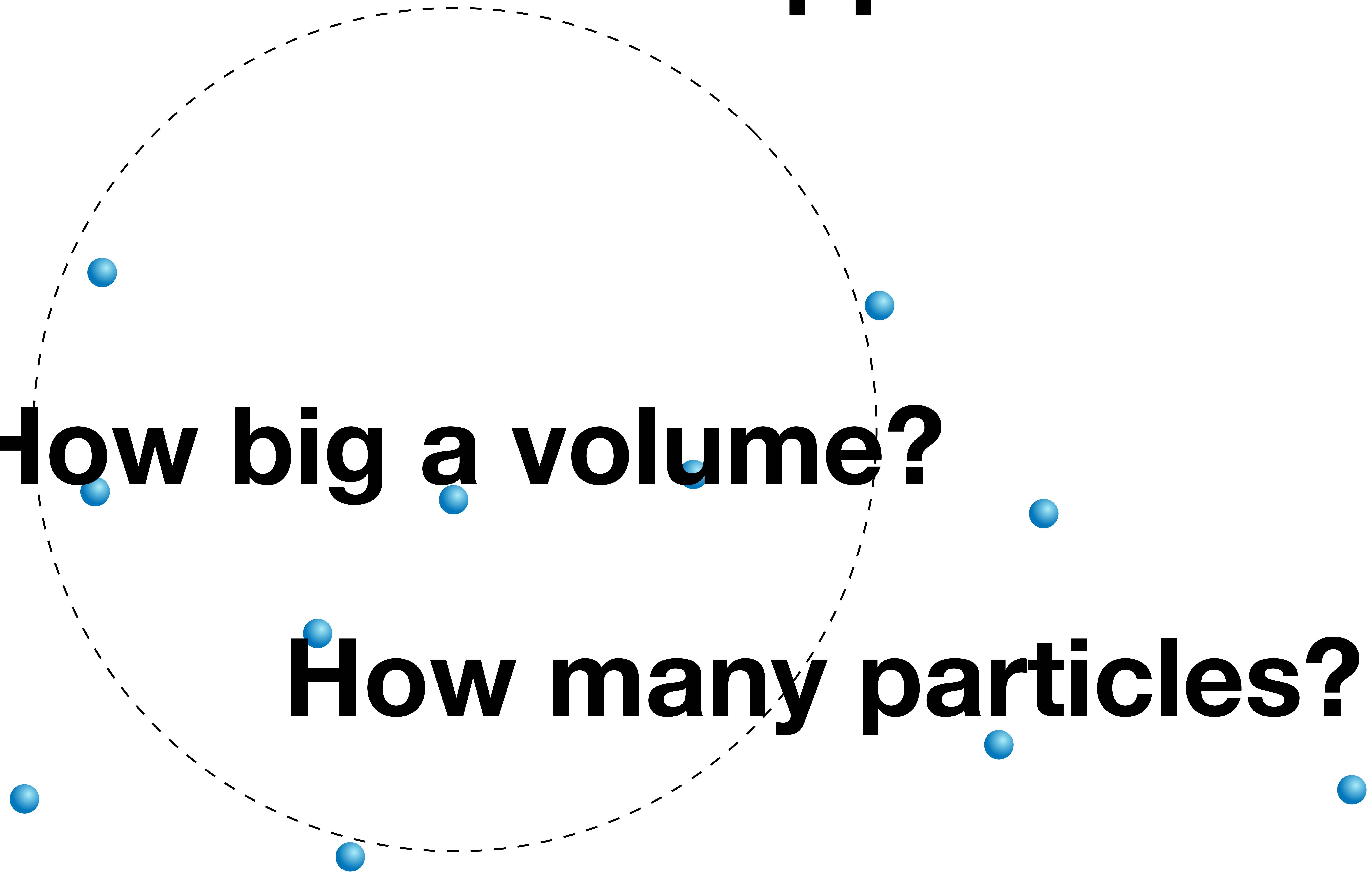
Fluid mechanics: spatial averaging over sufficiently large volume to approximate average quantities as continuous



Kinetic vs. Fluid Approach:



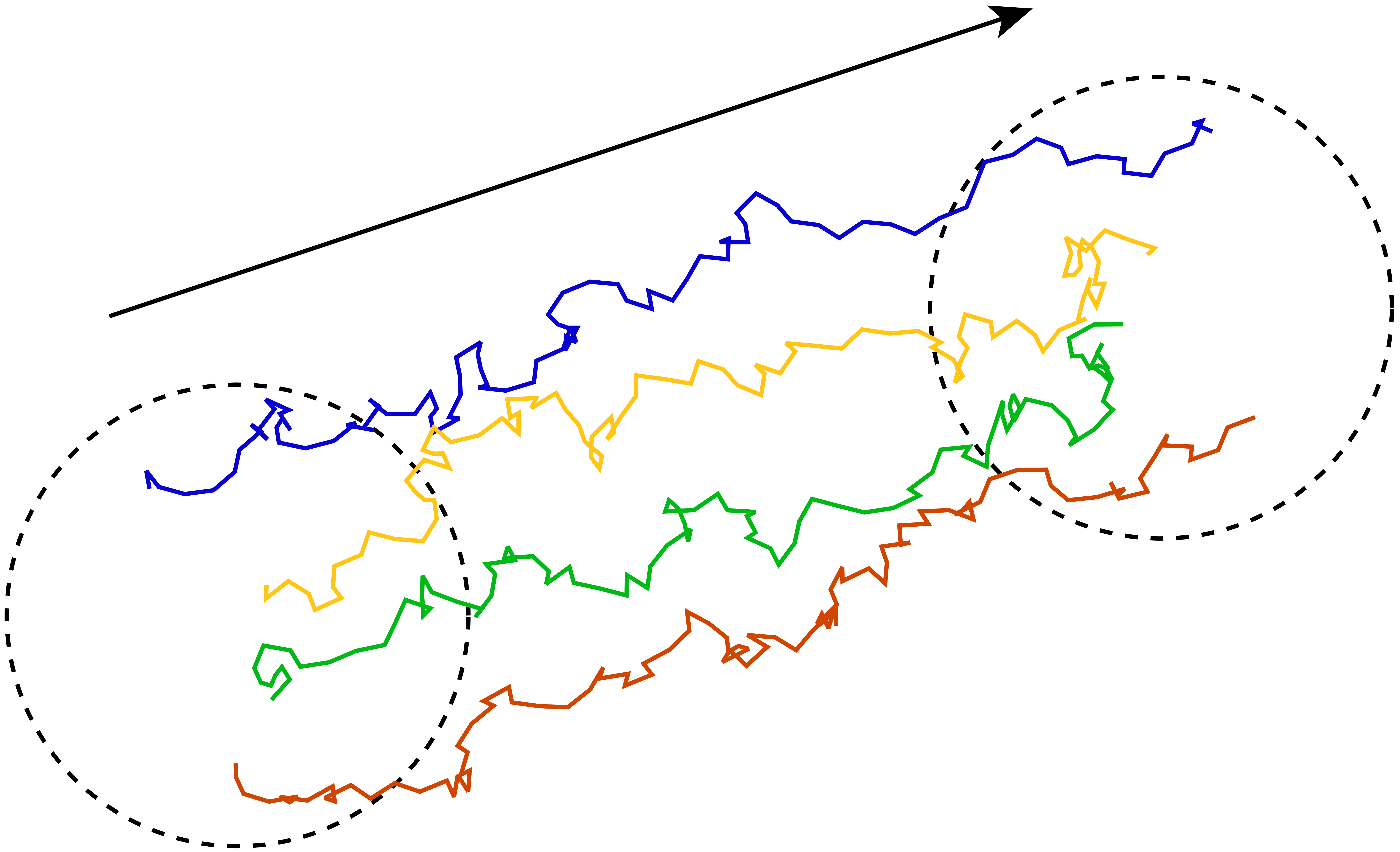
Kinetic vs. Fluid Approach:



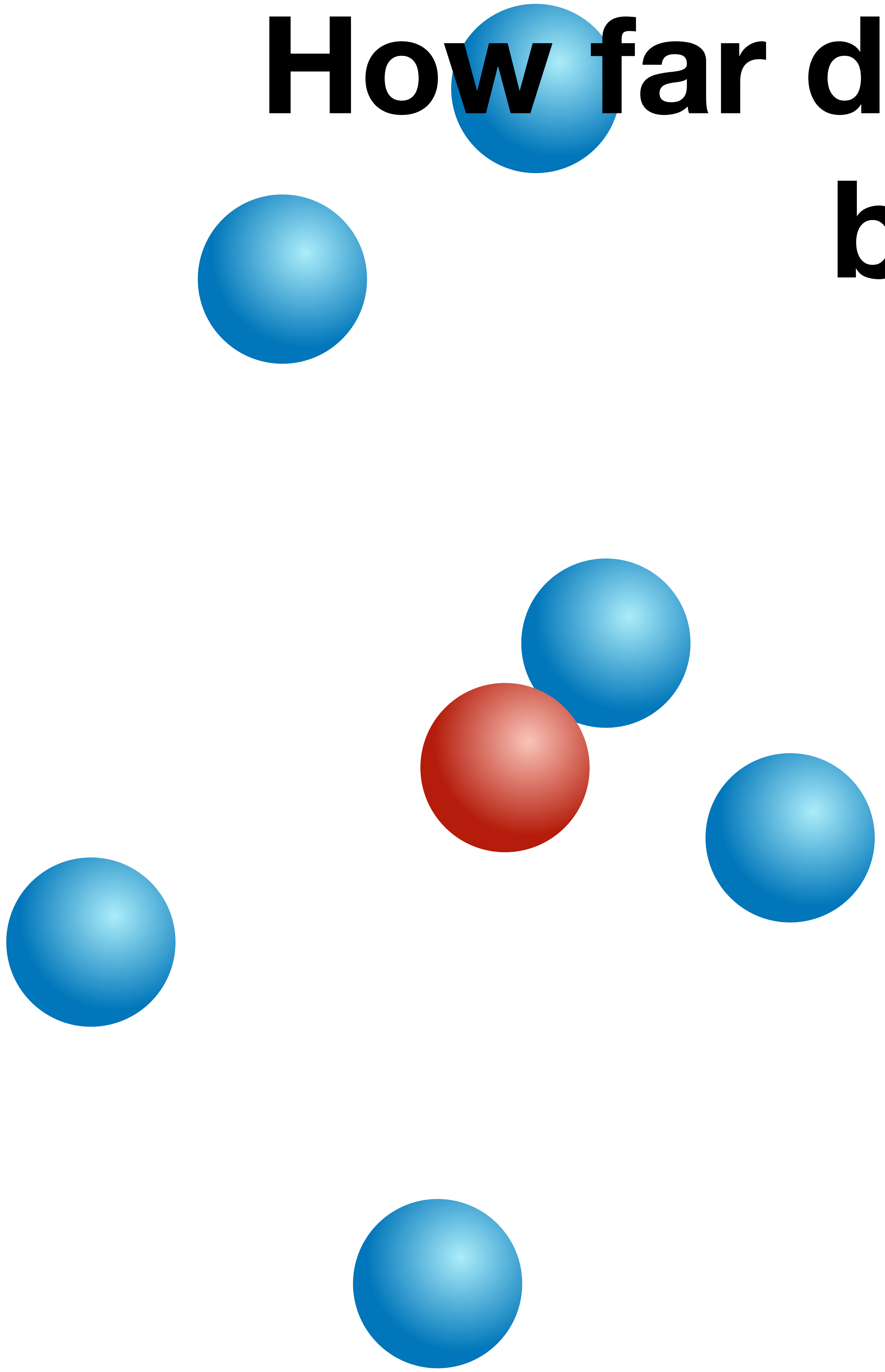
How big a volume?

How many particles?

Free-streaming vs. random walk

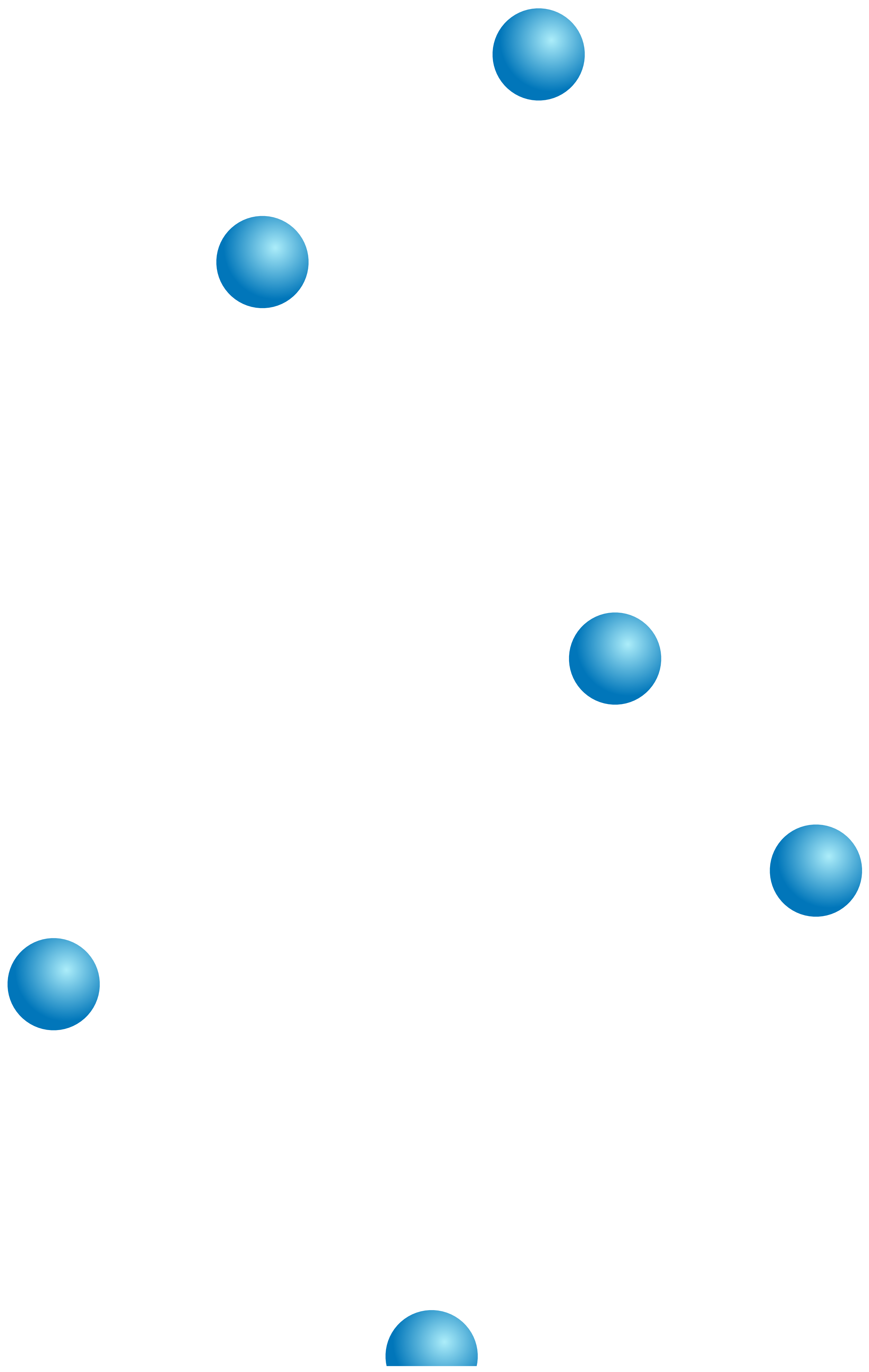


Particle mean free path: How far does a typical particle travel before interacting?



$$\lambda_{\mathrm{mfp}}$$

Estimate $\lambda_{\rm mfp}$ for Neutral ISM



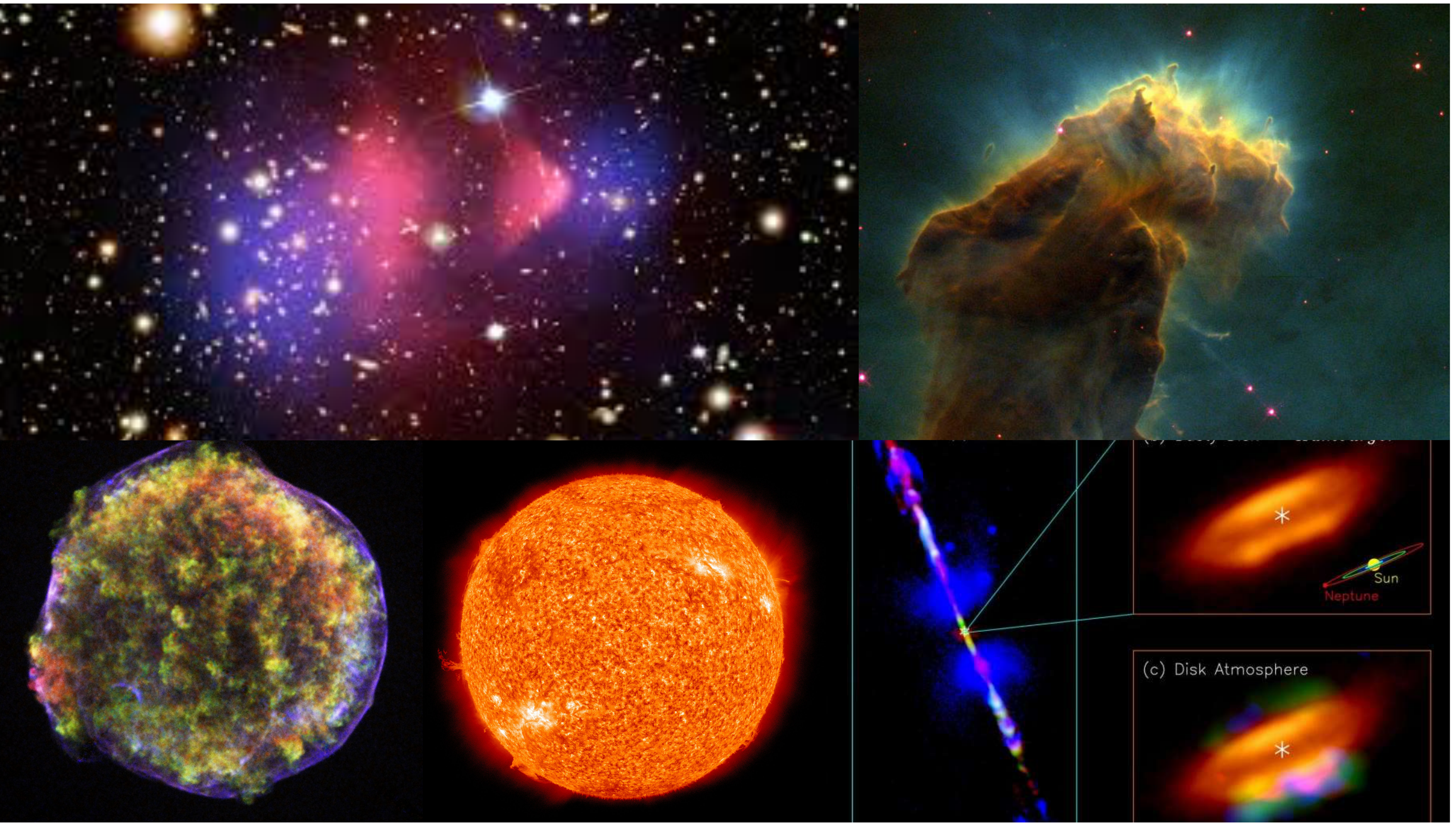
$$\lambda_{\rm mfp} = \frac{1}{n\sigma}$$

$$\sigma \sim \pi R_{\rm atom}^2$$

$$R_{\rm atom} \sim 5 \times 10^{-9}\,{\rm cm}$$

$$\lambda_{\rm mfp} \sim 10^{16}\,{\rm cm}$$

Estimate $\lambda_{\rm mfp}$ for Neutral ISM



$$\lambda_{\rm mfp} = \frac{1}{n\sigma}$$

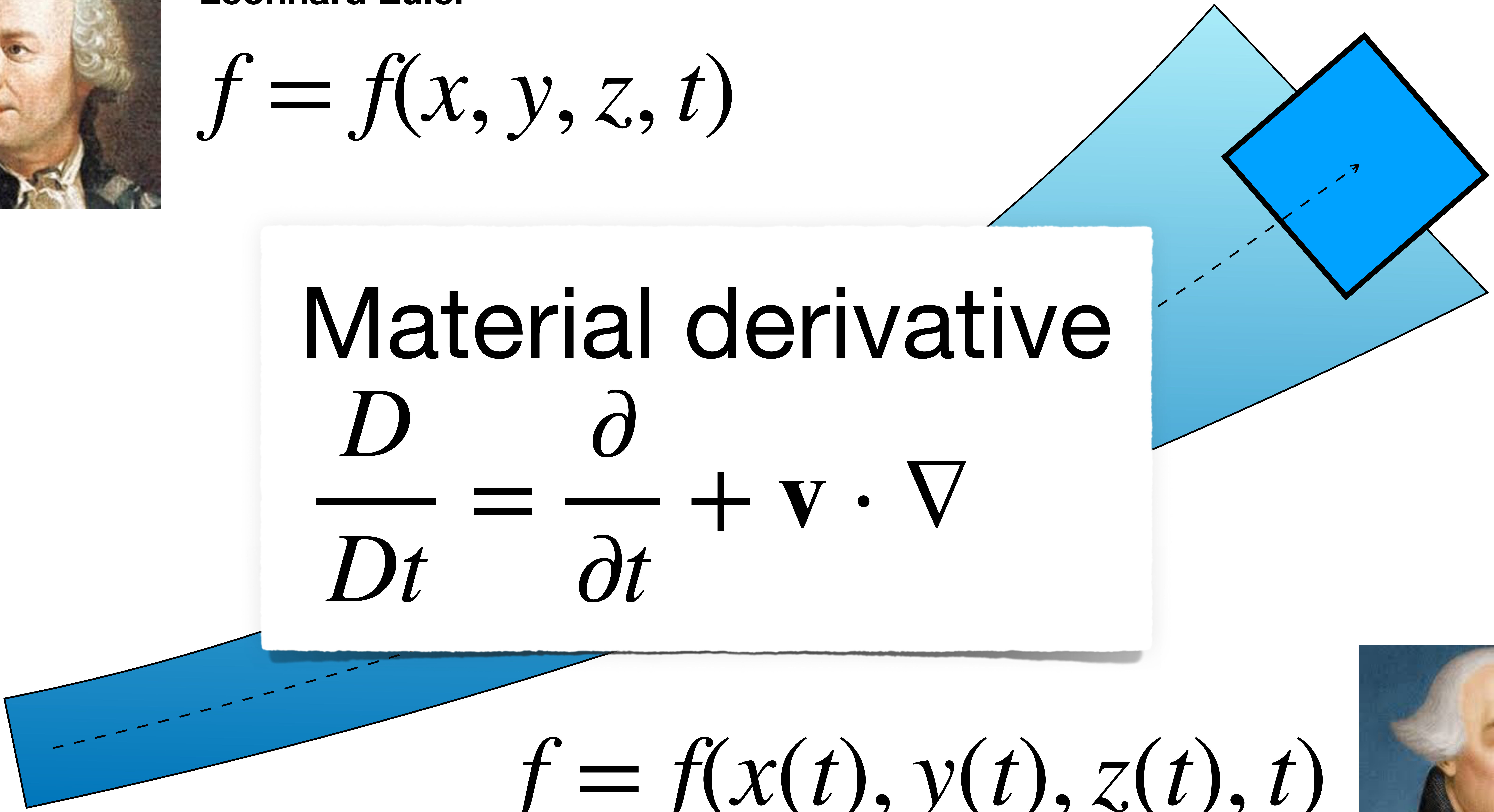
$$\sigma \sim \pi R_{\rm atom}^2$$

$$R_{\rm atom} \sim 5 \times 10^{-9}\,{\rm cm}$$

$$\lambda_{\rm mfp} \sim 10^{16}\,{\rm cm}$$

Leonhard Euler

$$f = f(x, y, z, t)$$

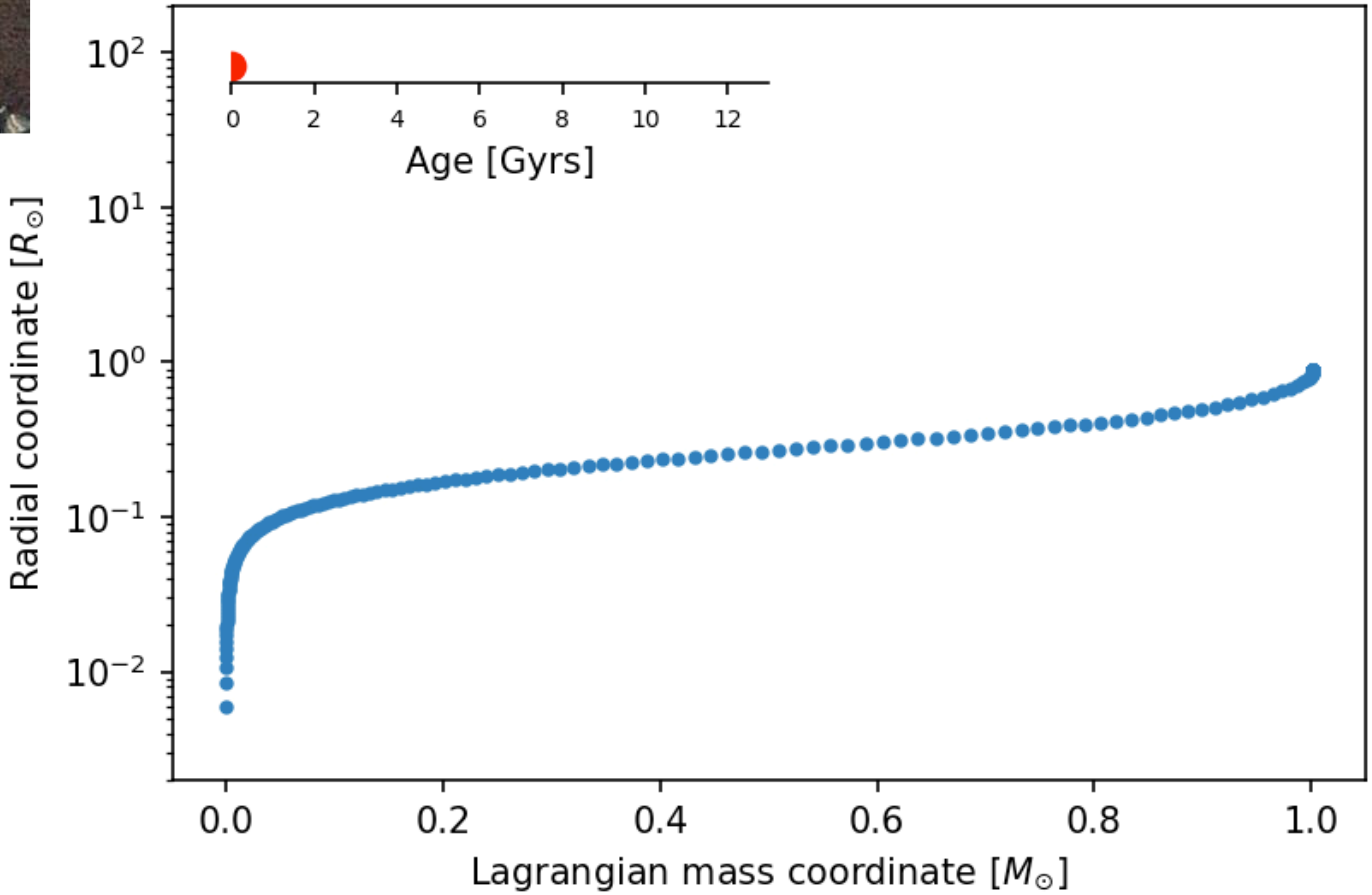


Material derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

$$f = f(x(t), y(t), z(t), t)$$

Joseph-Louis Lagrange

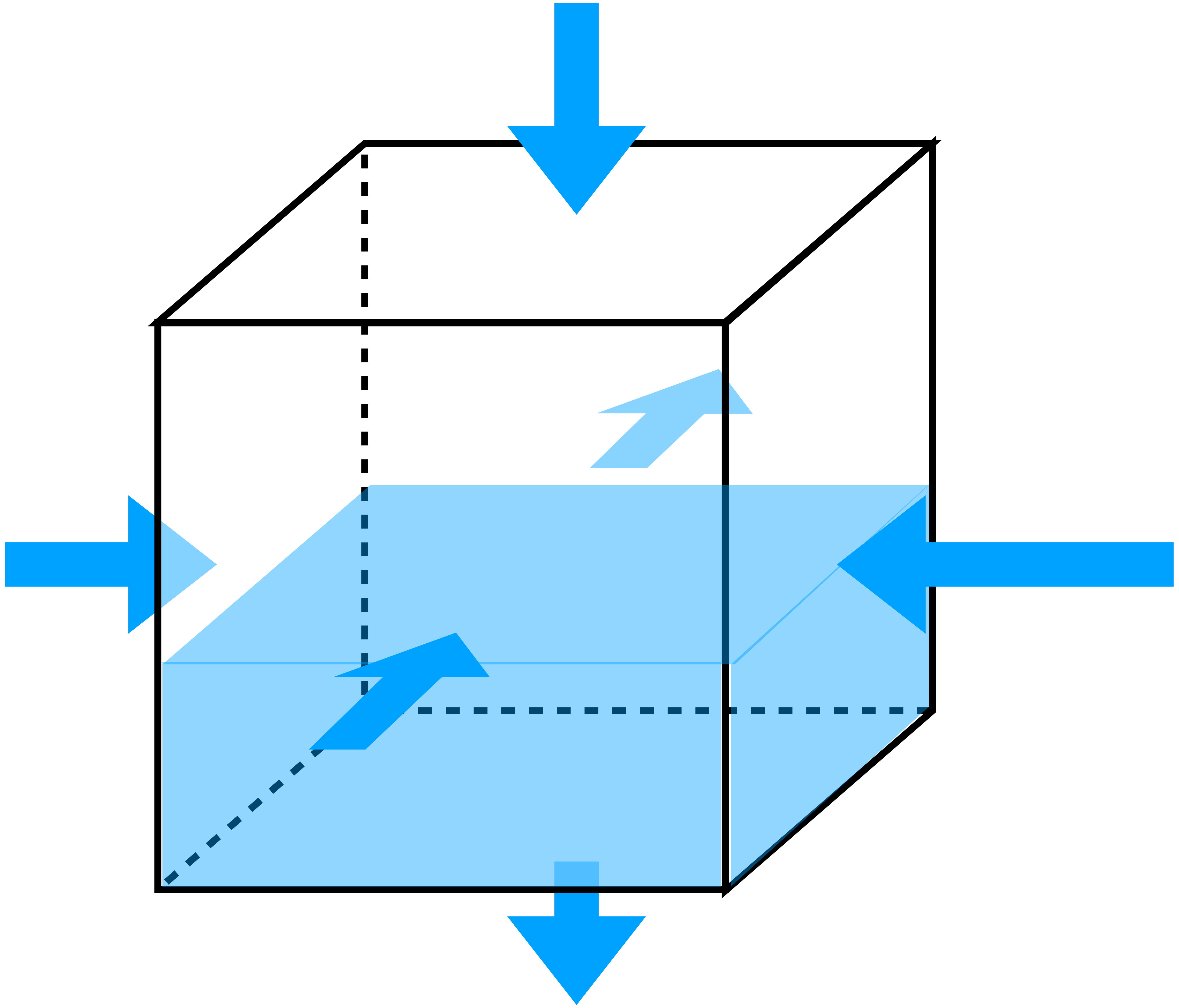


Age [Gyrs]
0
2
4
6
8
10
12
10^2
10^1
10^0
10^{-1}
10^{-2}
Radial coordinate [$R_\odot$]
0.0
0.2
0.4
0.6
0.8
1.0
Lagrangian mass coordinate [$M_\odot$]

In the Eulerian sense, what is the rate of change of water in the box?

Flux density of water:

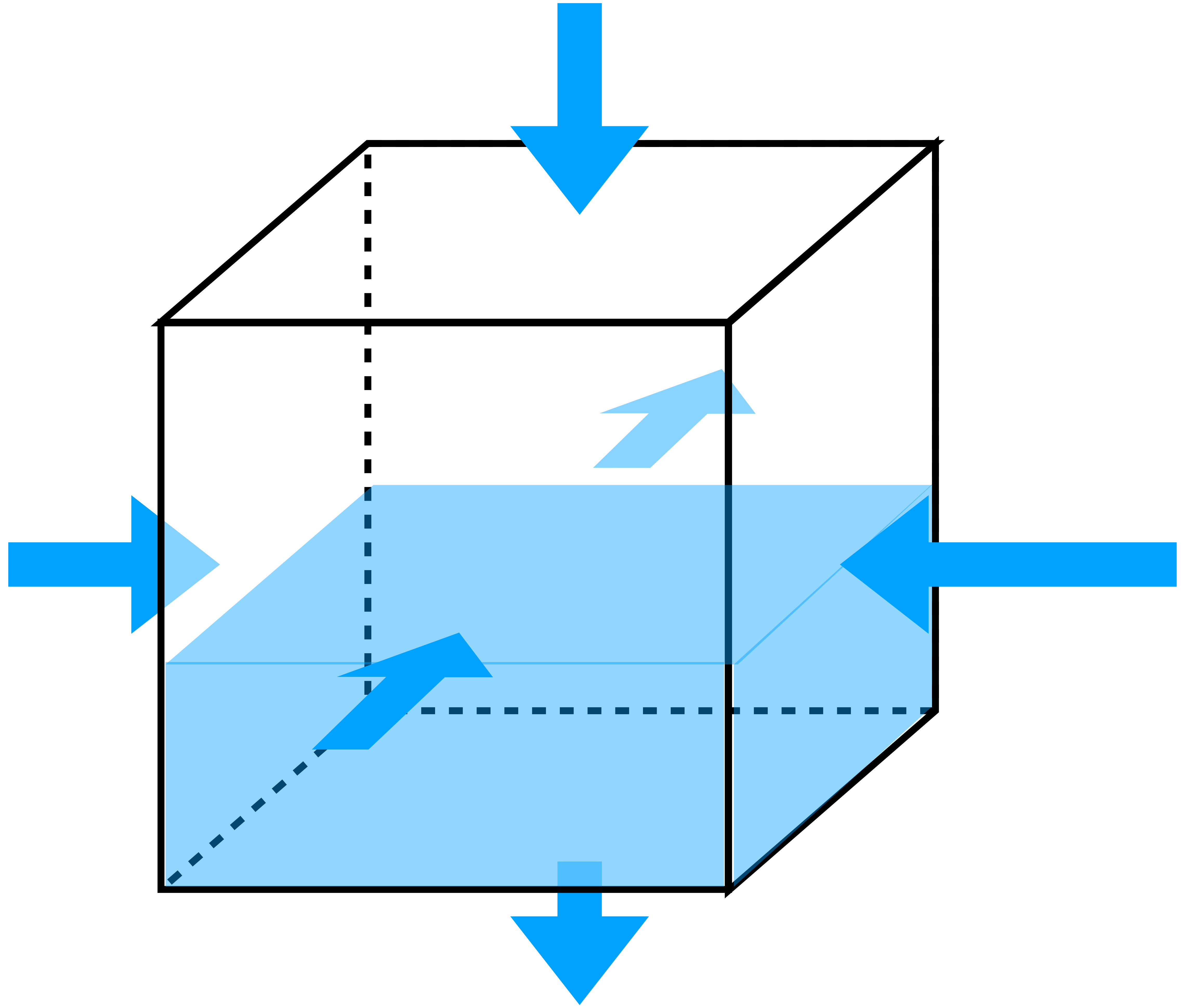
$$\vec{j}_{\mathrm{M}} = \rho \overrightarrow{v}$$



$$\frac{\partial M}{\partial t} = \int dV \frac{\partial \rho}{\partial t} = -\oint dA \mathbf{j} \cdot \mathbf{n} = -\int dV \nabla \cdot \mathbf{j}$$

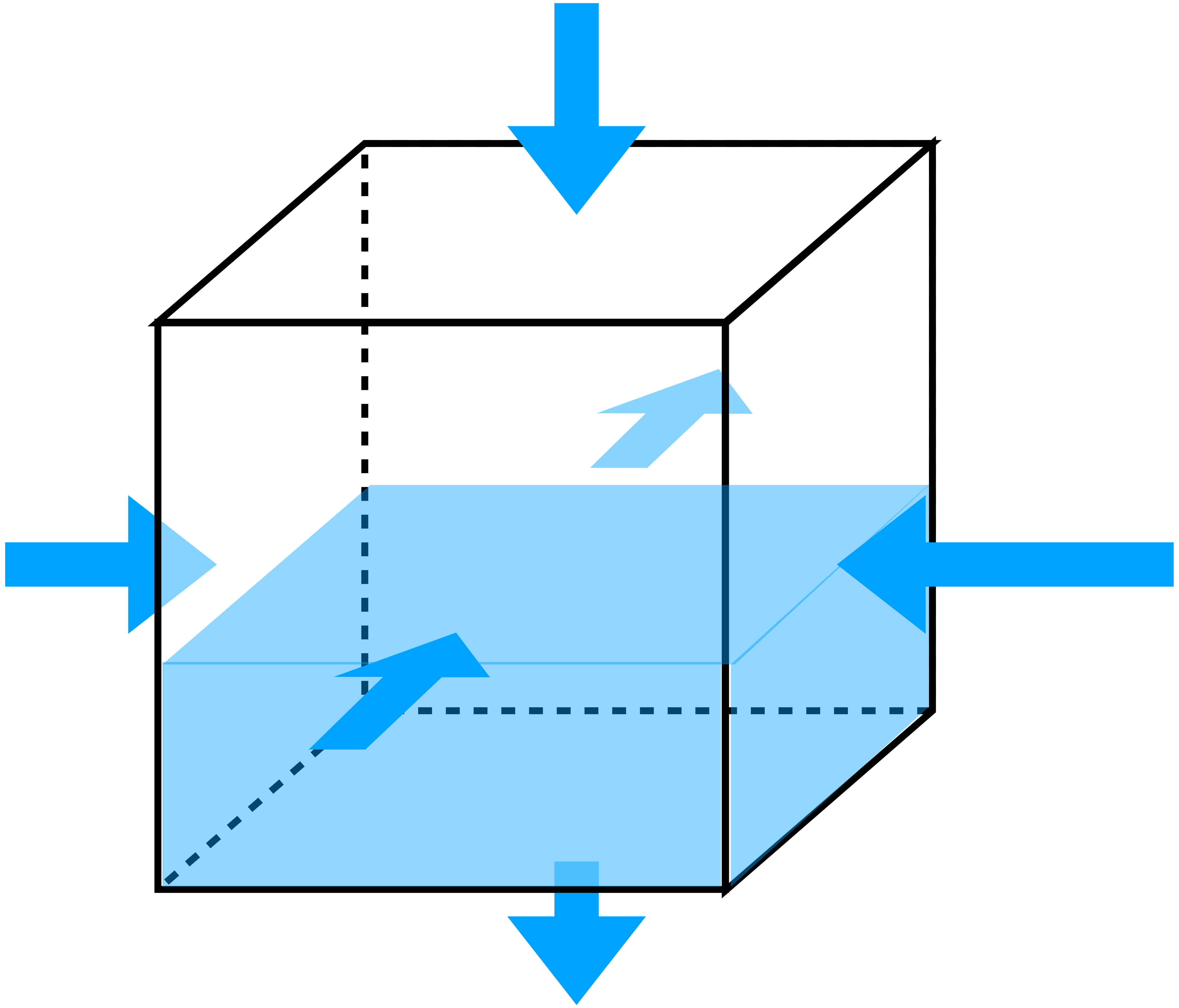
$$\int dV \nabla \cdot \mathbf{j} = \oint dA \mathbf{j} \cdot \mathbf{n}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \mathbf{v} \right) = 0$$



This is the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \mathbf{v}\right) = 0$$

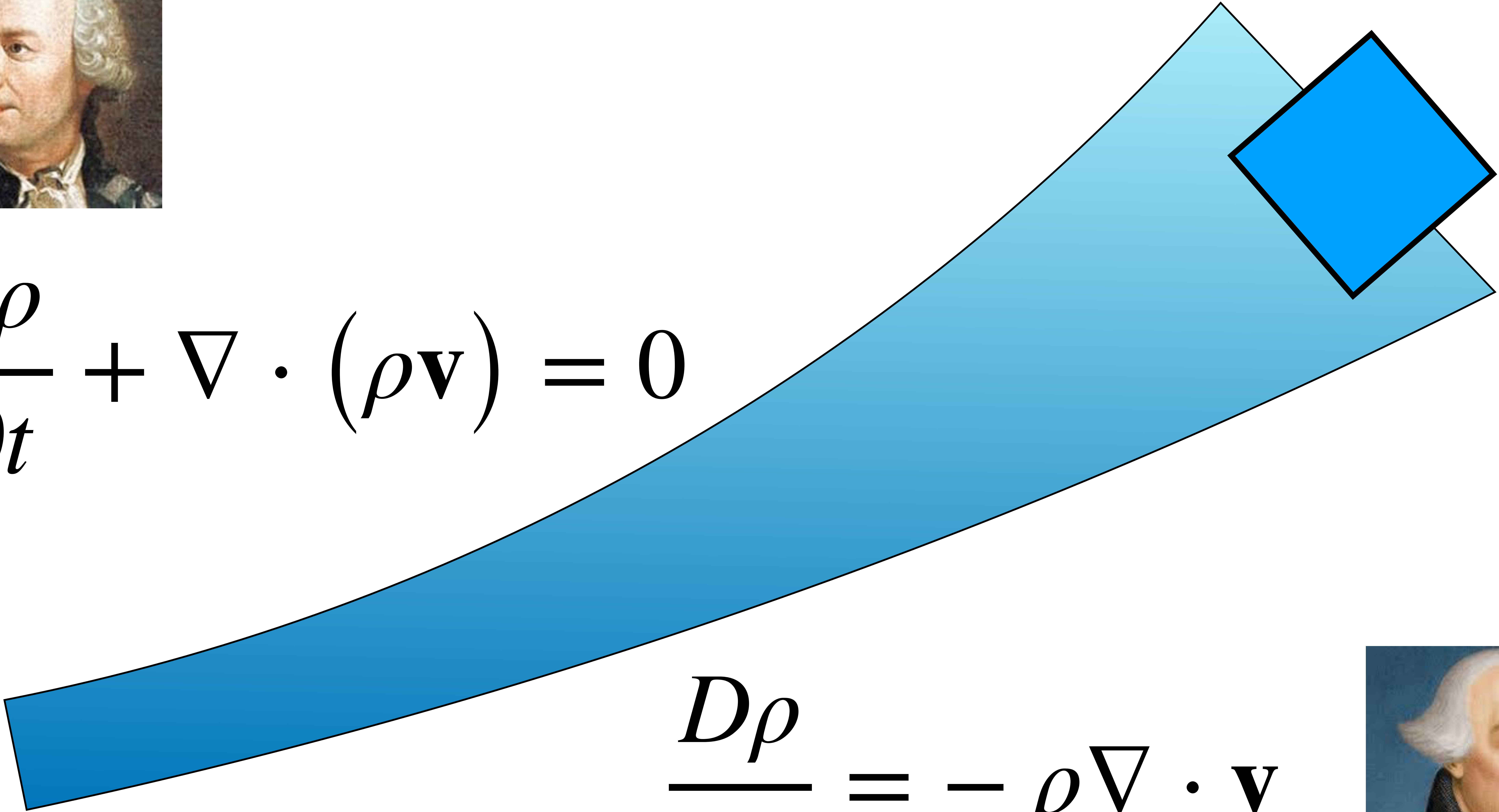


This is the continuity equation

Conservation law form:

$$\frac{\partial \rho_a}{\partial t} + \nabla \cdot \mathbf{j}_a = S_a$$

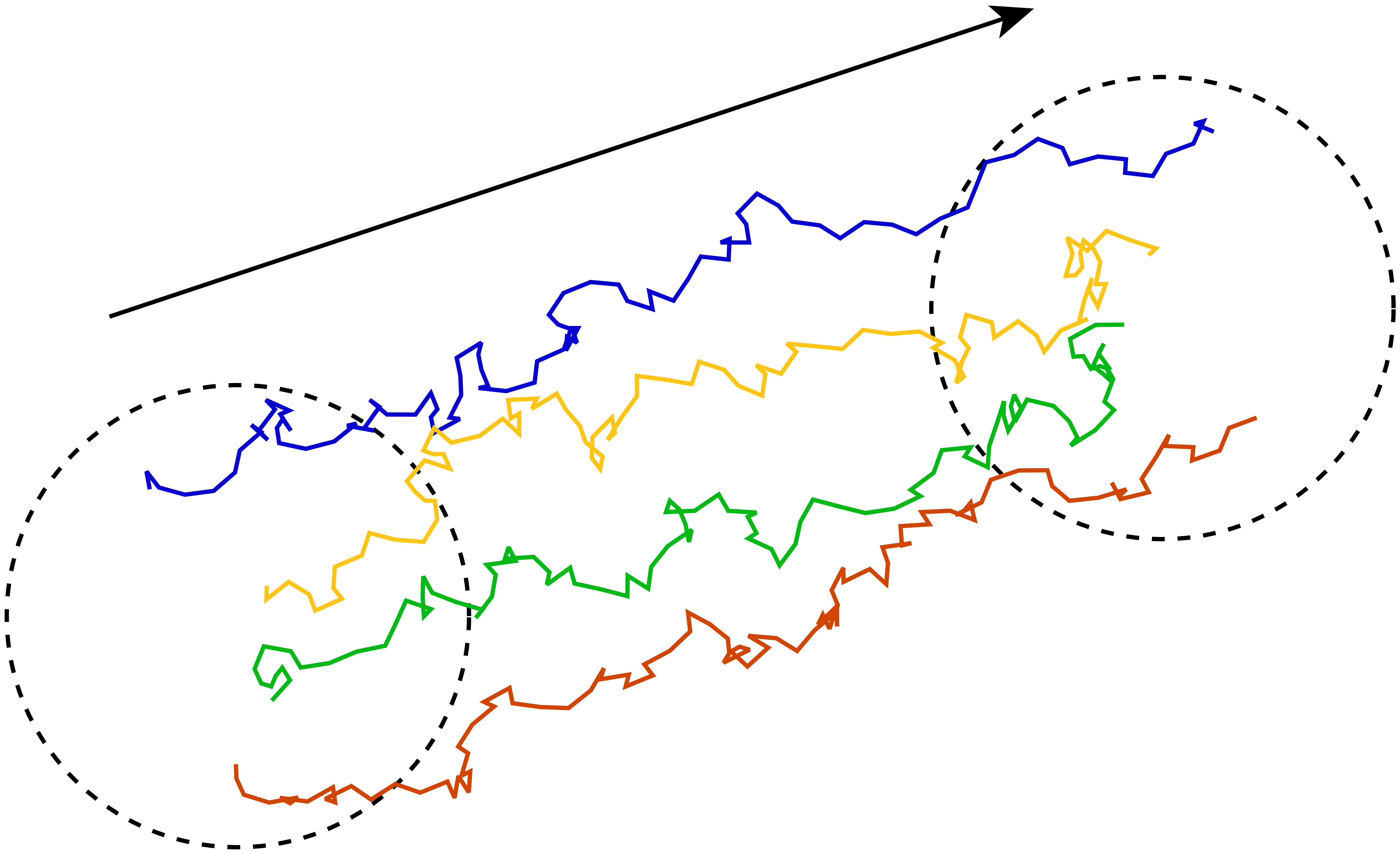
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$



$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v}$$

Conservation Laws

- Mass
- Momentum
- Energy



Next: Momentum Conservation

- Momentum flux density (for the i-momentum):

$$\mathbf{j}_{\mathrm{P,i}} = \rho v_{\mathrm{i}} \mathbf{v} + \overrightarrow{\pi}_{\mathrm{i}}$$

$$\frac{\partial \rho v_{\mathrm{i}}}{\partial t} + \nabla \left(\rho v_{\mathrm{i}} \mathbf{v} \right) + \nabla P = f_{\mathrm{ext,i}} + f_{\mathrm{visc,i}}$$

The in***viscid*** Euler equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \mathbf{v}\right) = 0$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \left(\mathbf{v} \cdot \nabla\right) \mathbf{v} + \nabla P = \mathbf{f}(\mathbf{v}, \mathbf{x}, \ldots)$$

$$\rho \frac{\partial e}{\partial t} + \rho \left(\mathbf{v} \cdot \nabla\right) e = - P \nabla \cdot \mathbf{v}$$

Specific internal energy (energy per unit mass)

Closure: An equation of state

$$e = e(P)$$

Adiabatic equation of state

$$e = \frac{1}{\gamma_{\rm ad} - 1} P$$

$$\gamma_{\rm ad} = \frac{C_P}{C_V}$$