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Astrophysical Shocks and Particle Acceleration

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Intended Learning Objectives

- Shocks in astrophysics
- Shock jump conditions
- Diffusive shock acceleration
- Energy distribution of high energy particles

What is a shock?

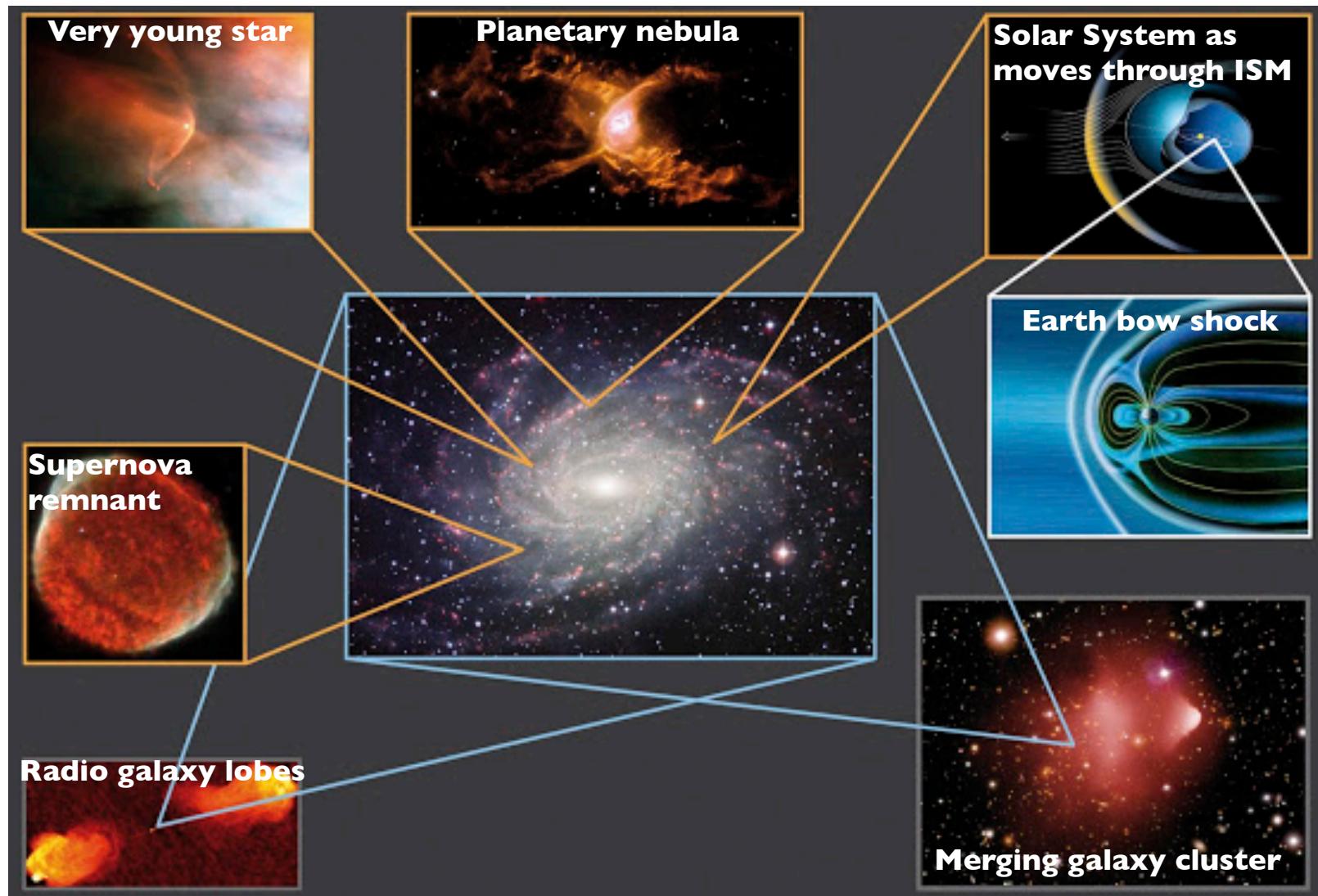
Shocks

- A shock occurs when a disturbance moves through a medium faster than the medium sound speed.
- The shock wave carries energy and propagates through the medium.
- A sudden change in bulk velocity, density, medium temperature occurs across the shock.
No forewarning for the material hit by the shock.



Example of shock: sonic boom

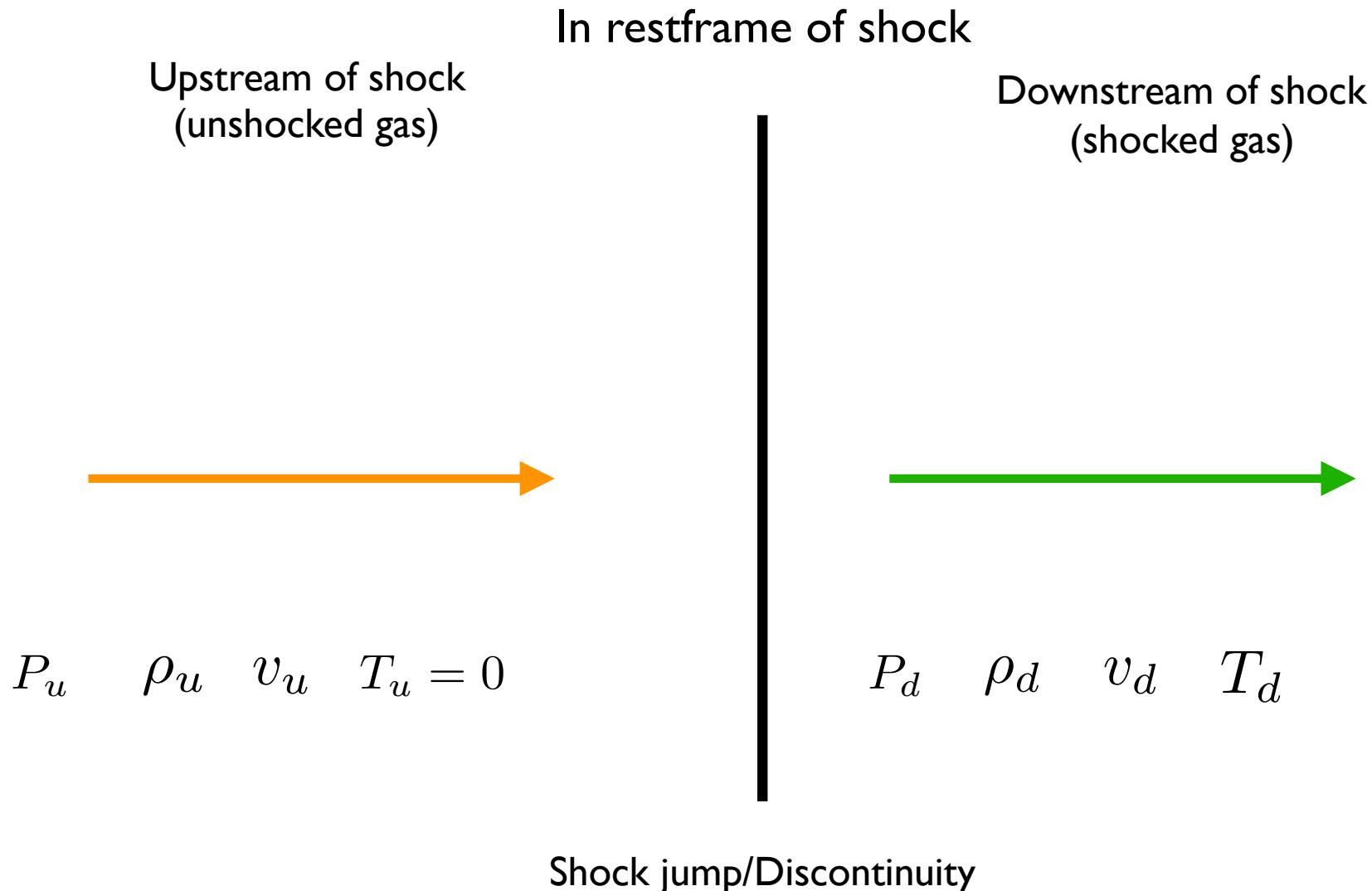
Shocks in Astrophysics



- Astrophysical shocks occur on all scales from planets to clusters of galaxies.
- Astrophysical shocks are collisionless (the shock front is thinner than the particle mean free path).

Shock Physics

To understand the shock conditions, let's go into the instantaneous rest frame of the shock.



Shock Physics

Assumptions

- Monoatomic gas (e.g. fully-ionized hydrogen plasma).
- Thermal (internal) energy of unshocked gas is negligible.
- Pressure and temperature of unshocked gas are approximately zero.

Conservation laws

Conservation of mass (the mass per unit area flowing across the shock is conserved)

$$v_d \rho_d = v_u \rho_u$$

Conservation of momentum (shock does not accelerate in its rest frame; the difference between upstream and downstream pressures is provided by the gas pressure downstream)

$$v_d^2 \rho_d = v_u^2 \rho_u + P_d$$

Conservation of energy (PdV is the work being done on the gas at the shock; the rate per unit area at which this work is done is $v_d P_d$)

$$v_u \left(\frac{1}{2} \rho_u v_u^2 \right) - v_d \left(\frac{1}{2} \rho_d v_d^2 + \frac{3}{2} P_d \right) = v_d P_d$$

Shock Physics

If we use the fact that $v_d \rho_d = v_u \rho_u$ and remove P_d , we obtain

$$\frac{\rho_d^2}{\rho_u^2} v_u^3 - 5 \frac{\rho_d}{\rho_u} v_u^3 + 4 v_u^3 = 0$$

This can be re-written as

$$\left(\frac{\rho_d}{\rho_u} - 4 \right) \left(\frac{\rho_d}{\rho_u} - 1 \right) = 0$$

The equation above has the trivial solution $v_u = v_d$, and otherwise leads to the **strong shock jump conditions**

$$\frac{\rho_d}{\rho_u} = 4 \quad \frac{v_u}{v_d} = 4$$

This is a highly simplified case; however, it is a reasonable approximation in many astrophysical environments. For a full derivation, see Longair or Blundell & Blundell.

Characteristics of Shocks

In order to characterize the shock, the Mach number is adopted

$$M = \frac{u_u}{c_u}$$

i.e., the ratio between the speed of the shock and the speed of sound in the unperturbed medium. For the sake of simplicity, we just derived the jump conditions for $M \gg 1$.

The temperature of the interstellar medium (ISM) is $T \simeq 10^4$ K and its sound speed is

$$c_s = \left(\frac{\gamma k T}{m} \right)^{1/2} \simeq 12 \text{ km/s}$$

- Stars have velocities 15 km/s; therefore, stellar bow shocks have Mach number $M \simeq 1.2$ (weak shocks).
- Some stars have fast stellar winds (100 – 3000 km/s); the wind meets the ISM in bow shocks with Mach number $M = 8 – 250$ (strong shocks).
- Supernova ejecta have velocities 10^4 km/s; therefore, the ejecta meet the ISM in shocks with Mach number $M = 10^3$ (very strong shocks).

Astrophysical Shocks

Shocks at the edge of SNR Cygnus Loop

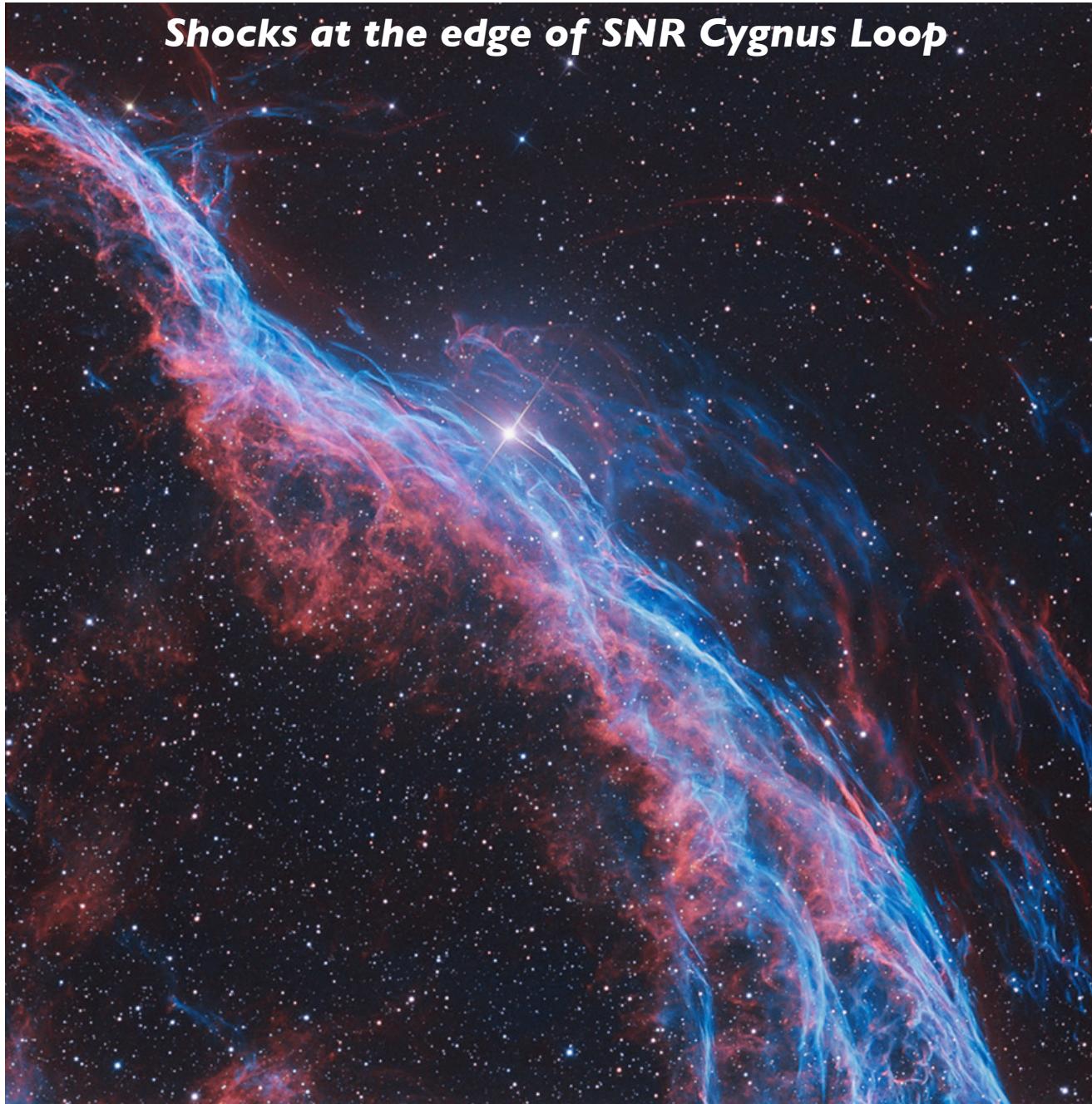


Image credits: <https://apod.nasa.gov/apod/ap130529.html>

What is the shape of the energy distribution of particles?

Power Law Energy Distribution

We will now explore the mechanisms behind the power-law energy distributions of particles in astrophysical environments

$$N(E) \propto E^\alpha$$

with α being a constant. This energy distribution is a non-thermal one because it differs from the Maxwellian distribution.

Energy Distribution

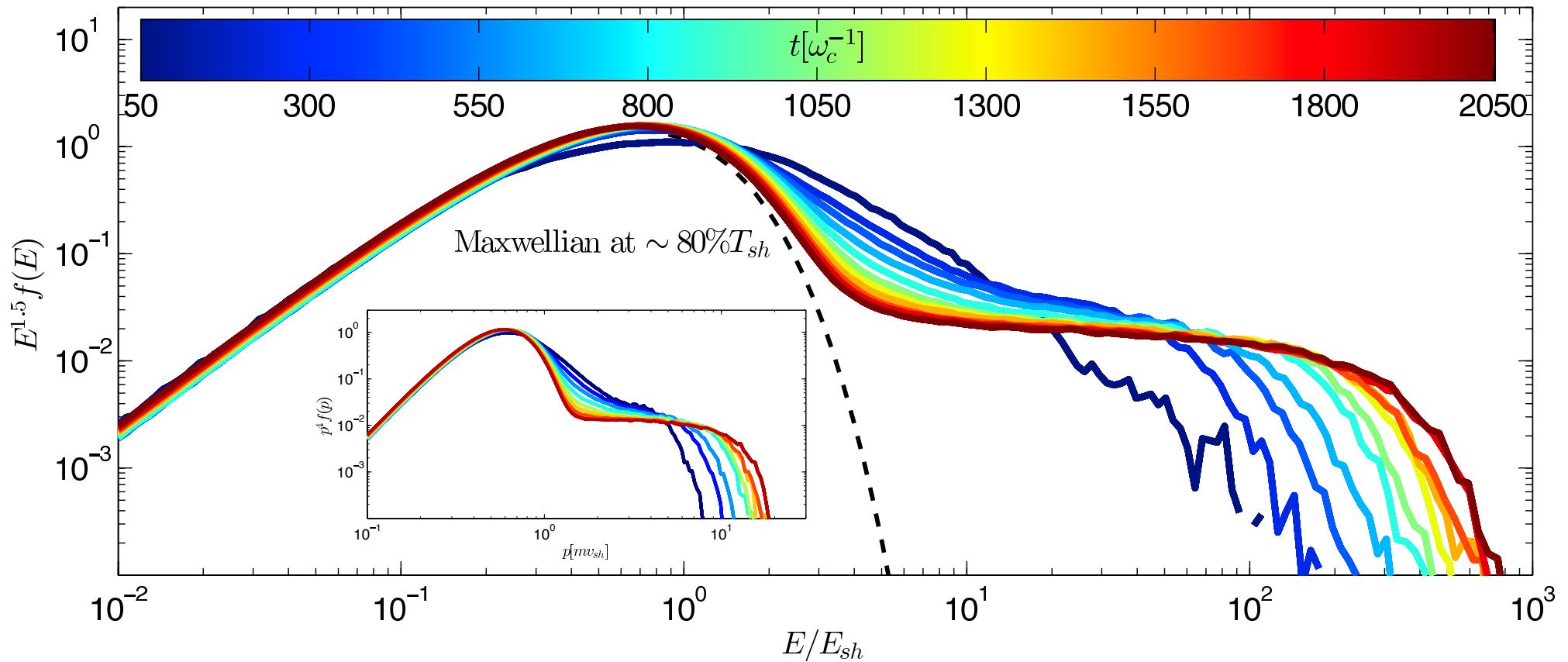
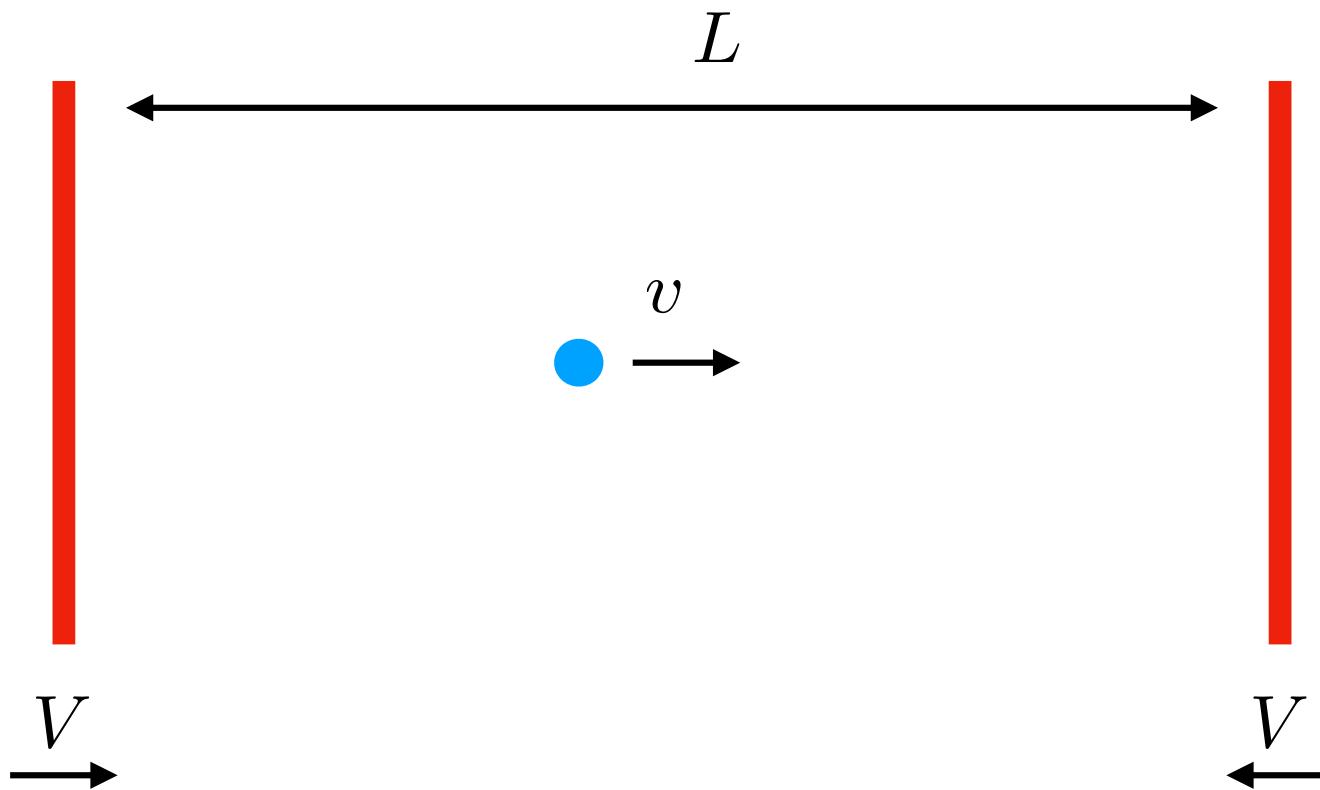


Figure 1. Downstream ion energy spectrum at different times, as in the legend. It is possible to note the thermal distribution, well-fitted by a Maxwellian with temperature about 80% the one expected for Mach number 20 shock that does not accelerate particles (dashed line), and a non-thermal power-law tail extending to larger energies at later times. The spectrum is plotted multiplied by $E^{1.5}$, to emphasize the agreement with the energy scaling predicted by DSA, which reads p^{-4} in momentum (see inset and eq. 3).

How can we obtain a power law?

Intuitive Picture



A particle of mass m is traveling at mildly relativistic speed v between two scattering surfaces at a distance L from each other. The two surfaces are approaching each other with speed $V \ll c$.

Intuitive Picture

If the particle collides alternately head-on with each scatterer, it will gain energy at a rate

$$\frac{dE}{dt} = \text{rate of collisions} \times \text{energy change per collision}$$

If the particle is relativistic, the momentum increases of γmV after each collision. The total energy increase is $E = pc$. Therefore, taking $v \simeq c$, we have

$$\frac{dE}{dt} \simeq \frac{v}{L} \times \gamma m V c \simeq \frac{\gamma m c^2 V}{L} \simeq \frac{EV}{L}$$

This implies that the particle gains energy as

$$\boxed{\frac{dE}{dt} = \frac{E}{\tau}}$$

where τ is the timescale defining the crossing time between scatterers. The particle energy will increase exponentially.

Intuitive Picture

This example tells us that we can look for a mechanism allowing multiple scatterings.

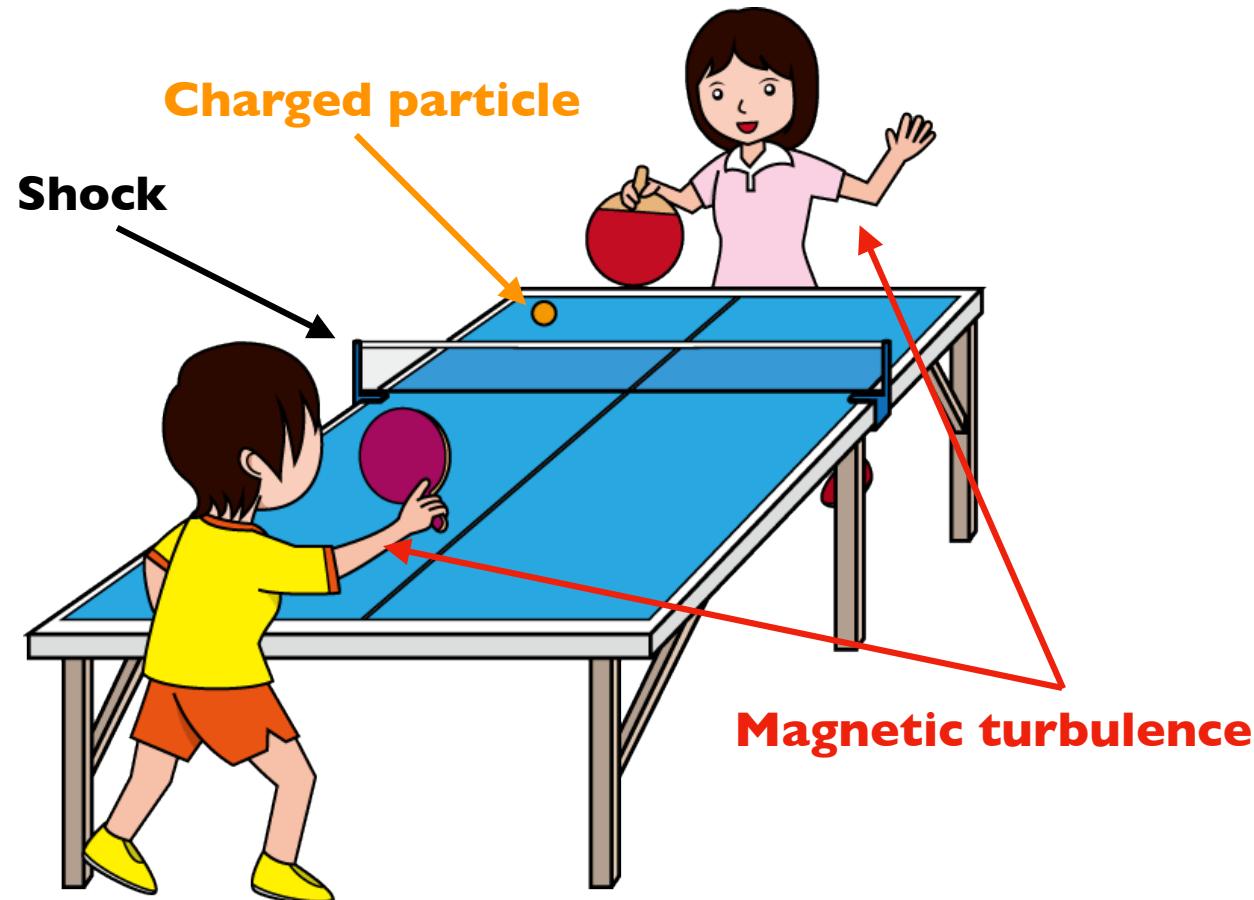
We can use shocks!

Diffusive Shock Acceleration

Diffusive shock acceleration (first order Fermi acceleration) leads to a power-law energy distribution.

Let us assume to have plasma on both sides of the shock and an isotropic distribution of charged particles (obtained via diffusion in the magnetic field).

Charged particles (or cosmic rays) cross the shock back and forth multiple times, each time gaining energy.

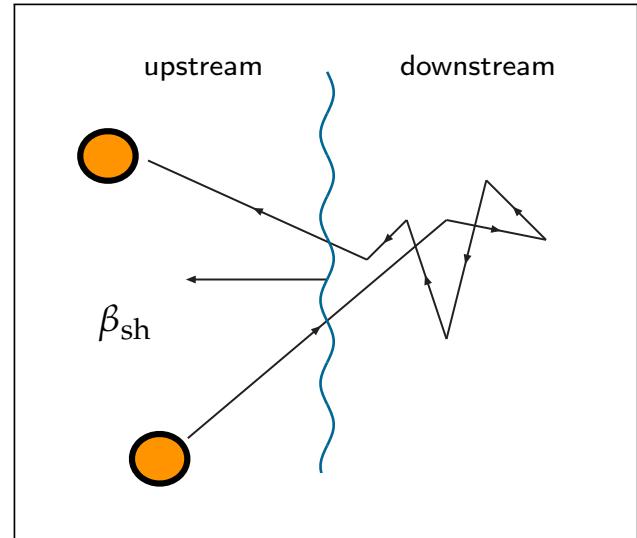


Diffusive Shock Acceleration

A particle crosses the shock from upstream to downstream and back.

The energy in the downstream region in the shock frame ($\beta = V/c$)

$$E'_d = \gamma E_d (1 - \beta \cos \theta_d)$$



The elastic scattering in the downstream region conserves energy ($E'_d = E'_u$). The direction is isotropized.

The emitted energy in the lab frame will then be

$$E_u = \gamma E'_u (1 + \beta \cos \theta'_u)$$

Therefore the energy gain per cycle is

$$\frac{\Delta E}{E_d} = \frac{E_u - E_d}{E_d} = \gamma^2 (1 + \beta \cos \theta'_u) (1 - \beta \cos \theta_d) - 1$$

The energy gain is always positive since $\cos \theta_d < 0$ and $\cos \theta'_u > 0$.

Diffusive Shock Acceleration

The distributions of θ_d and θ'_u (relative to v_{sh}) follow projection onto the shock

$$\frac{dn}{d \cos \theta_d} = -2 \cos \theta_d (\cos \theta_d < 0) \quad \text{and} \quad \frac{dn}{d \cos \theta'_u} = -2 \cos \theta'_u (\cos \theta'_u > 0)$$

Averaging each angle on its own distribution, we obtain

$$\langle \cos \theta'_u \rangle = \frac{2}{3} \quad \text{and} \quad \langle \cos \theta_d \rangle = -\frac{2}{3}$$

Hence the average energy gain is

$$\frac{\langle \Delta E \rangle}{E_d} = \gamma^2 (1 + \beta \langle \cos \theta'_u \rangle) (1 - \beta \langle \cos \theta_d \rangle) - 1 \simeq \frac{4}{3} \beta$$

On average then

$$\frac{\langle \Delta E \rangle}{E_d} \propto \beta$$

Acceleration at a planar shock is first order in shock velocity.

Each “encounter” (i.e., a pair of in and out crossings) results in an energy gain.

Exercise

Let us now proof that the diffusive shock acceleration leads to a power-law in the particle energy distribution.

Hint 1: After one collision, a particle with initial energy E_0 has an average energy $E = \beta E_0$.

Hint 2: Assume an initial ensemble of N_0 particles. Assume that the particles will not continue to cross the shock indefinitely. The net momentum flux of the shocked gas downstream will carry them away after a certain time. Hence, there will be a probability P of remaining in the shock-crossing region after each crossing.

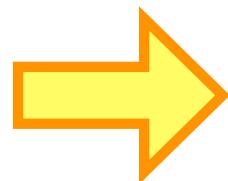
After n crossings, the particle will have energy

$$E = E_0 \beta^n$$

After n crossings, the particle number will be

$$N = N_0 P^n$$

$$\frac{\log(N/N_0)}{\log(E/E_0)} = \frac{\log(P)}{\log(\beta)}$$



$$N(E)dE \propto E^{-1 + \frac{\log P}{\log \beta}}$$

Diffusive Shock Acceleration

From the exercise above, we have recovered a power law energy distribution

$$N(E)dE \propto E^{-k}dE$$

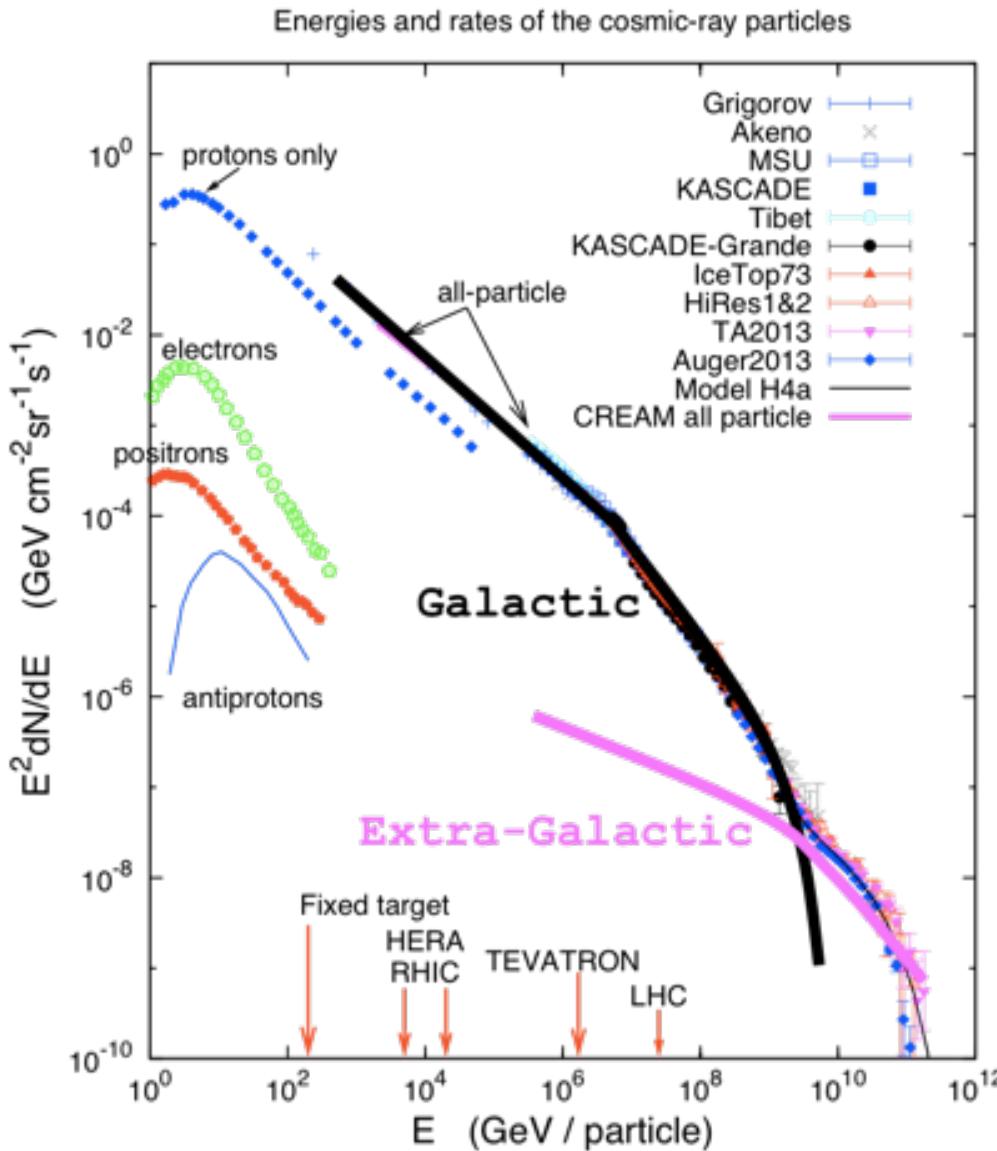
with $k = 1 - (\ln P / \ln \beta)$. It can be proved that

$$\frac{\ln P}{\ln \beta} = -1$$

therefore $N(E)dE \propto E^{-2}dE$.

This is close to the spectral index observed in cosmic rays and other cosmic accelerators such as AGNs. In reality the index is slightly steeper than 2 and this requires a more refined treatment.

Cosmic Ray Energy Spectrum



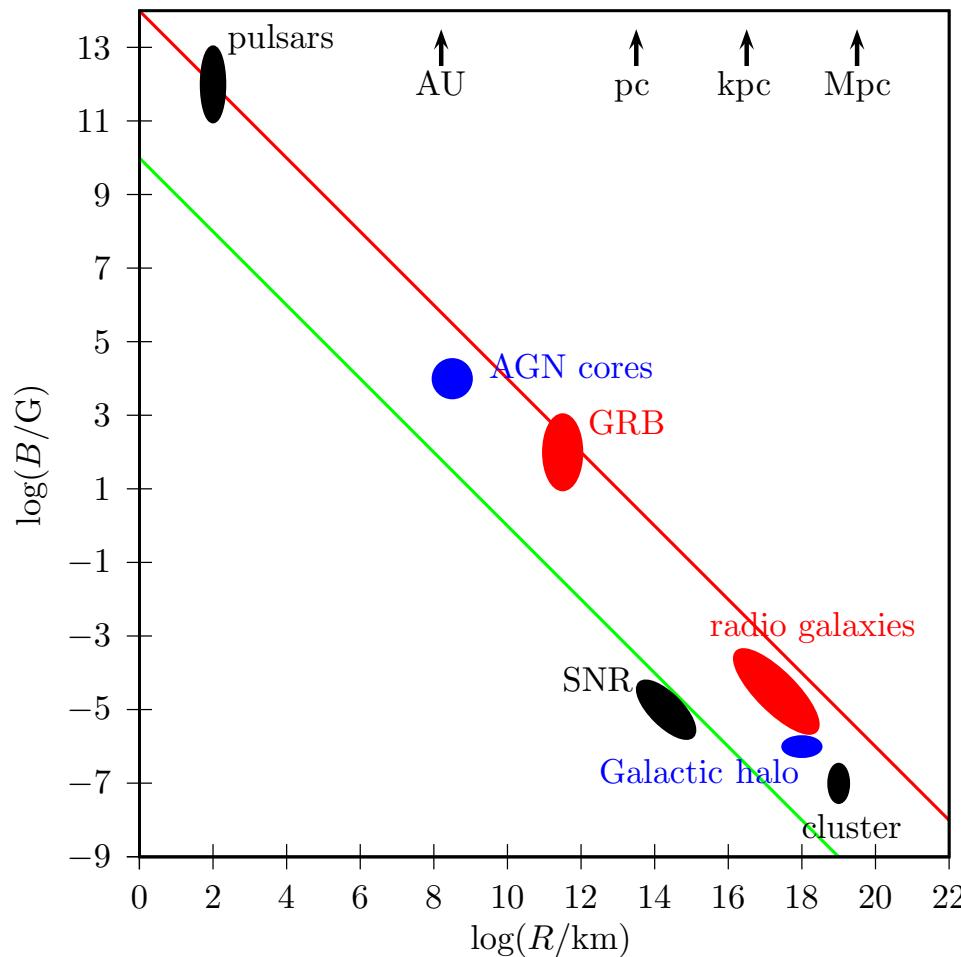
Cosmic rays include particles such as electrons, protons, and heavier nuclei. Their sources are not yet known, but we expect them to be both galactic and extragalactic.

Hillas Criterion

If a particle escapes the acceleration region, it will be unable to gain more energy. This imposes a limit on its maximum energy

$$E_{\max} = qBR$$

We need to demand that the Larmor radius of the particle does not exceed the acceleration region.



Hillas Plot

Sources above the red line accelerate protons up to 10^{21} eV, sources above the green line accelerate iron up to 10^{20} eV.

Take Home Messages

- A shock occurs when a disturbance moves through a medium faster than the medium sound speed.
- The jump conditions describe the variation of the quantities characterizing the shock in its proximity.
- Multiple scatterings of particles across the shock lead to a power law in the particle energy distribution.
- The maximum energy can be estimated through the Hillas criterion.

References

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- Blundell and Blundell, Concepts in thermal physics, Oxford University Press.
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