Optimization problem to Hamiltonian

matrix that represents the energy of a system

$$H_c = \sum_{ij} Q_{ij} Z_i Z_j + \sum_i b_i Z_j$$

Steps to take QAOA problem to Hamiltonian

1. Replace binary variables x_i to new set of variables $z_i \in \{-1, 1\}$ via $x_i = \frac{1 - z_i}{2}$

   - if $x_i = 0$ then $z_i = 1$ Do the math.

   Replace binary to spin variables in max-cut pair indicator equation

2. Now with our new equation (x_i's) for z_i's in optimization problem

optimization problem equation

$$\vec{x}^T Q x = \sum_{ij} Q_{ij} x_i x_j$$

$$= \frac{1}{4} \sum_{ij} Q_{ij} (1 - z_i)(1 - z_j)$$

math

$$= \frac{1}{4} \sum_{ij} Q_{ij} Z_i Z_j - \frac{1}{4} \sum_{ij} (Q_{ij} + Q_{ji}) z_i + \frac{n^2}{4}$$

3. Have $b_i = -\sum_j (Q_{ij} + Q_{ji})$, remove prefactor and constant n^2 term

$$\min_{x \in \{0,1\}^n} x^T Q x \leftrightarrow \min_{x \in \{-1,1\}^n} z^T Q z + b^T z$$

to obtain quantum formulation of the problem, promote z_i variables to Pauli Z matrix such as $Z_i = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

substitute the matrices to obtain following Hamiltonian

$$H_c = \sum_{ij} Q_{ij} Z_i Z_j + \sum_i b_i Z_i$$