MSLT equations, BAU

Inputs are $\mathrm{ACMR}(a, t_0)$ and $APC(a, t)$. Interventions are evaluated by comparing the business-as-usual values of $\mathrm{PY}_{adj}(a, t)$ and $\mathrm{LE}_{adj}(a, t)$ to their intervention-specific values.

$$
\begin{aligned}
\mathrm{ACMR}(a, t+1) &= \mathrm{ACMR}(a, t) \times \left[1 + \frac{\mathrm{APC}(a, t)}{100}\right] \\
\mathrm{ACMR}(a, t_0+n) &= \mathrm{ACMR}(a, t_0) \times \prod_{k=0}^{n-1} \left[1 + \frac{\mathrm{APC}(a, t_0+k)}{100}\right] \\
\mathrm{PD}(a, t) &= P(t < T_{death,a} < t+1 | T_{death,a} > t) = 1 - e^{-\mathrm{ACMR}(a,t)} \\
\mathrm{PD}_{cum}(a, t_0, n) &= \prod_{k=0}^{n} \left[1 - e^{-\mathrm{ACMR}(a+k, t_0+k)}\right] \\
\mathrm{Deaths}(a, t_0) &= \mathrm{Pop}(a, t_0) \times \mathrm{PD}(a, t_0) \\
\mathrm{Deaths}_{cum}(a+n, t_0+n) &= \mathrm{Pop}(a, t_0) \times \mathrm{PD}_{cum}(a, t_0, n) \\
\mathrm{Pop}(a+1, t_0+1) &= \mathrm{Pop}(a, t_0) - \mathrm{Deaths}(a, t_0) \\
&= \mathrm{Pop}(a, t_0) \times (1 - \mathrm{PD}(a, t_0)) \\
\mathrm{Pop}(a+n, t_0+n) &= \mathrm{Pop}(a, t_0) \prod_{k=0}^{n-1} \left[1 - \mathrm{PD}(a+k, t_0+k)\right] \\
\mathrm{PY}(a, t_0) &= \mathrm{Pop}(a, t_0) \times \left(1 - \frac{\mathrm{PD}(a, t_0)}{2}\right) \\
\mathrm{PY}(a+n, t_0+n) &= \mathrm{Pop}(a, t_0) \prod_{k=0}^{n-1} \left[1 - \mathrm{PD}(a+k, t_0+k)\right] \times \left(1 - \frac{\mathrm{PD}(a+k+1, t_0+k+1}{2}\right) \\
\mathrm{LE}(a, t) &= \sum_{k=0}^{a_{\max}-a} \frac{\mathrm{PY}(a+k, t+k)}{\mathrm{Pop}(a+k, t+k)} \\
\mathrm{YLDrate} &: a \mapsto \mathrm{YLDrate} \\
\mathrm{PY}_{adj}(a, t) &= \mathrm{PY}(a, t) \times [1 - \mathrm{YLDrate}(a)] \\
\mathrm{LE}_{adj}(a, t) &= \sum_{k=0}^{a_{\max}-a} \frac{\mathrm{PY}_{adj}(a+k, t+k)}{\mathrm{Pop}(a+k, t+k)}
\end{aligned}
$$

Lung cancer equations

$$
S_i(t+1) = \frac{2 \cdot (v_i - w_i) \cdot [S_i(t) \cdot (r_i + f_i + m_i) + C_i(t) \cdot r_i] + S_i(t) \cdot (v_i \cdot (q_i - l_i) + w_i \cdot (q_i + l_i)}{2 \cdot q_i}
$$

Symbol	Definition
ACMR	All-cause mortality rate
APC	Annual percent change in ACMR
PD	Probability of death in a cohort over a single year
Pop	Number of individuals in a cohort
PY	Person-years in a cohort over a single year
LE	Life expectancy, relative to current age
YLDrate	Year-life disability discount rate
PY_{adj}	Person-years, adjusted for YLD
LE_{adj}	Life expectancy, relative to current age and adjusted for YLD

Table 1: Definition of symbols used in equations.

Incorporating effects of an NCD into the life table

Interventions affect the ACMR and the life-year disability rate indirectly, because they directly affect disease prevalence.

The equations for disease incidence and mortality come from Barendregt et al., as circulated in my previous email.

This allows you to calculate the number of people alive after reaching age a, and the person-years (mean of numbers alive at a and at a+1).

The prevalence rate is then calculated.

The mortality risk is the number of people who died during the year (of being aged a) divided by the number of people alive at the start of that year.

The intervention scenario is identical to the business-as-usual scenario, except that incidence rates are different. Remission and case fatality rates remain the same.

To evaluate the effect of the intervention, calculate:

- The difference in prevalence rate;
- The change in mortality rate (not mortality risk).
  - Note to self: mortality rate is - log(1 - mortality risk).
- The change in PYLD rate: change in prevalence multiplied by the original PYLD rate for that disease.
- The change in mortality rate and PYLD rate are fed back into the life table.

There are also acute events, such as injuries or lower respiratory tract infections, which are handled differently. It is possible, if not sensible, to implement them using the same approach and selecting a very large value for the remission rate.

Intervention effect on a disease

An intervention's effect is defined by the "Intervention (1 - PIF)", which is an input into the disease worksheet. It has a value for every year of each cohort's life, and is generally close to unity.

The next column is the "Cumulative PIF" (CPIF):

$$\mathrm{CPIF}(t) = \mathrm{CPIF}(t-1) + (1 - \mathrm{InterventionPIF}(t))$$

The "Final (1 - PIF)" $(1 - \mathrm{PIF}_{\mathrm{Final}})$ is:

$$\begin{aligned} 1 - \mathrm{PIF}_{\mathrm{Final}} &= \frac{1 - (\mathrm{CPIF}(t_{\mathrm{upr}} - \mathrm{CPIF}(t_{\mathrm{lwr}})}{t_{\mathrm{upr}} - t_{\mathrm{lwr}}} \\ t_{\mathrm{upr}} &= t_0 + X \quad \text{(e.g., 10 years before any effect)} \\ t_{\mathrm{lwr}} &= t_{\mathrm{upr}} - \min(t_0, t_{\mathrm{upr}} - Y) \quad \text{(e.g., maximal effect takes 20 years)} \end{aligned}$$

We're effectively applying a mean PIF over N years. It's not a serious problem, there is no knowledge as to what shape this curve should take. The "Final (1 - PIF)" is then multiplied by BAU incidence to give the new incidence under the intervention.

Intervention on a risk factor

The risk factor may then generate a (1 - PIF) across any number of diseases.

The "Intervention (1 - PIF)" is average RR **after** the intervention, divided by the average RR **before** the intervention.

The average RR is the dot product of (a) the proportion of the population in each categorical bin (for each age × gender cohort); and (b) the relative risk associated with each bin.

The inputs to the intervention are:

- Some description of the business-as-usual population in relation to a risk factor. For example, physical activity is divided into 7 different bins.
  - The proportion of people in each categorical bin.
  - The average value of the risk for the people in each categorical bin; this is typically available as/estimated from population data.
- An intervention can, e.g., increase physical activity across all bins (or only some bins, e.g., lowest level of activity). It may instead move people between bins (e.g., quitting smoking).
  - So a risk may change, or quantities of people in a bin may change. Or perhaps even both may occur?

– At baseline, work out the relative risk of being in that bin. Some bin will be a chosen reference and have a relative risk of 1. This decision is typically determined by the source of RR data.

If you plot relative risk (y-axis) vs risk factor (x-axis), the response could conceivably have *any shape*. So we will have *some function* that characterises this relationship, based on population data. You then need to work out the *average value of the relative risk* across each of the categorical bins, for each of your demographic population bins. For our purposes, what matters is that this function can be evaluated over a reasonably dense set of points, in order to approximate the average RR for each categorical bin.

This relationship is typically categorical (i.e., already binned) or exponential, but in Anja's model it may also take the form $RR \cdot \text{exposure}^{0.25}$.

You then evaluate this functional form for your intervention, which may result in lower RRs in some bins (e.g., increased physical activity without moving people to a different bin) or may not change the RRs at all (if the effect of the intervention is simply to move people between bins).

Then we can calculate the average RR under the intervention (reminder: the dot product of (a) the proportion of the population in each categorical bin (for each age × gender cohort); and (b) the relative risk associated with each bin).

Diabetes

An intervention may have some effect on diabetes prevalence (i.e., there is still an intervention PIF). But diabetes is *also* a risk factor for CHD and stroke. Aside: you have to reduce the CFR for diabetes because some will die of CHD and stroke instead.

So for CHD and stroke, the same stuff happens as for other intervention effects on disease (see above). But you have an additional reduction because the intervention may also reduce diabetes prevalence, which indirectly reduces CHD and stroke prevalence.

People with diabetes have a higher risk of contracting another disease (in this case, CHD and/or stroke), relative to people without diabetes. We then compare how the overall risk of contracting this disease has changed as a result of the intervention, based on baseline prevalence (Prev) and prevalence after the intervention (Prev′):

$$\frac{\text{Prev}'(\text{diabetes}) \cdot RR_{\text{db}\mapsto\text{X}} + (1 - \text{Prev}'(\text{diabetes}))}{\text{Prev}(\text{diabetes}) \cdot RR_{\text{db}\mapsto\text{X}} + (1 - \text{Prev}(\text{diabetes}))}$$

The final or effective (1 - PIF) is the product of the (1 - PIF) *without diabetes*, and this *diabetes-specific* factor.