1 Wheeler Property Defined

A graph with the Wheeler property is said to be a **Wheeler Graph**

Wheeler Graph Definition

A **Wheeler Graph** is an edge-labeled directed[2] graph if nodes can be ordered such that:

(1)

0 in-degree nodes[1] come before others

> The first nodes in the graph must have zero for their label
>
> *Include animation example?*

For all pairs of edges e = (u,v), e' = (u',v') labeled a, a':

(2)

a $\prec^3$ a' $\Longrightarrow^4$ $v < v'$

> An edge labeled **a** being alphabetically less than an edge labeled **a'** implies the lesser edge's destination should proceed the greater edge's destination.
>
> *Include animation example?*

(3)

(a = a') $\wedge^5$ $(u < u') \Longrightarrow v \leq v'$

> An edge labeled a being alphabetically less than an edge labeled a' implies the lesser edge's destination should proceed the greater edge's destination.
>
> *Include animation example?*

Notes:

1) An in-degree node is a node with the arrow pointing towards another node. This may also be called a source node. A 0 in-degree node is simply a source node labeled with a zero
2) A directed graph is a graph that includes the directions of edges (i.e. node 1 has an arrow pointing towards node 2, as opposed to node 1 and node 2 being connected by an edge without an arrow representing a direction)
3) This symbol represents that one character is lexicographically smaller.
4)This symbol represents the mathematical notion of 'implies'. See here for more information: https://mathworld.wolfram.com/Implies.html#:~:text=%22Implies%22%20is%20the%20connective%20in,13).

5) This symbol is a mathematical representation for AND.

[Langmead(2021)]

2 Wheeler Graph Motivation

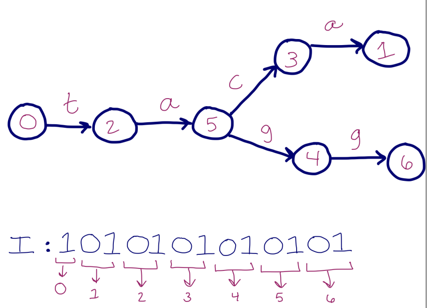
Wheeler graphs are desirable because they have the **path coherence** property, meaning that for any string, we can order nodes consecutively from the initial state to process that string. This property allows for the graph to be stored compactly. //
Given a graph G = (V,E), where V = set of vertices and E = set of edges, and σ is the size of the alphabet. $|V|$ = n, $|E|$ = e. Then, the graph may be stored in $2(e+n) + e \log \sigma + \sigma \log e + o(n + e \log \sigma)$ bits.
[Gibney and Thankachan(2022)]

Storage

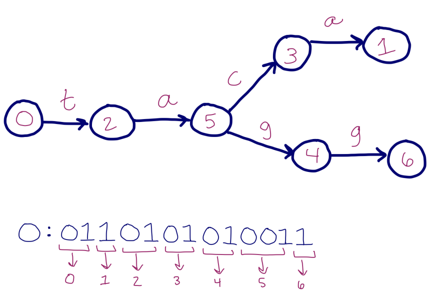
Given the unique properties of Wheeler graphs, most notably path coherence, we can store the graph using bitvectors.

Bitvectors One way to store a Wheeler Graph is by using unary encodings to storing the in- and out-degree nodes in bitvectors. We may also wish to store the edge labels.
To encode a given number, n, in **Unary**, we simple write n 0s, followed by a singular 1.
For example,
0 is encoded with 1
7 is encoded with 00000001
See here for more information on unary encoding:
https://en.wikipedia.org/wiki/Unary_coding

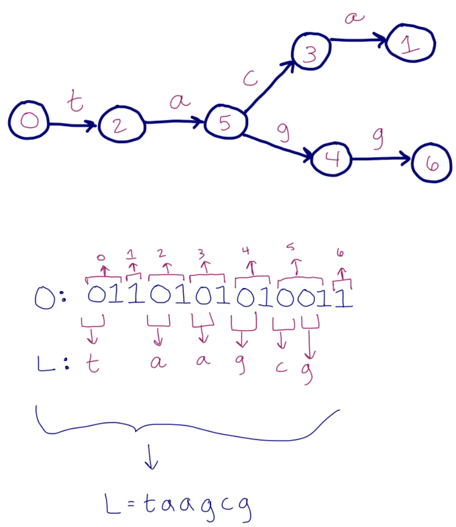
In-degree vector From left to right, we create a unary encoding for the number of in-degree nodes for the 0th to nth node in a Wheeler Graph with n nodes.



Out-degree vector From left to right, we create a unary encoding for the number of out-degree nodes for the 0th to nth node in a Wheeler Graph with n nodes.



Edge vector Observe that the number of 0s in the in or out-degree vector is the same as the number of edges. Observe that the number of 1s in the in- or out- degree vector is the same as the number of nodes. We map the 0s to edges from the out-degree vector. We can see that this makes sense, because each edge can only be outgoing from one node. It is important to be consistent with the ordering of edges that coordinate with the same node.



Pattern Matching Inquiry

TODO

References

[Gibney and Thankachan(2022)] Daniel Gibney and Sharma V. Thankachan. On the complexity of recognizing wheeler graphs, 2022. URL https://link.springer.com/article/10.1007/s00453-021-00917-5.

[Langmead(2021)] Benjamin Langmead. Wheeler graphs, part 3: Definition, Oct 2021. URL https://www.youtube.com/watch?v=C17g1j2SiI8.