Lecture 09: Phylogenetic Analyses (II)

CSCI-478/CSCI-578/BIOL-510 Bioinformatics, Fall 2021



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The UPGMA method — an example



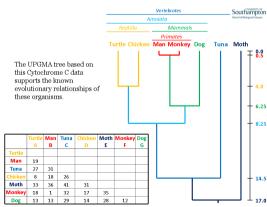
The UPGMA method

- A critical assumption of the UPGMA approach is that the rate of nucleotide or amino acid substitution is constant for all the branches in the tree, that is, the molecular clock applies to all evolutionary lineages.
- If this assumption is true, branch lengths can be used to estimate the dates of divergence, and the sequence-based tree mimics a species tree.
 - Edge lengths correspond to times measured by a clock.
 - Divergences of (sequences) assumed to occur at constant rate at all points in the tree.
 - Sum of times down a path to the leaves from any node is the same, whatever choice of path





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The molecular clock hypothesis

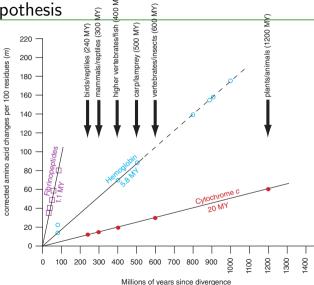
- In the 1960s, sequence data were accumulated for small, abundant proteins such as globins, cytochromes c, and fibrinopeptides.
 - □ Some proteins appeared to evolve slowly, while others evolved rapidly.
- Linus Pauling, et al. proposed the hypothesis of a molecular clock:

The molecular clock hypothesis

For every given protein, the rate of molecular evolution is approximately constant in all evolutionary lineages.



The molecular clock hypothesis





The molecular clock hypothesis and ultrametric data

If the molecular clock assumption holds, then the data is said to be ultrametric:

The ultrametric tree

The strong triangle inequality:

$$d(x,z) \le \max(d(x,y),d(y,z)) .$$

which implies that for any triplet of sequences i, j, k, the distances are either

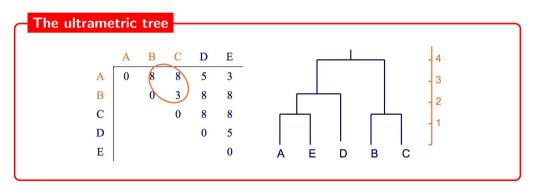
- all equal, or
- two are equal and one is smaller.

That is, (1) branch length represent time, and (2) the length from the root to the leaves are the same.



The molecular clock hypothesis and ultrametric data

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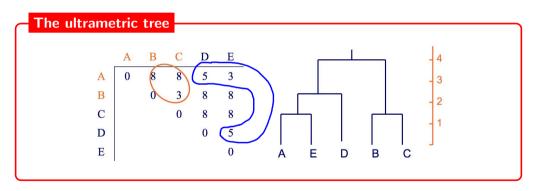


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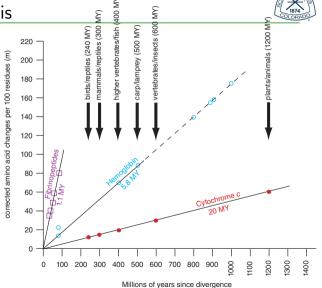
Yet, the molecular clock assumption does **not** always hold ...

The molecular clock hypothesis

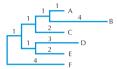
Yet, the molecular clock assumption does **not** always hold ...

Selection pressures, thereby evolution speed, vary across

- time periods,
- organisms,
- genes within an organism,
- □ regions within a gene
- . . .







Distance matrix

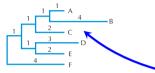
```
B 5 C D E

B 7 7 10 7

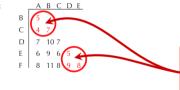
E 6 9 6 5

F 8 11 8 9 8
```





Distance matrix

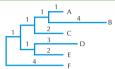


The data is **NOT** ultrametric, because

$$7 > \max(5,4) = 5$$
,

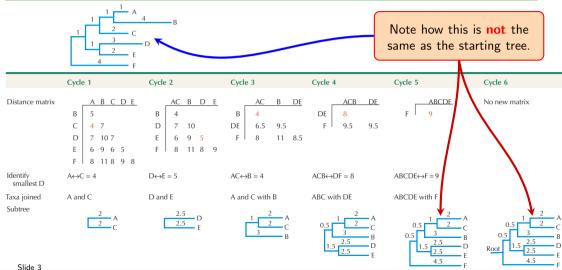
$$9 > \max(8,5) = 8$$
.



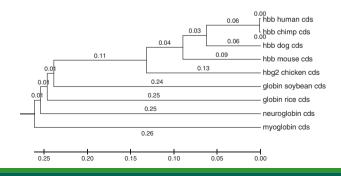


	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Cycle 6
Distance matrix	B 5 C D E C 4 7 D 7 10 7 E 6 9 6 5 F 8 11 8 9 8	AC B D E B 4 D 7 10 E 6 9 5 F 8 11 8 9	AC B DE B 4 DE 6.5 9.5 F 8 11 8.5	ACB DE B B B B B B B B B B B B B	ABCDE F 9	No new matrix
Identify smallest D	$A \leftrightarrow C = 4$	$D \leftrightarrow E = 5$	$AC \leftrightarrow B = 4$	$ACB \leftrightarrow DF = 8$	$ABCDE \leftrightarrow F = 9$	
Taxa joined	A and C	D and E	A and C with B	ABC with DE	ABCDE with F	
Subtree	2 A C	2.5 2.5 E	1 2 A C B	0.5 1 2 A C C 3 B D 2.5 D E	0.5 1 2 A 0.5 3 B 1.5 2.5 D 1.5 E	0.5 3 B Root 1.5 2.5 D 4.5 E
Slide 3					4.5 F	4.5 F

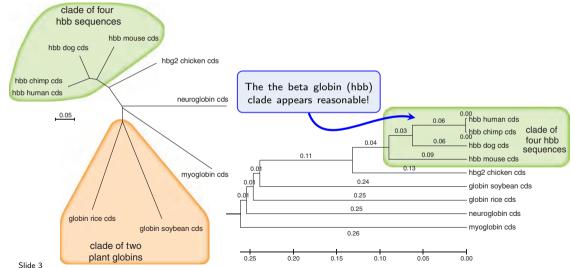




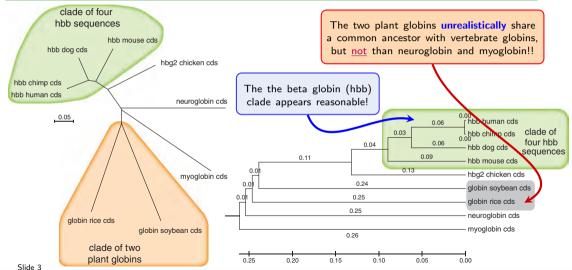
















The UPGMA algorithm

- assumes the constant molecular clock and ultrametric distances, and
- implicitly assumes additivity.

Additivity



The UPGMA algorithm

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Additivity



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- implicitly assumes additivity: the distance between any pair of leaves is the sum of the lengths of the edges on the path connecting them.

Can we design an algorithm that only assumes the additivity?



Additivity and the neighbour-joining algorithm

The UPGMA algorithm

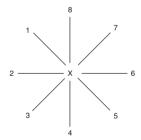
- assumes the constant molecular clock and ultrametric distances, and
- implicitly assumes additivity: the distance between any pair of leaves is the sum of the lengths of the edges on the path connecting them.

Can we design an algorithm that only assumes the additivity?

- Yes! We have the neighbour-joining algorithm.
- Like UPGMA, it constructs a tree by iteratively joining subtrees, but it is different in that:
 - □ How pair of subtrees to be merged is selected on each iteration
 - How distances are updated after each merge



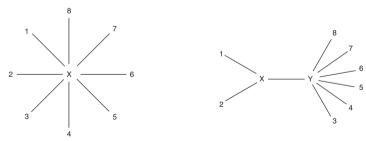
- **Initialization**: The sequences are first clustered in a starlike tree.
 - Neighbors are defined as OTUs that are connected by a single, interior node in an unrooted, multifurcating tree.
- **Repeat**: The two **closest** sequences are identified, such as sequences 1 and 2. These neighbors are connected to the other sequences via the internal branch XY.



But, how to identify the two closest sequences?



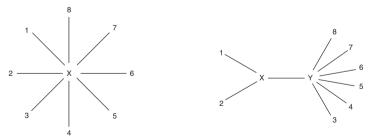
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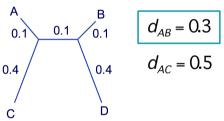
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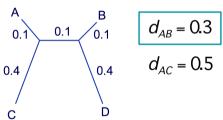


- At each step, pick a pair of nodes to join; should we pick a pair with minimal d_{ij} ?
- Suppose that the real tree looks like this and we're picking the first pair of nodes to join. Shall we pick up A and B? or shall we pick up A and C?





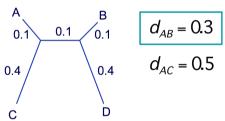
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- Wrong decision to join A and B!!
 - \Box A and C are neighbors, but not A and B, because they are connected via two internal nodes.



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- Wrong decision to join A and B!!
 - \Box A and C are neighbors, but not A and B, because they are connected via two internal nodes.
- We need to consider distance of pair to other leaves!



To avoid this, given N sequences, we pick a pair to join based on D_{ij} (Saitou & Nei '87, Studier & Kepper '88):

$$D_{ij} = d_{ij} - (S_i + S_j)$$
 , where $S_i = \frac{1}{N-2} \sum_{k \neq i} d_{ik}$.

Selecting nodes i and j by smallest D_{ij} , we join them via a **new** internal node k.



Updating distances in the neighbour-joining algorithm

Given a new internal node k, the distance to another node m is given by:

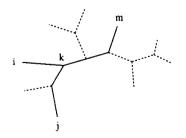
$$d_{km}=rac{1}{2}\left(d_{im}+d_{jm}-d_{ij}
ight) \ .$$



Updating distances in the neighbour-joining algorithm

Given a new internal node k, the distance to another node m is given by:

$$d_{km} = \frac{1}{2} (d_{im} + d_{jm} - d_{ij})$$
.



Using the additivity, we have:

$$\begin{cases} d_{im} = d_{ik} + d_{km} , \\ d_{jm} = d_{jk} + d_{km} , \\ d_{ij} = d_{ik} + d_{jk} , \end{cases}$$

by which we can derive that:

$$d_{km}=\frac{1}{2}\left(d_{im}+d_{jm}-d_{ij}\right) .$$



The neighbour-joining algorithm

- Define the tree T = set of leaf nodes.
- Set *L* = *T*
- While more than two subtrees in T:
 - \square Pick the pair *i* and *j* in *L* with minimal D_{ij} .
 - \square Add to T a new node k that joins i and j
 - Determine new distances:

$$egin{cases} d_{ik} &= rac{1}{2} \left(d_{ij} + S_i - S_j
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 \square Remove *i* and *j* from *L* and insert *k* (treat it like a leaf)



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Slide 8



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The

Distance matrix

C D F

Cycle 1

C D E

Cycle 2

Cycle 3

$$\begin{array}{c|cccc}
U_2 & U_3 \\
U_3 & 2 \\
F & 6 & 6
\end{array}$$

Cycle 4



Step 1

3 Calculations
$S_x = (\text{sum all } D_x)/(N_x)$
where N is the # o
OTUs in the set.

$$S_A = (5+4+7+6+8)/4 = 7.5$$

 $S_B = (5+7+10+9+11)/4 = 10.5$
 $-2)$, $S_C = (4+7+7+6+8)/4 = 8$
if $S_D = (7+10+7+5+9)/4 = 9.5$
 $S_E = (6+9+6+5+8)/4 = 8.5$
 $S_C = (8+11+8+9+8)/4 = 11$

$$S_{U1} = (3+6+5+7)/3 = 7$$

 $S_C = (3+7+6=8)/3 = 8$
 $S_D = (6+7+5+9)/3 = 9$
 $S_E = (5+6+5+8)/3 = 8$
 $S_C = (7+8+9+8)/3 = 10.6$

$$S_{U1} = (3+3+7)/2 = 6.5$$

 $S_C = (3+4+8)/2 = 7.5$
 $S_{U2} = (3+4+6)/2 = 6.5$
 $S_F = (7+8+6)/2 = 10.5$

$$S_{U_2} = (2+6)/1 = 8$$

 $S_{U_3} = (2+6)/1 = 8$
 $S_F = (6+6)/1 = 12$

Because
$$N-2=0$$
, we cannot do this calculation.

Step 2

Calculate pair with
smallest (
$$M$$
), where
 $M_{ii} = D_{ii} - S_i - S_i$.

Smallest are
$$M_{AB} = 5 - 7.5 - 10.5 = -13$$
 $M_{DE} = 5 - 9.5 - 8.5 = -13$
Choose one of these (AB here).

Smallest is
$$M_{CU_1} = 3 - 7 - 8 = -12$$

 $M_{DE} = 5 - 9 - 8 = -12$
e). Choose one of these (DE here).

Smallest is
$$M_{CL} = 3 - 6.5 - 7.5 = -11$$

$$M_{U2F} = 6 - 8 - 12 = -14$$

 $M_{U3F} = 6 - 8 - 12 = -14$
 $M_{U2U3} = 2 - 8 - 8 = -14$
Choose one of these (M_{U3U} here).

Step 3

Create a node (U) that
joins pair with lowest
$$M_{ij}$$
 such that
 $S_{0.1} = D_{ii}/2 + (S_i - S_i)/2$.

$$U_1$$
 joins A and B:
 $S_{AU_1} = D_{AB}/2 + (S_A - S_B)/2 = 1$
 $S_{BU_1} = D_{AB}/2 + (S_B - S_A)/2 = 4$

$$U_2$$
 joins D and E:
 $S_{DU2} = D_{DE}/2 + (S_D - S_E)2 = 3$
 $S_{EU2} = D_{DE}/2 + (S_E - S_D)/2 = 2$

$$\begin{array}{lll} U_2 \text{ joins D and E:} & U_3 \text{ joins C and } U_1: & U_4 \text{ joins } U_2 \text{ and } U_3: & \text{For last pair, c S} \\ S_{012} = D_{01} (2^2 + (S_1 - S_1)) 2 & 3 & S_{C12} = D_{C1} (2^2 + (S_1 - S_1)) 2 & 2 & S_{C24} & D_{12} \cup 1/2 + (S_{12} - S_{12}) 2 & 1 \\ S_{12} \cup 2 D_{01} (2^2 + (S_1 - S_2)) 2 & 2 & S_{11} \cup 3 & D_{12} (2^2 + (S_1 - S_2)) 2 & 1 \\ \end{array} \\ \begin{array}{lll} & V_4 \text{ in } F_1 & V_2 & V_3 & V_4 & V_2 & V_3 & V_4 & V_2 & V_3 \\ S_{11} & S_{11} \\ S_{11} & S_{11} \\ S_{11} & S_{11} \\ S_{11} & S_{11} \\ S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} \\ S_{11} & S_{11} & S_{11} &$$

$$U_2$$
 joins D and E: U_3 joins C and U_4 : U_4 joins U_2 and U_3 : For last pair, connect $S_{DU_2} = D_{DU}/2 + (S_D - S_D)/2 = 3$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 2$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$ $S_{CU_3} = D_{CU_3}/2 + (S_C - S_U)/2 = 1$

For last pair, connect

$$2 = 1$$
 U₄ and F with branch
 $2 = 1$. length = 5.

Step 4

Join i and i according to S above and make all other taxa in form of a star. Branches in black are of unknown length. Branches in red are of known length.

Step 5

Calculate new distance matrix of all other taxa to U with $D_{sti} = D_{ix} + D_{ix} - D_{ii},$ where i and i are those Slide Aselected from above.











Comments

Note this is the same tree we started with (drawn in unrooted form here).

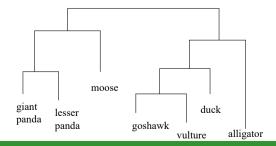


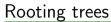
Rooting trees

Slide 9

Finding a root in an unrooted tree is sometimes accomplished by using an **outgroup**: a species known to be **more distantly** related to remaining species than they are to each other.

■ In other words, finding a species that **diverged earlier** than the rest species in the tree.

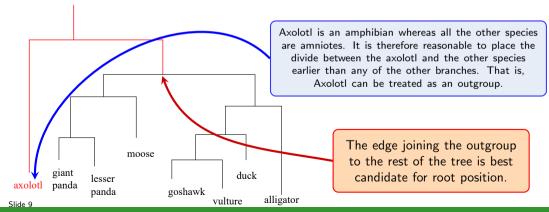




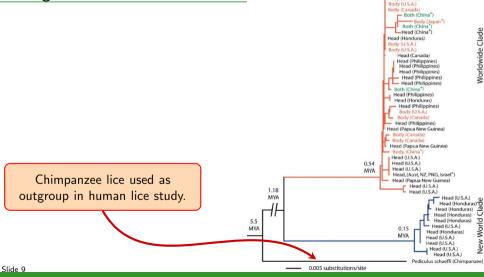


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Rooting trees





Head (Philippines)

Head, (Israel*)

Body (Canada)

Body (Canada)

Body, (China*)

Head (Honduras) Body (Canada)

Both, (Aus, Chn, Hng, Isr, Jap, Ken, PNG, USA*) Head (Philippines) Body (Yemen)



Distance-based methods — a summary

- If the given distance data is ultrametric (and these distances represent real distances), then UPGMA will identify the correct tree.
- If the data is additive only (and these distances represent real distances), then neighbor joining will identify the correct tree.
- Otherwise, the methods may not recover the correct tree, but they may still be reasonable heuristics.
- Neighbor joining is commonly used!!



Approaches to phylogenetic trees

Three general types of methods:

- **Distance**: find tree that accounts for estimated evolutionary distances
- Parsimony: find tree that requires minimum number of changes to explain the data
- Maximum likelihood: find tree that maximizes the likelihood of the data



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Building the phylogentic trees using parsimony — the motivations

The motivations

- The main idea behind maximum parsimony is that the best tree is that with the shortest branch lengths possible (Czelusniak et al., 1990).
- According to maximum parsimony theory, having fewer changes to account for the way a group of sequences evolved is preferable to more complicated explanations of molecular evolution.



Building the phylogentic trees using parsimony — the task

Task definition

Given:

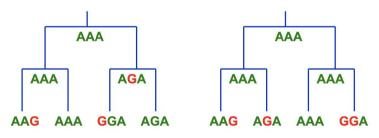
Character-based data

Do:

- □ Find the tree that explains the data with the minimal number of changes
- □ Focus on finding the right topology, not on estimating branch lengths

Building the phylogentic trees using parsimony — the explanations

There are various trees that could explain the phylogeny of sequences AAG, AAA, GGA, AGA including these two:



Parsimony prefers the left tree because it requires fewer substitution events: 3 substitutions *vs.* 4 substitutions.



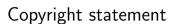
Parsimony-Based Approaches

Usually these approaches involve two separate components:

- A procedure to find the minimum number of changes needed to explain the data (for a given tree topology)
- A search through the space of trees

Basic assumptions

- Any state (e.g. nucleotide, amino acid, etc.) can convert to any other state
- The "costs" of these changes are uniform
- Positions are independent; we can compute the min number of changes for each position separately





This slide set is designed only for teaching CSCI-478/CSCI-578/BIOL-510 Bioinformatics at the Department of Computer Science of Colorado School of Mines in Fall 2021.

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