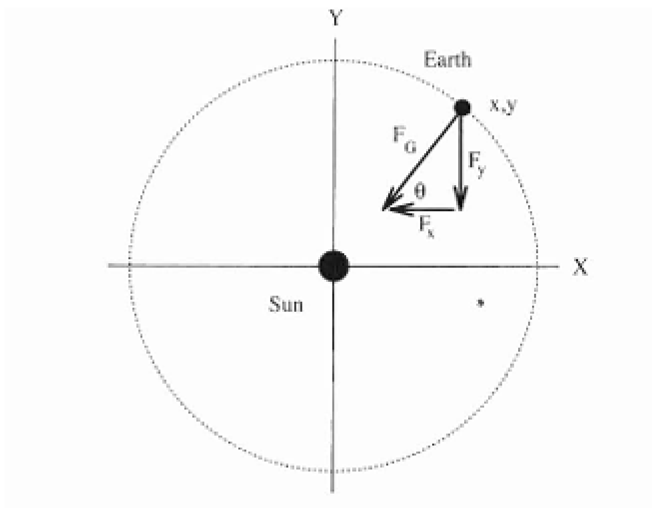
- This chapter primarily seeks to explore Kepler's laws and what use they have when analyzing astrological processes. Additionally, it explores astrological phenomena and quantifies various aspects of the process.
- Understanding the Forces and how they affect the position in different circumstances is crucial in this chapter.

Chapter 4.1

$$F_G = \frac{G\, M_S\, M_E}{r^2},$$

Force of the gravitational attraction between earth and sun.



Force diagram for this force.

$$\begin{aligned}
v_{x,i+1} &= v_{x,i} - \frac{4\,\pi^2\, x_i}{r_i^3}\,\Delta t \\
x_{i+1} &= x_i + v_{x,i+1}\,\Delta t \\
v_{y,i+1} &= v_{y,i} - \frac{4\,\pi^2\, y_i}{r_i^3}\,\Delta t \\
y_{i+1} &= y_i + v_{y,i+1}\,\Delta t\,,
\end{aligned}$$

Converting the equations of motion into difference equations

planet	mass (kg)	radius (AU)	eccentricity
Mercury	2.4×10^{23}	0.39	0.206
Venus	4.9×10^{24}	0.72	0.007
Earth	6.0×10^{24}	1.00	0.017
Mars	6.6×10^{23}	1.52	0.093
Jupiter	1.9×10^{27}	5.20	0.048
Saturn	5.7×10^{26}	9.54	0.056
Uranus	8.8×10^{25}	19.19	0.046
Neptune	1.03×10^{26}	30.06	0.010
Pluto	$\sim 6.0 \times 10^{24}$	39.53	0.248

Useful data table of values for planets(minus Pluto of course) in our solar system.

planet	T^2/a^3 ($\mathrm{yr}^2/\mathrm{AU}^3$)
Venus	0.997
Earth	0.998
Mars	1.005
Jupiter	1.010
Saturn	0.988

Result proving Kepler's third law. Contains the orbital periods of several planets.

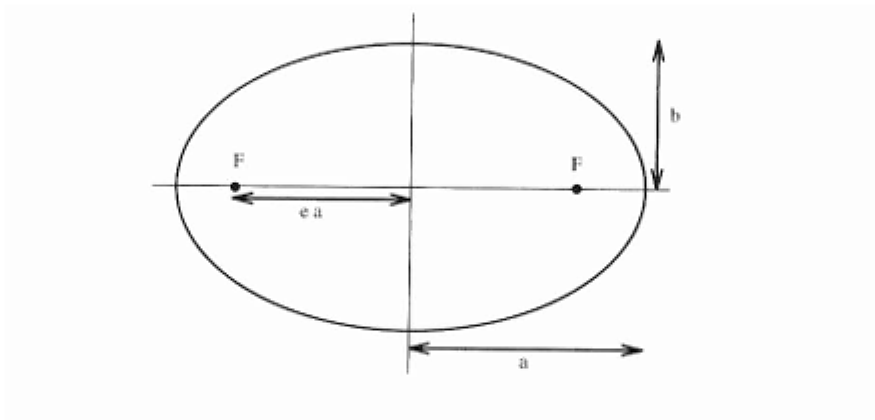
Chapter 4.2

The orbital trajectory for a body of reduced mass μ is given in polar coordinates by

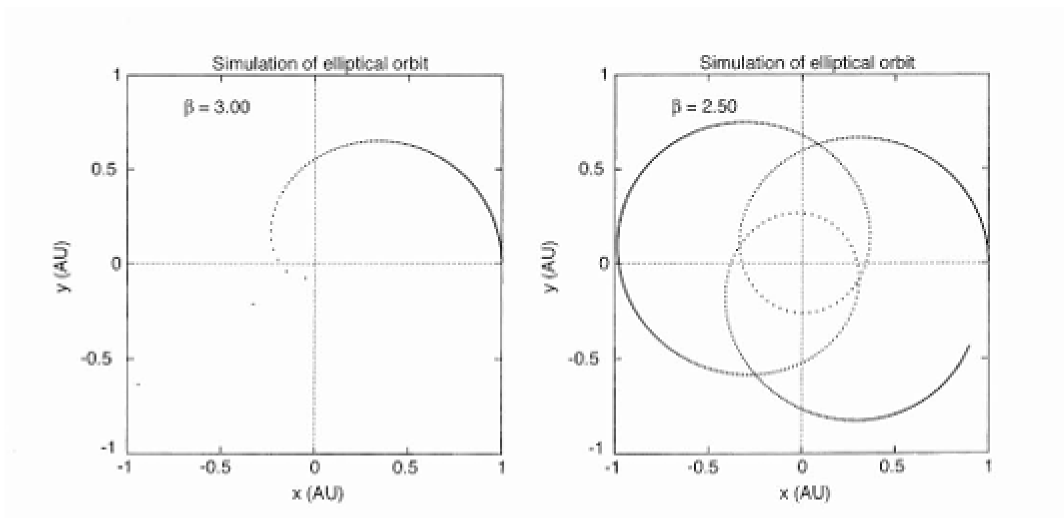
$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{L^2}F(r)\,, \tag{4.8}$$

Where r is

$$\frac{1}{r} = \left(\frac{\mu G M_S M_P}{L^2}\right)\left[1 - e\cos(\theta + \theta_0)\right]\,,$$

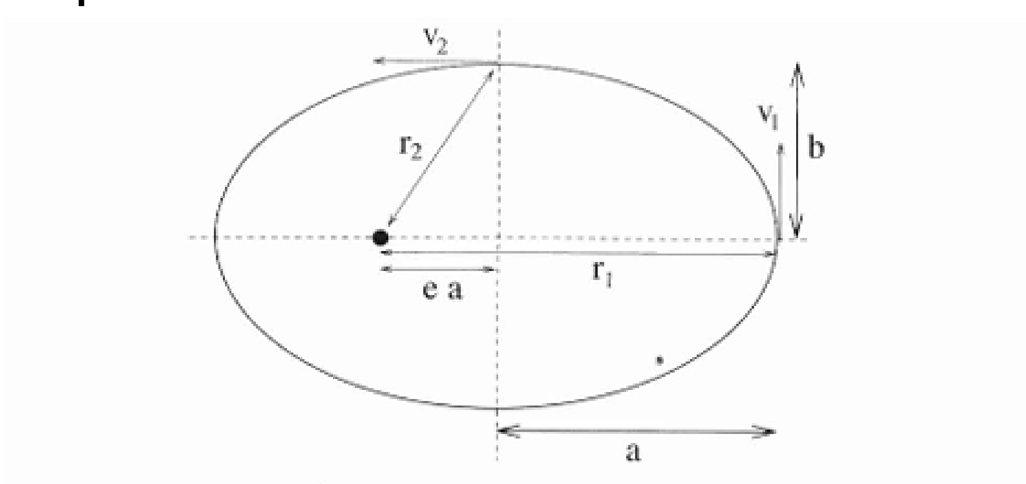


This is a hypothetical elliptical orbit.



Looking at how the elliptical orbit can change when changing the value of B even slightly.

Chapter 4.3



Simulation of Mercurys orbit.

Conservation of total energy (kinetic plus potential of the planet, ignoring the kinetic energy of the Sun) implies that the energies at points 1 and 2 in Figure 4.7 are the same. Thus

$$-\frac{G\,M_S\,M_M}{r_1} + \frac{1}{2}\,M_M\,v_1^2 = -\frac{G\,M_S\,M_M}{r_2} + \frac{1}{2}\,M_M\,v_2^2\,. \qquad (4.14)$$

at point 1 is equal to that at point 2, which yields

$$r_1\,v_1 = b\,v_2\,. \qquad (4.15)$$

We thus have two equations involving the unknowns v_1 and v_2. After several lines of algebra (we'll leave that to you) we find

$$\begin{aligned} v_1 &= \sqrt{2\,G\,M_S\left[\frac{b^2}{a^2(1+e)^2 - b^2}\right]\left[\frac{1}{\sqrt{e^2a^2 + b^2}} - \frac{1}{a + ea}\right]} \\ &= \sqrt{\frac{G\,M_S\,(1-e)}{a\,(1+e)}}\,, \end{aligned} \qquad (4.16)$$

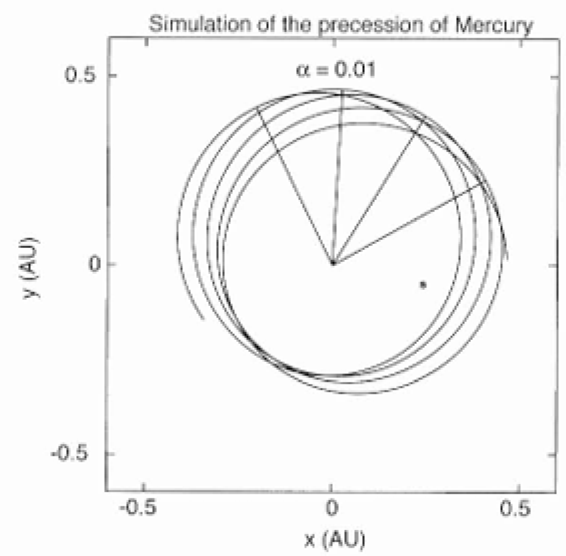


FIGURE 4.8: Simulated orbit for Mercury orbiting the Sun. The force law (4.13) was used, with $\alpha = 0.01$. The time step was 0.0001 yr. The program was stopped after several orbits. The solid lines emanating from the Sun (i.e., the origin) are drawn to the points on the orbit that are farthest from the Sun, so as to show the precession of the orientation of the orbit.

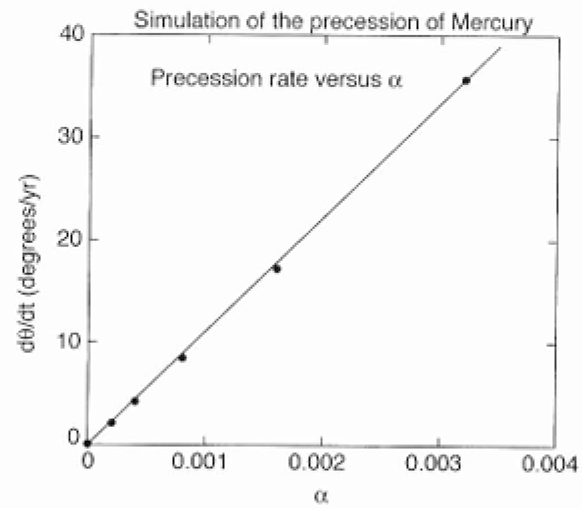


FIGURE 4.10: Precession rate of Mercury as a function of α. The solid line is a least-squares fit, which yielded a slope of 1.11×10^4 degrees per year per unit α.

Chapter 4.4
The effect of Jupiter on Earth.

$$F_{E,J} = \frac{G\, M_J\, M_E}{r_{EJ}^2} \,,$$

The magnitude of the effect of Jupiter on Earth.

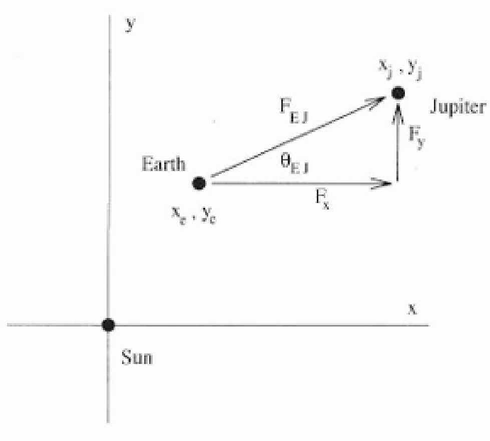


FIGURE 4.11: Components of the gravitational force due to Jupiter, located at x_j, y_j, with Earth at x_e, y_e. The Sun is at the origin.

$$F_{EJ,x} = -\frac{G\, M_J\, M_E}{r_{EJ}^2} \cos\theta_{EJ} = -\frac{G\, M_J\, M_E\, (x_e - x_j)}{r_{EJ}^3} ,$$

Fej in terms of components

$$\frac{dv_{x,e}}{dt} = -\frac{G\, M_S\, x_e}{r^3} - \frac{G\, M_J\, (x_e - x_j)}{r_{EJ}^3} ,$$

Equation of motion for the x component.

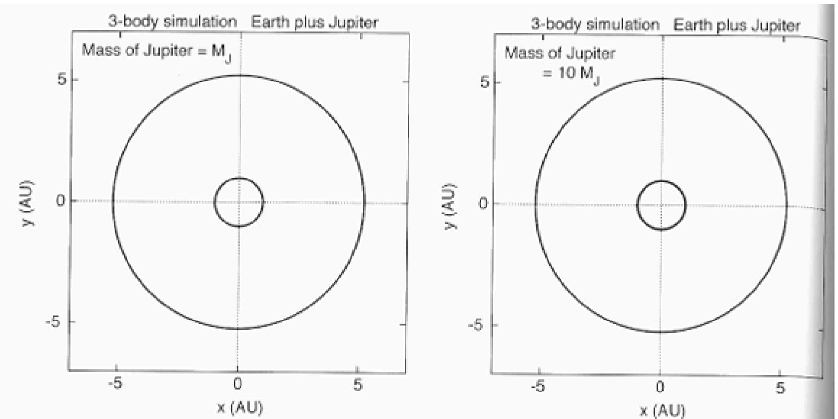


FIGURE 4.12: Simulation of a solar system with two planets, Earth and Jupiter. Left: Jupiter has its true mass; right: the mass of Jupiter has been set to 10 times its true mass.

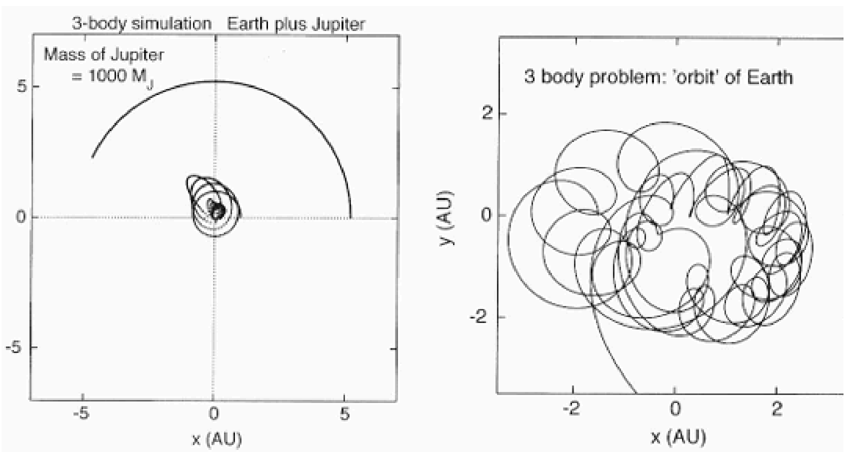


FIGURE 4.13: Simulation of a solar system with two planets, Earth and Jupiter. Left: the mass of Jupiter has been set to 1000 times its true mass, and we have used the routine `jupiter-earth`, which does *not* take into account the motion of the Sun. Here we stopped the simulation before Jupiter had completed even half an orbit, as the motion of Earth was unstable. Right: Typical results for a true 3-body simulation, in which the motions of Earth, Jupiter, and the Sun were all computed. Here we show only the motion of the Earth. The origin is now the center of mass of the 3-body system.