Collin

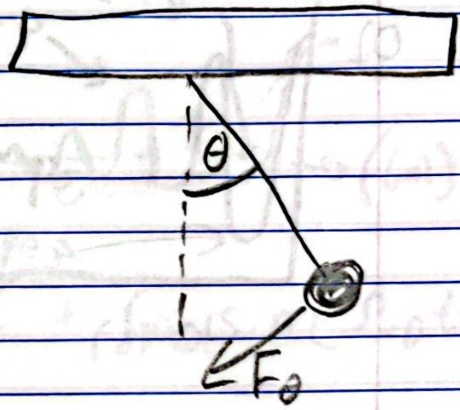
Ch3 - Oscillatory Motion & Chaos

ex of Osillatory phenomenon

- Motion of electrons in atoms
- Currents & voltages in electronic circuits
- Planetary orbits

3.1 Simple harmonic motion

$F_\theta = -m \cdot g \sin\theta$

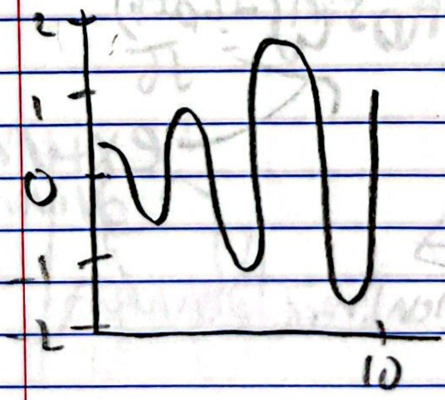


$\theta = \theta_0 \sin(\Omega t + \phi)$

$\Omega = \sqrt{g/l}$

$\theta_{i+1} = \theta_i + \omega_i \Delta t$

Simple Pendulum - Euler Method

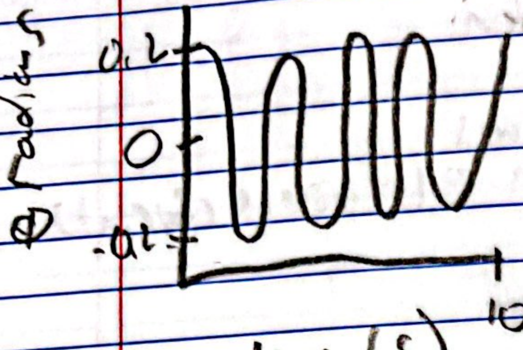


$E = \frac{1}{2} m \omega^2 l^2 + mgl(1 - \cos\theta)$

$E_{i+1} = E_i + \frac{1}{2} mgl\left(\omega_i^2 + \frac{g}{l}\theta_i^2\right)\Delta t^2$

Scanned with CamScanner

Simple Pendulum - Euler-Cromer Method

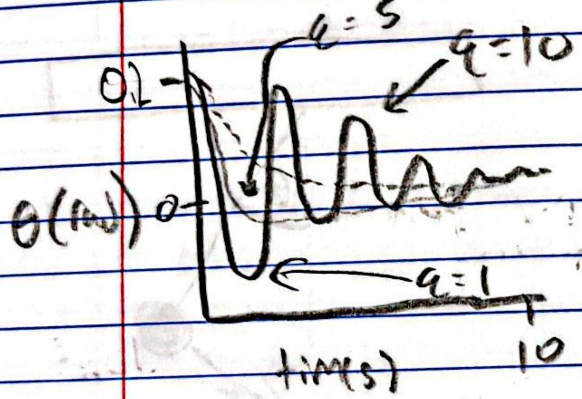


$$\frac{d^2x}{dt^2} = -kx^{\alpha}$$

$$\theta(t) = \theta_0 e^{-qt/2} \sin\left(\sqrt{\Omega^2 - q^2/4}\, t + \phi\right)$$

3.2

damped Pendulum



$$\theta(t) = \theta_0 e^{-\left(q/2 \pm \sqrt{q^2/4 - \Omega^2}\right)t}$$

Between the two extremes

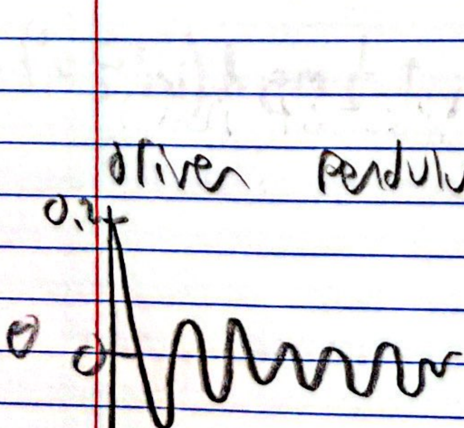
$$\theta(t) = (\theta_0 + Ct)e^{-qt/2}$$

equation of motion

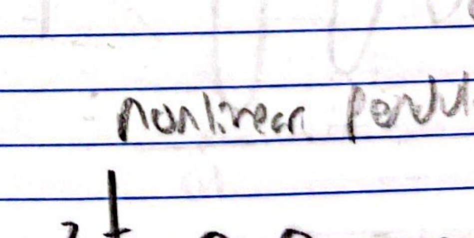
$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt} + F_0 \sin(\Omega_0 t)$$

external driving force

driven pendulum



nonlinear pendulum



Steady State Solution:

$$\theta(t) = \theta_0 \sin(\Omega_D t + \phi)$$

where:

$$\theta = \frac{F_D}{\sqrt{(\Omega^2 - \Omega_D^2)^2 + (q\Omega_D)^2}}$$

Now the equation of Motion =

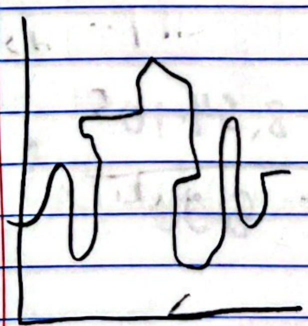
$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$$

3.3 - Driven Non-linear Pendulum equation of motion

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F_D\sin(\Omega_D t)$$

$$\frac{d\omega}{dt} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F_D\sin(\Omega_D t)$$

$$\frac{d\theta}{dt} = \omega \checkmark$$



← example of non-linear Chaotic pendulum