ANOVA Part 2

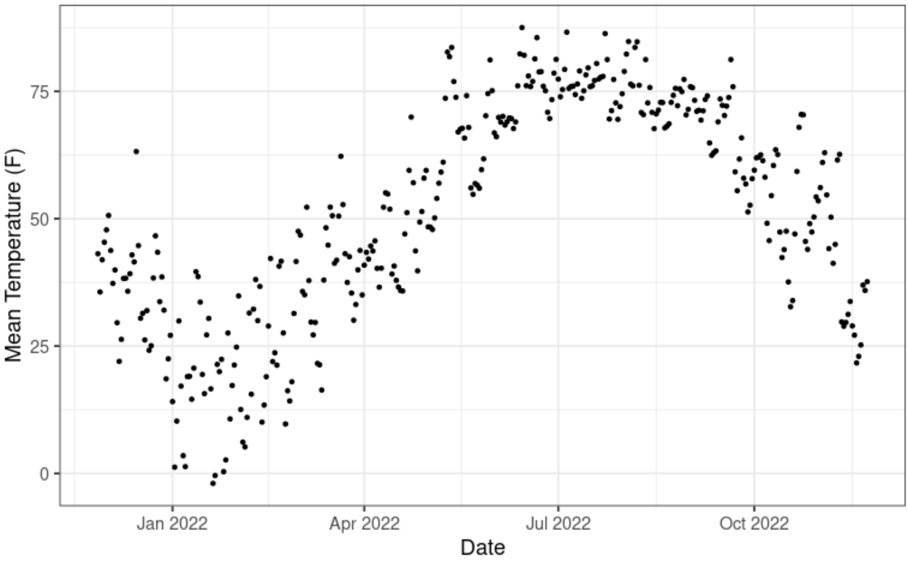
Grinnell College

December 3, 2025

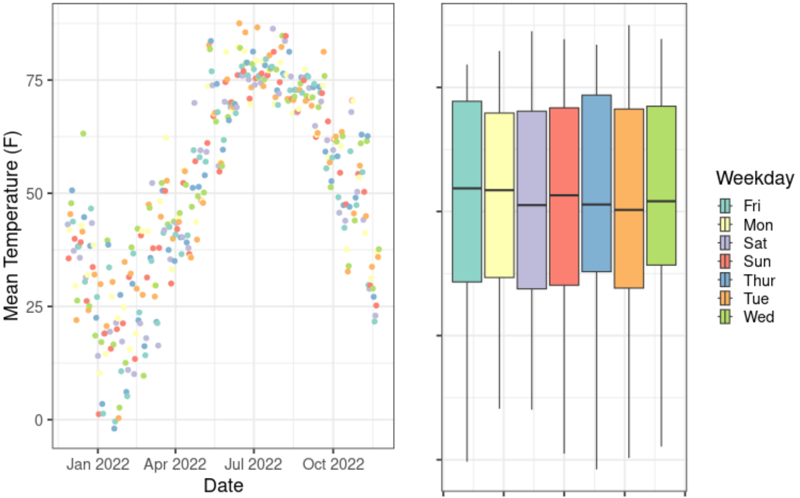
Review

Suppose we had daily average temperatures for Grinnell for the period of one calendar year

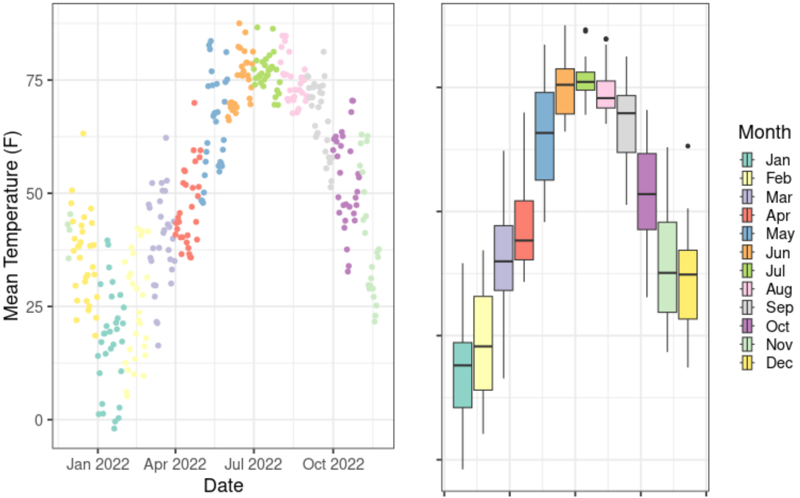
- What makes day of the week a poor candidate for predicting mean values?
- What makes month of the year a good candidate for predicting mean values?
- What kind of attributes make a categorical variable a good or bad candidate? (general conversation question)



Mean Temperature (F)
75
50
25
0
Jan 2022
Apr 2022
Jul 2022
Oct 2022
Date



	Df	Sum Sq	Mean Sq	F value	*p*-value
Weekday	6	342.71	57.12	0.12	0.9939
Residuals	355	168524.83	474.72		



	Df	Sum Sq	Mean Sq	F value	*p*-value
Month	11	138048.06	12549.82	142.52	<0.0001
Residuals	350	30819.48	88.06		

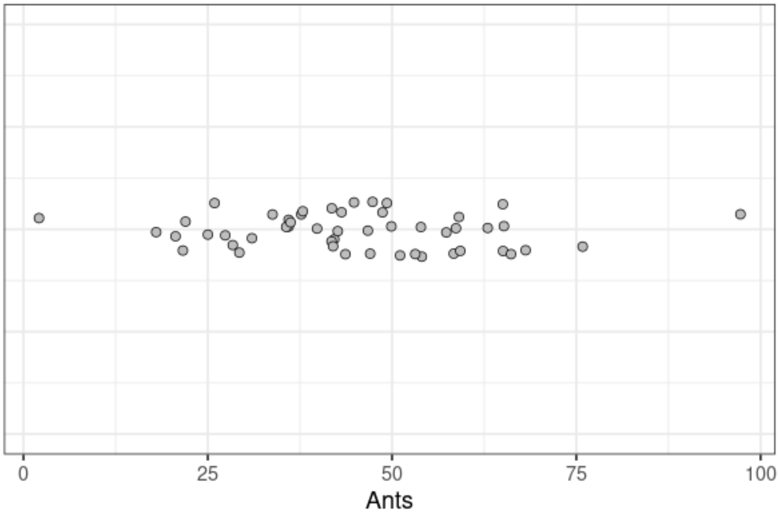
Formulas

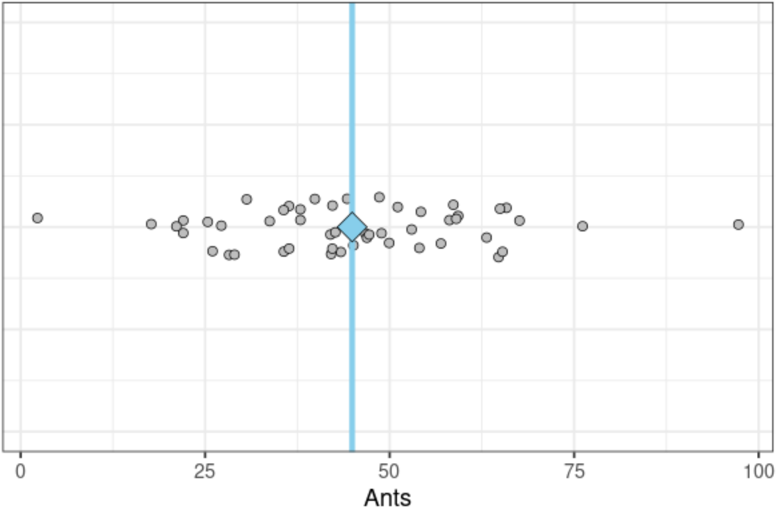
$$\underbrace{\sum_{i}^{n}(x_{ij}-\overline{x})^2}_{\text{SST}} = \underbrace{\sum_{i=1}^{n}(x_{ij}-\overline{x}_j)^2}_{\text{SSE}} + \underbrace{\sum_{j=1}^{k} n_j(\overline{x}_j-\overline{x})^2}_{\text{SSG}}$$

- SST = SSE + SSG
- SSE = sum of squares *within groups*
- SSG = sum of squares *between groups*
- $MSG = \frac{SSG}{k-1}$
- $MSE = \frac{SSE}{n-k}$
- $F = \frac{MSG}{MSE} = \left(\frac{n-k}{k-1}\right)\frac{SSG}{SSE}$

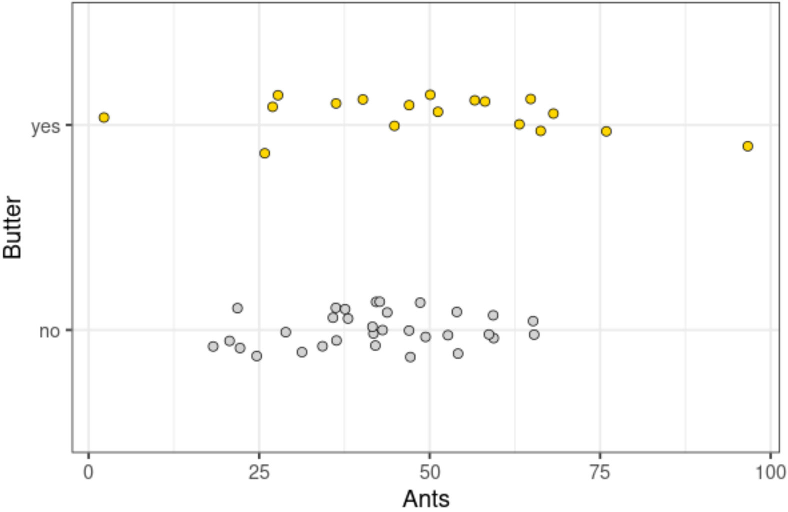
Source	df	Sum Sq	Mean Sq	F value	Pr(>F) / *p*-value
Group	k-1	SSG	$MSG = \frac{SSG}{k-1}$	$F = \frac{MSG}{MSE}$	Upper tail
Error	n-k	SSE	$MSE = \frac{SSE}{n-k}$		
Total	n - 1	SSTotal			

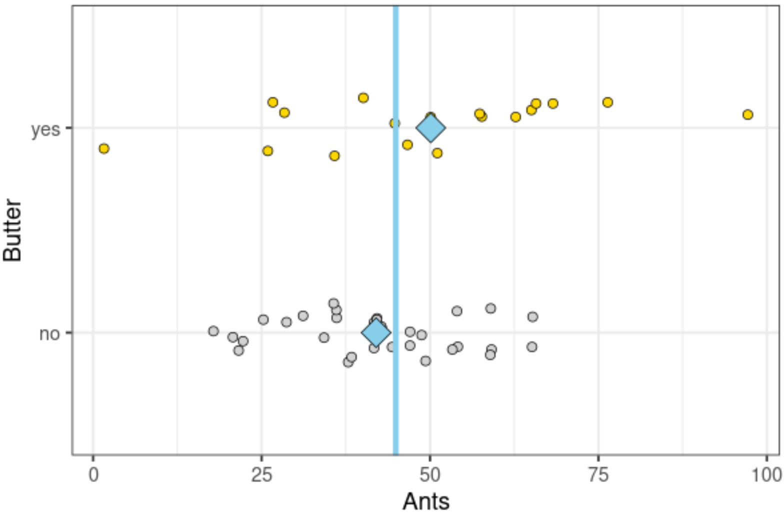
Ants and sandwiches


0
25
50
75
100
Ants

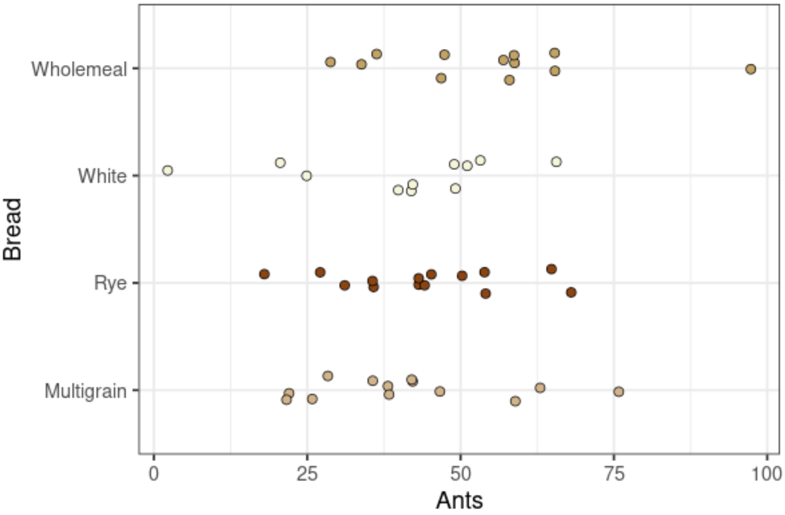


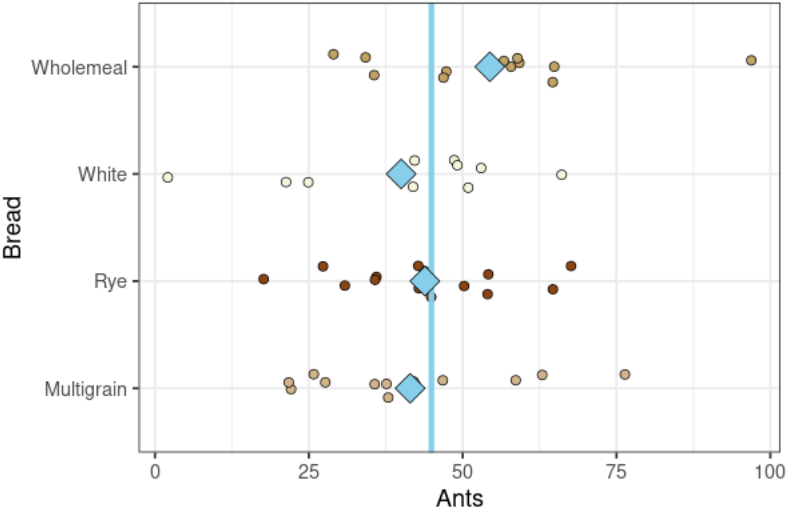
	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Residuals	49	14041.68	286.56		


yes
no
Butter
0
25
50
75
100
Ants

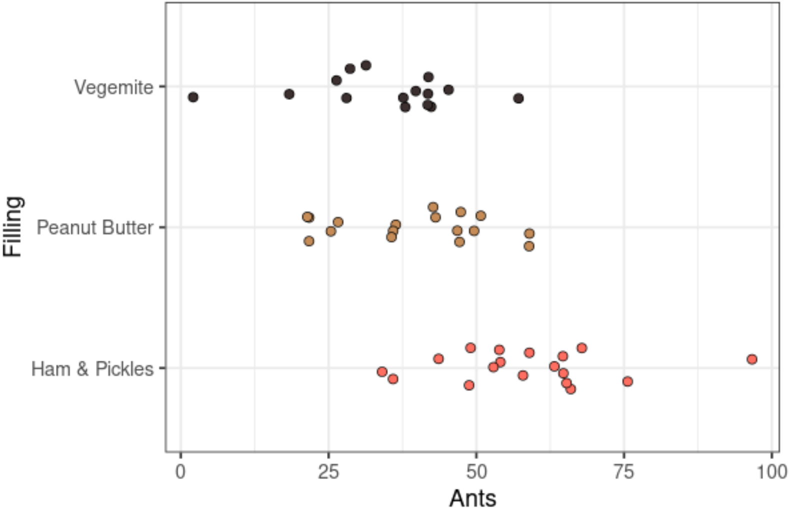


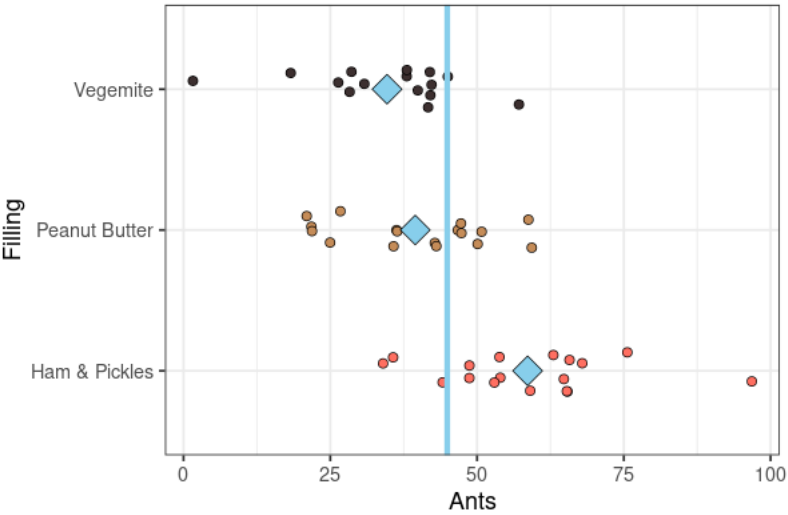
	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Butter	1	757.90	757.90	2.74	0.1045
Residuals	48	13283.78	276.75		


Wholemeal
White
Rye
Multigrain
Bread
0
25
50
75
100
Ants

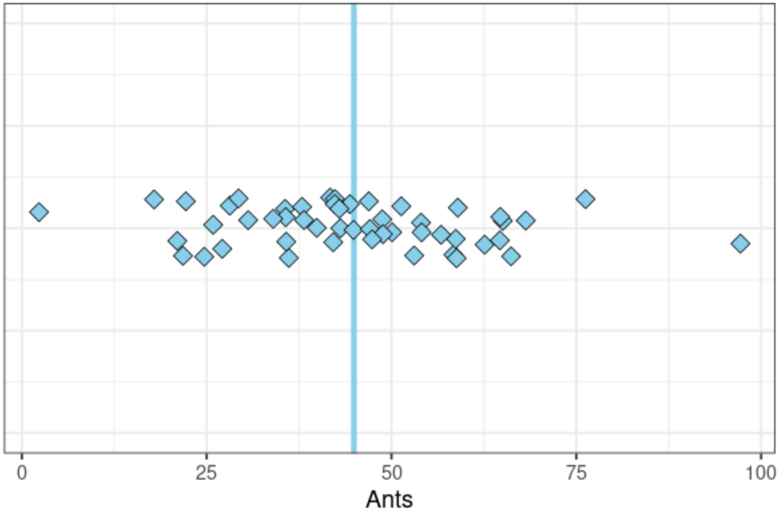


	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Bread	3	1519.82	506.61	1.86	0.1494
Residuals	46	12521.86	272.21		

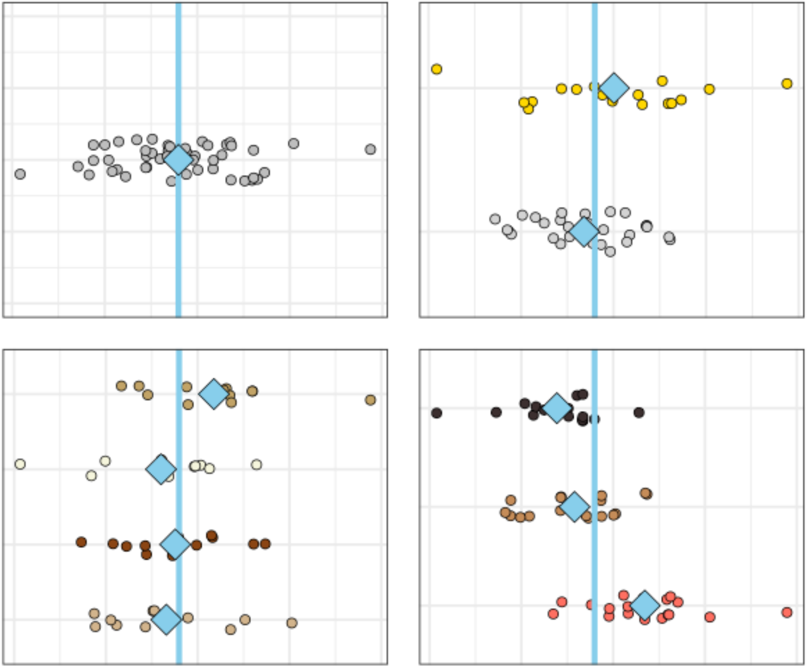

Vegemite
Peanut Butter
Ham & Pickles
Filling
0
25
50
75
100
Ants



	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Filling	2	5455.83	2727.92	14.93	0.0000095
Residuals	47	8585.85	182.68		



	Df	Sum Sq	Mean Sq
Individual	49	14041.68	286.56



	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Residuals	49	14041.68	286.56		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Butter	1	757.90	757.90	2.74	0.1045
Residuals	48	13283.78	276.75		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Bread	3	1519.82	506.61	1.86	0.1494
Residuals	46	12521.86	272.21		

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Filling	2	5455.83	2727.92	14.93	0.0000095
Residuals	47	8585.85	182.68		

To t or Not to t

Recall that for ANOVA we are testing the null hypothesis that *all* of our means our equal

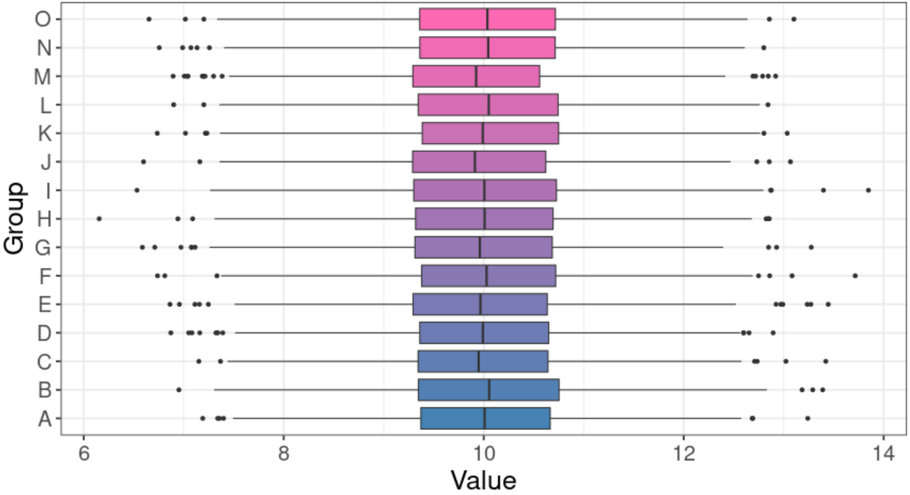
$$H_0 : \mu_A = \mu_B = \mu_C$$

Why not instead just stick with our t-test, doing

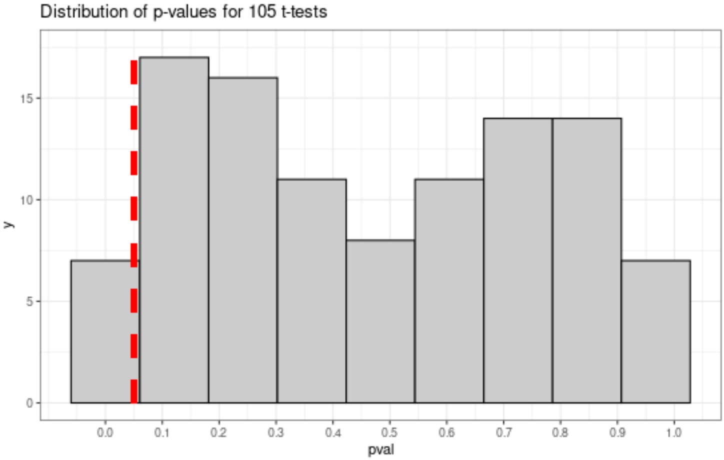
$$H_0 : \mu_A = \mu_B, \ \mu_A = \mu_C, \text{ and } \mu_B = \mu_C$$

Multiple tests

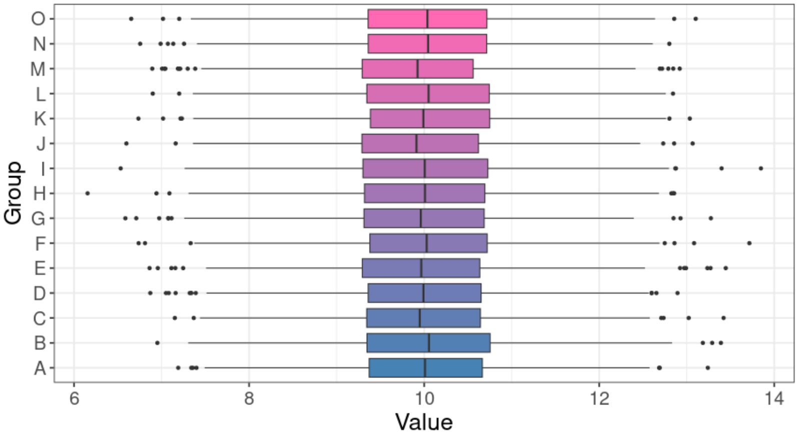
15 groups, all generated with the same mean value:



- 105 pair-wise tests
- 6 with p-value < 0.05



Multiple tests

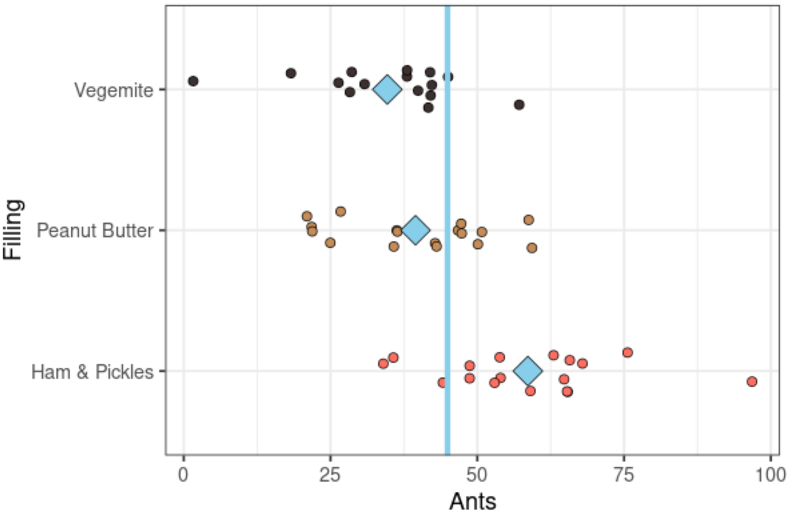


	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Group	14	15.40	1.10	1.10	0.3504
Residuals	14985	14964.85	1.00		

Post-hoc Tests

ANOVA only tells us *that* a difference exists, not where it is or to what degree

If our ANOVA test is such that we rejet the null hypothesis, we can use *post-hoc* testing via the **Tukey Range Test** or the **Tukey Honest Significant Difference Test** to identify any statistically significant pair-wise differences



	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Filling	2	5455.83	2727.92	14.93	0.0000095
Residuals	47	8585.85	182.68		

```
> aov(Ants ~ Filling, sandwich) %>% TukeyHSD()
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Ants ~ Filling, data = sandwich)

$Filling
                                diff     lwr      upr   p adj
Peanut Butter-Ham & Pickles -19.1405 -30.203  -8.0780 0.00036
Vegemite-Ham & Pickles      -23.9444 -35.380 -12.5090 0.00002
Vegemite-Peanut Butter       -4.8039 -16.391   6.7834 0.57845
```

Review

- ANOVA allows us to test equality of many means
  - By comparing ratio of between-group and within-group means
- Ameliorates problem of multiple testing
- *Post-hoc* testing can be done to determine which groups are different
- Tukey Honest Statistical Difference (TukeyHSD)