Bootstrapped differences in timeseries

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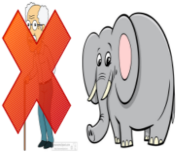
Introduction

- Problem
- Context
- Solution

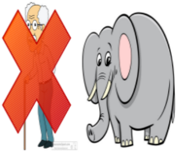
el

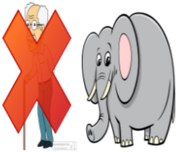
 

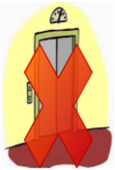
el → ele

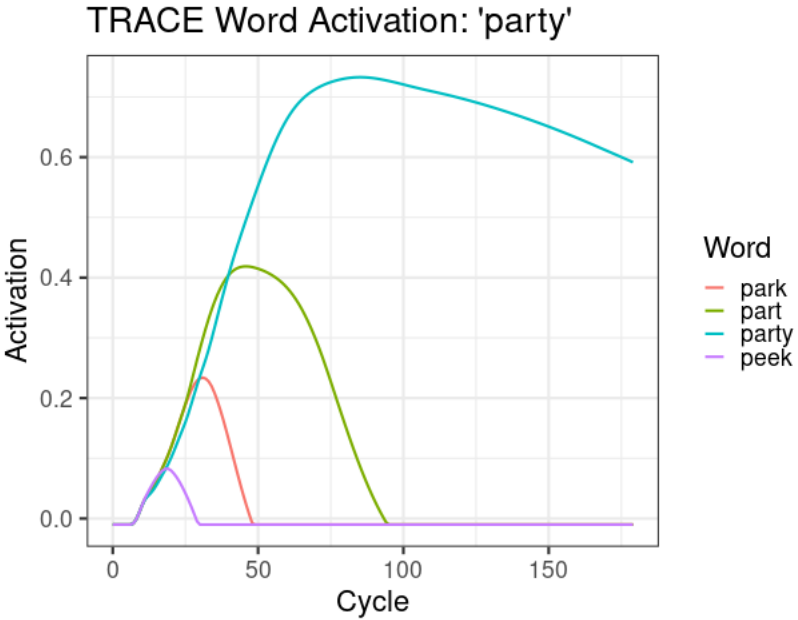


el → ele → elephant








TRACE Word Activation: 'party'
Activation
0.6
0.4
0.2
0.0
0
50
100
150
Cycle
Word
park
part
party
peek

Why do we care?

Typically interested in comparing activation between groups or conditions

- Normal Hearing (NH) vs Cochlear Implants (CI)
- Differentiating cognitive, specific, and non-specific impairments

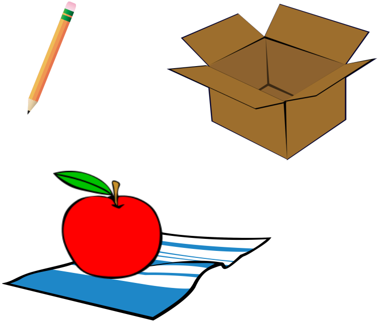
How do we measure this?

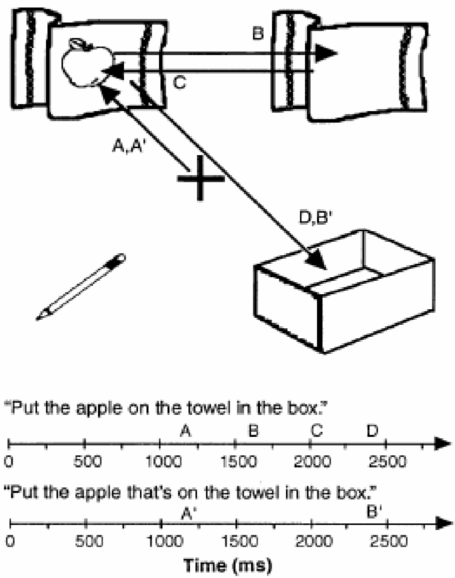
Visual World Paradigm

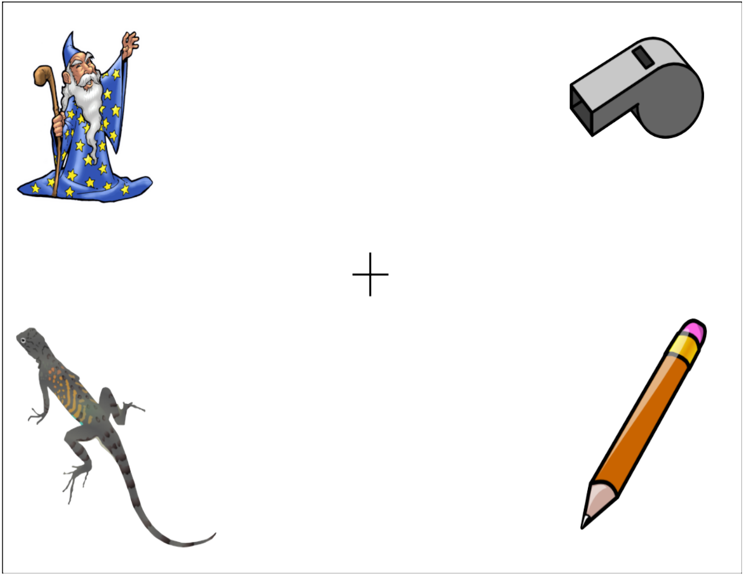
Visual World Paradigm (VWP) introduced in 1995

Eye tracking in conjunction with spoken sentence

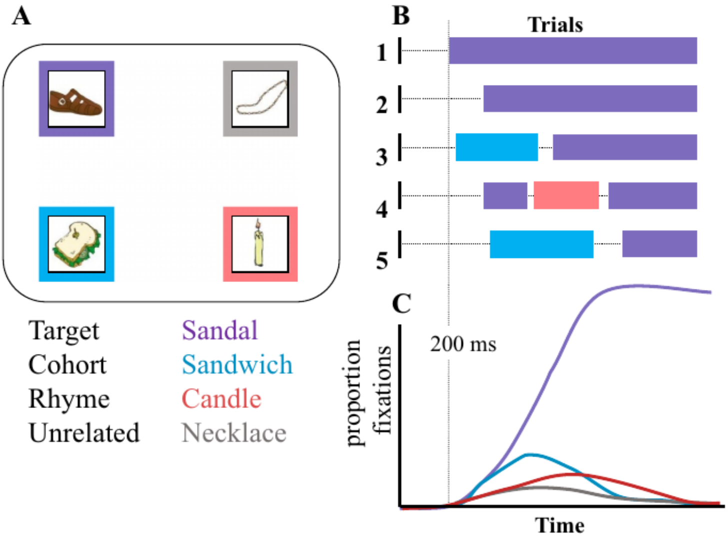
"Put the apple on the towel in the box"

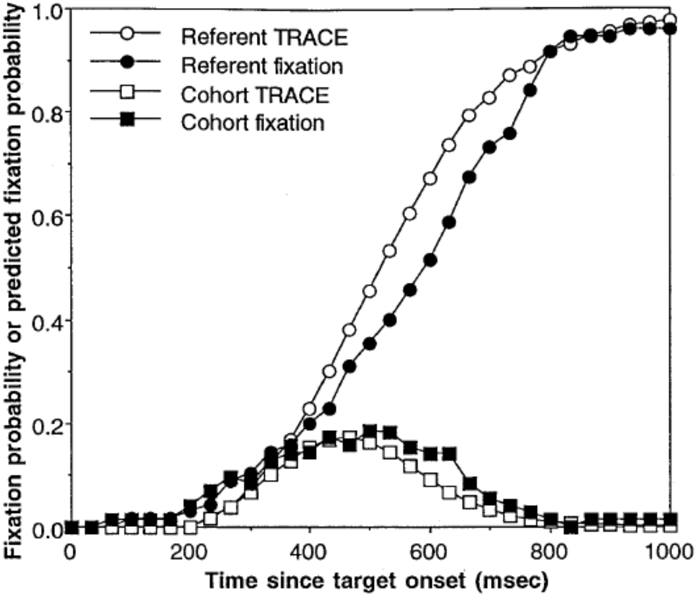



B
C
A,A'
D,B'
"Put the apple on the towel in the box."
A
B
C
D
0
500
1000
1500
2000
2500
"Put the apple that's on the towel in the box."
A'
B'
0
500
1000
1500
2000
2500
Time (ms)



VWP Trials


A
Target
Cohort
Rhyme
Unrelated
Sandal
Sandwich
Candle
Necklace
B
Trials
1
2
3
4
5
C
200 ms
proportion fixations
Time


Referent TRACE
Referent fixation
Cohort TRACE
Cohort fixation
Fixation probability or predicted fixation probability
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
Time since target onset (msec)

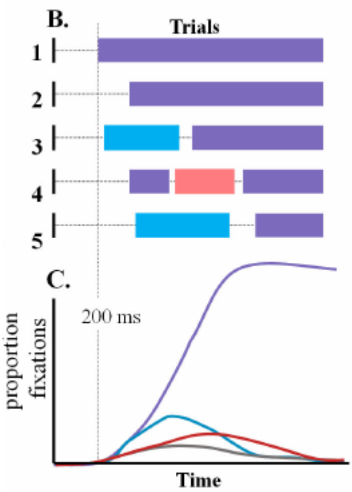
Relation to VWP

Letting z_{jt} represent an indicator of fixation at time t for trial $j = 1, \ldots, J$, we have empirical curve

$$y_t = \frac{1}{J} \sum_j z_{jt}$$

and find

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}\, \mathcal{L}(f_\theta, y)$$



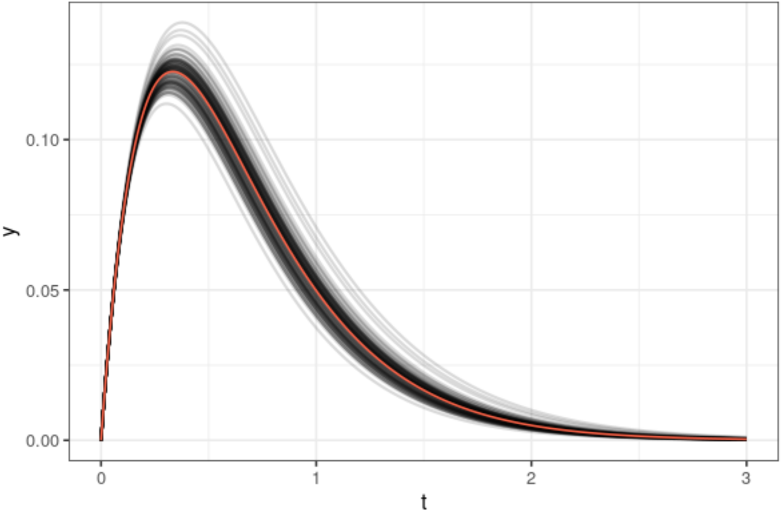
Stating the problem

For experimental groups $g = 1, \ldots, G$, with subjects $i = 1, \ldots, n$, we assume *subject-specific* parameters follow distribution

$$\theta_i \sim N(\mu_g, V_g)$$

We can then use this distribution to estimate distribution of functions $f(\cdot|\theta)$, giving representation of temporal changes in group characteristics

$$\lambda \sim N(3, \sigma), \qquad f(t|\lambda) = e^{t\lambda}$$



What are we doing?

- Create estimate of this distribution
- Main things are within + b/w subject var (useful for CI)
- Use this specifically for alpha adjustment style
- alternative algorithm uses permutation testing (borrow from functional analysis, no AR)

Subject specific distribution

For each subject, begin by fitting observed data to nonlinear curve fitter to construct sampling distribution

$$\hat{\theta}_i \sim N(\theta, s_i^2)$$

Standard errors from the curve fitting process serve as the standard deviation for the sampling distribution of $\hat{\theta}_i$

Estimating Group Distribution

For $b = 1, \ldots, B$:

1. Sample n subjects from group *with replacement*. For each subject i, draw set of parameters from subject-specific distribution

$$\theta^*_{ib} \sim N(\hat{\theta}_i, s_i^2)$$

2. Find average parameter across all bootstrapped θ^*_{ib} to construct bth group bootstrap θ^*_{gb} where

$$\theta^*_{gb} \sim N\left(\mu_g,\ \frac{1}{n} V_g + \frac{1}{n^2} \sum s_i^2\right)$$

   providing an estimate of *group-level* sampling distribution with additional term to account for uncertainty in model estimates

Collection of $\{\theta^*_{gb}\}_{b=1}^{B}$ used to constuct sample of population curves, $f(\cdot|\theta^*_{gb})$

Method for testing

Letting $\overline{p}_{gt}$ and s_{gt}^2 serve as the average and variance of the bootstrapped function estimates for groups g at timepoint t, we construct test statistics

$$T_t^{(B)} = \frac{(\overline{p}_{1t} - \overline{p}_{2t})}{\sqrt{s_{1t}^2 + s_{2t}^2}}$$

The AR(1) process can be characterized as

$$T_t = \rho T_{t-1} + \epsilon_t, \quad t = 1, \ldots, T.$$

Based on this, we have the relation

$$\begin{aligned} T_t | T_{t-1} &\sim N(\rho T_{t-1}, 1-\rho^2) \\ T_t &\sim N(0,1) \end{aligned}$$

For a given α^*, the actual FWER α can be calculated under the null as

$$\begin{aligned} \alpha &= 1 - P\left(\bigcap_{t=1}^{N} I_t\right) \\ &= 1 - P(I_1)\prod_{t=2}^{N} P\left(I_t | I_{t-1}\right) \\ &= \begin{cases} 1 - P(I_1) & \text{when } \rho = 1 \\ 1 - P(I_1)^N & \text{when } \rho = 0 \end{cases} \end{aligned}$$

where I_t presents the event that $|T_t| < z_{\left(1-\frac{\alpha^*}{2}\right)}$ (i.e., $P(I_1) = 1 - \alpha^*$)

Permutation Alternative

Instead of identifying $\theta^*_{gb} = \frac{1}{n}\sum \theta^*_{ib}$ at each bootstrap to estimate

$$f_b^*(\cdot|\theta_g) = f\left(\cdot \mid \frac{1}{n}\sum \theta^*_{ib}\right)$$

(i.e., a function of the distribution of θ), we instead find a distribution of functions at each permutation based on subject-level curves

$$f_p^*(\cdot) = \frac{1}{n}\sum f\left(\cdot|\theta^*_{ib}\right)$$

Our permutation test statistic is the absolute value (kinda) of the bootstrapped statistic

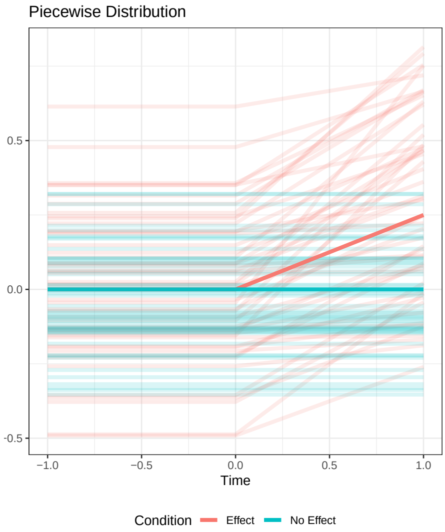
$$T_t^{(P)} = \frac{|\overline{p}_{1t} - \overline{p}_{2t}|}{\sqrt{s_{1t}^2 + s_{2t}^2}} = \left|T_t^{(B)}\right|$$

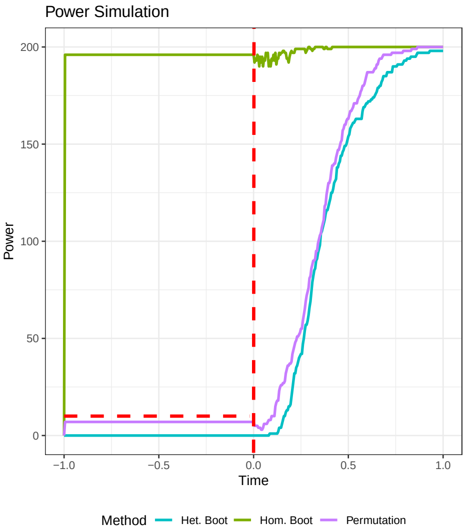
For $p = 1, \dots P$,

1. Permute group assignments for all observations
2. Redraw θ^*_{ib} for each subject to capture within-subject variability in estimate of $f(\cdot|\theta^*_{ib})$
3. Recalculate the test statistic $T_t^{(p)}$, recording the *maximum* value for each permutation

The collection of maximum statistics from each permutation serves as our null distribution, $\widetilde{T}$. Let $\widetilde{T}_\alpha$ be the $1 - \alpha$ quantile of $\widetilde{T}$

Time points where $T_t^{(P)} > \widetilde{T}_\alpha$ are designated significant


Piecewise Distribution
0.5
0.0
-0.5
−1.0
−0.5
0.0
0.5
1.0
Time
Condition
Effect
No Effect

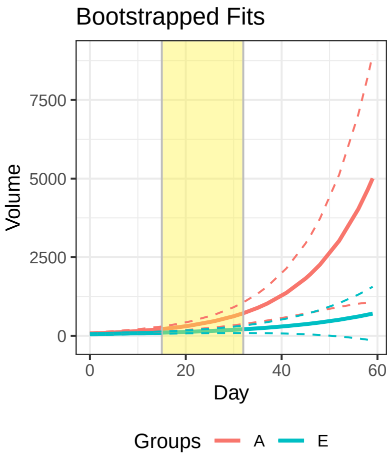

Power Simulation
Power
200
150
100
50
0
−1.0
−0.5
0.0
0.5
1.0
Time
Method
Het. Boot
Hom. Boot
Permutation

Bootstrapped differences in time series – `bdots`

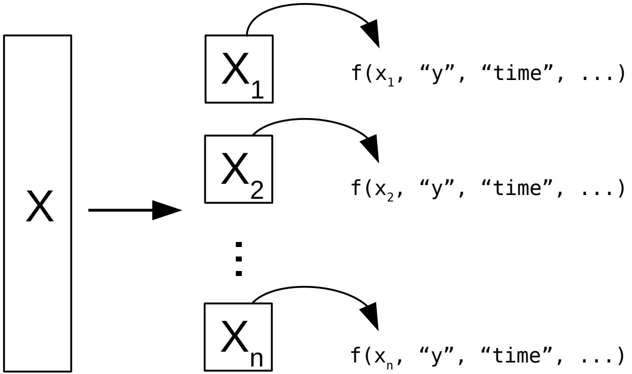
```
fit <- bfit(data = dat,
            y = “Volume”,
            subject = “ID”,
            time = “Day”,
            group = “Treatment”,
            curveFun = expCurve())


boot <- bboot(Volume ~ Treatment(A, E),
              bdObj = fit)


plot(boot)
```



```
fit ← bfit(data = X, y = “y”, time = “time”, curveFun = f(...))
```



```
①  linear <- function (dat, y, time, params = NULL, ...) {

        linearPars <- function(dat, y, time) {
            time <- dat[[time]]
②           y <- dat[[y]]
            if (var(y) == 0) {
                return(NULL)
            }
            mm <- (max(y) - min(y))/max(time)
            bb <- mean(y) - mm * mean(time)
            return(c(intercept = bb, slope = mm))
        }

③       if (is.null(params)) {
            params <- linearPars(dat, y, time)
        }
        if (is.null(params)) {
④           return(NULL)
        }
        y <- str2lang(y)
        time <- str2lang(time)
⑤       ff <- bquote(.(y) ~ slope * .(time) + intercept)
        attr(ff, "parnames") <- names(params)
        return(list(formula = ff, params = params))
    }
```

thanks

References

Magnuson, James S. **Fixations in the visual world paradigm: where, when, why?** 2019-09 *Journal of Cultural Cognitive Science*, Vol. 3, No. 2 Springer Science and Business Media LLC p. 113-139

McMurray, Bob **I'm not sure that curve means what you think it means: Towards a [more] realistic understanding of the role of eye-movement generation in the visual world paradigm** 2022 *Psychonomic Bulletin & Review* p 1-45

Oleson, Jacob J; Cavanaugh, Joseph E, McMurray, Bob; Brown, Grant **Detecting time-specific differences between temporal nonlinear curves: Analyzing data from the visual world paradigm** 2017 *Statistical Methods in Medical Research*, Vol. 26, No. 6 p 2708-2725

Paul D. Allopenna, James S. Magnuson, Michael K. Tanenhaus **Tracking the Time Course of Spoken Word Recognition Using Eye Movements: Evidence for Continuous Mapping Models** 1998 *Journal of Memory and Language*, Vol. 38, Issue 4 p 419-439

The streets