Odds

Grinnell College

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What we do today

- Introduce odds and probabilities
- Contingency tables
- Odds ratios

Odds and Probability

When dealing with a *binary* event, we often speak in terms of **odds**, a *ratio* of "number of successes" to "number of failures"

$$\#\text{ success} : \#\text{ failure}$$

This is distinct from the idea of **probabilities**, which give a ratio of the "number of successes" to the number of possible outcomes

$$\begin{aligned}\#\text{ success} &: \#\text{ total outcomes} \\ &: \#\text{ success} + \#\text{ failure}\end{aligned}$$

Odds

Suppose we have a 6-sided die, and we are interested in rolls that land on either 1 or 2 (note how we have turned six distinct outcomes into two "events").

$$\text{Die} = \{1, 2, 3, 4, 5, 6\}$$

- The *probability* of rolling a 1 or 2 is $1/3$
  1. There are 6 possible outcomes
  2. There are 2 possible successes
  3. Probably is $2 \ / \ 6 = 1/3$
- The *odds* of rolling a 1 or 2 are 2:4 (or 1:2)
  1. There are 2 possible successes
  2. There are 4 possible failures
  3. The odds of success are 2:4 (or 1:2)

Contingency Tables

A **contingency table** is a special two-way table in which both categorical variables have a binary response

	Event	Non-Event
Exposure	A	B
No Exposure	C	D

Specifically, we have event and non-event (order matters)

Odds Ratio

An **odds ratio** is the ratio of odds between two groups

	Event	Non-Event
Exposure	A	B
No Exposure	C	D

- The odds of an event for the exposure group are A:B (or A/B)
- The odds of an event for the no exposure group are C:D (or C/D)

The **odds ratio** for these groups is then the ratio of their odds:

$$OR = \frac{A:B}{C:D} = \frac{A/B}{C/D} = \frac{A \times D}{B \times C}$$

Why Ratios?

Situation 1:

	Event	Non-Event
Exposure	6	2
No Exposure	3	2

Situation 2:

	Event	Non-Event
Exposure	103	2
No Exposure	100	2

1. Difference in odds for each situation?
2. Ratio of odds for each situation?

Event vs Non-Event

Which column is our "Event" changes how we report our results

Case 1:

	Enjoy Ice Cream	
Age	Yes	No
Child	16	4
Adult	4	8

Case 2:

	Enjoy Ice Cream	
Age	No	Yes
Child	4	16
Adult	8	4

Group Rows

The same is true for which group is in the first row

Case 1:

	Enjoy Ice Cream	
Age	Yes	No
Child	16	4
Adult	4	8

Case 2:

	Enjoy Ice Cream	
Age	No	Yes
Adult	4	8
Child	16	4

Odds Ratio Summary

- Odds and probabilities
- Column/row order matters
- Odds ratios
- $OR > 1$, $OR = 1$, $OR < 1$
  - $OR = 1$ implies no association. Why?

Example 1

A report published in 1988 summarizes results of a Harvard Medical School clinical trial determining effectiveness of asprin in preventing heart attacks in middle-aged male physicians

Treatment Status	Myocardial Infarction	
	Attack	No Attack
Placebo	189	10,845
Asprin	104	10,933

- Odds of having a heart attack for placebo:
- Odds ratio for treatment and infarction:
- Associated?

Example 2

The table below shows the results for drivers and passengers in auto accidents in Florida in 2008, according to whether or not the individual was wearing a seat belt.

Sealt-Belt Use	Injury	
	Fatal	Nonfatal
No	1085	55,623
Yes	703	441,239

- *Probability* of wearing seatbelt conditional on fatality status:
- *Odds* of fatality conditional on seat-belt use:
- Associated?