

Meta-Heuristics

Collin Price & Ron Bond

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Overview

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Optimization

Metaheuristics

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Why?
Classifications

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Objective

- Minimize/maximize an objective output whose value(s) depend on a set of input variables
- Function often referred to as "Fitness Function"

Landscape

- The n-dimensional mapping of input to output values:
- inputs are parameters to be optimized
- outputs are the "fitness" or score of the given input configuration

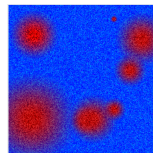


Figure:
Fitness
Landscape

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Metaheuristics

- Partial search algorithm
- Often stochastic in nature
- Do not guarantee a globally optimal solution

Why Metaheuristics?

- Incomplete or imperfect information
- Limited computation capacity

Classifications

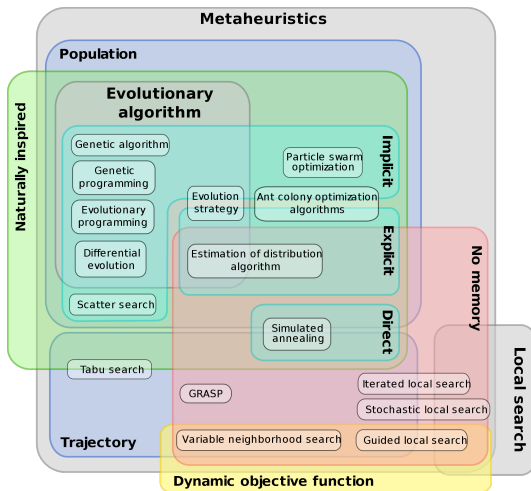


Figure: Metaheuristic Classifications

Iterative Framework Overview

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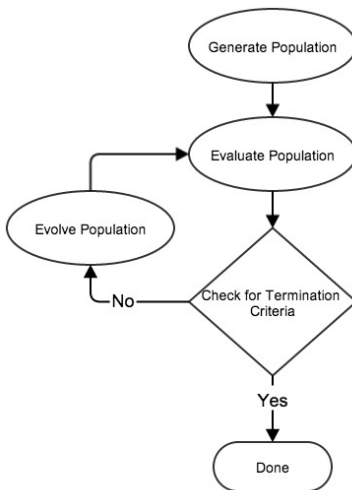
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Genetic Algorithm

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- A search heuristic that mimics the process of natural selection.
- Individuals are combined and modified to create new ones.

Individuals

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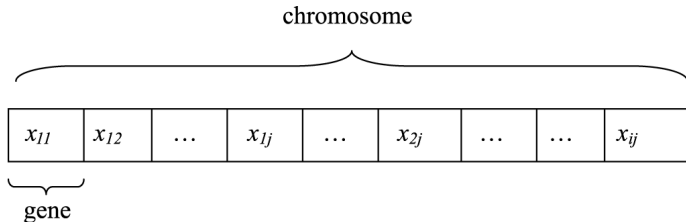
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- Represents a proposed solution to the problem.
- Commonly represented as an array.
- Representations can vary depending on the problem.



Crossover

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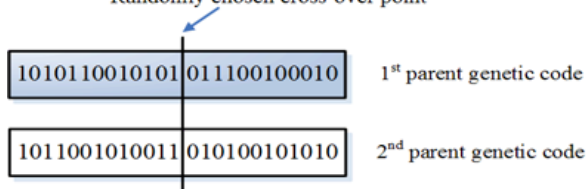
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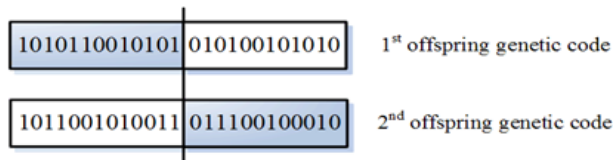
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Randomly chosen cross-over point



Situation after cross-over



Mutation

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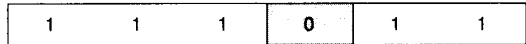
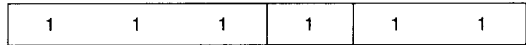
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Gene selected for mutation



Mutated gene

Selection

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- Method used to select individuals from the population for breeding.
- Example: Tournament Selection
 - Randomly choose n -individuals from the population.
 - Select the individual with the best fitness.

Parameters

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- There are several parameters that can be tuned to change the results of your genetic algorithm.
- There is no optimal set of parameters. Parameter values are empirically found.
- Parameters:
 - Number of Generations
 - Population Size
 - Crossover Rate
 - Mutation Rate
 - Elitism?

Particle Swarm Optimization

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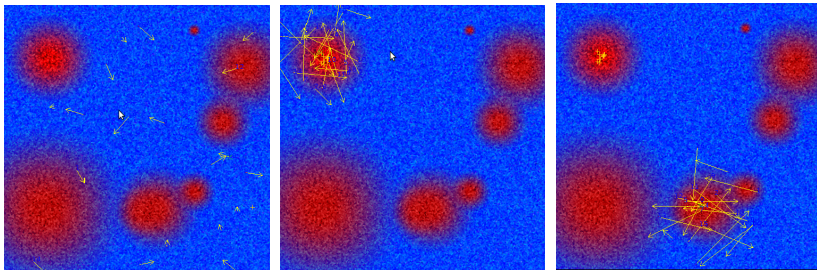
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$$v_{i,d} = \boxed{w} * v_{i,d} + \boxed{C_1 * Rnd(0, 1) * [pb_{i,d} - x_{i,d}]} + \boxed{C_2 * Rnd(0, 1) * [gb_d - x_{i,d}]}$$

Inertia Weight *cognitive influence* *social influence*

Figure: Original PSO Behaviour

Genetic Programming

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- Specialization of a genetic algorithm.
- Each individual is a computer program.
- Traditionally represented as tree structures.

Individuals

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- Individuals randomly generated from a list of operators and terminals.
- Example Operators: addition, subtraction, multiplication, cosine.
- Example Terminals: ephemeral random constants, problem specific numbers.

Crossover & Mutation

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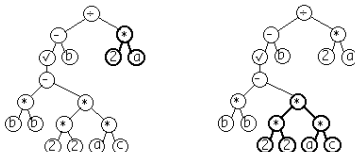
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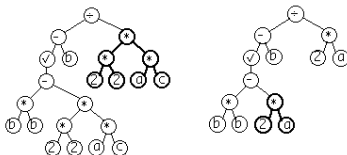
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Crossover Operation with Identical Parents

Parents



Children



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- Performance of Metaheuristics are limited to the information given
- "information" is encoded in
 - Solution Representation
 - Fitness Function



Representation - Efficiency

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- Search space grows with the size of the language
- Minimizing the language reduces search complexity
- Example: Language 1 (left) vs Language 2 (right)

- $\text{add}(x,y)$

- $\text{sub}(x,y)$

- $\text{mult}(x,y)$

- $\text{div}(x,y)$

- $\text{log}(x)$

- $\text{max}(x,y)$

- $\text{sub}(x,y)$

- $\text{mult}(x,y)$

- $\text{max}(x,y)$

Representation - Language

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- Language components offer different information
- Derivative components can offer "new information" and reduce search space
- Example: Simple (left) vs Derived (right)
 - $\text{sub}(x,y)$
 - $\text{mult}(x,y)$
 - $\text{max}(x,y)$
 - Edge Filters
 - Geometric Filter
 - ...

Representation

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Fitness Function

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- Produces values on which to compare generated solutions
- Defines the "desired solution"
- Fitness function will therefore impact performance



Figure: $f(x) = \text{Max}(\text{true positives})$

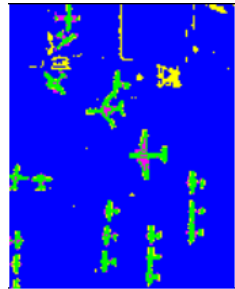


Figure: $f(x) = \text{Min}(\text{false positives} + \text{false negatives})$

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- NERO - Evolved Neural Networks
- Starcraft 2 build orders - Genetic Algorithm
- Evolved Antenna - Evolutionary Algorithm
- Boxcar 2D - Genetic Algorithm
- Evolving Virtual Creatures

No Free Lunch theorem

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No Free Lunch - Wolpert/Macready 1997

For all possible performance measures, no search algorithm is better than another when its performance is averaged over all possible discrete functions.

- No silver bullet
- Does not state that algorithms cannot generalize to categories of problems

Optimizing Optimizers

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Optimizing Optimizers

- Metaheuristics are general because they can be tuned to a specific problem
- Tuning is itself an optimization problem
- Trading one search space for another

Apply Another Metaheuristic

- Same problem exists for the higher-level heuristic
- computation increases exponentially

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Questions?