Homework 2

Advanced Machine Learning

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Exercise 1

Exercise 1 part a

$\mu = (1/3, 1/2)$
$\pi = (1/2, 1/2)$

To calculate responsibilities we first calculate the $P(x|\mu)$ based on h and t, use this to calculate the $P(x)$ and use both to calculate the responsibility.

$$P(x|\mu)_k = \binom{h+t}{h} * \mu_k^h * (1-\mu_k)^t$$

$$P(x) = \sum^k \pi_k * P(x|\mu)_k$$

$$\gamma_k = \frac{\pi_k * P(x|\mu)_k}{P(x)}$$

	gamma(Zn1)	gamma(Zn2)
(1, 4)	0.6781456953642385	0.32185430463576153
(3, 2)	0.34501347708894875	0.6549865229110512
(4, 1)	0.20846905537459282	0.7915309446254072
(2, 3)	0.5130260521042086	0.4869739478957915

Exercise 1 part b

In order to get the new optimal mu we need to set the derivative of the expected value of log likelihood function with respect found latent variable distributions (the coin responsibilities)

We first find the N values, the estimated times the coins were used by summing over the responsibilities for that coin

We then calculate μ by finding the average percentage value of the flips (percentage of heads).

$$mu = \sum^j \left(\gamma_k^j * \frac{h^j}{h^j + t^j} \right) * \frac{1}{N_k}$$

where the sum sums over all the trials.

We can calculate the new pi values using:

$$\pi_k = \frac{N_k}{\sum N}$$

Table 2: estimated times the coins were used

N_0	N_1
1.7446542799319884	2.2553457200680116

Table 3: new mu values, $P(heads)$

coin_1 prob_heads	coin_2 prob_head
0.4096071632572573	0.5699246452972468

Table 4: new pi values

coin_1 prob_use	coin_2 prob_use
0.4361635699829971	0.5638364300170029

Exercise 2

Exercise 2 part a

forward pass

$$\alpha_1(k) = \pi_k * B_k(T),$$
$$\alpha_{t+1}(k) = (\alpha_t \cdot A)_k * B_k(Obs)$$

t	$alpha_1$	$alpha_2$
1	0.35	0.1
2	0.14	0.05
3	0.0258	0.0832
4	0.018072	0.039008

backward pass

$$\beta_4(k) = 1,$$
$$\beta_t(k) = (A \cdot B(Obs_{t+1}))_k * \beta_{t+1}(k)$$

t	$beta_1$	$beta_2$
4	0.11952	0.15248
3	0.312	0.268
2	0.6	0.5
1	1	1

Exercise 2 part b

sum over all alpha_4 (note alpha_4 is actually a square matrix)

$alpha_{4,1}$	$alpha_{4,2}$
0.018072	0.039008

P(O|M) = 0.057080000000000006

Exercise 2 part c

Observation 1

$P(0) = 0.05708$

Gamma 1

	gamma1	
gamma1_1	0.7328661527680447	0.26713384723195516
gamma1_2	0.7652417659425367	0.2347582340574632
gamma1_3	0.27119831814996487	0.7288016818500349
gamma1_4	0.3166082690960056	0.6833917309039943

Eta 1

$$eta1 = \begin{bmatrix} 0.53566924 & 0.19719692 \\ 0.22957253 & 0.03756132 \end{bmatrix}$$
$$eta2 = \begin{bmatrix} 0.17659425 & 0.58864751 \\ 0.09460406 & 0.14015417 \end{bmatrix}$$
$$eta3 = \begin{bmatrix} 0.05423966 & 0.21695865 \\ 0.26236861 & 0.46643308 \end{bmatrix}$$