ASTR 3830: Astrophysics 2 – Galactic and Extragalactic
Final Exam Review Guide

The Students of ASTR 3830

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Contents

1 Exam 1 Content **5**
1.1 The Milky Way (C&O Chapter 24) . . . 5
1.1.1 Determining Morphology of the Milky Way . . . 5
1.1.2 Filter Systems . . . 5
1.1.3 Differential Star Count . . . 6
1.1.4 Integrated Star Count . . . 6
1.1.5 Obscuration . . . 7
1.1.6 Milky Way Components . . . 7
1.1.7 Milky Way Kinematics . . . 7
1.1.8 Galaxy Rotation Curves . . . 8
1.1.9 Evidence for Dark Matter . . . 8
1.1.10 MACHOs, WIMPs . . . 8
1.1.11 NFW Profile . . . 8
1.1.12 Galactic Center Observations . . . 9
1.1.13 Evidence for a SMBH in the Galactic Center . . . 9
1.1.14 Virial Theorem . . . 9
1.1.15 Sgr A* Luminosity Function . . . 9
1.2 The Nature of Galaxies (C&O Chapter 25) . . . 9
1.2.1 The Great Debate Over Spiral Nebulae . . . 9
1.2.2 Galaxy Morphologies, Hubble Classification Scheme . . . 10
1.2.3 Galaxy Surface Brightness, Sérsic Profile . . . 10
1.2.4 How to Measure Rotation in Other Galaxies . . . 11
1.2.5 Tully-Fisher Relation . . . 11
1.2.6 Types of Spiral Arms . . . 12
1.2.7 Winding Problem, Density Wave Theory . . . 12

2 Exam 2 Content **13**
2.1 The Nature of Galaxies . . . 13
2.1.1 Spiral Galaxies versus Elliptical Galaxies: Major Differences . . . 13
2.1.2 Stellar Velocity Dispersion . . . 13
2.1.3 Faber-Jackson Relation . . . 14
2.1.4 The Fundamental Plane . . . 14
2.1.5 Galaxy Luminosity Function . . . 14
2.1.6 The K-Correction . . . 15
2.2 Galactic Evolution . . . 15
2.2.1 Major versus Minor Galaxy Mergers . . . 15
2.2.2 Tidal Stripping and Tidal Tails . . . 15
2.2.3 Dynamical Friction . . . 16
2.2.4 Initial Mass Function . . . 16
2.2.5 Eggen, Lynden-Bell, and Sandage Collapse Model . . . 17
2.2.6 Hierarchical Merger Model . . . 17
2.3 The Structure of the Universe . . . 18

2.3.1 The Cosmological Distance Ladder . . . 18
2.3.2 Expanding Universe, Hubble's Law . . . 21
2.3.3 Peculiar Velocity . . . 21
2.3.4 The Age of the Universe . . . 21
2.3.5 Distribution of Galaxies in the Universe . . . 21
2.3.6 Groups and Cluster . . . 22
2.3.7 Evidence for Dark Matter from Cluster Masses . . . 23
2.3.8 Intracluster Gas . . . 23

3 Final Exam Content **25**
3.1 Active Galaxies . . . 25
3.1.1 Active Galactic Nuclei . . . 25
3.1.2 AGN Structure . . . 26
3.1.3 AGN Luminosity . . . 27
3.1.4 Gravitational Lensing . . . 28
3.2 Cosmology . . . 29
3.2.1 Models . . . 29
3.2.2 Cosmic Microwave Background . . . 30
3.2.3 Big Bang Nucleosynthesis . . . 31
3.2.4 Evolution of the Universe . . . 32
3.2.5 Distances (Observational Cosmology) . . . 34

Topic-Lecture Index **35**

Potentially Useful Equations **37**

Chapter 1

Exam 1 Content

1.1 The Milky Way (C&O Chapter 24)

1.1.1 Determining Morphology of the Milky Way

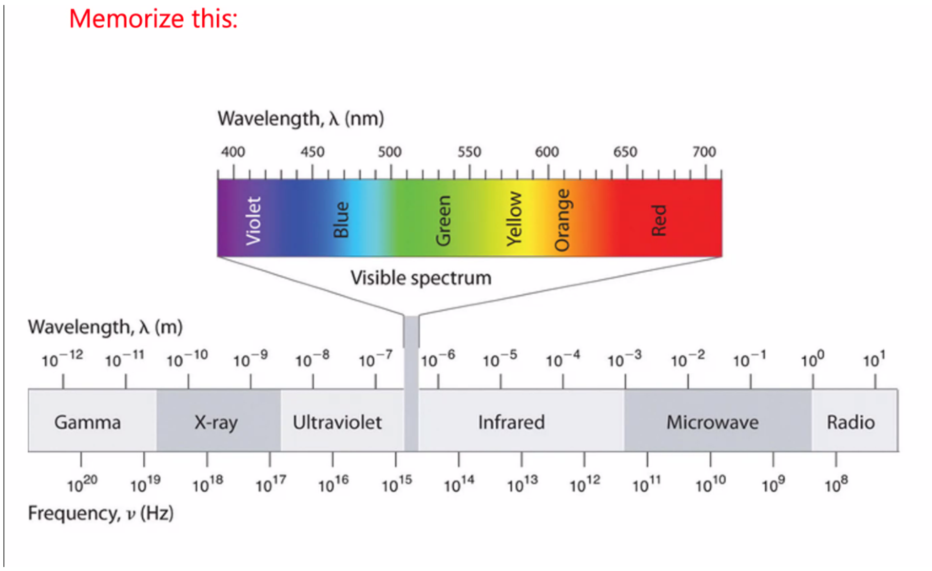
Magnitudes

- M: Absolute magnitude, magnitude as seen at 10 pc?
- m: Apparent magnitude, magnitude as seen at observer's distance
- Small magnitude = brighter
- Big mag = dimmer
- It's backwards
- Pro tip: when looking at a plot, take your time to orient yourself (which is left, which is right, up, down, brighter, dimmer, more massive, less massive, etc) because astronomers are out of their minds and don't make graphs like normal people.

The Milky Way is a SBb-SBc bar spiral.

1.1.2 Filter Systems

- F(###) - filter (centered at) note to watch out for inferred numbers
  - Example: U365
- Standard UBV System (Johnson system):
  - U - ultraviolet centered at 365 nm
  - B - blue centered at 440 nm
  - V - visual centered at 550 nm
- Extension: UBVRI
  - R - red
  - I - Infrared



1.1.3 Differential Star Count

- Counts the number of stars with an absolute magnitude between M and $M = dM$ that are found within a solid angle Ω and have apparent magnitudes in the range between m and $m + dm$:

$$A_M(M, S, \Omega, m)\ dM\ dm \equiv \frac{d\bar{N}_M(M, S, \Omega, m)}{dm}\ dM\ dm \tag{C\&O 24.4}$$

- In special case where we assume no interstellar extinction ($A = 0$) and infinite universe of uniform stellar density (i.e. $n_M(M, S, \Omega, r) = n_M(M, S) = \text{constant}$),

$$\begin{aligned} A_M(M, S, \Omega, m) &= \frac{d\bar{N}_m(M, S, \Omega, m)}{dm} \\ &= \frac{\ln 10}{5}\Omega n_m(M, S) 10^{3(m-M+5)/5} \\ &= \frac{3\ln 10}{5}\bar{N}_M(M, S, \Omega, r) \end{aligned} \tag{C\&O 24.5}$$

1.1.4 Integrated Star Count

- Counts the total number of stars with absolute magnitudes in the range M to $M + dM$ that appear brighter than the limiting magnitude, m (replaces the limiting distance r - see C&O 24.1):

$$n(S, \Omega, r) = \int_{-\infty}^{\infty} n_M(M, S, \Omega, r)\ dM \tag{C\&O 24.2}$$

  where $n_M(M, S, \Omega, r)\ dM$ is the number density of stars with attribute S that lie within a solid angle Ω in a specific direction.

- In the special case where we assume no interstellar extinction ($A = 0$) and infinite universe of uniform stellar density (i.e. $n_M(M, S, \Omega, r) = n_M(M, S) = \text{constant}$),

$$\begin{aligned} \bar{N}_M(M, S, \Omega, m) &= \frac{\Omega}{3} n_M(M, S) 10^{3(m-M+5)/5} \\ &= \frac{\Omega}{3} n_M(M, S) \exp\left(\ln 10^{3(m-M+5)/5}\right) \\ &= \frac{\Omega}{3} n_M(M, S) e^{[3(m-M+5)/5]\ln 10} \end{aligned}$$

1.1.5 Obscuration

- Use distance to solve for extinction A_λ:

$$d = 10^{(m_\lambda - M_\lambda - A_\lambda + 5)/5} \implies A_\lambda = m_\lambda - M_\lambda + 5 - 5\log_{10} d \qquad \text{(C\&O 24.1)}$$

- Observe using infrared (longer wavelengths) to penetrate dust

1.1.6 Milky Way Components

TABLE 24.1 Approximate Values for Various Parameters Associated with the Components of the Milky Way Galaxy. Definitions and details are discussed in the text.

Disks	Neutral Gas	Thin Disk	Thick Disk
M (10^{10} $M_\odot$)	0.5^a	6	0.2 to 0.4
L_B (10^{10} $L_\odot$)^b	—	1.8	0.02
M/L_B ($M_\odot/L_\odot$)	—	3	—
Radius (kpc)	25	25	25
Form	e^{-z/h_z}	e^{-z/h_z}	e^{-z/h_z}
Scale height (kpc)	< 0.1	0.35	1
σ_w (km s^{-1})	5	16	35
[Fe/H]	$> +0.1$	-0.5 to $+0.3$	-2.2 to -0.5
Age (Gyr)	$\lesssim 10$	8^c	10^d

Spheroids	Central Bulge^e	Stellar Halo	Dark-Matter Halo
M (10^{10} $M_\odot$)	1	0.3	$190^{+360\,f}_{-170}$
L_B (10^{10} $L_\odot$)^b	0.3	0.1	0
M/L_B ($M_\odot/L_\odot$)	3	~ 1	—
Radius (kpc)	4	> 100	> 230
Form	boxy with bar	$r^{-3.5}$	$(r/a)^{-1}(1 + r/a)^{-2}$
Scale height (kpc)	0.1 to 0.5^g	3	170
σ_w (km s^{-1})	55 to 130^h	95	—
[Fe/H]	-2 to 0.5	< -5.4 to -0.5	—
Age (Gyr)	< 0.2 to 10	11 to 13	~ 13.5

[a] $M_{\text{dust}}/M_{\text{gas}} \simeq 0.007$.

[b] The total luminosity of the Galaxy is $L_{B,\text{tot}} = 2.3 \pm 0.6 \times 10^{10}$ $L_\odot$, $L_{\text{bol,tot}} = 3.6 \times 10^{10}$ $L_\odot$ ($\sim$ 30% in IR).

[c] Some open clusters associated with the thin disk may exceed 10 Gyr.

[d] Major star formation in the thick disk may have occurred 7–8 Gyr ago.

[e] The mass of the black hole in Sgr A* is $M_{\text{bh}} = 3.7 \pm 0.2 \times 10^6$ $M_\odot$.

[f] $M = 5.4^{+0.2}_{-3.6} \times 10^{11}$ $M_\odot$ within 50 kpc of the center.

[g] Bulge scale heights depend on age of stars: 100 pc for young stars, 500 pc for old stars.

[h] Dispersions increase from 55 km s^{-1} at 5 pc to 130 km s^{-1} at 200 pc.

1.1.7 Milky Way Kinematics

- Does not follow Keplerian model, Flat velocity curve

- Density wave theory: explains the winding problem and how stars move in and out of the spiral arms, the spiral arms originate from quasi-static density waves – traffic build up example. Each star has its tilted stellar orbits
- Perigalacticon/Apogalacticon calculation

1.1.8 Galaxy Rotation Curves

- Doesn't follow Keplerian model
- Flattens out at higher radii, follows Keplerian at smaller radii
- Rotation curve is determined by the mass enclosed (light matter and dark matter)
- Inner region (up to ~ 5 kpc), density of stars goes as $\sim 1/r$, velocity goes as $\sim \sqrt{2}$
- Outer region, need density go as $\sim 1/r^2$ to create flat rotation curve, velocity goes as $\sim 1/\sqrt{r}$
  - M_r becomes constant

1.1.9 Evidence for Dark Matter

- It is not gas. Hot gas would have emission lines, cold gas would have absorption lines
- It is not asteroids, rocks or dust: there isn't enough
- Flat rotation curves
- Mass-to-light ratio: the ratio between the measured luminosity and the estimated mass. It does not account for the rotation of the stars.
- Gravitational lensing

1.1.10 MACHOs, WIMPs

- MACHO: massive compact halo objects: brown dwarfs, white dwarfs, neutron stars, black holes, etc.
- WIMPs: weakly interacting massive particles (neutrinos)
- Both possible explanations for dark matter
- Does not explain all the mass expected

1.1.11 NFW Profile

- NFW density profile

$$\rho_{NFW}(r) = \frac{\rho_0}{(r/a)(1+r/a)^2} \qquad \text{(C\&O 24.52)}$$

- Averages out to be $1/r^2$ over much of the halo to explain that velocity = constant on the rotation curves
- Shallow near the center ($\sim 1/r$) near the center
- Steeper ($\sim 1/r^3$) near the edge of the halo
- Total mass contained within NFW profile is still not bound (like problem 4 in homework)
- For galaxies, $a \sim 10 - 30$ kpc (scale radius)

1.1.12 Galactic Center Observations

- 1 m wavelength $\implies$ radio
- Adaptive optics: Keck Telescope using adaptive optics (AO) to observe $Sgr\ A^*$

1.1.13 Evidence for a SMBH in the Galactic Center

- Rotation of S2 and other stars around the center of the galaxy. (Seen through the microwave)
- No other explanation besides a SMBH.
- X-ray emission.
- Used orbits of stars to calculate mass of center = $3.7 \pm 0.2 \times 10^6\ M_\odot$

1.1.14 Virial Theorem

$$\langle E \rangle = \langle K \rangle + \langle U \rangle$$

- See Lecture 3, Section 2.4 of C&O
- $-2\langle K \rangle = \langle U \rangle$
  - Note: This form applies to special case where galaxy is in equilibrium and gravitationally bound

1.1.15 Sgr A* Luminosity Function

- Luminosity comes from accretion
- Using the virial theorem: $\langle E \rangle = \frac{1}{2}\langle U \rangle$
- Luminosity is dE/dt

1.2 The Nature of Galaxies (C&O Chapter 25)

1.2.1 The Great Debate Over Spiral Nebulae

- Side 1) The mysterious "spiral nebulae" are nebulae within our own galaxy
- Side 2) They are outside our galaxy - "island universes"
- Shapley: supported idea of nebulae being in our Galaxy - argued using apparent magnitudes of novae - argued that if the disk of Andromeda were as large as the Milky Way, then its angular size in the sky would imply a distance to the nebula so large that luminosities of novae would be greater than those found in Milky Way. Also argued the points of Maanen: proper-motion measurements of M101 suggest angular rotation of 0.02" yr^{-1}, if diameter similar to Milky Way then why rotational speed not larger?
- Curtis: supported the idea of spiral nebulae Being outside our Galaxy - argued that the novae observed must be at least 150kpc away in order to have intrinsic brightnesses comparable to those in the Milky Way. Also argued that the large radial velocities measured for spiral nebulae indicated they could not remain gravitationally bound within a Kapteyn-model Milky Way. (Lecture 1)

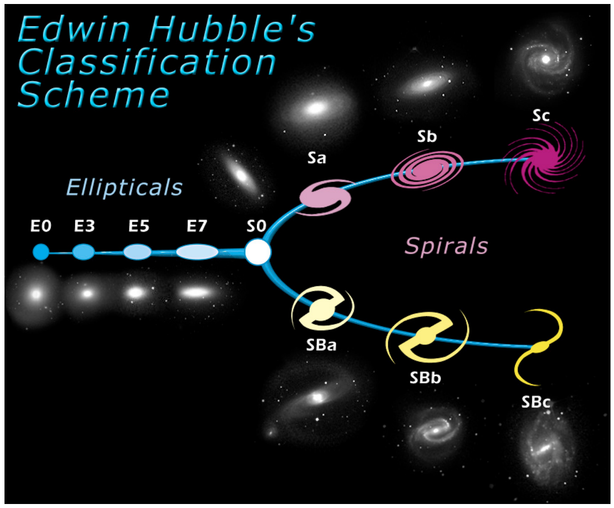
Curtis was proven correct when Hubble detected Cepheid variable stars in M31. Using apparent magnitudes to determine absolute magnitudes, and then using the period-luminosity relation:

$$M_{\langle V \rangle} = -2.81 \log_{10}(P_d) - 1.43 \qquad \text{(C\&O 14.1)}$$

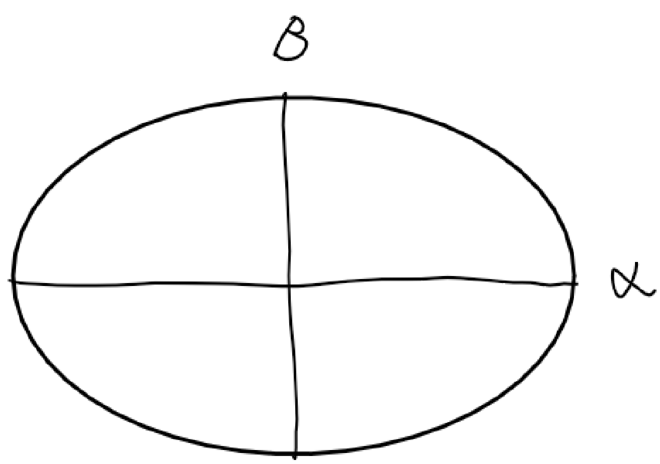
where $M_{\langle V \rangle}$ is the average absolute V magnitude and P_d is the pulsation period in units of days.

Hubble was able to approximately calculate the distance to Andromeda to be outside the Milky Way Galaxy.

1.2.2 Galaxy Morphologies, Hubble Classification Scheme



- 3 types: ellipticals, spirals, irregulars
- E0 → E7: greater ellipticity
- S0: disk, bulge, no arms
- Sa → Sc: lower bulge-to-disk size and luminosity ratios, spiral arms less tightly wound
- NOT an evolutionary sequence
- Observed ellipticity: $\epsilon = 1 - \beta/\alpha$



1.2.3 Galaxy Surface Brightness, Sérsic Profile

- A galaxy's surface brightness μ is the amount of flux from the galaxy per square arcsecond on the sky
- Calculate luminosity, then total flux from all stars
- Surface brightness is unrelated to distance
- Observer units: μ units: mag arcsec^{-2}
- Theorist units: I units: $L_\odot \ pc^{-2}$

- K-correction: de-redshifted
- de Vaucouleurs Profile: surface brightness $I \sim r^{1/4}$ for spiral galaxy bulges and elliptical galaxies:

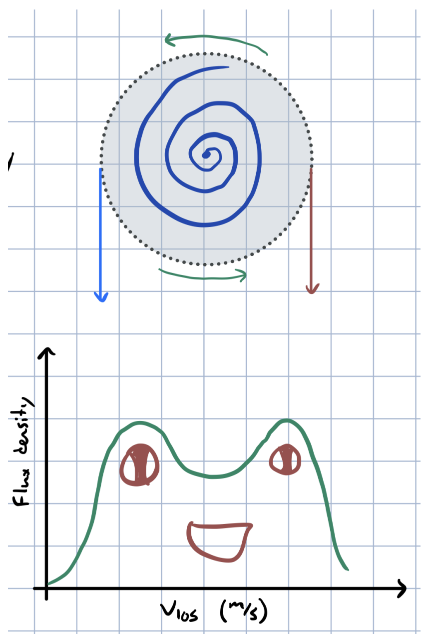
$$\log_{10}\left[\frac{I(r)}{I_e}\right] = -3.3307\left[\left(\frac{r}{r_e}\right)^{1/4} - 1\right] \qquad \text{(C\&O 24.13)}$$

where I is the surface brightness measured in units of $L_\odot\ pc^{-2}$, r_e is a reference radius (called the effective radius), and I_e is the surface brightness at r_e. r_e is defined to be that radius within which one-half of the bulge's light is emitted.

  - **Note**: the equation above uses I as the surface brightness which is consistent with C&O, but in class we have used μ as the surface brightness.

1.2.4 How to Measure Rotation in Other Galaxies

- Neutral H in the interstellar medium
- The spin flip transition of HI emits photons of wavelength 21 cm



(Art courtesy of James l'Artiste.) Find v_{max} with right peak - middle or left peak + middle

1.2.5 Tully-Fisher Relation

- Tully and Fisher studied 21-cm emission lines in spiral galaxies and found that the absolute magnitude is related to the rotation
- v_{max} is easiest to find, so measure v_{max}, use Tully-Fisher to get M (absolute magnitude), and then distance - you also need m (apparent magnitude)

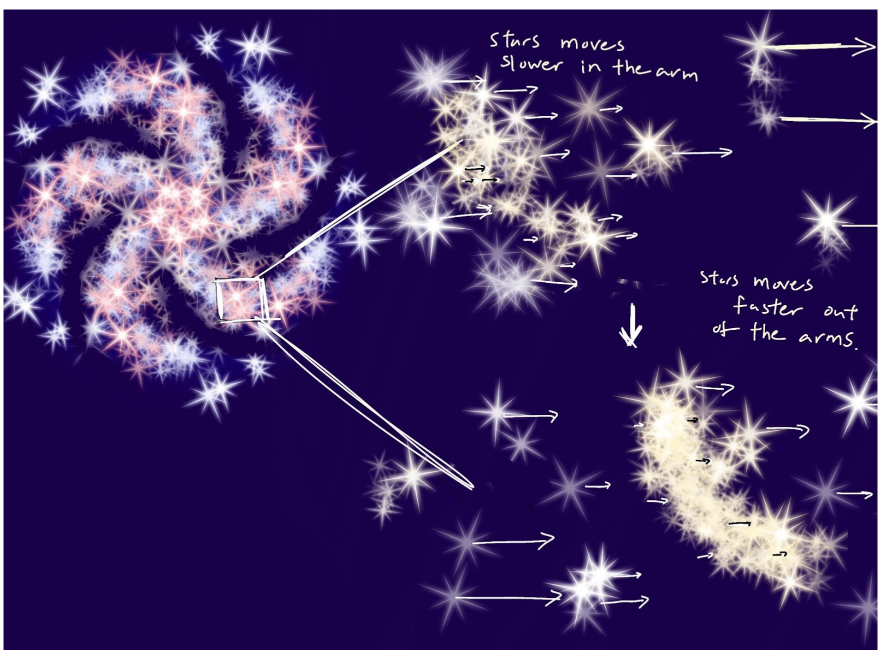
$$\boxed{\begin{aligned} M_B &= -9.95\log_{10} V_{\max} + 3.15 \qquad \text{(Sa)}, \\ M_B &= -10.2\log_{10} V_{\max} + 2.71 \qquad \text{(Sb)}, \\ M_B &= -11.0\log_{10} V_{\max} + 3.31 \qquad \text{(Sc)}. \end{aligned}} \qquad \begin{aligned} (25.5) \\ (25.6) \\ (25.7) \end{aligned}$$

1.2.6 Types of Spiral Arms

- Grand-design spiral: 2 large arms - 10% of the spiral galaxy
- Multiple-arm: > 2 large arms - 60% of the spiral galaxy
- Flocculent spiral: Not well defined arms - 30%

1.2.7 Winding Problem, Density Wave Theory

- Traffic jam produced by slow moving truck (density wave) while cars (stars) slow down while moving around the truck.
- If we assume that all the stars in the galaxy stay together as they orbit, the spiral arms would wind up until we eventually have no spiral arms.
- This assumption is wrong, instead the stars constantly speed up or slow down as they enter and exit the spiral arms.



Chapter 2

Exam 2 Content

2.1 The Nature of Galaxies

2.1.1 Spiral Galaxies versus Elliptical Galaxies: Major Differences

Spiral Galaxies

- Late-type in Hubble classification scheme
- Blue (which means less star formation!)
- Generally less luminous than ellipticals
- More abundant in the field
- Mass: 10^9 to $10^{12} M_\odot$ (less massive on average)

Elliptical Galaxies

- Early-type in Hubble classification scheme
- "Red and dead" (not forming new stars)
- Generally more luminous (brighter)
- More abundant in clusters
- Mass: 10^9 to $10^{14} M_\odot$ (more massive on average)

Bonus: Irrigular Galaxies!

- Sort of in between
- Usually the result of mergers

2.1.2 Stellar Velocity Dispersion

- "Statistical dispersion of stellar velocities around the man stellar velocity in a galaxy"
- Derived from the Virial Theorem to derive mass: $-2\langle K\rangle - \langle U\rangle$
  - Assuming spherical distribution, $\langle v_r^2\rangle = \sigma_r^2$ where σ_r is the "radial velocity dispersion"
- Side note: **virial mass** $M \approx 5R\sigma_r^2/G$ can be used to calculate the total mass of an elliptical galaxy or the bulge of a spiral galaxy but not the whole spiral galaxy

- Measure stellar absorption lines, add them all up to make a galaxy spectrum and measure the velocity dispersion to get the total mass of a galaxy. The wider the absorption line, the more massive the galaxy.

2.1.3 Faber-Jackson Relation

- Correlation between central velocity dispersion and luminosity
- Derived from the virial theorem
- For elliptical galaxies and spiral bulges (note: Tully-Fisher was for spiral galaxies)
- Brighter galaxies have larger velocity dispersions
- $L \propto \sigma^4 \propto L_\odot 10^{M_\odot/2.5} 10^{-M/2.5}$ where σ is the velocity dispersion and L is the luminosity
- $\log(\sigma_r) \propto -M$

2.1.4 The Fundamental Plane

- σ, luminosity, and size are the fundamental parameters of an elliptical galaxy or spherical bulge
- We often see 2D projections of the fundamental plane (i.e. Faber-Jackson relation)

2.1.5 Galaxy Luminosity Function

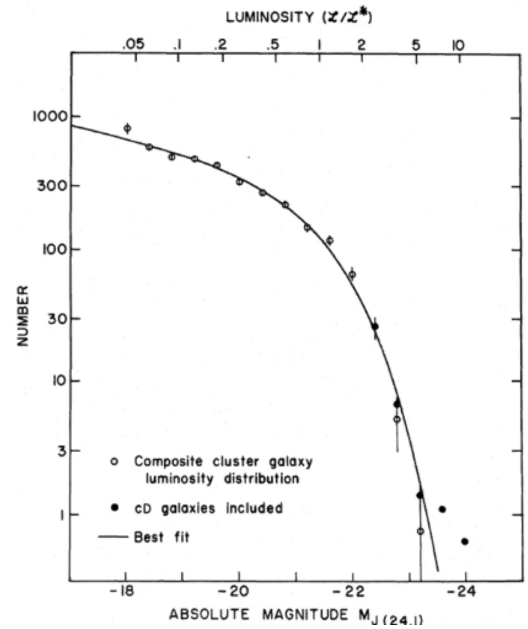
- Relative number of galaxies at each luminosity
- Number density of galaxies in a particular sample that have luminosities between L and $L + dL$:

$$\Phi(L)\ dL = \frac{\sigma^*}{L^*}\left(\frac{L}{L^*}\right)^\alpha e^{-L/L^*}\ dL$$

- When $L \ll L^*$, $L \to 0$
- In magnitudes:

$$\Phi(M)dM \approx 10^{0.4(\alpha+1)M} \exp\left(-10^{0.4(m^*-M)}\right)\ dM$$

- The "knee" is the turnover point where $\alpha = 1$
- To measure the luminosity function:
  1. Measure the apparent magnitudes for all galaxies in the sample
  2. Convert to absolute magnitudes
  3. Calculate K-correction
  4. Count the number of galaxies in each K-corrected absolute magnitude bin, then divide te number of galaxies by the volume surveyed.



2.1.6 The K-Correction

- De-redshift

- K-correction "corrects" for the fact that sources observed at different redshifts are compared with each other at different rest wavelength bands

- Calculating the absolute magnitude of galaxies requires making corrections to their observed apparent magnitudes if we are to properly account for the effect of extinction, both within the Milky Way and within the target galaxy.

2.2 Galactic Evolution

2.2.1 Major versus Minor Galaxy Mergers

- Major merger: mass ratio of merging galaxies is between 3:1 and 1:1

- Minor merger: mass ratio of merging galaxies is below 3:1

2.2.2 Tidal Stripping and Tidal Tails

- Tidal stripping:

$$F = \frac{2GMmR}{r^3}$$

  where F is the tidal force on the galaxy of mass m, has radius R, and is a distance r from a galaxy with mass M.

- Tidal tails are a result of tidal stripping as the tidal forces unbind gas and stars from galaxies

- The Milky Way is currently stripping material from the Large and Small Magellanic Clouds!

2.2.3 Dynamical Friction

- As an object of mass M moves through a galaxy, a high-density "wake" forms behind it. This wake exerts net gravitational force on M that opposes its forward motion, slowing it down.
  - This is the reason for mergers, otherwise objects would just pass through each other.
- Force due to dynamical friction is given by

$$F_d = -4\pi \ln(\Lambda) \left(\frac{G^2 M^2 \rho}{v^2} \right)$$

  where $\Lambda = b_{max}/m_{min}$ (depends on the material/density of material the galaxy is moving through) and $\rho = nm$
- In class, we assumed the density of the dark matter halo to be

$$\rho(r) = \frac{v^2}{4\pi G r^2}$$

2.2.4 Initial Mass Function

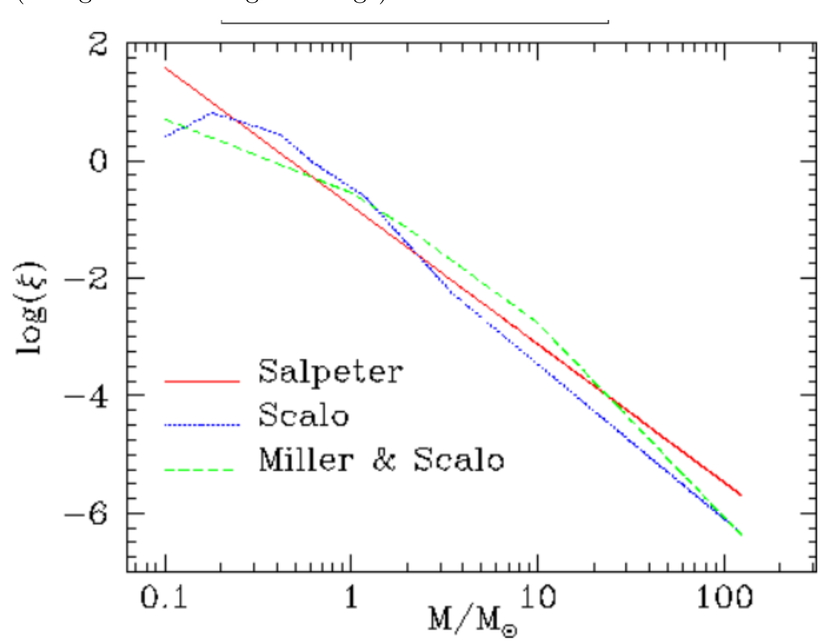
- Only gives distribution of stellar masses immediately after stars are born (doesn't give mass distribution of, for example, stars in the Milky Way today).
-

$$\xi(M) = \frac{dN}{dM} = CM^{-(1+x)}$$

  where N is the number of stars, M is the mass of the stars, and C is a normalization constant. For stars with masses in the range $7M_\odot < M < 35M_\odot$, $x = 1.8$.
- The above leads directly to

$$N = \int_0^\infty \xi(M)\, dM$$

  for *all* masses (change bounds for given range).



2.2.5 Eggen, Lynden-Bell, and Sandage Collapse Model

- Galaxy forms all at once from direct collapse of a proto-galactic nebula
- "Top-down" model

2.2.6 Hierarchical Merger Model

- Stars in the stellar halo were a part of stellar clusters in initial proto-galaxies (some clusters survived to become globular clusters)
- Proto-galactic gas clouds collided and settled toward the center, forming a thick disk
- Gas continued to settle onto the midplane, forming a thin disk
- Stripped gas from satellite galaxies in recent mergers settled toward the galactic center accounting for the young stars in the bulge
- "Bottom-up" model: small galaxies merge to larger galaxies
- Future gas for bulge: tidal stripping of LMC and SMC

Galaxy Structure

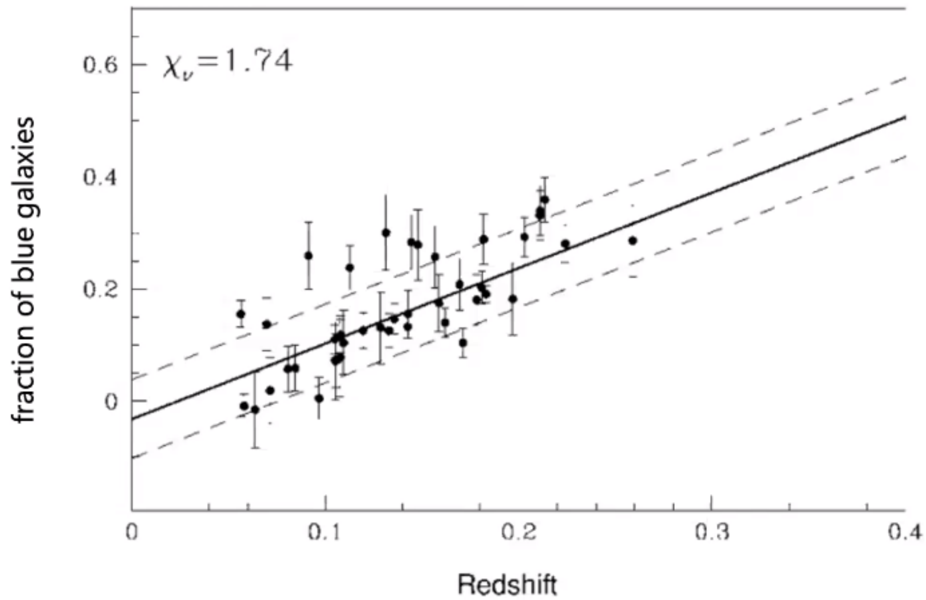
1. **Globular Clusers and Stellar Halos:** stellar clusters formed in proto-galaxies and merged to form galaxies. Some stars tidally got stipped away to become stars of stellar halo and some stellar clusters survived to become globular clusters.
2. **Thick Disk:** proto-galactic gas clouds collided and settled towards the center of the galaxy and then cooled to form new stars
3. **Thin Disk:** After the thick disk formation, gas continued to settle onto a galactic midplane and formed new stars
4. **Young Stars in the Bulge:** recent mergers with satellite galaxies stripped gas and settled towards the galactic center
5. **Future Gas for the Bulge:** tidal stripping of LMC and SMC

Morphology-Density Relation

Due to more mergers occurring in dense environments (clusters) and the transformation of spirals into ellipticals during mergers, elliptical galaxies are more abundant in clusters.

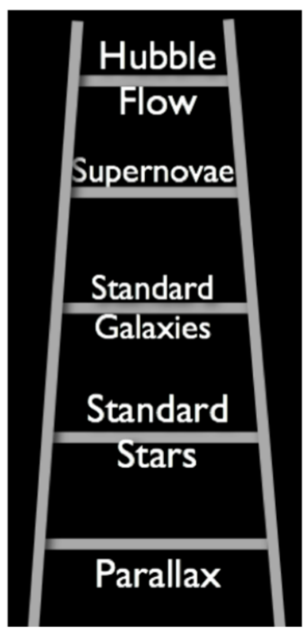
Butcher-Oemler Effect

- Galaxies becoming redder over time
- If a $z = 0$ galaxy cluster has 40% ellipticals, a $z = 2$ galaxy cluster has $< 40\%$ ellipticals and more blue spirals. Note: redshift of 2 corresponds to most merger events



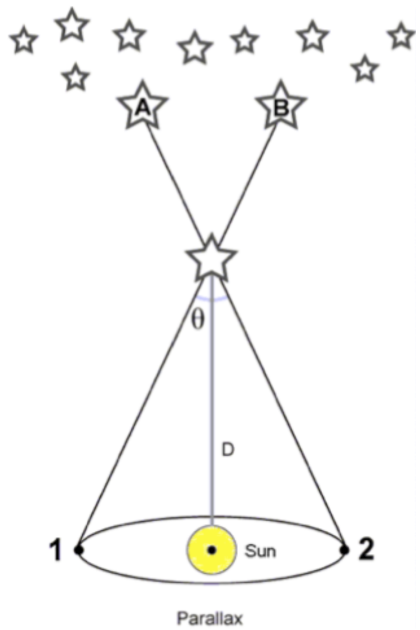
2.3 The Structure of the Universe

2.3.1 The Cosmological Distance Ladder



Parallax

- Only out to one kiloparsec



Cepheid Variable Stars

- Out to 30 Mpc
- Cepheids pulsate rapidly and we can measure the period of pulsations
- Period-luminosity relationship for Cepheids:

$$M_V = -3.53 \log(P_d) - 2.13 + 2.13(B - V)$$

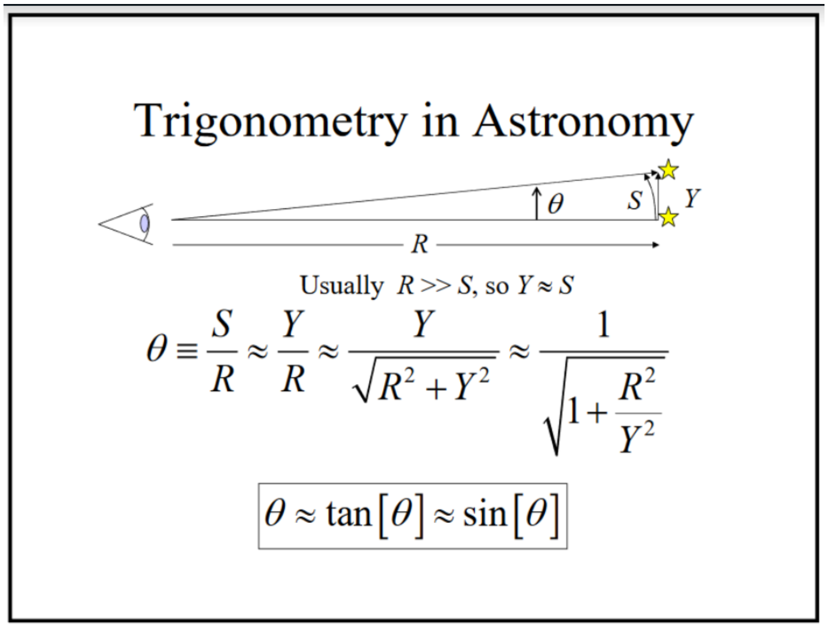
  where M_V is the absolute visual magnitude, P_d is the period in days, and $B - V$ is the color index.
- What can we find?
  - Observe P_d and $B - V$, infer M_V
  - Observe m_V, use M_V from above to get distance ($d = 10^{(m-M+5)/5}$ parsecs)
- Notes: Blue Cepheids are brighter, longer period means brighter

Tully-Fisher Relation

- Out to 100 Mpc
- Only works for spiral galaxies
- What can we find?
  - Observe v_{max} (distance between center and peak of graph), infer M_B
  - Observe m_B, infer distance

Supernovae

- Out to $> 1000 Mpc$
- Measure the size of a nearby supernova's photosphere



- Type Ia light curves:
  - The maximum brightness of a supernova is inversley correlated with the rate of light curve decline (bright supernovae decline more slowly)
  - What can we find?
    - Observe rate of decline, infer M
    - Observe m, combine with peak M to get distance

Hubble Flow

- Most galaxies exhibit redshifts in their spectra: $v = cs$ for $v \ll c$.
  - Farther galaxy means larger redshift means moving faster means "Hubble flow"
- Hubble's Law: $v = H_0 d$ where v is the galaxy's velocity along the line of sight, d is the distance to the galaxy in Mpc, and H_0 is Hubble's constant (current value of 71 km/sec/Mpc - controversial)
- Highest rung on the distance ladder - most galaxies

Method	Uncertainty for Single Galaxy (mag)	Distance to Virgo Cluster (Mpc)	Range (Mpc)
Cepheids	0.16	$15 - 25$	29
Novae	0.4	21.1 ± 3.9	20
Planetary nebula luminosity function	0.3	15.4 ± 1.1	50
Globular cluster luminosity function	0.4	18.8 ± 3.8	50
Surface brightness fluctuations	0.3	15.9 ± 0.9	50
Tully–Fisher relation	0.4	15.8 ± 1.5	> 100
D–σ relation	0.5	16.8 ± 2.4	> 100
Type Ia supernovae	0.10	19.4 ± 5.0	> 1000

2.3.2 Expanding Universe, Hubble's Law

- The further a galaxy is from Earth, the fast it is moving away and the larger its redshift z
- For non-relativistic motion:
$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{v}{c}$$
- For relativistic motion:
$$z = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1 \rightarrow \frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$
- Hubble's Law $v = H_0 d$ where H_0 is the Hubble constant. $H_0 = 100h$ km/s/Mpc
- Most galaxies are red-shifted and moving away, but some are blue-shifted, for example, Andromeda (M31)

2.3.3 Peculiar Velocity

- A galaxy's own velocity through space (as opposed to recessional velocity which is the velocity of the expanding universe carrying the galaxy along)
- If the recessional velocity is less than the peculiar velocity, the object is coming towards you!

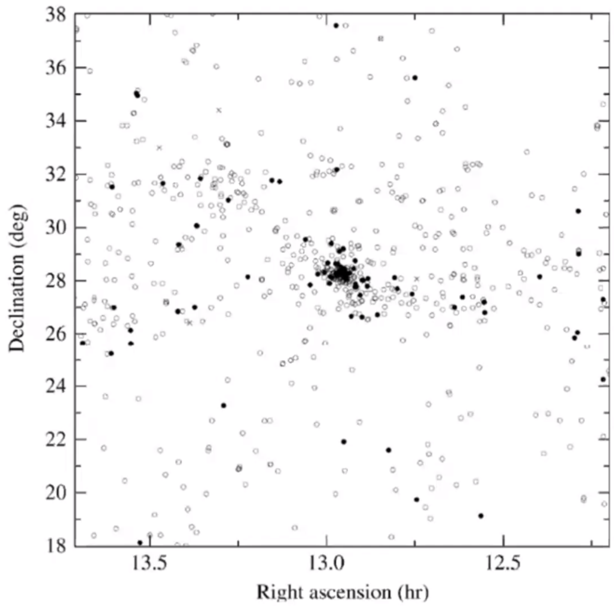
2.3.4 The Age of the Universe

- Assume constant rate of expansion
- $t = d/v = d/(H_0 \times d) = 1/H_0$
- Gives estimate of around 13 billion years which isn't too far off!

2.3.5 Distribution of Galaxies in the Universe

- More ellipticals near center of galaxy cluster due to being more dense and more likely for galaxies to merge and form ellipticals
- More likely for spiral galaxies to be on outer edge where mergers less likely

- Spirals more abundant in field, ellipticals more abundant in clusters



2.3.6 Groups and Cluster

- Most galaxies are found in groups or clusters: gravitationally bound associations of galaxies

- Groups have less than 50 members, diameter $1.4h^{-1}$ Mpc, velocity dispersion 150 km/s, and mass $2 \times 10^{13} h^{-1} M_{\odot}$.

- Clusters have between 50 and 1000 members (poor to rich, respectively) with diameter $6h^{-1}$ Mpc, velocity dispersion between 800 and 1000 km/s, and mass $10^{15} M_{\odot}$.

	Groups	Clusters
Number of members	<50	50 – 1000s (poor to rich)
Diameter (h^{-1} Mpc)	1.4	6

- Local Group: Milky Way and Andromeda, M33, and Pinwheel galaxy

- Nearest galaxy clusters:
  - Virgo Cluster: 16 Mpc away, 250 larger galaxies, 2000 smaller galaxies, diameter 3 Mpc
  - Coma Cluster: 90 Mpc away, roughly 10000 member galaxies, diameter 6 Mpc

- There are more elliptical galaxies at the center of a galaxy cluster

- Virial Mass for a galaxy cluster (lecture 12 boardwork):

$$M = \frac{5R\sigma^2}{G}$$

2.3.7 Evidence for Dark Matter from Cluster Masses

- Fritz "Nuclear Goblins Guy" Zwicky measured the velocity dispersion and estimated the cluster mass
- Compared to the total mass of galaxies in the cluster
- Total mass of galaxies did not account for all the mass in the cluster
- "Missing mass" turned out to be dark matter and hot intracluster gas

2.3.8 Intracluster Gas

- Roughly 90% of the baryonic mass of a galaxy cluster is in the form of ionized gas
- Gas radiates via thermal bremsstrahlung: free-free emission of x-ray photons
- Using ideal gas law and HSE, can derive galaxy cluster mass as a function of r entirely from hot intracluster gas quantities:
$$M_r = -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r}\right)$$
where μ is a constant.
- To find the total mass, evaluate $M_r(r = R)$.
- The hotter the gas, the more massive the cluster.

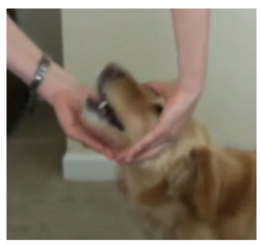
Chapter 3

Final Exam Content

3.1 Active Galaxies

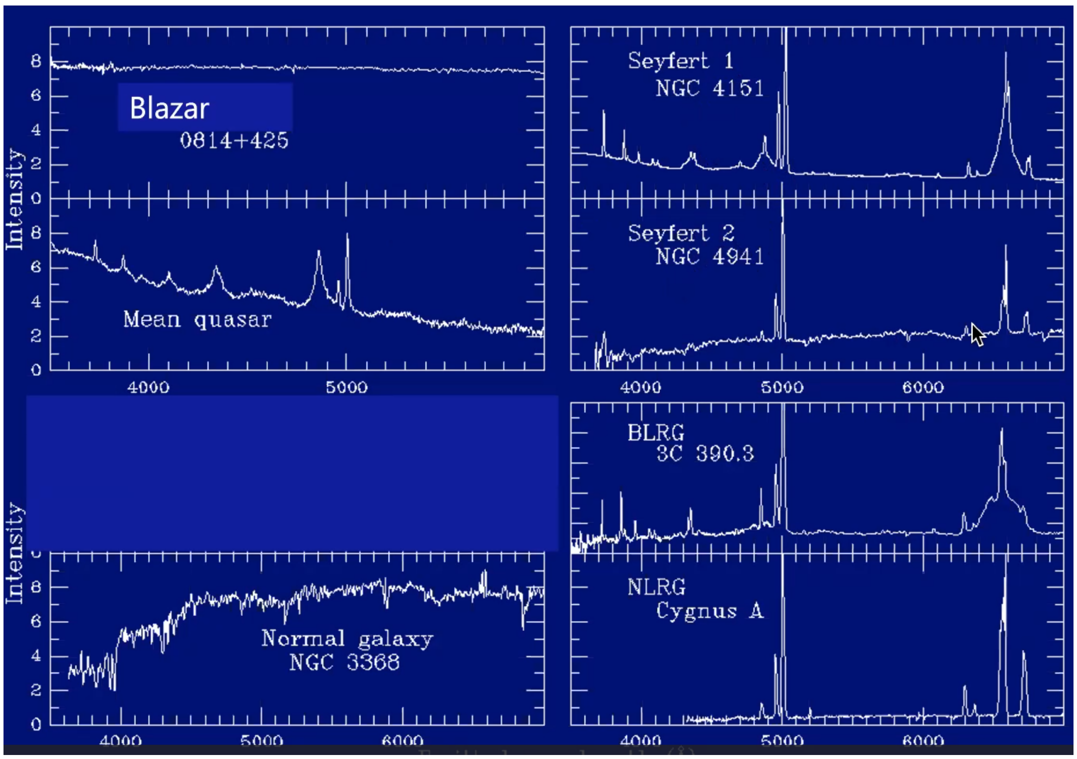
3.1.1 Active Galactic Nuclei

- Active Galactic Nuclei: Supermassive black holes that are active/accreting gas (most of them are inactive). Their host galaxy is called an Active galaxy
  - FYI: never put your nose across the event horizon boundary

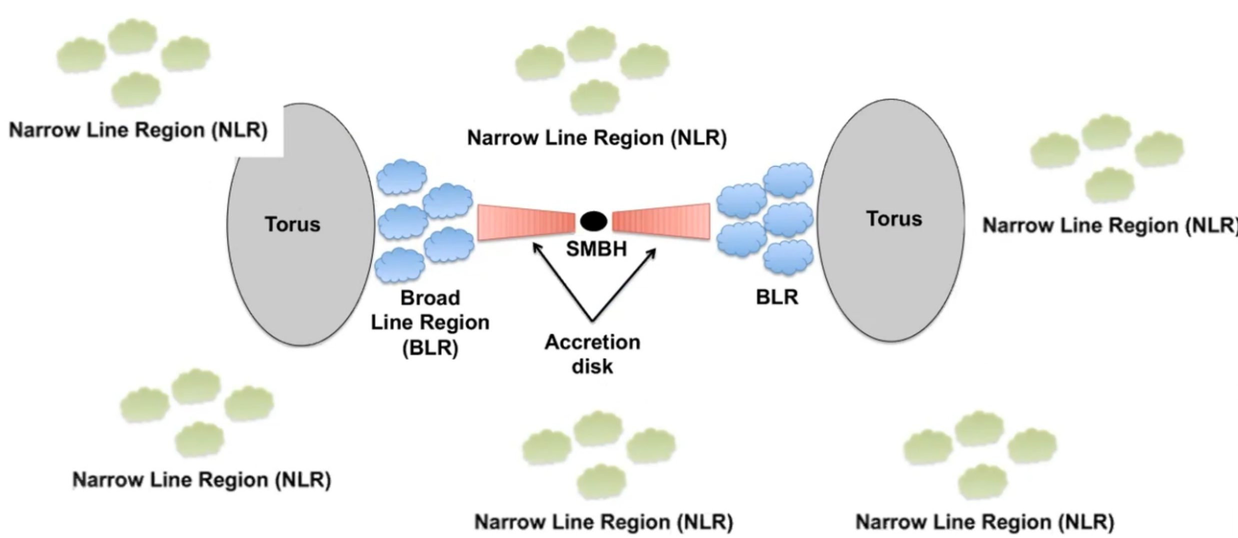


  - Calculating if another galaxy has a SMBH by measuring spectra of the gas at the center, get a velocity map and calculate the mass
- Types:
  - Quasars
    - Quasi-stellar radio sources
    - Very luminous (most luminous AGN) and visible from far away
    - Have both broad and narrow emission lines in their spectra
    - Quasars are radio-loud, QSOs are radio-quiet
    - Their energy comes from accretion onto supermassive black holes
    - Even 1 solar mass of accreting gas can generate enough energy to outshine the galaxy
    - To find out if a source is a star or an AGN, look at the spectra
  - Seyferts
    - Most common types of AGNs (weaker AGN)
    - Usually found in spiral galaxies
    - Defined by their spectra:
      - Type 1 Seyfert: has both broad and narrow emission lines
      - Type 2 Seyfert: has narrow emission lines only
  - Radio Galaxies
    - Have strong radio emission (from synchrotron radiation)

    * Usually found in elliptical galaxies
    * Two categories:
        · BLRG = broad and narrow emission lines
        · NLRG = narrow emission lines only
    * Radio jets
- Blazars
    * Have almost no emission lines
    * Strong radio emission
    * Usually found in elliptical galaxies



3.1.2 AGN Structure



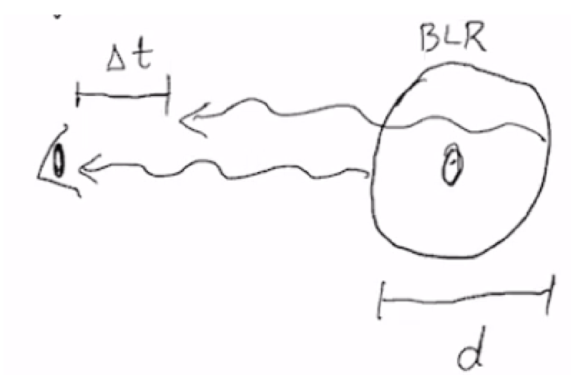
More specifics on how this works and what angles to see, etc. in Lecture 16, slides 6-10.

3.1.3 AGN Luminosity

- Accretion luminosity:
$$L_{disk} = \eta \dot{M} c^2 \tag{C\&O 28.6}$$
where η is the efficiency of the process, usually aroun 10%.
- Not all the energy of the accretion disk goes into luminosity, some goes into heat
- There is an upper limit to the luminosity you can get: the Eddington Luminosity Limit is given by
$$L_{Ed} = \frac{4\pi G c}{\bar{\kappa}} M \simeq 1.5 \times 10^{31} \frac{M}{M_\odot}\ W \tag{10.114}$$
and represents the maximum radiative luminosity that a star can have and still remain in hydrostatic equilibrium.

AGN Variability

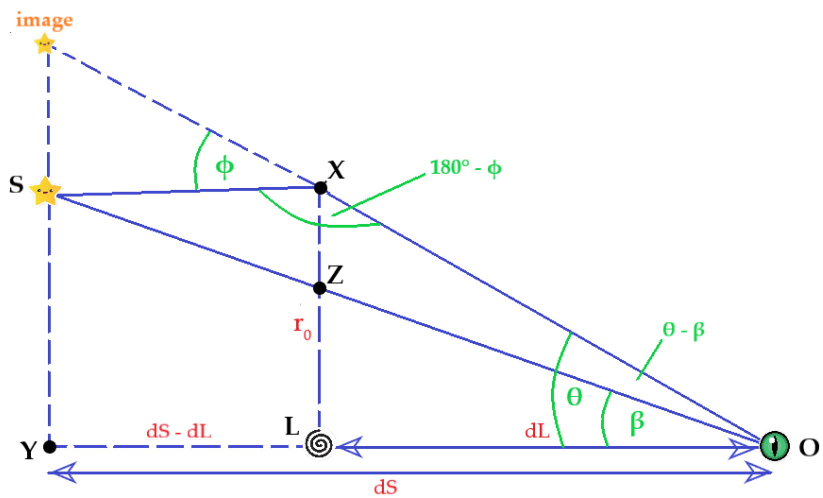
- Luminosity can vary on timescales as short as months, weeks or days
- Broad emission lines are variable but narrow emission lines have little variation
- Calculate the size of the Broad Line Region (BLR), using the timescale of variability: $d = c\Delta t$



- Calculate the size of the Narrow Line Region (NLR), using the Stromgren radius:
$$r_{NLR} \approx \left(\frac{3N}{4\pi\alpha_{qm}\epsilon}\right)^{1/3} \frac{1}{n_e^{2/3}} \tag{C\&O 28.13}$$
- Flickering AGN (burping): supermassive black hole that accretes some gas (became an AGN), and created radiation. Then the AGN stopped accreting, then later on it started accreting gas again. Caused by galaxy mergers. Fermi bubble: echo from Sgr A*.
- Voorwerp: echos of emission from an AGN that has since turned off
- Spectral Energy Distribution (SED) of an AGN: spectrum that covers all wavelengths
  - Radio: radio-loud and radio quiet AGN
  - IR bump: reradiated light from the dusty torus
  - Big blue bump: in UV, direct thermal emission from all the gas in the accretion disk
  - X-ray: Compton scattering of the accretion disk photons
- Monochromatic Energy Flux: AGN can be approximated by
$$F_\nu \propto \nu^{-\alpha} \tag{C\&O 28.1}$$

3.1.4 Gravitational Lensing

- Massive objects warp spacetime, more massive, more curved
- Gravitational wave:
  - Any massive object rolling around creates gravitational waves
  - Gravitational waves stretch and compress Earth as they pass by
  - LIGO has detected 70 mergers
  - LISA to detect gravitational waves in space using laser interferometry between three free-flying spacecraft
- Gravitational Lensing



  - Light from a background source traveling through curved space-time near a massive lens can create multiple observed images of the source. The angular deviation of a photon passing a distance r_0 from a mass M is
$$\phi = \frac{4GM}{r_0 c^2} \text{ rad} \qquad \text{(C\&O 28.20)}$$
  - Lens equation:
$$\theta^2 - \beta\theta - \frac{4GM}{c^2}\left(\frac{d_s - d_L}{d_s d_L}\right) = 0 \qquad \text{(C\&O 28.21)}$$
  - Mass of the lens:
$$M = -\frac{\theta_1 \theta_2 c^2}{4G}\left(\frac{d_S d_L}{d_S - d_L}\right) \qquad \text{(C\&O 28.23)}$$
  - Einstein ring: when the source is exactly behind the lens, it creates a perfect circle around the lens
$$\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)} \text{ rad} \qquad \text{(C\&O 28.24)}$$
  - Uses of gravitational lensing:
    * Measure masses of lenses (assuming a NFW profile)
    * Search for dark matter via MACHO lensing (microlensing)
  - Regimes
    * Microlensing: lens not very massive (i.e. MACHOs), flickering
    * Strong lensing: lens is massive
    * Weak lensing: multiple images not produced, instead tiny distortion of the shape of the object

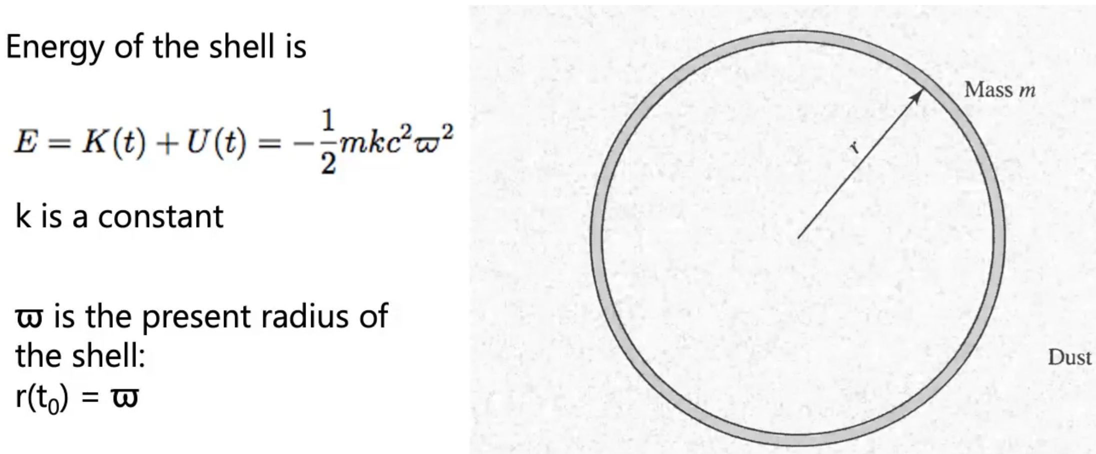
3.2 Cosmology

Cosmological principle: the universe is isotropic and homogeneous.

3.2.1 Models

Pressureless Dust Model

- Only one component: pressureless dust



- ϖ is the co-moving coordinate
- For $k > 0$, $\Omega_0 > 1$, universe is closed and will collapse on itself
- For $k < 0$, $\Omega_0 < 1$, universe is open and will expand forever
- For $k = 0$, $\Omega_0 = 1$, universe is flat and will slow down and come to a halt as $t \to \infty$
- $R(t)$ is the scale factor that describes the expansion (dimensionless)

$$r(t) = R(t)\varpi \qquad \text{(C\&O 29.3)}$$

- R and the redshift z are related by

$$R = \frac{1}{1+z} \qquad \text{(C\&O 29.4)}$$

- Today, $R = 1$ ($z = 0$). At $z = 2$, the size of the universe was $\frac{1}{3}$ what it is today.
- Hubble Law:

$$v(t) = H(t)r(t) = H(t)R(t)\varpi \qquad \text{(C\&O 29.7)}$$

- The value of the density that will result in a value of $k = 0$ is known as the critical density,

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G} \qquad \text{(C\&O 29.12)}$$

- The density parameter $\Omega(t)$ is the ratio of a measured density to the critical density:

$$\Omega_0 = \frac{\rho_0}{\rho_c} = \frac{8\pi G\rho_0}{3H_0^2}$$

- For a flat universe ($k = 0$, $\rho_0 = \rho_{c,0}$, and $\Omega_0 = 1$),

$$R_{flat} = \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3} \qquad \text{(C\&O 29.31)}$$

  – the universe was essentially flat

Two-Component Model

- Accounts for the relativistic equivalence of mass and energy from CMB on expansion

- Includes both the total density of matter (baryonic and dark), ρ_m, and the equivalent mass density of relativistic particles (such as neutrinos and CMB photons), ρ_{rel}

- Equation of state:

$$P = wu \qquad \text{(C\&O 29.52)}$$

  - matter: $w_m = 0$
  - relativistic particle: $w_{rel} = 1/3$
  - dark energy: $w_\Lambda = -1$

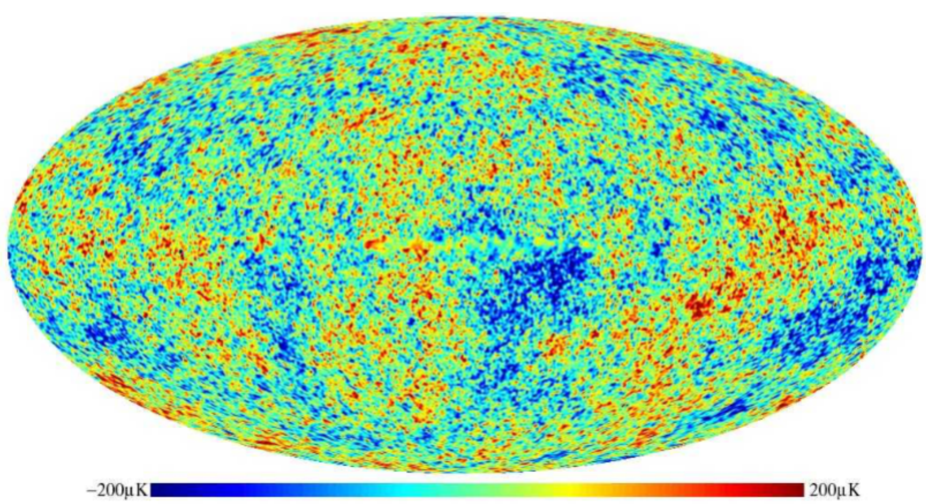
- Density evolves as

$$R^{3(1+w)}\rho = \text{constant} = \rho_0$$

Three-Component (ΛCDM) Model

See Early Universe section below.

3.2.2 Cosmic Microwave Background



- Temperature

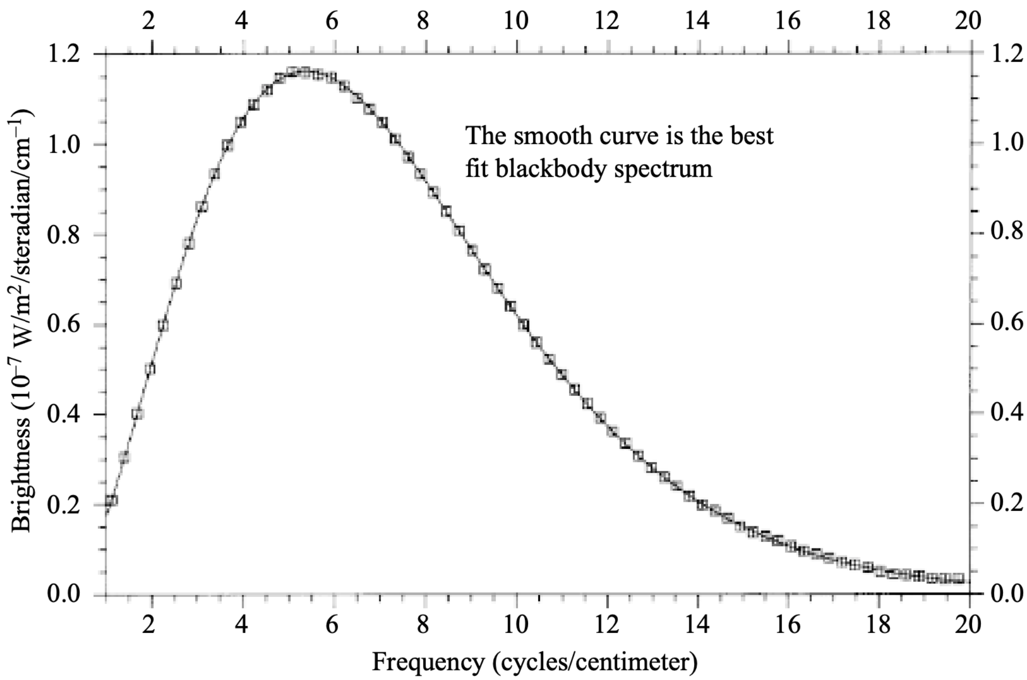


FIGURE 29.9 The COBE measurement of the spectrum of the cosmic microwave background, which is that of a blackbody with a temperature of 2.725 K. The horizontal axis (frequency) is actually $1/\lambda$ (cm^{-1}); the spectrum peaks at a frequency of 160 GHz (5.35 cycles per centimeter). (Figure adapted from Mather et al., *Ap. J. Lett*, *354*, L37, 1990. Courtesy of NASA/GSFC and the COBE Science Working Group.)

- $[T_0]_{\text{WMAP}} = 2.725 \pm 0.002\ K$
- $T(z) = T_0(1+z)$

- Origin
  - Early universe was opaque, hot, dense
  - Recombination: as it expanded, temperature cooled enough for electrons to combine with photons to form hydrogen and helium
  - Decoupling/surface of last scattering: photons could stream freely for the first time Universe was 380,000 years old

3.2.3 Big Bang Nucleosynthesis

- Baryonic mass: 74% Hydrogen, 24% Helium, 2% heavier elements
- Amount of heavy elements increases with time (more in galaxies with lots of stars/supernova), amount of light elements stars constant with time
- For 17 min, Hydrogen and Helium and the other light elements were created via fusion
- Boltzmann equation (C&O 8.6) gives the equilibrium ration of the number density of neutrons, n_n, to the number density of protons, n_p, as

$$\frac{n_n}{n_p} = \exp\left[-(m_n - m_p)c^2/kT\right]$$

  - Going back in time, $n_n/n_p \to 1$

  - As the universe expands, n_n/n_p decreases as T decreases

- Timeline
  - $t \sim 10^{-4}$ s and $T \sim 10_{12}$ K: loose particles, universe consisted of protons, neutrons, photons, electrons, positrons, electron and muon neutrons (and their antiparticles). $n_n/n_p = 0.985$
  - $t \sim 0.01$ s and $T \sim 10^{10}$ K: no long in equilibrium, neutrinos decay into protons. $n_n/n_p = 0.223$ (freeze-out)
  - $t \sim 2.9$ min and $T \sim 10^9$ K: hydrogen and helium can form. $n_n/n_p = 0.176$
  - $t \sim 3 - 20$ min: lighter elements (H, He, Be, Li) fuse (still hot enough, after this it was too cool to fuse)

3.2.4 Evolution of the Universe

- Friedmann equation: description of the dynamic evolution for the universe (isotropic, homogeneous)

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right] R^2 = -kc^2 \qquad \text{(C\&O 29.107)}$$

- With cosmological Constant Λ, originally to force a static universe

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right] R^2 = -kc^2 \qquad \text{(C\&O 29.108)}$$

- Equivalent mass density of dark energy:

$$\rho_L ambda \equiv \frac{\Lambda c^2}{8\pi G} = \text{constant} = \rho_{\Lambda,0} \qquad \text{(C\&O 29.113)}$$

  and corresponding Friedmann equation

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G(\rho_m + \rho_{rel} + \rho_\Lambda)\right] R^2 = -kc^2 \qquad \text{(C\&O 29.114)}$$

- Dark matter energy density parameter:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda c^2}{3H^2} \qquad \text{(C\&O 29.118)}$$

- Total density parameter:

$$\Omega \equiv \Omega_m + \Omega_{rel} + \Omega_\Lambda$$

  where

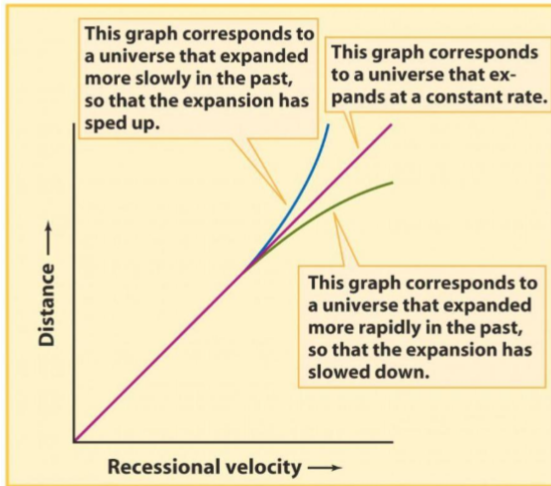
$$\boxed{\begin{aligned} [\Omega_{m,0}]_{\text{WMAP}} &= 0.27 \pm 0.04, \\ \Omega_{\text{rel},0} &= 8.24 \times 10^{-5}, \\ [\Omega_{\Lambda,0}]_{\text{WMAP}} &= 0.73 \pm 0.04, \end{aligned}}$$

- Adding these results from the Wilkinson Microwave Anisotropy Probe reveals that

$$\Omega_0 = \Omega_{m,0} + \Omega_{rel,0} + \Omega - \Lambda, 0 = 1.$$

  that is, the universe is flat ($z = 0$), and dark energy now dominates the expansion of the universe.

Acceleration of the Expansion



- Universe is accelerating in its expansion
- Expansion was decelerating until $z = 0.76$ but it has been accelerating ever since
- Source of acceleration is dark energy
- Deceleration parameter $q(t)$: a useful dimensionless quantity that describes the acceleration of the universal expansion

$$q(t) \equiv -\frac{R(t)[d^2R(t)/dt^2]}{[dR(t)/dt]^2} \tag{C\&O 29.54}$$

which can also be written as

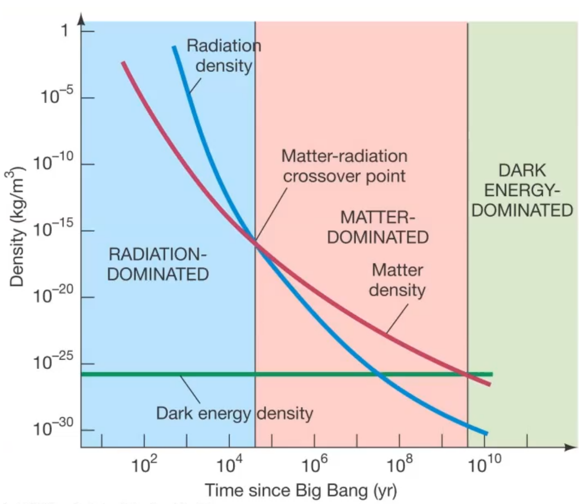
$$q(t) = \frac{1}{2}\sum_i (1+3w_i)\Omega_i(t) \tag{C\&O 29.123}$$

where w is the coefficient from the equation of state $P_i = w_i\rho_i c^2$ and the "i" subscript identifies one of the components of the universe. Using $w_m = 0$, $w_{rel} = 1/3$, and $w_\Lambda = -1$, we obtain

$$q(t) = \frac{1}{2}\Omega_m(t) + \Omega_{rel}(t) - \Omega_\Lambda(t) \overset{\text{WMAP}}{=} -0.60 \tag{C\&O 29.124}$$

where the minus sign indicates that the expansion of the universe is now accelerating ($d^2R/dt^2 > 0$)!

Dominant Components of the Universe



3.2.5 Distances (Observational Cosmology)

For use in upcoming formulas, we define

$$I(z) \equiv \int_0^z \frac{dz'}{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{rel,0}(1+z')^4 + \Omega_{\Lambda,0} + (1-\Omega_0)(1+z')^2}} \qquad \text{(C\&O 29.168)}$$

and

$$\begin{aligned} S(z) &\equiv I(z) \qquad (\Omega_0 = 1) & (29.173) \\ &\equiv \frac{1}{\sqrt{\Omega_0 - 1}} \sin\left[I(z)\sqrt{\Omega_0 - 1}\right] \qquad (\Omega_0 > 1) & (29.174) \\ &\equiv \frac{1}{\sqrt{1-\Omega_0}} \sinh\left[I(z)\sqrt{1-\Omega_0}\right] \qquad (\Omega_0 < 1), & (29.175) \end{aligned}$$

- Proper distance
  - Measuring the distance between two locations at the exact moment of time
  - How far it is today, not when light was emitted
  - Obtain an equation for proper distance by solving the Robertson-Walker metric, which gives the space-time interval between two events in a homogeneous isotropic universe
  - The present proper distance is

    $$d_{p,0} = \frac{c}{H_0} I(z) \qquad \text{(C\&O 29.169)}$$

    or, for $z \ll 1$,

    $$d_{p,0} \simeq \frac{cz}{H_0}\left[1 - \frac{1}{2}(1+q_0)z\right] \qquad \text{(C\&O 29.181)}$$

- Astronomers frequently translate a measured redshift z into the radial velocity a galaxy would have *as if* if had a peculiar velocity (moving through space) instead of its actual recessional velocity (moving along with expanding space) (relativistic Hubble distance):

  $$d \simeq \frac{c}{H_0} \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \qquad \text{(C\&O 27.7)}$$

  or, if $z \ll 1$, we have the non-relativistic Hubble distance:

  $$d = \frac{cz}{H_0}$$

- The Luminosity Distance is defined as

  $$d_L^2 \equiv \frac{L}{4\pi F} \qquad \text{(C\&O 29.182)}$$

  so for the

-

Topic-Lecture Index

Topic	Lecture
Welcome to ASTR 3830	1
Dark Matter	2
Galactic Center	3
Galaxy Morphologies	4
Surface Brightness	5
Spiral Galaxies	6
Elliptical Galaxies	7
Luminosity Function of Galaxies	8
Galactic Evolution	9
Galactic Evolution	10
Extragalactic Distance Scale	11
Extragalactic Distance Scale	12
Galaxy Clusters	13
Galaxy Clusters	14
Active Galaxies	15
Active Galaxies	16
Active Galaxies	17
Gravitational Lensing	18
Cosmology	19
Cosmology	20
Cosmology	21
Cosmology	22
Cosmology	23
Cosmology	24
Cosmology	25
Cosmology	26

Potentially Useful Equations

- Integrated Star Count:

$$n(S, \Omega, r) = \int_{-\infty}^{\infty} n_M(M, S, \Omega, r)\ dM \qquad \text{(C\&O 24.2)}$$

where $n_M(M, S, \Omega, r)$ is the number density of of stars with absolute magnitudes between M and $M + dM$ and attribute S that lie within a solid angle Ω in a specific direction at a distance r from the observer (S could be composition or the Morgan-Keenan spectral class, for example).

- Integrated star count written in terms of limiting distance, d:

$$N_M(M, S, \Omega, d)\ dM = \left[\int_0^d n_M(M, S, \Omega, r)\Omega r^2\ dr\right]\ dM \qquad \text{(C\&O 24.3)}$$

- Differential Star Count:

$$A_M(M, S, \Omega, m)\ dM\ dm \equiv \frac{d\bar{N}_M(M, S, \Omega, m)}{dm} dM\ dm \qquad \text{(C\&O 24.4)}$$

where A_M is the number of stars with an absolute magnitude between M and $M + dM$ that are found within a solid angle Ω and have apparent magnitudes in the range between m and $m + dm$.

- In the special case where we assume no interstellar extinction ($A = 0$) and infinite universe of uniform stellar density (i.e. $n_M(M, S, \Omega, r) = n_M(M, S) =$ constant),

$$\begin{aligned}
\bar{N}_M(M, S, \Omega, m) &= \frac{\Omega}{3} n_M(M, S) 10^{3(m-M+5)/5} \\
&= \frac{\Omega}{3} n_M(M, S) \exp\left(\ln 10^{3(m-M+5)/5}\right) \\
&= \frac{\Omega}{3} n_M(M, S) e^{[3(m-M+5)/5]\ln 10}
\end{aligned}$$

- In special case where we assume no interstellar extinction ($A = 0$) and infinite universe of uniform stellar density (i.e. $n_M(M, S, \Omega, r) = n_M(M, S) =$ constant),

$$\begin{aligned}
A_M(M, S, \Omega, m) &= \frac{d\bar{N}_m(M, S, \Omega, m)}{dm} \\
&= \frac{\ln 10}{5} \Omega n_m(M, S) 10^{3(m-M+5)/5} \\
&= \frac{3 \ln 10}{5} \bar{N}_M(M, S, \Omega, r) \qquad \text{(C\&O 24.5)}
\end{aligned}$$

- Interstellar Extinction:

$$m_\lambda = M_\lambda + 5 \log_{10} d - 5 + A_\lambda \qquad \text{(C\&O 12.1)}$$

where d is the distance in pc and $A_\lambda > 0$ represents the number of magnitudes of interstellar extinction present along the line of sight. We also use two other forms of this equation:

$$\begin{aligned}
d &= 10^{(m_\lambda - M_\lambda - A_\lambda + 5)/5} \\
A_\lambda &= m_\lambda - M_\lambda + 5 - 5 \log_{10} d \qquad \text{(C\&O 24.1)}
\end{aligned}$$

- Perigalacticon (closest approach):

$$r \equiv \frac{a(1-\epsilon^2)}{1+\epsilon\cos\theta}$$

where the ellipticity

$$\epsilon = 1 - \beta/\alpha \qquad \text{(C\&O 25.1)}$$

and α is the apparent semi-major axis and β is the apparent semi-minor axis.

- NFW density profile

$$\rho_{NFW}(r) = \frac{\rho_0}{(r/a)(1+r/a)^2} \qquad \text{(C\&O 24.52)}$$

where rho_0 and a are choices.

- Mass conservation equation and interior mass integral:

$$\frac{dM_r}{dr} = 4\pi r^2\rho \implies M_r = \int_0^r 4\pi r^2 \rho dr \qquad \text{(C\&O 10.7)}$$

- Virial theorem: for a system that has reached an equilibrium or steady-state configuration,

$$-2\langle 2\rangle = \langle U\rangle \qquad \text{(C\&O 2.46)}$$

The theorem may also be expressed in terms of the total energy of the system by using the relation $\langle E\rangle = \langle K\rangle + \langle U\rangle$:

$$\langle E\rangle = \frac{1}{2}\langle U\rangle$$

- Flux is related to an object's luminosity by

$$F = \frac{L}{4\pi r^2} \qquad \text{(C\&O 3.2)}$$

We can also define the flux ratio of two stars:

$$\frac{F_2}{F_1} = 100^{(m_1-m_2)/5}$$