ECEN5463 | HW 1

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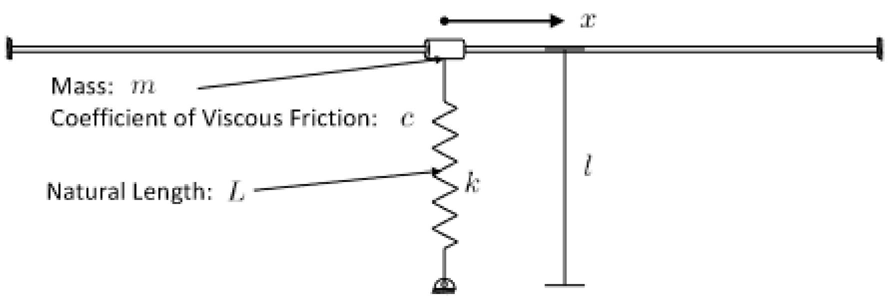
Problem 1

Problem 2

Problem 3

Problem 4



$m = 1kg \quad b = 0.1Nsm^{-1} \quad k = 1Nm^{-1} \quad L = 1m$

4-1 Derive equations of motion wrt x

Newtons 2nd: $m\ddot{x} = \sum_i F_i$

Def: $d = \sqrt{x^2 + l^2}$ (spring distance)

Derive forces:

- Spring force: $f_s = -k(d - x)$
- Along x-axis: $f_s^{(x)} = -k(d - L)sin(\theta) = -k(d - L)\frac{x}{d} = -kx\left(1 - \frac{L}{d}\right)$
- Friction force: $f_f^{(x)} = -c\dot{x}$

Equation of motion:

- $m\ddot{x} + c\dot{x} + kx\left(1 - \frac{L}{d}\right) = 0$

Statespace form:

- $$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$
- $$f(t,x) = \dot{x} = \begin{bmatrix} x_2 \\ -\dfrac{\mathrm{cx}_2 + \mathrm{kx}_1\left(1 - \dfrac{L}{\sqrt{x_1^2 + l^2}}\right)}{m} \end{bmatrix}$$

4-2 Show the system undergoes a supercritical pitchfork bifurcation at r=0

Rewrite state space in terms of $r = l - L$

$$f(t,x) = \begin{bmatrix} x_2 \\ -\dfrac{\mathrm{cx}_2 + \mathrm{kx}_1\left(1 - \dfrac{L}{\sqrt{x_1^2 + (r+L)^2}}\right)}{m} \end{bmatrix}$$

```
syms x1 x2 c k m L r

x = [x1;x2];
f = [x2;-(c*x2+k*x1*(1-L/sqrt(x1^2+(r+L)^2)))/m];
f = subs(f, [c,k,m,L], [0.1,1,1,1]);

eq_points = solve(f==0, x, 'ReturnConditions', true);
eq_points = [eq_points.x1, eq_points.x2]
```

eq_points =

$$\begin{pmatrix} 0 & 0 \\ \sqrt{-r\ (r+2)} & 0 \\ -\sqrt{-r\ (r+2)} & 0 \end{pmatrix}$$

1st eq. point ([0,0]) is always valid.

2nd & 3rd eq. points only valid on $r \in (-2, 0)$

<u>This implies a pitchfork bifurcation</u>

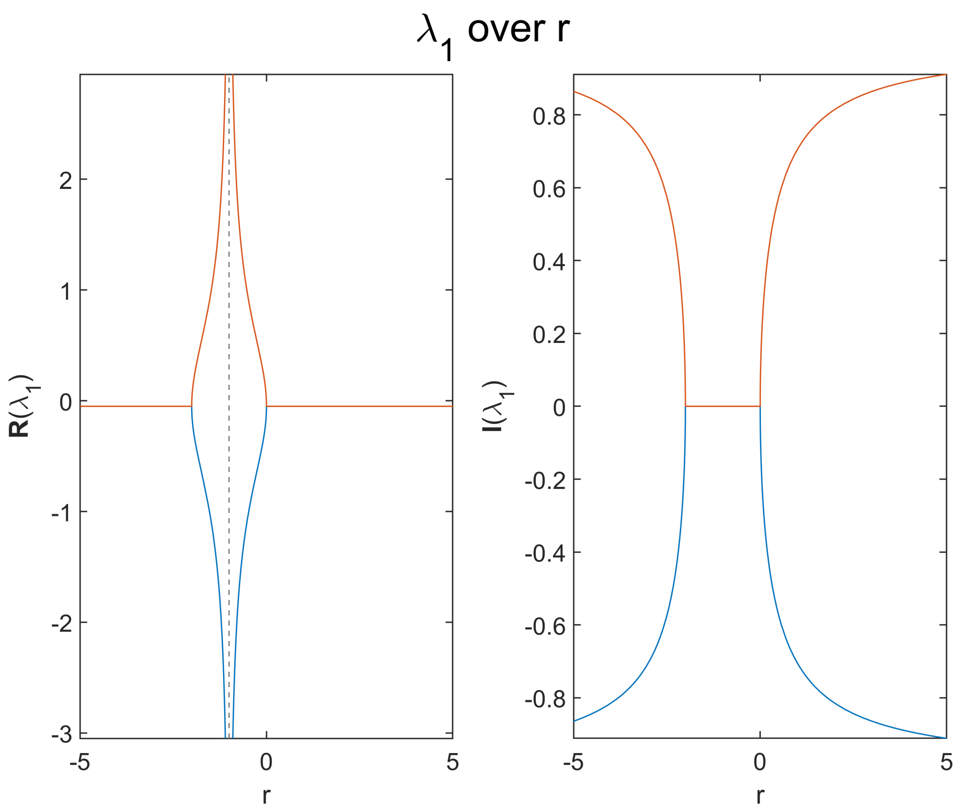
Now, must determine whether this is super- or sub- critical

```
jac = jacobian(f,x);
eig1 = eig(subs(jac, x, eq_points(1,:)'));
figure()
sgtitle("\lambda_1 over r")

subplot(1,2,1)
fplot(real(eig1), [-5,5])
```

```
xlabel("r")
ylabel("\bf{R}\rm(\lambda_1)")

subplot(1,2,2)
fplot(imag(eig1), [-5,5])
xlabel("r")
ylabel("\bf{I}\rm(\lambda_1)")
```



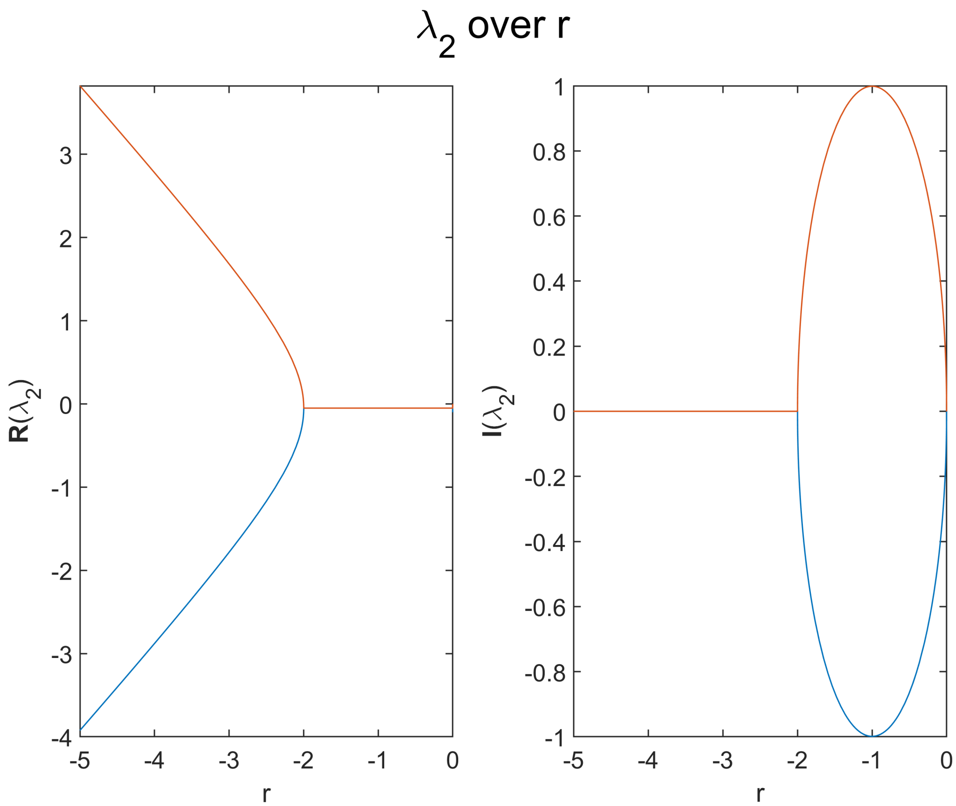
$\mathbb{R}\{\lambda_1(r), r \in (-2,0)\}$ contains both a positive and negative component while $\mathbb{I}\{\lambda_1(r), r \in (-2,0)\} = 0$. **This is a saddle point - not stable.**

$\mathbb{I}\{\lambda_1(r), r \in (0,5)\}$ contains both a positive and negative component while $\mathbb{R}\{\lambda_1(r), r \in (0,5)\} = 0$. **This is a stable focus - stable.**

```
eig2 = eig(subs(jac, x, eq_points(2,:)'));
figure()
sgtitle("\lambda_2 over r")

subplot(1,2,1)
fplot(real(eig2), [-5,0])
xlabel("r")
ylabel("\bf{R}\rm(\lambda_2)")

subplot(1,2,2)
fplot(imag(eig2), [-5,0])
xlabel("r")
ylabel("\bf{I}\rm(\lambda_2)")
```

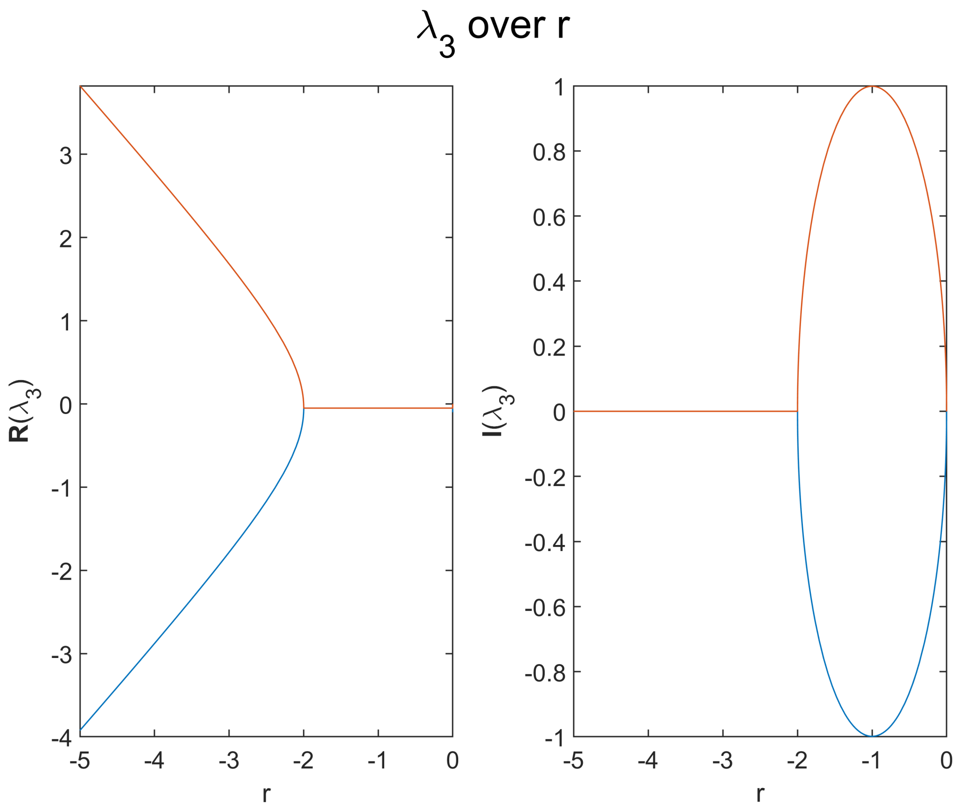


$\mathbb{R}\{\lambda_2(r), r \in (-2,0)\} = 0$ while $\mathbb{I}\{\lambda_2(r), r \in (-2,0)\}$ contains both positive and negative components. **This is a stable focus - stable.**

```
eig3 = eig(subs(jac, x, eq_points(3,:)'));
figure()
sgtitle("\lambda_3 over r")

subplot(1,2,1)
fplot(real(eig3), [-5,0])
xlabel("r")
ylabel("\bf{R}\rm(\lambda_3)")

subplot(1,2,2)
fplot(imag(eig3), [-5,0])
xlabel("r")
ylabel("\bf{I}\rm(\lambda_3)")
```



$\mathbb{R}\{\lambda_3(r), r \in (-2,0)\} = 0$ while $\mathbb{I}\{\lambda_3(r), r \in (-2,0)\}$ contains both positive and negative components. **This is a stable focus - stable.**

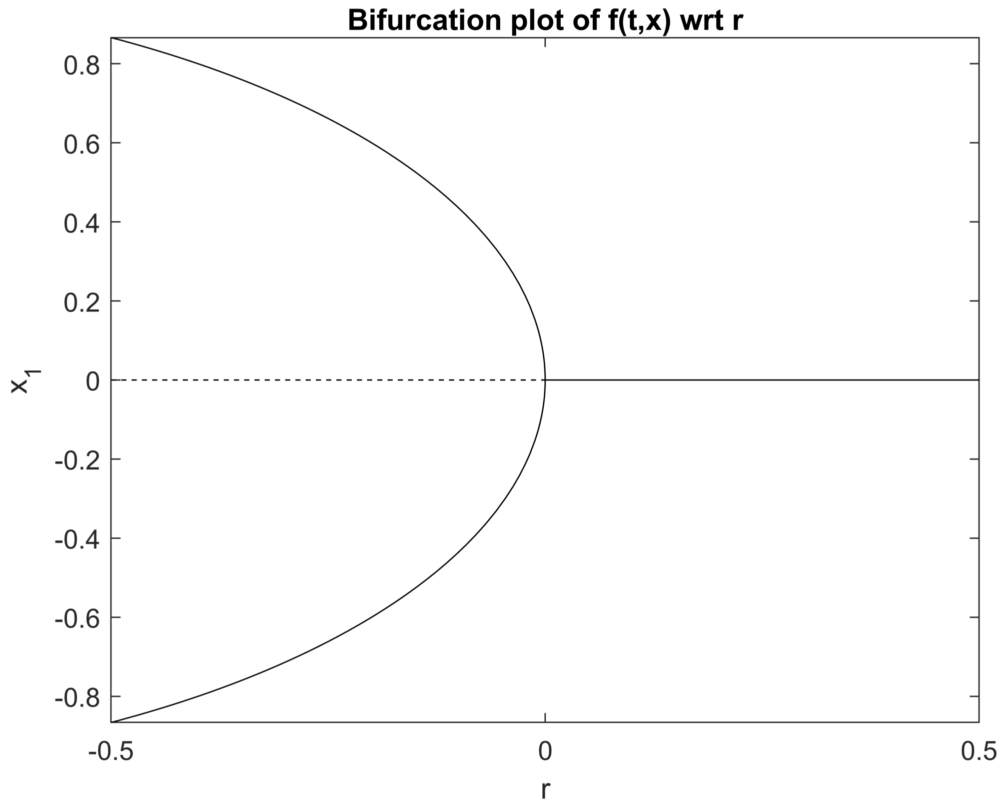
<u>Summary:</u>

- λ_1 is unstable on $r \in (-2,0)$ & stable on $r \in (0,2)$
- $\lambda_{2,3}$ are stable on $r \in (-2,0)$ & DNE on $r \in (0,2)$
- Thus, there exists a supercritical pitchfork bifurcation at $r = 0$.

4-3 Plot the bifurcation on $r \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

```
figure()

fplot(eq_points(1,1), [-0.5,0], '--k')
hold on;
fplot(eq_points(2:3,1), [-0.5,0], '-k')
fplot(eq_points(1,1), [0,0.5], '-k')
title("Bifurcation plot of f(t,x) wrt r")
xlabel("r")
ylabel("x_1")
```



Note that x_2 is always stable at 0