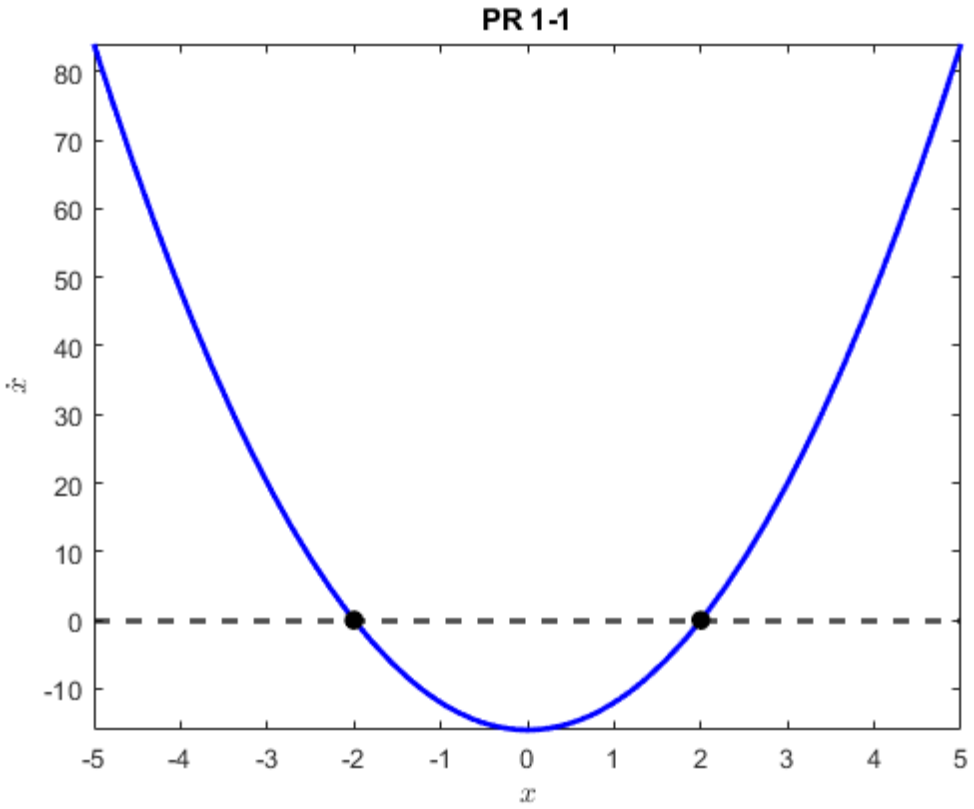
```
clear all;
close all;
clc;
```

Problem 1

```
syms y x
```

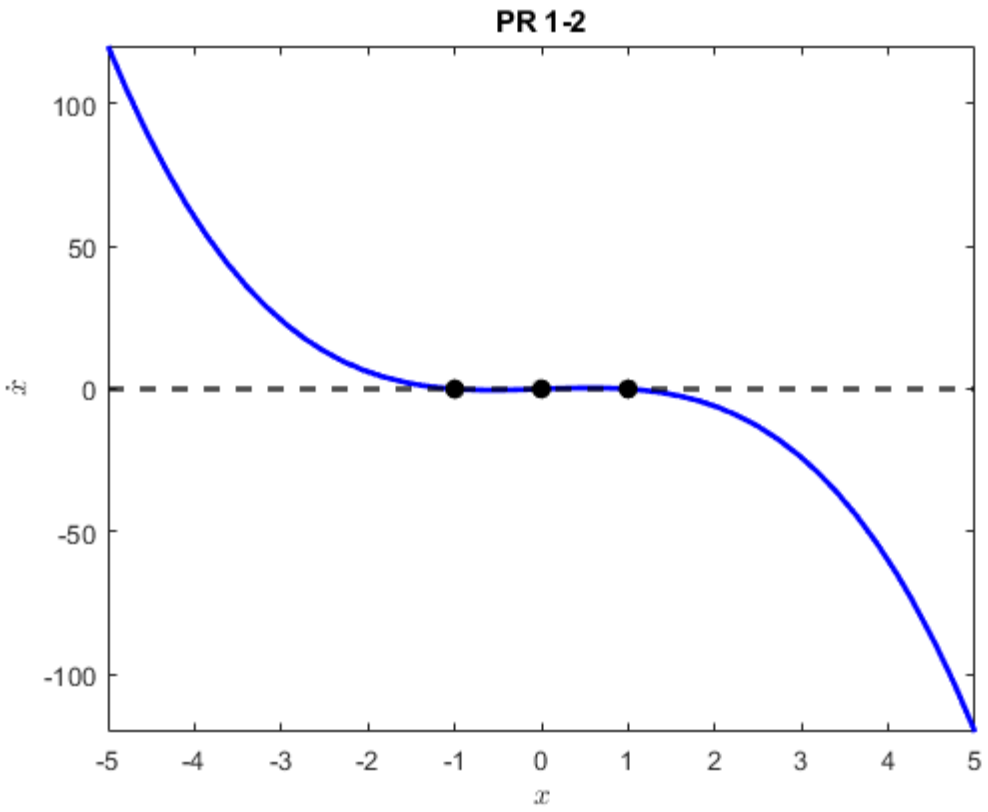
1-1

```
figure(1)
y = 4*x^2 - 16;

fplot(y, '-b', "LineWidth", 2)
hold on
yline(0, '--k', "LineWidth", 2)

xint = solve(y==0, "Real", true);
if ~isempty(xint)
    plot(xint, 0, '.k', 'MarkerSize', 25);
end

title("PR 1-1")
xlabel("$x$", "Interpreter", "latex")
ylabel("$\dot x$", "Interpreter", "latex")

hold off
```



1-2

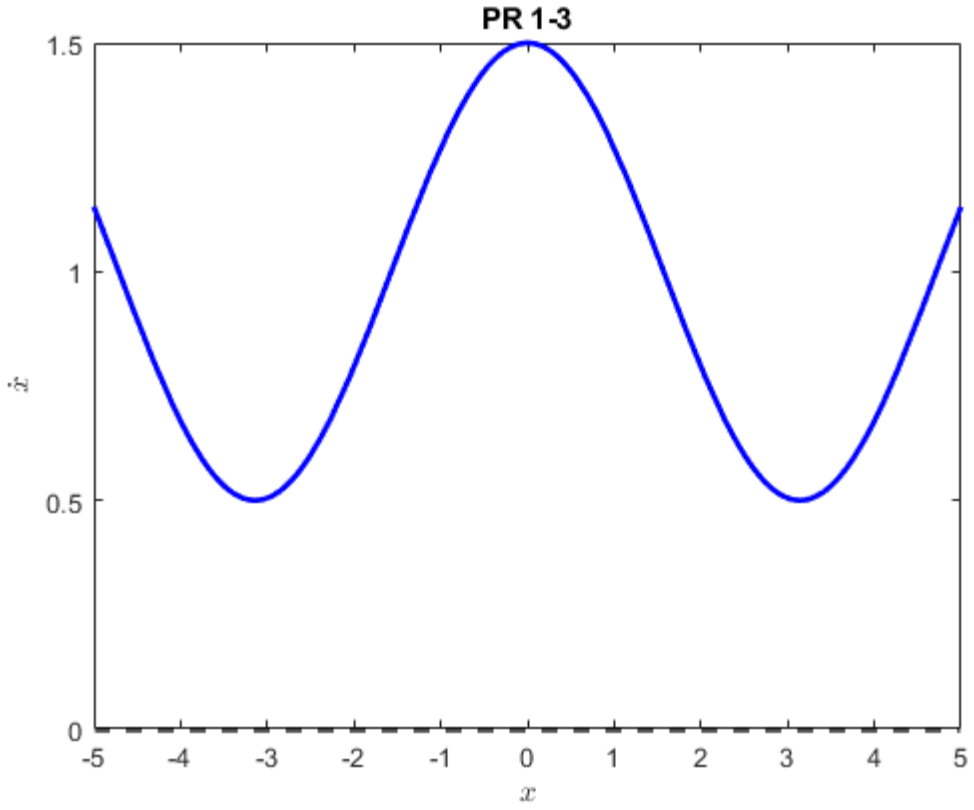
```
figure(2)
y = x - x^3;

fplot(y, '-b', "LineWidth", 2)
hold on
yline(0, '--k', "LineWidth", 2)

xint = solve(y==0, "Real", true);
if ~isempty(xint)
    plot(xint, 0, '.k', 'MarkerSize', 25);
end

title("PR 1-2")
xlabel("$x$", "Interpreter", "latex")
ylabel("$\dot x$", "Interpreter", "latex")

hold off;
```



1-3

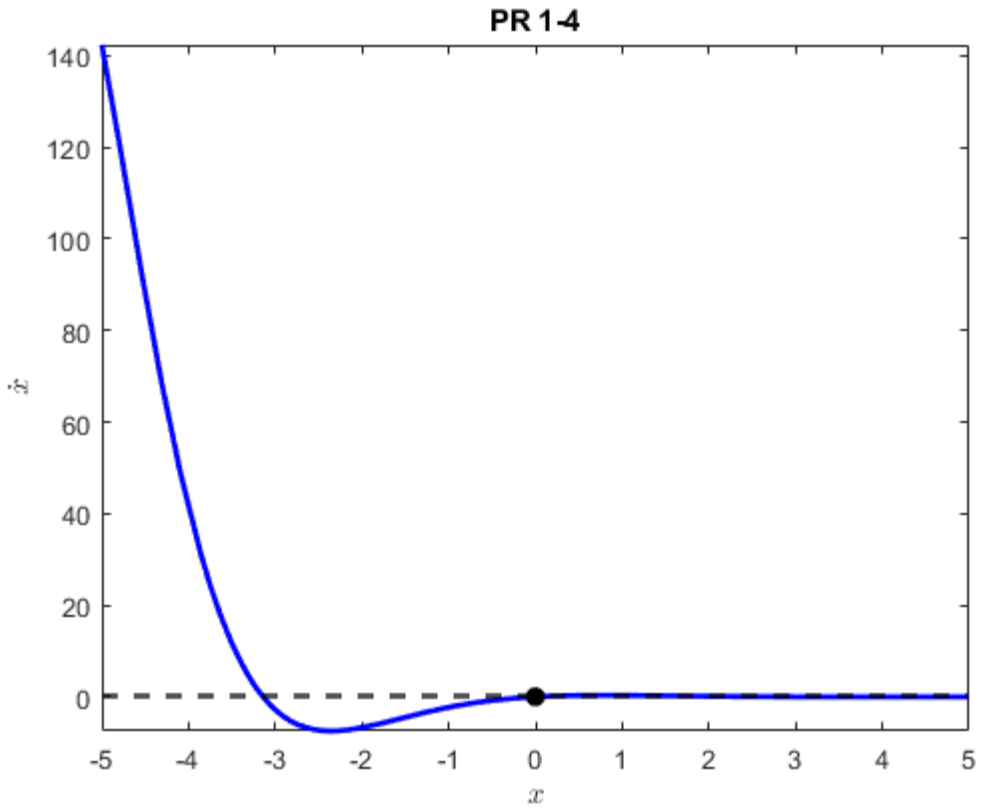
```
figure(3)
y = 1+1/2*cos(x);

fplot(y, '-b', "LineWidth", 2)
hold on;
yline(0, '--k', "LineWidth", 2);

xint = solve(y==0, "Real", true);
```

```
if ~isempty(xint)
    plot(xint, 0, '.k', 'MarkerSize', 25);
end

title("PR 1-3")
xlabel("$x$", "Interpreter", "latex")
ylabel("$\dot x$", "Interpreter", "latex")

hold off;
```



1-4

```
figure(4)
y = exp(-x)*sin(x);

fplot(y, '-b', "LineWidth", 2);
hold on;
yline(0, '--k', "LineWidth", 2);

xint = solve(y==0, "Real", true);
if ~isempty(xint)
    plot(xint, 0, '.k', 'MarkerSize', 25);
end

title("PR 1-4")
xlabel("$x$", "Interpreter", "latex")
ylabel("$\dot x$", "Interpreter", "latex")

hold off;
```



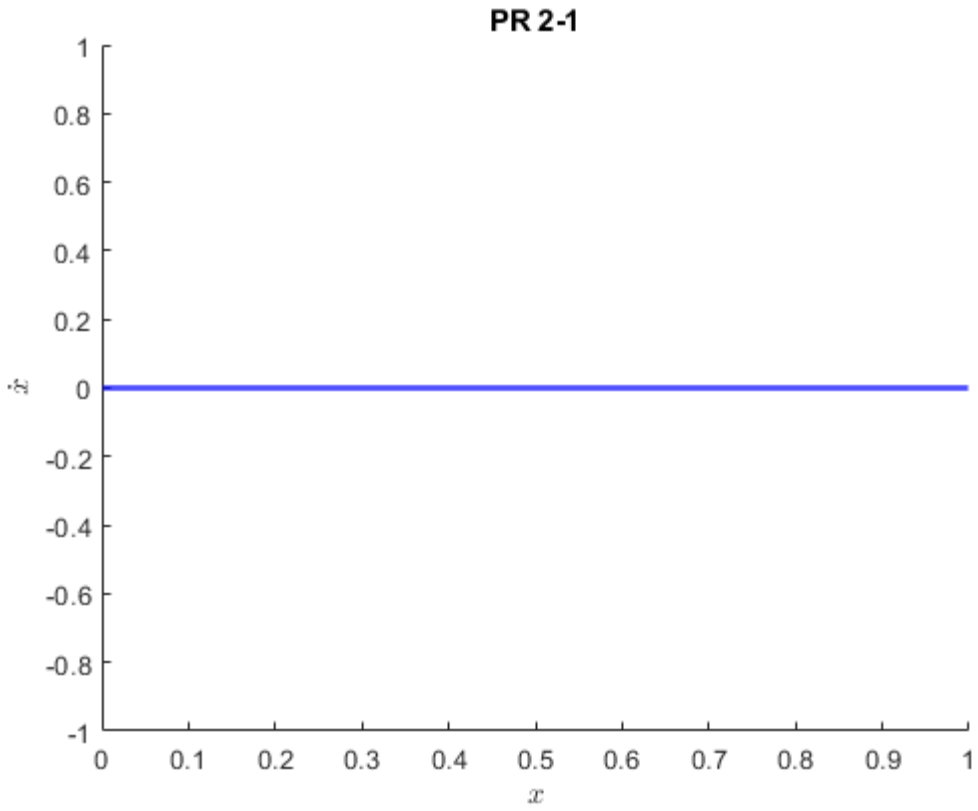
Problem 2

2-1

```
figure(5);
y = 0
```

```
y = 0
```

```
%fplot(y, '-b', "LineWidth", 2);
yline(0, '-b', "LineWidth", 2);

title("PR 2-1")
xlabel("$x$", "Interpreter","latex");
ylabel("$\dot x$", "Interpreter", "latex");

hold off;
```

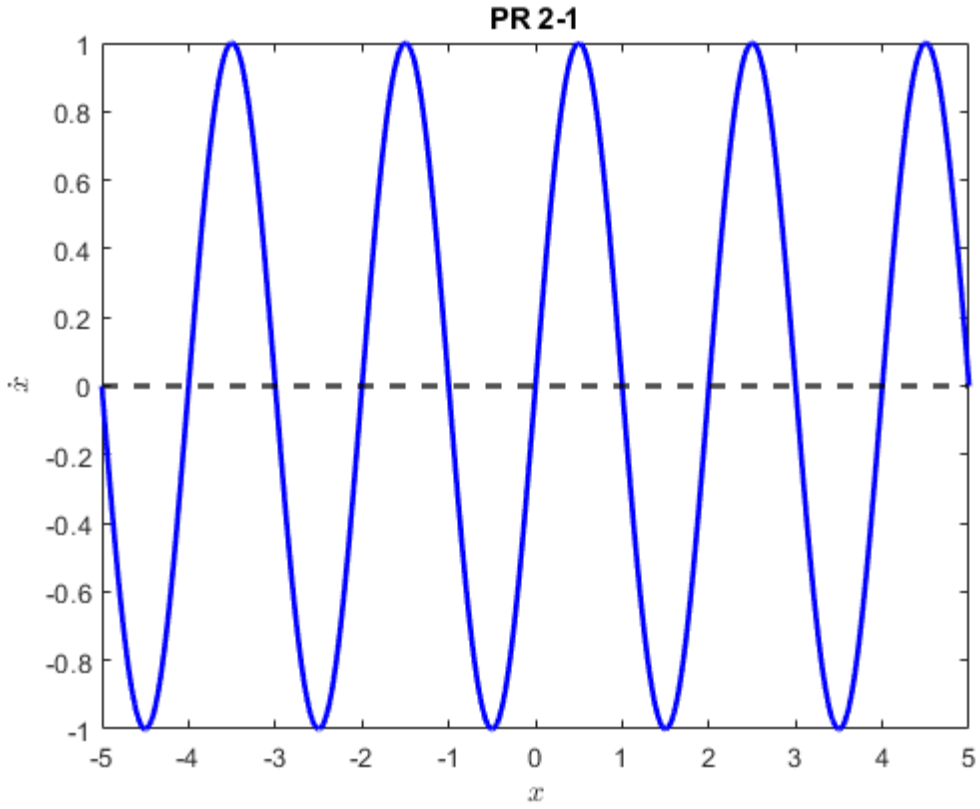


2-2

```
figure(6)
y = sin(pi*x)
```

$$y = \sin(\pi\, x)$$

```
fplot(y, '-b', "LineWidth", 2);
hold on;
yline(0, '--k', "LineWidth", 2);

xint = solve(y==0, x, "Real", true);

title("PR 2-1")
xlabel("$x$", "Interpreter","latex");
ylabel("$\dot x$", "Interpreter", "latex");

hold off;
```



2-3

```
figure(7)
% Not possible in 1D
```

2-4

```
% ??
```

Problem 3

```
syms x y z s r b

f1 = s*(y-x);
f2 = r*x - y - x*z;
f3 = x*y - b*z;
f = [f1;f2;f3];

jac = jacobian(f, [x,y,z]);
eq = solve(f==0, [x,y,z]);

% Compute linearization for each eq point

lin = cell(length(eq.x),1);
eigs = cell(length(eq.x),1);

disp("Jacobians")
```

Jacobians

```
for idx = 1:length(eq.x)
   lin{idx} = subs(jac, [x,y,z], [eq.x(idx),eq.y(idx),eq.z(idx)]);
   eigs{idx} = eig(lin{idx});
   disp(vpa(lin{idx}))
end
```

$$\begin{pmatrix} -1.0\,s & s & 0 \\ r & -1.0 & 0 \\ 0 & 0 & -1.0\,b \end{pmatrix}$$

$$\begin{pmatrix} -1.0\,s & s & 0 \\ 1.0 & -1.0 & \sqrt{b\,(r-1.0)} \\ -1.0\,\sqrt{b\,(r-1.0)} & -1.0\,\sqrt{b\,(r-1.0)} & -1.0\,b \end{pmatrix}$$

$$\begin{pmatrix} -1.0\,s & s & 0 \\ 1.0 & -1.0 & -1.0\,\sqrt{b\,(r-1.0)} \\ \sqrt{b\,(r-1.0)} & \sqrt{b\,(r-1.0)} & -1.0\,b \end{pmatrix}$$

```
disp("Case I")
```

Case I

```
for idx = 1:length(eq.x)
    disp(vpa(subs(eigs{idx}, [r,b,s], [0.5,1,1]),2))
end
```

$$\begin{pmatrix} -1.0 \\ -1.7 \\ -0.29 \end{pmatrix}$$

$$\begin{pmatrix} 0.37 \\ -1.4 \\ -2.0 \end{pmatrix}$$

$$\begin{pmatrix} 0.37 \\ -1.4 \\ -2.0 \end{pmatrix}$$

```
disp("Case II")
```

Case II

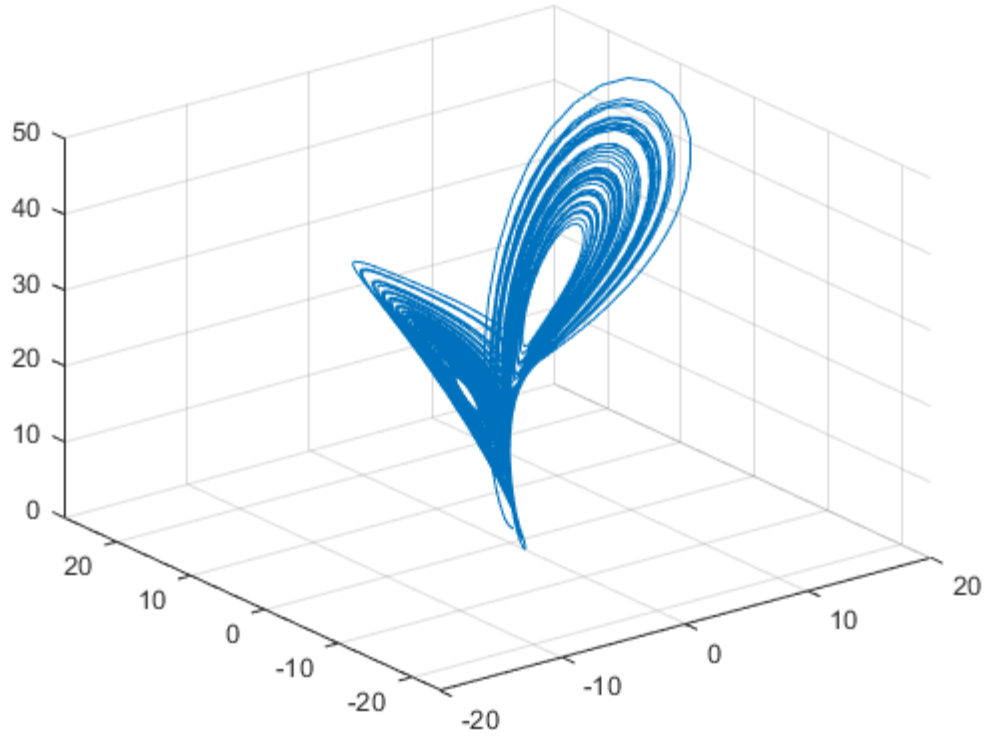
```
for idx = 1:length(eq.x)
    disp(vpa(subs(eigs{idx}, [r,b,s], [2,3,5]), 2))
end
```

$$\begin{pmatrix} -3.0 \\ -6.7 \\ 0.74 \end{pmatrix}$$

$$\begin{pmatrix} -1.3 - 1.7\,\mathrm{i} \\ -1.3 + 1.7\,\mathrm{i} \\ -6.5 - 2.3\mathrm{e}\text{-}10\,\mathrm{i} \end{pmatrix}$$

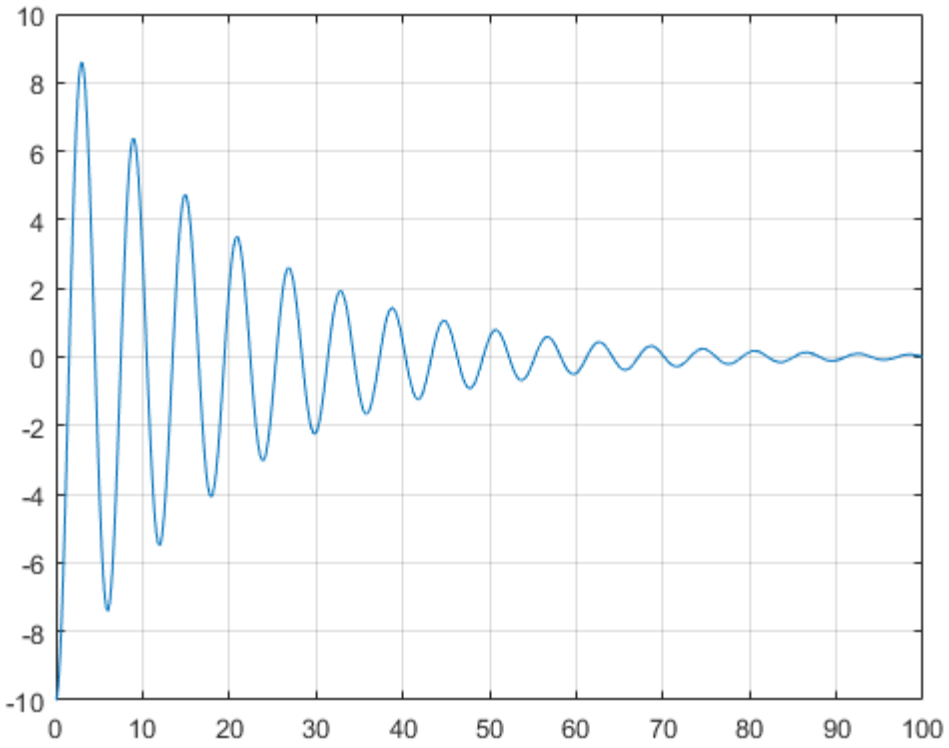
$$\begin{pmatrix} -1.3 - 1.7\,\mathrm{i} \\ -1.3 + 1.7\,\mathrm{i} \\ -6.5 - 2.3\text{e-}10\,\mathrm{i} \end{pmatrix}$$

```
[t,X] = ode45(@(t,x)LorenzSystem(t,x,10,28,8/3), [0,100],[1;1;1]);
plot3(X(:,1), X(:,2), X(:,3));
grid
```



Problem 4

```
syms r L k m c x1 x2

f1 = x2;
f2 = -c/m*x2 - (k*r)/(m*(r+L))*x1;
f = [f1;f2];

r = -10;

[t,X] = ode45(@(t,x)PitchforBifurcation(t,x,0.1,1,1,r,1), [0,100], [-10;0]);
plot(t,X(:,1));
grid;
```




```
function XD=LorenzSystem(t,X,sigma,r,b)
    XD = [sigma*(X(2)-X(1));
          r*X(1)-X(2)-X(1)*X(3);
          X(1)*X(2)-b*X(3)];
end

function XD=PitchforBifurcation(t,X,c,m,k,r,L)
    XD = [X(2);
          -c/m*X(2) - (k*r)/(m*(r+L))*X(1)];
end
```