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ITU-R
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Recommendation ITU-R P.528-4
(08/2019)

A propagation prediction method for aeronautical mobile and radionavigation services using the VHF, UHF and SHF bands

P Series

Radiowave propagation

ITU
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Note: *This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

RECOMMENDATION ITU-R P.528-4[*]

A propagation prediction method for aeronautical mobile and radionavigation services using the VHF, UHF and SHF bands

(Question ITU-R 203/3)

(1978-1982-1986-2012-2019)

Scope

This Recommendation contains a method for predicting basic transmission loss in the frequency range 125 MHz to 15.5 GHz for aeronautical and satellite services. It provides a step-by-step method to compute the basic transmission loss. The only data needed for this method are the distance between antennas, the heights of the antennas above mean sea level, the frequency, and the time percentage.

This Recommendation, also, gives the calculations for the expected protection ratio or wanted-to-unwanted signal ratio exceeded at the receiver for at least 95% of the time, R (0.95). This calculation requires the following additional data for both the wanted and unwanted signals: the transmitted power, the gain of transmitting antenna, and the gain of receiving antenna.

The ITU Radiocommunication Assembly,

considering

a) that there is a need to give guidance to engineers in the planning of radio services in the VHF, UHF, and SHF bands;

b) that the propagation model given in Annex 2 is based on a considerable amount of experimental data (see Annex 1);

c) that the aeronautical service often provides a safety of life function and therefore requires a higher standard of availability than many other services;

d) that a time availability of 0.95 should be used to obtain more reliable service,

recommends

1 that the integral software in this Recommendation should be used to generate basic transmission loss values and curves for terminal heights, frequencies, and time percentages likely to be encountered in the aeronautical services;

2 that the following Notes should be regarded as part of this Recommendation.

NOTE 1 – It should be emphasized that the generated values are based on data obtained mainly for a continental temperate climate.

NOTE 2 – The method gives basic transmission loss, that is, the loss between ideal loss-free isotropic antennas. Where surface reflection multipath at the ground station or the facility has been mitigated using counterpoises or a directional vertical radiation pattern suitable antenna radiation patterns should be included within the analysis.

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[*] This Recommendation is brought to the attention of Study Group 5.

Annex 1

Development and application of the model

Transmission loss prediction methods have been developed that determine the basic transmission losses for time percentages from 1% to 99% of the time for antenna heights applicable to the aeronautical services. These methods are based on a considerable amount of experimental data, and extensive comparisons of predictions with data have been made. In performing these calculations, a smooth (terrain parameter $\Delta h = 0$) Earth with an effective earth radius factor k of 4/3 (surface refractivity $N_s = 301$) was used along with compensation for the excessive ray bending associated with the $k = 4/3$ model at high altitudes. Constants for average ground horizontal polarization, isotropic antennas, and long-term power fading statistics for a continental temperate climate were also used. Although these parameters may be considered either reasonable or worst-case for many applications, the computed values should be used with caution if conditions differ drastically from those assumed.

With the exception of a region 'near' the radio horizon, values of median basic transmission loss for 'within-the-horizon' paths were obtained by adding the attenuation due to atmospheric absorption (in decibels) to the transmission loss corresponding to free-space conditions. Within the region 'near' the radio horizon, values of the transmission loss were calculated using geometric optics, to account for interference between the direct ray and a ray reflected from the surface of the Earth.

The two-ray interference model was not used exclusively for within-the-horizon calculations, because the lobing structure obtained from it for short paths is highly dependent on surface characteristics (roughness as well as electrical constants), atmospheric conditions (the effective earth radius is variable in time), and antenna characteristics (polarization, orientation and gain pattern). Such curves would often be more misleading than useful, i.e. the detailed structure of the lobing is highly dependent on parameters that are difficult to determine with sufficient precision. However, the lobing structure is given statistical consideration in the calculation of variability.

For time availabilities other than 0.50, the basic transmission loss, L_b, user-generated values do not always increase monotonically with distance. This occurs because variability changes with distance can sometimes overcome the median level changes. Variability includes contributions from both hourly-median or long-term power fading and within-the-hour or short-term phase interference fading. Both surface reflection and tropospheric multipath are included in the short-term fading.

The basic transmission loss, $L_b(0.05)$ values may be used to estimate L_b values for an unwanted interfering signal that is exceeded during 95% (100% − 5%) of the time. Median (50%) propagation conditions may be estimated with the $L_b(0.50)$ values. The $L_b(0.95)$ values may be used to estimate the service range for a wanted signal at which service would be available for 95% of the time in the absence of interference.

The expected protection ratio or wanted-to-unwanted signal ratio exceeded at the receiver for at least 95% of the time, $R(0.95)$, can be estimated as follows:

$$R(0.95) = R(0.50) + Y_R(0.95) \tag{1}$$

$$R(0.50) = [P_t + G_t + G_r - L_b(0.50)]_{Wanted} - [P_t + G_t + G_r - L_b(0.50)]_{Unwanted} \tag{2}$$

and

$$Y_R = -\sqrt{[L_b(0.95) - L_b(0.50)]^2_{Wanted} + [L_b(0.05) - L_b(0.50)]^2_{Unwanted}} \tag{3}$$

In equation (2), P_t is the transmitted power, and G_t and G_r are the isotropic gains of the transmitting and receiving antennas expressed in dB.

Additional variabilities could easily be included in equation (3) for such things as antenna gain when variabilities for them can be determined. Continuous (100%) or simultaneous channel utilization is implicit in the $R(0.95)$ formulation provided above so that the impact of intermittent transmitter operation should be considered separately.

The integral software to generate basic transmission loss values and curves is made available in the supplement zip file R-REC-P.528-4-201908-P1, with documentation. In addition, there are select tabulated basic transmission loss values available in supplement zip file R-REC-P.528-4-201908-P2.

Annex 2

Step by step method

This Annex uses the convention that variables describing the low terminal will be represented with the subscript '1' (i.e. the low terminal height h_{r1}) while variables for the high terminal will be represented with the subscript '2' (i.e. the high terminal height h_{r2}).

1   Introduction

This Annex describes a step-by-step method for calculating the basic transmission loss for user-specified path, defined by:

–   terminal heights, h_{r1} and h_{r2}, in km above mean sea level, with $0.0015 \leq h_{r1,2} \leq 20$ (1.5 metres to 20 000 metres)

–   frequency, f, in MHz, with $125 \leq f \leq 15\,500$ MHz

–   time percentage, q, with $0.01 \leq q \leq 0.99$

–   path distance, d, in km.

2   Assumptions, definitions and conventions

Recommendation ITU-R P.528 assumes the following values:

N_s :   surface refractivity, in N-Units. Set to 301 N-Units

a_0 :   actual radius of the Earth. Set to 6 370 km

a_e :   effective radius of the Earth. Set to 8 493 km (corresponding to a surface refractivity of 301 N-Units)

ϵ_r :   relative dielectric constant. Set to 15 (corresponding to Average Ground)

σ :   conductivity. Set to 0.005 S/m (corresponding to Average Ground).

Additionally, terminal antennas are assumed to be horizontally polarized.

3 Step-by-step method

Step 1: Compute the geometric parameters associated with each terminal. This requires using the steps presented in § 4 for both the low terminal and the high terminal. Once completed, proceed to Step 2. Use § 4 as follows:

given:

$h_{r1,2}$: real height of terminal above mean sea level (user input), in km;

find:

$d_{1,2}$: arc length to the smooth Earth horizon distance, in km

$\theta_{1,2}$: incident angle of the ray from the terminal to the smooth Earth horizon distance, in radians

$h_{1,2}$: adjusted height of the terminal above mean sea level used in subsequent calculations, in km

$\Delta h_{1,2}$: terminal height correction term, in km.

Step 2: Determine the maximum line-of-sight distance, d_{ML}, between the two terminals.

$$d_{ML} = d_1 + d_2 \quad \text{(km)} \tag{4}$$

Step 3: Smooth Earth diffraction is modelled linearly in Recommendation ITU-R P.528. This is done by selecting two distances well beyond d_{ML}, computing the smooth Earth diffraction loss at these distances, and constructing a smooth Earth diffraction line that passes through both of these points.

Step 3.1: Compute two distances, d_3 and d_4, that are well beyond the maximum line-of-sight distance d_{ML} from the above equation (4).

$$d_3 = d_{ML} + 0.5(a_e^2/f)^{1/3} \quad \text{(km)} \tag{5}$$

$$d_4 = d_{ML} + 1.5(a_e^2/f)^{1/3} \quad \text{(km)} \tag{6}$$

Step 3.2: Compute the diffraction losses A_{d3} and A_{d4} at the corresponding distances d_3 and d_4. This will require using § 6 twice – once for each path distance, $d_{3,4}$. Once completed proceed to Step 3.3. Use the method in § 6 as follows:

given:

$d_{3,4}$: the path distance of interest, d_0, as required for § 10, in km

$d_{1,2}$: arc lengths to the smooth Earth horizon distance of the terminals, h_1 and h_2, in km, as determined in Step 1 above

f : frequency, in MHz

find:

$A_{d3,4}$: smooth Earth diffraction loss, A_d, in dB, corresponding to the distance $d_{3,4}$.

Step 3.3: Create the smooth Earth diffraction line from the two distances, d_3 and d_4, and their respective losses, A_{d3} and A_{d4}, by computing the slope M_d and intercept A_{d0}.

$$M_d = (A_{d4} - A_{d3})/(d_4 - d_3) \quad \text{(dB/km)} \tag{7}$$

$$A_{d0} = A_{d4} - M_d d_4 \quad \text{(dB)} \tag{8}$$

Step 3.4: Compute the diffraction loss at the distance d_{ML} and the distance d_d, in km, at which the diffraction line predicts 0 dB loss.

$$A_{dML} = M_d d_{ML} + A_{d0} \quad \text{(dB/km)} \tag{9}$$

$$d_d = -(A_{d0}/M_d) \quad \text{(km)} \tag{10}$$

Step 4: Determine if the propagation path is in the line-of-sight region or transhorizon for the desired distance d. If $d < d_{ML}$ the path is line-of-sight and go to Step 5. Else, the path is transhorizon and go to Step 6.

Step 5: Refer to § 5 for the line-of-sight region calculations.

Step 6: In the transhorizon region $(d \geq d_{ML})$, as distance increases, the propagation path will begin with smooth Earth diffraction and transition into troposcatter. Physically, the models for smooth Earth diffraction and troposcatter need to be consistent at the transition point. Physical consistency implies there is no discontinuity at the transition point. The following iterative process ensures that the transition between the two models occurs without discontinuity.

> *Step 6.1*: Let d' and d'' be the iterative test distances and be initialized to:

$$d' = d_{ML} + 3 \quad \text{(km)} \tag{11}$$

$$d'' = d_{ML} + 2 \quad \text{(km)} \tag{12}$$

Step 6.2: Compute the troposcatter losses $A_s^{d'}$ and $A_s^{d''}$ at distances d' and d'' respectively. Use § 7 as follows:

given:

> d : representing the path distance of interest d' and d'', in km
>
> $d_{1,2}$: arc length to the smooth Earth horizon distance of the terminals, in km
>
> f : frequency, in MHz
>
> $h_{1,2}$: adjusted height of the terminal above mean sea level used in subsequent calculations, in km

find:

> $A_s^{d',d''}$: troposcatter loss, A_s, in dB.

Step 6.3: Compute the slope, M_s, of the line containing the two troposcatter points $(d', A_s^{d'})$ and $(d'', A_s^{d''})$ from Step 6.2. This line is approximately tangent to the troposcatter loss at distance d'.

$$M_s = \frac{A_s^{d'} - A_s^{d''}}{d' - d''} \quad \text{(dB/km)} \tag{13}$$

Step 6.4: Compare the slope M_s to the slope of the diffraction line, M_d, from equation (7). If $M_s > M_d$, then increase d' and d'' by 1 km and return to Step 6.2 to continue iterating. Else, proceed to Step 6.5.

Step 6.5: With $M_s \leq M_d$, the distance d' represents the approximate distance where either:

> Case 1: Smooth Earth diffraction losses are predicted to be less than the losses due to troposcatter, and the diffraction model is guaranteed to intersect the troposcatter model at some distance $\geq d'$. The transhorizon region propagation loss is physically consistent.

> Case 2: The diffraction line is parallel to the tangent of the troposcatter model. However, the transhorizon region propagation loss could be not physically consistent, i.e., there is a potential discontinuity.

To determine which of the above cases are true, find the diffraction loss at distance d''.

$$A_d^{d''} = M_d d'' + A_{d0} \quad \text{(dB)} \tag{14}$$

If $A_s^{d''} \geq A_d^{d''}$, then Case 1 in Step 6.5 is true and calculation proceeds to Step 7. Else, the slope of the diffraction line should be adjusted to the tangent point d', ensuring physical consistency. The adjustment is done by pinning one end of the diffraction line at

$(d_{ML}, A_{d_{ML}})$ and the other end at $(d'', A_s^{d''})$, then re-compute the new smooth Earth diffraction line.

$$M_d = \frac{A_s^{d''} - A_{d_{ML}}}{d'' - d_{ML}} \quad \text{(dB/km)} \tag{15}$$

$$A_{d0} = A_s^{d'} - M_d d' \quad \text{(dB)} \tag{16}$$

At this point the transhorizon region is physically consistent. Proceed to Step 7.

Step 7: Compute, A_T, the loss not represented by free space loss and atmospheric absorption. This is determined based on the diffraction and troposcatter models, including any adjustments that were performed in Step 6.

   Step 7.1: Compute the predicted smooth Earth diffraction loss, A_d, for the path distance d.

$$A_d = M_d d + A_{d0} \quad \text{(dB)} \tag{17}$$

   Step 7.2: Compute the troposcatter loss, A_s, for the path distance d. Use § 7 as follows:

   given:

d:	path distance of interest, in km
$d_{1,2}$:	arc length to the smooth Earth horizon distance of the terminals, in km
f:	frequency, in MHz
$h_{1,2}$:	adjusted height of the terminal above mean sea level used in subsequent calculations, in km

   find:

A_s:	troposcatter loss, in dB
h_v:	height to the common volume, in km
d_s:	the scattering distance, in km
d_z:	half the scattering distance, in km
θ_A:	cross-over angle

   Step 7.3: Select the loss value based on the following:

   If $d < d'$ (with d' originating from the final iteration in Step 6), then:

$$A_T = A_d \quad \text{(dB)} \tag{18}$$

   Else, depending on whether Case 1 or Case 2 was true in Step 6.5:

$$A_T = \begin{cases} \text{Min}(A_d, A_s), & \text{Case 1 is TRUE} \\ A_s, & \text{Case 2 is TRUE} \end{cases} \quad \text{(dB)} \tag{19}$$

Step 8: Compute the free space loss, A_{fs}, in dB, for the path:

$$r_{1,2} = \left[h_{r1,2}^2 + 4(a_0 + h_{r1,2}) * a_0 \sin^2(0.5\, d_{1,2}/a_0) \right]^{0.5} \quad \text{(km)} \tag{20}$$

$$r_{fs} = r_1 + r_2 + d_s \quad \text{(km)} \tag{21}$$

$$A_{fs} = -32.45 - 20 \log_{10} f - 20 \log_{10} r_{fs} \quad \text{(dB)} \tag{22}$$

Step 9: Compute the atmospheric absorption loss, A_a, for a transhorizon path, using § 13. Then proceed to Step 10. Use the method presented in § 13 as follows:

   given:

$h_{1,2}$:	terminal heights, in km
$d_{1,2}$:	terminal horizon distances, in km

$\theta_{1,2}$:   terminal grazing ray take off angle, in radians

f: frequency, in MHz

h_v: height to the common volume, in km, from Step 7.2

θ_A: cross-over angle, from Step 7.2

d_z: half the scattering distance, in km, from Step 7.2

find:

A_a: atmospheric absorption loss, in dB

Step 10: Compute the long-term variability loss, $Y_{total}(q)$, for the time quantile q. Use § 10. Then proceed to Step 11.

Given:

$h_{r1,2}$:   actual terminal heights, in km

d: path distance of interest, in km

f: frequency, in MHz

q: time percentage

find:

$Y(q)$:   Long-term variability loss, in dB.

Step 11: Compute the basic transmission loss, A, in dB.

$$A = A_{fs} + A_a + A_T + Y(q) \quad \text{(dB)} \tag{23}$$

This completes the step by step procedure for the given user defined input parameters.

4   Terminal geometry

This section computes the following geometric parameters associated with a terminal.

Given:

h_r: real terminal height above mean sea level, in km

find:

d: arc length to the smooth Earth horizon distance, in km

θ: incident angle of the ray from the terminal to the smooth Earth horizon, in radians

h: adjusted height of the terminal above mean sea level used in subsequent calculations, in km

Δh: terminal height correction term, in km

As previously defined, the effective radius of the Earth, a_e, is 8 493 km.

Step 1: Use ray tracing, as defined in § 5, to determine the following:

given:

h_r: real height of terminal above mean sea level (user input), in km

N_s: surface refractivity of 301 N-Units

find:

d_r: real arc distance (real smooth Earth horizon), in km

θ_r: ray incident angle on the terminal, in radians

Step 2: Compute the effective height of the terminal, h_e, in km, using the small angle approximation if required.

$$\phi = \frac{d_r}{a_e} \quad \text{(rad)} \tag{24}$$

$$h_e = \begin{cases} d_r^2/2a_e, & \phi \leq 0.1 \\ \frac{a_e}{\cos\phi} - a_e, & \phi > 0.1 \end{cases} \quad \text{(km)} \tag{25}$$

Step 3: When the effective height, h_e, is greater than the real height, h_r, the effect of ray bending can be overestimated. Therefore, compare h_e with h_r to determine the values of h, d, and θ to be used to define the terminal's geometric parameters.

$$h = \begin{cases} h_e, & h_e \leq h_r \\ h_r, & h_e > h_r \end{cases} \quad \text{(km)} \tag{26}$$

$$d = \begin{cases} d_r, & h_e \leq h_r \\ \sqrt{2a_e h_r}, & h_e > h_r \end{cases} \quad \text{(km)} \tag{27}$$

$$\theta = \theta_r \quad \text{(rad)} \tag{28}$$

Step 4: Compute the terminal height correction term, Δh.

$$\Delta h = h_r - h \quad \text{(km)} \tag{29}$$

Step 5: If $\Delta h = 0$ km, then make the following adjustments to θ and d:

$$\theta = \sqrt{2h_r/a_e} \quad \text{(rad)} \tag{30}$$

$$d = \sqrt{2h_r a_e} \quad \text{(km)} \tag{31}$$

This concludes the section to compute the terminal geometry.


5   Ray tracing

Radio waves traveling through the atmosphere bend due to changes in the atmospheric refractivity. In traditional terrestrial models, this is normally accounted for using the standard "4/3 Earth" method, which models a linear atmospheric refractivity gradient and is a valid approximation for paths near the surface. However, the actual atmospheric gradient is exponential in nature and in air-to-ground propagation paths, the use of a linear model can incur substantial errors.

Recommendation ITU-R P.528 utilizes ray tracing techniques to compute the path of the ray through the atmosphere. The atmosphere is modelled as a set of concentric atmospheric shells, with exponentially decreasing refractivity. Applications of Snell's Law in a spherical environment, shown in equation (32), are then applied to trace the ray.

$$n_i r_i \cos\theta_i = n_{i+1} r_{i+1} \cos\theta_{i+1} \tag{32}$$

Figure 1 presents a generalized geometry of a ray through a single layer of the atmosphere.

FIGURE 1

Geometry of tracing a ray through a layer of atmosphere

P.0528-01
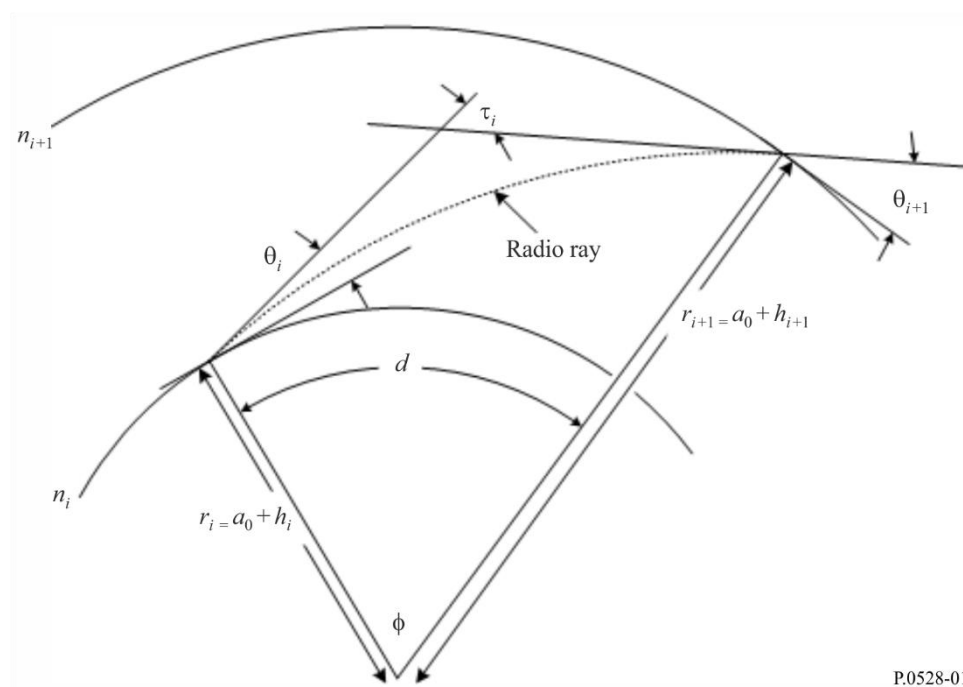
For the atmospheric model, Recommendation ITU-R P.528 uses the 25-layer reference atmosphere shown in Table 1. Rays above 475 km are assumed to travel in a straight line.

TABLE 1

Details of the 25-layer reference atmosphere

Layer, i	Height (AGL), h_i	Layer, i	Height (AGL), h_i	Layer, i	Height (AGL), h_i
0	0 km	9	1.00 km	18	50.00 km
1	0.01 km	10	1.524 km	19	70.00 km
2	0.02 km	11	2.00 km	20	90.00 km
3	0.05 km	12	3.048 km	21	110.00 km
4	0.10 km	13	5.00 km	22	225.00 km
5	0.20 km	14	7.00 km	23	350.00 km
6	0.305 km	15	10.00 km	24	475.00 km
7	0.50 km	16	20.00 km		
8	0.70 km	17	30.48 km		

Given:

  h_r: the real terminal height above ground, in km

  N_s: the surface refractivity, in N-Units

find:

  d_r: arc length to the smooth Earth horizon distance, in km

  θ_r: the incident angle of the grazing ray at the terminal, in radians

As previously defined, the actual radius of the Earth, a_0, is 6 370 km.

Step 1: Compute the scale factor, ΔN:

$$\Delta N = -7.32 \, e^{0.005577 \, N_s} \tag{33}$$

Step 2: Compute the constant, C_e:

$$C_e = \log\left(\frac{N_s}{N_s + \Delta N}\right) \tag{34}$$

Step 3: Ray tracing through the atmosphere is an iterative process, starting at the surface of the Earth and tracing up through each atmospheric layer until the height of the terminal is reached. Repeat the following sub-steps until the ray has been traced to h_r. The use of subscripts i and $i+1$ designate the lower and upper limits of the current iteration's atmospheric layer, respectively, as labelled in Fig. 1. For the first iteration ($i = 0$), let $\theta_0 = 0$ radians (representing a grazing ray).

    Step 3.1: Compute the refractivity, $N_{i,i+1}$, the refractive index, $n_{i,i+1}$, and the radial from the center of the Earth, $r_{i,i+1}$ for the current atmospheric layer:

$$r_{i,i+1} = a_0 + h_{i,i+1} \quad \text{(km)} \tag{35}$$

$$N_{i,i+1} = N_s * \exp\left(-C_e h_{i,i+1}\right) \quad \text{(N-Units)} \tag{36}$$

$$n_{i,i+1} = 1 + (N_{i,i+1} * 10^{-6}) \tag{37}$$

    Step 3.2: If $h_{i+1} > h_r$, corresponding to the terminal existing within the current atmospheric layer, then clamp the current layer iteration parameters to the terminal height and recompute the refractivity and refractive index:

$$r_{i+1} = a_0 + h_r \quad \text{(km)} \tag{38}$$

$$N_{i+1} = N_s * \exp(-C_e h_r) \quad \text{(N-Units)} \tag{39}$$

$$n_{i+1} = 1 + (N_{i+1} * 10^{-6}) \tag{40}$$

    Step 3.3: Compute the ray exit angle, θ_{i+1}:

$$\theta_{i+1} = \cos^{-1}\left(\frac{r_i n_i}{r_{i+1} n_{i+1}} \cos \theta_i\right) \quad \text{(rad)} \tag{41}$$

    Step 3.4: Compute the atmospheric layer's bending contribution, τ_i:

$$A_i = \frac{\log n_{i+1} - \log n_i}{\log r_{i+1} - \log r_i} \tag{42}$$

$$\tau_i = (\theta_{i+1} - \theta_i)\left(\frac{-A_i}{A_i + 1}\right) \quad \text{(rad)} \tag{43}$$

    Step 3.5: Repeat Step 3 for the next atmospheric layer until either (a) the height of the terminal has been reached or (b) the ray has escaped the atmosphere, i.e. reached a height of 475 km.

Step 4: If the ray has reached a height of 475 km and has still not reached the terminal height, h_r, compute the incident angle, by applying a final iteration of Snell's Law with $n_{i+1} = 1$, $r_i = a_0 + 475$ km, and $r_{i+1} = a_0 + h_r$ km. Else, proceed to Step 5.

$$\theta_{i+1} = \cos^{-1}\left(\frac{(a_0 + 475) \, n_i}{a_0 + h_r} \cos \theta_i\right) \quad \text{(rad)} \tag{44}$$

Step 5: With the ray now traced from the surface of the Earth to the terminal, the incident angle, θ_r, is:

$$\theta_r = \theta_{i+1} \quad \text{(rad)} \tag{45}$$

Step 6: The total bending angle, τ, is the sum of the bending contributions of each layer traced:

$$\tau = \sum_i \tau_i \quad \text{(rad)} \tag{46}$$

Step 7: Compute the arc distance along the surface of the Earth that the ray travelled by using the central angle ϕ.

$$\phi = \theta_r + \tau \quad \text{(rad)} \tag{47}$$

$$d_r = \phi a_0 \quad \text{(km)} \tag{48}$$

This completes the section on ray tracing.

6   Line-of-sight region

This section describes the step stake to compute the propagation loss for a line-of-sight path.

Given:

d_{ML}:   maximum line-of-sight distance, in km

d_d: distance, in km

$h_{1,2}$:   terminal heights, in km

$d_{1,2}$:   terminal horizon distances, in km

f: frequency, in MHz

A_{dML}:   diffraction loss at distance d_{ML}, in dB

q: time percentage of interest

d: path distance of interest

find:

A: the basic transmission loss, in dB

K: value used in later variability calculations

Step 1: Compute the wavelength, λ.

$$\lambda = 0.2997925/f \tag{49}$$

Step 2: The calculations to compute the loss in the line-of-sight region do not have a closed-form solution and thus require multiple iterations in order to converge on the correct result. To assist this process, it is useful to build a table of tuples $(\psi, \Delta r, d)$, which can be reference throughout this section as a means to interpolate. In this table, ψ is the reflection angle of the indirect ray in radians, Δr is the difference in ray length between the direct ray and the indirect ray, and d is the path distance between the two terminals. The following sub-steps provide useful points to populate this reference table with.

Step 2.1: Add the tuple $(0, 0, d_{ML})$ to the table, representing the maximum extent of the line-of-sight region.

Step 2.2: Add a set of tuples which are based on fractions of λ. Let $\mathbb{R}$ be the set of constants $\left\{0.06, 0.1, \frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}\right\}$. For each constant $r \in \mathbb{R}$, compute the angle ψ:

$$\psi = \sin^{-1}((\lambda r)/(2h_{e1})) \quad \text{(rad)} \tag{50}$$

Then use the ray optics method described in Section 7 to determine the values Δr and d for the reflection angle ψ. Add this tuple $(\psi, \Delta r, d)$ to the table. Proceed to Step 2.3 after all 10 tuples have been computed and added to the table. Use Section 7 as follows:

given:

ψ :   the ray reflection angle, in radians;

$h_{r1,2}$:   actual terminal heights, in km;

$\Delta h_{1,2}$:   terminal height correction terms, in km;

find:

Δr: ray length distance between the direct ray and the indirect ray, in km;

d: distance between the terminals corresponding to a reflection angle of ψ, in km.

Step 2.3: Add another set of tuples which again are based on fractions of λ. Using the same $\mathbb{R}$ defined in Step 2.2, compute the angle ψ for each constant $r \in \mathbb{R}$:

$$\psi = \sin^{-1}((\lambda r)/(2d_1)) \quad \text{(rad)} \tag{51}$$

Then use the ray optics method described in § 7 to determine the values Δr and d for the reflection angle ψ. Add this tuple $(\psi, \Delta r, d)$ to the table. Proceed to Step 2.4 after all 10 tuples have been computed and added to the table. Use § 7 as follows:

given:

ψ: the ray reflection angle, in radians

$h_{r1,2}$:   actual terminal heights, in km

$\Delta h_{1,2}$:   terminal height correction terms, in km

find:

Δr: ray length distance between the direct ray and the indirect ray, in km

d: distance between the terminals corresponding to a reflection angle of ψ, in km

Step 2.4: Generate a set of tuples based on the following set, $\mathbb{S}$, of reflection angles ψ. Let $\mathbb{S} = \{.2, .5, .7, 1, 1.2, 1.5, 1.7, 2, 2.5, 3, 3.5, 4, 5, 6, 7, 8, 10, 20, 45, 70, 80, 85, 88, 89\}$ degrees. For each $s \in \mathbb{S}$, compute the angle ψ, in radians,

$$\psi = s\frac{\pi}{180} \quad \text{(rad)} \tag{52}$$

Then use the ray optics method described in § 7 to determine the values Δr and d for the reflection angle ψ. Add this tuple $(\psi, \Delta r, d)$ to the table. Proceed to Step 2.5 after all 24 tuples have been computed and added to the table. Use § 7 as follows:

given:

ψ: the ray reflection angle, in radians

$h_{r1,2}$:   actual terminal heights, in km

$\Delta h_{1,2}$:   terminal height correction terms, in km

find:

Δr: ray length distance between the direct ray and the indirect ray, in km

d: distance between the terminals corresponding to a reflection angle of ψ, in km

Step 2.5: Add the final tuple $(\frac{\pi}{2}, 2h_1, 0)$ to the table.

Step 3: Use the generated table to interpolate and determine the distance $d_{\lambda/2}$, the distance corresponding to the distance at which $\Delta r = \lambda/2$. This is the minimum distance at which Recommendation ITU-R P.528 considers the effects of destructive interference through a two-ray model.

Step 4: Determine ψ_{limit}, the reflecting angle corresponding to the distance $d_{\lambda/2}$ by again using the generated table and interpolating.

Step 5: Use the generated table to determine the distance $d_{\lambda/6}$, the distance at which the difference in path lengths between the direct and reflected wave is $\lambda/6$ metres.

Step 6: Determine the distance d_0 in km.

If $d \geq d_d$ or $d_d \geq d_{ML}$,

$$d_o = \begin{cases} d_1, & d > d_{\lambda/6} \text{ or } d_{\lambda/6} > d_{ML} \\ d_{\lambda/6}, & else \end{cases} \quad \text{(km)} \tag{53}$$

Else if $d_d < d_{\lambda/6}$ and $d_{\lambda/6} < d_{ML}$,

$$d_o = \begin{cases} d_{\lambda/6}, & d_d < d_{\lambda/6} \text{ and } d_{\lambda/6} < d_{ML} \\ d_d, & else \end{cases} \quad \text{(km)} \tag{54}$$

Step 7: The current value of d_0 can be a course approximation for certain paths. In order to tune it, iteratively convert the distance into a reflection angle ψ using the table and compute the ray optics as defined in § 7. If the resulting d from § 7 is greater than or equal to the original d_0, or if increasing the distance 1 metre causes it to be beyond d_{ML}, use the ray optics output distance as the value for d_0. Else, increase the distance by 1 meter and re-compute the ray optics method.

Step 8: Compute the line-of-sight loss at the distance d_0. Use the table to generate the corresponding reflection angle ψ_{d0}. Then use § 7 to compute the ray optics for ψ_{d0}. Lastly, use § 8 to determine the loss, A_{d0}.

Step 9: Convert the desired distance d into its corresponding reflection angle ψ using the table. Then apply the ray optics calculations in § 7. In most cases, the resulting distance from the ray optics calculations will be different from the desired distance d. This is a source of error in the final result. To reduce this error, apply a small modification to ψ. An increase in ψ causes a decrease in the resulting ray optics distance. Iterate until the difference between the ray optics resulting distance and the desired distance is within the acceptable range of error. In general, $\epsilon \approx 1$ metre is sufficient for all cases. Let the final resulting ray optics distance be called d_{ro}.

Step 10: Use § 8 to compute the line-of-sight loss. Then proceed to Step 11. Use § 8 as follows:

given:

ψ: the ray reflection angle, in radians

$h_{r1,2}$: actual terminal heights, in km

$\Delta h_{1,2}$: terminal height correction terms, in km

find:

Δr: ray length distance between the direct ray and the indirect ray, in km

d: distance between the terminals corresponding to a reflection angle of ψ, in km

Step 11: Compute the atmospheric absorption loss for the path. The effective thicknesses of the absorbing layer for oxygen and water vapour are different. For oxygen, the effective thickness of the absorbing layer is $T_{eo} = 3.25$ km. For water vapour, the effective thickness of the absorbing layer is $T_{ow} = 1.36$ km.

Step 11.1: Compute the effective ray lengths through the oxygen and water vapour absorbing layers for the path using the steps described in § 8. This requires applying Section 8 twice: once for the oxygen absorbing layer and once for the water vapour absorbing layer. Then proceed to Step 11.2. Use § 8 as follows:

given:

z_1: the radial of the low point, in km

z_2: the radial of the high point, in km

a_a: the effective radius of the Earth, in km. Set to 8 493 km

r_0: the arc distance between the two points, in km, from equation (74)

θ_{h1}: the ray take off angle, in radians, from equation (77)

$T_{eo,ew}$: the thickness of the absorbing layer, T_e, in km, where $T_e = T_{eo} = 3.25$ km for the oxygen absorbing layer and $T_e = T_{ew} = 1.36$ km for the water vapour absorbing layer

find:

$r_{eo,ew}$: the effective ray length, r_e, in km. The ray length through the oxygen absorbing layer, r_{eo}, corresponds to $T_e = T_{eo} = 3.25$ km. The ray length through the water vapour absorbing layer, r_{ew}, corresponds to $T_e = T_{ew} = 1.36$ km.

Step 11.2: Determine the atmospheric absorption rates for oxygen, γ_{oo}, and water vapour, γ_{ow}, in dB/km using § 14. Then proceed to Step 11.3. Use § 14 as follows:

given:

f: frequency, in MHz

find:

γ_{oo}: oxygen absorption rate, in dB/km

γ_{ow}: water vapour absorption rate, in dB/km

Step 11.3: Compute the total atmospheric absorption loss, A_a, using the absorption rates γ_{oo} and γ_{ow} from Step 11.2 and the effective ray lengths r_{eo} and r_{ew} from equation (153).

$$A_a = \gamma_{oo}r_{eo} + \gamma_{ow}r_{ew} \quad \text{(dB)} \tag{55}$$

Step 12: Compute the free space loss, A_{fs}, in dB.

Step 12.1: Compute θ_{fs}.

$$\theta_{fs} = a_a(\theta_1 + \theta_2)/a_0 \quad \text{(rad)} \tag{56}$$

Step 12.2: Compute the radials, $z_{1,2}$, in km.

$$z_{1,2} = a_0 + h_{1,2} \quad \text{(km)} \tag{57}$$

Step 12.3: Compute the ray length, r_{FS}, in km.

$$r_{fs} = \max\left([(z_2 - z_1)^2 + 4z_1z_2 \sin^2(0.5\,\theta_{fs})]^{0.5}, z_2 - z_1\right) \quad \text{(km)} \tag{58}$$

Step 12.4: Compute the total free space loss, A_{fs}, in dB.

$$L_{bf} = -32.45 - 20\log_{10} f \quad \text{(dB)} \tag{59}$$

$$A_{fs} = L_{bf} - 20\log_{10} r_{fs} \quad \text{(dB)} \tag{60}$$

Step 13: Compute the contribution of variability to the total loss. Use § 16 to compute Y_{total}. Then proceed to Step 14. Use § 16 as follows:

given:

$h_{r1,2}$: actual terminal heights, in km

d: path distance of interest, in km

f: frequency, in MHz

q: time percentage

find:

$Y(q)$: long-term variability loss, in dB.

Step 14: Compute the basic transmission loss.

$$A = A_{fs} + A_a + A_{LOS} + Y(q) \quad \text{(dB)} \tag{61}$$

This concludes this section.

7 Line-of-sight ray optics

This section describes how to perform the geometric path parameters for two terminals within line-of-sight distance of each other using ray optics.

The inputs for this section are:

ψ: the ray reflection angle, in radians

$h_{r1,2}$: actual terminal heights, in km

$\Delta h_{1,2}$: terminal height correction terms, in km.

The output for this section is:

Δr: ray length distance between the direct ray and the indirect ray, in km

d: distance between the terminals corresponding to a reflection angle of ψ, in km.

Step 1: Compute the adjusted earth radius, a_a:

$$z = (a_0/a_e) - 1 \tag{62}$$

$$k_a = 1/(1 + \cos\psi) \tag{63}$$

$$a_a = a_0 k_a \quad \text{(km)} \tag{64}$$

Step 2: Compute the adjusted earth terminal height correction terms, $\Delta h_{a1,2}$.

$$\Delta h_{a1,2} = \Delta h_{1,2}(a_a - a_0)/(a_e - a_o) \quad \text{(km)} \tag{65}$$

Step 3: Compute the heights, $H_{1,2}$.

$$H_{1,2} = h_{r1,2} - \Delta h_{a1,2} \quad \text{(km)} \tag{66}$$

Step 4: Compute the terminal geometric parameters $z_{1,2}$, $\theta_{1,2}$, $D_{1,2}$, and $H'_{1,2}$.

$$z_{1,2} = a_a + H_{1,2} \quad \text{(km)} \tag{67}$$

$$\theta_{1,2} = \cos^{-1}(a_a \cos\psi/z_{1,2}) - \psi \quad \text{(rad)} \tag{68}$$

$$D_{1,2} = z_{1,2} \sin\theta_{1,2} \quad \text{(km)} \tag{69}$$

$$H'_{1,2} = \begin{cases} H_{1,2}, & \psi > 1.56 \\ D_{1,2}\tan\psi, & \psi \le 1.56 \end{cases} \quad \text{(km)} \tag{70}$$

Step 5: Compute Δz, the difference in the terminal radials.

$$\Delta z = |z_1 - z_2| \quad \text{(km)} \tag{71}$$

Step 6: Compute the path distance between the two terminals corresponding to a reflection angle of ψ.

$$d = \max(a_a(\theta_1 + \theta_2), 0) \quad \text{(km)} \tag{72}$$

Step 7: With the geometric parameters computed, determine the length of the direct ray, r_0, and the indirect ray, r_{12}.

$$\alpha = \tan^{-1}((H'_2 - H'_1)/(D_1 + D_2)) \quad \text{(rad)} \tag{73}$$

$$r_0 = (D_1 + D_2)/\cos\alpha \quad \text{(km)} \tag{74}$$

$$r_{12} = (D_1 + D_2)/\cos\psi \quad \text{(km)} \tag{75}$$

Step 8: Compute the difference in the length between the two rays.

$$\Delta r = 4\, H_1' H_2' / (r_0 + r_{12}) \quad \text{(km)} \tag{76}$$

Step 9: Compute the angles $\theta_{h1,2}$.

$$\theta_{h1} = \alpha - \theta_1 \quad \text{(rad)} \tag{77}$$

$$\theta_{h2} = -(\alpha + \theta_2) \quad \text{(rad)} \tag{78}$$

This completes this section.

8   Line-of-sight loss calculations

This section describes how to compute the loss calculations for a line-of-sight path.

Given:

> d: path distance of interest, in km
>
> ψ: the ray reflection angle, in radians
>
> d_0:   path distance at which diffraction begins to affect the line-of-sight region, in km
>
> f: frequency, in MHz

find:

> A_{LOS}:   line-of-sight loss, in dB.

Step 1: If the path distance, d, is greater than d_0, then the path is within the LOS-Diffraction blending region and the loss A_{LOS} is determine using equation (79). Else, $d \leq d_0$ and skip to Step 2.

$$A_{LOS} = ((d - d_0)(A_{dML} - A_{d0})/(d_{ML} - d_0)) + A_{d0} \quad \text{(dB)} \tag{79}$$

Step 2: If the reflection angle $\psi < \psi_{limit}$, then set $A_{LOS} = 0$ dB and return, as Recommendation ITU-R P.528 does not consider a 2-ray model within this region. Else, $\psi \geq \psi_{limit}$ and skip to Step 3.

Step 3: The path length d, is such that Recommendation ITU-R P.528 uses a 2-ray model. Compute the wavelength, λ.

$$\lambda = 0.2997925/f \tag{80}$$

Step 4: Compute the complex reflection coefficients, R_g and ϕ_g using § 9.

Step 5: The divergence factor, D_v, takes into account that a reflection from a smooth curved Earth surface is less efficient than from a flat Earth. Compute the divergence factor for the path.

$$D_v = \left[1 + \frac{2R_r(1 + \sin^2 \psi)}{a_a \sin \psi} + \left(\frac{2R_r}{a_a} \right)^2 \right]^{-1/2} \tag{81}$$

Step 6: The ray length factor, F_r, takes into account geometries in which the direct ray is significantly larger (and shorter) than the indirect ray, such as when both terminals are high and close in the case of two aircraft. Compute F_r:

$$F_r = \min(r_0/r_{12},\ 1) \tag{82}$$

Step 7: Compute the effective reflection coefficients R_{Tg} and ϕ_{Tg}.

$$R_{Tg} = R_g D_v F_r \tag{83}$$

$$\phi_{Tg} = (2\,\pi \Delta r/\lambda) + \phi_g \tag{84}$$

Step 8: Compute the loss, A_{LOS}.

$$R = R_{Tg} \cos \phi_{Tg} - R_{Tg} \sin \phi_{Tg} \tag{85}$$

$$W_{RL} = \min(|1 + R|, 1) \tag{86}$$

$$W_{R0} = W_{RL}^2 \tag{87}$$

$$A_{LOS} = 10 \log_{10} W_{R0} \tag{88}$$

This completes this section.

9 Ground reflection coefficients

This section describes the steps taken to compute the ground reflection coefficients.

Given:

    ψ: reflection angle, in radians

    f: frequency, in MHz

find:

    R_g: the real part of the reflection coefficient

    ϕ_g: the imaginary part of the reflection coefficient.

With the previously stated assumptions for the electrical properties of the ground of $\sigma = 0.005$ S/m and $\epsilon_r = 15$ (corresponding to Average Group).

Step 1: Compute the following values,

$$X = 18\,000\,\sigma/f \tag{89}$$

$$Y = \epsilon_r - \cos^2 \psi \tag{90}$$

$$T = [Y^2 + X^2]^{0.5} + Y \tag{91}$$

$$P = (0.5T)^{0.5} \tag{92}$$

$$Q = X/2P \tag{93}$$

$$B = 1/(P^2 + Q^2) \tag{94}$$

$$A = 2P/(P^2 + Q^2) \tag{95}$$

Step 2: Compute the real and imaginary parts of the reflection coefficient.

$$R_g = \left[\left((1 + B \sin^2 \psi) - A \sin \psi\right) / \left((1 + B \sin^2 \psi) + A \sin \psi\right)\right]^{0.5} \tag{96}$$

$$\phi_g = \tan2^{-1}(-Q, \sin \psi - P) - \tan2^{-1}(Q, \sin \psi + P) \tag{97}$$

This concludes this section.

10 Smooth Earth diffraction

This section describes the steps taken to compute the smooth Earth diffraction loss at a specified distance within the diffraction zone. The model assumes an "average ground" with a conductivity of 0.005 S/m and a relative dielectric constant of 15. The effective earth radius factor, k, of 4/3 is assumed (corresponding to a surface refractivity $N_s = 300$ N-Units). Horizontal polarization is assumed.

Given:

    d_0:   path distance of interest, in km

$d_{1,2}$:   horizon distance for the terminals, in km

f: frequency, in MHz

find:

A_d: smooth Earth diffraction loss, in dB.

Smooth Earth diffraction is computed using equation (98):

$$A_d = G(x_0) - F(x_1) - F(x_2) - 20 \quad \text{(dB)} \tag{98}$$

Step 1: Compute the normalized distances.

$$x_{0,1,2} = 1.607 f^{1/3} d_{0,1,2} \quad \text{(km)} \tag{99}$$

Step 2: Compute the distance dependent term for all three normalized distances.

$$G(x_{0,1,2}) = 0.05751 x_{0,1,2} - 10 \log_{10} x_{0,1,2} \quad \text{(dB)} \tag{100}$$

Step 3: Compute the term $y_{1,2}$

$$y_{1,2} = 40 \log_{10} x_{1,2} - 117 \quad \text{(dB)} \tag{101}$$

Step 4: Compute the height functions.

If $x_{1,2} \geq 2\,000$ km:

$$F(x_{1,2}) = G(x_{1,2}) \quad \text{(dB)} \tag{102}$$

Else if $200 < x_{1,2} < 2\,000$ km:

$$W_{1,2} = 0.0134 x_{1,2} e^{(-0.005 x_{1,2})} \tag{103}$$

$$F(x_{1,2}) = W_{1,2} y_{1,2} + (1 - W_{1,2}) G(x_{1,2}) \quad \text{(dB)} \tag{104}$$

Else, $x \leq 200$.

$$F(x_{1,2}) = y_{1,2} \quad \text{(dB)} \tag{105}$$

Step 5: With $G(x_0)$ and $F(x_{1,2})$ computed, use equation (98) above to compute the smooth Earth diffraction loss.

This completes this section.

11   Troposcatter

This section describes the steps taken to compute the troposcatter for a given distance. The computation of troposcatter loss is performed using mathematical techniques that consider the curved ray paths bounding the common volume of the two terminals. The earth radius factor k of 4/3 is assumed (surface refractivity $N_s = 301$ N-Units). Troposcatter is computed using equation (106) (described below).

$$A_s = S_e + S_V + 10 \, \log_{10}(\kappa \theta_s^3 / \ell) \quad \text{(dB)} \tag{106}$$

given:

$d_{1,2}$:   arc length along the surface of the smooth Earth, in km

$h_{1,2}$:   terminal heights, in km

f: frequency, in MHz

d: path distance of interest, in km

find:

A_s: troposcatter loss, in dB.

Step 1: Compute the scatter distance, d_s, in km.

$$d_s = d - d_1 - d_2 \quad \text{(km)} \tag{107}$$

Step 2: If $d_s = 0$, there is no common volume in the path geometry and thus propagation via troposcatter is not supported. Set the below results and return from § 7. Else $d_s > 0$ and continue to Step 3.

$$A_s = 0 \quad \text{(dB)} \tag{108}$$

$$h_v = 0 \quad \text{(km)} \tag{109}$$

$$d_z = 0 \quad \text{(km)} \tag{110}$$

$$\theta_s = 0 \quad \text{(rad)} \tag{111}$$

$$\theta_A = 0 \quad \text{(rad)} \tag{112}$$

Step 3: The smooth Earth arc distance from each terminal's grazing ray to the centre of the common volume, d_z, is:

$$d_z = 0.5 d_s \quad \text{(km)} \tag{113}$$

Step 4: Compute the atmospheric gradient parameters:

$$A_m = 1/a_0 \tag{114}$$

$$dN = A_m - 1/a_e \tag{115}$$

$$\gamma_e = N_s \times 10^{-6}/dN \tag{116}$$

Step 5: The following equations are used to determine the geometric parameters associated with troposcatter, including the height of the common volume, h_v, in km, and the ray slopes at the cross-over angle, θ_A, in radians.

$$z_a = \frac{1}{2a_e}\left(\frac{d_z}{2}\right)^2 \quad \text{(km)} \tag{117}$$

$$z_b = \frac{1}{2a_e}(d_z)^2 \quad \text{(km)} \tag{118}$$

$$Q_o = A_m - dN \tag{119}$$

$$Q_{a,b} = A_m - dN * e^{-z_{a,b}/\gamma_e} \tag{120}$$

$$Z_a = (7Q_o + 6Q_a - Q_b)\frac{d_z^2}{96} \quad \text{(km)} \tag{121}$$

$$Z_b = (Q_o + 2Q_a)\frac{d_z^2}{6} \quad \text{(km)} \tag{122}$$

$$Q_{A,B} = A_m - dN * e^{-Z_{a,b}/\gamma_e} \tag{123}$$

$$h_v = (Q_o + 2Q_A)\frac{d_z^2}{6} \quad \text{(km)} \tag{124}$$

$$\theta_A = (Q_o + 4Q_A + Q_B)\frac{d_z}{6} \quad \text{(rad)} \tag{125}$$

$$\theta_s = 2\,\theta_A \quad \text{(rad)} \tag{126}$$

Step 6: Compute the scattering efficiency term, S_e.

$$\epsilon_1 = (5.67 \times 10^{-6})N_s^2 - 0.00232N_s + 0.031 \tag{127}$$

$$\epsilon_2 = 0.0002N_s^2 - 0.06N_s + 6.6 \tag{128}$$

$$\gamma = 0.1424 \left(1 + \frac{\epsilon_1}{\exp[(h_v/4)^6]}\right) \tag{129}$$

$$S_e = 83.1 - \frac{\epsilon_2}{1+0.07716 h_v^2} + 20 \log_{10}[(0.1424/\gamma)^2 e^{\gamma h_v}] \quad \text{(dB)} \tag{130}$$

Step 7: Compute the scattering volume term, S_V.

$$X_{A1,2} = h_{1,2}^2 + 4(a_e + h_{1,2})a_e \sin^2\left(\frac{d_{1,2}}{2a_e}\right) \tag{131}$$

$$\ell_{1,2} = \sqrt{X_{A1,2}} + d_z \quad \text{(km)} \tag{132}$$

$$\ell = \ell_1 + \ell_2 \quad \text{(km)} \tag{133}$$

$$s = \frac{\ell_1 - \ell_2}{\ell} \tag{134}$$

$$\eta = \gamma \theta_s \ell / 2 \tag{135}$$

$$\kappa = f / 0.0477 \tag{136}$$

$$\rho_{1,2} = 2 \kappa \theta_s h_{1,2} \quad \text{(km)} \tag{137}$$

$$S_V = 10 \log_{10}\left(\frac{(A\eta^2 + B_S\eta)q_1 q_2}{\rho_1^2 \rho_2^2} + C_S\right) \quad \text{(dB)} \tag{138}$$

where:

$$X_{v1} = (1 + s)^2 \eta \tag{139}$$

$$X_{v2} = (1 - s)^2 \eta \tag{140}$$

$$q_1 = X_{v1}^2 + \rho_1^2 \tag{141}$$

$$q_2 = X_{v2}^2 + \rho_2^2 \tag{142}$$

$$A = (1 - s^2)^2 \tag{143}$$

$$B_S = 6 + 8s^2 + (8(1-s)X_{v1}^2\rho_1^2)/q_1^2 + (8(1+s)X_{v2}^2\rho_2^2)/q_2^2 + 2(1-s^2)(1 + 2X_{v1}^2/q_1)(1 + 2X_{v2}^2/q_2) \tag{144}$$

$$C_S = 12 \left(\frac{\rho_1 + \sqrt{2}}{\rho_1}\right)^2 \left(\frac{\rho_2 + \sqrt{2}}{\rho_2}\right)^2 \left(\frac{\rho_1 + \rho_2}{\rho_1 + \rho_2 + 2\sqrt{2}}\right) \tag{145}$$

Step 8: Compute the troposcatter loss using equation (106).

This completes this section.

12   Effective ray length

This section describes the steps to determine the effective ray length for use in the calculations of atmospheric absorption. This section uses a geometric model in which an absorbing layer sits upon the surface of the Earth and is defined by an effective thickness. Geometric calculations are then performed to determine the effective ray length, the portion of the ray path that is contained within the absorbing layer.

Three possible geometries exist, as showing in Fig. 2:

1   Scenario 1: Both endpoints are within the absorbing layer

2   Scenario 2: One endpoint is within the absorbing layer and one endpoint is above the absorbing layer

3   Scenario 3: Both endpoints are above the absorbing layer, although the ray path could still intersect the absorbing layer

FIGURE 2

The three possible geometries for determining the effective ray length relative to an absorbing layer

P.0528-02
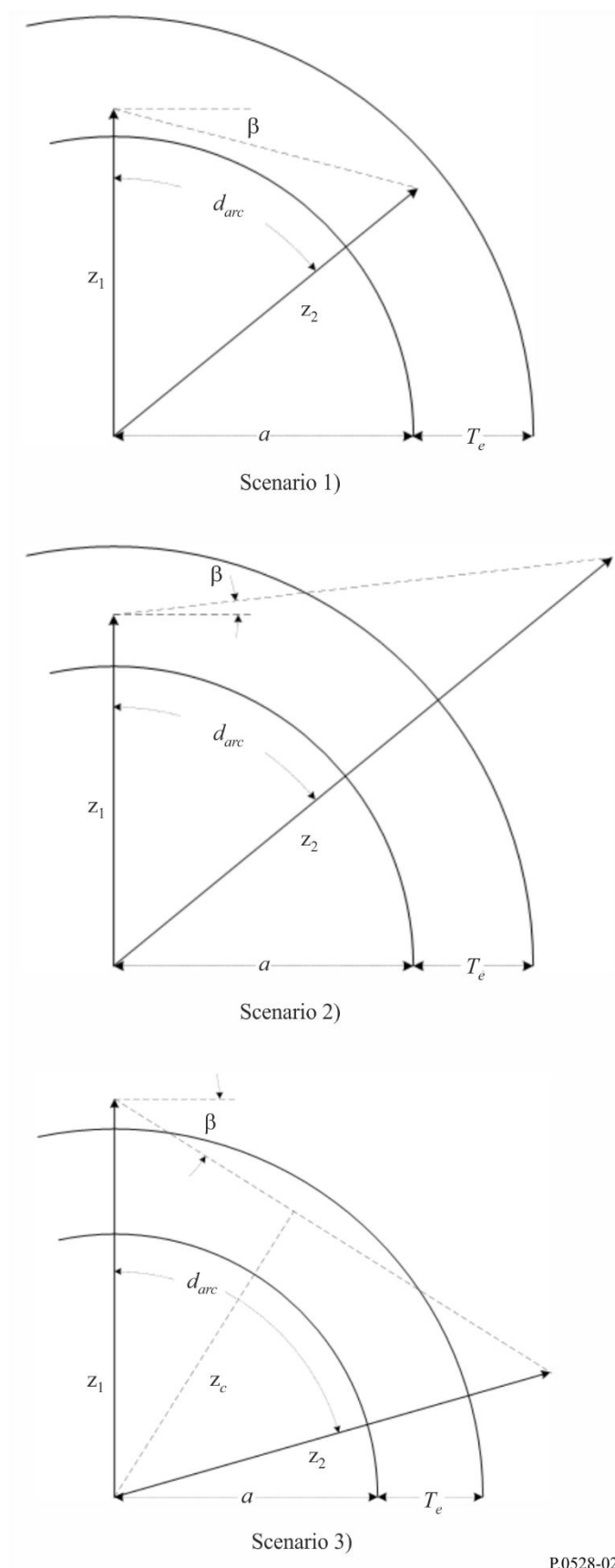
Given:

$z_{1,2}$: radials, in km

a: radius of the Earth, in km. The radius is dependent on the type of path

d_{arc}: arc length between the two radials along the surface of the Earth, in km

β: ray take off angle, relative to the tangent of the Earth surface, in radians

T_e: effective thickness of the absorbing layer, in km

find:

r_e: Effective ray length, in km.

Step 1: Compute the angle α and the radial z_T.

$$\alpha = (\pi/2) + \beta \quad \text{(rad)} \tag{146}$$

$$z_T = a + T_e \quad \text{(km)} \tag{147}$$

Step 2: It is necessary to determine the correct geometry based on the parameters provided. If $z_e \leq z_T$, then both endpoints are within the absorbing layer resulting in Scenario 1 in Fig. 2. Compute r_e using equation (148) and return. Else continue to Step 3.

$$r_e = d_{arc} \quad \text{(km)} \tag{148}$$

Step 3: If $z_T < z_1$, then both terminals are above the absorbing layer resulting in Scenario 3 in Fig. 2 and continue with this step. Else, proceed to Step 4.

To determine if the ray path intersects with the absorbing layer, compute they radial z_c, the lowest point along the ray path. Then use equation (150) to determine the correct value for r_e.

$$z_c = z_1 \sin \alpha \quad \text{(km)} \tag{149}$$

$$r_e = \begin{cases} 0, & z_T \leq z_c \\ 2\,z_T \sin(\cos^{-1}(z_c/z_T)), & \text{else} \end{cases} \quad \text{(km)} \tag{150}$$

Step 4: The geometry is determined to be that of Scenario 2 in Fig. 2, with the low terminal being within the absorbing layer and the high terminal being above it. The following equations are used to determine the portion of the ray path that is within the absorbing layer.

$$A_q = \sin^{-1}(z_1 \sin(\alpha)/z_T) \quad \text{(rad)} \tag{151}$$

$$A_e = \pi - \alpha - A_q \quad \text{(rad)} \tag{152}$$

$$r_e = \begin{cases} z_T - z_1, & A_e = 0 \\ (z_1 \sin A_e)/\sin A_q, & A_e \neq 0 \end{cases} \quad \text{(km)} \tag{153}$$

This completes this section.

13 Atmospheric absorption loss for transhorizon paths

This section describes the steps to compute the loss due to atmospheric absorption for transhorizon paths.

Given:

$h_{1,2}$: terminal heights, in km

$d_{1,2}$: horizon distances, in km

f: frequency, in MHz

h_v: height to the common volume, in km

θ_A: cross-over angle at the common volume, in radians

find:

A_a: atmospheric absorption loss, in dB.

Step 1: Compute the radials z_1, z_2, and z_v from the centre of the Earth to the low terminal, the high terminal, and the common volume, respectively.

$$z_1 = h_1 + a_e \quad \text{(km)} \tag{154}$$

$$z_2 = h_2 + a_e \quad \text{(km)} \tag{155}$$

$$z_v = h_v + a_e \quad \text{(km)} \tag{156}$$

Step 2: Focus on the portion of the path from the low terminal to the common volume, noting that the height of the common volume can be above or below that of the low terminal. To compute the effective ray length for this portion of the path, set the parameters z_{low1}, z_{high1}, and β_1 so that the geometric parameters are consistent with how the calculations for effective ray length is presented.

$$z_{low1} = \begin{cases} z_v, & z_v < z_1 \\ z_1, & z_v \geq z_1 \end{cases} \quad \text{(km)} \tag{157}$$

$$z_{high1} = \begin{cases} z_1, & z_v < z_1 \\ z_v, & z_v \geq z_1 \end{cases} \quad \text{(km)} \tag{158}$$

$$\beta_1 = \begin{cases} -\tan^{-1}\theta_A, & z_v < z_1 \\ -\theta_1, & z_v \geq z_1 \end{cases} \quad \text{(rad)} \tag{159}$$

Step 3: Now focus on the portion of the path from the common volume to the high terminal, noting that the height of the common volume can be above or below that of the high terminal. To compute the effective ray length for this portion of the path, set the parameters z_{low2}, z_{high2}, and β_2 so that the geometric parameters are consistent with how the calculations for effective ray length is presented.

$$z_{low2} = \begin{cases} z_v, & z_v < z_2 \\ z_2, & z_v \geq z_2 \end{cases} \quad \text{(km)} \tag{160}$$

$$z_{high2} = \begin{cases} z_2, & z_v < z_2 \\ z_v, & z_v \geq z_2 \end{cases} \quad \text{(km)} \tag{161}$$

$$\beta_2 = \begin{cases} -\tan^{-1}\theta_A, & z_v < z_2 \\ -\theta_2, & z_v \geq z_2 \end{cases} \quad \text{(rad)} \tag{162}$$

Step 4: Compute the arc distances for each of the two above portions of the path.

$$d_{arc1} = d_1 + d_z \quad \text{(km)} \tag{163a}$$

$$d_{arc2} = d_2 + d_z \quad \text{(km)} \tag{163b}$$

Step 5: The effective thicknesses of the absorbing layer for oxygen and water vapour are different. For oxygen, the effective thickness of the absorbing layer is $T_{eo} = 3.25$ km. For water vapour, the effective thickness of the absorbing layer is $T_{ow} = 1.36$ km. Compute the effective ray lengths through the oxygen and water vapour absorbing layers for the first portion of the path (from the low terminal to the common volume) using the steps described in § 8. This requires applying § 8 twice: once for the oxygen absorbing layer and once for the water vapour absorbing layer. Then proceed to Step 6. Use § 8 as follows:

given:

z_{low1}:   the radial of the low point, in km, from equation (157)

z_{high1}:   the radial of the high point, in km, from equation (158)

a_e: the effective radius of the Earth, in km. Set to 8 493 km

d_{arc1}:   the arc distance between the two points, in km, from equation (163a)

β_1:   the ray take off angle, in radians, from equation (159)

$T_{eo,ew}$:   the thickness of the absorbing layer, T_e, in km, where $T_e = T_{eo} = 3.25$ km for the oxygen absorbing layer and $T_e = T_{ew} = 1.36$ km for the water vapour absorbing layer

find:

$r_{eo1,ew1}$:   the effective ray length, r_e, in km. The ray length through the oxygen absorbing layer, r_{eo}, corresponds to $T_e = T_{eo} = 3.25$ km. The ray length through the water vapour absorbing layer, r_{ew}, corresponds to $T_e = T_{ew} = 1.36$ km.

Step 6: Compute the effective ray lengths through the oxygen and water vapour absorbing layers for the second portion of the path (from the height of the common volume to the high terminal), in the same manner as was done in Step 5 but using the geometric parameters associated with the path segment from the common volume to the high terminal. Then proceed to Step 7. Use § 8 as follows:

given:

z_{low2}:   the radial of the low point, in km, from equation (160)

z_{high2}:   the radial of the high point, in km, from equation (161)

a_e: the effective radius of the Earth, in km. Set to 8 493 km

d_{arc2}:   the arc distance between the two points, in km, from equation (163b)

β_2:   the ray take off angle, in radians, from equation (162)

$T_{eo,ew}$:   the thickness of the absorbing layer, T_e, in km, where $T_e = T_{eo} = 3.25$ km for the oxygen absorbing layer and $T_e = T_{ew} = 1.36$ km for the water vapour absorbing layer

find:

$r_{eo2,ew2}$:   the effective ray length, r_e, in km. The ray length through the oxygen absorbing layer, r_{eo}, corresponds to $T_e = T_{eo} = 3.25$ km. The ray length through the water vapour absorbing layer, r_{ew}, corresponds to $T_e = T_{ew} = 1.36$ km.

Step 7: Compute the total effective ray lengths for the oxygen absorbing layer, r_{eo}, and for the water vapour absorbing layer, r_{ew}, in km.

$$r_{eo} = r_{eo1} + r_{eo2} \quad (\text{km}) \tag{164a}$$

$$r_{ew} = r_{ew1} + r_{ew2} \quad (\text{km}) \tag{164b}$$

Step 8: Determine the atmospheric absorption rates for oxygen, γ_{oo}, and water vapour, γ_{ow}, in dB/km using § 14. Then proceed to Step 9. Use § 14 as follows:

given:

f :   frequency, in MHz

find:

γ_{oo}:   oxygen absorption rate, in dB/km

γ_{ow}:   water vapour absorption rate, in dB/km.

Step 9: Compute the total atmospheric absorption loss, A_a, using the absorption rates γ_{oo} and γ_{ow} from Step 8 and the effective ray lengths r_{eo} and r_{ow} from equation (164).

$$A_a = \gamma_{oo} r_{eo} + \gamma_{ow} r_{ew} \quad (\text{dB}) \tag{165}$$

This concludes the calculation of atmospheric absorption for a transhorizon path.

14 Atmospheric absorption rates

This section describes the steps taken to determine the absorption rate of oxygen, γ_{oo}, and water vapour, γ_{ow}, in dB/km.

Given:

f : frequency, in MHz

find:

γ_{oo}: oxygen absorption rate, in dB/km

γ_{ow}: water vapour absorption rate, in dB/km.

TABLE 2

Data on absorption rates vs frequency

f (MHz)	γ_{oo} (dB/km)	γ_{ow} (dB/km)	f (MHz)	γ_{oo} (dB/km)	γ_{ow} (dB/km)	f (MHz)	γ_{oo} (dB/km)	γ_{ow} (dB/km)
100	0.000 19	0	550	0.002 5	0	4 000	0.010	0.0001 7
150	0.000 42	0	700	0.003	0	4 900	0.011	0.003 4
205	0.000 70	0	1 000	0.004 2	0	8 300	0.014	0.002 1
300	0.000 96	0	1 520	0.005	0	10 200	0.015	0.009
325	0.001 3	0	2 000	0.007	0	15 000	0.017	0.025
350	0.001 5	0	3 000	0.008 8	0	17 000	0.018	0.045
400	0.001 8	0	3 400	0.009 2	0.000 1			

Step 1: Use the values in Table 2 to interpolate for γ_{oo} and γ_{ow}. Let frequencies f' and f'' be selected from the table such that $f' < f < f''$. Likewise for $\gamma'_{oo} < \gamma_{oo} < \gamma''_{oo}$ and $\gamma'_{ow} < \gamma_{ow} < \gamma''_{ow}$.

Step 2: Compute the interpolation scale factor, R.

$$R = \frac{\log_{10}(f) - \log_{10}(f')}{\log_{10}(f'') - \log_{10}(f')} \tag{166}$$

Step 3: Interpolate the value γ_{oo}.

$$X = R(\log_{10}(\gamma''_{oo}) - \log_{10}(\gamma'_{oo})) + \log_{10}(\gamma'_{oo}) \tag{167}$$

$$\gamma_{oo} = 10^X \quad \text{(dB/km)} \tag{168}$$

Step 4: Interpolate the value γ_{ow}. Note that the first 13 values of γ_{ow} in Table 2 are 0 dB/km. Thus care should be taken when interpolating. If $f < 3400$, then $\gamma_{ow} = 0$ dB/km. Else:

$$Y = R(\log_{10}(\gamma''_{ow}) - \log_{10}(\gamma'_{ow})) + \log_{10}(\gamma'_{ow}) \tag{169}$$

$$\gamma_{ow} = 10^Y \quad \text{(dB/km)} \tag{170}$$

This completes this section.

15 Total variability for transhorizon paths

This section defines how to compute the total contribution of variability to the median basic transmission loss for a transhorizon path.

Given:

$h_{r1,2}$:   real terminal heights, in km

q: time percentage of interest. Model input variable

f: frequency, MHz

d: path distance of interest, in km

A_T: loss predicted by either diffraction or troposcatter, in dB

θ_s: the scattering angle, in radians

find:

$Y_{total}(q)$:   total variability loss, in dB.

Step 1: Compute the contribution of long term variability for the time percentage q by using § 17. Then proceed to Step 2. Use § 17 as follows:

given:

$h_{r1,2}$:   real terminal heights, in km

d: path distance of interest, in km

f: frequency, MHz

q: time percentage of interest

$f_{\theta h}$:   set to the value $f_{\theta h} = 1$

A_T: loss predicted by either diffraction or troposcatter, in dB

find:

$Y_e(q)$:   long term variability loss, in dB.

Step 2: In order to correctly combine the effects of long term variability and tropospheric multipath, both of which are distributions, the mean value of the long term variability distribution is required. Compute the contribution of long term variability for time percentage 0.50 by using § 17. Then proceed to Step 3. Use § 17 as follows:

given:

$h_{r1,2}$:   real terminal heights, in km

d: path distance of interest, in km

f: frequency, MHz

0.50:   the mean time percentage ($q = 0.50$)

$f_{\theta h}$:   set to the value $f_{\theta h} = 1$

A_T: loss predicted by either diffraction or troposcatter, in dB

find:

$Y_e(0.50)$:   long term variability loss, in dB.

Step 3: In order to smoothly transition the effects of tropospheric multipath from the line-of-sight region to the transhorizon region, the value of K, from which tropospheric multipath is determined, should be determined at transition point between line-of-sight and non-line-of-sight. Compute the line-of-sight loss calculations, as described in § 6. Then proceed to Step 3. Use § 6 as follows:

given:

d_{ML}:   maximum line-of-sight distance, in km

d_d: distance, in km

$h_{1,2}$:   terminal heights, in km

$d_{1,2}$:   horizon distances, in km

f: frequency, in MHz

A_{dML}:   diffraction loss at distance d_{ML}, in dB

q: time percentage of interest

d: path distance of interest

find:

A: the basic transmission loss, in dB

K: value used in later variability calculations.

Step 4: Compute the value K_t which is used to determine the effects of tropospheric multipath. Let $\theta_{1.5} = 0.02617993878$ radians (1.5 degrees).

$$K_t = \begin{cases} 20, \theta_s \geq \theta_{1.5} \\ K_{LOS}, \theta_s \leq 0 \\ (\theta_s(20 - K_{LOS})/\theta_{1.5}) + K_{LOS}, 0 < \theta_s < \theta_{1.5} \end{cases} \qquad (171)$$

Step 5: Compute the contribution of tropospheric multipath for time percentage q using § 18. Then proceed to Step 6. Use § 18 as follows:

given:

K_t: the value K

q: time percentage of interest

find:

$Y_\pi(q)$:   contribution of tropospheric multipath at time percentage q, in dB.

Step 6: Combine the effects long term variability with tropospheric multipath to get the total variability contribution, $Y_{total}(q)$, using the previously computed values $Y_e(q)$, $Y_e(0.50)$, and $Y_\pi(q)$. The mean of tropospheric multipath is $Y_\pi(0.50) = 0$.

$$Y_{total}(0.50) = Y_e(0.50) + Y_\pi(0.50) \qquad (172)$$

$$Y = [(Y_e(q) - Y_e(0.50))^2 + (Y_\pi(q) - Y_\pi(0.50))^2]^{0.5} \qquad (173)$$

$$Y_{total} = \begin{cases} Y_{total}(0.50) + Y, \ q < 0.50 \\ Y_{total}(0.50) - Y, \ q \geq 0.50 \end{cases} \quad \text{(dB)} \qquad (174)$$

This completes this section.

16   Total variability for line-of-sight paths

This section defines how to compute the contribution of variability to the median basic transmission loss.

Given:

$h_{r1,2}$:   real terminal heights, in km

q: time percentage of interest. Model input variable

f: frequency, MHz

d: path distance of interest, in km

A_T: loss predicted, in dB

θ_s: the scattering angle, in radians

$f_{\theta h}$:   input value

find:

$Y_{total}(q)$:   Total variability loss, in dB.

Step 1: Compute the value $f_{\theta h}$ using the value of θ_{h1} from the ray optics calculations previously.

$$f_{\theta h} = \begin{cases} 1, \theta_{h1} \leq 0 \\ 0, \theta_{h1} \geq 0 \\ \max(0.5 - 0.31831\tan^{-1}(20\log_{10}(32\,\theta_{h1})), 0), else \end{cases} \tag{175}$$

Step 2: Compute the contribution of long term variability for the time percentage q by using § 17. Then proceed to Step 3. Use § 17 as follows:

given:

$h_{r1,2}$:   real terminal heights, in km

d: path distance of interest, in km

f: frequency, MHz

q: time percentage of interest

$f_{\theta h}$:   input value to this section

A_T: loss predicted, in dB

find:

$Y_e(q)$:   long term variability loss, in dB.

Step 3: In order to correctly combine the effects of long term variability and tropospheric multipath, both of which are distributions, the mean value of the long term variability distribution is required. Compute the contribution of long term variability for time percentage 0.50 by using § 17. Then proceed to Step 3. Use § 17 as follows:

given:

$h_{r1,2}$:   real terminal heights, in km

d: path distance of interest, in km

f: frequency, MHz

0.50:   the mean time percentage ($q = 0.50$)

$f_{\theta h}$:   input value to this section

A_T: loss predicted, in dB

find:

$Y_e(0.50)$:   Long term variability loss, in dB.

Step 4: Compute the following value of K_{LOS}, used for determining the effects of tropospheric multipath, as follows:

$$F_{AY} = \begin{cases} 1, A_Y \leq 0 \\ 0.1, A_Y \geq 9 \\ (1.1 + 0.9\cos(\pi A_Y / 9))/2, else \end{cases} \tag{176}$$

$$F_{\Delta r} = \begin{cases} 1, \Delta r \geq \lambda/2 \\ 0.1, \Delta r \leq \lambda/6 \\ 0.5[1.1 - 0.9\cos((3\pi/\lambda)(\Delta r - \lambda/6))], else \end{cases} \tag{177}$$

$$R_s = R_{Ts}F_{\Delta r}F_{AY} \tag{178}$$

If r_{ew}, the effective ray path through the water vapour absorbing layer is 0 km, then $W_a = 0.0001$. Else, compute the value of $Y_\pi(0.99)$, as:

$$Y_\pi(0.99) = 10\log_{10}(f\,r_{ew}^3) - 84.26 \quad \text{(dB)} \tag{179}$$

Then use Table 7 to interpolate for the value of K corresponding to $Y_\pi(0.99)$, and use that value K to compute W_a as:

$$W_a = 10^{0.1K} \tag{180}$$

With W_a computed, complete the computation of K_{LOS} as:

$$W_R = R_s^2 + 0.01^2 \tag{181}$$

$$W = W_R + W_a \tag{182}$$

$$K_{LOS} = \begin{cases} 0, & W \leq 0 \\ 10\log_{10} W, & W > 0 \end{cases} \tag{183}$$

Step 5: Compute the contribution of tropospheric multipath for time percentage q using § 18. Then proceed to Step 6. Use § 18 as follows:

> given:
>
> > K_{LOS}: value set as K_T
> >
> > q: time percentage of interest
>
> find:
>
> > $Y_\pi(q)$: contribution of tropospheric multipath at time percentage q, in dB.

Step 6: Combine the effects long term variability with tropospheric multipath to get the total variability contribution, $Y_{total}(q)$, using the previously computed values $Y_e(q)$, $Y_e(0.50)$, and $Y_\pi(q)$. The mean of tropospheric multipath is $Y_\pi(0.50) = 0$.

$$Y_{total}(0.50) = Y_e(0.50) + Y_\pi(0.50) \tag{184}$$

$$Y = [(Y_e(q) - Y_e(0.50))^2 + (Y_\pi(q) - Y_\pi(0.50))^2]^{0.5} \tag{185}$$

$$Y_{total} = \begin{cases} Y_{total}(0.50) + Y, & q < 0.50 \\ Y_{total}(0.50) - Y, & q \geq 0.50 \end{cases} \quad \text{(dB)} \tag{186}$$

This completes this section.

17 Long term variability

This section describes the steps taken to compute the statistical distribution of long-term variability for the desired time percentage, q. Long term variability utilizes a normalized effective distance, d_e, on an effective earth radius of 9 000 km (corresponding to $N_s = 329$ N-Units). This section relies on statistical parameters that are based on long term empirical measurement data.

Given:

> $h_{1,2}$: terminal heights, in km
>
> q: time percentage of interest. Model input variable
>
> f: frequency, MHz
>
> d: path distance of interest, in km
>
> $f_{\theta h}$: previously calculated parameter
>
> A_T: loss predicted by either LOS, Diffraction, or Troposcatter models (previously calculated), in dB

find:

$Y_e(q)$:   long term variability loss, in dB.

Note: The inverse complementary cumulative normal distribution function, $Q^{-1}(q)$, is used in various places in this section. A technique to approximate its value for this step-by-step procedure is included in Recommendation ITU-R P.1057.

Step 1: Compute the smooth Earth horizon distances via ray tracing for each terminal. Let each terminals horizon be defined as $d_{Lq1,2}$. Use § 4 as follows:

given:

$h_{1,2}$:   terminal heights, in km

N_s: surface refractivity of 329 N-Units

find:

$d_{Lq1,2}$:   arc distance (smooth Earth horizon), in km.

Step 2: Compute d_e, the effective distance in kilometres between the two terminals.

$$d_{qs} = 60(100/f)^{1/3} \quad \text{(km)} \tag{187}$$

$$d_{Lq} = d_{Lq1} + d_{Lq2} \quad \text{(km)} \tag{188}$$

$$d_q = d_{Lq} + d_{qs} \quad \text{(km)} \tag{189}$$

$$d_e = \begin{cases} (130\,d)/d_q, & d \le d_q \\ 130 + d - d_q, & d > d_q \end{cases} \quad \text{(km)} \tag{190}$$

Step 3: Compute $g_{0.1}$ and $g_{0.9}$.

$$g_{0.1} = \begin{cases} 0.21\sin(5.22\log_{10}(f/200)) + 1.28, & f \le 1600 \\ 1.05, & f > 1600 \end{cases} \tag{191}$$

$$g_{0.9} = f(x) = \begin{cases} 0.18\sin(5.22\log_{10}(f/200)) + 1.23, & f \le 1600 \\ 1.05, & f > 1600 \end{cases} \tag{192}$$

Step 4: Compute $V(0.5)$, $Y_0(0.1)$, and $Y_0(0.9)$ using the below equations and the values from Table 3.

TABLE 3

Values to compute long term variability equations

	$V(0.5)$	$Y_0(0.1)$	$Y_0(0.9)$
c_1	1.59e-5	5.25e-4	2.93e-4
c_2	1.56e-11	1.57e-6	3.75e-8
c_3	2.77e-8	4.70e-7	1.02e-7
n_1	2.32	1.97	2.00
n_2	4.08	2.31	2.88
n_3	3.25	2.90	3.15
f_∞	0.0	5.4	3.2
f_m	3.9	10.0	8.2

$$f_2 = f_\infty + (f_m - f_\infty)\exp(-c_2 d_e^{n_2}) \tag{193}$$

$$\left.\begin{array}{l} V(0.5) \\ Y_0(0.1) \\ Y_0(0.9) \end{array}\right\} = \left[c_1 d_e^{n_1} - f_2\right] \exp\left(-c_3 d_e^{n_3}\right) + f_2 \quad \text{(dB)} \tag{194}$$

Step 5: Compute $Y_e(q)$, the variability associated with long-term (hour-to-hour) power fading, based on the desired time percentage, q.

If $q = 0.50$, then:

$$Y_q = V(0.5) \quad \text{(dB)} \tag{195}$$

If $q > 0.50$, then:

$$z_{0.9} = Q^{-1}(0.9) \tag{196}$$

$$z_q = Q^{-1}(q) \tag{197}$$

$$c_q = z_q/z_{0.9} \tag{198}$$

$$Y = c_q(-Y_0(0.9) * g_{0.9}) \quad \text{(dB)} \tag{199}$$

$$Y_q = Y + V(0.5) \quad \text{(dB)} \tag{200}$$

If $q < 0.50$, then additional steps should be taken. If $q \geq 0.10$, then:

$$z_{0.1} = Q^{-1}(0.1) \tag{201}$$

$$z_q = Q^{-1}(q) \tag{202}$$

$$c_q = z_q/z_{0.1} \tag{203}$$

$$Y = c_q(Y_0(0.1) * g_{0.1}) \quad \text{(dB)} \tag{204}$$

$$Y_q = Y + V(0.5) \quad \text{(dB)} \tag{205}$$

Else, $0.01 \leq q < 0.10$. Use Table 4 of values to linearly interpolate c_q from q. Then apply equations (204) and (205) to get Y_q.

TABLE 4

Low probability values for c_q

q	c_q
0.10	1.000 0
0.05	1.326 5
0.02	1.716 6
0.01	1.950 7

Step 6: Compute $Y_{0.10}$, the variability associated with long-term (hourly) power fading for $q = 0.10$.

$$Y_{0.10} = (Y_0(0.1) * g_{0.1}) + V(0.5) \quad \text{(dB)} \tag{206}$$

Step 7: Compute $Y_{eI}(q)$ and $Y_{eI}(0.1)$.

$$Y_{eI}(q) = f_{\theta h} Y_q \quad \text{(dB)} \tag{207}$$

$$Y_{eI}(0.1) = f_{\theta h} Y_{0.10} \quad \text{(dB)} \tag{208}$$

Step 8: Compute A_Y, which is used to prevent available signal powers from exceeding levels expected for free-space propagation by an unrealistic amount when the variability around the median is large and near its free-space level.

$$A_{YI} = Y_{eI}(0.1) - A_T - 3 \quad \text{(dB)} \tag{209}$$

$$A_Y = \max(A_{YI}, 0) \quad \text{(dB)} \tag{210}$$

Step 9: If $q \geq 0.10$, compute the total variability loss, completing this section. Else, proceed to Step 10 and continue.

$$Y_e(q) = Y_{eI}(q) - A_Y \quad \text{(dB)} \tag{211}$$

Step 10: For time percentages less than 10% ($q < 0.10$), an additional correction may be required. Compute the value Y_{temp}.

$$Y_{temp} = Y_{eI}(q) - A_Y - A_T \quad \text{(dB)} \tag{212}$$

Step 11: Use Table 5 to linearly interpolate c_{Yq} from q.

TABLE 5

Low probability correction values

q	c_{Yq}
0.10	0.00
0.05	–3.70
0.02	–4.50
0.01	–5.00

Step 12: Compute the total variability loss.

$$Y_e(q) = \begin{cases} -c_{Yq} + A_T, & Y_{temp} > -c_{Yq} \\ Y_{temp} + A_T, & else \end{cases} \tag{213}$$

This concludes the long term variability section.

18   Tropospheric multipath

This section describes how to compute the contribution to the total variability due to tropospheric multipath.

Given:

        K: input parameter

        q: time percentage of interest

find:

        $Y_\pi(q)$:   variability for q, in dB.

This section utilizes tabular data of the Nakagami-Rice distribution. Table 6 presents the data for $q < 0.50$ and Table 7 presents the data for $q > 0.50$. For all values of $q = 0.50$, $Y_\pi(q) = 0$ dB.

TABLE 6

Low time percentage values for the Nakagami-Rice distribution

K	$Y_\pi(0.01)$	$Y_\pi(0.02)$	$Y_\pi(0.05)$	$Y_\pi(0.10)$	$Y_\pi(0.15)$	$Y_\pi(0.20)$	$Y_\pi(0.30)$	$Y_\pi(0.40)$
−40	−0.1417	−0.1252	−0.1004	−0.1784	−0.0634	−0.0516	−0.0321	−0.0155
−25	−0.7676	−0.6811	−0.5496	−0.4312	−0.3487	−0.2855	−0.1764	−0.0852
−20	−1.3184	−1.1738	−0.9524	−0.7508	−0.6072	−0.5003	−0.3076	−0.1484
−18	−1.6264	−1.4508	−1.1846	−0.9332	−0.7609	−0.6240	−0.3888	−0.1878
−16	−1.9963	−1.7847	−1.4573	−1.1558	−0.9441	−0.7760	−0.4835	−0.2335
−14	−2.4355	−2.1829	−1.7896	−1.4247	−1.1664	−0.9613	−0.5989	−0.2893
−12	−2.9491	−2.6507	−2.1831	−1.7455	−1.4329	−1.1846	−0.7381	−0.3565
−10	−3.5384	−3.1902	−2.6408	−2.1218	−1.7471	−1.4495	−0.9032	−0.4363
−8	−4.1980	−3.7975	−3.1602	−2.5528	−2.1091	−1.7566	−1.0945	−0.5287
−6	−4.9132	−4.4591	−3.7313	−3.0307	−2.5127	−2.1011	−1.3092	−0.6324
−4	−5.6559	−5.1494	−4.3315	−3.5366	−2.9421	−2.4699	−1.5390	−0.7434
−2	−6.3811	−5.8252	−4.9219	−3.9366	−3.3234	−2.8363	−1.5390	−0.7434
0	−7.0246	−6.8861	−5.4449	−4.4782	−3.7425	−3.1580	−1.9678	−0.9505
2	−7.5228	−6.8861	−5.8423	−4.8088	−4.0196	−3.3926	−2.1139	−1.0211
4	−7.8525	−7.1873	−6.0956	−5.0137	−4.1879	−3.5218	−2.2007	−1.0630
6	−8.1435	−7.3588	−6.2354	−5.1233	−4.2762	−3.6032	−2.2451	−1.0845
20	−8.2238	−7.5154	−6.3565	−5.2137	−4.3470	−3.6584	−2.2795	−1.1011

TABLE 7

High time percentage values for the Nakagami-Rice distribution

K	$Y_\pi(0.60)$	$Y_\pi(0.70)$	$Y_\pi(0.80)$	$Y_\pi(0.85)$	$Y_\pi(0.90)$	$Y_\pi(0.95)$	$Y_\pi(0.98)$	$Y_\pi(0.99)$
−40	0.0156	0.0323	0.0518	0.0639	0.0790	0.1016	0.1270	0.1440
−25	0.0897	0.1857	0.2953	0.3670	0.4538	0.5868	0.7391	0.8421
−20	0.1624	0.3363	0.5309	0.6646	0.8218	1.0696	1.3572	1.5544
−18	0.2023	0.4188	0.6722	0.8373	1.0453	1.3660	1.7416	2.0014
−16	0.2564	0.5308	0.8519	1.0647	1.3326	1.7506	2.2463	2.5931
−14	0.3251	0.6730	1.0802	1.3558	1.7028	2.2526	2.9156	3.3872
−12	0.4123	0.8535	1.3698	1.7289	2.1808	2.9119	3.8143	4.4715
−10	0.5221	1.0809	1.7348	2.2053	2.7975	3.7820	5.0372	5.9833
−8	0.6587	1.3638	2.1887	2.3535	3.5861	4.9287	6.7171	8.1418
−6	0.8239	1.7057	2.7374	3.5494	4.5714	6.4059	8.9732	11.0972
−4	1.0115	2.0942	3.3610	4.4009	5.7101	8.1216	11.5185	14.2546
−2	1.1969	2.0942	3.9770	4.6052	6.7874	9.6278	13.4690	16.4258
0	1.3384	2.7709	4.4471	5.8105	7.5267	10.5553	14.5401	18.0527
2	1.4189	2.9376	4.7145	6.1724	8.0074	11.0005	15.0271	18.0527
4	1.4563	3.0149	4.8385	6.2705	8.0732	11.1876	15.2273	18.3573

TABLE 7 (*end*)

High time percentage values for the Nakagami-Rice distribution

K	$Y_\pi(0.60)$	$Y_\pi(0.70)$	$Y_\pi(0.80)$	$Y_\pi(0.85)$	$Y_\pi(0.90)$	$Y_\pi(0.95)$	$Y_\pi(0.98)$	$Y_\pi(0.99)$
6	1.8080	3.7430	6.0071	6.9508	8.1386	11.2606	15.3046	18.3361
20	1.4815	3.0672	4.9224	6.3652	8.1814	11.3076	15.3541	18.3864

Step 1: Using Tables 6 and 7, linearly interpolate to determine $Y_\pi(q)$ for the desired values K and q. Remember that $Y_\pi(0) = 0$.

This concludes this section.

Annex 3

Experimental results

Propagation tests at 930 MHz were conducted on air-to-ground paths in Japan in November 1982 and April and June 1983. According to the test results, propagation losses within line-of-sight agreed well with free space values. The line-of-sight distance as calculated from the measured data at an altitude of 10 000 m was shorter than the distance implied by the curves in Annex 3 (on hold).