

Simplified version of Hsiao 2024

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Individual i with county of origin $j(i)$ and age group $k(i)$. There is some schooling shock ϵ and choose education e (some school, completed high school, some college) to maximize utility.

$$u_{jk}(\epsilon) = \max_e \left\{ \overbrace{\bar{v}_{jk}(e)}^{\text{labor utility}} - \underbrace{c_{jk}(e, \epsilon)}_{\text{education costs}} \right\} \quad (1)$$

$$\bar{v}_{jk}(e) = \mathbb{E}[v_{jk}(e, \epsilon) \mid e], \quad c_{jk}(e, \epsilon) = e\tau_{jk}^e \epsilon \quad (2)$$

In the second stage, the individual considers locations l . They take education e as given and realize skill shocks $\epsilon = \{\epsilon_l\}$ across locations. They choose a destination to maximize labor utility:

$$v_{jk}(e, \epsilon) = \max_l \{v_{jkl}(e, \epsilon_l)\} \quad (3)$$

Labor utility is modeled in the following way:

$$v_{jkl}(e, \epsilon) = \frac{a_l w_{jkl}(e, \epsilon_l)}{\tau_{jkl}^m}, \quad w_{jkl}(e, \epsilon_l) = r_l h_{jkl}(e, \epsilon_l), \quad h_{jkl}(e, \epsilon_l) = e^\eta s_{jkl} \epsilon_l \quad (4)$$

Wages depend on the wage rates r_l per unit of human capital h_{jkl} . Human capital increases with education e subject to decreasing marginal returns $\eta < 1$. Human capital also increases in skill s_{jkl} and skill shock ϵ_l . The skill shocks follow the Frechet distribution:

$$F(\epsilon_1, \dots, \epsilon_L) = \exp\left\{-\sum_l \epsilon_l^{-\theta}\right\} \quad (5)$$

A high value of the Frechet parameter θ implies low skill dispersion. Migration costs are τ_{jkl}^m which are all the moving costs associated with moving away from home.

We can define how good a location is based on it's features other than idiosyncratic tastes by the following measure:

$$\tilde{v}_{jkl} = \frac{a_l r_l s_{jkl}}{\tau_{jkl}^m}, \quad MA_{jk} = \sum_l \tilde{v}_{jkl}^\theta \quad (6)$$

The reason for the weighting is as follows: when θ is high, there is relatively little skill dispersion, therefore the best locations will win out as locations with better fundamentals get more weight. The opposite happens when θ is small as there is more dispersion in skill and some people idiosyncratically like some locations worse or better.

$$\bar{e}_{jk} = \mathbb{E}[e] = \left(\frac{\gamma \eta M A_{jk}^{\frac{1}{\theta}}}{\tau_{jk}^e} \right)^{\frac{1}{1-\eta}} \bar{e} \quad (7)$$

$$\bar{w}_{jkl} = \mathbb{E}[w \mid \text{choose } l] = \left(\frac{\gamma \tau_{jkl}^m M A_{jk}^{\frac{1}{\theta}}}{a_l} \right) \left(\frac{\bar{e}_{jk}}{\bar{e}} \right)^\eta \tilde{e} \quad (8)$$

Then, we have the migration equation:

$$\bar{m}_{jkl} = \mathbb{P}[\tilde{v}_{jkl} \epsilon_l \geq \tilde{v}_{jkl'} \epsilon_{l'} \text{ for all } l'] = \frac{\tilde{v}_{jkl}^\theta}{\sum_{l'} \tilde{v}_{jkl'}^\theta} \quad (9)$$

On the production side, we have national production given by Y , which sums across locations. Perfectly competitive firms produce with human capital H_l subject to productivity A_l and production elasticity κ . The production function determines demand for human capital.

$$Y = \sum_l Y_l, \quad Y_l = A_l H_l^\kappa \quad (10)$$

Since firms are perfect competitors, they will equate the wage rate to the marginal product of human capital:

$$r_l = \kappa A_l H_l^{\kappa-1} = \frac{\partial Y_l}{\partial H_l} \quad (11)$$

Total human capital sums across people as does wages

$$H_l = \sum_{j,k} N_{j,k} \bar{m}_{jkl} \bar{h}_{jkl}, \quad W_l = \sum_{j,k} N_{j,k} \bar{m}_{jkl} \bar{w}_{jkl} \quad (12)$$

\bar{m}_{jkl} is the share of people from cohort k that migrate to l and have average human capital \bar{h}_{jkl} and average wages \bar{w}_{jkl} . Since $\bar{w}_{jkl} = r_l \bar{h}_{jkl}$, we have

$$Y_l = \frac{r_l H_l}{\kappa} = \frac{W_l}{\kappa} \quad (13)$$

Equilibrium is defined as the wage rates that clear the market in every labor market:

$$H_l^D(r_l) = H_l^S(r) \quad \forall l \quad (14)$$

Firms demand human capital. Production is only local, so for each location we have the following demand for human capital and demand can therefore be evaluated separately for every market:

$$H_l^D r_l = \left(\frac{\kappa A_l}{r_l} \right)^{\frac{1}{1-\kappa}} \quad (15)$$

Supply is upward sloping and since people are able to migrate, we have

$$H_l^S(r) = \frac{W_l(r)}{r(l)} \quad (16)$$