Simplified version of Hsiao 2024

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Individual i with county of origin $j(i)$ and age group $k(i)$. There is some schooling shock ϵ and choose education e (some school, completed high school, some college) to maximize utility.

$$u_{jk}(\epsilon) = \max_e \{ \overbrace{\bar{v}_{jk}(e)}^{\text{labor utility}} - \underbrace{c_{jk}(e, \epsilon)}_{\text{education costs}} \} \tag{1}$$

$$\bar{v}_{jk}(e) = \mathbb{E}[v_{jk}(e, \epsilon) \mid e], \quad c_{jk}(e, \epsilon) = e\tau^e_{jk}\epsilon \tag{2}$$

In the second stage, the individual considers locations l. They take education e as given and realize skill shocks $\epsilon = \{\epsilon_l\}$ across locations. They choose a destination to maximize labor utility:

$$v_{jk}(e, \epsilon) = \max_l \{v_{jkl}(e, \epsilon_l)\} \tag{3}$$

Labor utility is modeled in the following way:

$$v_{jkl}(e, \epsilon) = \frac{a_l w_{jkl}(e, \epsilon_l)}{\tau^m_{jkl}}, \quad w_{jkl}(e, \epsilon_l) = r_l h_{jkl}(e, \epsilon_l), \quad h_{jkl}(e, \epsilon_l) = e^\eta s_{jkl}\epsilon_l \tag{4}$$

Wages depend on the wage rates r_l per unit of human capital h_{jkl}. Human capital increases with education e subject to decreasing marginal returns $\eta < 1$. Human capital also increases in skill s_{jkl} and skill shock ϵ_l. The skill shocks follow the Frechet distribution:

$$F(\epsilon_1, \ldots, \epsilon_L) = \exp\{-\sum_l \epsilon_l^{-\theta}\} \tag{5}$$

A high value of the Frechet parameter θ implies low skill dispersion. Migration costs are τ^m_{jkl} which are all the moving costs associated with moving away from home.

We can define how good a location is based on it's features other than idiosyncratic tastees by the following measure:

$$\tilde{v}_{jkl} = \frac{a_l r_l s_{jkl}}{\tau^m_{jkl}}, \quad MA_{jk} = \sum_l \tilde{v}^\theta_{jkl} \tag{6}$$

The reason for the weighting is as follows: when θ is high, there is relatively little skill dispersion, therefore the best locations will win out as locations with better fundamentals get more weight. The opposite happens when θ is small as there is more dispersion in skill and some people idiosyncratically like some locations worse or better.

$$\bar{e}_{jk} = \mathbb{E}[e] = \left(\frac{\gamma \eta M A_{jk}^{\frac{1}{\theta}}}{\tau_{jk}^{e}} \right)^{\frac{1}{1-\eta}} \bar{\epsilon} \tag{7}$$

$$\bar{w}_{jkl} = \mathbb{E}[w \mid \text{choose } l] = \left(\frac{\gamma \tau_{jkl}^{m} M A_{jk}^{\frac{1}{\theta}}}{a_l} \right) \left(\frac{\bar{e}_{jk}}{\bar{\epsilon}} \right)^{\eta} \tilde{e} \tag{8}$$

Then, we have the migration equation:

$$\bar{m}_{jkl} = \mathbb{P}[\tilde{v}_{jkl}\epsilon_l \geq \tilde{v}_{jkl'}\epsilon_{l'} \text{ for all } l'] = \frac{\tilde{v}_{jkl}^{\theta}}{\sum_{l'} \tilde{v}_{jkl'}^{\theta}} \tag{9}$$

On the production side, we have national production given by Y, which sums across locations. Perfectly competitive firms produce with human capital H_l subject to productivity A_l and production elasticity κ. The production function determines demand for human capital.

$$Y = \sum_{l} Y_l, \quad Y_l = A_l H_l^{\kappa} \tag{10}$$

Since firms are perfect competitors, they will equate the wage rate to the marginal product of human capital:

$$r_l = \kappa A_l H_l^{\kappa-1} = \frac{\partial Y_l}{\partial H_l} \tag{11}$$

Total human capital sums across people as does wages

$$H_l = \sum_{j,k} N_{j,k} \bar{m}_{jkl} \bar{h}_{jkl}, \quad W_l = \sum_{j,k} N_{j,k} \bar{m}_{jkl} \bar{w}_{jkl} \tag{12}$$

$\bar{m}_{jkl}$ is the share of people from cohort k that migrate to l and have average human capital h_{jkl} and average wages $\bar{w}_{jkl}$. Since $\bar{w}_{jkl} = r_l \bar{h}_{jkl}$, we have

$$Y_l = \frac{r_l H_l}{\kappa} = \frac{W_l}{\kappa} \tag{13}$$

Equilibrium is defined as the wage rates that clear the market in every labor market:

$$H_l^{D}(r_l) = H_l^{S}(r) \quad \forall l \tag{14}$$

Firms demand human capital. Production is only local, so for each location we have the following demand for human capital and demand can therefore be evaluated separately for every market:

$$H_l^D r_l = \left(\frac{\kappa A_l}{r_l}\right)^{\frac{1}{1-\kappa}} \tag{15}$$

Supply is upward sloping and since people are able to migrate, we have

$$H_l^S(r) = \frac{W_l(r)}{r(l)} \tag{16}$$