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Accelerating the Navier-Stokes Equation with Neural Differential Equations

Project Overview

The Navier-Stokes equations are inherently a 3-dimensional nonlinear partial differential equation which describes the temperature and velocity of a fluid (Acheson D. J. 1990). The ability to accurate describe the mean behavior of the heat flow is the central component of climate models and have been able to successfully predict phenomena such as polar amplification (Holland 2003) which drive essential climate-related policy making. However, these climate models are notoriously difficult to scale, both making it difficult to perform high-fidelity uncertainty quantification (https://doi.org/10.1111/j.1539-6924.1988.tb01155.x) and, due to the tremendous amount of computing power used, becoming a source of high energy usage itself and thus contributing to the climate change problem. Thus the goal of this project is to accelerate the computational solution of the Navier-Stokes equations for understanding mean temperature dynamics.

Our approach will be to utilize a new mathematical/computational technique known as neural differential equations. Neural differential equations are a method which allows embedding neural networks into scientific models such as differential equations. The Julia Lab at MIT has developed the first (and only) toolbox for training neural differential equations (https://arxiv.org/abs/1806.07366,https://arxiv.org/abs/1907.07587). These tools have recently been developed to allow for neural networks to be GPU-accelerated within high order adaptive solvers for stiff differential equations, making it possible to define part of a partial differential equation with latent neural network structure and fit the missing component. While there are many use cases for neural differential equations, such as data-driven discovery of scientific laws, in this UROP we will focus on utilizing neural differential equations to accelerate climate models.

For this project, we are working in collaboration with Dr. Andre Souza, research scientist at the MIT Department of Earth, Atmospheric, and Planetary Sciences and member of the Climate Modeling Alliance. His past work showcased how the 3-dimensional Navier-Stokes equations could be condensed into a 1-dimensional system of diffusion-advection equations. However, this system leads to an infinite set, requiring the use of a closure relationship in order to arrive at a finite set of diffusion-advection equations. This previous work demonstrated that one could arrive at a closure relation by choosing a functional form of an unknown term, and thus arriving at a small system of 1-dimensional semilinear PDEs which approximate the full Navier-Stokes equations. For their publication, a specific functional form was chosen by hand and was shown to work for one part of the ocean, and it was left as an open problem as to how to generalize the result.

In our study, we will generalize the result by utilizing a neural network to automatically learn the functional form of the remaining term in a data-driven manner within the context of neural *diffusion-advection equations. The data will be generated by high-fidelity short runs of the Navier-Stokes equation, generated by the Oceanangians.jl package (https://github.com/climate-machine/Oceananigans.jl). By being fully-automated and data-driven, this approach should be able to learn a functional approximation which applies to many areas of the ocean, thus giving a more broadly applicable acceleration technique which could later be incorporated into the MIT-associated CLIMA climate modeling project.*

While this project exists as a multidisciplinary MIT collaboration, this UROP has been de-risked since this project is not reliant on other departments to appropriately schedule in order for success. While our collaborates Dr. Souza et al. in EAPS have committed to generating the data with Oceananigans, should that fall through we can generate sufficient data from the existing infrastructure ourselves. In addition, most of the work will likely fall into the implementation and optimization of a neural diffusion-advection integrator / training routine. Thus while it intersects with cross-MIT initiatives in a non-trivial manner and forwards the institutional goals of MIT, the core of this project can be completed in isolation without relying on others for data, resources, or time. This makes it both a project with a high chance of success with a high chance of impact.

Personal Role & Responsibilities

I will be providing 10 hours of work each week on this project. I will be responsible for implementing and optimizing the neural diffusion-advection integrator/ training routine and helping my advisor complete the research he has proposed. I will be working from the Julia Lab in the CSAIL department and will spend approximately 2 hours a day to accomplish my 10 hours a week.

Goals

I want to be able to publish a paper by the end of this project. I would also like to have a better understanding of what is required to be a mathematical and computer science researcher. I would also like to be able to successfully manage my time, so I am always able to maintain efficient and quality work even as the semester becomes difficult. I would also like to better develop my programming skills, add Julia to the list of programming languages I am comfortable using, and have a sufficient grasp on neural networks so I can feel confident using them in the future.

Personal Statement

I am a sophomore course 18 and would like to pursue a double major with 6-3. I am very excited to start the research position I have been offered. I want this opportunity to help me find the area of mathematics and computer science I most enjoy so I can find a focus and a passion to pursue in grad school and or the work force.