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Paper No. 794
RADIO SECTION

GROUND-WAVE PROPAGATION OVER AN INHOMOGENEOUS SMOOTH EARTH

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(*The paper was first received* 5th *July, and in revised form* 8th *November*, 1948.)

SUMMARY

The problem of ground-wave propagation over an inhomogeneous smooth earth is discussed in terms of the known solution for a homogeneous earth. The inhomogeneity refers only to changes in the earth constants from place to place, and the problem is idealized by assuming a wave radiated from a vertical dipole over a series of homogeneous annular sections. After a statement of some of the conditions the solution would be expected to obey, including the essential one of reciprocity, some fundamental results of the theory for a homogeneous earth are described in a form directly useful to the argument. The solution is first given for the short-wave limit, where it is complete except in the neighbourhood of a boundary. By an approximate consideration of the energy flow at different heights above the ground, the solution is extended to the case of intermediate wavelengths where the first and last boundaries are in the diffraction region of the transmitting and receiving points respectively. It is then shown that a well-known empirical method yields the same solution when it is made reciprocal by taking the geometric mean of the value it gives and the value that would be obtained with the transmitter and receiver interchanged. This method is formally used to obtain a tentative solution for the effect of the disturbance function in the neighbourhood of a boundary. It leads to the striking suggestion that on passing from a section of one value of conductivity to another of a higher one, there is a recovery in field-strength before the attenuation of the wave becomes characteristic of the new section. On crossing the boundary in the other direction, there is a correspondingly increased drop in field strength before the attenuation takes its new characteristic type. Owing to the lack of sufficiently controlled conditions, most of the existing experimental results are inconclusive with regard to these features at a boundary, but some evidence is given in support of them. Stress is laid on the need for further experiments specifically designed to study the field near a land-sea boundary. The paper deals briefly with the practical application of the method, and gives a specimen field-strength/distance curve for a route consisting of several land and sea sections. It concludes by pointing out that further research is needed, especially with regard to the phase relationships, as the argument has dealt only with field-strength values.

(1) INTRODUCTION

The problem of ground-wave propagation over a homogeneous smooth earth was first solved by Sommerfeld,[1] and a comprehensive statement of his theory in its most general form has been given by Norton.[2] This treatment is, however, based on the assumption that the earth is flat, and it is thus limited in practice to relatively short distances. The rigid solution for a spherical earth was first given by Watson,[3] though the explicit form of his analysis referred to a perfectly conducting earth. The essential nature of the solution for an imperfectly conducting earth was shown by T. L. Eckersley,[4] and detailed treatments of this generalized problem were given independently by Eckersley and Millington,[5] van der Pol and Bremmer,[6] and Wwedensky.[7]

In all the above work, the earth was assumed to be homogeneous, i.e. to be of constant conductivity and permittivity throughout. This is, however, a condition that is far from being realized in practice, and as the solution depends intimately upon the earth constants, it is obvious that this assumption is not justified. If the changes in the earth constants from place to place were very irregular and ill-defined, then probably it would be best to assume average values of the conductivity and of the permittivity determined by field-strength measurements. When, however, there are well-defined sections of differing earth constants, as in a transmission path that is partly over land and partly over sea, or which passes from arable land to desert or rock, it becomes necessary to study the propagation over each section and the nature of the transitions across the boundaries.

Unfortunately the analysis, already very complex, becomes intractable when the condition of homogeneity is waived, and so far no complete solution of the problem has been given. In order to make it feasible at all, the problem must still be idealized by assuming symmetry about a vertical dipole transmitter, i.e. by considering that the various sections are annuli about a vertical axis through the dipole, though this assumption may be far from true, for instance, for a wave passing over a small island in the middle of the sea, where the field strength beyond the island may be modified by energy flowing round it. It is interesting to note that the recent work of Grunberg[8] and Feinberg[9] has been directed towards this aspect of the problem, though it does not appear to be in a form that is applicable to the problem as a whole or amenable to numerical computation.

In view of these difficulties, resort has been made in the past to empirical solutions based on the known solution for the homogeneous earth, and perhaps the best known is the method published as long ago as 1930 by P. P. Eckersley.[10] Apart from its empirical nature, this method is open to the objection that it does not satisfy the reciprocity condition, and all such methods are subject to the criticism that a seemingly intelligent guess may be erroneous just because it is a guess and not based on a rigid argument. On the other hand there is an urgent need for the solution of this problem, for it has long been of great importance in medium-wave broadcasting, and interest in it has recently been stimulated with the development of medium- and long-wave navigational aids.

It seems, therefore, that the time is ripe for a review of the subject, and in this paper we shall discuss the general features that one would expect the rigid solution of the problem to possess, and show to what extent an empirical solution to be proposed fulfils them. It should be borne in mind that the results are admittedly speculative and open to revision in the light of future developments in the complete theory, but it is hoped that they will give estimates of field strength that are of the right order and provide a reliable guide for the communication engineer faced with practical examples of this difficult problem.

We shall adopt the symmetrical conditions already stated, and assume that the transmitter is a vertical dipole. A wave leaving the transmitter will then pass over a series of sections in each of which the conductivity and permittivity are fixed throughout. In practice the nature and extent of the sections may vary with direction, but the propagation conditions along any given radius

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will be assessed on the assumption that the same conditions hold along all the other radii. We shall then discuss to what extent this procedure is justified in practice.

It should be noted that we retain throughout the condition that the earth is smooth, so that the inhomogeneity under consideration refers only to the earth constants and not to possible irregularities in the terrain. The latter become of major importance on very short waves, but are in general of less significance on the longer waves, where the earth constants affect more intimately the ground-wave propagation characteristics, and the ground-wave range can extend far beyond the horizon. No account is taken of the ionosphere, as the consideration of the relative strengths of ground-wave and sky-wave at a given distance is regarded as a separate subject to be studied after the characteristics of the individual waves are known.

(2) GENERAL FEATURES TO BE EXPECTED IN THE SOLUTION

In the neighbourhood of a transmitter, but beyond the range of the induction field, the radiation initially obeys an inverse-distance law of field strength, but eventually, with increasing distance, the propagation becomes characteristic of the earth constants, assuming that the earth is homogeneous and smooth. This is true whether the earth is considered flat or spherical, but whereas for the flat earth the variation of field strength ultimately becomes of inverse-distance-squared type, for the spherical earth the law of attenuation with distance becomes of an exponential type with an attenuation coefficient that is dependent upon the earth constants, except in the limiting cases of very long or very short waves. Accompanying the attenuation with distance, there is a characteristic height-gain effect which, especially close to the ground, depends on the earth constants over the whole range of wavelengths.

If now we consider an inhomogeneous earth in which at a certain distance from the transmitter there is a boundary at which the wave crosses from one type of earth to another, and we suppose that the new section carries on to an indefinite distance, it seems reasonable to expect that at a sufficient distance beyond the boundary the propagation must become characteristic of the earth constants of the new section, both as regards the attenuation with distance and the height-gain relation near the ground. In other words, at a sufficient distance beyond the boundary the type of transmission becomes the same as if the new section had extended right back to the transmitter and the radiated power had been modified in an appropriate way.

At the boundary itself, some kind of disturbance must be set up to account for the transition of the propagation from one characteristic type to another. Presumably this disturbance must extend in some degree towards the transmitter, so that the wave must be somewhat modified before it reaches the boundary. It seems reasonable to suppose, however, that this effect will be small compared with the modification beyond the boundary and that it can be neglected in an approximate theory.

In suggesting these features as the probable conditions to be obeyed by the solution of the problem, we are admittedly adopting a speculative line of reasoning, but it is useful to form such preconceived ideas in order to see to what extent any actual suggested solution conforms to them. There is, however, one condition that can be established by rigid analysis, to which the true solution must comply, namely the condition of reciprocity, which for our purposes can be stated in the simple form that the received field strength is unaltered if the positions of the transmitter and receiver are interchanged. As mentioned above, it is the failure of the method of P. P. Eckersley to satisfy this essential condition that lays it open to criticism, quite apart from its essentially empirical nature, though it is designed to satisfy our preconceived idea with regard to the attenuation with distance well beyond the boundary.

In order to be able to discuss the probable nature of the solution more explicitly, it is necessary to know the salient features of the solution for the homogeneous earth as regards attenuation with distance and the height-gain relation, and it will be useful to summarize them in a form convenient for the purpose.

(3) SUMMARY OF GROUND-WAVE ANALYSIS FOR A HOMOGENEOUS EARTH

(3.1) The Diffraction Region

The original work of Watson, as generalized by the later workers mentioned above, showed that the solution of the problem of propagation from a vertical dipole over a homogeneous smooth spherical earth can be represented as the sum of an infinite number of terms, each of which contains a factor that is exponentially attenuated with distance. The attenuation coefficients of the successive terms form a set of increasing numbers derived from an eigen-value equation, and at a sufficient distance from the transmitter the first term becomes predominant.

It is convenient to refer to the region in which the total field can be represented adequately by the first term as the diffraction region, where the field as a function of the distance d may be expressed by

$$\mathscr{E}_D = \frac{k}{d^{\frac{1}{2}}}\epsilon^{-\alpha d}f(h_T)f(h_R) \quad . \quad . \quad . \quad . \quad (1)$$

where $f(h_T)$ and $f(h_R)$ are the height-gain functions for the transmitter and receiver at heights h_T and h_R respectively, defined to be unity for zero height. The value of k depends upon the radiated power, and it is a function of the earth constants and the wavelength. The coefficient α is a function of the wavelength and in general of the earth constants, but at the short-wave and long-wave limits it is simply proportional to $\lambda^{-\frac{1}{3}}$, where λ is the wavelength, i.e. $\alpha\lambda^{\frac{1}{3}}$ has upper and lower limiting values.*

The condition for the predominance of the first term of the diffraction formula is mainly controlled by the difference between the αd values for the first and second terms. It can be shown that their difference depends somewhat on the value of the earth constants, but that a useful criterion for all wavelengths for the diffraction region, when both the transmitter and receiver are on the ground, is $d > 200\,\lambda^{\frac{1}{3}}$, where d and λ are both measured in kilometres.† Where the transmitter and receiver are several wavelengths above the ground, as is usually the case on very high frequencies, this distance must be referred to the horizon as datum.

Throughout the analysis, the earth constants occur combined in a function that we shall call ζ, and that is defined differently according as the polarization is vertical or horizontal, i.e. according as the transmitting aerial is a vertical electric or magnetic dipole. For vertical polarization

$$\zeta = \frac{\kappa - j2\sigma\lambda c}{\sqrt{(\kappa - 1 - j2\sigma\lambda c)}} \quad . \quad . \quad . \quad . \quad (2)$$

and for horizontal polarization

$$\zeta = \frac{1}{\sqrt{(\kappa - 1 - j2\sigma\lambda c)}}$$

where κ is the permittivity of the earth referred to unity for air, and σ is the conductivity in e.m.u. The wavelength λ must here be measured in centimetres if the velocity of light c is reckoned in

* These limits occur where $\sigma^{\frac{1}{2}}\lambda^{\frac{5}{6}} < 10^{-8}$ and $> 10^{-6}$ respectively. (σ in e.m.u. and λ in km.)

† The value $200\lambda^{\frac{1}{3}}$ corresponds to an error of about 3 db in the diffraction formula as given by the first term only. For an error of 1 db this distance must be increased to about $300\lambda^{\frac{1}{3}}$.

centimetres per second. The time factor is assumed to be $\exp(j\omega t)$.

For the case of vertical polarization, the field $\mathscr{E}_D$ in eqn. (1) is the vertical component of electric field, or alternatively the transverse horizontal magnetic field, while for horizontal polarization the roles of the electric and magnetic field must be interchanged throughout. On the medium and long waves, where the problem we are investigating is of special importance, we shall be concerned only with vertical polarization, and shall assume that the polarization is vertical unless otherwise stated. It is, however, interesting to note that the whole of the argument will apply to horizontal polarization provided that we use the appropriate form of ζ and interchange the electric and magnetic fields.

(3.2) The Sommerfeld Region

The complete diffraction formula holds at all distances from the transmitter, but as the transmitter is approached, more and more terms of the series have to be included and the formula becomes impracticable for general use. Fortunately, for some distance from the transmitter the effect of the earth's curvature is small and the field strength can be adequately represented by the Sommerfeld formula for a flat earth. This region we shall therefore refer to as the Sommerfeld region, and between it and the diffraction region there is a transition region in which it is usual to obtain the field-strength/distance curve by interpolation by eye.

The Sommerfeld analysis, as expressed by Norton, represents the field as the sum of a space wave and a surface wave. The space wave is itself the sum of a direct free-space wave and a wave reflected from the earth, and is zero when both transmitter and receiver are on the ground. On very short waves the space wave very rapidly predominates over the surface wave as either the transmitter or receiver is raised above the ground, but on medium waves the surface wave is predominant, except at great heights, for vertical polarization.

Both the surface wave and the space wave have to be taken into consideration in studying the height-gain effects, though we shall be mainly concerned with the case where the transmitter and receiver are in effect on the ground. The field strength is then given by the inverse-distance value corresponding to a perfectly conducting earth, modified by an attenuation factor that is a function of the earth-constant factor ζ, the wavelength and the distance, combined in a quantity known as the numerical distance w given by

$$w = -j\frac{\pi d}{\lambda\zeta^2}$$

where d and λ are in the same units.

Denoting the field in the Sommerfeld region by $\mathscr{E}_S$, we have

$$\mathscr{E}_S = \frac{K}{d}F(w) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where K is a constant depending on the radiated power, and $F(w)$ is the attenuation factor. The properties of $F(w)$ are discussed in detail by Norton in terms of the modulus and phase of w. When $|w| < 0{\cdot}01$, $F(w)$ is effectively unity, but as w is increased, $F(w)$ decreases, and when $|w| > 20$, $F(w)$ approximates to $-1/2w$. We thus have

$$\mathscr{E}_S = \frac{K}{2\pi d^2}\lambda|\zeta|^2 \text{ when } d > \frac{20}{\pi}\lambda|\zeta|^2 \quad . \quad . \quad . \quad (4)$$

where we need to consider only the modulus of the field.

Under these conditions the field obeys an inverse-distance-squared law, and this law comes into operation where the field has dropped about 32 db below the inverse-distance value. We have, however, to remember that in practice the curvature of the earth may come into play before this condition is reached, depending on the relative sizes of the critical distances $200\,\lambda^{\frac{1}{3}}$ and $(20/\pi)\lambda|\zeta|^2$.

With increasing wavelength, the first of these distances becomes smaller than the second, and at the long-wave limit the transition is direct from the inverse-distance law to the diffraction region. On the other hand, at the short-wave limit the inverse-square law takes charge before the curvature of the earth appreciably modifies the flat-earth value of the field strength. The wavelength at which the field begins effectively to go straight over from the inverse-distance law to the diffraction region depends on the assumed earth constants. For transmission over sea, for which we may take $\kappa = 80$ and $\sigma = 4 \times 10^{-11}$ e.m.u., this type of transition occurs for $\lambda > 30$ m, while for transmission over land, with $\kappa = 5$ and $\sigma = 10^{-13}$ e.m.u., it occurs for $\lambda > 2\,000$ m.

(3.3) Height-Gain Relations for Small Heights

In the diffraction formula, each term of the series can be shown to be the resultant of an incident and a reflected wave. In the immediate neighbourhood of the ground these waves behave as though they were parts of infinite plane waves with reflection taking place according to the Fresnel formula. It was shown by Weyl[11] that the Sommerfeld analysis for the flat earth is equivalent to an infinite set of plane waves similarly reflected. In both cases the angles of incidence, or rather their direction cosines, are complex, implying amplitude variations over the wavefront. If we denote by n the sine of the angle of elevation of the incident wave, the reflection factor is given by

$$\rho = \frac{\zeta n - 1}{\zeta n + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Strictly speaking, the denominator of ζ in eqn. (2) should here be $\sqrt{(\kappa - 1 + n^2 - j2\sigma\lambda c)}$.

If we retain only the variation of the fields of the incident and reflected waves with height, we may write the relative values as $\exp[j(2\pi/\lambda)nh]$ and $\rho\exp[-j(2\pi/\lambda)nh]$, and for small heights the combined field is $(1 + \rho) + (1 - \rho)j2\pi nh/\lambda$, compared with the field $1 + \rho$ on the ground. The initial form of the height-gain factor is therefore

$$f(h) = 1 + \left(\frac{1-\rho}{1+\rho}\right)j\frac{2\pi nh}{\lambda}$$

From eqn. (5) we have

$$\frac{1-\rho}{1+\rho} = \frac{1}{\zeta n}$$

so that

$$f(h) = 1 + j\frac{2\pi h}{\lambda\zeta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This result is of particular interest because it is practically independent of the value of n, and is therefore true for any system of such incident and reflected waves. Eckersley and Millington showed directly from the diffraction analysis that this result for the initial height-gain holds for a single term of the formula, and it was confirmed by van der Pol and Bremmer. As it is true for the sum of any number of such terms, it must also hold in the transition and Sommerfeld regions so long as the angle of elevation is small, and it is not difficult to show from Norton's expressions for the space and surface waves in the inverse-distance-squared region that their sum does initially obey this relation.

For vertical polarization we can see from eqn. (2) that the phase angle of ζ lies between 0 and $-\pi/4$, and hence the modulus of $f(h)$ initially decreases. This phenomenon was pointed out by Wwedensky, who derived it by actual computation of the height-gain factor for a particular case, but we can see that fundamentally it arises as a property of Fresnel reflection.

At the short-wave limit, where from eqn. (2) ζ is wholly real, this initial drop disappears, while at the long-wave limit, where the phase angle of ζ is $-\pi/4$, the initial drop does not develop, partly because we cannot in practice make $2\pi h/\lambda\zeta$ comparable with unity, and partly because the assumption that the wave is plane would not hold over a sufficient range of height. The height-gain factor at this limit is in effect constant and equal to unity. On intermediate wavelengths, however, the initial drop can reach its maximum of about 3 db.

(3.4) Height-Gain in the Diffraction Region

It was first pointed out by T. L. Eckersley that the height-gain function for each term of the diffraction formula is intimately associated with a certain concentric fictitious sphere of radius somewhat greater than that of the earth. Above this sphere the height-gain function rapidly assumes an exponential form, and apart from a slowly varying factor, the height-gain due to an increase in the height is given by exp $(\alpha d')$, where d' is the increase in the horizon distance to the fictitious sphere, and α is the same coefficient as in the attenuation factor in eqn. (1). We must remember that above the fictitious sphere an increase in height is equivalent to increasing by the corresponding distance d' the limit of the diffraction region.

From this result Eckersley made the observation that above the fictitious sphere an increase in height was equivalent to remaining at the same height and moving back towards the transmitter by the corresponding distance d', and alternatively that along a tangent to the fictitious sphere, the wave behaves like an unattenuated progressive wave.

The height of the fictitious sphere above the earth is proportional to $\lambda^{2/3}$. It depends somewhat on the value of the earth-constant function ζ, but the dependence is small, and to a reasonable approximation the height, say h_0, may be taken for all conditions to be

$$h_0 = 50\lambda^{2/3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where h_0 and λ are both measured in metres.

On very short waves, h_0 may be small compared with the transmitter and receiver heights, but on long waves it plays a major part in the practical application of the theory in deciding whether there will be any appreciable height-gains.

(3.5) Height-Gain at the Short-Wave Limit

At the short-wave limit the value of k in eqn. (1) is proportional to ζ^2, and the attenuation coefficient α has a limiting form proportional to $\lambda^{-1/3}$ and is independent of the earth constants. Under these conditions, the form of the height-gain function given in eqn. (6) holds at heights where $2\pi h/\lambda\zeta \gg 1$, so that $f(h) \simeq j2\pi h/\lambda\zeta$. If we define a height h_1 as that at which this approximation to $f(h)$ becomes valid, we see that for heights greater than h_1, where eqn. (6) still holds, $f(h)$ becomes proportional to $1/\zeta$.

It can be shown that at greater heights above the fictitious sphere, where $f(h)$ becomes of exponential form, $f(h)$ is still proportional to $1/\zeta$; in fact at all heights greater than h_1, even where eqn. (6) no longer holds, this relationship is true. If both transmitter and receiver are raised to heights greater than h_1, the combined height-gain effect $f(h_T)f(h_R)$ in eqn. (1) is proportional to $1/\zeta^2$, and as $k \propto \zeta^2$, it follows that the field-strength is independent of ζ, i.e. of the earth constants, since α is similarly independent under our limiting conditions.

This argument is not confined to the diffraction region, for where the inverse-distance-squared law holds, the field $\mathscr{E}_S$ in eqn. (4), with $h_T = h_R = 0$, is proportional to ζ^2. At the short-wave limit this law comes into operation in the Sommerfeld region, while the initial form of the height-gain function in eqn. (6) holds at least up to the height h_1.

We thus reach the very general conclusion that at the short-wave limit the field strength becomes independent of the earth constants when both the transmitter and the receiver are above the height h_1, and that this result holds even beyond the horizon in the diffraction region. It is important to notice that at the short-wave limit, where $\kappa \gg 2\sigma\lambda c$ in eqn. (2), and the earth behaves as a pure dielectric, ζ is still different for, say, land and sea, and the ground-to-ground field strength (i.e. with $h_T = h_R = 0$) is different. The initial height-gain is, however, different, and above h_1, for both transmitter and receiver, these two factors compensate, and further increases in height give the same increase in field strength whatever the value of ζ.

(4) SOLUTION AT THE SHORT-WAVE LIMIT FOR THE INHOMOGENEOUS EARTH

Let us now consider a transmission path of length d made up of two sections d_1 and d_2 with earth-constant functions ζ_1 and ζ_2, shown in Fig. 1, drawn flat for convenience, with the transmitter

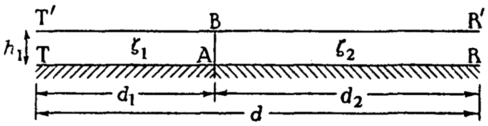

Fig. 1.—Single boundary for the short-wave limit, showing the height h_1.

and receiver both on the ground, at T and R. Confining our attention to the short-wave limit, let us imagine T and R both raised to a height h_1 at T′ and R′ respectively, where we choose the greater of the values for h_1 corresponding to ζ_1 and ζ_2. Then from the above argument, the received field strength at R′ from the transmitter at T′ is independent of the earth constants of the ground below, and hence of the fact that it consists of two sections of different ζ.

At the point B at the height h_1 above the boundary at A there is no discontinuity of field, as the attenuation with distance is the same for both sections, and above B the height-gain becomes independent of the earth constants. If $\mathscr{E}_1(d)$ is the ground-to-ground field strength on the assumption that the earth is homogeneous over the whole route and of type ζ_1, and $f_1(h)$ is the corresponding height-gain function, the field at R′ due to T′, say $\mathscr{E}_{T'R'}$, can be represented by

$$\mathscr{E}_{T'R'} = \mathscr{E}_1(d)[f_1(h_1)]^2 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Alternatively, assuming that the ground is all of type ζ_2, we may write

$$\mathscr{E}_{T'R'} = \mathscr{E}_2(d)[f_2(h_1)]^2 \quad . \quad . \quad . \quad . \quad (9)$$

We can see that these two values are equivalent, for we have shown that

$$\frac{\mathscr{E}_1(d)}{\mathscr{E}_2(d)} = \frac{\zeta_1^2}{\zeta_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and that for $h \geqslant h_1$,

$$\frac{f_1(h)}{f_2(h)} = \frac{\zeta_2}{\zeta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Now if the boundary at A is sufficiently far from both T and R, the height-gain at T will actually be of type $f_1(h)$ and at R it will be of type $f_2(h)$. Thus, if we move the transmitter and receiver back from T′ and R′ to T and R, the actual ground-to-ground field strength $\mathscr{E}(d)$ over the composite path may be found from

$$\mathscr{E}(d) = \frac{\mathscr{E}_{T'R'}}{f_1(h_1)f_2(h_1)}$$

and from eqns. (8) and (9) this may be written in the alternative forms

$$\mathscr{E}(d) = \mathscr{E}_1(d)\frac{f_1(h_1)}{f_2(h_1)}$$

and
$$\mathscr{E}(d) = \mathscr{E}_2(d)\frac{f_2(h_1)}{f_1(h_1)}$$

That these are equivalent again follows from eqns. (10) and (11), but by multiplying them together we obtain the simple relation

$$\mathscr{E}(d) = \sqrt{[\mathscr{E}_1(d)\mathscr{E}_2(d)]} \quad . \quad . \quad . \quad . \quad (12)$$

i.e. the ground-to-ground field strength is the geometric mean of the values obtained on the assumption that the ground is first all of type ζ_1 and then all of type ζ_2. This result has recently been given by T. L. Eckersley[12] specifically for the case where the boundary is in the diffraction region of the transmitter.

This method can obviously be extended to the case of n sections, provided that the first and last boundaries are sufficiently far from T and R respectively, the result being of the form

$$\mathscr{E}(d) = \sqrt{[\mathscr{E}_1(d)\mathscr{E}_n(d)]} \quad . \quad . \quad . \quad . \quad (13)$$

and depending only on the nature of the first and last sections.

Returning to the single boundary at A in Fig. 1, if we assume, as suggested earlier, that the nature of the propagation is unaffected before the boundary is reached, the height-gain from A to B will be of the type $f_1(h)$. The propagation beyond A towards R cannot become immediately characteristic of ζ_2 with a height-gain function $f_2(h)$. There must be a certain distance beyond A over which the field strength on the ground and the height-gain function adjust themselves in such a way that the height-gain eventually becomes of type $f_2(h)$, and that at the height h_1 the field beyond B is attenuated with distance as though there were no boundary to consider.

This adjustment may be attributed to some kind of disturbance function. We may say that above the height h_1 the wave to the right of the boundary is the same as if it had proceeded from a transmitter of modified power at T over ground wholly of type ζ_2, the modification in power being required to fit the wave at all heights above B on to the actual wave to the left of the boundary that has come from the actual transmitter at T over ground of type ζ_1. If at the boundary we attempt to carry these waves below h_1 to the ground, the different height-gain functions on the two sides of the boundary would mean that the field on the ground would change discontinuously at A, and we may regard the disturbance function as removing this discontinuity. We are assuming tentatively that the function does not extend back towards the transmitter and that it dies out at the height h_1. Of its extent beyond the boundary we can say little at this stage of the argument, but apart from this uncertainty we have obtained a complete solution in this limited case.

(5) THE GENERAL CASE IN THE DIFFRACTION REGION

At the long-wave limit, where in the diffraction region the attenuation coefficient again becomes independent of the earth constants, and where in the Sommerfeld region the inverse-distance law holds, the earth behaves as if it were a perfect conductor, and inhomogeneities in the earth constants do not affect the ground-wave propagation. It is at the intermediate wavelengths, where in the diffraction region the attenuation coefficient α is a function of the earth-constant function ζ, that the greatest difficulties in the theory arise. It is possible, for instance, on some wavelengths for α to be twice as great for propagation over land as for over sea, causing extreme differences in attenuation with distance. This is offset by the difference in the initial height-gain, and at great heights by the fact that above the fictitious sphere the height-gain function becomes of the type exp $(\alpha d')$. We see, however, that at a boundary we can no longer fit the two waves together at all heights above a certain height, both on account of the different height-gain relations and because of the different characteristic attenuation with distance on the two sides of the boundary.

In order to obtain an idea of the probable form of the solution in this general case, we shall restrict our argument and consider initially a single boundary in the diffraction region with respect to both the transmitting point T and the receiving point R as in Fig. 2, where both T and R are on the ground. Instead of con-

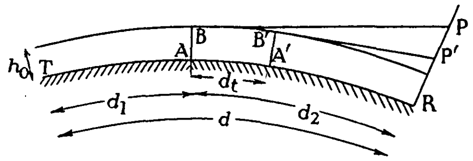

Fig. 2.—Single boundary in the diffraction region for the general case, showing the height h_0.

sidering the height h_1 as in Fig. 1, we use the height of the fictitious sphere h_0 given by eqn. (7), treating it as independent of the earth constants, i.e. of the functions ζ_1 and ζ_2 for the two sections, and we take B as the point on the fictitious sphere above the boundary A.

Assuming that from the transmitter to A the field at all heights is characteristic of the type ζ_1, it seems reasonable to suppose from the argument given in Section 3.4 that the field is unmodified beyond A at all points above the tangent to the fictitious sphere at B. Beyond A there will be a disturbance function, and we will suppose that it is necessary to go as far as A′ before the attenuation with distance has become characteristic of type ζ_2, and the height-gain function has become of type $f_2(h)$ for all points below B′ on the fictitious sphere. We then assume that beyond A′ the height-gain function is of type $f_2(h)$ for all points below the tangent to the fictitious sphere at B′.

In the case of the homogeneous earth we can construct from the results of Section 3.4 the physical picture in the diffraction region of a waveguide of width h_0, through the upper side of which energy flows out tangentially to supply the field in the space above the guide. The significance of the fictitious sphere as the upper boundary of a leaky waveguide was first pointed out by Booker.[13]

In the present argument, the fact that h_0 is approximately independent of the earth constants is used to justify a similar physical picture, and the redistribution of energy taking place in the guide between A and A′ affects the height-gain relationship at all heights between the tangents at B and B′. If these tangents cut the vertical through R at P and P′, we can say that above P the height-gain is of type $f_1(h)$ and that below P′ it is of type $f_2(h)$, while between P′ and P, as we go up from R, the height-gain is changing from type $f_2(h)$ to type $f_1(h)$.

At the short-wave limit the two functions $f_1(h)$ and $f_2(h)$ are of the same type at all points above h_0, since it can be shown that h_0 is there much greater than h_1, and the above physical picture becomes consistent with the argument of Section 4, though it is here formally restricted to the diffraction region. According to this picture the field strength at P can be calculated from the ground-to-ground field-strength $\mathscr{E}_1(d)$ for an earth all of type ζ_1 with a height-gain factor of type $f_1(h)$ for the height RP. From the analysis in Section 3.4 it follows that this height-gain is given approximately by $f_1(h_0)$ exp $(\alpha_1 d_2)$. Thus we have

$$\text{Field at P} = \mathscr{E}_1(d)f_1(h_0)\exp(\alpha_1 d_2)$$

For the reduction in field-strength in going from P to P′, corresponding to the transition distance AA′, say d_t, we shall assume that it is given by an exponential with an average coefficient $\frac{1}{2}(\alpha_1 + \alpha_2)$, so that we have

$$\text{Field at P}' = \mathscr{E}_1(d)f_1(h_0)\exp[\alpha_1 d_2 - \tfrac{1}{2}(\alpha_1 + \alpha_2)d_t]$$

Finally, to obtain the field at R, we must divide by the height-gain from R to P′ which we have seen is of type $f_2(h)$, and which to our approximation is given by $f_2(h_0)\exp[\alpha_2(d_2 - d_t)]$, so that the actual ground-to-ground field-strength is given by

$$\mathscr{E}(d) = \mathscr{E}_1(d)\frac{f_1(h_0)}{f_2(h_0)}\exp[\alpha_1 d_2 - \tfrac{1}{2}(\alpha_1 + \alpha_2)d_t - \alpha_2(d_2 - d_t)]$$

$$= \mathscr{E}_1(d)\frac{f_1(h_0)}{f_2(h_0)}\exp[(\alpha_1 - \alpha_2)d_2 - \tfrac{1}{2}(\alpha_1 - \alpha_2)d_t] \quad . \quad (14)$$

This value does not essentially obey the reciprocity condition, and if we interchange T and R and assume that the transition distance has the same value d_t, we have as an alternative value

$$\mathscr{E}(d) = \mathscr{E}_2(d)\frac{f_2(h_0)}{f_1(h_0)}\exp[(\alpha_2 - \alpha_1)d_1 - \tfrac{1}{2}(\alpha_2 - \alpha_1)d_t] \quad (15)$$

It is an argument in favour of the approximate correctness of our theory that these values are found by calculation to be nearly equal even in extreme cases. To make the result strictly reciprocal and to obtain agreement at the short-wave limit with the value in eqn. (12), we take the geometric mean, giving

$$\mathscr{E}(d) = \sqrt{[\mathscr{E}_1(d)\mathscr{E}_2(d)]}\exp[\tfrac{1}{2}(\alpha_1 - \alpha_2)(d_2 - d_1)] \quad . \quad (16)$$

As in Section 3.4, we have omitted a slowly varying factor in the height-gain function, but it can be shown that the small corrections its inclusion would make to eqns. (14) and (15) cancel out when the geometric mean is taken, leaving (16) unaltered. Further, in the transition region P′P we could assume for the average attenuation coefficient the more general form $l\alpha_1 + m\alpha_2$, where $l + m$ must be equal to unity to give the correct form at the limit when α_1 and α_2 are equal, and we should again find that it would not affect the value of the geometric mean.

We are therefore encouraged to believe that our admittedly approximate physical picture is reasonably good, and that it leads to a value of $\mathscr{E}(d)$ that is of the right order. It is only a matter of detail to generalize the method to the case of n sections, subject to the conditions laid down for the diffraction region, and the result is found to be

$$\mathscr{E}(d) = \sqrt{[\mathscr{E}_1(d)\mathscr{E}_n(d)]}$$
$$\exp[\tfrac{1}{2}(\alpha_1 + \alpha_n)d - \alpha_1 d_1 - \alpha_2 d_2 - \ldots - \alpha_n d_n] \quad . \quad (17)$$

which reduces to eqn. (16) when $n = 2$, and to (13) when the α's are all equal.

The special case of a single boundary given in eqn. (16) is contained in Eckersley's paper, but it is there deduced on the assumption that the height-gain functions are approximately of the same form, and that the waves on the two sides of the boundary can still be fitted together, except near the ground, as in the argument for the short-wave limit. For this reason he states that the method is definitely unsuitable for longer waves. The derivation given above waives the condition that the height-gain functions should be approximately of the same form, and is based on the concept of the energy flow above the fictitious sphere with a transition region extending above the fictitious sphere between the two tangents defined. It thus removes the restriction imposed by Eckersley, and justifies his tentative use of the method for longer waves.

We may note from (16) that for the case of the single boundary the correction factor, $\exp[\tfrac{1}{2}(\alpha_1 - \alpha_2)(d_2 - d_1)]$, to the geometric mean value $\sqrt{[\mathscr{E}_1(d)\mathscr{E}_2(d)]}$, is unity not only at the short-wave limit where $\alpha_1 = \alpha_2$, but also when $d_1 = d_2$ whatever the wavelength, i.e. when the boundary is mid-way between T and R. In his application of the method to the medium-wave broadcast band, Eckersley has taken the simple geometric mean, and the general agreement of his calculated values with the measured values for a number of continental stations received in England implies that for the paths considered the correction factor is of the order of unity.

If in eqn. (17) we give $\mathscr{E}_1(d)$ and $\mathscr{E}_n(d)$ their values

$$\mathscr{E}_1(d) = \frac{k_1}{d^{\frac{1}{2}}}\exp(-\alpha_1 d) \text{ and } \mathscr{E}_n(d) = \frac{k_n}{d^{\frac{1}{2}}}\exp(-\alpha_n d)$$

we have

$$\mathscr{E}(d) = \frac{\sqrt{(k_1 k_n)}}{d^{\frac{1}{2}}}\exp(-\alpha_1 d_1 - \alpha_2 d_2 - \ldots - \alpha_n d_n) \quad (18)$$

and as the distance d_n of the receiver from the last boundary is increased, the field as a function of d_n becomes proportional to $\exp(-\alpha_n d_n)$. Our solution thus conforms to our preconceived idea that at a sufficient distance from the last boundary the attenuation with distance becomes characteristic of the type of earth over which the wave is travelling.

Returning to the situation above the fictitious sphere in the case of the single boundary represented in Fig. 2, let us consider the change in field strength with distance at a given height shown by the chain line in Fig. 3. If this line cuts the tangents to the

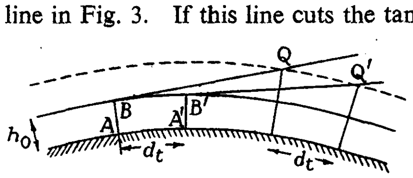

Fig. 3.—Single boundary in the diffraction region, showing the transition distance at a height above h_0.

fictitious sphere shown in Fig. 2 at Q and Q′, it will be seen that the movement from Q to Q′ corresponds to a distance along the ground equal to the transition distance AA′, whatever the height above the earth. It follows that the field-strength/distance curves at various heights above the fictitious sphere in the diffraction region become of the same shape, being laterally displaced with respect to one another by distances given by the differences in the horizon distance to the fictitious sphere. Each curve will have a transition region of length d_t over which the slope changes from the slope characteristic of one type of earth to that of the other type beyond the boundary.

In Fig. 4, a diagrammatic representation of a set of field-strength/distance curves for various heights above the fictitious sphere is shown for the case of a wave crossing a boundary to a section of higher conductivity, e.g. from land to sea. The transition region in each curve is shown as a broken line. The horizontal lines CD and C′D′ at the boundaries of the transition region correspond to the unattenuated propagation along the tangents at B and B′ in Figs. 2 and 3. Above CD and below C′D′ the curves have the characteristic slopes for the sections before and beyond the boundary respectively.

This diagram shows clearly how the difficulty of fitting together the waves with their different height-gain relations at all heights is overcome by means of the transition region defined by the tangents at B and B′. We can see that the greater height-gain at the vertical line U in Fig. 4, as compared with that shown at V, is offset by the steeper slope of the curves above CD as compared with the slope below C′D′, so that the curves for the different heights have the same shape as one another throughout their length, and may be obtained from one another by a lateral displacement.

Although the curves in Fig. 4 are meant only to be diagrammatic, they have been made to correspond to the case of a transmission on a wavelength of 100 m across a boundary from land of medium conductivity ($\sigma = 10^{-13}$ e.m.u.) to sea ($\sigma = 4 \times 10^{-11}$ e.m.u.). The curves above CD are obtained by applying the land height-gain to the overland ground curve, while for the curves below C′D′ the sea height-gain has been added to the ground curve given by eqn. (16). For a reason to

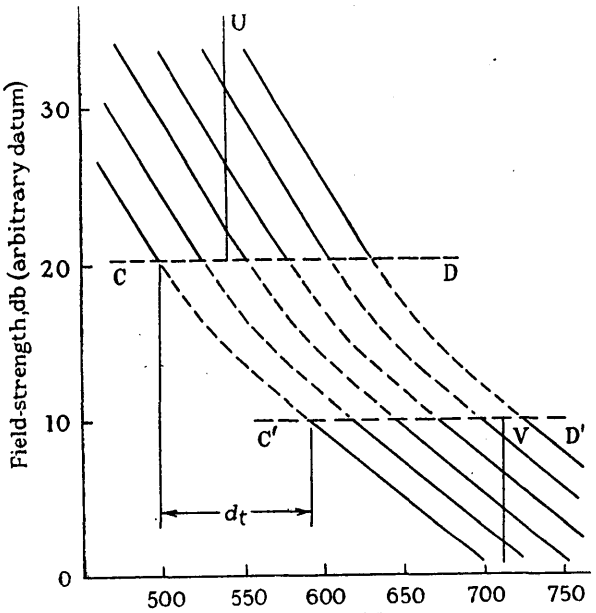

Fig. 4.—Field-strength/distance curves at heights well above h_0.

$\lambda = 100$ m.
$\sigma = 10^{-13}$ e.m.u. (land).
$= 4 \times 10^{-11}$ e.m.u. (sea).

be given later, the transition distance d_t has been taken as the critical distance $200\lambda^{\frac{1}{3}}$ defined above in connection with the limit of the diffraction region. It is interesting to note that the portion of a curve above CD can be joined to the portion below C′D′ with a smooth curve in the transition region, suggesting that (16) does give a reasonable value for the ground-to-ground curve at a sufficient distance beyond the boundary in the diffraction region.

The above picture should be modified slightly to include the slowly varying factor in the height-gain function and the $1/d^{\frac{1}{2}}$ term in the diffraction formula. Moreover, the exponential form of the height-gain function does not come fully into operation until one is well above the fictitious sphere, but by concentrating on the major factors we have obtained an assessment of the situation above the fictitious sphere that would seem to be correct in its essential features.

The problem remains of investigating the nature of the transition of the field strength on the ground from the one type of transmission to the other, e.g. from a curve with a slope characteristic of overland transmission to one characteristic of oversea transmission. If we take again a wavelength of 100 m, for which the fictitious sphere is at a height, from eqn. (7), of about 1 100 m, the oversea height-gain presents an initial drop and does not become positive until a height of nearly 2 000 m, whereas the overland height-gain becomes positive at about 150 m and is 25 db at 2 000 m. On the assumption that the field is unmodified at all heights until the boundary is reached, we can construct Fig. 5, in which the field-strength/distance curves are shown to the left of the boundary for various heights up to 5 000 m. Beyond the transition region, the ground curve given by eqn. (16) is shown on the right, and the corresponding curves for the heights shown on the left actually lie below it by amounts ranging up to 3 db, decreasing to zero at 2 000 m. A tentative curve for the transition at 2 000 m is shown, and it is obvious that below this height the form of the transition must change very rapidly as we pass through and below the fictitious sphere. At the height of 5 000 m, shown in Fig. 5, the curve is approaching the type in Fig. 4 in the region where the curves have the same shape and are merely laterally displaced.

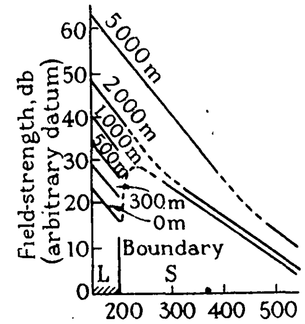

Fig. 5.—Field-strength/distance curves suggesting the transition from small to great heights.

$\lambda = 100$ m.
$\sigma = 10^{-13}$ e.m.u. (land).
$= 4 \times 10^{-11}$ e.m.u. (sea).

A transition curve joining up the two parts of the ground curve is also shown in Fig. 5. This is admittedly speculative, and in order to explain how it has been derived, we must now turn to an empirical solution, based on the observation, as we shall see, that the method proposed by P. P. Eckersley, referred to above, is in fact related to the solution we have derived.

(6) AN EMPIRICAL SOLUTION IN THE NEIGHBOURHOOD OF A BOUNDARY

P. P. Eckersley's method was to plot the field-strength/distance curves for propagation over various types of earth on a logarithmic scale of field strength, i.e. a linear scale in decibels, and then to build up a composite curve by moving the sections of the appropriate curves up or down until they fitted together at the various boundaries. Suppose, for instance, that we have a transmission over a distance d_1 of sea followed by a section d_2 of land. We start along the sea curve, as in Fig. 6, as far as the

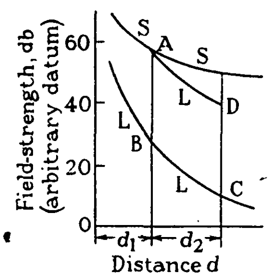

Fig. 6.—Empirical method (P. P. Eckersley).

point A at a distance d_1 from the transmitter. If we had used the land curve instead, we should have come to a point B at the same distance, and by continuing a further distance d_2 along this curve, should have arrived at the point C. We now move the section BC of the land curve up until B falls on A and C becomes the point D. The method then assumes that the composite curve is given by the sea curve as far as A, followed by the curve AD. The generalization of the method to more than one boundary is obvious.

Fig. 6 is only diagrammatic, but it is chosen to represent roughly the case of $\lambda = 100$ m considered above. The oversea curve is effectively the inverse-distance curve until the transition to the diffraction region takes place, whereas the overland curve first goes over rapidly to the inverse-distance-squared curve before the transition to a diffraction curve much steeper than the oversea curve. It is not difficult to see that this method is not reciprocal, and in Fig. 7 the curves for 20 km of sea followed by

20 km of land, and vice versa, are shown. The resulting fields at X and Y, respectively, differ by more than 20 db, the discrepancy being largely attributable to the extreme difference of the land and sea curves in the neighbourhood of the transmitter.

In one respect this empirical method fulfils our preconceived ideas in that at a sufficient distance from a boundary the field is characteristic of the section concerned as regards attenuation with distance. It is certainly wrong, however, in satisfying this condition immediately the boundary is passed, as we have seen that there must exist a disturbance function extending for some distance beyond the boundary.

On the assumption that the true field should lie somewhere between the values at X and Y in Fig. 7, we are led by our

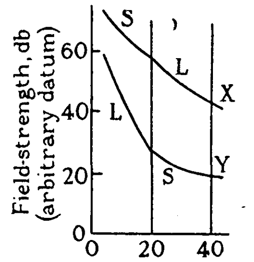

Fig. 7.—Non-reciprocity of the P. P. Eckersley method.

previous arguments to consider their geometric mean (or arithmetic mean if the fields are expressed in decibels relative to some datum). In other words we make the method strictly reciprocal by taking the geometric mean of the value it gives and the value it would give with the transmitter and receiver interchanged. In terms of our previous notation, the two fields in question for the general case of n sections would be

$$\mathscr{E}_1(d_1)\frac{\mathscr{E}_2(d_1+d_2)}{\mathscr{E}_2(d_1)}\frac{\mathscr{E}_3(d_1+d_2+d_3)}{\mathscr{E}_3(d_1+d_2)}\cdots\frac{\mathscr{E}_n(d)}{\mathscr{E}_n(d_1+\ldots+d_{n-1})} \quad . \quad . \quad . \quad (19)$$

and $$\mathscr{E}_n(d_n)\frac{\mathscr{E}_{n-1}(d_n+d_{n-1})}{\mathscr{E}_{n-1}(d_n)}\cdots\frac{\mathscr{E}_1(d)}{\mathscr{E}_1(d_n+\ldots+d_2)} \quad . \quad (20)$$

For the case when all the boundaries are in the diffraction regions of T and R, so that all the fields are of the type in eqn. (1), these values reduce to

$$\frac{k_1}{d^{\frac{1}{2}}}\exp(-\alpha_1 d_1 - \alpha_2 d_2 - \ldots - \alpha_n d_n)$$

and $$\frac{k_n}{d^{\frac{1}{2}}}\exp(-\alpha_1 d_1 - \alpha_2 d_2 - \ldots - \alpha_n d_n)$$

giving a geometric mean value of

$$\mathscr{E}(d) = \frac{\sqrt{(k_1 k_n)}}{d^{\frac{1}{2}}}\exp(-\alpha_1 d_1 - \alpha_2 d_2 - \ldots - \alpha_n d_n)$$

This is identical with the value in eqn. (18) and hence with (17). Thus the method, when made reciprocal, gives the same result as we have already obtained under these conditions.

If we apply the method formally to a flat earth under conditions in which all the fields involved are of the inverse-distance-squared type given in eqn. (4), the curves over any given section have the same shape, differing only in absolute magnitude depending on the value of ζ. Thus the composite curve for transmission from T to R coincides with that for transmission over earth all of the type of the first section, giving a field-strength value of $\mathscr{E}_1(d)$. The reciprocal geometric mean will then be $\sqrt{[\mathscr{E}_1(d)\mathscr{E}_n(d)]}$, giving under these special conditions the value in eqn. (13) without the short-wave limit restriction. In general, however, the transmission paths are not confined to the Sommerfeld region, but may extend into the diffraction region, and alternatively in the Sommerfeld region itself the paths may not extend into the inverse-distance-squared region.

It is interesting to note that when there is a single boundary midway between T and R, we have to take the geometric mean of

$$\mathscr{E}_1(d_1)\frac{\mathscr{E}_2(d)}{\mathscr{E}_2(d_1)} \text{ and } \mathscr{E}_2(d_2)\frac{\mathscr{E}_1(d)}{\mathscr{E}_1(d_2)}$$

where $d_1 = d_2$, which is $\sqrt{[\mathscr{E}_1(d)\mathscr{E}_2(d)]}$. Thus this particular result, which was shown to hold for the diffraction region, is given by the present method regardless of the shape of the $\mathscr{E}_1(d)$ and $\mathscr{E}_2(d)$ curves.

The method, as used by P. P. Eckersley, applied only to the Sommerfeld region and specifically to the case where $2\sigma\lambda c \gg \kappa$, but as a technique it is clearly general, as it can be regarded as a graphical process applicable whatever the shape of the sections of the curves from which the composite curve is constructed. It can therefore be used formally at all distances even when the transmitter or receiver may be close to a boundary.

Although there may be no theoretical justification for using the method in this way at the boundary, we have done so to obtain the transition region of the ground curve in Fig. 5. We obtain the remarkable suggestion that on crossing the boundary from land to sea there can be a considerable recovery of field strength before the curve settles down to the over-sea type. We can see, however, that this result is not physically impossible in view of the large redistribution of energy that must take place below the fictitious sphere in the transition region to account for the change in type of the height-gain function. It appears that energy is fed from higher levels down to the ground, causing the signal strength to increase with distance for some way beyond the boundary.

If the boundary is at a distance d_1 from the transmitter and the receiver is at a distance d_2 beyond, the method gives for the field strength $\mathscr{E}(d_1 + d_2)$ the value

$$\mathscr{E}(d_1+d_2) = \sqrt{\left[\mathscr{E}_1(d_1)\frac{\mathscr{E}_2(d_1+d_2)}{\mathscr{E}_2(d_1)}\mathscr{E}_2(d_2)\frac{\mathscr{E}_1(d_1+d_2)}{\mathscr{E}_1(d_2)}\right]}$$

In the diffraction region we have

$$\mathscr{E}_1(d_1+d_2) = \frac{d_1^{1/2}}{(d_1+d_2)^{1/2}}\exp[-\alpha_1 d_2]\mathscr{E}_1(d_1)$$

and $$\mathscr{E}_2(d_1+d_2) = \frac{d_1^{1/2}}{(d_1+d_2)^{1/2}}\exp[-\alpha_2 d_2]\mathscr{E}_2(d_1)$$

so that

$$\frac{\mathscr{E}(d_1+d_2)}{\mathscr{E}_1(d_1)} = \sqrt{\left[\frac{d_1}{(d_1+d_2)}\frac{\mathscr{E}_2(d_2)}{\mathscr{E}_1(d_2)}\right]}\exp[-\tfrac{1}{2}(\alpha_1+\alpha_2)d_2] \quad (21)$$

giving the ratio of the field beyond the boundary to the field at the boundary.

As d_2 increases from zero under the conditions of Fig. 5, $\mathscr{E}_2(d_2)$ becomes much greater than $\mathscr{E}_1(d_2)$, as these functions correspond to the field strengths at a distance d_2 that is small and in the Sommerfeld region. Initially this effect more than compensates for the decrease due to the term $\exp[-\frac{1}{2}(\alpha_1+\alpha_2)d_2]$, thus accounting for the rise in field strength.

The field given by eqn. (21) does not settle down to the characteristic type $\exp[-\alpha_2 d_2]$ until d_2 exceeds the value $200\lambda^{1/3}$, and $\mathscr{E}_2(d_2)$ and $\mathscr{E}_1(d_2)$ are given by their diffraction forms proportional to $\exp[-\alpha_2 d_2]$ and $\exp[-\alpha_1 d_2]$, when

$$\sqrt{\left[\frac{\mathscr{E}_2(d_2)}{\mathscr{E}_1(d_2)}\right]}\exp[-\tfrac{1}{2}(\alpha_1+\alpha_2)d_2] \text{ becomes proportional to } \exp[-\alpha_2 d_2]$$

It was for this reason that the transition distance d_t was taken to be $200\lambda^{1/3}$ in constructing Fig. 4. At the short-wave limit in the Sommerfeld region, d_t is given by $(20/\pi)\lambda|\zeta|^2$, i.e. the distance at which the inverse-distance-squared law comes into operation for the larger of the ζ values on the two sides of the boundary.

If we use the method similarly in the neighbourhood of a boundary from sea to land, we obtain, instead of an increase, a correspondingly rapid drop in field strength beyond the boundary before the curve settles down to a slope characteristic of overland transmission. Bearing in mind the fact that the reciprocity condition must be fulfilled, even in the neighbourhood of the boundary, we could explain away the proposed increase on the one hand only by a still more rapid drop on the other as the boundary is crossed. In order to obtain an idea of what the reciprocity condition implies, it is instructive to apply our method formally to construct a set of land-sea and sea-land curves for various positions of the boundary; the result for our example with $\lambda = 100$ m is shown in Fig. 8.

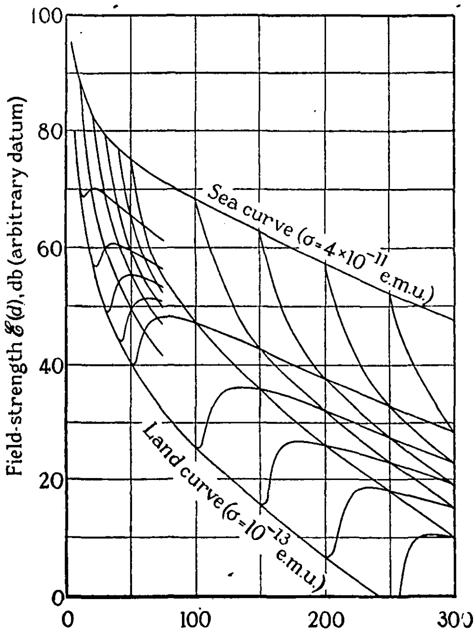

Fig. 8.—Field-strength/distance curves for various positions of a land-sea or sea-land boundary, obtained by the reciprocal empirical method.

$\lambda = 100$ m.

It will be seen that, wherever the curves intersect, the reciprocity condition is obeyed. The land-to-sea recovery develops in the Sommerfeld region as soon as the overland curve departs appreciably from the inverse-distance law and goes over to the inverse-distance-squared type. The effect becomes a maximum in the diffraction region where the limiting form given by eqn. (21) is reached. On inspecting these curves, it is difficult to avoid the conclusion that there must be some such recovery if the reciprocity condition is to be met everywhere, though admittedly this does not justify the method we have adopted to obtain the curves. Rather, we must regard the method as a tentative approach to the study of the disturbance function in an attempt to reveal the general features we should expect the rigid solution to present.

(7) EXPERIMENTAL EVIDENCE

It is natural to ask to what extent our conclusions are borne out by experiment, especially with regard to the land-to-sea recovery. Unfortunately it appears that practically no specially controlled experiments have been carried out across a land-to-sea boundary, so that the issue may still be regarded as open. The author is indebted, however, to the B.B.C. for permission to refer to the fact that they have found indications of a recovery in field strength where a wave has crossed from ground of low to ground of medium conductivity. Fig. 9 is a reproduction of one of their

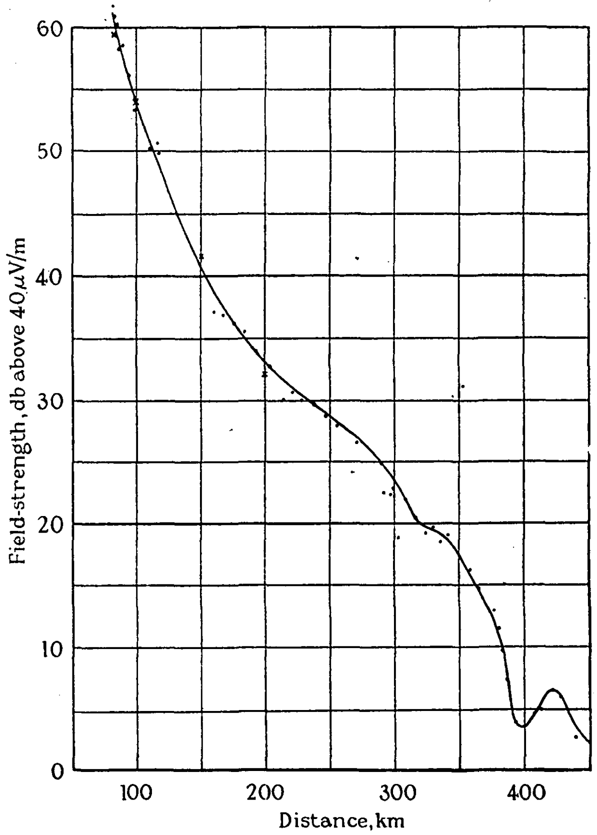

Fig. 9.—Experimental curve over inhomogeneous land, showing a small recovery effect.

$\lambda = 286$ m.
Dots—observed.
Crosses—calculated.

curves obtained by measurements on a test transmitter at Start Point ($\lambda = 286$ m) taken along a line to Happisburgh on the Norfolk coast. It is known that there are local regions of poor conductivity in East Anglia, and it will be noticed that the recovery is preceded by an especially steep section of the curve.

The fact that the recovery is small may be accounted for partly by the relatively small change in conductivity involved, and partly because the change may not be sudden, as in our idealized picture, or extend far enough for the effect to be fully developed. By itself, therefore, it can throw little light on the order of accuracy of the empirical method, though it supports the contention that a recovery of some kind is to be expected. It draws attention to the need for a specific experiment under controlled conditions across a land-sea boundary where the sea stretches well into the diffraction region with respect to the boundary.*

The B.B.C. experiments also support the view upon which our argument has been based, that the modification to the field strength is negligible until the boundary is reached. In the Start Point experiments, for instance, they have found that the initial transmission across the sea gives a field at the Dorset coast agreeing with the calculated all-oversea value. The curve in Fig. 9 also shows an increased rate of drop in field strength on crossing a sea-to-land boundary, the crosses shown being calculated according to our method, assuming $\sigma = 10^{-13}$ e.m.u.

* A predicted recovery of 12 db has since been obtained on a wavelength of 4 m.—G. M. (10.12.48).

for the land, though this evidence is not conclusive without a local survey to justify the use of this average value for the conductivity. Further B.B.C. experiments with a transmitter at Rampisham near Dorchester do, however, support this choice of σ for the initial part of the land path in the Start Point tests.

Other B.B.C. measurements agree with those of T. L. Eckersley, quoted above, in supporting the proposed solution over part-land and part-sea routes where the detailed nature of the field in the neighbourhood of a boundary is not under consideration. The surveys which they have carried out in connection with the service areas of their various medium-wave stations also throw valuable light on the problem of differing conditions along different radials from a transmitter. In Fig. 10 are shown some field-strength contours round the transmitter at Washford Cross, and it will be seen how profound an effect the estuary of the River Severn has on the shape of the contours.

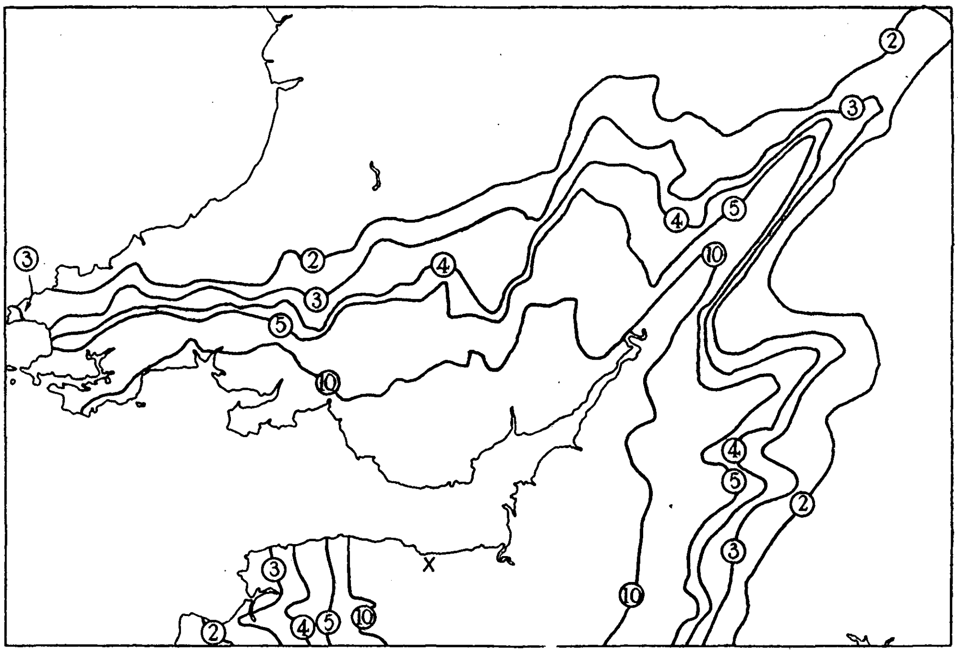

Fig. 10.—Experimental field-strength contours (in mV/m), showing the radial effect.
$\lambda = 373 \cdot 1$ m. Power = 70 kW.

These contours suggest that our idealizing assumption for working out the field-strength variations along a radial is roughly true, and though the contours of equal field strength must be rounded off where they cross a land-sea boundary, they do in fact undergo very rapid changes in direction. Even where the wave is transmitted almost parallel to a coast-line, the method gives a good indication of the variation of field strength inland from the coast at different distances from the transmitter, as, for instance, has been found in the case of the service area of Start Point along the South Coast towards Brighton.

This aspect of the problem is obviously associated with the size of the distances concerned relative to the wavelength, and it is here that the treatment of Grunberg and Feinberg may be of special importance. There also arises the allied question of coastal refraction, but this is a subject we cannot enter into here, where we have been considering only the field strength, though it is of equal concern to those interested in the application of medium and long waves to navigational aids.

There is one further point on which it would be most useful to have experimental evidence. According to the curves in Fig. 8, it should be possible under some conditions for a transmitter radiating initially over sea to produce a stronger signal over the succeeding land than it would if it were placed on the land itself. For instance, we see that a transmission over 50 km of sea and then over 50 km of land gives a field strength about 7 db greater than for a transmission over 50 km of land alone, the reason presumably being that the initial sea path has reduced the heavy earth losses in the neighbourhood of the transmitter.

An experiment to investigate this possibility would help greatly towards proving whether our method of representing the field in the neighbourhood of a boundary is reasonably accurate or not. In any case, the reciprocity requirement emphasizes a point that is not always fully appreciated, that not only should the wave be given a good start by placing the transmitter on a site of high conductivity, if it is to be used to the best advantage, but also the receiving site should be made equally conducting. In other words, the earth losses are important not only in the neighbourhood of the transmitter but also at the receiver, a fact which is implied in the identical form of the height-gain factors $f(h_T)$ and $f(h_R)$.

(8) PRACTICAL CONSIDERATIONS

There remains the problem of interpreting the method we have discussed in a practical form for the computation of the field strength for any given route of two or more sections. We have been primarily concerned with the development of the fundamental ideas upon which the method is based in an attempt to reveal the features that a complete solution would present, so that we cannot enter here into great practical detail. We will, however, conclude with a few general suggestions that may be useful to the engineer.

Even if we do not wish to employ the method in the neighbourhood of a boundary where it is admittedly speculative, the fact that it agrees with our earlier arguments for the regions well beyond the boundary enables us to base our method of computation on eqns. (19) and (20) rather than on (17). This is in general preferable because all the quantities involved are directly obtainable from standard ground-wave propagation curves for various conductivities, and it is not limited strictly to the diffraction region. There is not space here to present such ground-wave curves, and it must be assumed that reference can be made to existing sets. It is desirable to use curves provided with a decibel scale relative, for example, to 1 μV/m as datum, so that the computation of eqns. (19) and (20) and their geometric mean can be turned into a purely additive process.

If a number of individual and unrelated cases have to be worked out, it is probably simplest to compute each one in turn directly with the aid of eqns. (19) and (20) expressed in decibel form. Suppose, however, that a detailed investigation has to be made of the field-strength contours round a given transmitter on a fixed wavelength, so that the field-strength/distance curves have to be constructed along a large number of radials. It is then probably worth while to adopt the following procedure.

We make use of the fact that the standard field-strength curves on a linear scale of distance eventually become effectively linear to extrapolate the linear portion back to the ordinate $d = 0$, where it will make an intercept $2p$, as in Fig. 11, on the decibel scale. If the slope of this straight line is q decibels per unit distance, the field strength at a distance d in decibels can then be written

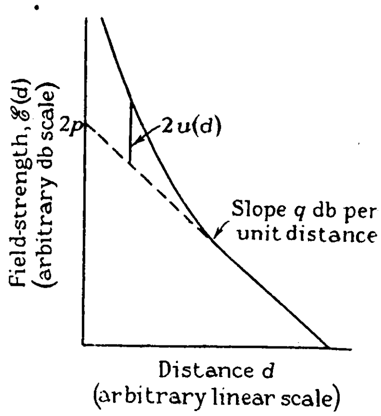

Fig. 11.—Practical resolution of a field-strength/distance curve.
$\mathscr{E}(d) = 2p - qd + 2u(d)$.

$$\mathscr{E}(d) = 2p - qd + 2u(d)$$

where $2u(d)$ is the departure of the actual curve from the straight line in Fig. 11. This function $u(d)$ tends to zero with increasing d, and to infinity as d approaches 0. If, however, we have two types of earth for which separately we have

$$\mathscr{E}_1(d) = 2p_1 - q_1 d + 2u_1(d) \quad . \quad . \quad . \quad . \quad (22)$$

and

$$\mathscr{E}_2(d) = 2p_2 - q_2 d + 2u_2(d) \quad . \quad . \quad . \quad . \quad (23)$$

then $\mathscr{E}_1(d)$ and $\mathscr{E}_2(d)$ both tend to the inverse-distance value as d approaches 0. It therefore follows that

$$u_2(d) - u_1(d) \to p_1 - p_2, \text{ as } d \to 0$$

Thus if we define a function ${}_2u_1(d)$ by

$$ {}_2u_1(d) = u_2(d) - u_1(d) \quad . \quad . \quad . \quad . \quad (24)$$

it will approach a finite limit $p_1 - p_2$ as d approaches 0, and will tend to zero as d increases.

In terms of this notation, our required $\mathscr{E}(d)$ over a path of n sections may be reduced to the form

$$\begin{aligned}\mathscr{E}(d) = p_1 + p_n + {}_1v_n(d) - [q_1d_1 + q_2d_2 + \ldots + q_nd_n] \\ + [{}_1u_2(d_1) + {}_2u_3(d_1 + d_2) + \ldots + {}_{n-1}u_n(d_1 + d_2 \\ + \ldots + d_{n-1}) + {}_nu_{n-1}(d_n) + {}_{n-1}u_{n-2}(d_n + d_{n-1}) \\ + \ldots + {}_2u_1(d_n + d_{n-1} + \ldots + d_2)] \quad (25)\end{aligned}$$

where

$$ {}_1v_n(d) = u_1(d) + u_n(d) \quad . \quad . \quad . \quad . \quad . \quad (26)$$

From eqn. (24) we have

$$ {}_1u_2(d) = - {}_2u_1(d), \text{ etc.} \quad . \quad . \quad . \quad . \quad (27)$$

while from (26) it follows that ${}_nv_1(d) = {}_1v_n(d)$, and from (22) and (23) that

$$p_1 + p_n + {}_1v_n(d) \to I(d), \text{ as } d \to 0 \quad . \quad . \quad . \quad (28)$$

where $I(d)$ is the inverse-distance value.

From the relations in eqns. (27) and (28) we can see that $\mathscr{E}(d)$ in (25) approaches, as it should, the inverse-distance value $I(d)$ when the total distance d is indefinitely small.

The advantage of this method of computation is that in the term $[q_1d_1 + q_2d_2 + \ldots + q_nd_n]$, any sections with the same q value can be grouped together as a single section of the same total length, while the u functions are in general small corrections which are zero when the boundaries concerned are at a sufficient distance from the terminal with respect to which they are reckoned. The function ${}_1v_n(d)$ involves only the total distance, and is zero in many practical cases.

This method implies, as a preliminary, that we should prepare from the standard propagation curves for the types of earth involved a table of constants of the form $p_1 + p_n$, a set of straight lines through the origin giving q_1d, q_2d, etc., for the computation of $[q_1d_1 + q_2d_2 + \ldots + q_nd_n]$, and curves of the various functions ${}_2u_1(d)$, etc., and ${}_1v_n(d)$. Very often, in practice, there are only two types of earth involved, e.g. land and sea, though sometimes the land may have to be subdivided, e.g. into medium conductivity ($\sigma = 10^{-13}$ e.m.u.) and poor conductivity ($\sigma = 10^{-14}$ e.m.u.), so that the method is not so complicated as it may seem at first sight.

As an example of the basic curves derived from the standard propagation curves, those for $\lambda = 100$ m are shown in Fig. 12 for a part land and part sea route where for land σ is everywhere 10^{-13} e.m.u. In this case the constants required are $2p_L$ and $2p_S$ for routes both beginning and ending with land and with sea respectively, and $p_L + p_S$ where there is land at one end of the route and sea at the other. Their values will depend upon the radiated power and the datum level for the field-strength decibel scale, whereas the functions shown in Fig. 12 are independent of them, being essentially decibel differences by definition.

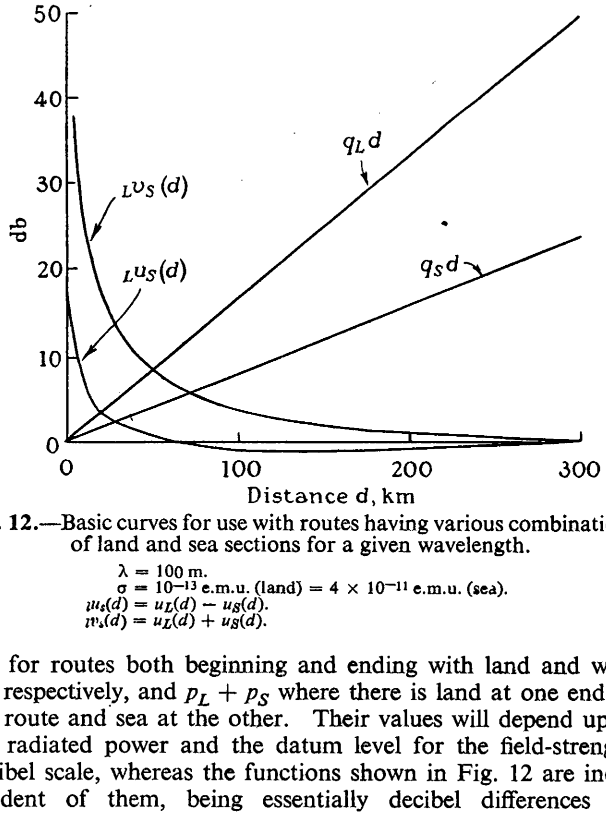

Fig. 12.—Basic curves for use with routes having various combinations of land and sea sections for a given wavelength.
$\lambda = 100$ m.
$\sigma = 10^{-13}$ e.m.u. (land) $= 4 \times 10^{-11}$ e.m.u. (sea).
${}_Lu_S(d) = u_L(d) - u_S(d)$.
${}_Lv_S(d) = u_L(d) + u_S(d)$.

We cannot enter further here into the details of this method of computing the field strength, but it is found to lead to an easy and rapid technique after a little experience in handling it has been acquired.

As an example of the method, a field-strength/distance curve is given in Fig. 13 for a 1-kW transmission on a wavelength of 1 000 m over 200 km of land ($\sigma = 10^{-13}$ e.m.u.), 200 km of sea, 200 km of land ($\sigma = 10^{-14}$ e.m.u.) followed by sea to an

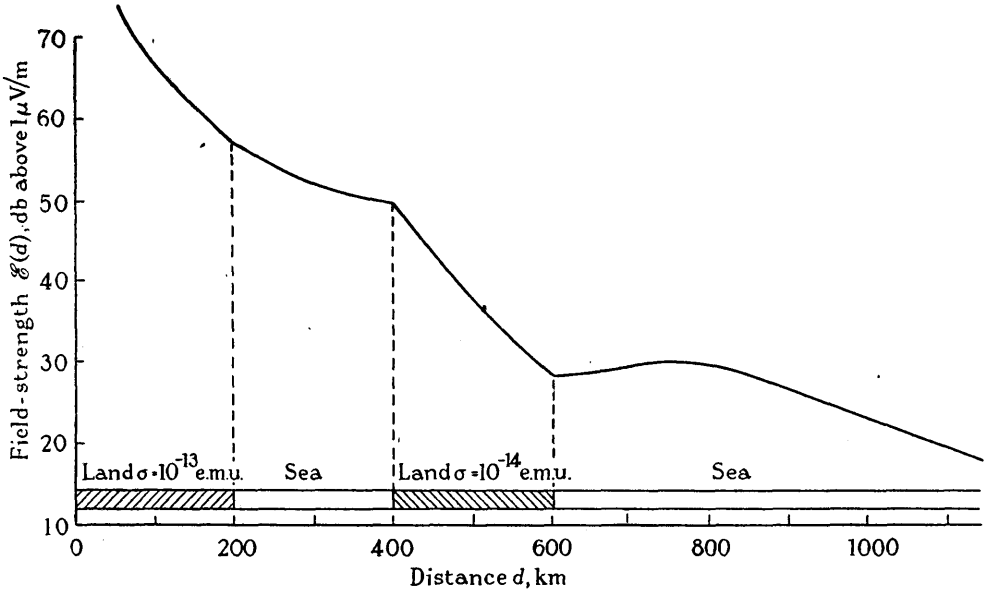

Fig. 13.—Practical example showing several boundaries.

λ = 1 000 m. Power = 1 kW.

indefinite distance. On this wavelength the land-sea recovery in field strength is not so pronounced, owing to the approach towards the long-wave limit, but beyond the boundary at 600 km the field strength holds up for 300 km, and the curve illustrates well the practical implications of the solution here proposed. It is clearly a matter of great importance to the engineer that the problem should be subjected to further theoretical and experimental study.

(9) CONCLUSIONS

In conclusion it is necessary to stress again the imperfections of the solution. It is difficult to assess the approximations involved in the argument based on the properties of the wave above the fictitious sphere, though it is felt that the physical picture it presents is essentially sound, even if it needs modifying in detail. We can therefore feel justified in using it to support an empirical method that gives the same result under the same conditions. The extension of this method to the neighbourhood of a boundary is admittedly speculative, but it has served to draw attention to some of the features we may expect a rigid solution to reveal.

The inadequacy of the method is most clearly shown in the fact that no consideration is taken of the phase of the wave at the boundary. It therefore leaves unanswered the important question of coastal refraction and the distortion of the surfaces of constant phase, for which a complete solution of the boundary problem is needed. It is interesting that the formula for ground-to-ground propagation has been based on a study of the height-gain effects, but this is reasonable in view of the redistribution of energy that must take place above a boundary.

In the diffraction region, the appropriate height-gains can be applied for a raised transmitter or receiver in accordance with the picture that has been given, but the empirical method that has been applied formally only to the ground-to-ground case can be extended to study the effect of raising a terminal that is near a boundary. In many applications of the theory to medium- and long-wave propagation the height-gain effects are negligible, though it should not be overlooked that quite considerable effects may be involved in the case of high-flying aircraft employing medium-wave navigational aids.

It is hoped that this paper may stimulate others to pursue this most important subject on the theoretical side, and to carry out controlled experiments designed to test the views that have been put forward.

(10) ACKNOWLEDGMENTS

The author is indebted to Marconi's Wireless Telegraph Co., Ltd., for permission to publish the paper and to members of the Research Division for help and encouragement in its preparation. He also gratefully acknowledges the co-operation received from the research staff of the British Broadcasting Corporation and the permission to publish the experimental results given in Figs. 9 and 10 and described in Section 7.

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