
Contents

	Preface	xiii
1	<i>The ionosphere and magnetosphere</i>	1
1.1	The earth's atmosphere	1
1.2	Plane and spherical radio waves	3
1.3	Waves in ion plasmas	3
1.4	Relation to other kinds of wave propagation	5
1.5	Height dependence of electron concentration: the Chapman layer	7
1.6	Collision frequencies	10
1.7	Observations of the ionosphere	12
1.8	The structure of the ionosphere	14
1.9	The magnetosphere	17
1.10	Disturbances of the ionosphere and magnetosphere	19
	<i>Problems 1</i>	20
2	<i>The basic equations</i>	22
2.1	Units and symbols	22
2.2	Definitions of electric intensity \mathbf{e} and magnetic intensity \mathbf{h}	23
2.3	The current density \mathbf{j} and the electric polarisation \mathbf{p}	24
2.4	The electric displacement \mathbf{d} and magnetic induction \mathbf{b}	25
2.5	Harmonic waves and complex vectors	25
2.6	Maxwell's equations	27
2.7	Cartesian coordinate system	28
2.8	Progressive plane waves	29
2.9	Plane waves in free space	31
2.10	The notation \mathcal{H} and \mathbf{H}	31
2.11	The power input to the plasma from a radio wave	32
2.12	The flow of energy. The Poynting vector	33
2.13	Complex refractive index	34
2.14	Evanescent waves	35
2.15	Inhomogeneous plane waves	35
	<i>Problems 2</i>	37

3	<i>The constitutive relations</i>	38
3.1	Introduction	38
3.2	Free, undamped electrons	38
3.3	The Lorentz polarisation term	40
3.4	Electron collisions. Damping of the motion	42
3.5	The Debye length	44
3.6	Effect of the earth's magnetic field on the motion of electrons	45
3.7	Effect of the magnetic field of the wave on the motion of electrons	46
3.8	Electric neutrality of the plasma. Plasma oscillations	48
3.9	The susceptibility matrix	49
3.10	Complex principal axes	50
3.11	Properties of principal axis elements of the permittivity. Effect of ions	53
3.12	Collisions. The Sen–Wyller formulae	57
3.13	Electron–electron collisions. Electron–ion collisions	61
	<i>Problems 3</i>	62
4	<i>Magnetoionic theory 1. Polarisation and refractive index</i>	66
4.1	Plane wave and homogeneous plasma	66
4.2	Isotropic plasma	67
4.3	Anisotropic plasma. The wave polarisation	68
4.4	Properties of the polarisation equation	71
4.5	Alternative measure of the polarisation. Axis ratio and tilt angle	73
4.6	Refractive index 1. The dispersion relation	74
4.7	Longitudinal component of electric polarisation and electric field	75
4.8	The flow of energy for a progressive wave in a magnetoplasma	76
4.9	Refractive index 2. Alternative derivations and formulae	77
4.10	Zeros and infinity of refractive index. Equal refractive indices	79
4.11	Dependence of refractive index on electron concentration 1. $Y < 1$	80
4.12	Dependence of refractive index on electron concentration 2. $Y > 1$	84
4.13	Effect of collisions included	86
4.14	The transition collision frequency	86
4.15	The terms 'ordinary' and 'extraordinary'	88
4.16	Dependence of refractive index on electron concentration 3. Collisions allowed for	89
4.17	Approximations for refractive indices and wave polarisations	94
	<i>Problems 4</i>	99
5	<i>Magnetoionic theory 2. Rays and group velocity</i>	103
5.1	Introduction	103
5.2	Refractive index surfaces	104
5.3	The ray. Ray surfaces	108
5.4	Properties of ray surfaces	111
5.5	Crystal optics	113
5.6	Classification of refractive index and ray surfaces. C.M.A. type diagrams	116
5.7	Dependence of refractive index on frequency	124
5.8	Group velocity	128

5.9	Properties of the group velocity	131
5.10	Effect of electron collisions on the group refractive index	137
	<i>Problems 5</i>	139
6	<i>Stratified media. The Booker quartic</i>	141
6.1	Introduction	141
6.2	The variable q	142
6.3	The Booker quartic. Derivation	144
6.4	Some properties of the Booker quartic	146
6.5	Some special cases of the Booker quartic	151
6.6	The discriminant of the Booker quartic	152
6.7	The Booker quartic for east–west and west–east propagation	153
6.8	The Booker quartic for north–south and south–north propagation	155
6.9	Effect of electron collisions on solutions of the Booker quartic	160
6.10	The electromagnetic fields	162
	<i>Problems 6</i>	163
7	<i>Slowly varying medium. The W.K.B. solutions</i>	165
7.1	Introduction	165
7.2	The differential equations for an isotropic ionosphere	166
7.3	The phase memory concept	167
7.4	Loss-free medium. Constancy of energy flow	168
7.5	W.K.B. solutions	169
7.6	The W.K.B. method	170
7.7	Discrete strata	172
7.8	Coupling between upgoing and downgoing waves	174
7.9	Liouville method and Schwarzian derivative	175
7.10	Conditions for the validity of the W.K.B. solutions	177
7.11	Properties of the W.K.B. solutions	178
7.12	W.K.B. solutions for oblique incidence and vertical polarisation	180
7.13	Differential equations for anisotropic ionosphere	181
7.14	Matrix theory	183
7.15	W.K.B. solutions for anisotropic ionosphere	187
7.16	The matrices \mathbf{S} and \mathbf{S}^{-1}	190
7.17	W.K.B. solutions for vertical incidence	192
7.18	Ray theory and ‘full wave’ theory	193
7.19	The reflection coefficient	194
	<i>Problems 7</i>	195
8	<i>The Airy integral function and the Stokes phenomenon</i>	197
8.1	Introduction	197
8.2	Linear height distribution of electron concentration and isolated zero of q	197
8.3	The differential equation for horizontal polarisation and oblique incidence	199
8.4	The Stokes differential equation	200
8.5	Qualitative discussion of the solutions of the Stokes equation	201
8.6	Solutions of the Stokes equation expressed as contour integrals	202

8.7	Solutions of the Stokes equation expressed as Bessel functions	204
8.8	Tables of the Airy integral functions. Computing	205
8.9	Zeros and turning points of $Ai(\zeta)$ and $Bi(\zeta)$	205
8.10	The W.K.B. solutions of the Stokes equation	206
8.11	Asymptotic expansions	206
8.12	The Stokes phenomenon of the 'discontinuity of the constants'	209
8.13	Stokes lines and anti-Stokes lines	209
8.14	The Stokes diagram	211
8.15	Definition of the Stokes multiplier	212
8.16	Furry's derivation of the Stokes multipliers for the Stokes equation	212
8.17	The range of validity of asymptotic approximations	213
8.18	The choice of a fundamental system of solutions of the Stokes equation	214
8.19	Connection formulae, or circuit relations	215
8.20	Stratified ionosphere. Uniform approximation	216
8.21	The phase integral method for reflection	218
8.22	The intensity of light near a caustic	223
	<i>Problems 8</i>	227
9	<i>Integration by steepest descents</i>	229
9.1	Introduction	229
9.2	Some properties of complex variables and complex functions	229
9.3	Saddle points	231
9.4	Error integrals and Fresnel integrals	233
9.5	Contour maps	237
9.6	Integration by the method of steepest descents	238
9.7	Application to solutions of the Stokes equation	241
9.8	The method of stationary phase	247
9.9	Higher order approximation in steepest descents	249
9.10	Double steepest descents	251
	<i>Problems 9</i>	252
10	<i>Ray tracing in a loss-free stratified medium</i>	254
10.1	Introduction	254
10.2	The ray path	255
10.3	Wave packets	257
10.4	Equations of the ray path	259
10.5	The reversibility of the path	261
10.6	The reflection of a wave packet	261
10.7	An example of a ray path at oblique incidence	263
10.8	Poeverlein's construction	264
10.9	Propagation in magnetic meridian plane. The 'Spitze'	266
10.10	Ray paths for the extraordinary ray when $Y < 1$	268
10.11	Extraordinary ray when $Y > 1$	270
10.12	Lateral deviation at vertical incidence	273
10.13	Lateral deviation for propagation from (magnetic) east to west or west to east	275
10.14	Lateral deviation in the general case	276
10.15	Calculation of attenuation, using the Booker quartic	277

10.16	Phase path. Group or equivalent path	278
10.17	Ray pencils	279
10.18	Caustics	281
10.19	The field where the rays are horizontal	285
10.20	The field near a caustic surface	286
10.21	Cusps. Catastrophes	287
10.22	The skip distance	288
10.23	Edge focusing	289
	<i>Problems 10</i>	292
11	<i>Reflection and transmission coefficients</i>	295
11.1	Introduction	295
11.2	The reference level for reflection coefficients	295
11.3	The reference level for transmission coefficients	297
11.4	The four reflection coefficients and the four transmission coefficients	298
11.5	Reflection and transmission coefficient matrices	299
11.6	Alternative forms of the reflection coefficient matrix	300
11.7	Wave impedance and admittance	301
11.8	Reflection at a sharp boundary 1. Isotropic plasma	304
11.9	Properties of the Fresnel formulae	306
11.10	Reflection at a sharp boundary 2. Anisotropic plasma	307
11.11	Normal incidence. Anisotropic plasma with free space below it	308
11.12	Normal incidence. Two anisotropic plasmas	309
11.13	Probing the ionosphere by the method of partial reflection	311
11.14	Spherical waves. Choice of reference level	313
11.15	Goos-Hänchen shifts for radio waves	315
11.16	The shape of a pulse of radio waves	320
	<i>Problems 11</i>	325
12	<i>Ray theory results for isotropic ionosphere</i>	328
12.1	Introduction	328
12.2	Vertically incident pulses	329
12.3	Effect of collisions on phase height $h(f)$ and equivalent height $h'(f)$	330
12.4	Equivalent height for a parabolic height distribution of electron concentration	332
12.5	Effect of a 'ledge' in the electron height distribution	336
12.6	The calculation of electron concentration $N(z)$, from $h'(f)$	337
12.7	Ray paths at oblique incidence	342
12.8	Equivalent path P' at oblique incidence	345
12.9	Maximum usable frequency, MUF	348
12.10	The forecasting of MUF	349
12.11	Martyn's theorem for attenuation of radio waves	352
	<i>Problems 12</i>	353
13	<i>Ray theory results for anisotropic plasmas</i>	356
13.1	Introduction	356
13.2	Reflection levels and penetration frequencies	357

13.3	The calculation of equivalent height, $h'(f)$	359
13.4	Ionograms	362
13.5	Topside sounding	365
13.6	The calculation of electron concentration $N(z)$ from $h'(f)$	368
13.7	Faraday rotation	372
13.8	Whistlers	376
13.9	Ion cyclotron whistlers	380
13.10	Absorption, non-deviative and deviative	391
13.11	Wave interaction 1. General description	393
13.12	Wave interaction 2. Outline of theory	395
13.13	Wave interaction 3. Kinetic theory	397
	<i>Problems 13</i>	398
14	<i>General ray tracing</i>	400
14.1	Introduction	400
14.2	The eikonal function	402
14.3	The canonical equations for a ray path	403
14.4	Properties of the canonical equations	405
14.5	The Haselgrove form of the equations	407
14.6	Fermat's principle	409
14.7	Equivalent path and absorption	412
14.8	Signal intensity in ray pencils	414
14.9	Complex rays. A simple example	417
14.10	Real pseudo rays	422
14.11	Complex rays in stratified isotropic media	424
14.12	Complex rays in anisotropic absorbing media	425
14.13	Reciprocity and nonreciprocity with rays 1. The aerial systems	428
14.14	Reciprocity and nonreciprocity with rays 2. The electric and magnetic fields	431
14.15	Reciprocity and nonreciprocity with rays 3. Applications	433
	<i>Problems 14</i>	436
15	<i>Full wave solutions for isotropic ionosphere</i>	438
15.1	Introduction	438
15.2	Linear electron height distribution	439
15.3	Reflection at a discontinuity of gradient	441
15.4	Piecewise linear models	443
15.5	Vertical polarisation at oblique incidence 1. Introductory theory	446
15.6	Vertical polarisation 2. Fields near zero of refractive index	448
15.7	Vertical polarisation 3. Reflection coefficient	450
15.8	Exponential electron height distribution	453
15.9	Parabolic electron height distribution 1. Phase integrals	456
15.10	Parabolic electron height distribution 2. Full wave solutions	460
15.11	Parabolic electron height distribution 3. Equivalent height of reflection	464
15.12	The differential equations of theoretical physics	466
15.13	The hypergeometric equation and its circuit relations	467
15.14	Epstein distributions	470

15.15	Reflection and transmission coefficients for Epstein layers	472
15.16	Ionosphere with gradual boundary	473
15.17	The 'sech ² ' distribution	475
15.18	Other electron height distributions	476
15.19	Collisions. Booker's theorem	477
	<i>Problems 15</i>	479
16	<i>Coupled wave equations</i>	480
16.1	Introduction	480
16.2	First order coupled equations	482
16.3	Coupled equations near a coupling point	485
16.4	Application to vertical incidence	489
16.5	Coupling and reflection points in the ionosphere	492
16.6	Critical coupling	495
16.7	Phase integral method for coupling	499
16.8	The Z-trace	502
16.9	Additional memory	505
16.10	Second order coupled equations	507
16.11	Försterling's coupled equations for vertical incidence	509
16.12	Properties of the coupling parameter ψ	510
16.13	The method of 'variation of parameters'	514
16.14	The coupling echo	517
	<i>Problems 16</i>	518
17	<i>Coalescence of coupling points</i>	520
17.1	Introduction	520
17.2	Further matrix theory	521
17.3	Coalescence of the first kind, C1	523
17.4	Coalescence of the second kind, C2	525
17.5	Ion cyclotron whistlers	530
17.6	Radio windows 1. Coalescence	532
17.7	Radio windows 2. Formulae for the transparency	534
17.8	Radio windows 3. Complex rays	540
17.9	Radio windows 4. The second window	542
17.10	Limiting polarisation 1. Statement of the problem	543
17.11	Limiting polarisation 2. Theory	546
18	<i>Full wave methods for anisotropic stratified media</i>	550
18.1	Introduction	550
18.2	Integration methods	552
18.3	Alternative methods 1. Discrete strata	553
18.4	Alternative methods 2. Vacuum modes	556
18.5	Alternative methods 3. The matrizant	558
18.6	Starting solutions at a great height	560
18.7	Finding the reflection coefficient	562
18.8	Allowance for the earth's curvature	563

18.9	Admittance matrix as dependent variable	566
18.10	Other forms, and extensions of the differential equations	569
18.11	Numerical swamping	574
18.12	Reciprocity	576
18.13	Resonance	579
	<i>Problems 18</i>	581
19	<i>Applications of full wave methods</i>	583
19.1	Introduction	583
19.2	Vertical incidence and vertical magnetic field	584
19.3	Oblique incidence and vertical magnetic field	587
19.4	Resonance and barriers	591
19.5	Isolated resonance	593
19.6	Resonance tunnelling	596
19.7	Inversion of ionospheric reflection measurements	602
19.8	Full wave solutions at higher frequencies	606
	<i>Answers to problems</i>	609
	<i>Bibliography</i>	612
	<i>Index of definitions of the more important symbols</i>	643
	<i>Subject and name index</i>	652