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Radio frequency scattering from a heated ionospheric volume, 3, Cross section calculations

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The proper normalization procedure for determining per-unit-volume properties of aspect-sensitive scattering media from experimental measurements is discussed. This is complicated by the fact that, because of the aspect sensitivity, it is not clear *a priori* what the per-pulse scattering volume actually is. In general, the correct normalization requires quantitative knowledge of the aspect sensitivity which may not be possible to determine because of fundamental limitations imposed by the spatial-resolution capability of the measurement system on the extent to which this property of the scattering medium can actually be determined. The identical problem exists in radio-auroral measurements. This problem is considered here, and an exact expression from which the desired per-unit-volume quantities of interest can be obtained from measurements is given. It is shown that proper choice of the experimental parameters leads to a simplification whereby the normalization can be carried out without knowledge of the aspect sensitivity being necessary. The results are applied to calculating from experimental data the per-unit-volume scattering properties of a heated ionospheric volume as a function of frequency, from which a transverse scale size for the scattering medium of 3 m is estimated. For these experiments, under maximum heating conditions, a representative value of 1% for the rms fractional electron density deviation is calculated; an upper bound is also established showing that values as large as 4 or 5% were probably never achieved. It is shown that, for an electron density distribution axially symmetric with respect to the geomagnetic field, **B**, the cross section for backscatter within a plane containing **B** uniquely determines the cross section for bistatic scattering axially around **B**. Experimental results for small axial-bistatic angles are presented showing good agreement between calculated and measured values.

1. INTRODUCTION

In what follows, the RF scattering cross section of an ionospheric volume subjected to heating by means of high power HF illumination [*Utlaut,* 1970] is calculated on the basis of experimental results [*Minkoff et al.*, 1974*a*; *Fialer,* 1974]. Similar calculations have been carried out [*Rao and Thome,* 1974] by integrating the total power scattered over the entire range extent of the heated volume under certain assumptions regarding the form of the mathematical functions entering into the calculations. In the procedure followed below, a generalized approach is developed in which no assumptions regarding the specific mathematical form of any of the functions involved are necessary.

As discussed by *Minkoff et al.* [1974*a*] two scattering modes for the heated volume have been observed. Only center-line scattering is considered here, which is observed to be highly aspect sensitive with respect to the direction of the earth's magnetic field **B**. For this mode, on the basis of arguments presented by *Minkoff* [1973*a*], the per-unit-volume differential cross section derived by *Booker* [1956] can be written in the form

$$\sigma = r_e^2 \langle |\Delta n|^2 \rangle S_L S_T \tag{1}$$

where r_e is the classical radius of the electron, e^2/mc^2, $\langle |\Delta n|^2 \rangle$ is the variance of the electron density distribution, and S_L and S_T describe the spatial frequency content of the electron density distribution parallel and perpendicular to the geomagnetic field, **B**. The wave number spectra S_L and S_T are in turn characterized by the longitudinal and transverse coherence lengths or scale sizes, L and T, which are interpreted as the average distances parallel and perpendicular to **B** over which collective scattering effects take place, and which define, in some sense, the widths of S_L along the k_x axis, and S_T in the (k_y, k_z) plane, as $2\pi/L$ and $2\pi/T$, respectively. The arguments for writing S in separable form as $S = S_L S_T$ are based on the experimentally observed fact that the scattering takes place from field-aligned structures very long in comparison with their width ($L/T \gg 1$) [*Minkoff et al.*, 1974*b*].

Complete specification of the scattering properties of the heated volume is not possible for these experiments since, as shown by *Minkoff* [1973*a*, *b*], measurement of the aspect sensitivity of the scattering, S_L, requires extremely high spatial resolution capability on the part of the radar system, the most stringent condition being that the vertical antenna aperture, D, be much larger than L. *Minkoff et al.* [1974*a*] have shown that because of this it is possible to infer only that L is larger, and probably much larger, than the maximum antenna diameter used in these experiments which was 85 ft. Because of the aspect sensitivity, determination of $\langle|\Delta n|^2\rangle$ and S_T from the measurements is also not straightforward. These quantities are determined from measurement of scattered power vs. radar wavelength, λ. However, since the strength of the received signal at any instant in time also depends on the size of the per-pulse scattering volume, and further, since only per-unit-volume quantities can provide a meaningful generalized description of the scattering properties of the heated volume, the measurements must be properly normalized with respect to the size of the scattering volume if the quantities of interest are actually to be obtained. However, because of the aspect sensitivity, the size of the actual scattering volume will in general be very different from the net illuminated volume, which may be presumed known under beam-filling conditions, and in fact, as is discussed below, the actual size of the effective scattering volume remains an unknown quantity unless S_L, or at least L, is known. Thus the proper normalization of the data to obtain $\langle|\Delta n|^2\rangle$ and S_T in the most general case may not be possible.

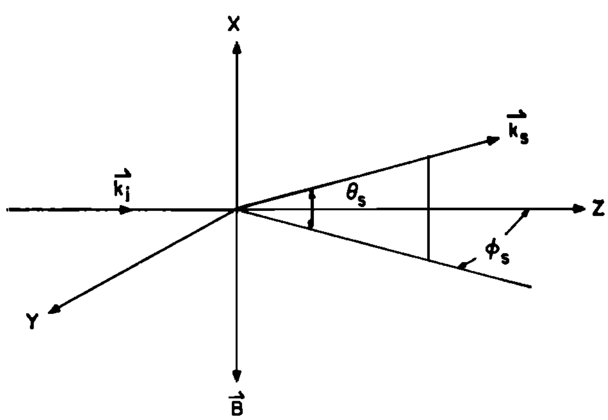

Fig. 1. Generalized scattering geometry for radar line of sight, $\mathbf{k}_i$, perpendicular to $\mathbf{B}$. The line of sight to the receiver is $\mathbf{k}_s$.

This problem arises in observations of the radio aurora and in all such measurements of aspect-sensitive scattering. As shown below, however, the difficulties can be evaded if the radar parameters are chosen properly. The necessary conditions are that $D < L$ and, (referring to Figure 2) that $\tau < R \cot \phi_\perp / fL$ where τ is the transmitted pulse width and f is the radar frequency. The approach taken here is to show that, under these conditions, a generalized radar equation for aspect-sensitive scattering can be derived in which the received power can be expressed in terms of $\langle|\Delta n|^2\rangle$ and S_T, independently of S_L or L. Because the data considered below consist of both backscatter and bistatic measurements essentially within the plane of the magnetic meridian, both of these cases are considered. From these results an estimated value for T of 3 m is obtained. From calculations of $\langle|\Delta n|^2\rangle$ for these experiments, a representative value for the rms fractional deviation, $(\langle|\Delta n|^2\rangle/n^2)^{1/2}$, under maximum heating conditions is 1%; an upper bound on this quantity is also established showing that values as large as 4% or 5% were probably never achieved. It is also shown below that the cross section for bistatic scattering axially around $\mathbf{B}$ can be determined from backscatter measurements in a plane containing $\mathbf{B}$; comparisons between predicted theoretical values and experimental results are presented here and by *Fialer* [1974].

2. THE SCATTERING CROSS SECTION

Consider the situation shown in Figure 1 in which the incident wave vector $\mathbf{k}_i$, representative of the radar line of sight, is normal to $\mathbf{B}$ (zero angle of incidence) and, for convenience, the wave is vertically polarized. In such experiments it is reasonable to assume that $\lambda/L \ll 1$, in which case it can be shown [*Minkoff*, 1973*a*] that, for arbitrary scattering wave vector, $\mathbf{k}_s$, where $|\mathbf{k}_s| = |\mathbf{k}_i| = k = 2\pi/\lambda$, the cross section can be written as:

$$\sigma = r_e^2\langle|\Delta n|^2\rangle S_L(k\theta_s)S_T[k \sin \phi_s, k(\cos \phi_s - 1)] \quad (2)$$

Thus, for $\lambda \ll L$, the cross section takes the convenient form in which the θ_s and ϕ_s dependence is confined separately to S_L and S_T respectively. For backscatter, the received power at any instant in time corresponding to some slant range R contains contributions from all the scatterers within the radar range-resolution cell centered at R. Over the scattering region (Figure 2) $\mathbf{B}$ approximately coincides with a circle of radius R,

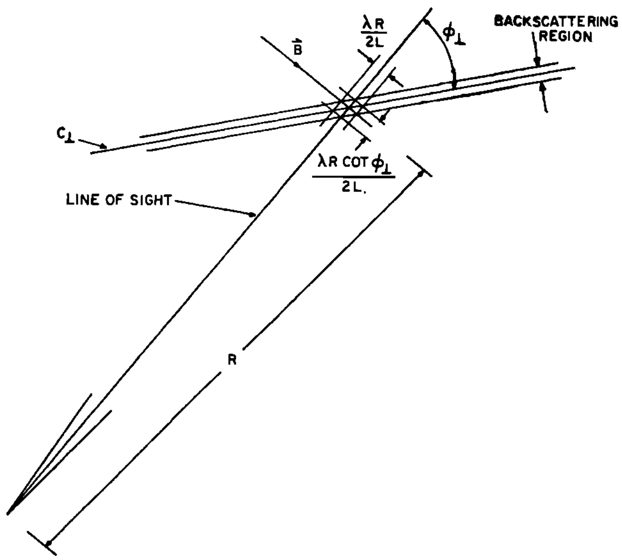

Fig. 2. Backscatter geometry. The radar line of sight is in the magnetic meridian plane. Maximum backscatter takes place along the perpendicularity contour, $C_\perp$, defined as the locus of points over which the transmitted rays are perpendicular to **B**. The line of sight makes an angle $\phi_\perp$ with $C_\perp$. The thickness of the scattering region, defined as the distance along **B** over which, because of the aspect sensitivity, backscatter is actually produced, can be shown to be about $\lambda R/2L$. For $\lambda/L \ll 1$ this distance is approximately coincident with a circle of radius R, and defines the locus of scattering points which determine the value of received power corresponding to a delay time $2R/c$. The necessary condition on τ for equations 5, 6, and 7 to be valid is equivalent to the condition that Δ be less than the effective range extent of the scattering region, $\lambda R \cot \phi_\perp/2L$.

and the transmitted signal is incident to **B** over a range of angles depending on the size of the antenna. For a ray whose angle of incidence is α, where α is not too large, the argument of S_L in (2) is $k(\theta_s + \alpha)$, which for backscatter becomes $2k\alpha$ where α ranges over the vertical antenna beamwidth.

Now consider the bistatic situation shown in Figure 3. We observe that, over the scattering region, the ellipse which defines the locus of points of constant time delay between the two sites, as before, approximately coincides with **B**. It is easily shown that for every transmitted ray which makes an angle α with the transmitter line of sight (defined by γ), the angle of incidence to **B** is $\gamma \pm \alpha$ and, at this point, the angle of reflection between the ray scattered to the receiver site and **B** is $\gamma \mp \zeta$, where ζ is the angle between the scattered ray and the receiver line of sight. Thus, over any given ellipse which defines a fixed value of time delay between the two sites, the observer sees the longitudinal spectrum in the form $S_L[k(\alpha + \zeta)]$. It is also evident that for any value of γ the scattering takes place in a cone consisting of the locus of all such specularly reflected wave vectors, with **B** as the cone axis and $\pi/2 - \gamma$ as the cone half-angle, and that the magnitude of the scattering measured axially around **B** is described by S_T. The transverse spectrum is seen to be a function of two independent variables, $k_y = k \sin \phi_s$ and $k_z = k(\cos \phi_s - 1)$. This shows that, in general, in order to determine S_T, measurements at at least two azimuthally bistatic sites corresponding to two different values of ϕ_s, as well as over a range of values of k, are necessary. However, if the statistical properties of the random electron density distribution are assumed to be axially symmetric with respect to **B**, then it can be shown [*Minkoff*, 1973*a*] that σ takes the form:

$$\sigma = r_e^2\langle|\Delta n|^2\rangle S_L[k(\alpha + \zeta)]S_T[2k \sin (|\phi_s|/2)] \quad (3)$$

In this case S_T is a function of a single variable and can be determined either by measurements at a single frequency over a range of values of ϕ_s or by measurements for a single value of ϕ_s over a range of frequencies. For transmitter and receiver within the meridian plane $\phi_s = \pi$ and, for purposes of calculating values of received power at any fixed instant in time, the per-unit-volume differential cross

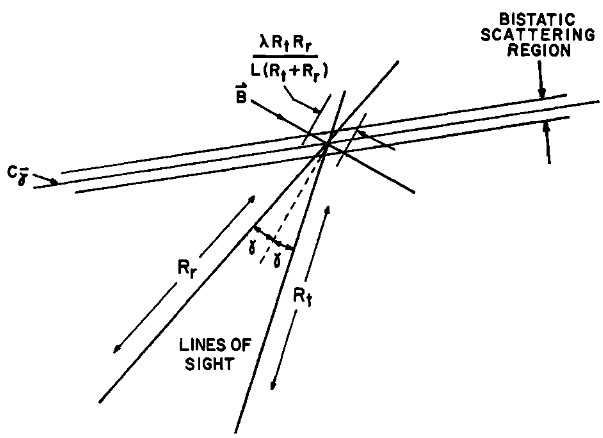

Fig. 3. Bistatic scattering geometry. Lines of sight from the transmitter, R_t, and the receiver, R_r, are in the magnetic meridian plane. Maximum scattering from t to r takes place along the contour, $C_{\bar{\gamma}}$, over which rays from t are specularly reflected to r. As shown, $\gamma = \bar{\gamma} - \pi/2$ is the angle between R_t and R_r and the normal to **B** at the point of intersection of R_t and R_r over the heater. The thickness of the bistatic scattering region, defined as the distance along **B** over which scattering from t to r takes place, can be shown to be approximately $\lambda R_t R_r/L(R_t + R_r)$. Bistatic scattering is equivalent to backscatter in that, for $\lambda/L \ll 1$, the ellipse of constant time delay approximately coincides with the thickness.

section for both bistatic and backscatter measurements is

$$\sigma = r_e^2\langle|\Delta n|^2\rangle S_L[k(\alpha + \zeta)]S_T(2k) \quad (4)$$

where α and ζ range over the vertical beamwidths of the transmitting and receiving antennas (for backscatter, $\zeta = \alpha$).

3. THE RADAR EQUATION FOR ASPECT SENSITIVE SCATTERING

In order to calculate at any instant in time the actual value of received power for backscatter or bistatic measurements, σ must be integrated over the per-pulse scattering volume, V, with the rays at each scattering point within V weighted by S_L depending on their angles of incidence to **B**. However, it is shown by *Minkoff* [1973*a*] for backscatter that, for $D < L$ and $\tau < R \cot \phi_\perp/fL$, this integration can be carried out independently of S_L, in which case:

$$b(k) = r_e^2\langle|\Delta n|^2\rangle S_T(2k)$$
$$= 2R^2P_k/[P_T\lambda^2/(4\pi)^2]G_\alpha^2\lambda\Delta \int_{\beta(R)} G_\beta^2(\beta)\, d\beta \quad (5)$$

where P_k = received (peak) power for a given value of k, Δ = range-resolution cell = $c\tau/2$, c = speed of light, P_T = transmitted power, α and β range over the elevation and azimuth antenna patterns and $b(k)$ is denoted as the backscatter coefficient. In this expression the antenna gain is written separately as $G(\Omega) = G_\alpha(\alpha)G_\beta(\beta)$ and $\int_{\beta(R)} G_\beta^2(\beta)\, d\beta$ is the integral of the square of the azimuthal antenna pattern over $\beta(R)$, the azimuthal extent of the scattering volume at the range R. For dish antennas such as those used in the experiments described by *Minkoff et al.* [1974*a*], and assuming beam filling in azimuth, the radar equation takes the convenient form

$$P_k = (\pi/8)P_TD^3b(k)\Delta/R^2 \quad (6)$$

where D is the antenna diameter; for $D < L$ beam filling in elevation is of course irrelevant since in the vertical direction the scattering region appears as a point target, which leads to the D^3 dependence.

Equations 5 and 6 can be understood simply by recognizing that the per-pulse scattering volume V has the dimensions: $R\beta(R)$ in azimuth, $\Delta = c\tau/2$ in range, and the thickness (that is, the distance measured along the direction of **B** from which, because of the aspect sensitivity, backscatter is actually received) is, as discussed by *Minkoff* [1973*b*], $\lambda R/2L$. In general, because of the aspect sensitivity, σ will vary over Δ and over the thickness; the variation over $R\beta(R)$ is negligible as long as $\beta(R)$ is not too large. However, for $\tau < R \cot \phi_\perp/fL$ it is easily seen that the maximum variation in angle of incidence to **B** across Δ will be less than the scattering lobe width λ/L and this variation can therefore be ignored. With regard to the final dimension, the thickness, if $D < L$ then the antenna beam width will be larger than the scattering beam width with the result that, over any azimuthal range, all the scattered energy within S_L is collected by the antenna and the aspect sensitivity therefore becomes integrated out. This can be expressed by assigning an average value to S_L, defined as:

$$\int_{-\infty}^{\infty} S_L(k_x)\, dk_x = 2\pi = (2\pi/L)\langle S_L\rangle$$

since the width of $S_L(k_x)$ is $2\pi/L$. Hence $\langle S_L\rangle = L$, and the product σV becomes:

$$\sigma V = b(k)LR\beta(R)(\lambda R/2L)\Delta$$

which shows that, although σ increases with L since the scattering thereby becomes more directive, the size of the scattering volume at the same time decreases with L in such a way that the two effects tend to cancel, with the net result that the received power becomes independent of the aspect sensitivity. Thus, if the above conditions on D and τ are satisfied, and $\langle|\Delta n|^2\rangle$ is assumed constant over V, by applying the usual radar equation, with the expression for the radar cross section replaced by $4\pi\sigma V$, we obtain:

$$P_k = [P_T\lambda^2/(4\pi)^2]G_\alpha^2G_\beta^2[\beta(R)\lambda b(k)\Delta/2R^2] \quad (7)$$

which is exactly equivalent to (5) if we let

$$\int_{\beta(R)} G_\beta^2(\beta)\, d\beta \rightarrow \beta(R)G_\beta^2$$

It may be noted that for purposes of this calculation the scattering can be thought of as emanating from an infinitely narrow plane whose projection in the magnetic meridian plane is $C_\perp$, and for which the dimensionless quantity $(\lambda R/2)b(k)$ serves as the backscatter cross section per unit area.

For bistatic scattering within the meridian plane the cross section, as shown above, is the same as for backscatter but the size of the effective per-pulse scattering volume is somewhat different. The range resolution cell is $\Delta/\cos \gamma$ and the thickness, that is, the distance measured along **B** over which, because of the aspect sensitivity, scattering between the transmitter (t) and the receiver (r) actually takes place, can be shown to be $\lambda R_tR_r/L(R_t + R_r)$. For these experiments $2\gamma \simeq 5°$, $\Delta/\cos \gamma \simeq \Delta$, and the required conditions on D and τ, which in general can be shown to be somewhat different in the bistatic case, also turn out to be essentially the same as for backscatter. The radar equation for bistatic scattering within the meridian plane is then:

$$P_k = [P_T\lambda^2/(4\pi)^2] \cdot G_\alpha^{(t)} G_\alpha^{(r)} G_\beta^{(t)} G_\beta^{(r)} b(k) W\lambda\Delta/R_t R_r (R_t + R_r) \quad (8)$$

where W is the azimuthal extent (in units of length) of the scattering volume common to both sites.

4. CALCULATION OF THE BACKSCATTER COEFFICIENT

Using data obtained from the experiments reported by *Fialer* [1974] between 40 and 88 MHz, and measurements at 157.5 and 435 MHz as described by *Minkoff et al.* [1974*a*], a curve of $b(f) = b(ck/2\pi)$, representative of the per-unit-volume backscatter cross section of the heated volume as a function of frequency was calculated using (7) and (8). The 40 to 88 MHz data were obtained in bistatic measurements in which the transmitting and receiving sites were at Los Lunas and Socorro, New Mexico, respectively ($R_t \simeq 621$ km, $R_r \simeq 716$ km); the 157.5 and 435 MHz data were obtained in backscatter measurements ($R \simeq 925$ km) using the RAM radar at the White Sands Missile Range (WSMR). With regard to the necessary conditions on D and τ in order for (7) and (8) to apply, since $L > 85$ ft, the necessary requirement that $D < L$ was satisfied in all cases. With regard to the requirement on τ, some reasonable guesses must be made. For these measurements $\phi_\perp \simeq 15°$. For the 40 to 88 MHz data ($\Delta = 1.125$ km), it can be shown that the requirement on τ was satisfied for $L \lesssim 10$ km; for the 157.5 and 435 MHz data ($\Delta = 4.5$ and 1.5 km) the condition was satisfied for $L \lesssim 1$ km. It is seen from the discussion in section 3 that, using this procedure, for any given value of τ, too large a value of L results in an underestimation of $b(f)$.

In generating the curve of $b(f)$ presented below, maximum values of received power, corresponding to optimum conditions for scattering over the Los Lunas-Socorro path and for backscatter at RAM, were used. In the former case optimum conditions require maximum heating altitude, h_f, in the vicinity of the specular reflection point between the two sites which took place at approximately 240 km above the heater. For backscatter at RAM, maximum signals were observed for $h_f \simeq 290$ km at which point the radar line of sight was perpendicular to **B** above the heater; optimum conditions also include maximum HF heater power (about 2 Mw CW). Over the two-year period during which these experiments were carried out, the maximum signal levels were found to be quite repeatable, say to within ±3 db, under optimum conditions. In combining these data it must be assumed that maximum heating produces the same value of $\langle|\Delta n|^2\rangle$ at 240 km as it does at 290 km.

Using these data the curve of $b(f)$, the back-

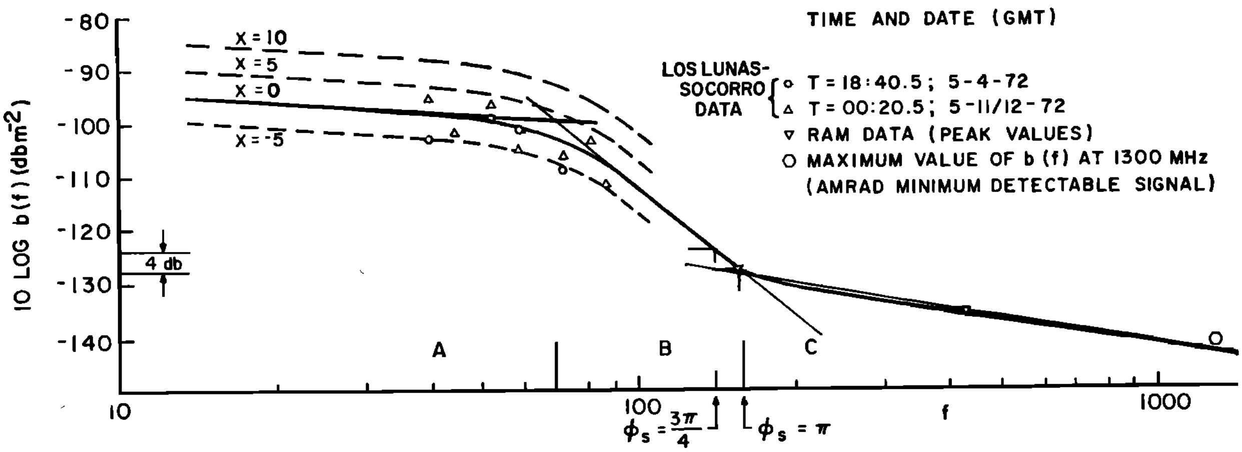

Fig. 4. The backscatter coefficient $b(f)$ as determined from experimental data using equations 6 and 8. The width of the curve is nominally 100 MHz from which $T \simeq 3$ m. Varying $\langle|\Delta n|^2\rangle$ results in vertical displacement of $b(f)$, the shape presumably remaining the same. By using this procedure upper and lower bounds on $\langle|\Delta n|^2\rangle$ for these data are calculated from the dashed curves corresponding to $x = \pm 5$ db respectively; the results of this analysis are summarized in Table 1. The values of $\phi_s = \pi$ and $\phi_s = 3\pi/4$ refer to a comparison of the measured backscatter and axially-bistatic cross sections as discussed in section 6. As indicated in the figure, the theoretically predicted increase in cross section for the axially-bistatic case is 4 db.

scatter coefficient of the heated volume as a function of frequency, is presented in Figure 4. Measurements below 40 MHz have not been included since refraction effects evidently become significant in this range. Following the above discussion, in order to determine the proper shape of $b(f)$, for consistency only those data obtained during times of maximum scattering are used at the different frequency points, for which the corresponding representative curve is shown as a solid line corresponding to $x = 0$. Taking 100 MHz as representative of the width of the curve the estimated value of T is 3 m. As discussed by *Minkoff et al.* [1974*a*], no echoes from the heated volume were ever observed in backscatter by the AMRAD L-band radar at White Sands. Hence a maximum value of $b(f)$ at 1300 MHz can also be estimated from knowledge of the minimum detectable AMRAD cross section and this point is also included in the figure. From this curve, the transverse spectrum S_T can be determined, if necessary, once $\langle|\Delta n|^2\rangle$ has been calculated. This is discussed in the following section.

5. CALCULATION OF $\langle|\Delta n|^2\rangle$ AND $[\langle|\Delta n|^2\rangle/n^2]^{1/2}$

Since the spatial autocorrelation function of the random electron density distribution must be unity at the origin then the wave number spectrum, S, is normalized such that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(k_x, k_y, k_z)\, dk_x\, dk_y\, dk_z = (2\pi)^3 \qquad (9)$$

As discussed at the beginning of section 1, the spectrum is written as $S(k_x, k_y, k_z) = S_L(k_x)S_T(k_y, k_z)$. If the random electron density fluctuations are assumed to be axially symmetric around **B**, then using polar coordinates in the (k_y, k_z) plane, (9) becomes:

$$\int_{-\infty}^{\infty} S_L(k_x)\, dk_x = 2\pi \text{ and } \int_0^{\infty} kS_T(k)\, dk = 2\pi \qquad (10)$$

Hence if

$$m = \int_0^{\infty} kb(k)\, dk \qquad (11)$$

then

$$\langle|\Delta n|^2\rangle = 2m/\pi r_e^2 \qquad (12)$$

In order to calculate m from the curve in Figure 4, let $b(f)$ for $x = 0$ be approximated by three straight lines in the regions A, B, and C as shown in the figure where

A: $0 \le f < 70$ MHz

$$10 \log b(f) = -55.1 - 5.6 \log f$$

B: $70 \le f < 157.5$ MHz

$$10 \log b(f) = 525 - 79.5 \log f$$

C: $157.5 \le f \le 30{,}000$ MHz

$$10 \log b(f) = 21.6 - 18.1 \log f$$

In calculating $\langle|\Delta n|^2\rangle$ the curve in region C has been arbitrarily cut off at a frequency corresponding to a 1-cm Debye length at 300 km, beyond which no collective scattering takes place. The integration is straightforward, giving $m = 2.52 \times 10^{-2}$ from which $\langle|\Delta n|^2\rangle = 2.0 \times 10^{19}$ (el m^{-3})^2.

The fractional deviation will depend on the value chosen for the mean electron density n. This can be related to the heater frequency f_h since maximum heating takes place where f_h is equal to the local plasma frequency f_p. Taking 6 MHz as a representative value, for which $n = 4.5 \times 10^{11}$ el m^{-3}, we obtain $\langle|\Delta n|^2\rangle = 10^{-4}$ for an rms fractional deviation of 1%. Since for all the experimental results presented here the minimum value of f_h was about 5 MHz we obtain the maximum fractional deviation by choosing the corresponding value of n, 3.1×10^{11} el m^{-3}, for which the rms fractional deviation is 1.5%.

Now we note that variations in $\langle|\Delta n|^2\rangle$ result in an upward or downward shift in the curve of $b(f)$, the shape presumably remaining the same. This enables upper and lower bounds on $\langle|\Delta n|^2\rangle$ for these experiments to be easily estimated, since a shift by x db results simply in a change in $\langle|\Delta n|^2\rangle$ by a factor of $10^{0.x}$. Choosing for x the values $\pm$ 5 and + 10 we obtain the dashed curves shown in Figure 4. The results of this analysis are summarized in Table 1. It is seen that, unless S_T takes a sudden unexpected increase below 40 MHz, for these experiments a reasonable upper bound on the rms fractional electron density deviation under maximum heating conditions is 2.6%, and that values as large as 4% or 5% were probably never achieved.

6. CROSS SECTIONS FOR AXIALLY BISTATIC SCATTERING

As discussed in section 2, assuming that the electron density distribution is axially symmetric with respect to **B**, for bistatic measurements out of the meridian plane, where $\phi_s \neq \pi$, for any given transmitted frequency, f, the cross section is determined by

TABLE 1. Effects of vertical displacement of $b(f)$ on values of $\langle|\Delta n|^2\rangle^{1/2}$ and $(\langle|\Delta n|^2\rangle/n^2)^{1/2}$.

x(db)	$\langle\lvert\Delta n\rvert^2\rangle^{1/2}$ (el m^{-3})	$(\langle\lvert\Delta n\rvert^2\rangle/n^2)^{1/2}$ (percent) f_h = 6 MHz	f_h = 5 MHz (max values)	
10	1.4×10^{10}	3.2	4.6	Too large
5	8.0×10^9	1.8	2.6	Upper bound
0	4.5×10^9	1.0	1.5	Average value
−5	2.5×10^9	0.6	0.8	Lower bound

evaluating the backscatter coefficient $b(f)$ at an effectively lower frequency $\hat{f} = f \sin(|\phi_s|/2)$ (Equations 3 and 4). Hence, because of the shape of $b(f)$ in Figure 4, the cross section for scattering out of the meridian plane exceeds that for scattering within the meridian plane. Furthermore, using the backscatter coefficient, which is based on measurements for which $\phi_s = \pi$, it should be possible to predict axially bistatic signal levels for arbitrary f and ϕ_s.

For the experiment described in section 4 of *Minkoff et al.* [1974*b*], VHF signals transmitted by the RAM radar at WSMR, scattered from the heated volume, were received at a bistatic site at El Centro, California. A diagram of the RAM-Platteville-El Centro geometry is presented in the same reference, showing that the axial bistatic scattering angle, ϕ_S, was about $3\pi/4$. It is also seen in this figure that, for this path, the specular reflection criterion necessary for maximum bistatic scattering was satisfied for a radar elevation angle at RAM of 14°, which is very close to the angle required for maximum backscatter (15°). It is therefore consistent in this experiment to compare maximum signal levels received bistatically with those received in backscatter, since for both cases the scattering took place in very nearly the same region within the heated volume. Furthermore, because of the small bistatic angle ϕ_S, the size of the per-pulse scattering volume was also essentially the same in both cases.

Typical values of peak signal levels received at RAM and El Centro during this experiment were −86 db m and −102 db m respectively. To compare the bistatic and backscatter cross sections we scale these values as follows. The antenna configuration at El Centro consisted of two horizontally stacked Yagis for which the factory-specified gain per antenna at 157.5 MHz is 13 db; the total gain is therefore assumed to be 16 db which is 14 db below the RAM UHF antenna gain of 30 db. The slant ranges from RAM and El Centro to the scattering volume are taken to be 900 and 1275 km, respectively, which results in a range correction of about 3 db. Finally there is also a maximum polarization correction of 6 db since RAM, which experiences virtually no polarization loss, transmits circular polarization of which, assuming no Faraday rotation, El Centro receives the horizontal component only multiplied by $\sin^2 \chi$, where χ is the angle between the polarization vectors of the incident and scattered signals, which in this case is 1/2. Therefore, subtracting 14 + 3 + 6 = 23 db m from the −86 db m signal received at RAM gives −109 db m which is 7 db below the −102 db m El Centro signal level.

To compare this result with theory we refer to Figure 4 where, for a bistatic angle of $3\pi/4$ an increase in the scattering coefficient of 4 db is predicted as shown. It is seen that the measured result, as scaled above, exceeds the predicted value by 3 db. This may be explained, at least in part, by effects due to Faraday rotation as follows. We note that for the vertically polarized component of the scattered signal the factor $\sin^2 \chi$ is unity. Hence the scattered signal is elliptically polarized with the minor horizontal axis one half of the vertical major axis. Thus, because of the orientation of the receiving antenna, any Faraday rotation decreases the 6-db polarization-loss factor used above thereby bringing the predicted and measured result more nearly into agreement; for a 90° rotation the agreement is exact. These results, therefore, show, for small angles ϕ_S, a reasonable agreement between measured and predicted values. Other such comparisons between predicted and measured results are presented by *Fialer* [1974].

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