Multi-hop, High-Frequency Radio Propagation Near Japan

Team 93463

February 12, 2018

To whom it may concern,

When reading our solution, we hope that you are can read our document in Adobe Acrobat Reader as we have `.gif` files that can only be played with that particular PDF reader. Please visit `https://get.adobe.com/reader/` to download it if need be.

We are trying to put our best foot forward and we would appreciate if you receive our work as intended. It is worth it. We promise. Thank you.

Respectfully,
Team 93463

Abstract

We modeled high frequency radio-waves and their interactions with the ionosphere, turbulent and calm ocean, and smooth and rugged terrain. We did this to

Hi team, here is what we *need* to have in our report (according to the MCM overlords):

- Restatement and clarification of the problem
- Explain assumptions and rationale/justification
- Include your model design and justification
- Describe model testing and sensitivity analysis
- Discuss the strengths and weaknesses

They also claim to judge the quality of our writing. So remember our good friend Williams. ¯_(ツ)_/¯[10]

1 Introduction

High frequency (HF) radio-waves (defined as 3–30 MHz) can travel through the atmosphere by multiple reflections off of the Earth's ionosphere and the its surface unless the frequencies are larger than the maximum usable signal (MUF), then they pass through the ionosphere and are lost.[1]

Empirically, radio-waves reflect off of the ocean (or terrain) differently depending on whether the ocean is turbulent (rugged) or calm (smooth), impacting the distance the signal can faithfully.[1] We chose Tokyo, Japan as a location of study because it an ideal location to study radio-wave interactions as it is an island and is "mostly rugged and mountainous."[6]

The goal of our model was to find the following: (1) find the number of "skips" a HF signal could have with ocean before losing signal integrity, (2) the same but with rugged terrain rather than ocean, and (3) how a boat in the ocean could receive signals on turbulent waters.

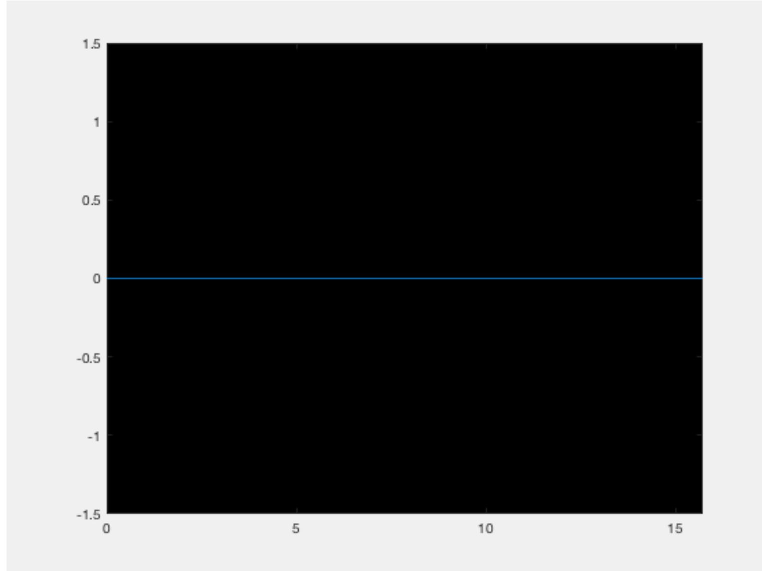

Figure 1: A test `.gif` file. Click image to see animation.

You need to use Adobe Acrobat Reader to view this `.gif` file. If you do not have it, please visit `https://get.adobe.com/reader/` to download it. It is worth it.

2 Model

2.1 HF Signals

As radio-waves propagate over a certain distance, we expect them to pick up some noise as they go, degrading the integrity of the information they are trying to transmit. To simulate this, we additive white Gaussian noise (AWGN).[9,7]

2.2 The Ionosphere

The ionosphere consists of roughly three layers that lie between 75–1000 km above the Earth's surface: (1) the F-region, (2) the E-region, and (3) the D-region; each of these regions has charge-density dependent on the time of year, the number of sunspots present, the time of day, and the movement of the charged particles. The ionosphere interacts heavily with radio waves, mainly through the interaction of these free electrons.[3]

To model the ionosphere, we modify the Chapman Law,[5,4,3,2] which describes the electron density, and make it dependent on the time of day and year. We found a simple, analytic model for the ionosphere, $\overline{N}(z,t)$, which is a function of altitude and time which was given by

$$\overline{N}(z,t) = T(t)N(z) \tag{2.1}$$

where $N(z) = N_0 \exp\left[\frac{1}{2}\left(1 - \frac{z-z_0}{\kappa} - \sec(\alpha)e^{-(z-z_0)/\kappa}\right)\right]$ and $T(t) = (1+d(t)+s(t))$ and $d(t) \in [0,1]$ is the daytime contribution of the ionosphere and $s(t) \in [0,4]$ is the seasonal contribution.

To understand how to radio-waves will interact with the ionosphere, we must found an expression for the refractive index, n, of light through the plasma. For an isotropic ionosphere where electrons do not collide and for purely transverse waves, we found $n^2 = 1 - \overline{N}e^2/\epsilon_0 m\omega^2$, which is independent in of the direction of the HF signal but is dependent on its frequency and some fundamental constants like permitivity, electron mass and charge.

2.3 The Ocean

Sea water essentially reflects HF signals perfectly.[8] However, turbulent and choppy waves will disrupt the signal. Using a simple square wave we were able to show that a quasi-index of refraction for the ocean, m is given by $m = n(1-\sqrt{R})/(1+\sqrt{R})$ where $R = \cos^2(\phi/2)$ and ϕ is the phase difference created by a HF signal with wavelength λ. We derived

$$\phi = \frac{2}{\lambda}\sqrt{A^2 + (\lambda_s/4)^2}. \tag{2.2}$$

where A and λ_s are the amplitude and wavelengths of the ocean waves, respectively.

3 Results

3.1 Problem I: A Turbulent Ocean

3.2 Problem II: The Japanese Alps

3.3 Problem III: A Message to a Boat

4 Conclusion

Acknowledgments

We would like to thank our professors for lending us resources and pointing us in the direction of others. We would like to thank our College for financial support allowing us to enter this competition. Without it, we would not have been able to have this much fun.

References

[1] 2018 MCM: Problem A: Multi-hop HF Radio Propagation, February 2018.

[2] Kenneth George Budden. *The propagation of radio waves: the theory of radio waves of low power in the ionosphere and magnetosphere*. Cambridge University Press, 1988.

[3] KG Budden. *Radio Waves in the Ionospere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers*. Cambridge University Press, 1961.

[4] S Chapman. The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth part ii. grazing incidence. *Proceedings of the physical society*, 43(5):483, 1931.

[5] Sydney Chapman. The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth. *Proceedings of the Physical Society*, 43(1):26, 1931.

[6] CIA Factbook. The world factbook. *See also: https://www.cia.gov/library/publications/the-world-factbook*, 2010.

[7] Thomas Kailath. An innovations approach to least-squares estimation–part i: Linear filtering in additive white noise. *IEEE transactions on automatic control*, 13(6):646–655, 1968.

[8] William M. Adams Larry K. Lepley. Reflectivity of electromagnetic waves at an air-water interface for pure and sea water.

[9] Claude E Shannon. Communication in the presence of noise. *Proceedings of the IEEE*, 72(9):1192–1201, 1984.

[10] John S Townsend. *A modern approach to quantum mechanics*. University Science Books, 2000.