
Index of definitions of the more important symbols

A	coefficient of n^4 in dispersion relation for cold magnetoplasma (also used with other meanings)	78
A_j ($j = 1, 2, 3, 4$)	quantities used in S_{ij} (also used with other meanings)	190
A	2×2 admittance matrix	302
A_{ij} ($i, j = 1, 2$)	elements of A	303
A, A_{ij}	used with different meanings in §13.6.	371
a	half thickness of parabolic layer (also used with other meanings)	332, 456
$a_1, \dots, a_6,$ $\bar{a}_1, \dots, \bar{a}_6$	quantities used in elements of S and S^{-1}	190, 191
B	magnetic induction of earth's magnetic field (used with different meanings in §§1.1, 13.6)	23, 45
B	coefficient of $-2n^2$ in dispersion relation for cold magnetoplasma (also used with other meanings)	78
B	4×4 matrix whose trailing diagonal elements are unity	184
b	(real) magnetic induction	25
C	$\cos \theta$, cosine of angle of incidence coefficient of n^0 in dispersion relation for cold magnetoplasma	142 78
\mathcal{C}	cylinder function; solution of Bessel's equation	598
$\mathcal{C}_p(S)$	\mathcal{C} integrals	59
c	speed of light in a vacuum (also used with other meanings)	22

D	dispersion of a whistler	378
	function used in a_1, \dots, a_6	190
	function used in dispersion relation	403
	horizontal range	342
	(also used with other meanings)	
D_G	horizontal range measured over earth's curved surface	349
\mathbf{D}	4×4 diagonal matrix	506
D	complex electric displacement in wave with harmonic time dependence	26, 29
	(also used as 2×2 matrix)	
D_x, D_y, D_z	components of electric displacement D	26, 28
\mathcal{D}	Schwarzian derivative	176
d	(real) electric displacement	25
	(also used with other meanings)	
E	complex electric intensity in wave with harmonic time dependence	26
E_x, E_y, E_z	components of E (also used with superscripts)	26
E	magnitude of E	26
	short for E_y	457
E_1, E_2, E_3	contravariant components of E in complex principal axes (used with other meanings in §§14.13–14.14).	51
\mathcal{E}	eikonal function	402
e	exponential	
e	charge on electron (a negative number)	39, 46
e	(real) electric intensity	23
\mathbf{e}	column matrix with elements $E_x, -E_y, \mathcal{H}_x, \mathcal{H}_y$	162, 182
$F(q)$	left side of Booker quartic equation	144
F	function used in equation of ray surface (also used to denote other functions)	410
$F_1, F_2,$ F_3, F_4	$F_1 = (q_1 - q_2)(q_1 - q_3)(q_1 - q_4)$ etc.	191
F_H	proton gyro-frequency	383
$\mathcal{F}_O, \mathcal{F}_E, \mathcal{F}$	field variables in Försterling's equations	510, 597
f	frequency (also used with other meanings)	39
f_c	critical coupling frequency	496
f_{cr}	crossover frequency	383
f_{crs}	f_{cr} at satellite	384
f_H, f_{He}	gyro-frequency for electrons	46, 54
f_{Hi}	gyro-frequency for ions of species i	55

f_L	lower hybrid frequency	57
f_N, f_{Ne}	plasma frequency for electrons	39, 55
f_{Ni}	plasma frequency for ions of species i	55
f_p	penetration frequency (also used with superscripts)	332, 357
f_R	value of f_N where reflection occurs	360
f_s	with various superscripts: transition frequencies of the plasma near a satellite	366
f_U	$(f_N^2 + f_H^2)^{\frac{1}{2}}$; upper hybrid resonance frequency	54
\mathbf{f}	column matrix with four elements	163, 188
f_1, f_2, f_3, f_4	elements of \mathbf{f} ; amplitudes of the four characteristic waves	163, 188
G	$\frac{1}{2}(\epsilon_1 + \epsilon_2) - \epsilon_3$ function used in dispersion relation (also used with other meanings)	52 408
\mathcal{G}	$\epsilon_3(\epsilon_1 - \epsilon_2)/\{\epsilon_1\epsilon_2 - \frac{1}{2}\epsilon_3(\epsilon_1 + \epsilon_2)\}$	383
H	scale height of atmosphere rate of heating of unit volume of plasma	7 396
\mathbf{H}	complex magnetic intensity in wave with harmonic time dependence	26
H_x, H_y, H_z	components of \mathbf{H}	28
\mathcal{H}	$Z_0\mathbf{H}$, alternative measure of magnetic intensity	31
$\mathcal{H}_x, \mathcal{H}_y,$ \mathcal{H}_z	components of \mathcal{H}	31
$h, h(f)$	phase height	329
h_0	height of base of ionosphere	331
$h', h'(f)$	equivalent (or group) height of reflection also used with subscripts, O for ordinary or X for extraordinary	329, 359 362
\mathbf{h}	(real) magnetic intensity	23
i	$\sqrt{(-1)}$	
i	integer label to indicate species of positive ion	55
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors parallel to x, y, z axes respectively	28
J	$\frac{1}{2}\epsilon_3(\epsilon_1 + \epsilon_2) - \epsilon_1\epsilon_2$ (also used with other meanings)	52
$J_{ij}(i, j$ $= x, y, z)$	3×3 tensor	415
\mathbf{j}	(real) current density	24
K	Boltzmann's constant (also used with other meanings)	7, 45
$K_{ij}(i, j$ $= x, y, z)$	3×3 tensor	415

k	$\omega/c = 2\pi f/c$. Propagation constant in a vacuum	29
k_0	predominant value of k in wave packet	258
k_p	value of k at the penetration frequency	465
l_D	Debye length	45
l_x, l_y, l_z	direction cosines of Y , antiparallel to the earth's magnetic field (also used for components of other unit vectors I)	144
I	unit vector in direction of wave normal	36
M	effective refractive index	446
\mathbf{M}	4×4 matrizant	558
\mathbf{M}	3×3 susceptibility matrix of magnetoplasma with elements $M_{ij}(i, j = x, y, z)$	50
\mathcal{M}	$n \cos \alpha$; ray refractive index	111
m	mass of electron	39
m_i	mass of ion of species i	55
N, N_e	concentration of electrons	23, 55
N_m	N at maximum of ionospheric layer	9
N_i	concentration of ions of species i (N alone or with various subscripts is also used with other meanings)	55
\mathcal{N}_0	power per unit solid angle in ray pencil	280
n	refractive index (in general complex)	29, 66
n_o, n_e	n for ordinary and extraordinary wave, respectively	80, 81
\mathbf{n}	refractive index vector (in general complex)	78, 103
n_i	Cartesian components of \mathbf{n} ($i = x, y, z$) ($i = \xi, \eta, \zeta$)	78 103
	(n is also used with other subscripts, and with superscripts, for the refractive index in specified conditions)	
n'	$\partial(fn)/\partial f$; group refractive index (in general complex)	131
$O(x)$	quantity such that $\lim_{x \rightarrow 0} \{O(x)/x\}$ is bounded	
\mathbf{P}	complex electric polarisation in wave with harmonic time dependence	26
P_x, P_y, P_z	Cartesian components of \mathbf{P}	46
P_1, P_2, P_3	contravariant components of \mathbf{P} in complex principal axes	52
P	phase path (usually for oblique incidence)	278
P'	equivalent path (usually for oblique incidence)	278
\mathbf{p}	(real) electric polarisation	24
p	gradient of X in linear model of ionosphere	262, 439

Q	4 × 4 diagonal matrix whose 4 elements are solutions q of Booker quartic	183
Q	average thermal energy per electron in plasma	396
	effective value of q	446
q	solution of Booker quartic equation	143
q_i	($i = 1, 2, 3, 4$) the four values of q	183
$q_0, q_1, \dots, q_m, \dots$	values of q in successive strata (q alone, or with subscripts, is also used with other meanings)	172
R	reflection coefficient	194
R_0, R_1, \dots, R_D	values of R in specified conditions	296, 297
\mathbf{R}	2 × 2 reflection coefficient matrix (also used with subscripts 0, c) (also used with other meanings)	299
\mathcal{R}	2 × 2 reflection matrix for ordinary and extraordinary component waves	573
R_{ij}	($i, j = 1, 2$) elements of \mathbf{R} for linearly polarised fields	298
	($i, j = 1, r$) elements of \mathbf{R}_0 for circularly polarised fields	301
	($i, j = O, E$) elements of \mathbf{R}_e for ordinary and extraordinary waves	310
r	average displacement of an electron	39
r	radius from centre of earth (also used to denote an integer, and with other meanings)	260
S	$\sin \theta$, sine of angle of incidence	146, 342
S_A, S_B, S_C	transition values of $\sin \theta$	156
S_R	square root in Appleton–Lassen formula	88
S_1, S_2	x and y direction cosines of wave normal of incident wave in free space	142
S_{10}, S_{20}	predominant values of S_1, S_2 in wave packet	257, 258
S	4 × 4 matrix of eigen columns of T	162, 183
S_{ij}	($i, j = 1, 2, 3, 4$) elements of S	190, 192
\mathcal{S}	energy per unit volume in a plasma	33
$\mathcal{S}_E, \mathcal{S}_M$	electric and magnetic parts, respectively, of \mathcal{S}	32
$\mathcal{S}_A(\zeta), \mathcal{S}_B(\zeta)$	solutions of Stokes equation	444
s	distance along ray path	409
	kz ; height measured in units of $\lambda/2\pi$ (also used with other meanings)	516

\mathbf{s}_i	($i = 1, 2, 3, 4$) eigen column of \mathbf{T}	183
T	temperature	7, 58
	time of travel of wave along a ray	378
	transmission coefficient	297
	(also used with other meanings)	
$T_0, T_1, \dots,$ T_m, \dots	values of transmission coefficient T in discrete strata	172, 173
\mathbf{T}	2×2 transmission coefficient matrix	299
$T_{ij} (i, j = 1, 2)$	elements of \mathbf{T}	299
\mathbf{T}	4×4 matrix in basic equations (7.80)	182
$T_{ij} (i, j = 1,$ $2, 3, 4)$	elements of \mathbf{T}	182
\mathbf{T}	\mathbf{T}^T is transpose of any matrix \mathbf{T}	183
t	time	25
	(also used with other meanings)	
\mathbf{U}	2×2 unitary matrix	300
	(also used with other meanings)	
\mathbf{U}	4×4 transforming matrix	486
U	$1 - iZ$ for electrons	44
U_e, U_i	U for electrons and for ions of species i respectively	55
\mathcal{U}	3×3 unitary matrix	51
	group velocity vector	130
\mathcal{U}	magnitude of group velocity	130
\mathcal{U}_z	component of group velocity parallel to wave normal	129, 130
u	$\frac{1}{2}mv^2/KT$ for electrons	58
	(also used with other meanings)	
\mathbf{V}	ordered part of electron's velocity	10
	ray velocity vector	111
$V_\xi, V_\eta, V_\zeta;$ V_x, V_y, V_z	components of ray velocity \mathbf{V}	112, 410
V	ray velocity c/\mathcal{M} ; magnitude of \mathbf{V}	111
	volume	33
	(also used with other meanings)	
\mathbf{V}	4×4 transforming matrix	526
v	speed of an electron	11, 57
	wave velocity	111
	(also used with other meanings)	
W	bilinear concomitant (stratified medium)	184, 185
	(also used with other meanings)	

W	bilinear concomitant vector	427
X	$Ne^2/(\epsilon_0 m \omega^2)$	39
X_e, X_i	X for electrons and ions of species i respectively	54, 55
X_∞	value of X where one value of n or q is infinite	79, 149
x	Cartesian coordinate	28, 29, 141
Y	$eB/(m\omega)$ (antiparallel to earth's magnetic induction B)	49
Y	magnitude of Y	49
Y_e, Y_i, Y_e, Y_i	Y, Y for electrons and ions of species i respectively	54, 55
y	Cartesian coordinate	28, 29, 141
Z	v/ω for electrons	44
Z_e, Z_i	Z for electrons and for ions of species i respectively	55
Z_t	transition value of Z	71
Z_0	$(\mu_0/\epsilon_0)^{1/2}$; characteristic impedance of a vacuum	31
z	Cartesian coordinate (height in stratified medium)	28, 29, 141
z_0	level of reflection; (complex) value of height z where $n = 0$ or $q = 0$; 'true' height of reflection	195, 262, 329
z_p, z_q	(complex) values of height z at coupling points	387, 520–2
z_m	height of maximum N in ionospheric layer	9, 332
α	coefficient of q^4 in Booker quartic	145
	coefficient of z in exponent, for exponential height distribution	331
	angle between wave normal and ray	110
	$\frac{1}{2}(q_1 + q_2)$	486
	(also used with other meanings)	
β	coefficient of q^3 in Booker quartic	145
	angle between ray and Y	110
	$\frac{1}{2}(q_1 - q_2)$	486
	(also used with other meanings)	
Γ	4×4 coupling matrix	188
Γ_{ij}	$(i, j = 1, 2, 3, 4)$ elements of Γ	482
γ	coefficient of q^2 in Booker quartic	145
Δ	discriminant of Booker quartic equation	152
	(also used with other meanings)	
δ	coefficient of q in Booker quartic	145
	(also used to indicate arbitrarily small quantity)	
ϵ	3×3 electric relative permittivity tensor of magnetoplasma	33
ϵ_{ij}	$(i, j = x, y, z)$ elements of ϵ	33

ε_{ijk}	($i, j = x, y, z$) isotropic tensor or rank three	415
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	diagonal elements of ε in complex principal axes	52
$\varepsilon_a, \varepsilon_b, \varepsilon_c$	real elements of diagonalised ε for biaxial crystal	113
ε_0	permittivity of a vacuum	22
ε	relative electric permittivity; n^2 for isotropic plasma	29
	coefficient of q^0 in Booker quartic	145
ζ	Cartesian coordinate, parallel to Y	103
	scaled value of height z (the method of scaling depends on the problem)	9, 200 332, 440, 457
	(also used with other meanings)	
η	Cartesian coordinate, perpendicular to wave normal and to earth's magnetic field	103
η_1, η_2, η_3	parameters in Epstein theory	470
Θ	angle between wave normal and the vector Y	68
Θ_r	value of Θ where one refractive index is infinite (resonance)	105
Θ_s	value of Θ at Storey cone	106
Θ_t	transition value of Θ	90
	(Θ alone or with subscripts is also used with other meanings)	
θ	angle of incidence; angle between wave normal and vertical in reference free space	142, 342
	(also used with subscripts to indicate specified conditions, and with other meanings)	
κ	$k\mathbf{n}$, $\omega\mathbf{n}/c$ wave propagation vector, magnitude κ	130
κ_i	($i = x, y, z$ or ξ, η, ζ) components of κ	130
λ	wavelength in free space c/f	289
λ_e	mean free path of electron	11, 58
$\lambda_1, \lambda_2, \lambda_3$	Stokes multipliers	212
μ	real part of refractive index	34
μ_0	magnetic permeability of a vacuum	22
$\nu, \nu(v)$	average collision frequency for electron with speed v	10
ν_{eff}	effective collision frequency for electrons	10
ν	short for ν_{eff}	43
ν_{av}	average collision frequency for electrons	58
ξ	Cartesian coordinate (coplanar with wave normal and earth's magnetic field)	103
	scaled value of height z	447

Π	real Poynting vector	34
Π_{av}	time average of Poynting vector	34
Π	magnitude of Π_{av}	415
Π_i	($i = x, y, z$) components of Π_{av}	415
ρ	wave polarisation	68
ρ, ρ_0, ρ_F	electric charge density in plasma	48
ρ_O, ρ_E	polarisation ρ of ordinary and extraordinary waves respectively	71
ρ, ρ_0	density of air	7
σ	scaling factor in Epstein distributions (also used with other meanings)	470
τ	variable that increases as a ray is traversed	403
Φ	azimuth angle of wave normal	103
ϕ	azimuth angle of spherical polar coordinates generalised phase of wave in isotropic ionosphere (also used with other meanings)	408 170
φ	(complex) phase of wave (also used with other meanings)	109
χ	minus imaginary part of refractive index n zenith angle of sun's radiation (also used with other meanings)	34 7
ψ	coupling parameter angle between wave normal and earth's radius angle between wave normal and vertical (also used with other meanings)	484, 510 260 264
Ω	angular modulation frequency	396
ω	$2\pi f$; angular frequency	26
$\omega_c, \omega_{c1}, \omega_{c2}$	angular cut-off frequencies where one value of refractive index is zero	118, 126
ω_H	$2\pi f_H$; angular gyro-frequency for electrons	46
ω_N	$2\pi f_N$; angular plasma frequency	39
ω_t	transition value of collision frequency	71
ω_U	$2\pi f_U$; angular upper hybrid resonance frequency	124
ω_∞	angular frequency where one refractive index is infinite	127
ω	with one or two subscripts: transition values of ω or ω_N	124