

# MULTI-HOP, HIGH-FREQUENCY RADIO PROPAGATION NEAR JAPAN

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## Abstract

We attempted to model high frequency radio-waves and their interactions with the ionosphere, turbulent and calm oceans, and smooth and rugged terrain. To do this, we used a simple and isotropic model for the ionosphere, modifying the Chapman Law to make it time-dependent, i.e. dependent on the time of day and the time of year. To model the ocean, we had a simple model in which ocean wave amplitude and wavelength are varied. By doing this we are able to change its index of refraction.

Using this we found...

## 1 Introduction

High frequency (HF) radio-waves (defined as 3–30 MHz) can travel through the atmosphere by multiple reflections off of the Earth’s ionosphere and the its surface unless the frequencies are larger than the maximum usable signal (MUF), then they pass through the ionosphere and are lost.<sup>[1]</sup>

Empirically, radio-waves reflect off of the ocean (or terrain) differently depending on whether the ocean is turbulent (rugged) or calm (smooth), impacting the distance the signal can faithfully.<sup>[1]</sup> We chose Tokyo, Japan as a location of study because it is an ideal location to study radio-wave interactions as it is an island and is “mostly rugged and mountainous.”<sup>[6]</sup>

The goal of our model was to find the following: (1) find the number of “skips” a HF signal could have with ocean before losing signal integrity, (2) the same but with rugged terrain rather than ocean, and (3) how a boat in the ocean could receive signals on turbulent waters.

## 2 The Model

### 2.1 HF Signals

As radio-waves propagate over a certain distance, we expect them to pick up some noise as they go, degrading

**Figure 1:** A test .gif file. Click image to see animation. (You need to use Adobe Acrobat Reader to view this .gif file. If you do not have it, please visit <https://get.adobe.com/reader/> to download it. It is worth it.)

the integrity of the information they are trying to transmit. To simulate this, we additive white Gaussian noise (AWGN).<sup>[9,7]</sup>

### 2.2 The Ionosphere

The ionosphere consists of roughly three layers that lie between 75–1000 km above the Earth’s surface: (1) the F-region, (2) the E-region, and (3) the D-region; each of these regions has charge-density dependent on the time of year, the number of sunspots present, the time of day, and the movement of the charged particles. The ionosphere interacts heavily with radio waves, mainly through the interaction of these free electrons.<sup>[3]</sup>

To model the ionosphere, we modify the Chapman Law,<sup>[5,4,3,2]</sup> which describes the electron density, and make it dependent on the time of day and year. We found a simple, analytic model for the ionosphere,  $\bar{N}(z, t)$ , which is a function of altitude and time which

was given by

$$\bar{N}(z, t) = T(t)N(z) \quad (2.1)$$

where  $N(z) = N_0 \exp \left[ \frac{1}{2} \left( 1 - \frac{z-z_0}{\kappa} - \sec(\alpha) e^{-(z-z_0)/\kappa} \right) \right]$  with arbitrary constants  $z_0$  and  $\kappa$ . The time dependence is given by  $T(t) = (1 + d(t) + s(t))$  and  $d(t) \in [0, 1]$  is the daytime contribution of the ionosphere and  $s(t) \in [0, 4]$  is the seasonal contribution.

To understand how to radio-waves will interact with the ionosphere, we must found an expression for the refractive index,  $n$ , of light through the plasma. For an isotropic ionosphere where electrons do not collide and for purely transverse waves, we found  $n^2 = 1 - \bar{N}e^2/\epsilon_0 m \omega^2$ , which is independent in of the direction of the HF signal but is dependent on its frequency and some fundamental constants like permitivity, electron mass and charge.

### 2.3 The Ocean

Sea water essentially reflects HF signals perfectly.<sup>[8]</sup> However, turbulent and choppy waves will disrupt the signal. Using a simple square wave we were able to show that a quasi-index of refraction for the ocean,  $m$  is given by  $m = n(1 - \sqrt{R})/(1 + \sqrt{R})$  where  $R = \cos^2(\phi/2)$  and  $\phi$  is the phase difference created by a HF signal with wavelength  $\lambda$ . We derived

$$\phi = \frac{2}{\lambda} \sqrt{A^2 + (\lambda_s/4)^2}. \quad (2.2)$$

where  $A$  and  $\lambda_s$  are the amplitude and wavelengths of the ocean waves, respectively.

## 3 Results

### 3.1 Problem I: A Turbulent Ocean

### 3.2 Problem II: The Japanese Alps

### 3.3 Problem III: A Message to a Boat

## 4 Conclusion

## Acknowledgments

We would like to thank our professors for lending us resources and pointing us in the direction of others. We would like to thank our College for financial support allowing us to enter this competition. Without it, we would not have been able to have this much fun.

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