Multi-hop, High-Frequency Radio Propagation Near South America

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Abstract

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MCM Requirements

Hi team, here is what we *need* to have in our report (according to the MCM overlords):

- Restatement and clarification of the problem
- Explain assumptions and rationale/justification
- Include your model design and justification
- Describe model testing and sensitivity analysis
- Discuss the strengths and weaknesses

They also claim to judge the quality of our writing. So remember our good friend Williams.

1 Motivation

2 Introduction

3 Model

3.1 Radiowaves

3.2 The Ionosphere

The ionosphere consists of roughly three layers that lie between 75–1000 km above the Earth's surface: (1) the F-region, (2) the E-region, and (3) the D-region; each of these regions has charge-density dependent on the time of year, the sunspots number of sunspots present, the time of day, and the movement of the charged particles[1](Figure 1). X-rays, ultraviolet light, ejected plasma, and other high energy particles that are released by the sun interact with the atoms in the atmosphere and strip them of electrons. [1] The ionosphere interacts heavily with radio waves, mainly through the interaction of these free electrons. [3]

Early experiments demonstrated that the electrons in the ionosphere are arranged in approximately horizontal layers, meaning that the number density is a function of height above the Earth's surface. Presumably, these cations are stripped atoms or molecules. Because the ratio between the mass of a proton (the smallest positive charge we could have in the atmosphere) and an electron is on the order of 2×10^3, we would expect there must be approximately $5 \times 10{-}4$ more cations than electrons.[2] If this were the case, the ionosphere would be unstable, due to the large repulsive forces of this unbalanced positive charge. However, due to the ionosphere's empirically determined and stable layers, the ionosphere must be nearly electrically neutral, i.e. there must be an equal number of positive and negative charges per unit volume. [3] Consequently, we assume that the radio-waves only interact with the free electrons in the ionosphere.

[1] The study of which has the impressive sounding name of magnetohydrodynamics.

[2] This is because the electric field of the propagating radio-waves will have 5×10^{-4} of the effect on a proton than an electron because $F \propto m$.

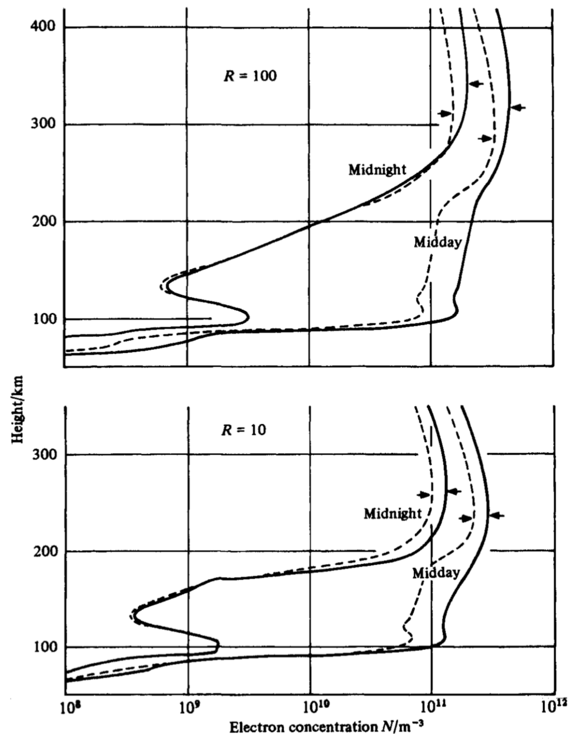

Figure 1: The charge density as a function of height in the ionosphere. It also shows how the ionosphere changes at midnight and noon. Dashed lines are for January while solid lines are for June. Also given is the number of sunspots, R, at the time of data collection. Figure is taken from K.G. Budden.[2]

As stated above, at a first approximation, we expect that the electron density, N, is only a function of the eight above the earth's surface height, z. More compactly, $N = N(z)$.

3.2.1 Electron Density, $N(z)$

Before we can calculate how radio-waves are reflected off of the ionosphere, we must model the ionosphere's electron density. In the early 1930's, a simple model for the electron density as a function was produced called the Chapman Law.[4,5]

If we assume the Earth is constant in composition and temperature (as well as neglect the curvature of the earth[3]), then the density of the atmosphere, $\rho = \rho(z)$, is given by

$$\rho(z) = \rho_0 e^{-Mgz/RT} = \rho_0 e^{-z/\kappa} \tag{3.1}$$

where $\kappa \equiv RT/Mg$, $\rho_0 \equiv \rho(0)$, M is the average molecular weight, g is the gravitational acceleration (assumed constant), R is the universal gas constant,[4] and T is the temperature.

[3]See `https://www.tfes.org/` for more information.

[4]Since all of us are "physicists," we feel the need to note the definition of R: $R \equiv k_B N_A$ where k_B is Boltzmann's constant and N_A is Avogadro's number.

The result of (3.1) comes by balancing the forces acting on infinitesimal layers of atmosphere that, on a whole, is taken to be static and solving the barometric equation. [7]

Next, we must find the intensity of the sun that passes through the earth's atmosphere. The intensity of the sun—not surprisingly—decreases as it goes through the atmosphere. [3] Since, the sun enters at an angle from the zenith, α. By drawing a simple diagram,[5] we can see that the area normal to the incident radiation of the sun will be a factor of secant too large than the area that the ground actually receives, i.e. $dA = dA_\perp \cos\alpha$. Since intensity is inversely proportional to dA, we will have the relationship of

$$dI = I\sigma\rho \sec(\alpha)\, dz. \tag{3.2}$$

where σ is a mass coefficient of absorption of the atmosphere. If we plug (3.1) into (3.2) and integrate, we get

$$I(z) = I_0 \exp\left[-\sigma\kappa\rho_0 \sec(\alpha) e^{-z/\kappa}\right] = I_0 \exp\left[-\sec(\alpha) e^{-(z-z_0)/\kappa}\right] \tag{3.3}$$

if we (cleverly) define $z_0 \equiv \kappa \ln(\sigma\kappa\rho_0)$. The rate of absorption of energy at height z is $\cos(\alpha)(dI/dz)$. If we assume that the rate of production of electrons, q, is proportional to this, then we get

$$q(z) = q_0 \exp\left[1 - \frac{z-z_0}{\kappa} - \sec(\alpha) e^{-(z-z_0)/\kappa}\right]. \tag{3.4}$$

Empirically—at least in the D and E-layers of the ionosphere[6]—the variation of charge density is given by [3]

$$\frac{dN}{dt} = q - aN^2 \tag{3.5}$$

where a is a "recombination constant." In the steady-state solution, where $dN/dt \approx 0$, we have $N = \sqrt{q/a}$ or

$$N(z) = N_0 \exp\left[\frac{1}{2}\left(1 - \frac{z-z_0}{\kappa} - \sec(\alpha) e^{-(z-z_0)/\kappa}\right)\right] \tag{3.6}$$

where N_0 is a constant we can gather from experimental data.

3.3 The Ocean

4 Results

5 Conclusion

Acknowledgments

[5]Remember, we have a flat Earth; this is a subtle but important point in this derivation.

[6]In the F-layer, this rate law may be equal to $dN/dt = q - bN$

References

[1] Tracking Solar Flares. URL `http://solar-center.stanford.edu/SID/activities/ionosphere.html`.

[2] Kenneth George Budden. *The propagation of radio waves: the theory of radio waves of low power in the ionosphere and magnetosphere*. Cambridge University Press, 1988.

[3] KG Budden. *Radio Waves in the Ionospere: The Mathematical Theory of the Reflection of Radio Waves from Stratified Ionised Layers*. Cambridge University Press, 1961.

[4] S Chapman. The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth part ii. grazing incidence. *Proceedings of the physical society*, 43(5):483, 1931.

[5] Sydney Chapman. The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth. *Proceedings of the Physical Society*, 43 (1):26, 1931.

[6] John David Jackson. *Classical electrodynamics*. Hamilton Printing Company, 3rd edition, 1999.

[7] Daniel V Schroeder. An introduction to thermal physics, 1999.

[8] John S Townsend. *A modern approach to quantum mechanics*. University Science Books, 2000.