Crary and Ewing[2] have shown that the buoyant response of the vertical seismograph to changes in the atmospheric pressure can be an important source of long period seismic noise. It is also known that small variations in temperature can cause considerable long period noise. Extreme care was taken during the installation of the seismic system to reduce these effects to a minimum. The surface seismometers are heavily insulated and the mine seismometers are in a thermally stable environment. The vertical seismometers were sealed in rigid steel cases with time constants of several hours. These seismometers were placed in sealed vaults with time constants ranging from 10 to 26 h. Tests showed that external pressure variations of about 100 μbar at a period of 200 s produced pressure variations within the vault which were of the same order as the system noise of the microbarograph ($\sim$0.02 μbar).

The mine, which has only one entrance, acts as a low pass filter to surface pressure variations. The time constant of the mine is greater than 4 min. Nevertheless seismic disturbances which were practically identical were recorded both at the surface and in the mine during the passage of the acoustic wave. The coherences between the base station microbarograph and the surface and mine vertical seismographs, which are shown in Fig. 3, indicate that at periods from 20 to 100 s the seismic disturbance is closely related to the acoustic wave. In view of the precautions taken during the installation of the seismographs, we believe that the disturbance recorded by the seismographs represents true Earth motion.

It has been shown[3-5] that although the energy transfer between the atmosphere and the Earth is very slight a disturbance in one medium can cause motions in the other. Bolt[6] observed a dispersed acoustic signal from the Great Alaskan Earthquake some 2 h 40 min after the earthquake. The signal resembled the acoustic waves generated by nuclear explosions.

Our results are in general agreement with unpublished theoretical studies by G. G. S. of the response of the Earth to slowly moving plane pressure waves. These studies show that in a half-space of reasonably competent rock, a pressure wave moving at 330 ms^{-1} would generate a vertical displacement at the surface of the Earth of 10 to 15 nm per μbar in the period range from 20 to 100 s. These studies also show that the amplitudes of the seismic disturbance recorded at a depth of 183 m should be within 5 per cent of the amplitudes recorded at the surface.

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Received August 17, 1970.

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Determination of Ocean Wave Spectra from Doppler Radio Return from the Sea Surface

RADIO backscatter from the sea surface has attracted considerable attention because of its potentiality for remote measurement of sea state. Ward[1], for example, has shown that with ionospheric propagation and time-gating techniques relatively small patches of the ocean ($\sim$100 km^2) can be sampled successively at ranges extending up to several thousand kilometres. The ocean wave origin of the sea echo has been established clearly[1-6], but the success of the method rests ultimately on the ability to recover the ocean wave spectrum —or at least significant parameters of the spectrum—from the properties of the returned radio signal.

Following Crombie[2], several workers[3-6] have shown that the Doppler spectrum of sea echo contains a dominant line which arises from selective Bragg scattering by particular components of the surface wave spectrum. Unfortunately, at the usual h.f. frequencies the Bragg waves are short compared with the dominant waves of the sea and therefore lie in Phillips's[7] saturation range of the spectrum, which is essentially independent of the local wind–wave conditions. The Doppler spectra, however, contain additional information in the form of a detailed background structure. I wish to show in this communication that, in contrast to the Bragg line, the continuous structure is directly related to the wind sea. It is identical, to a good approximation, with the two sided image of the wave frequency spectrum, folded at the Bragg line as virtual frequency origin.

According to lowest order scattering theory[8-10], an incident electromagnetic wave with frequency ω_i and horizontal wavenumber $\mathbf{k}_i$ is scattered by a surface gravity wave of frequency ω_g and wavenumber $\mathbf{k}_g$ into two waves with frequencies and horizontal wavenumbers given by the Bragg (resonant interaction) conditions

$$\omega_s = \omega_i \pm \omega_g, \quad \mathbf{k}_s = \mathbf{k}_i \pm \mathbf{k}_g \tag{1}$$

The vertical wavenumber component, and thus the scattering angle, follow from the known phase velocity c. In this case, the propagation is very nearly horizontal and the scattering is backward, so that $\mathbf{k}_s = -\mathbf{k}_i$, $\mathbf{k}_g = \pm 2\mathbf{k}_i$ and

$$\omega_g = \sqrt{gk_g} \sim \sqrt{\frac{2g\omega_i}{c}}$$

Thus the horizontal radio return arises from interactions with two gravity wave components, with wavelengths half that of the incident waves, propagating radially towards and away from the radio source. For a continuous surface wave spectrum $F(\mathbf{k})$, the Doppler backscatter spectrum consists of two lines with energies

$$E_{\pm} = \alpha T_{(1)} F(\pm 2\mathbf{k}_i) \tag{2}$$

at the frequencies $\omega_d = \omega_s - \omega_i = \mp \omega_g$, where $T_{(1)}$ is proportional to the backscatter cross-section and α is a transmission factor. In most cases, the gravity wave spectrum contains only one of the two Bragg components, so that the Doppler spectrum reduces to a single line.

The shifts of the dominant lines observed in the Doppler spectra generally agree quite well with the theoretical values. The lowest order theory, however, fails to explain the existence or structure of the background continuum.

It has been suggested that the continuum is due to the finite extent of the scattering wave groups[2] or to higher interference orders[1], by analogy with the scattering by a refraction grating. It should be noted, however, that both these effects are related only to the representation of the scattering field. In lowest order scattering theory, they are automatically accounted for by correct spectral representation of the surface wave field. For example, the higher interference orders of a diffraction grating arise because the Fourier representation of a typical square tooth diffraction grating contains not only the fundamental wavenumber $\mathbf{k}_0$, but all higher harmonics $n\mathbf{k}_0$. The scattering conditions (1) and the relation (2) still apply for each interference order (because the scattering spectrum is a Dirac comb, (2) implies that the return signal in this case is zero unless $\mathbf{k}_i$ is a half multiple of $\mathbf{k}_0$). For any scattering spectrum, the wavenumber of the scattering wave component is uniquely determined—except for the sign—by the condition (1), and the Doppler shift is given by the frequency of that component.

A continuous Doppler spectrum arises, however, if the interaction analysis is extended to quadratic order in the amplitude of the scattering field. The interaction of an incident electromagnetic wave with two gravity wave components 1 and 2 yields four scattered waves which satisfy the second order Bragg conditions

$$\omega_s = \omega_i + s_1\omega_1 + s_2\omega_2,\ \mathbf{k}_s = \mathbf{k}_i + s_1\mathbf{k}_1 + s_2\mathbf{k}_2\ (s_1, s_2 = \pm) \quad (3)$$

The corresponding second order Doppler power spectrum $E_{(2)}(\omega_d)$ $(\omega_d = \omega_s - \omega_i)$ is obtained by integrating over all interactions satisfying (3) for fixed ω_d

$$E_2(\omega_d) = \alpha \sum_{s_1, s_2} \int T_{(2)} F(s_1\mathbf{k}_1) F(s_2\mathbf{k}_2) \delta(\omega_d - s_1\omega_1 - s_2\omega_2) d\mathbf{k}_1$$

where $T_{(2)}$ is a transfer function determined by the wave–wave coupling coefficients. These include both hydrodynamic[10] and electromagnetic interactions[8]. If the surface wave spectrum is peaked fairly sharply at a wavenumber which is small compared with the wavenumber $\mathbf{k}_i$ of the incident radiation (as is the case for h.f. waves), the integral can be shown to reduce to the simple form

$$E_{(2)}(\omega_d) = \alpha T_{(2)} F(2s\mathbf{k}_i) E_g(\omega_d + s\omega_g) \quad (4)$$

where $E_g(\omega)$ is the two sided frequency spectrum of the surface waves, $T_{(2)}$ is a slowly varying transfer function and $s = \pm$ is the sign occurring in the lowest order Bragg-line interaction. (If both Bragg lines are present, equation (4) should be replaced by the sum over s.)

The transmission coefficient α can be eliminated by dividing by the energy of the Bragg line (2)

$$E_g(\omega_d + s\omega_g) = \frac{T_{(1)}}{T_{(2)}} \frac{E_2(\omega_d)}{E_s} \quad (5)$$

Because $T_{(1)}/T_{(2)}$ can be determined from theory, equation (5) yields the wave spectrum free from calibration and transmission factors.

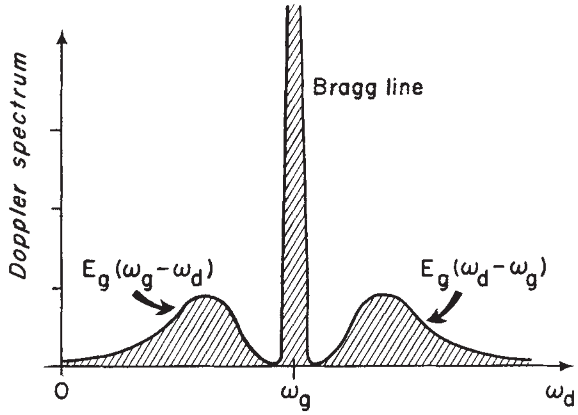

Fig. 1 Theoretical Doppler spectrum for h.f. backscatter from the sea surface. Second order interactions yield a two sided image of the surface wave spectrum E_g on either side of the first order Bragg line at ω_g.

Physically, the continuum (4) is due to the modulation of the lowest order scattering at short, low amplitude waves $\mathbf{k}_1 \approx 2s_1\mathbf{k}_i$ through long waves $\mathbf{k}_2$ of relatively high amplitude A_2. This leads to a splitting of the lowest order Bragg frequency $\omega_i + s_1\omega_1$ into two frequencies $\omega_i + s_1\omega_1 \pm \omega_2$. Summing over all long waves, one obtains a two sided image of the long wave frequency spectrum on either side of the Bragg line (Fig. 1).

The coupling coefficient of the second order interaction is of order $k_1 A_2 \tan\theta$, where θ is the depression angle of the incident wave. This is normally sufficiently large to yield appreciable sideband power levels. For example, in Ward's measurements[1], $k_1 \sim 4\pi/13m$, so that a long wave amplitude $A_2 \sim 2m$ and a depression angle $\theta = 10°$ yields $k_1 A_2 \approx 1/3$. For still shorter incident waves, the perturbation parameter becomes so large that the wave–wave interaction expansion breaks down and other models have to be used (refs. 11 and 12 and unpublished work of K. H. and M. Schieler).

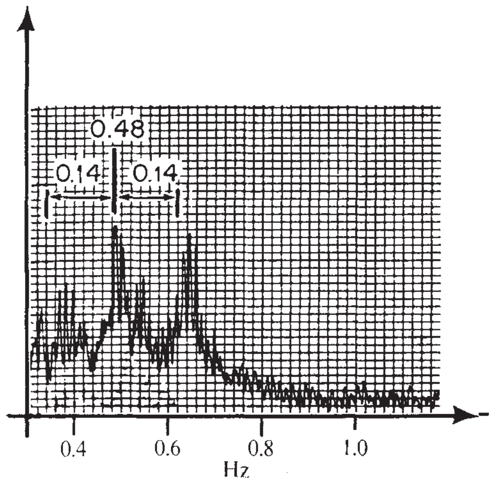

Fig. 2 Observed Doppler spectrum, from Ward[1]. The theoretical Bragg frequency is 0.48 Hz. The side lobes are consistent with an ocean wave spectrum peaked at 0.14 Hz.

Fig. 2 shows an example of a Doppler spectrum obtained by Ward[1]. The recording duration of 1 min was too short for statistical resolution of the ocean wave spectrum, so that a quantitative test of the relation (5) is not possible. The distribution is, however, suggestive of a two sided spectrum superimposed on a Bragg line at the theoretically predicted frequency of 0.48 Hz. The difference frequency between the Bragg line and the secondary maxima is of the order 0.14 Hz, which corresponds to a local wind velocity for a fully developed Pierson–Moskowitz[13] spectrum of about 10 ms^{-1}. The other spectra shown by Ward[1] are similar in structure, although the symmetry is not always so well pronounced. Because of the small number of degrees of freedom, this is not inconsistent with the present interpretation. Further measurements with longer sampling times would be very desirable to decide whether Doppler power spectral analysis of h.f. radio sea return can be used for quantitative remote measurement of ocean wave spectra.

This work was supported in part by the US Office of Naval Research.

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Received July 13; revised September 4, 1970.

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