# Answers to problems

**1.1.** (a) 70 s. (b) 4 years.

**2.2.** (a) Circular. Area $\pi$. Normal 0, 0, 1.
(b) Ellipse. Area $\pi 2\sqrt{2}$, Axis ratio $2^{-\frac{1}{2}}$, Normal $\propto (1, 1, 0)$.
(c) Ellipse. Area $12\pi$, Axis ratio $\frac{1}{2}$, Normal $\propto (-1, -1, 1)$.
(d) Linear. $|E| = \frac{1}{2}\sqrt{33}$. Direction $\propto (4, 4, 1)$.
(e) Circular. Area $2\pi$, Normal 0, 0, 1.
(f) Ellipse. Area $\pi ab$, Axis ratio $a/b$, Normal 0, 1, 0.

**3.1.** Ordered: $0.5\,\mathrm{km\,s^{-1}}$. Thermal $150\,\mathrm{km\,s^{-1}}$

**3.8.** (a) $\sim \omega_N^{-1}$. (b) $2 \times 10^{15}$ s for temperature 800 K.

**4.1.** $\left\{\dfrac{1 + |R^2| - |1 + R^2|}{1 + |R^2| + |1 + R^2|}\right\}^{\frac{1}{2}}, \quad \frac{1}{2}|E_x^2|(1 + |R^2| + |1 + R^2|).$

**4.2.** $|(\rho - R)/(1 - R\rho)|$.

**4.4.** Smallest 0, 45°. Greatest $(1 - a)/(1 + a)$, 90°.

**4.5.** (a) Elliptical. Axis ratio $\frac{1}{3}$. (b) Partially circularly polarised. Energy flux ratio (circular component)/(unpolarised component) $= \frac{3}{2}$.

**5.2.** (a) $V = c$.

**5.4.** $Gn^2 + J = 0$. See (3.51) for meaning of $G$ and $J$. If this is true for any $\Theta$, it is true for all $\Theta$.

**5.5.** $\tan\Theta = \{1 \pm (1 - 8\tan^2\beta)^{\frac{1}{2}}\}/2\tan\beta$. See fig. 5.19 for an example.

**6.3.** $$q^2 = C^2 - \frac{X(1 - X)}{1 - X - \frac{1}{2}Y^2S^2 \pm Y\{(1 - X)(C^2 - X) + \frac{1}{4}Y^2S^4\}^{\frac{1}{2}}}.$$

**7.1.** $\mathrm{d}^2y/\mathrm{d}x^2 + y\omega^2 m/T = 0$.

$$y \sim m^{-\frac{1}{4}} \exp\left(\pm \mathrm{i}\omega T^{-\frac{1}{2}} \int^x m^{\frac{1}{2}}\,\mathrm{d}x\right).$$

Amplitude proportional to $m^{-\frac{1}{4}}$.

**7.2.** ($M$ is the mean molecular mass, $K$ is Boltzmann's constant and $\gamma$ is the ratio of

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the specific heats)

$$\frac{d^2p}{dz^2}-\frac{T}{P}\frac{d}{dz}\left(\frac{P}{T}\right)\frac{dp}{dz}+\frac{\omega^2M}{\gamma KT}p=0,$$

$$\frac{d^2\zeta}{dz^2}+\frac{1}{P}\frac{dP}{dz}\frac{d\zeta}{dz}+\frac{\omega^2M}{\gamma KT}\zeta=0.$$

(a) $\zeta\sim T^{\frac{1}{4}}\exp\left\{\pm i\omega\left(\frac{M}{\gamma K}\right)^{\frac{1}{2}}\int^z T^{-\frac{1}{2}}\,dz\right\},$

$p\sim\mp i\omega\gamma P\left(\frac{M}{\gamma KT}\right)^{\frac{1}{2}}\zeta.$

(b) $p\sim P^{\frac{1}{2}}\exp\left\{\pm i\omega\left(\frac{M}{\gamma KT}\right)^{\frac{1}{2}}z\right\},$

$\zeta\sim\frac{i}{\omega P}\left(\frac{KT}{\gamma M}\right)^{\frac{1}{2}}p,$

**7.3.** $a\propto L^{\frac{1}{4}}$, $T\propto L^{\frac{1}{2}}$. $E\propto L-{\frac{1}{4}}$.

**7.5.** 0.25 km approx.

**8.1.** (a) $a_0a_1$, (b) $a_0\mathrm{Ai}'(0)$, (c) $-a_1\mathrm{Ai}(0)$.

**8.3.** (a) $\pi\{\mathrm{Bi}(0)\mathrm{Ai}(z)-\mathrm{Ai}(0)\mathrm{Bi}(z)\}$.
(b) $\pi\{\mathrm{Bi}'(0)\,\mathrm{Ai}(z)-\mathrm{Ai}'(0)\,\mathrm{Bi}(z)\}$.
(c) $\{\mathrm{Bi}(-1)\,\mathrm{Ai}(z)-\mathrm{Ai}(-1)\,\mathrm{Bi}(z)\}/\{\mathrm{Bi}(-1)\,\mathrm{Ai}(0)-\mathrm{Ai}(-1)\,\mathrm{Bi}(0)\}$.
(d) No solution; homogeneous boundary conditions.
(e) $\mathrm{Ai}(z)/\mathrm{Ai}(0)$.
(f) As for (d)
(g) Any multiple of $\mathrm{Ai}(z)$.
(h) $\mathrm{Ai}(z)/\mathrm{Ai}'(-a)$.
(i) No solution; conditions incompatible

**8.4.** (a) $a$ is smallest $x$ that makes $\mathrm{Ai}(-x)=0$. $y=\mathrm{Ai}(z-a)$. Homogeneous.
(b) $a$ is any solution of $\mathrm{Ai}(-a)/\mathrm{Bi}(-a)=\mathrm{Ai}(1-a)/\mathrm{Bi}(1-a)$. $y=\mathrm{Bi}(-a)\mathrm{Ai}(z-a)-\mathrm{Ai}(-a)\mathrm{Bi}(z-a)$. Homogeneous.
(c) $a$ can have any value provided $\mathrm{Ai}(-a)\neq 0$. $y=\mathrm{Ai}(z-a)/\mathrm{Ai}(-a)$. Inhomogeneous.
(d) $a$ is solution of $\mathrm{Ai}'(-a)=0$. $y=\mathrm{Ai}(z-a)$. Homogeneous.
(e) $a$ is solution of $\mathrm{Ai}'(-a)/\mathrm{Bi}'(-a)=\mathrm{Ai}(1-a)/\mathrm{Bi}(1-a)$. $y=\mathrm{Bi}(1-a)\mathrm{Ai}(z-a)-\mathrm{Ai}(1-a)\mathrm{Bi}(z-a)$. Homogeneous
(f) $a$ can have any value provided denominator of $y$ is not zero. $y=[\mathrm{Bi}(1-a)\mathrm{Ai}(z-a)-\mathrm{Ai}(1-a)\mathrm{Bi}(z-a)]/$
$[\mathrm{Bi}(1-a)\mathrm{Ai}'(-a)-\mathrm{Ai}(1-a)\,\mathrm{Bi}'(-a)]$. Inhomogeneous.

**8.5.** $\frac{d^2u}{dz^2}-\frac{1}{z}\frac{dz}{dz}-zu=0.$

**8.6.** (a) $y\sim z^{-\frac{1}{2}}\exp(\pm\frac{1}{4}iz^2)$.

(b) $\arg z = \pm\frac{1}{4}\pi,\ \pm\frac{3}{4}\pi$.
(c) $\arg z = 0,\ \pi,\ \pm\frac{1}{2}\pi$.
(d) $\mathrm{i}\sqrt{2}$ on all four Stokes lines.

**9.1.** $(2\pi/a)^{\frac{1}{2}}\left(1 - \dfrac{1}{8a} + \dfrac{9}{128a^2} - \cdots\right)$.

$(2\pi/a)^{\frac{1}{2}}\left(1 - \dfrac{5}{8a} + \dfrac{129}{128a^2} - \cdots\right)$.

In each case method (a) gives first term only.

**9.2.** $(\pi/a)^{\frac{1}{2}}\left(1 - \dfrac{1}{2a} + \dfrac{3}{4a^2} - \dfrac{3.5}{8a^3} + \cdots\right)$.

**11.1.** For reflected wave $E_y/\mathscr{H}_y = R_{21}/R_{11} = R_{22}/R_{12}$. Zero reflection when incident wave has $E_y/\mathscr{H}_y = R_{11}/R_{12} = R_{21}/R_{22}$.

**11.2.** $\boldsymbol{R}_0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

**11.3.** $(R_{11} - R_{22})^2 = -4R_{12}R_{21}$.

**11.4.** Incident wave $E_x = 12$, $E_y = 0$. Reflected wave circular $E_y = -\mathrm{i}E_x = 5$.

**11.5.** Electric fields of incident and reflected waves have same direction. Amplitude ratio $\frac{1}{3}$.

**11.6.** $R_{11} = (n + Z_0\tau/\rho - 1)/(n + Z_0\tau/\rho + 1)$ where $n$ is refractive index, $\tau$ is thickness and $\rho$ is resistivity. $\tau \approx 5 \times 10^{-10}$ m (about two atomic diameters).

**11.7.** $R_{11} = \dfrac{n^2\cos\theta - (n^2 - \sin^2\theta)^{\frac{1}{2}} - \gamma\sin\theta}{n^2\cos\theta + (n^2 - \sin^2\theta)^{\frac{1}{2}} + \gamma\sin\theta}$

where

$$n^2 = \frac{(U-X)^2 - Y^2}{U(U-X) - Y^2}, \quad \gamma = \frac{\mathrm{i}XY}{U(U-X) - Y^2}.$$

**11.10.** Angle shift $(z_0 - z_1)D/(D^2 + z_0^2)$, Range shift $2z_0(z_0 - z_1)(D^2 + Z_0^2)^{-\frac{1}{2}}$, for observer on ground at distance $D$ from transmitter.

**12.4.** (a) $f_N^2 = \begin{cases} 0 & 0 \leqslant z < h_0 \\ f_1^2 + \alpha(z - h_0) & h_0 \leqslant z \end{cases}$

(b) $f_N^2 = \begin{cases} 0 & 0 \leqslant z \leqslant h_0 \\ \alpha(z - h_0) & h_0 \leqslant z \leqslant h_0 + f_1^2/\alpha \\ f_1^2 + \beta(z - h_0 - f_1^2/\alpha) & h_0 + f_1^2/\alpha \leqslant z \end{cases}$

**14.4.** The problem of the last sentence has no solution.

**18.2.** $\gamma = -(1 + \rho_2\rho_1^*)/(1 + \rho_1\rho_1^*)$.
(a) $\gamma = 0$. Solution (b) is the penetrating mode.
(b) $\gamma = -1$. In solution (b) the penetrating mode has been completely swamped. Its fields at the bottom cannot be found from the data.
(c) $\gamma = -1$. At the bottom the penetrating mode is linearly polarised with $E_y = \rho_2$, $\mathscr{H}_y = -C\rho_2$.