



(Deep) Generative Models for Multi-Variate Time-Series

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Member of the Committee: Prof. Full Name 3

Month Year

What I cannot create, I do not understand.

- Richard Feynman

Acknowledgments

A few words about the university, financial support, research advisor, dissertation readers, faculty or other professors, lab mates, other friends and family...

Resumo

Inserir o resumo em Português aqui com o máximo de 250 palavras e acompanhado de 4 a 6 palavras-chave...

Palavras-chave: palavra-chave1, palavra-chave2,...

Abstract

Multi-variate time-series data have become ubiquitous in the past decades, due to the time

Keywords: Multi-variate Time-series, Deep Generative Models, Deep Learning, Variational Inference, Approximate Inference

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Nomenclature

Greek symbols

α	Angle of attack.
β	Angle of side-slip.
κ	Thermal conductivity coefficient.
μ	Molecular viscosity coefficient.
ρ	Density.

Roman symbols

C_D	Coefficient of drag.
C_L	Coefficient of lift.
C_M	Coefficient of moment.
p	Pressure.
\mathbf{u}	Velocity vector.
u, v, w	Velocity Cartesian components.

Subscripts

∞	Free-stream condition.
i, j, k	Computational indexes.
n	Normal component.
x, y, z	Cartesian components.
ref	Reference condition.

Superscripts

*	Adjoint.
T	Transpose.

Glossary

- CFD** Computational Fluid Dynamics is a branch of fluid mechanics that uses numerical methods and algorithms to solve problems that involve fluid flows.
- CSM** Computational Structural Mechanics is a branch of structure mechanics that uses numerical methods and algorithms to perform the analysis of structures and its components.
- MDO** Multi-Disciplinary Optimization is an engineering technique that uses optimization methods to solve design problems incorporating two or more disciplines.

Chapter 1

Introduction

Insert your chapter material here...

1.1 Motivation

Relevance of the subject...

1.2 Topic Overview

Provide an overview of the topic to be studied...

1.3 Objectives

Explicitly state the objectives set to be achieved with this thesis...

1.4 Thesis Outline

Briefly explain the contents of the different chapters...

Chapter 2

Multi-variate Time-series

Insert your chapter material here...

2.1 Theoretical Overview

Some overview of the underlying theory about the topic...

2.2 Theoretical Model 1

The research should be supported with a comprehensive list of references. These should appear whenever necessary, in the limit, from the first to the last chapter.

A reference can be cited in any of the following ways:

- Citation mode #1 - [1]
- Citation mode #2 - Jameson et al. [1]
- Citation mode #3 - [1]
- Citation mode #4 - Jameson, Pierce, and Martinelli [1]
- Citation mode #5 - [1]
- Citation mode #6 - Jameson et al. 1
- Citation mode #7 - 1
- Citation mode #8 - Jameson et al.
- Citation mode #9 - 1998
- Citation mode #10 - [1998]

Several citations can be made simultaneously as [2, 3].

This is often the default bibliography style adopted (numbers following the citation order), according to the options:

```
\usepackage{natbib} in file Thesis_Preamble.tex,  
\bibliographystyle{abbrvnat} in file Thesis.tex.
```

Notice however that this style can be changed from numerical citation order to authors' last name with the options:

```
\usepackage[numbers]{natbib} in file Thesis_Preamble.tex,  
\bibliographystyle{abbrvnat} in file Thesis.tex.
```

Multiple citations are compressed when using the `sort&compress` option when loading the `natbib` package as `\usepackage[numbers,sort&compress]{natbib}` in file `Thesis_Preamble.tex`, resulting in citations like [4–7].

2.3 Theoretical Model 2

Other models...

Chapter 3

Probabilistic Graphical Models

Probabilistic Graphical Models are a language used to describe a set of random variables and their interactions, namely in the form of dependencies.

3.1 Theoretical Overview

Some overview of the underlying theory about the topic...

3.2 Theoretical Model 1

The research should be supported with a comprehensive list of references. These should appear whenever necessary, in the limit, from the first to the last chapter.

A reference can be cited in any of the following ways:

- Citation mode #1 - [1]
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- Citation mode #7 - 1
- Citation mode #8 - Jameson et al.
- Citation mode #9 - 1998
- Citation mode #10 - [1998]

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Chapter 4

Approximate Inference

For most interesting probabilistic models, the posterior distribution is intractable. This intractability arises from the integral in the denominator of the fraction obtained via Bayes' rule:

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} \quad (4.1)$$

$$= \frac{p(D|\theta)p(\theta)}{\int p(D|\theta')p(\theta')d\theta'} \quad (4.2)$$

This problem requires the use of methods that enable an approximate computation of the posterior. Two of the most used families of such methods are MCMC methods and Variational methods.

4.1 MCMC Methods

Markov-chain Monte-Carlo methods work by devising a scheme that allows for sampling from a distribution close to the one of interest. They do so by defining a Markov-Chain with a transition function that will make it converge asymptotically to the distribution of interest, given some constraints (ergodicity...)

4.2 Variational Methods

Variational methods work by turning the problem of integration into one of optimization. They propose a family of parametric distributions, and then optimize the parameters so as to minimize the "distance" between the approximate (normally called "variational") distribution and the distribution of interest.

The "distance" metric most commonly used for this is called Kullback-Leibler divergence, and is given by:

$$KL(p||q) = \sum p \log \frac{q}{p} \quad (4.3)$$

Chapter 5

TRCRP

Insert your chapter material here...

5.1 Numerical Model

Description of the numerical implementation of the models explained in Chapter 3...

5.2 Verification and Validation

Basic test cases to compare the implemented model against other numerical tools (verification) and experimental data (validation)...

Chapter 6

Variational TRCRP

Insert your chapter material here...

6.1 Numerical Model

Description of the numerical implementation of the models explained in Chapter 3...

6.2 Verification and Validation

Basic test cases to compare the implemented model against other numerical tools (verification) and experimental data (validation)...

Chapter 7

Results

Insert your chapter material here...

7.1 Problem Description

Description of the baseline problem...

7.2 Baseline Solution

Analysis of the baseline solution...

7.3 Enhanced Solution

Quest for the optimal solution...

7.3.1 figures

Insert your section material and possibly a few figures...

Make sure all figures presented are referenced in the text!

Drawings

Insert your subsection material and for instance a few drawings...

The schematic illustrated in Fig. 7.1 can represent some sort of algorithm.

7.3.2 Equations

Equations can be inserted in different ways.

The simplest way is in a separate line like this

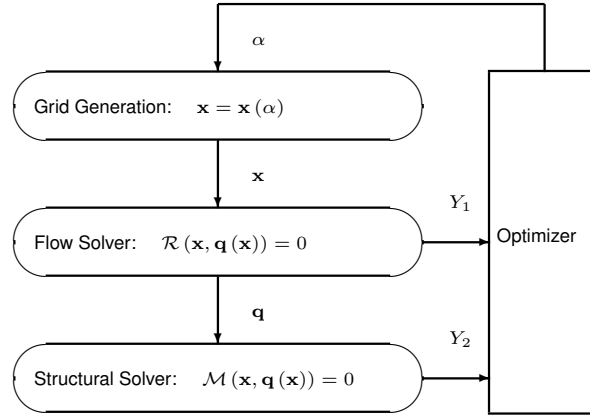


Figure 7.1: Schematic of some algorithm.

$$\frac{dq_{ijk}}{dt} + \mathcal{R}_{ijk}(\mathbf{q}) = 0. \quad (7.1)$$

If the equation is to be embedded in the text. One can do it like this $\partial \mathcal{R} / \partial \mathbf{q} = 0$.

It may also be split in different lines like this

$$\begin{aligned} &\text{Minimize} && Y(\alpha, \mathbf{q}(\alpha)) \\ &\text{w.r.t.} && \alpha, \\ &\text{subject to} && \mathcal{R}(\alpha, \mathbf{q}(\alpha)) = 0 \\ &&& C(\alpha, \mathbf{q}(\alpha)) = 0. \end{aligned} \quad (7.2)$$

It is also possible to use subequations. Equations 7.3a, 7.3b and 7.3c form the Navier–Stokes equations 7.3.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0, \quad (7.3a)$$

$$\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j + p \delta_{ij} - \tau_{ji}) = 0, \quad i = 1, 2, 3, \quad (7.3b)$$

$$\frac{\partial}{\partial t} (\rho E) + \frac{\partial}{\partial x_j} (\rho E u_j + p u_j - u_i \tau_{ij} + q_j) = 0. \quad (7.3c)$$

7.3.3 Tables

Insert your subsection material and for instance a few tables...

Make sure all tables presented are referenced in the text!

Follow some guidelines when making tables:

- Avoid vertical lines

- Avoid “boxing up” cells, usually 3 horizontal lines are enough: above, below, and after heading
- Avoid double horizontal lines
- Add enough space between rows

Model	C_L	C_D	C_{My}
Euler	0.083	0.021	-0.110
Navier–Stokes	0.078	0.023	-0.101

Table 7.1: Table caption.

Make reference to Table 7.1.

Tables 7.2 and 7.3 are examples of tables with merging columns:

	Virtual memory [MB]	
	Euler	Navier–Stokes
Wing only	1,000	2,000
Aircraft	5,000	10,000
(ratio)	5.0×	5.0×

Table 7.2: Memory usage comparison (in MB).

		$w = 2$			$w = 4$		
		$t = 0$	$t = 1$	$t = 2$	$t = 0$	$t = 1$	$t = 2$
$dir = 1$							
c		0.07	0.16	0.29	0.36	0.71	3.18
c		-0.86	50.04	5.93	-9.07	29.09	46.21
c		14.27	-50.96	-14.27	12.22	-63.54	-381.09
$dir = 0$							
c		0.03	1.24	0.21	0.35	-0.27	2.14
c		-17.90	-37.11	8.85	-30.73	-9.59	-3.00
c		105.55	23.11	-94.73	100.24	41.27	-25.73

Table 7.3: Another table caption.

An example with merging rows can be seen in Tab.7.4.

ABC	header			
	1.1	2.2	3.3	4.4
IJK	group	0.5		0.6
		0.7		1.2

Table 7.4: Yet another table caption.

If the table has too many columns, it can be scaled to fit the text width, as in Tab.7.5.

Variable	a	b	c	d	e	f	g	h	i	j
Test 1	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
Test 2	20,000	40,000	60,000	80,000	100,000	120,000	140,000	160,000	180,000	200,000

Table 7.5: Very wide table.

Chapter 8

Conclusions

Insert your chapter material here...

8.1 Achievements

The major achievements of the present work...

8.2 Future Work

A few ideas for future work...

Bibliography

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Appendix A

Vector calculus

In case an appendix is deemed necessary, the document cannot exceed a total of 100 pages...

Some definitions and vector identities are listed in the section below.

A.1 Vector identities

$$\nabla \times (\nabla \phi) = 0 \tag{A.1}$$

$$\nabla \cdot (\nabla \times \mathbf{u}) = 0 \tag{A.2}$$

