

Mealy Machine

- Output depends on the present state as well as present input.

If input change, output also changes

less state is required

More hardware requirement for circuit implementation.

React faster to input

Asynchronous output generation

Output is placed on transitions
difficult to design.

Ans. • \mathcal{G} is a sequence of pattern that defines a string.

- Ques Describe Theory of automata?

Ans

- It is a theoretical branch of computer science.
- It is a study of abstract machine & computer can be solved using these machines.
- The abstract machine is called automata.

- Main motivation behind developing the automata theory was to develop method to describe & analyse the dynamic behaviour of discrete systems.

- Automata is a machine which takes string as input and input goes through finite no. of state and may enter in final state.

- It is consist of states & transitions.

Ques Difference between NP-hard & NP-complete.

(H)

NP-hard

- NP-hard problems (X) can be solved if & only if there is a NP-complete problem (Y), that can be reducible into X in polynomial time.

- To solve problem it do not have to be in NP

- Do not have to be decision problem.

- Ex:- Hitting problem, vertex cover problem.

NP-complete.

NP-complete problems can be solved by Non-deterministic Algo / Turing machine in polynomial time.

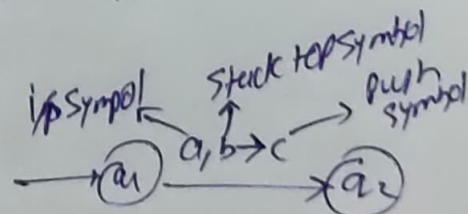
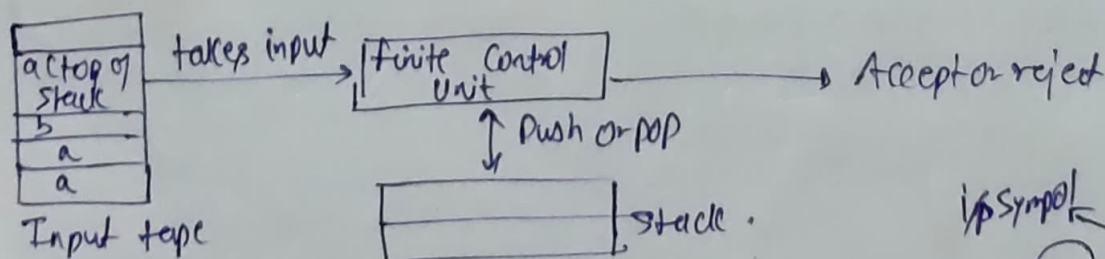
To solve such problem it has to be in both NP and NP-hard.

It is exclusively a decision problem.

ex: circuit satisfiability problem.

Ques what is Pushdown Automata?

- It is a way to implement context free grammar in a similar way to design DFA (deterministic finite automata) for a regular grammar.
- DFA can remember finite amt. of info but PDA can remember ∞ amt. of info.
- PDA follows "finite state machine + stack".
- PDA has 3 component:
 - Input tape
 - Control unit
 - A stack with ∞ size.



- It has 7 tuples: $(Q, \Sigma, S, \delta, q_0, I, F)$ — Set of accept state.
- finite no of state — Q
- i/p Alphabet — Σ
- Stack Symbol — S
- transition function — δ
- initial state — q_0
- initial state top symbol — I
- final state — F

- Programmable TM is called UTM.
- It provides a solⁿ to problems that are computable.
- It minimizes space complexity.
- UTM is subset of all TM.
- Transition funⁿ is $Q \times T \rightarrow Q \times T \times \{L, R\}$ | Q is the set of states, T is the set of tape symbols.

- Although developed for theoretical reasons, it helped in development of stored program computers.

Ans :: Acc to Chomsky, there are 4 types of grammars - Type-0
Type-1
Type-2

Recursively enumerable

Context-sensitive

Context-free

Regular

Type-0 Production

- $\alpha \rightarrow \beta$
- α cannot be null
- β is a string of terminal & non-terminal
- α is a string $\rightarrow / / \text{---}$
 $// \text{---}$ with at least one non-terminal

Type-1

Type-3

Production
 $X \rightarrow a$ or $X \rightarrow aY$

$x, y \in \text{Non Term}$.
 $a \in \text{Term}$.

Produktion

$$\alpha A\beta \rightarrow \alpha \gamma \beta$$

- $\alpha \in \beta$ may be empty
- γ must non-empty

Type-2

Production

$$A \rightarrow Y$$

- A (non-terminal)
- γ (string of terminal & non-terminal)

Ques Describe Context Free grammar? (R)

Ans. CFG is used to generate all possible strings in given formal language.
 • can be described as 4Tuples:

$$G = (V, T, P, S)$$

∴ G : Grammar
 V : finite set of non-terminal symbol
 T : Π ——— terminal symbol
 P : Set of production rules.
 S : Start symbol.

• CFG can be classified on the basis of following two properties:

① Based on no. of strings it generates:-

- If it generate finite no. of strings, then CFG is non-Recursive grammar.
- If Π ——— infinite no. of strings, then CFG is Recursive grammar.

② Based on no. of derivation tree:-

- If there is only 1 derivation tree, then CFG is unambiguous.
- If there is more than 1 derivation tree, then CFG is ambiguous.

Ques Define Turing Machine?

Ans: • It is an accepting device which accepts the languages generated by type 0 grammars.

• (TM) is a mathematical model which consist of an ∞ length tape divided into cells on which input is given.

• Described as a 7 tuple $(Q, X, \Sigma, \delta, q_0, B, F)$ set of final states.
 finite set of states tape alphabet input alphabet transition function initial state Blank symbol

- It consist of head which reads the input
- A state register which stores the state of Turing Machine.

Machine	Stack Data Structure	Deterministic?
finite Automata	N.A	Yes
Pushdown Automate	LIFO	No
Turing Machine	Infinite Tape	Yes

- Time Complexity, $T(n) = O(n \log(n))$
 Space Π ——— $S(n) = O(n)$

Ques Diff. b/w DFA & NFA

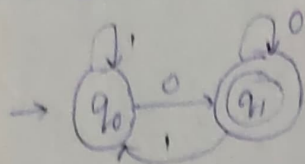
DFA

- refers to Deterministic Finite Automata
- Said to DFA if corresponding to an i/p symbol there is single resultant state i.e. only one transition.
- All DFA is NFA
- DFA is more difficult to construct
- Requires more space.
- Backtracking is allowed in DFA

NFA

- refers to Non-Deterministic Finite Automata
- Said to NFA if there is more than one possible transition from one state to same i/p symbol
- Not all NFA are DFA.
- NFA is easier to construct
- Requires less space
- Backtracking is not allowed always. in NFA

Transition functions


$$\begin{aligned} \delta(q_0, 0) &= q_1 \\ \delta(q_1, 1) &= q_0 \\ \delta(q_1, 0) &= q_1 \\ \delta(q_1, 1) &= q_0 \end{aligned}$$

Mathematisch model q + 1 cl

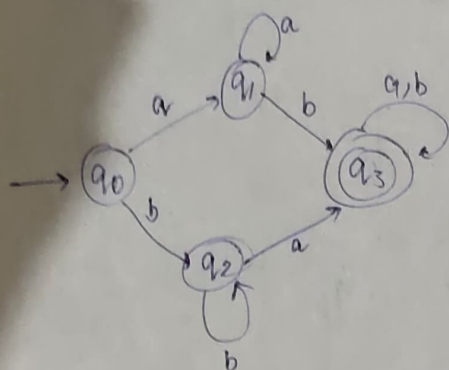
$Q = (q_0, q_1)$ set of states

Σ or $i/p = (0, 1)$

q_0 = start state

$q_1 = \text{final state}$

Transition dig \rightarrow transition table



spiele

	input	
	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_1	q_3
q_2	q_3	q_2
q_3	q_3	q_3

finite state Automata

→ finite no. of nodes

→ 5 tuples

→ Two types of FSA

L, Tuples

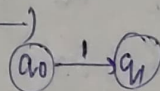
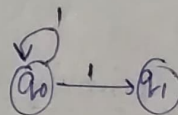
Q = Set of states

→ Σ = set of input alphabet

$\rightarrow T = \text{trans}^{\text{fun}} = Q^{\text{tr}} / p > \text{state}$

Initial state is 00

), final Mark. F

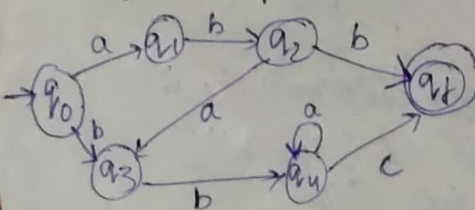


NFA (non-deterministic finite state Automata)

→ DFA (deterministic finite automata)

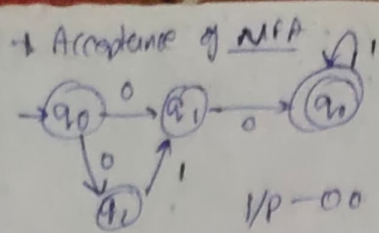
Acceptance of string by DFA.

→ Rule for acceptance of string is that on traversing from initial state to final state then string is accepted.



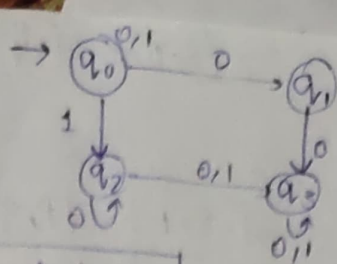
Strij's ~~ababac~~ ac

$\delta(q_0, \text{ababaaac}) \rightarrow \delta(q_1, \text{babaaac}) \rightarrow \delta(q_2, \text{abaaac}) \rightarrow$
 $\delta(q_3, \text{baaac}) \rightarrow \delta(q_4, \text{aaac}) \rightarrow \delta(q_5, \text{aac}) \rightarrow$
 $\delta(q_6, \text{ac}) \rightarrow \delta(q_7, \text{c}) \rightarrow \delta(q_8, \epsilon) \text{ accepted}$



$\delta(q_0, 0) \rightarrow \delta(q_1, 0) \rightarrow \delta(q_2) \rightarrow$ final state reached

$\delta(q_0, 0) \rightarrow \delta(q_2, 0) \rightarrow$ X not reached.

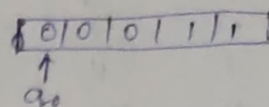


$\delta(q_0, 00011) \rightarrow \delta(q_1, 0011) \rightarrow \delta(q_3, 011)$

$\rightarrow \delta(q_3, 11) \rightarrow \delta(q_3, 1) \rightarrow \delta(q_3, \text{null})$

- Accepted

I/P 00011



→ Conversion of NFA to DFA

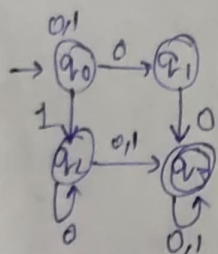
→ for every NFA we have a Equivalent DFA

→ equivalent DFA, → accept same type of string as accepted by NFA

→ For conversion of NFA to DFA two different state which are behaving in similar manner will be combined into single state and new state will be born, and the behavior of new state will be similar to both state.

$$(P, r) \rightarrow q$$

$$\delta(P, 0) \cup \delta(r, 0) \approx \delta(q, 0)$$



Step-1 (Make transition table for given NFA)

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	$\{q_3\}$	-
q_2	$\{q_2, q_3\}$	$\{q_3\}$
q_3	$\{q_3\}$	$\{q_3\}$

Step-2 (Group similar state / Make TT of DFA)

	0	1
q_0	$\{q_0, q_1\} = p$	$\{q_0, q_2\} = r$
p	$\delta(p, 0)$	$\delta(p, 1)$
r	$\delta(r, 0)$	$\delta(r, 1)$

$$\delta(p, 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \\ = \{q_0, q_1\} \cup \{q_3\} \\ = \{p, q_3\}$$

$$\delta(p, 1) = \delta(q_0, 1) \cup \delta(q_2, 1) \\ = r, \emptyset \\ = r$$

$$\delta(r, 0) = \delta(q_2, 0) \cup \delta(q_3, 0) \\ = p \cup \{q_2, q_3\} \\ = \{p, q_2, q_3\} = t$$

$$\delta(r, 1) = \delta(q_2, 1) \cup \delta(q_3, 1) \\ = r \cup q_3 \\ = r \cup q_3$$

	0	1
q_0	p	r
p	$\{p, q_3\} = s$	r
s	$\{p, q_2, q_3\} = t$	$\{r, q_3\} = u$

	0	1
q_0	p	s
p	s	s
s	t	u
t	s	u
u	$V = q_0, q_1, q_3$	u
v	v	u

$$\delta(s, 0) = (p, q_3) \rightarrow \delta(p, 0) \cup \delta(q_3, 0) \\ p \cup q_3 \cup q_3 \\ (p, q_3)$$

$$\delta(s, 1) = (p, q_3) \rightarrow \delta(p, 1) \cup \delta(q_3, 1) \\ r \cup q_3 = u$$

$$\delta(t, 0) = (p, a_2, a_3) \rightarrow \delta(p, 0) \cup \delta(a_2, 0) \cup \delta(a_3, 0) \\ \rightarrow (q_0, 0) \cup (q_1, 0) \cup (q_2, 0) \cup (q_3, 0) \\ \rightarrow a_0, a_1, a_2, a_3 = v$$

$$\delta(t, 1) = (p, a_2, a_3) \rightarrow \delta(p, 1) \cup (a_2, 1) \cup (a_3, 1) \\ \rightarrow \delta(a_0, 1) \cup (a_1, 1) \cup (a_2, 1) \\ \cup (a_3, 1)$$

$$(a_0, a_2) \cup p \cup a_3 \cup q_3 \\ a_0, a_2, a_3$$

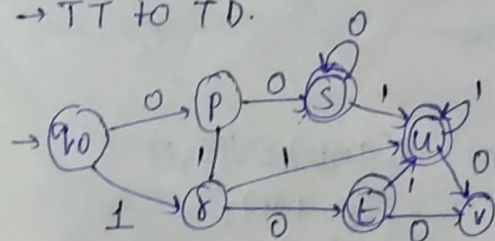
$$\delta(v, 0) \\ = (a_0, a_2, a_3 \cup 0) \\ a_0, a_2, a_3 \cup (a_3 \cup v(a_2, a_3)) \\ v(a_3)$$

$$\delta(v, 1) \\ = a_0, a_2, a_3 \cup 1 \\ a_0, a_2 \cup p, a_3 \cup a_3 \\ = u$$

$$\delta(u, 0) = (a_0, a_2, a_3 \cup 0) \quad \textcircled{T} \\ = \delta(a_0, 0) \cup (a_2, 0) \cup (a_3, 0) \\ a_0, a_1 \cup a_2, a_3 \cup e_3$$

$$\delta(u, 1) = \bar{v} (a_0, a_2, a_3 \cup 1) \\ = \delta(a_0, 1) \cup \delta(a_2, 1) \cup (a_3, 1) \\ = a_0, a_2 \cup a_3 \cup a_3 \\ = a_0, a_2, a_3 \\ = u$$

→ TT to TD.

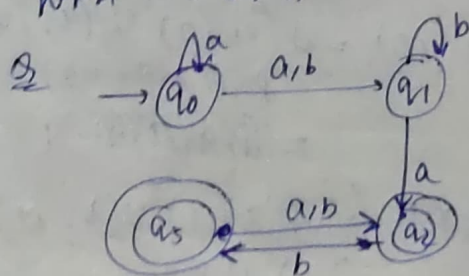


~~00000~~ 000110 → check string acceptn.

TT of NFA

	a	b
q_0	q_0, q_1	q_1
q_1	q_2	q_1
q_2	$-$	a_3
q_3	q_2	q_2
a_0, a_1	a_0, a_2	q_1
q_0, a_1, q_2	q_0, a_2	q_1, q_3
q_1, q_3	q_2	q_1, q_2
q_1, q_2	q_2	q_1, q_3

Ques NFA → DFA

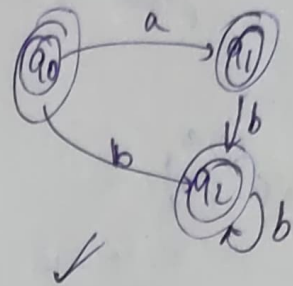


$$q_0, a_1 = p \\ q_0, a_1, a_2 = s \\ a_1, a_3 = u \quad q_1, a_2 = t$$

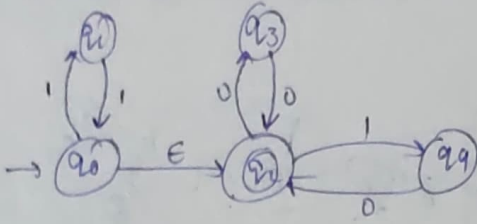
$$\begin{aligned} \delta'(q_2, b) &= \text{E-clos}(\delta(\text{E-clos}(q_2), b)) \\ &= \text{E-clos}(\delta(q_2, b)) \\ &= \text{E-clos}(q_2) \\ \boxed{\delta'(q_2, b) &= q_2} \end{aligned}$$

TT after removal of ϵ moves.

	a	b
q_0	q_1, q_2	\emptyset
q_1	\emptyset	q_2
q_2	q_2	q_2



Q Convert ϵ -NFA to NFA



- $\text{E-closure}(q_0) = \{q_0, q_2\}$
- $\text{E-clos}(q_1) = \{q_1\}$
- $\text{E-clos}(q_2) = \{q_2\}$
- $\text{E-clos}(q_3) = \{q_3\}$
- $\text{E-clos}(q_4) = \{q_4\}$

$$\boxed{\delta'(q, a) = \text{E-clos}(\delta(\text{E-clos}(q), a))}$$

$$\begin{aligned} \delta'(q_0, 0) &= \text{E-clos}(\delta(\text{E-clos}(q_0), 0)) \\ &= \text{E-clos}(\delta(q_0, q_2), 0) \\ &= \text{E-clos}(\delta(q_0, 0) \cup \delta(q_2, 0)) \\ &= \text{E-clos}(\emptyset \cup q_3) \\ &= \text{E-clos}(q_3) \end{aligned}$$

$$\boxed{\delta'(q_0, 0) = q_3}$$

$$\begin{aligned} \delta'(q_1, 1) &= \text{E-clos}(\delta(\text{E-clos}(q_1), 1)) \\ &= \text{E-clos}(\delta(q_1, 1)) \\ &= \text{E-clos}(q_0) \end{aligned}$$

$$\boxed{\delta'(q_1, 1) = q_0, q_2}$$

$$\begin{aligned} \delta'(q_0, 1) &= \text{E-clos}(\delta(\text{E-clos}(q_0), 1)) \\ &= \text{E-clos}(\delta(q_0, q_2), 1) \\ &= \text{E-clos}(\delta(q_0, 1) \cup \delta(q_2, 1)) \\ &= \text{E-clos}(q_1 \cup q_4) \end{aligned}$$

$$\boxed{\delta'(q_0, 1) = q_1, q_4}$$

$$\begin{aligned} \delta'(q_1, 0) &= \text{E-clos}(\delta(\text{E-clos}(q_1), 0)) \\ &= \text{E-clos}(\delta(q_1, 0)) \end{aligned}$$

$$\boxed{\delta'(q_1, 0) = \emptyset}$$

$$\begin{aligned} \delta'(q_2, 0) &= \text{E-clos}(\delta(\text{E-clos}(q_2), 0)) \\ \boxed{\delta'(q_2, 0) &= q_3} \end{aligned}$$

$$\begin{aligned} \delta'(q_2, 1) &= \text{E-clos}(\delta(\text{E-clos}(q_2), 1)) \\ &= \text{E-clos}(\delta(q_2, 1)) \end{aligned}$$

$$\boxed{\delta'(q_2, 1) = q_4}$$

$$\begin{aligned} \delta'(q_3, 0) &= \text{E-clos}(\delta(\text{E-clos}(q_3), 0)) \\ &= \text{E-clos}(\delta(q_3, 0)) \end{aligned}$$

$$\boxed{\delta'(q_3, 0) = q_2}$$

$$\begin{aligned} \delta'(q_4, 0) &= \text{E-clos}(\delta(\text{E-clos}(q_4), 0)) \\ \boxed{\delta'(q_4, 0) &= q_2} \end{aligned}$$

$$\delta'(q_3, 1) = \text{E-clos}(\delta(\text{E-clos}(q_3), 1))$$

$$\boxed{\delta'(q_3, 1) = \emptyset}$$

$$\delta'(q_4, 1) = \text{E-clos}(\delta(\text{E-clos}(q_4), 1))$$

$$\boxed{\delta'(q_4, 1) = \emptyset}$$

read only)

	0	1
q_0	q_3	$q_1, q_4 = q_A$
q_1	\emptyset	$q_0, q_2 = q_B$
q_2	q_3	q_4
q_3	q_2	\emptyset
q_4	q_2	\emptyset
q_A	q_2	q_B
q_B	q_3	q_A

	0	1
q_0	q_3	q_4
q_1	\emptyset	q_B
q_2	q_7	q_4
q_3	q_2	\emptyset
q_4	q_2	\emptyset
q_A	q_2	q_B
q_B	q_3	q_A

(9)

Regular Expression

$re \Rightarrow$ ① Special Symbols $\rightarrow +, *, (), \emptyset, /$

$re_1 + re_2 = re$

$re_1 \cup re_2 = re$

$re_1 \cdot re_2 = re$

$re(re_1 + re_2) = re \cdot re_1 + re \cdot re_2$

Ex

$\{ \epsilon, a, aa, aaa, aaaa, \dots \}$

$\in a^n, n \geq 0$

here null is allowed

To write this mathematical expression in terms of re.

$re \Rightarrow a^*$ where $*$ - Kleene closure
 $*$ - zero or more times.

$\{ a, aa, aaa, aaaa, \dots \}$

here null is not allowed

$a^n, n \geq 1$

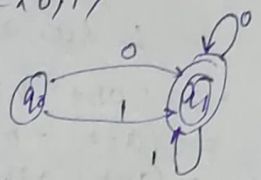
a^+

positive closure

a is will come one or more times

Ques write a regular expression made of $\{0, 1\}$.

$\Sigma = \{0, 1\}$



$re = (0+1)^*$

$(or) \rightarrow +$ (parallel path)

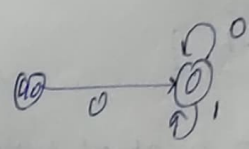
$\epsilon, 0, 1, 00, 10, \dots$

Q $\Sigma = \{0, 1\}$ Begin with zero

Restriction

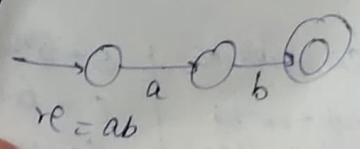
$\{ 0, 00, 01, 010, 0111, 0000, \dots \}$

$re = 0(0+1)^*$



$a \cdot b \rightarrow ab$

(meaning b followed by a)



$re = ab$

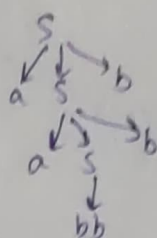
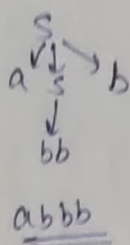
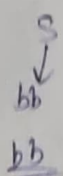
$(ab)^* = (ab, abab, ababab, \dots)$

(iv) $a^n b^{n+2}, n \geq 0$

$\lambda = 0$

$$S \rightarrow aSb \mid bb$$

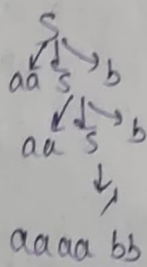
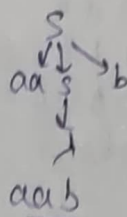
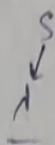
bb



aabbbb

(iv) $a^n b^n, n \geq 0$

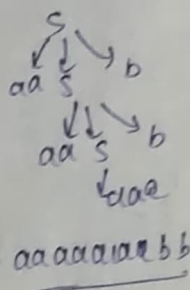
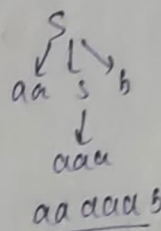
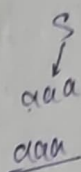
$$S \rightarrow a u S b \mid A$$



⑤ $a^{n+3} b^n, n \geq 0$

$\eta = 0$
aaa

$$S \rightarrow aaSb \mid aaa$$



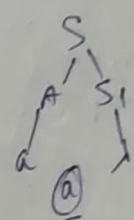
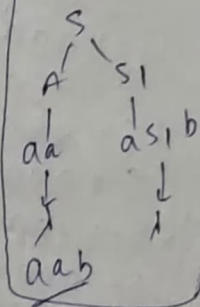
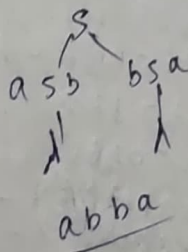
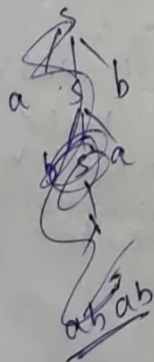
① $a^m b^n, m \geq n$
 $n \geq 0$

$$S_1 \rightarrow a S_1 b \mid \lambda$$

$$A \rightarrow aA|a$$

Q $\{w \mid n_a(w) = n_b(w)\}$ $S \rightarrow AS$

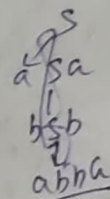
$$S \rightarrow asb \mid bsa \mid \cancel{oss} \mid \lambda$$



$$\underline{Q} \quad ww^Rv$$

$$w(a+b)w^R$$

$$S \rightarrow aSa \mid bSb \mid d \mid b$$



$$2 a^m b^m c^n, m, n \geq 0$$

$$S \rightarrow aS, b| \lambda$$

$C \rightarrow cc1d$

$$S \rightarrow SIC$$

Derivation

$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow bB \mid \lambda$$

Check whether

$$W = aaaa bb \in L(G)$$

$$V = \{A, B\}$$

$$S = \{S\}$$

$$T = \{a, b\}$$

$$S \rightarrow AB$$

$$S \rightarrow aaAB$$

$$S \rightarrow aaaaAB$$

$$S \rightarrow aaaa bB$$

$$S \rightarrow aaaa b bB$$

$$S \rightarrow aaaa bb$$

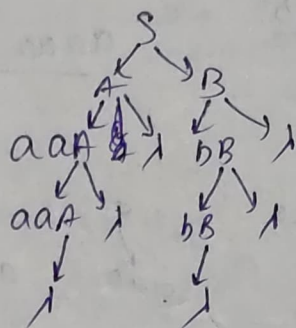
(using $A \rightarrow aaA$)

(using $A \rightarrow aaA$)

(using $A \rightarrow \lambda$)

(using $B \rightarrow bB$)

(Hence string is accepted)



aaaa bb

Ques

$$S \rightarrow AB$$

$$A \rightarrow aaA$$

$$S \rightarrow aB \mid bA$$

$$A \rightarrow a \mid as \mid bAA$$

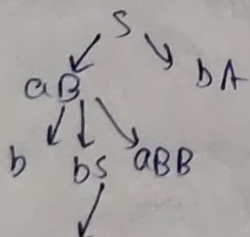
$$B \rightarrow b \mid bS \mid aBB$$

$$\text{string} = aaaa babbba$$

$$\text{variable } V = \{A, B\}$$

$$T = \{a, b\}$$

$$S = \{S\}$$



$$S \rightarrow aB$$

$$S \rightarrow aaBB \quad (\text{using } B \rightarrow aBB)$$

$$S \rightarrow aaaaBBB \quad (\text{using } B \rightarrow aBB)$$

$$S \rightarrow aaaa bBB \quad (\text{using } B \rightarrow b)$$

$$S \rightarrow aaaa b bB \quad (\text{using } B \rightarrow b)$$

$$S \rightarrow aaaa b b aBB \quad (\text{using } B \rightarrow aBB)$$

$$S \rightarrow aaaa b b a bB \quad (\text{using } B \rightarrow b)$$

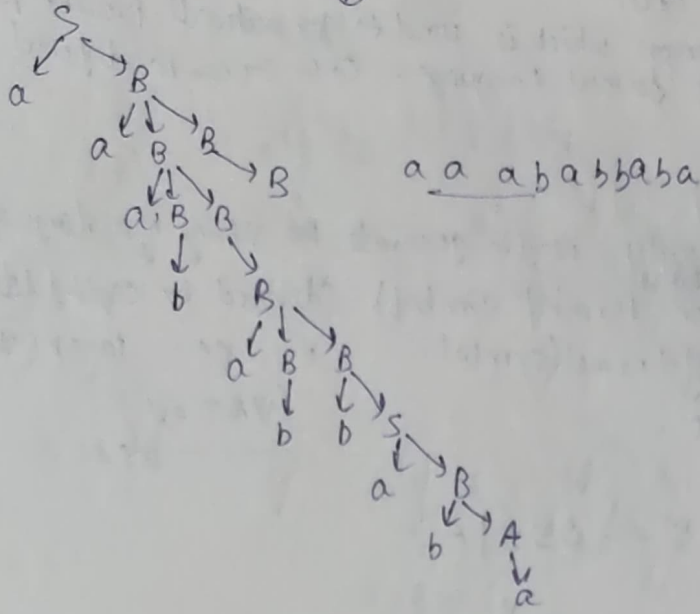
$$S \rightarrow aaaa b b a b bS \quad (\text{using } B \rightarrow bS)$$

$$S \rightarrow aaaa b b a b b aB \quad (\text{using } S \rightarrow aB)$$

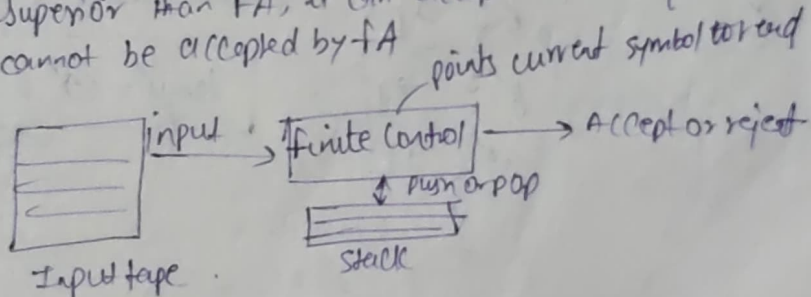
$$S \rightarrow aaaa b b a b b b a \quad (\text{using } S \rightarrow bA)$$

derivation tree

(P)

→ Push Down Automata

PDA is a way to implement a CFG. PDA can remember infinite amount of info. It is simply an NFA augmented with an external stack memory. PDA is more powerful than FA. PDA is more superior than FA, it can accept a class of language which even cannot be accepted by FA.



PDA is collection of 7 tuple. $(Q, \Sigma, \Gamma, q_0, Z, F, \delta)$

Q : finite set of states

Σ : input set

Γ : stack symbol

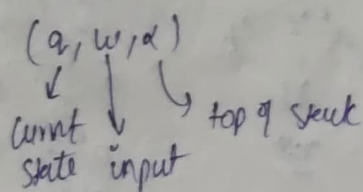
q_0 : initial state

Z : start symbol

F : final set state

δ : transition function.

Instantaneous Description (ID) : tells instant description of PDA



→ acceptance of string in PDA.
 $L = \{a^n b^n \mid n \geq 1\}$
 input string $w = aabbb$

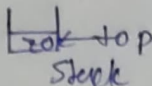
$$\begin{aligned} \delta(q_0, a, z_0) &= (q_0, az_0) \\ \delta(q_0, a, a) &= (q_0, aa) \\ \delta(q_0, b, a) &= (q_1, \epsilon) \\ \delta(q_1, b, a) &= (q_1, \epsilon) \\ \delta(q_1, \epsilon, z_0) &= (q_2, z_0) \end{aligned}$$

Solⁿ
 $\delta(q_0, aabbb, z_0) \vdash q_0, aabbb, az_0$ (push)
 $\vdash q_0, abbb, aaz_0$ (push)
 $\vdash q_0, bbb, aaaz_0$ (push)
 $\vdash q_1, bb, aaaz_0$ (pop)
 $\vdash q_1, b, aaaz_0$ (pop)
 $\vdash q_1, \epsilon, aaaz_0$ skip
 $\vdash q_2, z_0$ string is accepted.

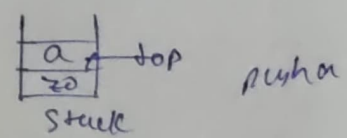
Q Construct PDA $L = \{a^n b^n \mid n \geq 1\}$

Solⁿ $L = \{ab, aabb, aaabbb, \dots\}$

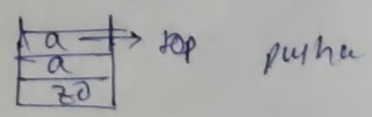
Let string be aabb

Step-1
 $w = aabb$
 $a, z_0 / az_0$ 
 $\rightarrow q_0$

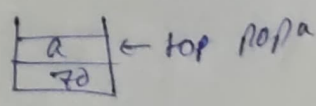
Step-2
 input symbol a $\rightarrow q_0$ $a, z_0 / az_0$



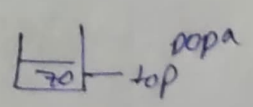
Step-3
 input symbol a $\rightarrow q_0$ $a, z_0 / az_0$
 $a, a / aa$



Step 4
 input symbol b $\rightarrow q_0$ $a, z_0 / az_0$
 $a, a / aa$
 $b, a / \epsilon \rightarrow q_1$



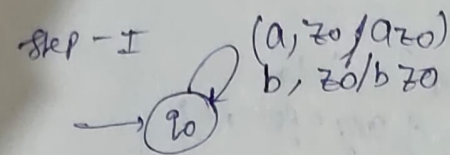
Step 5
 input symbol b $\rightarrow q_0$ $a, z_0 / az_0$
 $a, a / aa$
 $b, a / \epsilon \rightarrow q_1$
 $b, a / \epsilon$



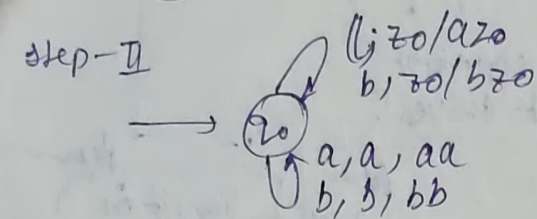
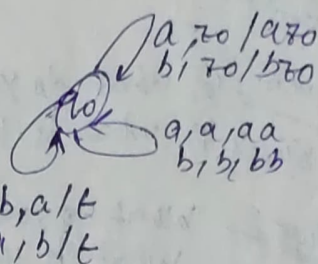
Step-6
 input end string $\rightarrow q_0$ $a, z_0 / az_0$
 $a, a / aa$
 $b, a / \epsilon \rightarrow q_1$
 $\epsilon, z_0 / z_0 \rightarrow q_2$

Construct PDA $L = \{ w \mid n_a(w) = n_b(w) \}$
 $\{ ab, ba, aabb, bbba, baaba \dots \}$

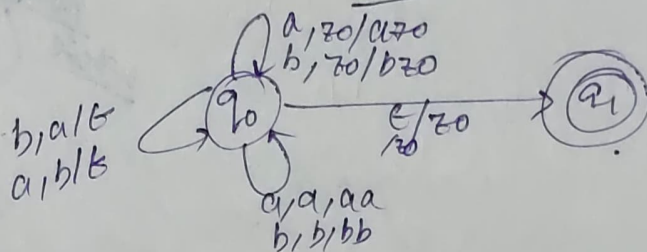
(4)



Step-3



Step-4



Transition fun.

$$\left. \begin{aligned} \delta(q_0, a, z_0) &= (q_0, a z_0) \\ \delta(q_0, b, z_0) &= (q_0, b z_0) \\ \delta(q_0, a, a) &= (q_0, aa) \\ \delta(q_0, b, b) &= (q_0, bb) \end{aligned} \right\} \text{Push}$$

$$\left. \begin{aligned} \delta(q_0, a, b) &= (q_0, \epsilon) \\ \delta(q_0, b, a) &= (q_0, \epsilon) \end{aligned} \right\} \text{Pop}$$

$$\delta(q_0, \epsilon, z_0) = (q_1, z_0) \text{ or step accepted, final state.}$$