Marco, M. Donatelli

Regularization by multigrid-type algorithms

Universit dell'Insubria
Dip Fisica e Matematica
Via Valleggio
11
22100 Como (CO) - Italy
marco.donatelli@uninsubria.it
Stefano, S. Serra-Capizzano

We consider the de-blurring problem of noisy and blurred images in the case of space invariant point spread functions. The use of appropriate boundary conditions leads to linear systems with structured coefficient matrices related to space invariant operators like Toeplitz, circulants, trigonometric matrix algebras etc. We combine an algebraic multigrid (which is designed ad hoc for structured matrices) with the low-pass projectors typical of the classical geometrical multigrid employed in a PDEs context. Thus, using an appropriate smoother, we obtain an iterative regularizing method (see [2, 1]) based on: projection in a subspace where it is easier to distinguish between the signal and the noise and then application of an iterative regularizing method in the projected subspace. Therefore any iterative regularizing method like conjugate gradient (CG), conjugate gradient for normal equation (CGNE), Landweber etc., can be used as smoother in our multigrid algorithm. The projector is chosen in order to maintain the same algebraic structure at each recursion level and having a low-pass filter property, which is very useful in order to reduce the noise effects. In this way, we obtain a better restored image with a flatter restoration error curve and also in less time than the auxiliary method used as smoother. As direct consequence the choice of the exact iteration where to stop is less critical than in other regularizing iterative methods. Furthermore, we can choose multigrid procedures which are extremely more efficient than classical techniques without losing accuracy in the restored image. Several numerical experiments show the effectiveness of our proposal.

A Theoretical analysis of multigrid methods is usually a difficult task and a first largely used approach considers a two grid method. In the same way, to proving the regularizing properties of our multigrid methods, we provide some estimations on the filter factor of the two level strategy.

Finally, it can be easily (by using a simple projection at every step) combined with nonnegativity constraints. Moreover we propose a possible generalization where the multigrid regularization is applied as a one-step method: now the only parameter to choose is the number of recursive calls.

Bibliography

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