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**Robust design of cardiovascular surgeries - a
derivative-free stochastic optimization approach**

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Recent advances in coupling novel optimization methods to large-scale computing problems have opened the door to tackling a diverse set of physically realistic engineering design problems. A large computational overhead is associated with computing the objective function for most practical problems involving complex physical phenomena. Such problems are also plagued with uncertainties in a diverse set of parameters. We present a computational framework for solving stochastic optimization problems with multiple constraints. We demonstrate the use of this method by coupling the optimization routine with a customized finite element flow solver for cardiovascular surgery design.

A novel and non-intrusive adaptive stochastic collocation [1] technique is used to account for uncertainties. Randomness is viewed as a separate dimension (akin to space and time) which is referred to as stochastic space. The stochastic space is numerically discretized using Chebyshev points and Lagrange interpolates. Function adaptive Smolyak sparse grids are computed using a novel algorithm that relies on hierarchical error indicators. Optimization is performed using a derivative-free technique called the Surrogate Management Framework (SMF), in which Kriging interpolation functions are used to approximate the objective function. The SMF method with mesh adaptive direct search polling strategy [2,3] is extended to stochastic optimization problems. The filter approach is used to incorporate non-linear probabilistic constraints. We show that the iterates generated by the algorithm are stationary in the stochastic sense using a moment-based approach [4]. The optimization algorithm is validated on systems characterized by partial differential equations such as thermal (heat-conduction) and solid mechanics problems with reliability constraints. In each optimization iteration, a set of PDEs are solved. The numerical efficiency of this method is compared with Monte-Carlo techniques. To further reduce the computational effort, a stochastic response surface (SRS) technique is developed to approximate higher order solution statistics. This results in a significant reduction in the number of simulations and hence, the computational time.

The stochastic optimization technique is applied to cardiovascular bypass graft

shape optimization problems of potential clinical interest. We optimize shape of BG surgeries to minimize adverse hemodynamic flow conditions such as low wall shear stresses. Numerical simulations of blood flow in the cardiovascular system are performed using custom finite element methods and specialized out-flow boundary conditions tailored for cardiovascular applications. We account for uncertainties in cardiovascular simulations which occur due to factors such as: (a) reproducing simulation models based on MRI images, and (b) boundary conditions including velocity, resistance and lumped parameter models. We compare and contrast the robust design solutions with results obtained using deterministic optimization. The SRS technique shows an order of magnitude improvement in the computational efficiency over the conventional stochastic collocation technique.

References:

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