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Block approximate inverse preconditioners using the Sherman-Morrison-Woodbury formula [1]

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In this work we consider the solution of nonsymmetric linear systems of the form

$$Ax = b\,,$$

by preconditioned Krylov iterations where $A \in \mathbf{R}^{n\times n}$ is a sparse, nonsingular matrix. We introduce a block approximate inverse preconditioner which is a generalization of the AISM preconditioner presented by Bru et al. [SIAM Journal on Scientific Computing, 25(2):701–715, 2003]. The computation of the preconditioner involves the well known Sherman-Morrison-Woodbury formula.

Consider the matrix A partitioned in block form:

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1p} \\ A_{21} & A_{22} & \dots & A_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ A_{p1} & A_{p2} & \dots & A_{pp} \end{bmatrix} \tag{1}$$

where $A_{ij} \in \mathbf{R}^{m_i\times m_j}$, $\sum_{k=1}^{p} m_k = n$. We denote the block columns of A by A_i, $i = 1, \dots, p$. That is,

$$A_i = [A_{1i}, A_{2i}, \dots, A_{pi}]^T.$$

Let $X = I_n$ and $Y = (A - sI_n)^T$ be matrices partitioned accordingly where I_n is the identity matrix of size n and s is a positive scalar. One has that,

$$A = sI_n + \sum_{k=1}^{p} X_k Y_k^T. \tag{2}$$

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By defining block column vectors $U_k, V_k,\ k = 1, \dots, p$ as,

$$U_k = X_k - \sum_{i=1}^{k-1} s^{-1} U_i T_i^{-1} V_i^T X_k, \tag{3}$$

$$V_k = Y_k - \sum_{i=1}^{k-1} s^{-1} V_i T_i^{-T} U_i^T Y_k, \tag{4}$$

where

$$T_k = I_{m_k} + s^{-1} V_k^T X_k = I_{m_k} + s^{-1} V_{kk}^T, \tag{5}$$

one obtains the following expression for the inverse of A,

$$A^{-1} = s^{-1} I_n - s^{-2} U T^{-1} V^T \,. \tag{6}$$

The matrices U, V have block columns $U_k, V_k,\ k = 1, \dots, p$, respectively, and T is a block diagonal matrix with diagonal blocks $T_k,\ k = 1, \dots, p$.

A sparse preconditioner is obtained by applying a dropping strategy during the computation of U_k and V_k. This strategy consists in removing off-diagonal nonzero entries which are less than a given threshold. Once the inexact factors $\bar{U}_k$ and $\bar{V}_k$ have been computed, two different preconditioning strategies can be defined:

$$M_1 := s^{-1} I_n - s^{-2} \bar{U} \bar{T}^{-1} \bar{V}^T,$$

and

$$M_2 := s^{-2} \bar{U} \bar{T}^{-1} \bar{V}^T.$$

It will be shown that the block preconditioner can be computed without breakdowns for $M-$matrices and $H-$matrices. The results of numerical experiments obtained for a representative set of matrices will be presented. Compared with point AISM it will be shown that the BiCGSTAB and GMRES methods preconditioned with block AISM converge in less iterations. Indeed, for some problems where point AISM fails to converge, as the UTM* matrices, the block version works successfully. In addition, the effect of some reorderings of the coefficient matrix on the performance of the preconditioner is also considered. The results will show that AISM can benefit from them.