Fast Minimization methods for Multiplicative Noise Removal

Wang Fan*

Abstract

Multiplicative noise and blur removal problems have attracted much attention in recent years. In this paper, we propose an efficient minimization method to recover the blurred and noisy image. We make use of the logarithm to transform multiplicative problems into additive problems and then employ l_1-norm to measure the data-fitting. The total variation is also used as a regularization to the recovered image. As the set of feasible solutions is nonconvex in the formulation, we propose to use approximation to make it to be convex, and therefore make sure the convergence of the proposed algorithm. Experimental results are report to demonstrate that the proposed algorithm performs better than the other existing methods.

1 Introduction

Image deblurring and denoising problem is important in signal and image processing fields. Most imaging systems capture an image x and return a degraded data f. A common model of the degradation process is given by

$$f = Hx + b \ , \tag{1.1}$$

where H is the blur operator, x is the original image and b is some noise. For this additive noise model, recovering x from f is usually an ill-posed inverse problem. Since the work of Rudin, Osher and Fasemi [14], the regularization methods based on total variation (TV) have known an important success, mostly due to their ability to preserve in the image.

In (1.1), the additive noise assumption is made. In recent years, many researchers studied the multiplicative noise assumption, see [13], [1], [9], [16]. The model of the degradation process is given by

$$f = Hx \circ b \ , \tag{1.2}$$

where $\circ$ refers to the component-wise multiplication. There are many applications for multiplicative noise removal problems in medical imaging, e.g., magnetic field inhomogeneity in MRI [7], [11], speckle noise in ultrasound [19], and speckle noise in synthetic aperture radar (SAR) images [1], [18].

1.1 Multiplicative Denoising

When H is the identity operator, this is a multiplicative denoising case. The first method is based on total variation regularization by Rudin, Lions and Osher [13] (the RLO model):

$$\begin{aligned} &\min \int_\Omega |\nabla x| \ , \\ &\text{subject to} \ \begin{cases} \int_\Omega \frac{f}{x} = 1 \ , \\ \int_\Omega \left(\frac{f}{x} - 1\right)^2 = \sigma^2 \ , \end{cases} \end{aligned} \tag{1.3}$$

where σ^2 is the variance of noise. They designed a gradient projection-based algorithm to find the minimizer.

The second method is given by Aubert and Aujol [1] (the AA model):

$$\min_x \int_\Omega \log x + \frac{f}{x} + \lambda |\nabla x| \ . \tag{1.4}$$

The objective function is derived by using Maximum a Posteriori (MAP) estimator approach. Although the objective function is nonconvex, Aubert and Aujol showed existence of minimizers of the objective

*Institute for Computational Mathematics and Department of Mathematics, Hong Kong Baptist University, Kowloon Tong, Hong Kong. School of Mathematics and Statistics, Lanzhou University, Gausu, P.R.China. Email : 09466029@hkbu.edu.hk

function, gave a sufficient condition ensuring uniqueness and showed that a comparison principle holds. They employed a gradient method to show the capability of their model on some numerical examples.

Recently, Shi and Osher [16] proposed to consider a noisy observation $\log f = \log x + \log b$ and derived the total variation minimization model for multiplicative noise removal problems. Bioucas-Dias and Figueiredo[2] used a variable splitting or split-Bregman method to solve the minimization problem. Huang et al. [9] also used the logarithm transformation $x = \exp z$ into (1.4), and considered the total variation regularization for z in the model. The resulting objective function is strictly convex. They developed an alternating minimization algorithm to find the minimizer of such objective function efficiently, and show the convergence of the minimizing method.

Durand et al. [5] proposed a method composed of several stages. They also used the log-image data and applied a reasonable under-optimal hard-thresholding on its curvelet transform, then they applied a variational method by minimizing a specialized hybrid criterion composed of an l_1 data-fidelity to the thresholded coefficients and a total variation regularization term in the log-image domain. The restored image can be obtained by using an exponential of the minimizer, weighted in a such way that the mean of the original image is preserved. Their restored images combine the advantages of shrinkage and variational methods and avoid their main drawbacks.

1.2 Multiplicative Deblurring

However, as far as we know, there exist very few papers addressing the deblurring problem of multiplicative noise. Indeed, this becomes extremely difficult when we add a blur in the original image since it brings another instable issues. Rudin, Lions and Osher [13] proposed the following model to handle the deblurring problem:

$$\begin{array}{rl} \min & \int_\Omega |\nabla x| \\ \text{subject to} & \int_\Omega \frac{f}{Hx} = 1 \ , \\ & \int_\Omega (\frac{f}{Hx} - 1)^2 = \sigma^2 \ , \end{array} \tag{1.5}$$

the model is proposed to be solved by the gradient projection method. Note that, as point out in [13], this model has been treated in [3] by homomorphic filtering. This method takes logarithm of f and treat it as a problem involving additive noise, and then filter and apply the exponential. This could be somewhat inappropriate since the distribution of logarithm of Gaussian noise is no longer Gaussian and is rather difficult to handle. The numerical results seem to be rather promising though in certain sense, it is somewhat limited.

Aubert and Aujol [1] had also extended their model to handle the deblurring problem in the case of multiplicative speckle noise modeling. The AA model reads:

$$\min_x \int_\Omega \log(Hx) + \frac{f}{Hx} + \lambda|\nabla x| \ . \tag{1.6}$$

This model is also solved by the gradient projection method in [1]. Let us mention that the gradient projection method may stick at some local minimizer and the restoration results strongly rely on the initial guess and the numerical schemes. Moreover, as the blurring operator H also involves another instable problem, it is rather difficult to get a steady state of the above models in practices. Their restored image is not as good as in the denoising case.

Setzer, Steidl and Teuber [15] proposed a method to handle the restoration of deblurring problem of multiplicative poissonian noise:

$$\min_x \int_\Omega Hx - f\log(Hx) + \lambda|\nabla x| \ . \tag{1.7}$$

They solve the task by minimizing an energy functional consisting of the I-divergence as similarity term and the TV regularization term, they use alternating split Bregman technique to deal with the minimizing algorithm, in such a way inner loops do not occur. In contrast to other algorithms, this method avoid the choice of additional step length parameters and appropriate stopping rules. Then the nonnegative restored images can be obtained and the convergence is guaranteed by known convergence results of the alternating split Bregman algorithm.

In this paper, we focus on the deblurring issues under multiplicative noise model. We try to reformulate the problem (1.2) into some optimization problems with favorably separable structures, we notice that the problem (1.2) can be solved efficiently via the well-developed alternating direction methods (ADM). Because the set of feasible solutions is nonconvex, we propose to use a convex set to approximate the nonconvex one, which makes the convergence of the proposed algorithm guaranteed. We can prove this set is a convex set and show that it is a good approximation of original constrain set using a specific example. The numerical examples show our algorithm is more efficient than the RLO model and the AA model to handle the deblurring problem under multiplicative noise.

Our paper is organized as follows. In section 2, we will show that how to apply the ADM method for solving the problem. In section 3, numerical examples are given to demonstrate the effectiveness of the proposed method. Conclusions are given in Section 4.

2 The proposed model

At first, we transform the multiplicative noise problem (1.2) into additive noise by

$$\log f = \log Hx + \log b \ . \tag{2.1}$$

We define the notation: $|x| = \sqrt{x_1^2 + x_2^2} \in \mathbb{R}^n$ and: $|||x|||_1 = ||(|x|)||_1$. Then we use the regularization method based on TV to recover x from f. It can be written as

$$\begin{aligned} \min \ & |||\nabla x|||_1 \\ \text{subject to } \ & x \in \mathbb{R}^n \\ & ||\log f - \log Hx||_1 \leq \alpha \ . \end{aligned} \tag{2.2}$$

Herein, n is the pixels number, α is a positive real number which means the trade-off between the fit to f and the amount of regularization; $\nabla : \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^n$ is a discrete version of the gradient; and the notation $||\cdot||_1$ refers to the standard definition of an l_1-norm.

The aim of this work is to provide fast schemes for solving the constrained problems (2.2). By reformulating the problem (2.2) into some optimization problems with favorably separable structures, we notice that the problem (2.2) can be solved efficiently via the well-developed alternating direction methods (ADM). Overall, the aim of this paper is to develop ADM-based numerical algorithms for solving the constrained problem (2.2). we refer to, e.g. [6, 12, 17, 21, 22], for some novel applications of ADM in the area of image processing.

2.1 Reformulation

We consider to recover x from f, and it is captured by the variational formulation (2.2). Now, let $p = \nabla x, z = Hx$ and assume that the set $Z := \{z \in \mathbb{R}^n, ||\log f - \log z||_1 \leq \alpha\}$ is not empty, we can get the following constrained optimization problem

$$\begin{aligned} \min \ & |||p|||_1 \\ \text{subject to } \ & x \in \mathbb{R}^n \\ & p \in \mathbb{R}^n \times \mathbb{R}^n, p = \nabla x \\ & z \in \mathbb{R}^n, z = Hx, z \in Z := \{z \in \mathbb{R}^n | \ ||\log f - \log z||_1 \leq \alpha\} \ . \end{aligned} \tag{2.3}$$

We want to use ADM methods to solve the problem (2.3). The good convergence quality of the method can be guaranteed when the constraint set is a convex set. But the constraint set Z can be written as:

$$Z := \{z \in \mathbb{R}^n | \sum_{i=1}^{n} |\log(\frac{f_i}{z_i})| \leq \alpha\} \ .$$

If $f_i \leq z_i$, it is a convex set, but if $f_i > z_i$, it is not a convex set. That means, it is not a convergence model if we use ADM method to solve the problem.

A method to overcome this difficulty is to use another convex set to approximate the original constraint set. When $f_i > z_i$, we use a linear function to approximate the logarithm function. In other words, we can define a set like this:

$$\bar{Z} := \{z \in \mathbb{R}^n | \sum_{i=1}^{n} |F(z_i)| \leq \alpha\}$$

and $F(z_i)$ is defined as:

$$F(z_i) = \begin{cases} \log(f_i/z_i) & \text{if } f_i < z_i, \\ f_i - z_i & \text{else.} \end{cases}$$

We try to use the set $\bar{Z}$ to approximate the set Z.

2.2 Analysis on the convex set

Now we are ready to analysis the convex attribute of the constraint set $\bar{Z}$:

$$\bar{Z} := \{z \in \mathbb{R}^n | \sum_{i=1}^{n} |F(z_i)| \leq \alpha\} \ ,$$

the function $F(z_i)$ is defined as:

$$F(z_i) = \begin{cases} \log(f_i/z_i) & \text{if } f_i < z_i, \\ f_i - z_i & \text{else.} \end{cases}$$

Since it is a connected set, and can be divided into different regions according to $f_i > z_i$ or $f_i \leq z_i$. It suffices to show that it is a convex set for each region.

Without loss of generality, we assume that the region is $f_i > z_i, (i = 1, 2, \cdots, k)$ and $f_j \leq z_j, (j = k+1, k+2, \cdots, n)$, here $k \in [0, n]$. The other regions can be proved that it is the same as this region. So we can rewrite the set as:

$$\bar{Z}' := \{z \in \mathbb{R}^n | \sum_{i=1}^{k} |f_i - z_i| + \sum_{j=k+1}^{n} |\log f_j - \log z_j| \leq \alpha, \text{if all } f_i > z_i \text{ and } f_j \leq z_j\} \ . \tag{2.4}$$

Then, let $z \in \mathbb{R}^n$, we define a function $g : \mathbb{R}^n \to \mathbb{R}$ as:

$$g(z) = \sum_{i=1}^{k} |f_i - z_i| + \sum_{j=k+1}^{n} |\log f_j - \log z_j|, \text{if all } f_i > z_i \text{ and } f_j \leq z_j \ . \tag{2.5}$$

We can compute Hessian matrix or second derivative $\nabla^2 g$ in the domain of g:

- For the part of the $g(z)$ equal to $|f_i - z_i|$, the $\frac{\partial^2 g}{\partial z_i^2}$ is 0;
- For the part of the $g(z)$ equal to $|\log f_i - \log z_i|$, the $\frac{\partial^2 g}{\partial z_i^2}$ is $\frac{1}{z_i^2}$;
- Since every part of $g(z)$ just have one variable of z_i, all the mixed partial derivatives are 0.

thus the Hessian or second derivative $\nabla^2 g$ is a diagonal matrix, and it is a positive semidefinite matrix. For example: if all $f_i \leq z_i, i = 1, 2, \cdots, n$, in this part of the constraint set, the Hessian matrix $\nabla^2 g$ is:

$$\begin{pmatrix} \frac{1}{z_1^2} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{z_2^2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{z_3^2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{1}{z_n^2} \end{pmatrix} \tag{2.6}$$

here if any $f_i > z_i$, we just replace $\frac{1}{z_i^2}$ by 0 in the Hessian matrix $\nabla^2 g$.

That means, in each part of the connected set, the function g is a convex function, thus we know the set $\bar{Z}$ is convex set in every region. So we can show the constraint set $\bar{Z}$ is a convex set.

Now we illustrate the difference of the question (2.3) use the original constraint set and the approximation constraint set. Without loss of generality, we take $x = (10, 10)'$ which are corrupted by a gamma noise that variance equals to 0.01, and we get the vector $g = (10.9055, 8.9281)$, $\alpha = 0.2001$, we use our method to recover x from g. At first, we take all the points in the square of $(5, 5)$, $(5, 18)$, $(18, 18)$ and $(18, 5)$. The gap is 0.01 to general gridding, so we can computer all of the points in the region to check whether the point in the constraint set. Then we plot the region of the set Z and $\bar{Z}$ using our definition respectively. The results are shown in the left and middle figures of Figure 1. We add a line in the left figure of Figure 1. From these two figures, we see that the constraint set Z is not a convex set, however the constraint set $\bar{Z}$ is a convex set.

In practice, the set $\bar{Z}$ may be not a good approximation of the origin constraint set. For further studies, when $f_i > z_i$, we use a piecewise linear function to approximate the logarithm function. That mean in every integer interval where $\frac{f_i}{z_i} > 1$, we use the straight-line equation passing the two points: $\lfloor \frac{f_i}{z_i} \rfloor$ and $\lfloor \frac{f_i}{z_i} \rfloor + 1$ to approximate the logarithm function. We define the set as:

$$\bar{\bar{Z}} := \{z \in \mathbb{R}^n | \sum_{i=1}^{n} |F(z_i)| \leq \alpha\}$$

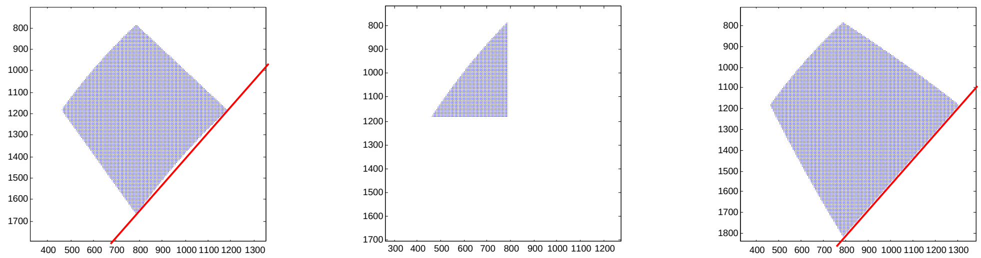


Figure 1: The left is the region of the set Z. The middle is the region of the set $\bar{Z}$. The right is the region of the set $\bar{\bar{Z}}$.

and $F(z_i)$ is defined as:

$$F(z_i) = \begin{cases} \log(f_i/z_i) & \text{if } f_i \leq z_i, \\ B(f_i/z_i) & \text{else.} \end{cases} \tag{2.7}$$

here

$$B(f_i/z_i) = \log\lfloor \frac{f_i}{z_i} \rfloor + (\log(\lfloor \frac{f_i}{z_i} \rfloor + 1) - \log\lfloor \frac{f_i}{z_i} \rfloor)(\frac{f_i}{z_i} - \lfloor \frac{f_i}{z_i} \rfloor)$$

We can prove that the constraint set $\bar{\bar{Z}}$ is a convex set following the similar arguments. And we also illustrate that the region of the constraint set $\bar{\bar{Z}}$ by the above example in the right figure of Figure 1. We also add a line in this figure, we can see that the constraint set $\bar{\bar{Z}}$ is not only a convex set, but also a good approximation of the origin constraint set Z.

2.3 The proposed algorithm

Thus the problem can be transformed as:

$$\begin{aligned} \min \ & |||p|||_1 \\ \text{subject to } \ & x \in \mathbb{R}^n \\ & p \in \mathbb{R}^n \times \mathbb{R}^n, p = \nabla x \\ & z \in \mathbb{R}^n, z = H\mathbf{x}, z \in \bar{\bar{Z}} := \{z \in \mathbb{R}^n | \sum_{i=1}^{n} |F(z_i)| \leq \alpha\} \ . \end{aligned} \tag{2.8}$$

Herein, $F(\cdot)$ can be defined as (2.7). Then let us show that ADM is applicable for solving the problem (2.8).

Let $\chi_{\bar{\bar{Z}}}$ be defined by

$$\chi_{\bar{\bar{Z}}} = \begin{cases} 0, , & \text{if } z \in \bar{\bar{Z}}; \\ \infty, & \text{otherwise.} \end{cases}$$

Then, it is easy to check that (2.8) is a special case of ADM methods where

$$\mathbf{x} := x, \ \mathbf{y} := \begin{pmatrix} p \\ z \end{pmatrix}, \ f_1(\mathbf{x}) := 0, \ f_2(\mathbf{y}) := |||p|||_1 + \chi_{\bar{\bar{Z}}}(z)$$

and

$$B := \begin{bmatrix} \nabla \\ H \end{bmatrix}, \ C := \begin{bmatrix} -I_{n\times n} \\ -I_{n\times n} \end{bmatrix}, \ \mathbf{b} = 0, \ X := \mathbb{R}^n \text{ and } Y := \mathbb{R}^n \times \mathbb{R}^n.$$

Thus the problem can be written as:

$$\begin{aligned} \min \ & f_2(\mathbf{y}) \\ \text{subject to } \ & B\mathbf{x} + C\mathbf{y} = \mathbf{b}, \\ & \mathbf{x} \in X, \mathbf{y} \in Y. \end{aligned}$$

The augmented lagrangian of this problem writes:

$$L(\mathbf{x}, \mathbf{y}, \lambda) = f_2(\mathbf{y}) + < \lambda, B\mathbf{x} - \mathbf{y} > + \frac{\beta}{2} ||B\mathbf{x} - \mathbf{y}||^2 \ .$$

Hence, according to ADM method, we immediately derive the ADM algorithmic framework for solving (2.8) as follows:

Algorithm 1: Alternating Direction Method for solving (2.8)

Input: The maximal number of iterations $ITER$; The starting point $\mathbf{x}^0 \in \mathrm{dom}(f_1)$, $\mathbf{y}^0 \in \mathrm{dom}(f_2)$ and $\lambda^0 \in \mathbb{R}^l$; The initial value of β.

Output: $\mathbf{x}^{ITER}$, an estimate of an element of $\mathbf{x}^*$ satisfying some stopping criterion.

begin

for $k = 0$ *to* $ITER - 1$ **do**

Step 1. Find $\mathbf{x}^{k+1} \in \begin{array}{rl} \arg\min & \langle \lambda^k, B\mathbf{x} - \mathbf{y}^k \rangle + \frac{\beta}{2} \left|\left| B\mathbf{x} - \mathbf{y}^k \right|\right|^2 \\ \text{subject to} & \mathbf{x} \in \mathbb{R}^n \end{array}$

Step 2. Find $\mathbf{y}^{k+1} \in \begin{array}{rl} \arg\min & f_2(\mathbf{y}) + \langle \lambda^k, B\mathbf{x}^{k+1} - \mathbf{y} \rangle + \frac{\beta}{2} \left|\left| \mathbf{y} - B\mathbf{x}^{k+1} \right|\right|^2 \\ \text{subject to} & \mathbf{y} \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \end{array}$

Step 3. $\lambda^{k+1} = \lambda^k + \beta(B\mathbf{x}^{k+1} + C\mathbf{y}^{k+1} - \mathbf{b})$.

We now elaborate on the solutions of the subproblems of the ADM method for (2.8). For the first subproblem, it is easy to see that it amounts to solving a system of linear equations. Thus, the analytical solution of this subproblem can be determined by solving a linear system:

$$\beta B^T B\mathbf{x} = B^T(\mathbf{y}^k - \lambda^k) \;\; \text{or} \;\; \beta(\nabla^T \nabla + H^T H)\mathbf{x} = \begin{bmatrix} \nabla \\ H \end{bmatrix}^T (\mathbf{y}^k - \lambda^k) \tag{2.9}$$

This linear system can be diagionalized by the DFT, so it makes no difficulty to get the solution $\mathbf{x}^{k+1}$.

For the second subproblem, first of all, the solution of this problem is unique as it is a convex problem. Second, We will also make use of the proximal operator made popular in signal processing in papers like [4]. Let f be a convex closed function.The proximal operator of f is defined for $x^0 \in \mathbb{R}^n$ by:

$$\mathrm{prox}_f(x^0) = \begin{array}{rl} \arg\min & f(x) + \frac{1}{2}||x - x^0||^2 \\ \text{subject to} & x \in \mathbb{R}^n \end{array} .$$

Then, we have

$$\begin{aligned} \mathbf{y}^{k+1} &= \begin{array}{rl} \arg\min & f_2(\mathbf{y}) + \langle \lambda^k, B\mathbf{x}^{k+1} - \mathbf{y} \rangle + \frac{\beta}{2} \left|\left| \mathbf{y} - B\mathbf{x}^{k+1} \right|\right|^2 \\ \text{subject to} & \mathbf{y} \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \end{array} \\ &= \begin{array}{rl} \arg\min & f_2(\mathbf{y}) + \frac{\beta}{2} \left|\left| \mathbf{y} - \left(B\mathbf{x}^{k+1} + \frac{\lambda^k}{\beta} \right) \right|\right|^2 \\ \text{subject to} & \mathbf{y} \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \end{array} \\ &= \mathrm{prox}_{f_2/\beta} \left(B\mathbf{x}^{k+1} + \frac{\lambda^k}{\beta} \right). \end{aligned}$$

We can write $\begin{pmatrix} p^k \\ z^k \end{pmatrix} = Bx^{k+1} + \frac{\lambda^k}{\beta}$ and decompose $\mathrm{prox}_{f_2/\beta}\left(\cdot \right)$ as:

$$\mathrm{prox}_{f_2/\beta} \begin{pmatrix} p \\ z \end{pmatrix} = \begin{pmatrix} \mathrm{prox}_{|||\cdot|||_1/\beta}\left(p \right) \\ \mathrm{prox}_{\chi_{\bar{\bar{Z}}}/\beta}\left(z \right) \end{pmatrix}$$

If we let shrink operation as:

$$\mathrm{shrink}_\beta(y^0) = \mathrm{prox}_{\beta|||\cdot|||_1}(y^0) = \begin{array}{rl} \arg\min & \beta|||y|||_1 + \frac{1}{2}||y - y^0||^2 \\ \text{subject to} & y \in \mathbb{R}^n \times \mathbb{R}^n \end{array} .$$

and

$$(\mathrm{shrink}_\beta(y^0))(i,j) = y^0 - \min(\beta, |y^0|) \cdot \frac{y^0}{|y^0|} .$$

where $\frac{y^0}{|y^0|}$ should be taken as 0 on places where $|y^0| = 0$. We can prove that $\mathrm{prox}_{|||\cdot|||_1/\beta}\left(p \right) = \mathrm{shrink}_{1/\beta}\left(p \right)$.

Now let us focus on

$$\begin{aligned} \mathrm{prox}_{\chi_{\bar{\bar{Z}}}/\beta}\left(z \right) &= \Pi_{\bar{\bar{Z}}}(z) \\ &= \begin{array}{rl} \arg\min & \frac{1}{2}\left|\left| z' - z \right|\right|^2 \\ \text{subject to} & z' \in \mathbb{R}^n \\ & \left|\left| F(z') \right|\right|_1 \leq \alpha \end{array} \end{aligned}$$

here, the function $F(\cdot)$ is defined by (2.7). This operation is a projection onto a weighted l_1-ball. We can compute $F(z')$ by definition (2.7) at first, then we can choose $\alpha = ||F(\frac{f}{Hx})||_1$, finally we project it on the weighted l_1-ball and take an exponential of it to get z^{k+1}. We refer the reader to the appendix of [20] for a detailed implementation.

3 Experimental Results

In this section, numerical simulations are performed to validate the efficiency of the proposed method for deblurring and multiplicative noise removal simultaneously. We compared our results with those obtained by the RLO model and the AA model. All experiments were performed under Windows XP and Matlab v7.7 running on a desktop with an Intel (R) Core (Tm)2 Quad CPU 2.66GHz and 4GB of RAM memory.

The images "Lena", "Rice", "Cameraman" (image size is 256×256) and "Barbara" (image size is 512×512) are used in our experiment. In addition, in the experiments of image restoration, the peak signal noise ratio (PSNR) and the relative error (ReEr) are used to show the restoring quality of the images. Suppose the image size is $m-$by$-n$, PSNR values is defined as follows:

$$\mathrm{PSNR} = 10 \log_{10}(\frac{\mathrm{V}^2}{\mathrm{MSE}}),$$

where $\mathrm{MSE} = \sum_{i=1}^{m} \sum_{j=1}^{m} (\tilde{w}(i,j) - w(i,j))^2/mn$, w and $\tilde{w}$ are the original image and the recovered image respectively, and $\mathrm{V} = \max_{i,j}(\tilde{w}(i,j), w(i,j))$. The relative error of the image $\tilde{w}$ is defined by

$$\mathrm{ReEr} = \frac{||\tilde{w} - w||_2^2}{||w||_2^2}.$$

The stopping criterion for all the experiments by the proposed model, the RLO model and the AA model is that the PSNR value of the restored image reaches its maximum.

In the experiments, the blurred images are degraded by a gamma noise n the mean is one. The probability density function is given by

$$P_\Gamma(n; L) = \begin{cases} \frac{L^L n^{L-1}}{\Gamma(L)} e^{-Ln}, & n > 0; \\ 0, & n \leq 0. \end{cases} \tag{3.1}$$

The level of the mean one gamma noise is determined by the parameter L since the variance of it equals the reciprocal value of L.

In the experiments, the original image corrupted by a motion blur (Matlab function is fspecial('motion', len, theta)), a gaussian blur (Matlab function is fspecial('gaussian', hsize, sigma)) or a disk blur (Matlab function is fspecial('disk', radius)) respectively and different level multiplicative gamma noise with variance 0.01 and 0.03. The relative error (ReEr), PSNRs of recovered images and the restoring time are shown in the table (1). We just show the result of the "Lena" in the figure (2-7) . From these figures and the table, we can see that our algorithm is very effective to remove the different blur and the multiplicative noise simultaneously.

4 Conclusion

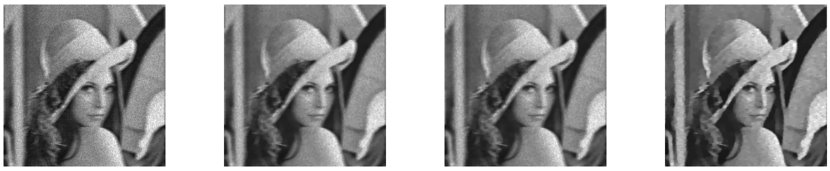
We have proposed a simple and very efficient algorithm for the restoration of blurred images corrupted with multiplicative gamma noise. We use alternating direction methods to handle the optimization problem. In order to get a good convergence we use a approximate convex set to replace the original nonconvex constraint set. So our algorithm's convergence is guaranteed by known convergence results of the alternating direction methods. In the numerical experiment, we show our method is more effective than the other methods by recovering blurred and noisy image simultaneously.

References

[1] G. Aubert and J.F. Aujol. A variational approach to remove multiplicative noise. *SIAM J. Appl. Math*, 68(4):925–946, 2008.

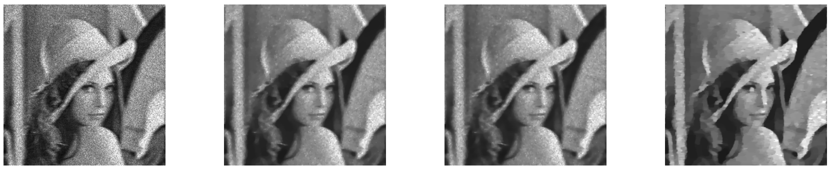
Image	Noise variance	Used method	motion blur			gaussian blur			disk blur		
			PSNR	ReEr	Time	PSNR	ReEr	Time	PSNR	ReEr	Time
Lena	0.01	AA	23.46	0.1463	9.11	22.69	0.1513	12.68	21.90	0.1715	12.63
		RLO	23.42	0.1471	17.44	22.74	0.1555	13.39	21.75	0.1717	26.50
		Our	25.36	0.1205	3.47	24.71	0.1312	1.57	23.41	0.1479	1.35
	0.03	AA	23.82	0.1543	20.60	23.90	0.1626	20.19	22.96	0.1822	18.76
		RLO	23.80	0.1556	37.41	23.87	0.1642	34.82	22.90	0.1839	31.34
		Our	25.04	0.1373	13.41	24.13	0.1513	1.89	24.13	0.1513	1.89
Cameraman	0.01	AA	22.80	0.1472	10.24	21.37	0.1637	8.03	20.81	0.1730	21.05
		RLO	22.74	0.1480	16.62	21.37	0.1642	13.66	20.78	0.1736	34.75
		Our	24.71	0.1234	6.42	22.97	0.1405	1.57	22.11	0.1547	1.49
	0.03	AA	22.42	0.1554	22.76	21.63	0.1709	20.01	21.33	0.1863	18.41
		RLO	22.18	0.1574	35.67	21.62	0.1719	33.21	21.32	0.1870	31.33
		Our	23.18	0.1422	2.57	22.00	0.1577	2.00	22.84	0.1699	1.92
Rice	0.01	AA	22.96	0.1701	6.74	21.42	0.2000	5.40	19.54	0.2375	3.20
		RLO	22.95	0.1704	11.29	21.37	0.2020	7.41	19.52	0.2389	4.13
		Our	25.31	0.1307	2.20	24.32	0.1441	2.21	23.01	0.1667	1.56
	0.03	AA	23.75	0.1804	19.01	22.32	0.2106	14.88	21.39	0.2448	12.70
		RLO	23.67	0.1827	30.11	22.24	0.2130	24.12	21.34	0.2468	20.22
		Our	25.20	0.1446	5.25	23.91	0.1760	1.99	23.23	0.1927	1.67
Barbara	0.01	AA	23.52	0.1582	41.21	23.13	0.1656	38.09	22.49	0.1741	34.84
		RLO	23.46	0.1591	69.05	23.12	0.1661	63.11	22.46	0.1747	56.58
		Our	23.78	0.1528	5.66	23.20	0.1582	5.60	22.71	0.1628	5.58
	0.03	AA	24.32	0.1650	96.06	24.17	0.1722	92.46	23.34	0.1806	85.39
		RLO	24.28	0.1657	161.03	24.17	0.1727	155.33	23.34	0.1811	146.02
		Our	24.75	0.1617	8.75	24.31	0.1694	8.03	23.48	0.1737	8.04

Table 1: The recovered results for the different blur and gamma noise.



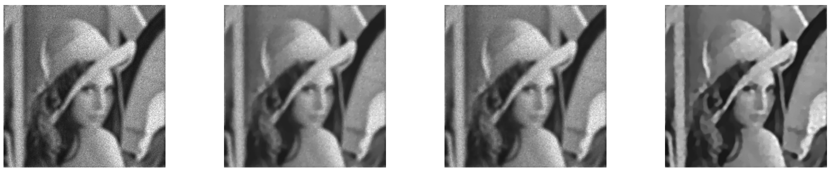
(a) Destoryed image (b) AA method (c) RLO method (d) Our method

Figure 2: Motion blur (fspecial(‘motion’,7)) and gamma noise variance is 0.01.



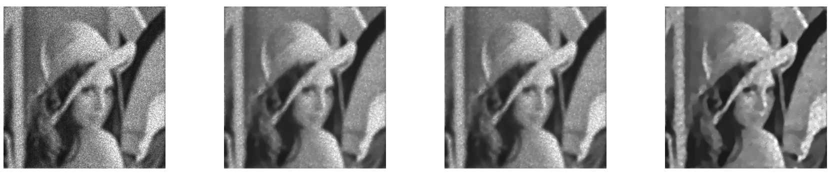
(a) Destoryed image (b) AA method (c) RLO method (d) Our method

Figure 3: Motion blur (fspecial(‘motion’,7)) and gamma noise variance is 0.03.



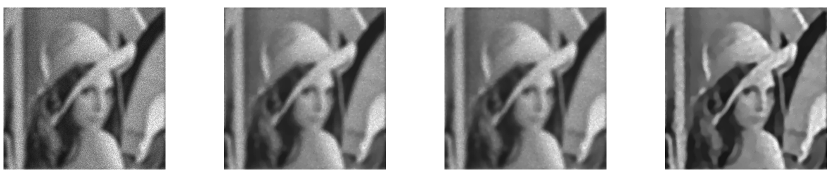
(a) Destoryed image (b) AA method (c) RLO method (d) Our method

Figure 4: Gaussian blur (fspecial(‘gaussian’,7,5)) and gamma noise variance is 0.01.



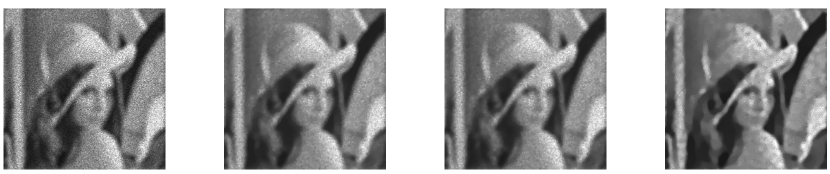
(a) Destoryed image (b) AA method (c) RLO method (d) Our method

Figure 5: Gaussian blur (fspecial(‘gaussian’,7,5)) and gamma noise variance is 0.03.



(a) Destoryed image (b) AA method (c) RLO method (d) Our method

Figure 6: Disk blur (fspecial(‘disk’,5)) and gamma noise variance is 0.01.



(a) Destoryed image (b) AA method (c) RLO method (d) Our method

Figure 7: Disk blur (fspecial(‘disk’,5)) and gamma noise variance is 0.01.

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