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**An AMG solver for the 3D time-harmonic Maxwell  
equations**

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We describe a parallel algebraic multigrid (AMG) method for the solution of the 3D Maxwells equations in the frequency domain in the quasi-static limit. This AMG method is intended as a preconditioner to a Krylov iterative method such as quasi-minimum residual, where the application of interest is the repeated solution of the forward modeling problem arising in geophysical subsurface imaging applications. The underlying formulation of the new frequency-domain AMG method leverages off of an AMG scheme for real values Maxwell problems arising from the time-dependent eddy current equations. The central components of the real valued AMG method are distributed relaxation for the smoother and a specialized grid transfer operator for the coarse grid correction. The key to this AMG method is the proper representation of the  $(\text{curl}, \text{curl})$  null space on coarse meshes. This is achieved by maintaining certain commuting properties of the inter-grid transfers. To adapt the real valued AMG scheme to complex arithmetic, the complex operator is first written as a  $2 \times 2$  real block matrix system, called an equivalent real form (ERF). The inter-grid transfers for the ERF are then generated via the real valued AMG algorithm. The smoother for the ERF is also adapted from the real value AMG method. This distributed relaxation on the ERF matrix in turn leads to a nice decoupling of the problem. To complete the method, a variety of smoothers, including complex polynomial, jacobi and one-step krylov smoothers are developed for use within the distributed relaxation process. While some care is required to develop these smoothers, they work well in parallel and avoid difficulties associated with parallel Gauss-Seidel. Numerical experiments are presented for some 3D problems arising in geophysical subsurface imaging applications. The experiments illustrate the efficiency of the approach on various parallel machines in terms of both convergence and parallel speed-up.