## Eric C. Cyr Approximate Block Factorization and Physics-based Preconditioning: Application to CFD and MHD

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Multiphysics applications are characterized by strongly-nonlinear coupled physical mechanisms that produce a solution with a wide-range of length and time scales. As a result, developing effective preconditioners for these systems can be a formidable challenge. One approach, which has demonstrated some success in a range of multiphysics applications (low Mach number CFD, reacting flow, drift-diffusion simulations and low Mach number resistive MHD), is a multigrid method with incomplete factorization smoothing. The coarsening is an aggressive graph-based aggregation applied to the nonzero block structure of the Jacobian matrix. These techniques have been shown to have favorable scaling properties for a number of nearly-elliptic systems. However, it is an open question on how robust and efficient these methods will be as the physics increases in complexity or progresses towards the hyperbolic limit.

An alternative approach is to use approximate block factorizations for preconditioning of multiphysics systems. These methods segregate the linear operator into different sub-matrices based on the components of the physics. These individual components are typically more amenable to black-box AMG technology. The difficulty with this approach is that an effective approximation of the physical coupling embodied in the Schur complement operator is required. For the Navier-Stokes equations this has been a very active area of study. The approaches have included the basic physics-based SIMPLE solution method to more sophisticated techniques based on commuting arguments, such as the pressure-convection diffusion or least-squares commutator preconditioners.

In this talk performance results for both preconditioning techniques are compared. Several applications are considered, including Navier-Stokes and compressible and incompressible Magnetohydrodyanamics. The specific Schur complement approximations as well as the implementation of these methods in the recently developed Trilinos software package "Teko" will be discussed.