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Solution of the nonlinear multifrequency radiation diffusion equation in a multiphysics, high energy density, AMR code

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We describe a scheme to solve the multifrequency radiation diffusion equation which is intended for a multiphysics, high energy density computer code with adaptive mesh refinement (AMR). In our code, AMR is implemented by refining in both space and time [1]. There may be several levels of refinement, which, going from fine to coarse, are nested within each other.

We time-advance as follows. Assume there are only two levels, one coarse, with domain Ω_c and boundary $\partial\Omega_c$, and one fine Ω_f with boundary $\partial\Omega_f$. Since the domains are nested, $\Omega_f \subseteq \Omega_c$. At the start of the time cycle, the equations are first updated on Ω_c using a timestep Δt_c, a process defined as a *level solve* on Ω_c. If the spatial grid on Ω_f is a twofold refinement of that discretizing Ω_c, we need two level solves on Ω_f, each with timestep $\Delta t_c/2$, in order to bring the Ω_f solution up to the advanced coarse level time. Boundary conditions (BC) are required on $\partial\Omega_f$. On parts of $\partial\Omega_f$ which do not extend to the physical boundary, BC are obtained by interpolating the coarse grid solution. For diffusion equations, e.g., $u_t = (Du_x)_x$, conventionally, one supplies Dirichlet data. This ensures that the coarse and fine grid solution is continuous across $\partial\Omega_f$. However, the flux $-Du_x$ may be discontinuous, which is unacceptable since this results in a loss of conservation. To remedy the defect, after the level solves, the coarse and fine grid solutions are *synced.* One solves a related, nearly homogeneous, problem for corrections on the union of discretizations of Ω_f and Ω_c. The sole non-homogeneity of the system for the corrections is the miss-match of the fluxes on $\partial\Omega_f$. When the corrections are added to the result of the level solves, one obtains a conservative solution, continuous and with continuous flux.

This paper describes the AMR implementation for the multigroup radiation diffusion and matter energy balance equations,

$$\partial_t u_g = \nabla \cdot D_g \nabla u_g + \kappa_g \, (B_g - u_g) , \quad g = 1, \ldots , G \tag{1}$$

$$\rho \, c_v \partial_t T = - \sum_{k=1}^{G} \Delta_k \, \kappa_k \, (B_k - u_k) . \tag{2}$$

In (1)–(2), u_g is the radiation energy density of the gth *group.* Groups arise by discretizing the frequency domain $0 \leq \nu \leq \infty$ into G intervals. In (1)–(2), D_g and κ_g are the diffusion and coupling coefficients, B_g is the Planck function, ρ the mass density, c_v the specific heat, and $\Delta_k = \nu_k - \nu_{k-1}$. The system is nonlinear; D_g and κ_g, which in addition to being strong functions of frequency, depend on ρ and T. For non-ideal gases, c_v depends on ρ and T. Equations (1)–(2) describe the evolution of the $G+1$ unknowns $\{u_k\}_{k=1}^{G}$ and T.

A single level solve of (1)–(2) is a formidable task in itself. For the advance, we use the procedure described by Shestakov [3], generalized for "real," multiple materials whose properties (c_v, k_g, etc.) are given in tabular form.

For simulations using AMR, after advancing on two levels, Ω_c and Ω_f, the solutions are synced using a generalization of the Howell and Greenough procedure (HG) [1], which may be directly applied to (1)–(2) if $G = 1$. However, if $G > 1$, the situation is more complicated since the energies u_g are coupled. We resolve the difficulty by applying concepts of the "Partial Temperature" scheme (PT) of Lund and Wilson [2]. As in PT, we cycle through the groups in random order. Each group is synced as in HG, but the correction to T is only a partial change. Only after all the groups have been addressed, do we obtain the final correction.

Our AMR procedure is implemented in a multiphysics code. Results will be presented. We simulate effects of strong explosions in air and compare multigroup results with runs where the frequency domain is not discretized, so-called gray diffusion. The simulations also use the hydro and heat conduction modules. In addition to the coarse level, there are two levels of refinement.

Bibliography

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[3] A. I. Shestakov, *8th Copper Mountain Conference on Iterative Methods,* March 2004.