Michael Overton Nonsmooth, Nonconvex Optimization: Theory, Algorithms and Applications

Courant Institute 251 Mercer St New York NY 10012 overton@cs.nyu.edu

Theory: there are two standard approaches to generalizing derivatives to nonsmooth, nonconvex optimization: the Clarke subdifferential (or generalized gradient), and the MIRW subdifferential (or subgradient sets), as expounded in Rockafellar and Wets (Springer, 1998). We briefly discuss these and mention their advantages and disadvantages. They coincide for an important class of functions: those that are locally Lipschitz and regular, which includes continuously differentiable functions and convex functions.

Algorithms: the usual approach is bundle methods, which are complicated. We describe some alternatives: BFGS (a new look at an old method), and Gradient Sampling (a simply stated method that, although computationally intensive, has solved some previously unsolved problems and has a nice convergence theory).

Applications: these abound in control, but surely in other areas too. Of particular interest to me are applications involving eigenvalues and singular values of nonsymmetric matrices. Sometimes even easily stated problems in a few variables are hard. Our new code HIFOO (H-Infinity Fixed-Order Optimization) is intended for use by practicing control engineers and has solved some open problems in control.

This is all joint work with James Burke and Adrian Lewis. HIFOO is also joint with Didier Henrion.