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**Design Optimization and Uncertainty Analysis by Using
Gradient/Hessian Enhanced Surrogate Model Approaches**

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Numerical design optimization in the field of aerospace engineering has attracted attention these days thanks to the maturity of Computational Fluid Dynamics (CFD) and the increase in computer performance. The major bottleneck of aerodynamic design optimization with global search methods, such as genetic algorithms (GA), is the huge number of required CFD function evaluations which can exceed several thousand. Recently, therefore, optimization approaches based on surrogate models have attracted attention because of the capability to reduce the number of CFD function evaluations within design optimization.

Recently, efficient gradient and approximate Hessian calculation methods using adjoint method and automatic differentiation (AD) have been developed in our group [1]. In these approaches, the computational costs for a gradient vector and Hessian matrix with respect to geometrical variables are respectively comparable to one CFD function evaluation. Thus it is promising to utilize the gradient and Hessian information within surrogate models to obtain more accurate response surface. Such enhanced response surface will promote the efficiency of design optimization process.

Surrogate models can be constructed using a variety of methods [2]. In this work, polynomial regression, radial basis function (RBF) and Kriging methods have been extended to be enhanced by gradient and Hessian information. Two gradient enhanced Kriging (direct and indirect cokriging) approaches have been developed and applied to design optimizations in the literature [3]. The indirect cokriging approach has been extended in this work to make use of gradient/Hessian information. This is because the number of sample points which have gradient/Hessian information can be easily limited in this approach, which results in the reduction of computational cost to construct surrogate model.

The developed gradient/Hessian enhanced surrogate model approaches have been firstly investigated for multi-dimensional analytical function fitting problems. The results showed that surrogate models enhanced by gradient/Hessian

information provided better fitting on the actual functions. These methods showed better efficiencies even if taking the computational cost for gradient/Hessian evaluation into consideration. Then, these methods were applied to a drag minimization problem of a two-dimensional transonic airfoil. A faster reduction of the objective function value, in other words faster convergence towards the global optimal design was confirmed with the gradient/Hessian enhanced model approaches. Since the computational costs for CFD gradient and Hessian evaluation are independent from the number of dimensionality of problem, the developed methods should be more capable in higher-dimensional complex problems.

Uncertainty analysis is also watched with keen interests these days [4-5] since high-fidelity CFD computations typically assume perfect knowledge of all parameters. In reality, however, there is much uncertainty due to manufacturing tolerances, in-service wear-and-tear, approximate modeling parameters and so on. The most straightforward and accurate method for uncertainty analysis is a full nonlinear Monte Carlo (MC) simulation. Although this method is easy to implement, it is still prohibitively expensive for high-fidelity CFD computations. Moment methods based on Taylor series expansions are alternative ways to assess uncertainty although these methods only give the mean and standard deviation of the output function. To apply surrogate model approaches is another positive way for uncertainty analysis which can reduce the computational cost dramatically. The computational advantage is evident because the surrogate model is an analytic representation of the design space. Its estimated function values can be used for an inexpensive Monte Carlo (IMC) simulation to obtain not only mean and standard deviation, but also an approximate probability density function (PDF) of the output function. Our accurate response surfaces enhanced by gradient and Hessian information can increase the accuracy of uncertainty analysis by using IMC. The results of uncertainty analysis will be shown in the presentation to compare the performances with full nonlinear MC simulation. This approach is promising since it can be easily extended to gRobust Designh approach because of the lower computational cost of the uncertainty analysis.

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