Ilya Lahuk STEEPEST DESCENT AND CONJUGATE GRADIENT METHODS WITH VARIABLE PRECONDITIONING

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We show that the conjugate gradient method with variable preconditioning in certain situations cannot give any improvement compared to the steepest descent method for solving a linear system with a symmetric positive definite (SPD) matrix of coefficients. We assume that the preconditioner is SPD on each step, and that the condition number of the preconditioned system matrix is bounded from above by a constant independent of the step number. Our proof is geometric and is based on the simple fact that a nonzero vector multiplied by all SPD matrices with a condition number bounded by a constant generates a circular cone.

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