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Parallel Preconditioned Iterative Solvers for Inductance Extraction

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The design phase of VLSI circuits needs to take into account the signal delays due to parasitic resistance (R), capacitance (C), and inductance (L). With increasing clock frequency and integration density, signal delays in next-generation circuits with millions of interconnect segments will be dominated by parasitic inductance. In this paper, we develop fast and accurate inductance extraction techniques that are critical to the design of future VLSI circuits. We present a parallel preconditioned iterative algorithm for solving the linear system arising in inductance extraction. The reduced system, obtained in divergence free subspace using solenoidal basis method, is solved efficiently by preconditioned GMRES method. Parallel implementations of the algorithm have also shown good scalability and speedup on multiprocessors.

For a set of interconnect segments, the inductance extraction problem consists of finding an impedance matrix that represents pairwise mutual impedance among the segments at a given frequency. Each interconnect segment is discretized using a uniform mesh whose edges represent current carrying filaments. The equation for the potential drop across each filament and Kirchoff's conservation law on node currents give rise to the following liner system [2]:

$$
\begin{bmatrix} \mathbf{Z} & -\mathbf{B} \\ \mathbf{B}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{I}_f \\ \mathbf{V}_n \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \end{bmatrix}, \tag{1}
$$

where $\mathbf{Z} = \mathbf{R} + j\omega\mathbf{L}$ is $n \times n$ impedance matrix, $\mathbf{R}$ is diagonal matrix of filament resistance, $\mathbf{L}$ is a dense, symmetric, positive definite inductance matrix denoting the inductive coupling between current carrying filaments. $\mathbf{B}^T$ is the $m \times n$ branch index matrix of filaments and nodes containing only two nonzero entries, -1 and 1, per row. $\mathbf{I}_f$ is the unknown filament current vector, and $\mathbf{V}_n$ is the unknown node potential vector. Here, j denotes the complex root of unity and ω represents the operational frequency. The coefficient matrix in (1) consists of a sparse submatrix and a dense submatrix. Entries in $\mathbf{L}$ are given by:

$$\mathbf{L}_{kl} = \frac{\mu}{4\pi} \frac{1}{a_k a_l} \int_{r_k \epsilon V_k} \int_{r_l \epsilon V_l} \frac{\mathbf{u}_k . \mathbf{u}_l}{||\mathbf{r}_k - \mathbf{r}_l||} dV_k dV_l,$$

The current $\mathbf{I}_f$ may be expressed as the matrix vector product $\mathbf{I}_f = \mathbf{Px}$ for arbitrary $\mathbf{x}$, where $\mathbf{P}$ is a basis for null space of $\mathbf{B^T}$. This leads to a reduced system of the form:

$$\mathbf{P^T ZPx = P^T F} \qquad (2)$$

which is solved using an iterative method such as GMRES. A sparse basis for the null space of $\mathbf{B^T}$ can be constructed using local solenoidal functions that represent a unit current circulating along the filaments of a single mesh cell. Each such local flow contributes a column to the matrix $\mathbf{P}$. (see, e.g., [4] for more details.)

The most time consuming step in the iterative solver is matrix vector product with $\mathbf{L}$ which appears in the reduced system (2). Use of fast hierarchical methods like Fast Multipole Method [1] makes the algorithm matrix-free and reduces the time complexity from $O(n^2)$ to $O(n)$. In absence of the system matrix, however preconditioning the iterative method becomes a challenging task.

We present a nearly optimal preconditioner [4] for the reduced system that demonstrates convergence independent of the problem parameters, namely the mesh size h and the frequency of operation ω. Preconditioning step is analogus to computing the mutual impedance of filaments placed at the center of each mesh cell. The preconditioner has a structure similar to $\mathbf{L}$, and is applied to a vector using fast hierarchical methods. Following table shows the number of right preconditioned GMRES iterations to compute the self-impedance of the ground plane conductor problem.

Mesh	Filament	Frequency		
Size	Length	1 GHz	10 GHz	100 GHz
65×65	2^{-6}cm	6	6	5
129×129	2^{-7}cm	7	6	6
257×257	2^{-8}cm	9	7	7
513×513	2^{-9}cm	12	8	7

On a set of benchmark problems, a serial implementation of this technique is up to 5 times faster than *FastHenry*, a popular inductance extraction software, with only one-fifth of memory requirements. Further enhancements to the implementation are achieved by exploiting the parallelism in the algorithm [3]. Following table shows the parallel efficiency of the code on the IBM p690 multiprocessor.

Ongoing work involves development of an MPI-based parallel object oriented code for the algorithm for distributed memory multiprocessors. Performance evaluation of a hybrid MPI-OpenMP implementation is also underway.

No. of Processors	Mesh Size					
	128×128		256×256		512×512	
	Time	Efficiency (%)	Time	Efficiency (%)	Time	Efficiency (%)
1	769.87	100.0	3727.04	100.0	17163.5	100.0
2	385.00	100.0	1864.51	100.0	8610.04	99.7
4	192.74	99.8	937.95	99.3	4315.18	99.4
8	102.72	93.8	482.45	96.6	2203.06	97.4
16	54.84	87.7	249.91	93.3	1192.77	89.9

Bibliography

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