Misha Kilmer A Regularized Gauss-Newton Method for Nonlinear Imaging Problems in Diffuse Optical Tomography

Mathematics Dept
Tufts Univ
503 Boston Ave
Medford
MA 02155
misha.kilmer@tufts.edu
Eric de Sturler

The goal in diffuse optical tomography for medical imaging is the joint reconstruction of parameterized images of absorption and scattering of light in the body. The reconstruction requires the approximate solution of a nonlinear least squares problem for the image parameters. While traditional approaches such as damped Gauss-Newton (GN) and Levenberg-Marquardt (LM) have been shown to be effective at solving the imaging problem in its various forms, a considerable number of additional function and Jacobian evaluations are involved in determining the correct step length and/or damping parameter. In 3D imaging problems, depending on the particular image model, the cost of one function evaluation is, at a minimum, the cost of a dense matrix-vector product and in the worst, but more realistic, case requires the solution of several large-scale PDEs. A Jacobian evaluation is even slightly more expensive. Therefore, it is crucial to keep the number of function and Jacobian evaluations to a minimum.

The ill-conditioning of the Jacobian, together with the presence of noise in the data, motivates us to devise a regularized, trust-region-based Gauss-Newton approach for determining search directions. Although LM can be thought of as a regularized analogue to determining the GN direction, LM has the property of damping possibly important contributions to the search direction in spectral components corresponding to small singular values. On the other hand, the Gauss-Newton direction is too influenced by components due to small singular values early on, causing the line search to work hard to refine the step length. We propose a method that systematically evaluates the potential contribution of each of the spectral components corresponding to the GN-direction and constructs the new direction relative to this contribution within the confines of a trust-region. Examples show the success of our method in minimizing function evaluations with respect to other well-known methods.