We consider the solution of large scale optimization problems governed by parabolic partial differential equations (PDEs). A quadratic functional containing a data misfit term, is minimized to approximately recover the parameter function. The resulting constrained optimization problem is solved by using the reduced Hessian approach. The conjugate gradient method (CG) is employed for the solution of the system involving matrix vector multiplications which are nontrivial. These matrix vector products do not need to be computed exactly. We investigate two approaches for determining the inner iteration tolerance at each CG step and provide numerical experiments comparing these approaches.