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A comparison of iterative solvers for the stochastic finite element method

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The stochastic finite element method is an important technique for solving stochastic partial differential equations (PDEs). This method approximates the solution of the PDE by a generalized polynomial chaos expansion. By using a Galerkin projection in the stochastic dimension, the stochastic PDE is transformed into a coupled set of deterministic PDEs. A finite element discretization converts the deterministic PDEs into a high dimensional algebraic system. Specialized iterative solvers are required to solve this system.

A number of specialized solvers have already been proposed, for example a conjugate gradient solver with multigrid preconditioning based on coarsening the spatial domain, or conjugate gradients combined with a Jacobi-based preconditioner. Here, we shall present an overview of solution approaches. We start from iterative methods based on a block splitting of the algebraic system matrices. Next, we extend these methods for use as preconditioner or for use in a multilevel context. Then, the various solvers are compared based on their convergence properties, computational cost and implementation effort.

Our findings are illustrated on two numerical problems. The first is a steady-state diffusion problem with a discontinuous random field as diffusion coefficient. The second is a deterministic diffusion problem defined on a random domain.