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A Comparison of Geometric and Algebraic Multigrid for the Discrete Convection-Diffusion Equation

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The purpose of this paper is to evaluate different solution strategies for the linear systems obtained from discretization of the convection-diffusion equation

$$\begin{cases} -\epsilon \triangle u + b \cdot \nabla u = f, \\ u = g \quad \text{on } \partial\Omega, \end{cases}$$

where b and f are sufficiently smooth and the domain Ω is convex with Lipschitz boundary $\partial\Omega$. We are interested in convection-dominated case, i.e. $|b| \gg \epsilon$. In this setting, the solution typically has steep gradients in some parts of the domain Ω. These may take the form of boundary layers caused by Dirichlet conditions on the outflow boundaries or internal layers caused by discontinuities in the inflow boundaries.

It is well known that the standard Galerkin finite element discretization on uniform grids produce inaccurate oscillatory solutions to these problems. On the other hand, with carefully chosen stabilization parameter, the streamline diffusion finite element discretization (SDFEM) is able to eliminate most oscillations and produce accurate solutions in the regions where no layers are present. To increase accuracy of the solutions in the regions where layers are present, an adaptive mesh refinement process can be used. The adaptive mesh refinement process in this paper consists of the following steps.

1. The posteriori error estimator proposed by Kay and Silvester is computed.
2. The maximum marking strategy is employed to select elements in which the values of the error estimator are large.
3. The selected elements are refined by regular refinement.

This adaptive mesh refinement process can be applied recursively until a certain tolerance of the error between the finite element solution and the true solution is met.

In this paper, we are concerned with the costs of solving the discrete system obtained when the effective discretization strategy SDFEM together with an adaptive mesh refinement process are used. We would like to explore the effectiveness of the geometric multigrid (GMG) and algebraic multigrid (AMG) methods for solving this linear system. Our numerical studies suggest that the Krylov-subspace accelerated AMG is a robust solver for the convection-diffusion problems.