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On the Manifold-Mapping Optimization Technique

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Some optimization problems in practice imply the computation of cost functions that rely on models which are very expensive to evaluate. An example is any simulation-based design with an accurate finite element discretization. As a consequence, many optimizations may require very long computing times. The space-mapping (SM) technique [1, 2] was developed as an alternative in these situations.

In SM terminology, the accurate but expensive-to-evaluate models are called *fine* models $\mathbf{f} : X \subset \mathbb{R}^n \to \mathbb{R}^m$. The SM method needs a second and computationally faster model, the *coarse* model $\mathbf{c} : Z \subset \mathbb{R}^n \to \mathbb{R}^m$, in order to speed-up the optimization process. The key element in this technique is a right-preconditioning, known as the *SM function* $\mathbf{p} : X \to Z$, that aligns the two model responses. The function $\mathbf{c}(\mathbf{p}(\mathbf{x}))$ corrects the coarse model and can be used as a surrogate for the fine model in the accurate optimization. In most cases the SM function is much simpler than the fine model, in the sense that it is easier to approximate. This fact endows the SM technique its well-reported efficiency. However, it not always converges to the right solution.

Defect-correct theory [3] helps to see that in order to achieve the accurate optimum, the SM function is generally insufficient and also left-preconditioning is needed. In [4] we introduce the mapping $\mathbf{s} : \mathbf{c}(Z) \to \mathbf{f}(X)$ and the associated manifold-mapping (MM) algorithm. MM employs $\mathbf{s}(\mathbf{c}(\bar{\mathbf{p}}(\mathbf{x})))$ as the fine model surrogate. In that context, the function $\bar{\mathbf{p}} : X \to Z$ is not the above SM function but an arbitrary (easy-to-compute) bijection. The MM algorithm is as efficient as SM but converges to the accurate optimal solution [4, 5].

In the first part of the presentation the MM algorithm will be briefly introduced and a proof of convergence will be given. The use of more than two models (multi-level approach) and the possibility of having a coarse model with a different dimension than the fine one ($X \subset \mathbb{R}^{n_\mathbf{f}}$ and $Z \subset \mathbb{R}^{n_\mathbf{c}}$ with $n_\mathbf{f} \neq n_\mathbf{c}$) will be the issues dealt in the second part of the talk.

Bibliography

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