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H(div) preconditioning for a mixed finite element formulation of the stochastic diffusion problem

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We study $H(div)$ preconditioning for the saddle-point systems that arise in a stochastic Galerkin mixed formulation of the steady-state diffusion problem with random data. The key ingredient is a multigrid V-cycle for a weighted, stochastic $H(div)$ operator, acting on a certain tensor product space of random fields with finite variance. The traditional deterministic Arnold-Falk-Winther multigrid algorithm is exploited by varying the spatial discretization from grid to grid whilst keeping the stochastic discretization fixed. We extend the deterministic analysis to accommodate the modified $H(div)$ operator and establish spectral equivalence bounds with a new multigrid V-cycle operator that are independent of the discretization parameters. We then implement multigrid within a block-diagonal preconditioner for the full stochastic saddle-point problem, summarize eigenvalue bounds for the preconditioned system matrices and investigate the impact of all the discretization parameters on the convergence rate of preconditioned MINRES.