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**MIQR: A Multilevel Incomplete QR Preconditioner for  
Large Sparse Least-Squares Problems**

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We present a Multilevel Incomplete QR (MIQR) factorization for solving large sparse least-squares problems. The algorithm builds the factorization by exploiting structural orthogonality in general sparse matrices. At any given step, the algorithm finds an independent set of columns, i.e., a set of columns that have orthogonal patterns. The other columns are then block orthogonalized against columns of the independent set and the process is repeated recursively for a certain number of levels on these remaining columns. The final level matrix is processed with a standard QR or Incomplete QR factorization. Dropping strategies are employed throughout the levels in order to maintain a good level of sparsity. A few improvements to this basic scheme are explored. Among these is the relaxation of the requirement of independent sets of columns. Numerical tests are proposed which compare this scheme with the standard incomplete QR preconditioner, the robust incomplete factorization (RIF) preconditioner, and ARMS (on the normal equations).