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A Novel Algebraic Multigrid-Based Approach for Solving Maxwell's Equations

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This talk presents a new algebraic multigrid-based method for solving the curl-curl formulation of Maxwell's equations discretized with edge elements. The ultimate goal of this approach is two-fold. The first is to produce a multiple-coarsening multigrid method with two approximately decoupled hierarchies branching off at the initial coarse level, one resolving the divergence-free error and the other resolving the curl-free error, i.e., a multigrid method that couples only on the finest level and mimics a Helmholtz decomposition on the coarse levels. The second consideration is to produce the hierarchies using a non-agglomerate coarsening scheme. To roughly attain this two-fold goal, this new approach constructs the first coarse level using topological properties of the mesh. In particular, a discrete orthogonal decomposition of the finest edges is constructed by dividing them into two sets, those forming a minimum spanning tree and the complement set forming the cotree. Since the cotree edges do not form closed cycles, these edges cannot support "complete" near-nullspace gradient functions of the curl-curl Maxwell operator. Thus, partitioning the finest level matrix using this tree/cotree decomposition, the cotree-cotree submatrix does not have a large near-nullspace. Hence, a non-agglomerate algebraic multigrid method (AMG) that can handle strong positive and negative off-diagonal elements can be applied to this submatrix. This cotree operator is related to the initial coarse-grid operator for the divergence-free hierarchy. The curl-free hierarchy is generated by a nodal Poisson operator obtained by restricting the Maxwell operator to the space of gradients. Unfortunately, because the cotree operator itself is not the initial coarse-grid operator for the divergence-free hierarchy, the multiple-coarsening scheme composed of the cotree matrix and its coarsening, and the nodal Poisson operator and its coarsening does not give an overall efficient method. Algebraically, the tree/cotree coupling on the finest level, which is accentuated through smooth divergence-free error, is too strong to be handled sufficiently only on the finest level. In this new approach, these couplings are handled using oblique/orthogonal projections onto the space of discretely divergence-free vectors. In the multigrid viewpoint, the initial coars-

ening from the target fine level to the divergence-free subspace is obtained using these oblique/orthogonal restriction/interpolation operators in the Galerkin coarsening procedure. The resulting coarse grid operator can be preconditioned with a product operator involving a cotree-cotree submatrix and a topological matrix related to a discrete Poisson operator. The overall iteration is then a multigrid cycle for a nodal Poisson operator (the curl-free branch) coupled on the finest grid to a preconditioned Krylov iteration for the fine grid Maxwell operator restricted to the subspace of discretely divergence-free vectors. Numerical results are presented to verify the effectiveness and difficulties of this new approach for solving the curl-curl formulation of Maxwell's equations.