Killian Miller

Top-level Acceleration of an AMG Method for Markov Chains Via the Ellipsoid Method

University of Waterloo
Department of Applied Mathematics
200 University Avenue West
Waterloo
Ontario N2L 3G1
Canada
killian.miller@gmail.com
Hans De Sterck
Geoff Sanders

In many application areas including information retrieval, networking systems and performance modeling of communication systems, the steady-state distribution vector of an irreducible Markov chain is of interest, and it is often difficult to compute. The steady-state vector is the solution to a nonsymmetric eigenproblem with known eigenvalue, $B\mathbf{x} = \mathbf{x}$, subject to the probability constraints $\|\mathbf{x}\|_1 = 1$ and $x_i \geq 0\ \forall i$, where B is a column-stochastic matrix. A relatively new approach to solving these eigenvalue problems has been the application of multigrid techniques. Recently, scalable multilevel methods based on *smoothed aggregation* [2] and *algebraic multigrid* [1] were proposed to solve such problems. The performance of these methods was investigated for a wide range of numerical test problems, and for most test cases, near-optimal multigrid efficiency was obtained.

In [3], it was shown how the convergence of these multilevel methods can be accelerated by the addition of an outer iteration, with the resulting accelerated algorithm similar in principle to a preconditioned flexible Krylov subspace method. The acceleration was performed by selecting a linear combination of previous fine-level iterates to minimize a functional $\mathcal{F}$ over the space of probability vectors $\mathcal{P}$. Only the m most recent fine-level iterates were used, where m is the *window size*. The functional was taken as the 2-norm of the residual, $\mathcal{F}_2(\mathbf{x}) = \|(I - B)\mathbf{x}\|_2$; consequently each acceleration step consisted of solving a small ($m \leq 5$) quadratic programming problem, for which both constrained and unconstrained variants were considered.

In this talk we consider a different functional, namely, $\mathcal{F}_1(\mathbf{x}) = \|(I - B)\mathbf{x}\|_1$. This gives rise to the following nonlinear convex programming problem (CPP) which must be solved at each acceleration step:

$$\begin{array}{ll} \text{minimize} & \mathcal{F}_1(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in (\mathcal{P} \cap \mathcal{V}), \end{array}$$

where $\mathcal{V}$ is the space spanned by the m most recent fine-level iterates. To solve this CPP we use a variation of the well-known *ellipsoid algorithm* from linear optimization. Our motivation for considering the functional $\mathcal{F}_1$ is from a numerical standpoint: the 1-norm optimization problem may be easier and faster to solve than the 2-norm problem. Moreover, since $\mathcal{F}_1(\mathbf{x}) \ll 1$ implies that $\mathcal{F}_2(\mathbf{x}) \ll 1$, the acceleration in the 1-norm case should be comparable to the acceleration in the 2-norm case. We quantify our approach by directly comparing our results with those obtained in [3], for a variety of test problems. For simplicity we focus solely on constrained acceleration of the algebraic multigrid method for Markov chains from [1].

References

[[1]]Hans De Sterck, Thomas A. Manteuffel, Stephen F. McCormick, Killian Miller, John Ruge, and Geoffrey Sanders, *Algebraic multigrid for Markov chains*, SIAM J. Sci. Comp., accepted, 2009. Hans De Sterck, Thomas A. Manteuffel, Stephen F. McCormick, Killian Miller, James Pearson, John Ruge, and Geoffrey Sanders, *Smoothed aggregation multigrid for Markov chains*, SIAM J. Sci. Comp., accepted, 2009. Hans De Sterck, Thomas A. Manteuffel, Killian Miller, and Geoffrey Sanders, *Top-level acceleration of adaptive algebraic multilevel methods for steady-state solution to Markov chains*, submitted to Advances in Computational Mathematics, Sept. 2009.