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Multigrid absolute value preconditioning

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For a given matrix A, its polar decomposition is $A = U|A|$, where $|A| = \sqrt{A^*A}$ and U is unitary. Let A be real symmetric and nonsingular, than the matrix absolute value $|A|$ is also nonsingular and U is the matrix sign of A, having only two distinct eigenvalues, plus and minus one. The matrix $T = |A|^{-1}$ is the *ideal* symmetric positive definite preconditioner for the linear system $Ax = b$, making the preconditioned MINRES to converge in at most two steps. We call T the *absolute value preconditioner*, if it is spectrally equivalent to $|A|^{-1}$.

If the matrix A is (block) strictly diagonally dominant, the preconditioner T can be chosen as the inverse to the absolute value of the (block) diagonal of A. Such a choice can, e.g., be efficient in plain-wave electronic structure calculations.

For a model problem, where A is a finite difference approximation of the shifted negative Laplacian, we construct an efficient geometric multigrid absolute value preconditioner, in which the smoothing is done using the action of A and $|A|^{-1}$ appears only on the coarsest grid. Our numerical tests demonstrate the effectiveness of such a preconditioning.