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Multigrid for Large Scale Time-Dependent Sylvester Equations

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We will discuss a multigrid solver for the numerical solution of the time-dependent Sylvester equation

$$\frac{dX}{dt} = AX + XB + F, \qquad \text{with } X \in \mathbf{R}^{N\times N},$$

where A and B are discretisations of an elliptic PDE operator. This type of equation occurs in optimal control problems and in uncertainty propagation algorithms for partial differential equations with random parameters. Common to these applications is the large number of unknowns even for problems with a reasonable mesh width.

Our multigrid solver is built on a recently developed algorithm for the stationary Sylvester equation [1]

$$AX + XB + F = 0.$$

We exploit the fact that the iterates can be well compressed when the right hand side F has a low-rank structure. If this compression is used throughout the multigrid cycle, a significant reduction in time and memory can be achieved for large scale problems. This is accomplished by approximating the unknown X by a low-rank matrix

$$X \simeq UV^T, \qquad \text{with } U, V \in \mathbf{R}^{N\times k}.$$

For a certain precision, this low-rank multigrid typically requires $\mathcal{O}(Nk^2) = \mathcal{O}(N\log^2(N))$ work whereas standard multigrid would require $\mathcal{O}(N^2)$. An adaptive strategy that gradually enlarges the rank to get better precision is explored.

Bibliography

[1] L. Grasedyck and W. Hackbusch: *A Multigrid Method to Solve Large Scale Sylvester Equations*, Technical Report 48, Max Planck Institute for Mathematics in the Sciences, 2004