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**Solution of the nonlinear multifrequency radiation  
diffusion equation in a multiphysics, high energy density,  
AMR code**

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We describe a scheme to solve the multifrequency radiation diffusion equation which is intended for a multiphysics, high energy density computer code with adaptive mesh refinement (AMR). In our code, AMR is implemented by refining in both space and time [1]. There may be several levels of refinement, which, going from fine to coarse, are nested within each other.

We time-advance as follows. Assume there are only two levels, one coarse, with domain  $\Omega_c$  and boundary  $\partial\Omega_c$ , and one fine  $\Omega_f$  with boundary  $\partial\Omega_f$ . Since the domains are nested,  $\Omega_f \subseteq \Omega_c$ . At the start of the time cycle, the equations are first updated on  $\Omega_c$  using a timestep  $\Delta t_c$ , a process defined as a *level solve* on  $\Omega_c$ . If the spatial grid on  $\Omega_f$  is a twofold refinement of that discretizing  $\Omega_c$ , we need two level solves on  $\Omega_f$ , each with timestep  $\Delta t_c/2$ , in order to bring the  $\Omega_f$  solution up to the advanced coarse level time. Boundary conditions (BC) are required on  $\partial\Omega_f$ . On parts of  $\partial\Omega_f$  which do not extend to the physical boundary, BC are obtained by interpolating the coarse grid solution. For diffusion equations, e.g.,  $u_t = (Du_x)_x$ , conventionally, one supplies Dirichlet data. This ensures that the coarse and fine grid solution is continuous across  $\partial\Omega_f$ . However, the flux  $-Du_x$  may be discontinuous, which is unacceptable since this results in a loss of conservation. To remedy the defect, after the level solves, the coarse and fine grid solutions are *synced*. One solves a related, nearly homogeneous, problem for corrections on the union of discretizations of  $\Omega_f$  and  $\Omega_c$ . The sole non-homogeneity of the system for the corrections is the miss-match of the fluxes on  $\partial\Omega_f$ . When the corrections are added to the result of the level solves, one obtains a conservative solution, continuous and with continuous flux.

This paper describes the AMR implementation for the multigroup radiation diffusion and matter energy balance equations,

$$\partial_t u_g = \nabla \cdot D_g \nabla u_g + \kappa_g (B_g - u_g), \quad g = 1, \dots, G \quad (1)$$

$$\rho c_v \partial_t T = - \sum_{k=1}^G \Delta_k \kappa_k (B_k - u_k). \quad (2)$$

In (1)–(2),  $u_g$  is the radiation energy density of the  $g$ th *group*. Groups arise by

discretizing the frequency domain  $0 \leq \nu \leq \infty$  into  $G$  intervals. In (1)–(2),  $D_g$  and  $\kappa_g$  are the diffusion and coupling coefficients,  $B_g$  is the Planck function,  $\rho$  the mass density,  $c_v$  the specific heat, and  $\Delta_k = \nu_k - \nu_{k-1}$ . The system is nonlinear;  $D_g$  and  $\kappa_g$ , which in addition to being strong functions of frequency, depend on  $\rho$  and  $T$ . For non-ideal gases,  $c_v$  depends on  $\rho$  and  $T$ . Equations (1)–(2) describe the evolution of the  $G + 1$  unknowns  $\{u_k\}_{k=1}^G$  and  $T$ .

A single level solve of (1)–(2) is a formidable task in itself. For the advance, we use the procedure described by Shestakov [3], generalized for “real,” multiple materials whose properties ( $c_v$ ,  $k_g$ , etc.) are given in tabular form.

For simulations using AMR, after advancing on two levels,  $\Omega_c$  and  $\Omega_f$ , the solutions are synced using a generalization of the Howell and Greenough procedure (HG) [1], which may be directly applied to (1)–(2) if  $G = 1$ . However, if  $G > 1$ , the situation is more complicated since the energies  $u_g$  are coupled. We resolve the difficulty by applying concepts of the “Partial Temperature” scheme (PT) of Lund and Wilson [2]. As in PT, we cycle through the groups in random order. Each group is synced as in HG, but the correction to  $T$  is only a partial change. Only after all the groups have been addressed, do we obtain the final correction.

Our AMR procedure is implemented in a multiphysics code. Results will be presented. We simulate effects of strong explosions in air and compare multigroup results with runs where the frequency domain is not discretized, so-called gray diffusion. The simulations also use the hydro and heat conduction modules. In addition to the coarse level, there are two levels of refinement.

# Bibliography

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