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**Implicit integration of 3D ice sheet flow using hybrid  
factorization/relaxation block preconditioning**

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Ice sheets are non-Newtonian free-surface flows with nonlinear slip boundary conditions. Their most dynamic coupling to other climate components is the point where ice becomes floating as it flows into the ocean, known as the grounding line. Grounding line stability is unknown for many important regions, thus introducing great uncertainty in sea level forecasts. Since this regime is fundamentally non-shallow, we present an ALE formulation in which the velocity is governed by Stokes equations with power-law rheology, and the mesh satisfies surface kinematic equations with elasticity in the interior to prevent tangling.

Semidiscretization using a high-order finite element method produces differential algebraic equations where only surface location and enthalpy are differential. Integrating this DAE implicitly using general linear methods, and solving the resulting algebraic system with Jacobian-free Newton-Krylov, necessitates an effective preconditioner for the coupled indefinite system. Since this system contains physics operating on multiple scales with very different spectral properties, we apply relaxation field splitting to separate the loosely coupled parts, and a factorization split for the heterogeneous Stokes problem, with preconditioners for the Schur complement derived from approximate commutators.