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**Uniformly Well-Conditioned Pseudo-Arclength Continuation**

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Numerical continuation is the process of solving nonlinear equations of the form

$$G(x, \lambda) = 0$$

for various real number parameter values, $\lambda$. The obvious approach, called natural parameterization, is to perturb $\lambda$ with each continuation step and find the corresponding solution $x$ via a nonlinear solver (Newton's method). While this approach is reasonable for paths containing only regular points (points $(x, \lambda)$ where the Jacobian matrix of $G$ is nonsingular), the approach breaks down at simple fold points where the Jacobian matrix of $G$ becomes singular and Newton's method fails. In order to remedy this, one may implement pseudoarclength continuation (PAC) which introduces a new parameter based on the arclength $s$ of the solution path. In order to implement PAC, one converts the old problem $G(x, \lambda) = 0$ to a new problem

$$F(x(s), \lambda(s)) = 0.$$

Using PAC on the new problem requires the Jacobian matrix of $F$, $F'$, which ought to be nonsingular at both regular points and simple folds if we have indeed bypassed the problem that natural parameterization presents. While the nonsingularity of $F'$ at regular points and simple folds is a known fact, we present a theorem that gives conditions under which $F'$ is uniformly nonsingular for a path containing simple folds. We do this by bounding the smallest singular value of $F'$ from below. The theorem justifies the use of PAC in a practical way for solution curves containing nothing "worse" than a simple fold.