
John Shadid
**Performance of Implicit Fully-coupled Solution Methods
for Large-scale Stabilized FE Simulations of Resistive
MHD**

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The magnetohydrodynamics (MHD) model describes the dynamics of charged fluids in the presence of electromagnetic fields. MHD models are used to describe important phenomena in the natural physical world and in technological applications. The mathematical basis for the continuum modeling of these systems is the solution of the governing partial differential equations (PDEs) describing conservation of mass, momentum, charge, and thermal energy augmented by Maxwell's equations for the electric and magnetic field. This system of PDEs is non-self adjoint, strongly coupled, highly nonlinear and characterized by multiple physical phenomena that span a very large range of length- and time-scales. These interacting, nonlinear multiple time-scale physical mechanisms can balance to produce steady-state behavior, nearly balance to evolve a solution on a dynamical time scale that is long relative to the component time-scales, or can be dominated by just a few fast modes. These characteristics make the scalable, robust, accurate, and efficient computational solution of these systems over relevant dynamical time scales of interest (or to steady-state solutions) extremely challenging. For multiple-time-scale systems, fully-implicit methods can be an attractive choice that can often provide unconditionally-stable time integration techniques. The stability of these methods, however, comes at a very significant price, as these techniques generate large and highly nonlinear sparse systems of equations that must be solved at each time step.

This talk describes recent progress on the development of a scalable fully-implicit stabilized unstructured finite element (FE) capability for low-Mach-number resistive MHD. The brief discussion considers the development of the stabilized FE formulation and the underlying fully-coupled preconditioned Newton-Krylov nonlinear iterative solver. To enable robust, scalable and efficient solution of the large-scale sparse linear systems generated by the Newton linearization,

fully-coupled multilevel preconditioners are employed. The multilevel preconditioners are based on two differing approaches. The first algebraic multilevel technique employs a graph-based aggressive-coarsening aggregation method applied to the nonzero block structure of the Jacobian matrix [1,2]. The second approach utilizes approximate block decomposition methods and physics-based preconditioning approaches that reduce the coupled systems into a set of simplified systems to which multilevel methods are applied [3,4].

The performance of the multilevel preconditioners is compared to standard variable overlap additive one-level Schwarz domain decomposition type preconditioners. Parallel performance results are presented for a set of challenging prototype problems that include the solution of an MHD Faraday conduction pump, a hydromagnetic Rayleigh-Bernard linear stability calculation, and a magnetic island coalescence problem. Initial results that explore the scaling of the solution methods are presented on up to 4096 processors for problems with up to 64M unknowns on a CrayXT3/4. Additionally, a large-scale proof-of-capability calculation for 1 billion unknowns for the MHD Faraday pump problem on 24,000 cores is also presented.

References

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