Andrei Drăgănescu

An abstract method for extending two-level preconditioners to multilevel preconditioners of comparable quality

Sandia National Laboratories
Optimization/Uncertainty Estimation Department
P O Box 5800
MS 1110
Albuquerque
87185-1110
aidraga@sandia.gov

We present an abstract method for designing a multilevel preconditioner given a two-level preconditioner for an operator with positive definite symmetric part.

If we denote by A_{fine} and A_{coarse} two discrete versions of a continuous operator, then a two-level preconditioner T_{fine} for A_{fine} can be described in general by a function $T_{\text{fine}} = \mathcal{F}(A_{\text{coarse}}^{-1}, A_{\text{fine}})$, where it is assumed that the evaluation of $\mathcal{F}$ requires a level-independent number of applications of A_{fine} and k applications of A_{coarse}^{-1} ($k = 1$ or 2). The natural extension to a multilevel preconditioner, consisting in replacing in T_{fine} the call to A_{coarse}^{-1} with a recursive call to $\mathcal{F}$, is known to sometimes produce multilevel preconditioners of lower quality (e.g., for certain types of inverse problems). Based on the idea that inverting A_{fine} essentially means to solve the nonlinear equation $X^{-1} - A_{\text{fine}} = 0$, we define our multigrid preconditioner to be the first Newton iterate of the map $X \mapsto X^{-1} - A_{\text{fine}}$ starting at the "natural" multilevel preconditioner. For $k = 1$, the resulting algorithm has a W-cycle structure, and differs only slightly from the textbook version of the W-cycle. Moreover, the method guarantees that the resulting preconditioner maintains the approximation quality of the initial two-level preconditioner. The quality of approximation is measured using a certain distance function, which determines the degree to which two operators with positive definite symmetric parts are spectrally equivalent. We apply this method to designing and analyzing a multigrid preconditioner for a linear advection-diffusion-reaction equation.

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed-Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000.