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Constructing M-matrix preconditioners for Finite Element Matrices

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The finite element method is one of the most popular numerical techniques for solving partial differential equations (PDEs), especially over complex domains. The finite element discretization of second order elliptic PDEs gives a large sparse linear system of equations that can be solved by iterative methods such as the preconditioned conjugate gradient method.

Support graph preconditioning is a relatively new technique to construct preconditioners. A support graph is a subgraph of the graph associated with the sparse matrix which is obtained by eliminating certain edges. The matrix corresponding to the support graph is used as a preconditioner. This approach has several attractive features. The scheme yields robust preconditioners whose effectiveness can be increased at the expense of additional memory during the edge elimination process itself. The quality of the preconditioner appears to be relatively insensitive to boundary conditions and domain characteristics such as anisotropy and inhomogeneity. In addition, there are theoretical results to quantify the behavior of the preconditioner. A recent study shows that this approach can be used to solve the system in almost linear time.

A major limitation of the original support graph approach was that the coefficient matrix A had to be a symmetric, positive semi-definite, and diagonally dominant M-matrix. In recent years, the support graph idea has been extended to a larger class of matrices: maximum weight bases preconditioner has been proposed for symmetric diagonally dominant matrices; techniques have been proposed for matrices arising from the finite element discretization of elliptic problems.

In this talk, we describe a technique to extend support graph preconditioners to symmetric positive definite matrices arising in the finite element discretization of elliptic problems. Our approach uses element-level coordinate transformations to construct a symmetric positive definite diagonally dominant M-matrix $\tilde{A}$ that can be used to approximate A. A support graph preconditioner M for $\tilde{A}'$

is constructed in the usual way, and used as a preconditioner for A. In order to have an effective preconditioner, it is necessary to construct an accurate approximation $\tilde{A}$ that guarantees that the condition number $\kappa(\tilde{A}^{-1}A)$ will be small. In our approach the condition number of matrix $\tilde{A}^{-1}A$ is bounded as a function of the mesh angles. We show that this property holds for the common triangular elements and quadrilateral elements, as well as their three-dimensional counterparts. The element-level coordinate transformations can be extended easily to other types of Lagrangian elements. Our approach is simple, effective, easy to analyze and is applicable to a large class of problems.