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A scalable Helmholtz solver combining the deflation with shifted Laplace preconditioner.

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Our object is to develop high performance iterative solution algorithm for solving the discrete Helmholtz equation modeling wave propagation on large scale. Ingredients in our work are the shifted Laplace preconditioner and deflation. The development of the shifted Laplace preconditioner for the Helmholtz equation was a breakthrough in the development of efficient solution techniques for the Helmholtz equation. The distinct feature of this preconditioner is the introduction of a complex shift, effective introducing damping of wave propagation in the approximate solve. This preconditioner was extensively discussed in various texts and applied in a number of different contexts. Although performant, the resulting algorithm is not truly scalable. The bigger the wavenumber, the more spectrum scatters away from one, hampering the convergence. Idea of projection has been used since long to deflate unfavorable eigenvalues. By inducting eigenvectors corresponding to unwanted eigenvalues, better convergence for CG and GMRES has been reported in various texts. We also combine the idea of deflation with shifted Laplace preconditioner, which leads to a scalable Helmholtz solver, in the sense iterations does not depend upon parameters. We provide a convergence analysis. We perform a Fourier two-grid analysis of one-dimensional model problem with Dirichlet boundary conditions discretized by a second order accurate finite difference scheme. The components analyzed are the shifted Laplace preconditioner used as smoother, full-weighting and linear interpolation intergrid transfer operators, and a Galerkin coarsening scheme. This Fourier analysis results in a closed form expression for the eigenvalues of the two-grid operator. This expressions shows that the spectrum is favourable for convergence of Krylov subspace methods. We apply the deflated shifted Laplace preconditioner to two-dimensional model problems method with constant and non-constant wave numbers and Sommerfeld boundary conditions discretized by second order accurate finite difference scheme on uniform meshes. Numerical results show that the number of GMRES iterations is wave-number independent.