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AMG for Meshfree Finite Difference Methods

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Lagrangian particle methods, such as Smoothed Particle Hydrodynamics, approximate the equations of fluid flow on a cloud of points that move with the flow. Thus, convective terms are treated exactly, which is a great advantage compared to Eulerian approaches. The fundamental challenge in particle methods are unstructured node distributions. A common strategy is to approximate differential operators by meshfree finite difference stencils. These can be derived for instance by the moving least squares method.

Incompressible flows can be simulated by a pressure correction in each time step. The arising Poisson equations can be approximated on the cloud of particles by meshfree finite differences. The generation and solution of the arising linear systems is often times very costly, thus multigrid methods are of great interest. A natural approach is Algebraic Multigrid, since geometric strategies to refine a fully unstructured point cloud are difficult to formulate.

The finite difference matrices arising from classical meshfree least squares approaches have three properties that are challenging with respect to AMG:

- Finite difference methods on unstructured data sets approximate the Laplace operator by non-symmetric matrices.
- The arising matrices are about six times as dense as finite difference matrices on rectangular grids.
- An M-matrix structure is not guaranteed, and is—unless the point geometry is sufficiently nice—in general violated.

While here the non-symmetry is in some sense weaker than coming from the approximation of a convection operator, it can not be overcome. Better sparsity and the M-matrix structure, however, *can* be achieved, by a new finite difference approach, that is based on linear sign-constrained optimization.

In this talk, I will outline meshfree finite difference methods, present the new finite difference approach, show results when applied in the context of AMG methods, and pose open questions. Numerical tests were done using SAMG (Fraunhofer SCAI, K. Stüben et al.), as well as AMLI (C. Mense and R. Nabben).