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An Efficient Matrix-Free Implementation of the Ensemble Kalman Filter

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The Kalman filter provides the basis for data assimilation in a wide variety of applications, including the prediction of extreme weather events (e.g., hurricane tracks), and the evolution of contaminant plumes in the subsurface. In these high-dimensional nonlinear models, statistical variants of the Extended Kalman Filter (EKF) are commonly used. Here we will focus on the Ensemble Kalman filter (EnKF), which uses a statistical approach to alleviate the high-cost of evolving the error covariance matrix. However, as more data becomes available for assimilation, potentially tens of thousands of data points are available in some applications today, a new bottleneck in the EnKF arises. This bottleneck is the storage and inversion of the matrix embedded in the computation of the Kalman gain matrix. In particular, if this matrix is formed explicitly it is dense and of dimension $n \times n$, where n is the number of data points. Typically, current implementations of the EnKF do form this dense matrix and then compute either the LU decomposition or the singular value decomposition to facilitate the explicit computation of the Kalman gain matrix. These methods exhibit a cost of $O(n^3)$ making them expensive and time consuming, moreover, the assimilation algorithm only requires the action of the Kalman gain matrix on a vector. In this work we propose a matrix-free preconditioned Krylov iteration to compute the action of the required inverse matrix on a vector. To provide the action of a spectrally equivalent approximate inverse for the preconditioning step, we express the matrix using the Sherman-Morrison identity and leverage an established inversion process. Thus, we avoid the cost of storing the matrix while providing an algorithm that scales optimally with the number of data points to be assimilated. Additionally this algorithm provides an readily parallelizable approach to this critical component of the EnKF method.