Andrei Drăgănescu

Multigrid preconditioning of linear systems for interior point methods applied to a class of box-constrained optimal control problems

Department of Mathematics and Statistics University of Maryland Baltimore County Baltimore MD 21250 draga@umbc.edu Cosmin Petra

In this work we construct and analyze multigrid preconditioners for operators of the form $\mathcal{D}_{\lambda} + \mathcal{K}^*\mathcal{K}$, where D_{λ} is the multiplication with a relatively "smooth" function $\lambda > 0$ and \mathcal{K} is a discretization of a compact linear operator. These systems arise when applying interior point methods to the distributed optimal control problem $\min_{u} \frac{1}{2}(\|\mathcal{K}u - f\|^2 + \beta \|u\|^2)$ with box constraints $\underline{u} \leq u \leq \overline{u}$ on the control u. The presented preconditioning technique is related to the one developed by Drăgănescu and Dupont in [1] for the associated unconstrained problem, and is intended for large-scale problems. As in [1], the quality of the resulting preconditioners is shown to increase as $h \downarrow 0$ at a rate that is optimal with respect to h if the meshes are uniform, but decreases as the smoothness of λ declines. We test this algorithm first on a Tikhnov-regularized backward parabolic equation with [0,1] constraints and then on the elliptic-constrained optimization problem

minimize
$$\frac{1}{2}\|y-f\|^2+\frac{\beta}{2}\|u\|^2$$
 subj. to
$$\Delta y=u\ ,\ y\in H^1_0(\Omega),\ \underline{u}\leqslant u\leqslant \overline{u}\ a.e.$$

In both cases it is shown that the number of linear iterations per optimization step, as well as the total number of fine-scale matrix-vector multiplications is decreasing with increasing resolution, thus showing the method to be potentially very efficient for truly large-scale problems.

This is joint work with Cosmin Petra from the Argonne National Laboratory.

Bibliography

[1] Andrei Drăgănescu and Todd F. Dupont. Optimal order multilevel preconditioners for regularized ill-posed problems, Math. Comp., 77 (2008), pp. 2001–2038.