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A Cell-Structure-Preserving Multigrid Method for the Diffusion Equation

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We consider the standard cell-centered discretization of the diffusion equation on a rectangle with discontinuous diffusion coefficient. Previously we considered an extension of black box multigrid with a coarsening factor of three. For a coarsening factor of two, the standard application of black box multigrid has been to coarsen the logically rectangular grid. This approach does not preserve the cell structure on coarser grids, as one would wish to do for local grid refinement and other applications. If variational coarsening is employed, with interpolation being a generalization of bilinear interpolation, it is well known that the difference operator on the next coarser grid has a twenty-five point stencil. In this work we extend the difference operator on the cell-centers of the finest grid to be a difference operator on the underlying grid consisting of cell- centers, cell-vertices and cell-edges. For this extended operator we derive an operator-induced interpolation, which is used to coarsen variationally to a difference operator on the grid of cell-vertices. This second operator is coarsened in the standard black box multigrid fashion to the grid of coarse-grid cell centers. The resulting operator has a nine point stencil, and the resulting interpolation operator from coarse-grid cell-centers to fine grid cell-centers has the same stencil as bilinear interpolation while the restriction operator from fine grid cell-centers to coarse-grid cell-centers has the same stencil as the transpose of bilinear interpolation.