
Merico E. Argentati
**Majorization-based convergence rate bounds for subspace
iterations and the block Lanczos method**

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The Rayleigh-Ritz method finds the stationary values, called Ritz values, of the Rayleigh quotient on a given trial subspace as approximations to eigenvalues of a Hermitian operator A . If the trial subspace is A -invariant, the Ritz values are some of the eigenvalues of A . Given two subspaces X and Y of the same finite dimension, such that X is A -invariant, the absolute changes in the Ritz values of A with respect to X compared to the Ritz values with respect to Y represent the absolute eigenvalue approximation error. A recent paper by M. Argentati et al. bounds the error in terms of the principal angles between X and Y using weak majorization. In this talk we derive a new majorization-type convergence rate bound for subspace iterations and combine it with the previous result to obtain a similar bound for the block Lanczos method. These majorization results imply a very general set of inequalities that involve all of the principal angles, not just the largest principal angle.

We proceed in two main steps. First, we prove new majorization-type convergence rate bounds for subspace iterations in terms of the principal angles between subspaces. Second, we assume that the Rayleigh-Ritz method is applied in subspace iterations, so we combine the subspace iterations convergence rate bounds for angles with our Rayleigh-Ritz method error bounds for eigenvalues. We consider polynomial-type subspace iterations and show that the Chebyshev polynomials are optimal in this context. Finally, we apply the convergence rate bound of the Chebyshev subspace iterations to the block Lanczos method. Weak majorization appears to be novel in this context.