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**A Novel Multigrid Method for Sn Discretizations of the
Mono-Energetic Boltzmann Transport Equation in the
Optically Thick and Thin Regimes with Anisotropic
Scattering**

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This talk presents a new multigrid method applied to the most common Sn discretizations (Petrov-Galerkin, diamond-differenced, corner-balanced, and discontinuous Galerkin) of the mono-energetic Boltzmann transport equation in the optically thick and thin regimes, and with strong anisotropic scattering. Unlike methods that use scalar DSA diffusion preconditioners for the source iteration, this multigrid method is applied directly to an integral equation for the scalar flux. Thus, unlike the former methods that apply a multigrid strategy to the scalar DSA diffusion operator, this method applies a multigrid strategy to the integral source iteration operator, which is an operator for 5 independent variables in 3-d and 4 in 2-d : 3 or 2 in space and 2 in angle. The core smoother of this multigrid method involves applications of the integral operator. Since the kernel of this integral operator involves the transport sweeps, applying this integral operator requires a transport sweep (an inversion of an upper triangular matrix) for each of the angles used. As the equation is in 5-space or 4-space, the multigrid approach in this talk coarsens in both angle and space, effecting efficient applications of the coarse integral operators. Although each V-cycle of this method is more expensive than a V-cycle for the DSA preconditioner, since the DSA equation does not have angular dependence, the overall computational efficiency is about the same for problems where DSA preconditioning *is* effective. This new method also appears to be more robust over all parameter regimes than DSA approaches. Moreover, this new method is applicable to a variety of Sn spatial discretizations, to problems involving a combination of optically thick and thin regimes, and more importantly, to problems with anisotropic scattering cross-sections, all of which DSA approaches perform poorly or are not applicable to. This multigrid approach is most effective in neutron scattering applications, where the total cross-section coefficient σ_t and spatial meshsize h satisfies $\sigma_t h \approx 1$. For this case, coarsening can be done aggressively. For problems with $\sigma_t h \approx 10$, this multigrid scheme requires a moderate coarsening rate.

An even slow coarsening is required for problems with $\sigma_t h$ ranges between 20 and 1000, which occur in high-energy photon applications.