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**A Space-Angle-Energy Multigrid Method for Sn  
Discretizations of the Multi-Energetic Boltzmann  
Equation**

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The multi-energetic Boltzmann equation is used to model neutron/photon transport. It is 7-dimensional in spatial 3-dimensions, and 6-dimensional in spatial 2-dimensions (1 in time, 1 in energy, 2 in angle, and 3 or 2 in space). Development of a fast multigrid solver for this multi-energetic equation has been rather slow, particularly because the energy coupling is described through scattering cross-sections that are highly oscillatory in energy. Thus, robust and efficient homogenization techniques for coarsening these cross-sections have been difficult, if not impossible, to develop. In this talk, we describe a multi-energetic extension of the space-angle semi-coarsening method for the mono-energetic Boltzmann equation. The resulting method is also a semi-coarsening scheme: semi-coarsening first in energy, then in space or angle. Coarsening in energy is algebraically based (i.e., it is based on the energy features of the near nullspace component of the multi-energetic Boltzmann operator), which permits simple and efficient handling of the highly oscillatory scattering cross-sections. In fact, the energy restriction operator is constant, and the coarse energy-grid selection is based on a simple binning of the original energy groups. At each energy level of the energy hierarchy, relaxation consists of a few sweeps of a space-angle semi-coarsening V-cycle, with the angles considered collectively over all the energy groups at that energy level. Spectral analysis and numerical results will be given.