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Computing dominant poles of transfer functions

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Recent work on power system stability, controller design and electromagnetic transients has used several advanced model reduction techniques. Although these techniques, such as balanced truncation, produce good results, they impose high computational costs and hence are only applicable to moderately sized systems. Modal model reduction is a cost-effective alternative for large-scale systems, when only a fraction of the system pole spectrum is controllable-observable for the transfer function of interest. Modal reduction produces transfer function modal equivalents from the knowledge of the dominant poles and their corresponding residues. In this talk a specialized eigenvalue method will be presented that computes the most dominant poles and corresponding residues of a SISO transfer function.

The transfer function of a single input single output (SISO) system is defined as

$$H(s) = \mathbf{c}^T(sI - A)^{-1}\mathbf{b} + d, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $\mathbf{b}, \mathbf{c} \in \mathbb{R}^n$, $d \in \mathbb{R}$ and $I \in \mathbb{R}^{n\times n}$ is the identity matrix and $s \in \mathbb{C}$. Without loss of generality, $d = 0$ in the following.

Let the eigenvalues (poles) of A and the corresponding right and left eigenvectors be given by the triplets $(\lambda_j, \mathbf{x}_j, \mathbf{v}_j)$, and let the right and left eigenvectors be scaled so that $\mathbf{v}_j^*\mathbf{x}_j = 1$. It is assumed that $\mathbf{v}_j^*\mathbf{x}_k = 0$ for $j \neq k$. The transfer function $H(s)$ (1) can be expressed as a sum of residues R_j over first order poles:

$$H(s) = \sum_{j=1}^{n} \frac{R_j}{s - \lambda_j},$$

where the residues R_j are

$$R_j = (\mathbf{x}_j^T\mathbf{c})(\mathbf{v}_j^*\mathbf{b}).$$

A *dominant* pole is a pole λ_j that corresponds to a residue R_j with large magnitude $|R_j|/|\mathrm{Re}(\lambda_j)|$, i.e. a pole that is well observable and controllable in the transfer function. This can also be observed from the corresponding Bode magnitude plot of $H(s)$, where peaks occur at frequencies close to the imaginary parts of the dominant poles of $H(s)$. An approximation of $H(s)$ that consists of $k < n$ terms with $|R_j|/|\mathrm{Re}(\lambda_j)|$ above some value, determines the effective transfer function behavior and is called the transfer function modal equivalent:

$$H_k(s) = \sum_{j=1}^{k} \frac{R_j}{s - \lambda_j},$$

The problem of concern can now be formulated as:

> Given a SISO linear, time invariant, dynamical system $(A, \mathbf{b}, \mathbf{c}, d)$, compute $k \ll n$ dominant poles λ_j and the corresponding right and left eigenvectors $\mathbf{x}_j$ and $\mathbf{v}_j$.

The algorithm to be presented, called Subspace Accelerated Dominant Pole Algorithm (SADPA)[1], combines a Newton algorithm[2] with subspace acceleration, a clever selection strategy and deflation to efficiently compute the dominant poles and corresponding residues. It can easily be extended to handle MIMO systems as well[3]. The performance of the algorithm will be illustrated by numerical examples of large scale power systems.

[1] J. Rommes and N. Martins, *Efficient computation of transfer function dominant poles using subspace acceleration*, 2005, UU Preprint 1340

[2] N. Martins, L.T.G. Lima and H.J.C.P. Pinto, *Computing dominant poles of power system transfer functions*,IEEE Trans. Power Syst.", vol. 11, nr. 1, pp 162–170, 1996

[3] J. Rommes and N. Martins, *Efficient computation of multivariable transfer function dominant poles using subspace acceleration*, 2006, UU Preprint 1344