Craig C. Douglas Isogeometric Multigrid

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In engineering design, geometry is a major bottleneck in obtaining finite element solutions for a particular problem. A design is usually born inside a CAD package and subsequently needs to be tessellated (or meshed) such that the geometry is approximated by a finite element space. This process, while semi-automatic in some cases, is not without pitfalls and often requires human interaction to verify the resulting mesh. Furthermore, mesh refinements require a return to the CAD system and a re-tessellation of the CAD object. This means that convergence studies are prohibitively expensive for complex geometries and seldom performed.

Isogeometric analysis [1] has been developed as a solution to this problem, simplifying, and in some cases eliminating, the problem of converting geometric discretizations in the engineering design process. Isogeometric analysis is an isoparametric finite element method that uses the Non-Uniform Rational B-spline basis (NURBS), which dominates the CAD market. It is hoped that in using this basis form, which is prevalent in the CAD community, that the bridge between analysis and design can be bridged.

A NURBS basis may be h-refined via a process known as knot insertion. This process can be likened to the splitting of elements in traditional finite elements, however knot insertion may also be used to control the continuity between elements. More importantly the knot insertion process may be captured as a linear operator that interpolates a vector from one function space to a refined space exactly. We exploit these operators to develop a multigrid approach for isogeometric systems.

In this talk we introduce isogeometric multigrid analysis and the NURBS basis in more detail as well as define the interpolation operator and finally show numerical results.

[1] T. J. R. Hughes, J. Cottrell, Y. Bazilevs, Isogeometric analysis: CAD, finite

elements, NURBS, exact geometry and mesh refinement, Computer Methods in Applied Mechanics and Engineering, 194 (2005), pp. 4135-4195.