Paul G. Constantine

A Stieltjes-Lanczos Method for Parameterized Matrix Equations

PO BOX 5800
MS 1318
Albuquerque
NM 87185
pconsta@sandia.gov
David F. Gleich

We develop a Stieltjes-Lanczos procedure for a symmetric, positive definite parameterized matrix $A(s)$ and a given parameterized vector $b(s)$. Each element in $A(s)$ and $b(s)$ is assumed to be a bounded and continuous function of a set of parameters $s \in \mathcal{D} \subset \mathbb{R}^d$. The method computes a *constant* tridiagonal matrix whose eigenvalues approximate the parameterized spectrum of $A(s)$. We show how this can be interpreted as constructing a Gaussian quadrature formula for a Riemann integral of the form $\langle b^T f(A) b \rangle$, where $f(\cdot)$ is an analytic function and $\langle \cdot \rangle$ denotes integration over the parameter space. We also apply this procedure to iteratively approximate the vector-valued function $x(s)$ that solves $A(s)x(s) = b(s)$; such problems commonly arise within discretizations of partial differential equations with stochastic inputs. Preliminary numerical experiments are provided which validate the theory and suggest future directions for research.