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Complex shifted-Laplace preconditioners for the Helmholtz equation

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In this paper, the time-harmonic wave equation in heterogeneous media is solved numerically. The underlying equation governs wave propagations and scattering phenomena arising in many area's, e.g. in aeronautics, geophysics, and optical problems. In particular, we look for efficient iterative solution techniques for the Helmholtz equation discretized by finite difference discretizations. Since the number of grid points per wavelength should be sufficiently large for accurate solutions, for very high wavenumbers the discrete problem becomes extremely large, thus prohibiting the use of direct solvers. However, since the coefficient matrix is sparse, iterative solvers are an interesting alternative.

In many geophysical applications that are of our interest, an unbounded domain is used. In our model we approximate such a domain by a bounded domain, where appropriate boundary conditions are used to prevent spurious reflections. As boundary conditions we compare the following possibilities: Dirichlet, Neumann, Sommerfeld, Absorbing Layer and Perfect Matched Layer. Due to the boundary conditions and damping in the heterogeneous medium, the coefficient matrix is complex-valued.

It appears that standard iterative solvers (ILU preconditioned Krylov solver,

Multigrid, etc.) fail for the Helmholtz equation, if the wavenumber becomes sufficiently high. In this paper we present a Bi-CGSTAB solution method combined with a novel preconditioner for high wavenumbers. The preconditioner is based on the inverse of an Helmholtz operator, where an artificial damping term is added to the operator. This preconditioner can be approximated by multigrid. This is somewhat surprising as multigrid, without enhancements, has convergence troubles for the original Helmholtz operator at high wavenumbers.

Currently, we are investigating the best choice of the damping term. If the damping term is small Bi-CGSTAB converges fast, but it is difficult to use multigrid for the preconditioner. On the other hand, if the damping term is large the multigrid approximation is very good, but the convergence of Bi-CGSTAB is slow. So a compromise is required to obtain an efficient solver. To find a good value of the damping term we study the spectral properties of the preconditioned matrix. It appears that an eigenvalue analysis of this matrix can be used to predict the convergence of GMRES. In practice it appears that these insights can also be used for the convergence of Bi-CGSTAB. We conclude that Bi-CGSTAB combined with the novel preconditioner converges satisfactorily for all choices of the boundary conditions.