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**Top-level Acceleration of an AMG Method for Markov  
Chains Via the Ellipsoid Method**

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In many application areas including information retrieval, networking systems and performance modeling of communication systems, the steady-state distribution vector of an irreducible Markov chain is of interest, and it is often difficult to compute. The steady-state vector is the solution to a nonsymmetric eigenproblem with known eigenvalue,  $B\mathbf{x} = \mathbf{x}$ , subject to the probability constraints  $\|\mathbf{x}\|_1 = 1$  and  $x_i \geq 0 \ \forall i$ , where  $B$  is a column-stochastic matrix. A relatively new approach to solving these eigenvalue problems has been the application of multigrid techniques. Recently, scalable multilevel methods based on *smoothed aggregation* [2] and *algebraic multigrid* [1] were proposed to solve such problems. The performance of these methods was investigated for a wide range of numerical test problems, and for most test cases, near-optimal multigrid efficiency was obtained.

In [3], it was shown how the convergence of these multilevel methods can be accelerated by the addition of an outer iteration, with the resulting accelerated algorithm similar in principle to a preconditioned flexible Krylov subspace method. The acceleration was performed by selecting a linear combination of previous fine-level iterates to minimize a functional  $\mathcal{F}$  over the space of probability vectors  $\mathcal{P}$ . Only the  $m$  most recent fine-level iterates were used, where  $m$  is the *window size*. The functional was taken as the 2-norm of the residual,  $\mathcal{F}_2(\mathbf{x}) = \|(I - B)\mathbf{x}\|_2$ ; consequently each acceleration step consisted of solving a small ( $m \leq 5$ ) quadratic programming problem, for which both constrained and unconstrained variants were considered.

In this talk we consider a different functional, namely,  $\mathcal{F}_1(\mathbf{x}) = \|(I - B)\mathbf{x}\|_1$ . This gives rise to the following nonlinear convex programming problem (CPP) which must be solved at each acceleration step:

$$\begin{aligned} &\text{minimize} && \mathcal{F}_1(\mathbf{x}) \\ &\text{subject to} && \mathbf{x} \in (\mathcal{P} \cap \mathcal{V}), \end{aligned}$$

where  $\mathcal{V}$  is the space spanned by the  $m$  most recent fine-level iterates. To solve this CPP we use a variation of the well-known *ellipsoid algorithm* from linear optimization. Our motivation for considering the functional  $\mathcal{F}_1$  is from a numerical standpoint: the 1-norm optimization problem may be easier and faster to solve than the 2-norm problem. Moreover, since  $\mathcal{F}_1(\mathbf{x}) \ll 1$  implies that  $\mathcal{F}_2(\mathbf{x}) \ll 1$ , the acceleration in the 1-norm case should be comparable to the acceleration in the 2-norm case. We quantify our approach by directly comparing our results with those obtained in [3], for a variety of test problems. For simplicity we focus solely on constrained acceleration of the algebraic multigrid method for Markov chains from [1].

## References

- [1 ] HANS DE STERCK, THOMAS A. MANTEUFFEL, STEPHEN F. MCCORMICK, KILLIAN MILLER, JOHN RUGE, AND GEOFFREY SANDERS, *Algebraic multigrid for Markov chains*, SIAM J. Sci. Comp., accepted, 2009.
- [2 ] HANS DE STERCK, THOMAS A. MANTEUFFEL, STEPHEN F. MCCORMICK, KILLIAN MILLER, JAMES PEARSON, JOHN RUGE, AND GEOFFREY SANDERS, *Smoothed aggregation multigrid for Markov chains*, SIAM J. Sci. Comp., accepted, 2009.
- [3 ] HANS DE STERCK, THOMAS A. MANTEUFFEL, KILLIAN MILLER, AND GEOFFREY SANDERS, *Top-level acceleration of adaptive algebraic multilevel methods for steady-state solution to Markov chains*, submitted to Advances in Computational Mathematics, Sept. 2009.