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Challenges for Multilevel Sampling in Statistical Inference

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Statistical inference through the Bayesian framework provides a powerful probabilistic approach to characterizing the solution of inverse problems. A canonical example is the recovery of the electrical conductivity of an object from measurements of potential and flux at the boundary, often called electrical impedance tomography (EIT). The mathematical model that maps the state (e.g., conductivity) to the measured data (e.g., potential and flux) is refered to as the forward model. In this statistical framework, solving the inverse problem corresponds to quantifying statistics about the posterior distribution of the state (e.g., conductivities) conditioned on measurements. Although this formulation is very flexible, the calculation of integrals with respect to the posterior distribution is typically done through Markov Chain Monte Carlo (MCMC) sampling, and hence it is a significant computational burden for complex multiscale forward models. To alleviate this burden applications often restrict their analysis to oversimplified forward models, in turn limiting the utility of their analysis.

To avoid these undesirable simplifications and additional uncertainty, considerable research has focused on schemes that improve the efficiency of the MCMC work horse, single-site Metropolis. In fact, a variety of approaches have been proposed to improve the efficiency of this rudimentary MCMC sampling scheme for high-dimensional posteriors. Multivariate updating approaches have been introduced that adjust multiple parameters simultaneously to construct a new proposal, while two-level delayed acceptance schemes rigorously allow the use of an approximate forward map (or approximate solve) in an initial screening step. In addition, high-level or templated-based priors (i.e., distributions of blobs or layers of conductivity) have been used to effectively reduce the dimensionality of the problem. Each of these techniques have demonstrated significant gains in efficiency for some problems, but achieving a significant improvement in scalability over a broad class of inverse problems seems to remain tied to reducing the dimensionality.

This desire to reduce the dimensionality creates an allure to leverage concepts from multilevel iterative solvers, which achieve their optimal scaling through the recursive use of coarser grids, with MCMC sampling. In this talk we highlight key issues in pursing this connection, such as the interpretation of the hierarchy of coarse-scale models, as well as the scale-dependent interpretation of the model parameters and their distributions. Specifically, we explore the use of multilevel solvers within a delayed acceptance scheme, and discuss the potential speedup of this approach. Then to combine the strengths of this approach with concepts from high-level proposals we explore a framework that directly samples the discrete hierarchy of models.