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# Defect correction with algebraic multigrid for the linearized steady Euler equations

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In the present work the linearized discrete compressible Euler equations are investigated. These equations are of importance for evaluation of solution derivatives, *a posteriori* error estimation, and frequency-domain methods for periodic flows. Moreover, their solution can be considered as a preliminary step towards developing algorithms for the non-linear equations. We focus on solution of 2nd-order accurate finite volume discretizations built on unstructured grids with either upwind or central convective fluxes. The goal is to reduce the CPU time required to obtain steady-state convergence for these given discretizations.

Nowadays, such problems are typically solved with geometric multigrid methods, employing either explicit Runge-Kutta or an approximate implicit scheme as a smoother. Despite the investigation of many variations on this theme over the years there has been a continued lack of success in obtaining satisfactory convergence for all but the simplest geometries. The situation is even worse for unstructured discretizations, where the additional complexity of definition of a proper coarse grid hierarchy arises.

In this work defect correction [1, 2] is applied to solve the linear problem. Defect correction allows by-passing direct solution of the 2nd-order accurate discretization, instead, a sequence of problems with a 1st-order discretization on the left-hand side is solved. On this better conditioned problems we apply AMG.

Since algebraic multigrid [4, 5, 6] does not rely on the geometry of the grid, it is an attractive alternative to geometric multigrid for problems discretized on unstructured grids. However, initially developed for scalar elliptic PDEs, algebraic multigrid needs special extensions to be applicable to systems of PDEs, one of those extensions being a point-based approach. The point-based AMG solver, implemented in the SAMG package [3], was investigated in this work. The solver, applied together with accelerator BiCGStab, proved to be very efficient for solution of the 1st-order accurate discretizations considered here in sub-, tran- and super-sonic flow regimes. One should mention that our attempts to apply AMG directly to 2nd-order discretizations were not much successful so far.

The defect correction approach combined with the AMG solver was applied to solve 2nd-order accurate discretizations and was seen to significantly outperform the geometric multigrid solver [7, 8] in terms of CPU time for all flow regimes. This is mainly by virtue of the efficency of AMG, which requires very few iterations to give sufficient accuracy of the inner problem at each defect correction step. Furthermore, as the 1st-order matrix is constant, only one AMG setup phase must be performed for the entire calculation.

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