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Least Squares Preconditioners for Stabilized Mixed Approximation of the Navier-Stokes Equations

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We consider the Navier–Stokes equations

$$\begin{aligned} -\nu\nabla^2\mathbf{u} + (\mathbf{u}\cdot\mathrm{grad})\,\mathbf{u} + \mathrm{grad}\,p &= \mathbf{f} \\ -\mathrm{div}\,\mathbf{u} &= 0 \end{aligned} \tag{1}$$

on $\Omega \subset \mathbb{R}^d$, $d = 2$ or 3. Here, $\mathbf{u}$ is the d-dimensional velocity field, which is assumed to satisfy suitable boundary conditions on $\partial\Omega$, p is the pressure, and ν is the kinematic viscosity, which is inversely proportional to the Reynolds number.

Linearization and discretization of (1) by finite elements, finite differences or finite volumes leads to a sequence of linear systems of equations of the form

$$\begin{bmatrix} F & B^T \\ B & -\frac{1}{\nu}C \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ p \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ g \end{bmatrix}. \tag{2}$$

These systems, which are the focus of this talk, must be solved at each step of a nonlinear (Picard or Newton) iteration. Here, B and B^T are matrices corresponding to discrete divergence and gradient operators, respectively and F operates on the discrete velocity space. For *div-stable* discretizations, $C = 0$. For mixed approximation methods that do not uniformly satisfy a discrete inf-sup condition, the matrix C is a nonzero *stabilization operator*. Examples of finite element methods that require stabilization are the mixed approximations using linear or bilinear velocities (trilinear in three-dimensions) coupled with constant pressures, as well as any discretization in which equal order discrete velocities and pressures are specified using a common set of nodes.

The focus of this talk is the Least Squares Commutator (LSC) preconditioner developed by Elman, Howle, Shadid, Shuttleworth and Tuminaro, and unveiled at the Copper Mountain Conference in 2004. This preconditioning methodology is one of several choices that are effective for Navier-Stokes equations, and it has the advantage of being defined from strictly algebraic considerations. The resulting preconditioning methodology is competitive with the pressure convection-diffusion preconditioner of Kay, Loghin and Wathen, and in some cases its performance is superior. However, the LSC approach has so far only been shown to be applicable to the case where $C = 0$ in (2). In this talk we show that the least squares commutator preconditioner can be extended to cover the case of mixed approximation that require stabilization. This closes a gap in the derivation of these ideas, and a version of the method can be also formulated from algebraic considerations, which enables the fully automated algebraic construction of effective preconditioners for the Navier-Stokes equations by essentially using only properties of the matrices in (2).

Our focus in this work is on steady flow problems although the ideas discussed generalize in a straightforward manner to unsteady flow.