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Multilevel Preconditioners for Nonselfadjoint or Indefinite Orthogonal Spline Collocation Problems

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We develop and study symmetric multilevel preconditionersfor thecomputation of the orthogonal spline collocation (OSC)solution of aDirichlet boundary value problem (BVP) with a nonselfadjointor anindefinite operator. The OSC solution is sought in the spaceofpiecewise Hermite bicubic spline functions defined on auniformpartition. We consider an additive and a multiplicative multilevelpreconditioners that are used with the preconditionedconjugategradient (PCG) method.Let Ω be a unit square $(0,1) \times (0,1)$ withthe boundary$\partial\Omega$, and let $x = (x_1, x_2)$. We consider aBVP

$$Lu \equiv \sum_{i,j=1}^{2} a_{ij}(x) u_{x_i x_j} + \sum_{i=1}^{2} b_i(x) u_{x_i} + c(x) u = f(x), \quad x \in \Omega, \quad u = 0 \text{ on } \partial\Omega. \tag{1}$$

Operator L could be non-selfadjoint or indefinite in L^2inner product. We assume that the principal part of$L$ satisfies theuniform ellipticity condition and that BVP (1)has a unique solution in $H^2(\Omega)$.Let π_0 be a uniform coarsest rectangular partitionof Ω.We obtain a set of partitions $\{\pi_k\}_{k=0}^{K}$ bystandardcoarsening, and let $V_0 \subset V_1 \subset \ldots_K \equiv V_h$be the set of corresponding nested spaces of piecewiseHermitebicubics that vanish on $\partial\Omega$. Let $\sum$denotethetwo-dimensional composite Gauss quadrature correspondingto partitionπ_h with 4 nodes in each element. Let $\mathcal{G}_h$ denotethecorresponding set of Gauss points. The OSC discretizationof BVP(1) is defined by

$$u_h \in V_h, \quad Lu_h(\xi) = f(\xi), \quad \xi \in \mathcal{G}_h, \tag{2}$$

and it can be written as the operator equation$L_h u_h = f_h$in theHilbert space V_h with the inner product$(v,w)_h = \sum vw$.Let $\{\psi_{k,j}\}_{j=1}^{N_k}$ be the standard finiteelement basisof V_k consisting of products of one-dimensional valueand slopebasis functions. Using space decomposition

$$V_h = V_0 + \sum_{k=1}^{J} \sum_{j=1}^{N_k} V_{k,j}, \quad V_{k,j} = \operatorname{span}(\psi_{k,j}),$$

we define and study multilevel additive$B_{\mathrm{a}}$and multiplicative B_{m}preconditionersfor solving the normal equation $L_h^* L_h u_h = L_h^* f_h$,where L_h^* isthe adjoint to L_h. The implementation of$B$ andB_{m} is based onrelationshipsbetween basis functions for two consecutive partitionsand the implementation of $B_{\mathrm{m}}$is similar to that forV(1,1)-cycle with the Gauss-Seidel smoothing. A problemon the coarsest partition is assumed sufficiently small,and it is solved exactly. The computational cost of thepreconditioning algorithms isO(N_K). The following is our main result.**Theorem.** *There are positive independent of* h *and* K *constants*α_{a},β_{a},α_{m},*and*β_{m},*such that*

$$\begin{aligned} \alpha_{\mathrm{a}}\,(B_{\mathrm{a}}v, v)_h \le (L_h^* L_h v, v)_h \le \beta_{\mathrm{a}}\,(B_{\mathrm{a}}v, v)_h, \quad v \in V_h, \\ \alpha_{\mathrm{m}}\,(B_{\mathrm{m}}v, v)_h \le (L_h^* L_h v, v)_h \le \beta_{\mathrm{m}}\,(B_{\mathrm{m}}v, v)_h, \quad v \in V_h. \end{aligned} \tag{3}$$

We present numerical results that demonstrate theefficiency of our preconditioning algorithms.