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AMG for Linear Systems Obtained by Local Elimination

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We consider algebraic solvers for linear systems arising in finite element simulations of scalar and electromagnetic diffusion applications, where a set of "interior" degrees of freedom have been eliminated to reduce the problem size. This elimination is assumed to be local, so the "interior" principal sub-matrix is block-diagonal, and the resulting Schur complement is still sparse. In order to be feasible, the elimination process should not only result in using less memory to store the matrix and vector, but also lead to an algebraic problem that can still be solved efficiently.

In this talk we investigate AMG-type solution algorithms applied to the assembled reduced problem, and we discuss the influence of the local elimination to solver-related properties such us element aspect ratios, operator complexity and near-nullspace preservation. For Nedelec discretizations of definite Maxwell problems, the reduction extends to the nodal variables and generalizes the concepts of discrete gradient and node-to-edge Nedelec interpolation matrices [1]. We also consider a modification of the reduction process that targets singular problems, which arise naturally in electromagnetic diffusion simulations with pure void (zero conductivity) regions. This case needs a special treatment, since the local "interior" matrices in the void elements have non-trivial kernel components.

We conclude the presentation with a number of 2D, 3D and axi-symmetric numerical simulations in the framework of [2]. The results demonstrate that the combination of an appropriately chosen local elimination with the use of the BoomerAMG and AMS solvers from [3] can lead to significant improvements in the overall solution time.

[1] T. Kolev and P. Vassilevski, *Parallel Auxiliary Space AMG for H(curl) Problems*, Journal of Computational Mathematics, (27) 2009, pp. 604-623.

[2] P. Bochev and A. Robinson, *Matching algorithms with physics: Exact sequences of finite element spaces*, in Collected Lectures on the Preservation of Stability under Discretization, D. Estep and S. Tavener, eds., SIAM, Philadelphia, 2001, pp. 145-165.

[3] *hypre*: a library of high performance preconditioners, *http://www.llnl.gov/CASC/hypre/*