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Multilevel Preconditioners for Nonselfadjoint or Indefinite Orthogonal Spline Collocation Problems

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We develop and study symmetric multilevel preconditionersfor the computation of the orthogonal spline collocation (OSC)solution of a Dirichlet boundary value problem (BVP) with anonselfadjoint or an indefinite operator. The OSC solution issought in the space of piecewise Hermite bicubic splinefunctions defined on a uniform partition. We consider anadditive and a multiplicative multilevel preconditionersthat are used with the preconditioned conjugate gradient (PCG)method.Let Ω be a unit square $(0,1)\times(0,1)$ with theboundary $\partial\Omega$, and let $x=(x_1,x_2)$. We consider aBVP

$$Lu \equiv \sum_{i,j=1}^{2} a_{ij}(x)u_{x_ix_j} + \sum_{i=1}^{2} b_i(x)u_{x_i} + c(x)u = f(x), \quad x\in\Omega, \quad u=0 \ \text{ on } \ \partial\Omega. \tag{1}$$

Operator L could be non-selfadjoint or indefinite in L^2inner product. We assume that the principal part of L satisfies the uniform ellipticity condition and that BVP (1) hasa unique solution in $H^2(\Omega)$.Let π_0 be a uniform coarsest rectangular partition ofΩ. We obtain a set of partitions $\{\pi_k\}_{k=0}^{K}$ bystandard coarsening, and let$V_0 \subset V_1 \subset \ldots \subset V_K \equiv V_h$ be the set ofcorresponding nested spaces of piecewise Hermite bicubics thatvanish on $\partial\Omega$. Let $\sum$ denote the two-dimensionalcomposite Gauss quadrature corresponding to partition $\pi_h$with 4 nodes in each element. Let $\mathcal{G}_h$ denote thecorresponding set of Gauss points. The OSC discretization of BVP(1) is defined by

$$u_h \in V_h, \quad Lu_h(\xi) = f(\xi), \quad \xi\in\mathcal{G}_h, \tag{2}$$

and it can be written as the operator equation $L_hu_h = f_h$ inthe Hilbert space V_h with the inner product $(v,w)_h = \sum vw$.Let $\{\psi_{k,j}\}_{j=1}^{N_k}$ be the standard finite elementbasis of V_k consisting of products of one-dimensional valueand slope basis functions. Using space decomposition

$$V_h = V_0 + \sum_{k=1}^{J}\sum_{j=1}^{N_k} V_{k,j}, \quad V_{k,j} = \operatorname{span}(\psi_{k,j}),$$

we define and study multilevel additiveB_{a} and multiplicativeB_{m} preconditioners for solving thenormal equation $L_h^* L_h u_h = L_h^* f_h$, where$L_h^*$ is the adjoint to L_h. The implementation of$B_{\mathrm{a}}$and B_{m} is based on relationshipsbetween basis functions for two consecutive partitions and theimplementation of $B_{\mathrm{m}}$is similar to that for V(1,1)-cycle with the Gauss-Seidelsmoothing. A problem on the coarsest partition is assumedsufficiently small, and it is solved exactly. The computationalcost of the preconditioning algorithms is $\mathrm{O}(N_K)$.The following is our main result.

Theorem 1 *There are positive independent of h and K constants*α_{a}*,*β_{a}*,*α_{m}*, and*β_{m}*, such that*

$$\begin{aligned} \alpha_{\mathrm{a}}\,(B_{\mathrm{a}}v, v)_h \le (L_h^* L_h v, v)_h \le \beta_{\mathrm{a}}\,(B_{\mathrm{a}}v, v)_h, \quad v \in V_h, \\ \alpha_{\mathrm{m}}\,(B_{\mathrm{m}}v, v)_h \le (L_h^* L_h v, v)_h \le \beta_{\mathrm{m}}\,(B_{\mathrm{m}}v, v)_h, \quad v \in V_h. \end{aligned} \tag{3}$$

We present numerical results that demonstrate the efficiency ofour preconditioning algorithms.