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Scaling models and data for solving large sparse linear systems : a comparison of methods

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As industrial problems may involve different kinds of physical parameters and different types of coupled equations, ill-conditionned sparse linear systems may arise from the discretization method. Let $Au = f$ be a nonsingular sparse linear system where $A \in \mathbb{C}^{n \times n}$, and $u, f \in \mathbb{C}^n$. If the spectral condition number $\kappa(A)$ is too far from one, direct solvers can lack of accuracy and iterative methods can fail to converge. An economical way of avoiding these difficulties is to find two diagonal matrices D_r and D_c such that $\kappa(D_r A D_c) \approx \min_{D_1, D_2} \kappa(D_1 A, D_2)$. Then, the solving process becomes

1. compute $\hat{u}$ such that $\hat{A}\hat{u} = \hat{f}$
2. compute $u = D_c \hat{u}$

where $\hat{A} = D_r A D_c$ and $\hat{f} = D_r f$. Numerical properties of $\hat{A}$ differ according to the scaling method : it can have normalized rows/columns[1,5] or it can be approximately doubly stochastic[3,7]. Other methods make the matrix have arbitrary row/column sums[4,6]. In this paper, we propose to make clear the interests of scaling corrections for supernodal and multifrontal direct solvers and for preconditioned iterative methods on industrial applications based on Maxwell equations (coupled problems, nonlinear materials, moving structures, transient problems) and discretized by means of nodal or edge finite elements.

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