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**Multigrid vs. Conjugate Gradient for Piece-Wise Smooth
Surface Reconstruction, a Case Study**

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An interesting problem from the field of computer vision is reconstruction of piece-wise smooth functions from noisy and incomplete data. The problem is more difficult than reconstructing smooth functions and algorithms typically employ an iterative relaxation method as a component.

We studied a particular algorithm, published twenty years ago by Leclerc [1], which reconstructs two-dimensional, piece-wise polynomial functions from noisy data. An objective function is derived from the premise that a description of the data in terms of a piece-wise polynomial signal plus noise from some known stochastic process is more likely to be correct the shorter it is. The objective, f , is not continuous and thus difficult to optimize.

Leclerc devises a continuation method by which a sequence of tentative solutions is sought, solutions corresponding to a sequence of smooth objective functions converging to f . For each step of the continuation method Leclerc uses the Jacobi method to relax the most recent tentative solution on the linear system that results from setting the gradient of the next objective in the sequence to zero.

In our experiments it turned out that Leclerc's algorithm finds satisfying solutions only for small problem sizes and performs poorly with realistic sizes. In several variations of Leclerc's continuation scheme we substitute other methods for the Jacobi relaxation to remedy the poor performance. In particular, we substitute a Conjugate Gradient method, an inexact Newton method, and a multigrid Gauss-Seidel relaxation method.

In this presentation we briefly review Leclerc's work and give a picture of the difficulties we encountered with larger problem sizes. We sketch our variations of Leclerc's continuation algorithm, present results for each method and discuss their merits and shortcomings in terms of accuracy, computational efficiency, and amount of work required for their implementation.

[1] Y. Leclerc: "Constructing simple stable descriptions for image partitioning",

IJCV (3), pp. 73-102, 1989.