Scott MacLachlan

An angular multigrid method for modeling charged-particle transport in Flatland

Department of Mathematics
Tufts University
Bromfield-Pearson Hall
503 Boston Ave
Medford
MA 02155
scott.maclachlan@tufts.edu
Christoph Börgers
Department of Mathematics, Tufts University

Beams of microscopic particles penetrating scattering background matter play an important role in several applications. In this work, we consider parameter choices that are motivated by the problem of electron-beam cancer therapy planning. Mathematically, a steady particle beam penetrating matter, or a configuration of several such beams, is modeled by a boundary value problem for a Boltzmann equation. Grid-based discretization of such a problem leads to a system of algebraic equations, which is typically very large because of the large number of independent variables in the Boltzmann equation (six if no dimension-reducing assumptions other than time independence are made). If grid-based methods are to be practical for these problems, it is necessary to develop fast solvers for the discretized problems.

For beams of mono-energetic particles interacting with a passive background, but not with each other, in two space dimensions, an angular domain decomposition was proposed by Börgers in 1997. In this talk, we discuss an angular multigrid algorithm for the same model problem, based on a careful choice of relaxation and coarse-grid correction processes. Our numerical experiments show rapid, grid-independent convergence for the forward-peaked scattering typical of electron beams. Unlike angular domain decomposition, the angular multigrid method works well even when the angular diffusion coefficient is fairly large.