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An efficient iterative scheme for the Helmholtz equation with deflation.

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The Helmholtz equation arises in many physical problems involving steady state (mechanical, acoustical, thermal, electromagnetic) oscillations. The complexity of the physical problem leaves no choices except to solve the equation numerically. The finite difference discretization leads to sparse, complex symmetric coefficient matrices that become indefinite for sufficient large wave number. The size of the problem increases with the wave number as a minimum number of grid points per wavelength is required to represent the physics correctly. Krylov subspace and Multigrid techniques have been succesfully applied in a wide range of applications. For symmetric positive definite problems, the conjugate gradient (CG) is the method of choice. For indefinite problems, more general Krylov subspace solvers are to be applied. The fast convergence of these methods requires some form of preconditioning. The straightforward application of multigrid as a preconditioner is hampered by slow convergence causes by eigenmodes corresponding to eigenvalues with a negative real part. To overcome this difficulty, Shifted-Laplace preconditioners have been developed by Yogi A. Erlangga [Tech. Report 03-18, DIAM Delft Univ. Tech. The Netherlands, 2003].The idea of shifted-Laplace preconditioners is to base the precondioner of the discrete Helmholtz operator with a modified shift (coefficient before the first order term) for with the multigrid preconditioner can be shown to satisfaction. It was shown that the resulting preconditioned GMRES solver has favourable convergence properties in the sense the required number of iterations only depends mildly on the wavenumber. Allowing some damping in the shifted Laplace operator in particalur proves to benefical for speeding up overall convergence. In more recent work by Yogi and Nabben[On a ultilevel Krylov method for the Helmholtz equation preconditioned by shifted Laplacian, In Press], the combined use of deflation and the shifted-Laplace precondioner was shown to result in a on more performant solver.In this paper we study the combined use of the shifted-Laplace precondioner with multigrid deflation. We show that deflation allows to remove unfavourable modes in the spectrum of the precondioned operator. This allows us to that the Krylov solver is close to optimal in the sense that the required number of iterations is almost independent of the wavenumber. Also the deflation matrix is sparse. Experimental results to support the claims are presented for a 2-D Helmholtz problem.