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Multilevel Preconditioners for Nonselfadjoint or Indefinite Orthogonal Spline Collocation Problems

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TheoremWe develop and study symmetric multilevel preconditioners for the computation ofthe orthogonal spline collocation (OSC) solution of a Dirichlet boundary value problem(BVP) with a nonselfadjoint or an indefinite operator.The OSC solution is sought in the space of piecewise Hermite bicubic spline functionsdefined on a uniform partition.We consider an additive and a multiplicative multilevel preconditionersthat are used with the preconditioned conjugate gradient (PCG) method.Let Ω be a unit square $(0,1) \times (0,1)$ with the boundary $\partial\Omega$, andlet $x = (x_1, x_2)$. We consider a BVP

$$Lu \equiv \sum_{i,j=1}^{2} a_{ij}(x)u_{x_i x_j} + \sum_{i=1}^{2} b_i(x)u_{x_i} + c(x)u = f(x), \ \ x \in \Omega, \quad u = 0 \ \text{ on } \ \partial\Omega.$$

(1)

Operator L could be non-selfadjoint or indefinite in L^2 inner product.We assume that the principal part of L satisfies the uniform ellipticity conditionand that BVP (1) has a unique solution in $H^2(\Omega)$.Let π_0 be a uniform coarsest rectangular partition of Ω. We obtain a set of partitions$\{\pi_k\}_{k=0}^{K}$ by standard coarsening, andlet $V_0 \subset V_1 \subset \ldots \subset V_K \equiv V_h$ be the set of corresponding nested spaces ofpiecewise Hermite bicubics that vanish on $\partial\Omega$.Let $\sum$ denote the two-dimensional composite Gauss quadrature corresponding to partitionπ_h with 4 nodes in each element. Let $\mathcal{G}_h$ denote the corresponding set of Gauss points.The OSC discretization of BVP (1) is defined by

$$u_h \in V_h, \quad Lu_h(\xi) = f(\xi), \quad \xi \in \mathcal{G}_h,$$

(2)

and it can be written as the operator equation $L_h u_h = f_h$ in the Hilbert space V_h with theinner product $(v, w)_h = \sum vw$.We define and study multilevel additiveB_{a} and multiplicative B_{m} preconditionersfor solving the normal equation $L_h^* L_h u_h = L_h^* f_h$, where L_h^* is the adjoint to L_h.The implementation of B_{a} and $B_{\mathrm{m}}$is based on relationships between basis functions for two consecutive partitionsand the implementation of B_{m} is similar to that forV(1,1)-cycle with the Gauss-Seidel smoothing.A problem on the coarsest partition is assumed

sufficiently small, and it is solved exactly.The computational cost of the preconditioning algorithms is $O(N_K)$.The following is our main result.There are positive independent of h and K constantsα_a,β_a,α_m,andβ_m,such that

$$\alpha_\mathrm{a} \left(B_\mathrm{a}v, v\right)_h \leq (L_h^* L_h v, v)_h \leq \beta_\mathrm{a} \left(B_\mathrm{a}v, v\right)_h, \quad v \in V_h, \tag{3}$$
$$\alpha_\mathrm{m} \left(B_\mathrm{m}v, v\right)_h \leq (L_h^* L_h v, v)_h \leq \beta_\mathrm{m} \left(B_\mathrm{m}v, v\right)_h, \quad v \in V_h.$$

We present numerical results that demonstrate the efficiency ofour preconditioning algorithms.