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Optimal Additive Schwarz Preconditioning for Minimal Residual Methods with Euclidean and Energy Norms

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For the solution of non-symmetric or indefinite linear systems arising from discretizations of elliptic problems, two-level additive Schwarz preconditioners are known to be optimal in the sense that convergence bounds for the preconditioned problem are independent of the mesh and the number of subdomains. These bounds are based on some kind of *energy norm*. However, in practice, iterative methods which minimize the Euclidean norm of the residual are used, despite the fact that the usual bounds are non-optimal, i.e., the quantities appearing in the bounds may depend on the mesh size; see [X.-C. Cai and J. Zou, *Numer. Linear Algebra Appl.*, 9:379–397, 2002]. In this paper, iterative methods are presented which minimize the same energy norm in which the optimal Schwarz bounds are derived, thus maintaining the Schwarz optimality. As a consequence, bounds for the Euclidean norm minimization are also derived, thus providing a theoretical justification for the practical use of Euclidean norm minimization methods preconditioned with additive Schwarz. Both left and right preconditioners are considered, and relations between them are derived. Numerical experiments illustrate the theoretical developments.