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A Preconditioned Newton-Krylov Strategy for Moving Mesh Adaptation

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We propose a new approach to adaptive mesh motion based on solving a coupled self-consistent system for the physics equations and for the grid generation equations. The key aspect of the new method is the solution of the non-linear coupled system with a preconditioned Newton-Krylov method. The present work described the approach and focuses on the preconditioning techniques.

Adaptive grids are becoming an ever more common tool for high performance scientific computing. We focus here on the type of adaptation achieved by moving a constant number of points according to appropriate rules, an approach termed moving mesh adaptation (MMA). The approach we consider here is based specifically on retaining a finite volume approach but allowing the grid to evolve in time according to a grid evolution equation obtained from minimization principles. The approach originates from the seminal papers by Brackbill and Saltzmann [2] and by Winslow [3].

In the present paper we consider the fundamental question in the application of MMA. Is it worth the effort? The literature is very rich and considerable results have been obtained in designing MMA approaches that provide grids that can indeed present the desired properties. But the question of whether once the adaptive grids are used the simulations are actually more cost effective remains largely unanswered.

We have revisited the question and have reached the conclusion that in order to obtain an effective MMA strategy, three ingredients need to be considered.

First is the effective formulation of the moving grid equations. In 1D the problem is benign, as error minimization leads to error equiditribution and to a rigorous and simple minimization procedure. In 2D and 3D the problem is more challenging but we have derived an effective approach based on the classic approach by Brackbill-Saltzmann-Winslow [2, 3]. The crucial ingredients of our approach are the formulation of the physics equations in a conservative form and of the formulation of the grid generation equations using harmonic mapping [1, 4]. The independent variables of the physics equations are changed from the physical to the logical space and the equations are rewritten in the logical space in a fully conservative form [4].

Second is the solution algorithm for the MMA method. Here we bring a new development. The moving mesh equations and the physics equations, derived by discretizing the problem under investigation on a moving grid, form a tightly coupled system of algebraic non-linear equations. Traditionally, the coupling is broken, the physics and grid equations being solved separately in a lagged time-splitting approach. Each time step is composed of two alternating steps: the physics equations are solved on the current grid, the grid equations are then solved using new information from the solution of the physics equations. However, in presence of sharp fronts or other moving features, breaking such coupling can lead to grid lagging with respect of the physics equations, with adaptation resulting behind rather than on the moving feature.

We avoid breaking the coupling and solve the full non-linear set of physics and grid equation using the preconditioned Newton-Krylov (NK) approach.

Third ingredient in a cost-effective grid adaptation is an efficient preconditioning technique. In 1D a simple block tridiagonal approach works effectively [5]. Each set of equations, for physics and for grid generation, is preconditioned with a tridiagonal matrix obtained by numerically approximating the corresponding diagonals in the Jacobian. In 2D, we rely on a multigrid preconditioning strategy where a crucial innovation is how to coarsen the information relative to the adaptivity in the harmonic grid generation equations [4]..

In the present paper we describe the approach followed and we report a number of examples to illustrate the performance of the new approach.

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