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# The effect of boundary conditions and inner solver accuracy within pressure convection-diffusion preconditioners for the incompressible Navier-Stokes equations.

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*Pressure convection–diffusion* preconditioners for solving the incompressible Navier–Stokes equations were first proposed by Kay, Loghin, and Wathen and Silvester, Elman, Kay, and Wathen. While numerous theoretical and numerical studies have demonstrated mesh independent convergence for these block methods on several problems and their overall efficacy, there are several potential weaknesses remaining in the practical use of these methods.

Perhaps one of the most poorly understood topics within this block preconditioner family is the influence of boundary conditions on overall algorithm convergence. The notion of differential commuting is the basis for all *pressure convection-diffusion* preconditioners. The main mathematical difficulty is that differential commuting does not hold at the boundaries. Thus, it is unclear what boundary conditions should be enforced in subblocks of the preconditioner. Heuristics have been developed that roughly account for boundary conditions associated with inflow, outflow, and no–slip. However, these rough heuristics often do not properly capture the boundary interactions, and can in fact lead to a degradation in convergence rates as the mesh is refined. We first explore the effect of having "ideal" boundary conditions within the preconditioner. While not computationally feasible, the ideal boundary condition results highlight the importance of choosing suitable boundary conditions. We then explore somewhat more practical approximations to the ideal conditions based on ILU factorizations and probing [Siefert and de Sturler, 2005].

Another important issue is the relationship between the accuracy of inner subproblem solves to the overall convergence rate of the outer iteration. It has been known for quite some time that when mesh-independent sub-problem solvers are used inexactly within this block preconditioner, the overall convergence of the outer iteration remains mesh independent for the Stokes equations [Silvester and Wathen, 1994]. The situation is not well understood for the Navier-Stokes equations, and it is also not well understood when the inner subblock solver has less than ideal behavior. We have observed a noticeable degradation in the outer iteration convergence rate when a sub-solver is terminated too quickly. We discuss possible solutions to this issue including reusing information from repeated linear solves.