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Boundary Conditions for Block Preconditioners Based on Approximate Commutators

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Pressure convection-diffusion preconditioners are based on an approximation to the inverse Schur complement where the true Schur complement is obtained by eliminating velocity variables from the incompressible Navier-Stokes equations. This inverse Schur complement is essentially approximated by the product of two matrices: a convection-diffusion operator and an approximation to the inverse Laplacian. These operators follow from commuting properties of certain differential operators away from boundaries. While these operators are now fairly well understood for the domain interior, appropriate boundary conditions for defining the preconditioning operators are significantly less clear.

In this talk, we look closely at boundary conditions. We show in particular that for problems with velocities satisfying inflow boundary conditions, the preconditioner can be defined so that discrete commutativity holds exactly at the inflow boundary. That is, in certain situations the inverse Schur complement approximation is exact, even at domain boundaries. This exact relationship motivates new boundary conditions which give rise to good preconditioners in more general settings. In addition to examining standard pressure convection-diffusion preconditioners, the analysis motivates a simple and practical modification to the related least squares commutator method (LSC). We demonstrate that the new boundary conditions have a significant impact on the convergence behavior of both the pressure convection-diffusion method and the LSC method. Further, mesh independent convergence rates are now obtained for both methods in cases where previously some convergence deterioration occurred as the mesh was refined.