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AMLS and Spectral Schur Complements

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In

the

last

few

decades,

Krylov

projection

methods

such

as

the

Lanczos

algorithm

and

its

variants,

have

dominated the scene of algorithms for eigenvalue problems. Recently, an alternative approach has emerged in structural engineering as a competitor to the standard shift-and-invert Lanczos approach. The algorithm, called Automated Multilevel Substructuring method (tt AMLS) is rooted in a domaindecomposition framework. It has been reported as being capable of computing t h o u s a n d s

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A

theoretical
framework
for
tt
AMLS
was
recently
presented
by
Benighof
and
Lehoucq
in
cite lehoucq.amls
from
the
point
of
view
of
domain
decomposition,
using
adequate
functional

spaces and operators on them. The goal of this talk is to present a complementary viewpoint, which is entirely algebraic.

tt

AMLS is essentially aSchur

complement
method.
Schur
complement
techniques
are
well
understood
for
solving
linear
systems
and
play
a
major
role
in
Domain
Decomposition
techniques.
Relatively
speaking,
the
formulation

of this method for eigenvalue problems has been essentially neglected so far. One could of course extend the approach used for linear systemsin order to

compute
eigenvalues,
by
formulating
a
Schur
complement
problem
for
each
different
eigen-pair,
(e.g.,
by
solving
the
eigenvalue
problem
as
a
sequence
of
linear
systems
through

shift-and-invert). This viewpoint was considered quite early on by Abramov cite Abramov1,Abramov2 and Chichov cite Chichov who presented what may termed a spectral Schur complement method. It can

easily

be

verified

that

a

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an

eigenvalue

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to those $lambda$ $'s$ that are not in the spectrum of B). This nonlinear eigenvalue problem may be solved by a Newton-type approach. Alternatively, one can also devisespecial iterative schemes based on the above observation. An approach of this type is clearly limited by the fact that a Schur complement (or several consecutive ones in an iterative process) is required for each different eigenvalue. It can, however, work well for computing one, or a few, eigenvalues or in some other special situations. For example, this nonlinear viewpoint led to the development of effectiveshifts of origin for the QR algorithm for t r i d i a g o n a l

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number

of

the

smallest

eigenvalues

it

with

only

one

Schur

complement.

To

achieve

this,

tt

AMLS

relies

on

clever

projection

techniques.

It

builds

good

bases

from one Schur complement, and expands them in aneffective way to bigger and bigger domains. In this talk we adopt a purely algebraic viewpoint and d e m o n s t r a t e

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a good subspace for a Rayleigh-Ritz projection process. This technique leads to approximations from a single Schur complement derived from a domain decomposition of the physical problem. Exploiting thisobservation, we have devised several possible enhancements in two main directions. The first introduces Krylov subspaces to the technique, and the second considers a more accurate (second order instead of first order) scheme, which is based on a quadratic eigenvalue problem. Finally, combinations of the above two strategies

have

been

considered

with

a

goal

of

enhancing

robustness.

Currently,

tt

AMLS

is

a

one-shot

algorithm

in

the

sense

that

certain

approximate

eigenvectors

are

build from the last level up to the highest level and no further refinements are made. The current framework does iteratively refine these approximations. We w i l l d i s c u s s

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