

---

Armando Coco  
**Multigrid technique for non-eliminated boundary  
conditions in arbitrary domain**

Via Procida  
54  
96012 Avola (SR)  
Italy  
`coco@dmi.unict.it`  
Giovanni Russo

Poisson equation in arbitrary domain (possibly with moving boundary) is central to many applications, such as diffusion phenomena, fluid dynamics, charge transport in semiconductors, crystal growth, electromagnetism and many others. We present a rather simple iterative method [2] to solve the Poisson equation in arbitrary domain  $\Omega$ , identified by a level set function  $\phi$  in such a way  $\Omega = \{x \in \mathbb{R}^d : \phi(x) < 0\}$ , and mixed boundary conditions. Such iterative scheme is just the building-block for a proper multigrid approach [3].

The method is based on ghost-cell technique for finite difference discretization on a regular Cartesian grid. The structure of ghost points is complex and elimination of discrete boundary conditions from the system is hard to perform. In addition, a simple Gauss-Seidel scheme for the whole system does not converge. Therefore, in order to provide a good smoother for the multigrid approach, we relax the whole problem introducing a fictitious time and looking for the steady-state solution. The iterations on the boundary are performed in order to provide smooth errors. The iterative scheme is proved to converge, at least for first order accurate discretization.

Multigrid techniques for ghost points are well-studied in literature, especially in the case of rectangular domain, where the restriction of the boundary is performed using a restriction operator of codimension 1, since ghost points are aligned with the Cartesian axis. In the case of arbitrary domain, ghost points have an irregular structure and we provide a reasonable definition of the restriction operator for the boundary.

We also show that a proper treatment of the boundary iterations can improve the rate of convergence of the multigrid and the cost of this extra computational work is negligible.

The method can also be extended to the interesting case of discontinuous coefficients across an interface. Some preliminary numerical results are provided in this talk.

# Bibliography

- [1] A. Brandt, Rigorous quantitative analysis of multigrid, I: constant coefficients two-level cycle with L2-norm, *SIAM Journal on Numerical Analysis*, 31 (1994), pp. 1695-1730.
- [2] A. Coco and G. Russo, A fictitious time method for the solution of poisson equation in an arbitrary domain embedded in a square grid, submitted to *Journal of Computational Physics*.
- [3] A. Coco and G. Russo, Multigrid approach for poisson's equation with mixed boundary condition in an arbitrary domain, *in preparation*.
- [4] A. Coco and G. Russo, Second order multigrid methods for elliptic problems with discontinuous coefficients on an arbitrary interface, I: one dimensional problems, submitted to *Numerical Mathematics: Theory, Methods, Application*, proceedings to European Multi-grid Conference EMG2010.
- [5] F. Gibou and R. Fedkiw, A second-order-accurate symmetric discretization of the poisson equation on irregular domains, *Journal of Computational Physics*, 176 (2002), pp. 205-227.
- [6] J. Papac, F. Gibou and C. Ratsch, Efficient Symmetric Discretization for the Poisson, Heat and Stefan-Type Problems with Robin Boundary Conditions, *Journal of Computational Physics*, 229 (2010), pp. 875-889.
- [7] U. Trottemberg, C. Oosterlee, and A. Schuller, *Multigrid*. Academic Press, (2000).