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BiCGStab, VPAStab and an adaptation to mildly nonlinear systems.

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BiCGStab [Van der Vorst, 1992] and GMRes [Saad and Schultz, 1986] are the names of two famous families of iterative methods for the solution of large linear systems. Subsequently, much research has focussed on appropriate generalizations for large nonlinear systems [Kelley, 1995]. Here, we will discuss a nonlinear generalization of BiCGStab, and see its performance on the ϕ-2 equation and Bratu equations in 2D, and for Burghers' equation.

The key equations of BiCGStab will be summarized, and then its connections with vector Padé approximation of the Richardson series will be briefly reviewed. These considerations lead naturally to the algorithm called VPAStab for stabilised vector-Padé approximation of a vector-valued function whose coefficients are linearly generated [Graves-Morris, 2003]. VPAStab, when applied to systems of linear equations, is very similar to BiCGStab. A generalization of the algorithm for the acceleration of convergence of a nonlinearly generated system of equations is proposed here using recursions of the form

$$\begin{aligned}
\mathbf{x}^{(2k)} &:= (1+\alpha_k)\mathbf{x}^{(2k-1)} - \alpha_k \mathbf{x}^{(2k-2)} \\
&\qquad + (1-\theta_k)[(1+\alpha_k)\mathbf{r}^{(2k-1)} - \alpha_k \mathbf{r}^{(2k-2)}], \\
\mathbf{x}^{(2k+1)} &:= (1+\beta_k)\mathbf{x}^{(2k)} - \beta_k \mathbf{x}^{(2k-1)} + (1+\beta_k)\mathbf{r}^{(2k)} \\
&\qquad - \beta_k(1-\theta_k)\mathbf{r}^{(2k-1)}.
\end{aligned}$$

In these formulas, $\mathbf{x}^{(i)}$ are accelerated estimates of the solution of the equations, and $\mathbf{r}^{(i)}$ are their (preconditioned) residuals. The initialization of the recursions and the other parameters α_k, β_k and θ_k are common to those of VPAStab. Some encouraging comparative numerical results will be shown.