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Preconditioning HyBR for Inverse Problems

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In this talk we consider large scale severely ill-conditioned linear systems that arise when solving ill-posed inverse problems. Computed solutions are very sensitive to errors in the data, and regularization is needed to stabilize the inversion process. There are many forms or regularization. In this talk we consider an iterative hybrid bidiagonalization regularization (HyBR) scheme that efficiently applies Tikhonov regularization. An advantage of the HyBR algorithm is that it can automatically estimate and refine regularization parameters at each iteration, and it can estimate a stopping iteration (both of these issues are very difficult for inverse problems). HyBR is based on Lanczos bidiagonalization, where the large scale problem is projected onto a small Krylov subspace. The "hybrid" part refers to the fact that regularization is not applied a priori to the large scale problem, but instead to the small projected problem. Preconditioning to accelerate convergence of HyBR is very difficult; standard approaches are likely to magnify data errors in the early iterations, making it impossible to recover a good approximation of the desired solution. In this talk we illustrate the difficulties of preconditioning HyBR, and describe an approach that can be used for certain classes of inverse problems. Computational examples include the inverse heat equation, as well as examples from image processing.