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Screening spectral information in preconditioning design

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It is well known that the convergence of the Conjugate Gradient algorithm for solving symmetric positive definite systems $Ax = b$ depends on the eigenvalue distribution of the iteration matrix. For a wide class of problems, the presence of isolated eigenvalues at the extreme of the spectrum slows down the convergence significantly. In this circumstance, the rate of convergence of CG can be greatly improved if components of the residual in the directions of eigenvectors associated to the isolated eigenvalues are deflated by means of a suitable preconditioner. Several adaptive preconditioning techniques have been proposed in the past few years that attempt to tackle this problem efficiently [3, 4]. The preconditioner M has generally the form of a low-rank matrix update, and is constructed using spectral information extracted from the coefficient matrix; the spectral information can be computed in advance, prior to the iterative solution, or gathered over the iterations from the Lanczos process.

In this talk, we perform a preliminary study on the effect of the accuracy of the eigencomputation on the quality of the spectral preconditioning; in our study, we consider in particular the spectral preconditioner proposed in [1, 2]. The analysis is based on the perturbation theory, and leads us to proposing a criterion for screening the spectral information in the design of the preconditioner. In particular, we extend this analysis to the case of solving a sequence of linear systems involved in a non-linear process, like those arising in Gauss-Newton process. We show results of numerical experiments from data assimilation problems in oceanography to illustrate the effectiveness of our approach in a real-life application.

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