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**Recycling Preconditioners and Krylov Subspaces for  
Quasi-Newton Methods**

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For many large systems of nonlinear equations or optimization problems the computation of the Jacobian (Hessian) may be very expensive. In addition, potentially large jumps in the coefficients of a system of partial differential equations and the highly nonlinear nature of some problems, for example, porous media flow, require robust Newton methods. As a result, such problems are often solved by Broyden-type methods or other quasi-Newton algorithms, and, for large problems, the resulting linear systems are solved using Krylov subspace methods. This leads to a sequence of related linear systems that change in a structured way although the changes need not be norm-wise small.

In this presentation we discuss how to solve such problems efficiently using preconditioned Krylov subspace methods. The key to efficient solvers is to update and reuse, hence *recycle*, preconditioners and the generated search spaces, and modify the Newton scheme such as to exploit this optimally.

Updating preconditioners reduces the often high computational cost of computing the preconditioner while maintaining or even improving the convergence rate. We consider very simple right preconditioner updates that can be combined directly with the recycling of an existing Krylov space or with the generation of a new Krylov space. Furthermore, we consider various types of recycling of search spaces to make the solution methods more efficient. In particular, we focus on an algorithm that combines two complementary types of recycling. The first type are the Krylov-Secant solvers proposed by Klie and Wheeler [1]. These methods force secant type updates to the Jacobian such that the range of the operator (over a given number of steps) remains in the Krylov space that already exists. The second type are the recycling solvers proposed by Parks, de Sturler et al. [2] and Wang, de Sturler, and Paulino [3] that significantly improve convergence rates for sequences of linear systems where the matrix changes slowly.

By modifying the (quasi-)Newton method we are able to make multiple non-linear iterations with a very modest number of (preconditioned) matrix-vector products in the linear solver, sometimes none. In experimental results it also appears that the number of nonlinear steps where a new accurate Jacobian must be computed is reduced.

## REFERENCES

1. H. Klie and M.F. Wheeler, *Nonlinear Krylov-Secant Solvers*, Technical Report ICES 06-02, Institute for Computational Engineering and Sciences, University of Texas at Austin, January 2006
2. M.L. Parks, E. de Sturler, G. Mackey, D.D. Johnson, S. Maiti. *Recycling Krylov Subspaces for Sequences of Linear Systems*. SIAM Journal on Scientific Computing, Vol. **28**, 1651-1674, 2006.
3. S. Wang, E. de Sturler, and G.H. Paulino. *Large-Scale Topology Optimization using Preconditioned Krylov Subspace Methods with Recycling*. International Journal for Numerical Methods in Engineering, Vol. **69**, 2441–2468, 2007.