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Multiple Restarted Arnoldi Methods to Solve Eigenproblem

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The restarted Arnoldi methods (RAMs) allow to compute some eigenpairs of large sparse non Hermitian matrices. However, the size of the subspace in these methods is chosen empirically. A poor choice of this size could lead to the non-convergence of the method. We propose a technique to remedy to this problem. This approach, called multiple restarted Arnoldi method (MRAM) is based on the projection of the problem on several Krylov subspaces instead of a single one. MRAM allows to update the restarting subspace of a RAM by taking into account the interesting eigen-information obtained in all subspaces. We present a general framework for this kind of methods and an adaptation of its concept to the explicitly/implicitly restarted Arnoldi method (ERAM/IRAM). We focus then on a particular case of MIRAM with nested subspaces (MIRAMns).

The subspaces in MIRAMns defer by their size while the subspaces in the general context of the MRA can defer by their size and their initial vectors. MIRAMns makes use of Arnoldi method to compute the Ritz elements of a large matrix A in a set of ℓ nested Krylov subspaces. If the accuracy of the desired Ritz elements calculated in none of these subspaces is not satisfactory, MIRAMns selects the "best" of these subspaces. This subspace is one that contains the "best" Ritz elements. Then a QR shifted algorithm will be applied to the $m_{best} \times m_{best}$ matrix which represents A in this m_{best}-size projection subspace. As these are the non desired eigenvalues which are chosen for shifts, the leading submatrix issued from QR algorithm concentrates the information corresponding to the desired eigenvalues. MIRAMns completes Arnoldi projections on ℓ nested Krylov subspaces starting with this submatrix whose size is the number of wanted eigenvalues. This approach can be considered as an IRAM with the largest subspace size in which to update the restarting vector the results of the projection on some smaller nested subspaces are taken into account.

One of the well known problems of the restarted iterative methods is the sensibility of the convergence in the small perturbation on the subspace size. Indeed, they can not converge with a subspace and converge with the same re-

duced/extended subspace with nearby sizes. MIRAMns allows to remedy to this problem by making choose of the "best" size between these subspace sizes. Another advantage of this technique is the better property of convergence with almost the same complexity relative to IRAM. We present some experiment results which show a very good acceleration of convergence with respect to the implicitly restarted Arnoldi method.

In point of view of computation, the MRA methods also called hybrid methods can be considered as synchronous or asynchronous methods. They are well suited to recent large-scale highly hierarchical distributed systems. These methods allow to couple synchronously or asynchronously several instances of a RAM (also known as co-methods) in order to decrease the number of iterations required to compute the solution. For a co-method of an asyncrounous hybrid method it is still possible to continue the computation even if the results of the other co-methods are not available. Moreover asynchronus methods are interesting because they provide built-in fault tolerance. As long as one of the co-methods is still running, the application will generate a result. Lastly, these methods can improve the convergence speed thanks to the heterogeneity of processors. Different architectures or processors can lead to different round errors due to differences in floating point operations hardware. Each method can use the result obtained from the others to decrease the effect of round errors. We will see how we can exploit the hybrid methods in the context of large scale distributed systems. We present then the results of some experiments on Grid'5000 which is a French national testbed dedicated to large scale distributed system experiments.