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**Design of Multigrid Methods for Traffic Models based on
Correlated Random Walks**

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We study the configuration of multigrid methods for computing statistics of traffic models. For this purpose we consider the available models for the mobility of vehicles based on different types of continuous-time Markov chains. Of these models we concentrate on the subset represented by Correlated Random Walks because we will show that they are amenable to solution by efficient multigrid methods.

The goal of the multigrid method is to compute statistics of “absorbing times” that represent the time needed for a vehicle to arrive at a certain destination from either a random or a fixed starting point. Within this framework we apply our recent results on the convergence of multigrid methods to design efficient inter-grid operators [1]. Thus, we are able to obtain an optimal method, with efficient parameters, to compute statistics on traffic models based on correlated random walks. We are also able to provide a complete understanding of the convergence issues in this setting.

A mobility model based on continuous-time Markov chains models the streets of a city as a finite grid where each point on the grid corresponds to an intersection of streets. Therefore, a vehicle moving in the city belongs to one point in the grid at a time and it jumps to other grid nodes at random times. The time instant at which the vehicle takes its steps is governed by a Poisson process, meaning that it moves from one intersection to a neighboring intersection with a randomly chosen velocity. In the simplest scenario, the transitions occur only between neighboring grid nodes, which represents a random walk mobility model. In the correlated random walk mobility model, the probability of a vehicle moving between grid nodes in the same direction as its previous step is different than the probability of moving in the opposite direction. Thus, correlated random walks are both more flexible and more accurate models of traffic because they can account for time dependency, geographical restrictions, and nonzero drift.

The statistics associated with absorbing times are important for the evaluation

of mobility models because they give an accurate description of the motion of vehicles on a grid. For example, knowledge of the times between a vehicle's contacts with other vehicles is important in mobile ad-hoc networks that create a communication network based only on wireless communications between vehicles. An important goal is to scale the network and maintain reliable communication. In this scenario, it is known that the mobility patterns have a significant impact on the network coverage and throughput-delay characteristics [2]. The statistics of absorbing times are thus important for the design of large mobile ad-hoc networks.

The statistics of absorbing times are difficult to obtain since they require the solution of sparse linear systems. The multigrid algorithm represents an optimal choice for this purpose because its complexity scales optimally. Nevertheless, the configuration of parameters needs special care to guarantee and optimize convergence. Although the structure of the systems is unusual for classic multigrid theory, we prove that it fits the assumptions of our previous results on convergence analysis. Using this analysis, we can design smoothing and inter-grid operators with precise adjustment of convergence properties. Thus, we obtain an efficient and robust algorithm for the calculation of absorbing times.

To apply our analysis, we first need to check some assumptions on the system. The strongest assumption is condensed in a single algebraic property, called the Harmonic Aliasing Property, that contains all the information needed from the geometry of the discretization and the modal structure of the system. Then, the starting point is to check this property on the correlated random walk mobility model. After proving this property holds, we apply our analysis to design efficient inter-grid operators and check their performance. By this means we obtain the exact rates at which groups of modal components of the error evolve and interact for systems of different sizes. This provides a complete understanding of the algorithm in terms of its convergence properties and how they scale with the system.

The multigrid configuration that we propose is not the only choice for this application. The most common choice for these type of problems is algebraic multigrid methods (AMGs), as they are designed to avoid almost any assumption on the system. Unfortunately, the convergence results obtained so far in the theory of AMG are not as strong as the well-established results for linear operators with constant stencil coefficients, known as Local Fourier Analysis (LFA). On the other hand, the use of LFA for this application is not possible because the linear systems needed to be solved have variable stencil coefficients. Thus, this application shows how our convergence analysis is more flexible than LFA as it allows us to obtain exact convergence rates for models where LFA is not applicable.

Finally, in our previous work it remained unknown how flexible our convergence analysis could be. We have shown that it is more general than LFA but at

the same time it is not as general as AMG. The last seems reasonable, as our analysis gives convergence rates which are as precise as the ones obtained by LFA. We have also provided examples of systems where our analysis works but LFA does not. These early examples were not, unlike the one in this paper, motivated by real applications. Thus, the application of our convergence analysis in traffic models gives new insight on the nature of the systems where our analysis is applicable. This application is therefore an important step towards understanding the flexibility and limitations of our new analytical approach.

[1]P. Navarrete, and E.J. Coyle, "A semi-algebraic approach that enables the design of inter-grid operators to optimize multigrid convergence," *Numerical Linear Algebra with Applications*, Vol. 15, No. 2-3, pp. 219-247, March 2008.

[2]S. Bandyopadhyay, E.J. Coyle, and T. Falck, "Stochastic Properties of Mobility Models in Mobile Ad-Hoc Networks," *IEEE Transactions on Mobile Computing*, Vol. 6, No. 11, pp. 1218-1229, November 2007.