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Multigrid Methods for Solving Hamilton-Jacobi-Bellman and Isaacs Equations Arising from Nonlinear Option Pricing Problems

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Many financial pricing problems can be formulated as optimal control problems, leading to nonlinear Hamilton-Jacobi-Bellman (HJB) partial differential equations (PDEs) or Hamilton-Jacobi-Bellman-Isaacs (HJBI) equations. In general, the HJB equations can be written as:

$$V_\tau = \sup_{q \in Q} \left\{ a(S,\tau,q) V_{SS} + b(S,\tau,q) V_S - c(S,\tau,q) V \right\},$$

where V typically represents the value of an option, S is the underlying asset price, τ is the time to expiry, and q is a control parameter. In the case of HJBI equations, they can be written as:

$$V_\tau = \sup_{q \in Q} \inf_{p \in P} \left\{ a(S,\tau,q,p) V_{SS} + b(S,\tau,q,p) V_S - c(S,\tau,q,p) V \right\},$$

where p is an additional control parameter. These two types of equations are highly nonlinear.

An implicit discretization yields a nonlinear set of equations which must be solved at each time step. Standard methods such as fixed point iterations can be very slow for fine grids. In the case of HJB equations, a Newton-type iteration scheme, often known as the Policy iteration in stochastic control, has been proposed. Quadratic convergence is observed when close to solution. However, in the case of HJBI equations, Policy iteration may not necessarily converge.

In this talk, we present multigrid methods for solving the HJB and HJBI equations. The use of Policy iteration and FAS multigrid will be discussed. The proposed multigrid methods are based on a relaxation scheme as smoother which is convergent for both HJB and HJBI equations. We show by Fourier analysis the smoothing property of the relaxation scheme. Linear interpolation and full weighting are used for intergrid transfer. We demonstrate numerically the effectiveness of the multigrid methods by examples of financial pricing problems.