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Multigrid preconditioning of linear systems for interior point methods applied to a class of box-constrained optimal control problems

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In this work we construct and analyze multigrid preconditioners for operators of the form $\mathcal{D}_\lambda + \mathcal{K}^*\mathcal{K}$, where D_λ is the multiplication with a relatively "smooth" function $\lambda > 0$ and $\mathcal{K}$ is a discretization of a compact linear operator. These systems arise when applying interior point methods to the distributed optimal control problem $\min_u \frac{1}{2}(\|\mathcal{K}u - f\|^2 + \beta\|u\|^2)$ with box constraints $\underline{u} \leqslant u \leqslant \overline{u}$ on the control u. The presented preconditioning technique is related to the one developed by Drăgănescu and Dupont in [1] for the associated unconstrained problem, and is intended for large-scale problems. As in [1], the quality of the resulting preconditioners is shown to increase as $h \downarrow 0$ at a rate that is optimal with respect to h if the meshes are uniform, but decreases as the smoothness of λ declines. We test this algorithm first on a Tikhnov-regularized backward parabolic equation with $[0, 1]$ constraints and then on the elliptic-constrained optimization problem

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}\|y - f\|^2 + \frac{\beta}{2}\|u\|^2 \\ \text{subj. to} & \Delta y = u\ ,\ y \in H_0^1(\Omega),\ \underline{u} \leqslant u \leqslant \overline{u}\ \ a.e. \end{array}$$

In both cases it is shown that the number of linear iterations per optimization step, as well as the total number of fine-scale matrix-vector multiplications is decreasing with increasing resolution, thus showing the method to be potentially very efficient for truly large-scale problems.

This is joint work with Cosmin Petra from the Argonne National Laboratory.

Bibliography

[1] Andrei Drăgănescu and Todd F. Dupont. *Optimal order multilevel preconditioners for regularized ill-posed problems*, Math. Comp., 77 (2008), pp. 2001–2038.