

---

Philipp Birken  
**Matrix-free preconditioned Newton-Krylov methods for  
nodal discontinuous Galerkin schemes**

University of Kassel  
Department of Mathematics  
Heinrich-Plett-Str 40  
34132 Kassel  
Germany  
`birken@mathematik.uni-kassel.de`  
Mark Haas

In recent years, discontinuous Galerkin (DG) methods have received ever increasing interest in the context of computational fluid dynamics, both for incompressible and compressible flows. Here, we will focus on unsteady compressible flows. DG methods can be seen as generalizations of first order finite volume schemes, that extend to higher orders in a natural way. However, they have more severe stability restrictions for explicit time integration and thus, implicit schemes are necessary for a large set of applications. Of interest for unsteady flows are higher order schemes and we will apply diagonally implicit Runge-Kutta (DIRK) methods. This means that on each stage, a nonlinear equation system has to be solved. However, so far efficient solution schemes in the context of DG methods are lacking and the design of such algorithms is currently one of the most important challenges for the successful application of DG schemes.

In this talk, we will consider preconditioned Newton-Krylov schemes for the solution of the resulting linear equation systems. The system matrices are non-normal block matrices, with block sizes of 16-120 for two dimensional flows, which makes special treatment of these blocks necessary and storing the whole matrix impossible for large problems. Therefore, we consider matrix-free methods that circumvent storing the Jacobian by approximating  $Ax$  via difference quotients. Furthermore, this allows to approximate the impact of the Jacobian by a cheap low order operator in a straight forward way. Several preconditioners suitable for matrix-free schemes will be considered, in particular Jacobi and SGS.