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**Using spectral low rank preconditioners for large electromagnetic calculations.**

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In electromagnetic calculations, a classic problem is the monostatic radar cross section calculation of an object. The procedure consists in taking a set of waves with the same wavelength but different incident angles that illuminate the object. For each of these waves we compute the electromagnetic field backscattered in the direction of the incident wave and then deduce the radar signature corresponding to this incident angle. This requires the solution of one linear system per incident wave. For a complete radar cross section calculation, from a few ten up to a few hundred of waves have to be considered. We have then to solve a sequence of linear systems having the same coefficient matrix but different right-hand sides. The problem can be written:

$$M_1 A(x_1, \dots, x_p) = M_1(b_1, \dots, b_p)$$

where $M_1$ is a left preconditioner and $A$ is a large dense complex symmetric matrix that arises from Boundary Element Method.

Our study starts from the observation that when the matrix $M_1 A$ has some eigenvalues near zero, the convergence of the Krylov methods is often slow. The following proposition from [1] shows that we can construct an update $\widetilde{M}_c$ from spectral information of $M_1 A$ to correct $M_1$ such as the new preconditioned system $M_2 A u = M_2 b$ no longer has eigenvalues in a certain neighbourhood of zero. Assume that $M_1 A$ is diagonalizable:

$$M_1 A = V \Lambda V^{-1},$$

with $\Lambda$ the diagonal matrix formed by the eigenvalues $\{\lambda_i\}_{i \in \{1,n\}}$ ordered by increasing magnitude, and $V$ the associated right eigenvectors. We consider the $k$ smallest eigenvalues and $V_k$ the associated right eigenvectors.

**Proposition 1.** *Let $W$ be such that $\tilde{A}_c = W^H A V_k$ has full rank, $\tilde{M}_c = V_k \tilde{A}_c^{-1} W^H$ and $M_2 = M_1 + \tilde{M}_c$. Then $M_2 A$ is similar to a matrix whose eigenvalues are*

$$\begin{cases} \eta_i = \lambda_i & \text{if} \quad i > k, \\ \eta_i = 1 + \lambda_i & \text{if} \quad i \leq k. \end{cases}$$

The matrix $\tilde{M}_c$ is defined as the Spectral Low Rank Update (SLRU) for the left preconditioner $M_1$.

To illustrate the efficiency of this approach we consider a set of large and challenging real life industrial problems. We perform experiments with a parallel fast multipole code [2] to compute the matrix-vector products involving A. For $M_1$ we choose the preconditioner developed in [3], suitable for implementation in a multipole framework on parallel distributed platforms. It is based on a sparse approximate inverse using a Frobenius norm minimization with an a priori sparsity pattern selection strategy. The spectral information is computed in a preprocessing phase by an external eigensolver: ARPACK [4].

In this talk, we present the gain in terms of times and matrix-vector products, for the complete monostatic calculations [6]. We also illustrate the effects on the convergence rate of GMRES [5] of parameters such as the dimension of the update, the accuracy of the spectral information, the quality of the original preconditioner or the size of the restart. We conclude with some comments on our on-going work where we combine the SLRU preconditioner and the Seed-GMRES or the GMRES-DR [7] solver.

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