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Implicit Interface Tracking: Marker-Redistancing Jacobian-Free Newton-Krylov (MR-JFNK) Method

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A new algorithm for high-fidelity interface tracking is introduced. It combines the *Lagrangian marker tracking* with the *Eulerian level-set based implicit re-distancing*, forging advantages of each of them in a rather simple and very accurate algorithm. The Lagrangian markers are the carriers of the interface information. In difference to the front-tracking method, the markers are not explicitly connected, which is a significant simplification in three dimensions. The interface topology/connectivity and geometry (normals and curvature) are provided by the Eulerian field (the signed distance, or the level set function). During each time step, the markers are moved with the velocity field (defined by either interface topology, in the curvature-driven applications; by material velocity, in the fluid-dynamics applications; or by their combination, in the phase-change fluid dynamics applications). Then, the signed distance function is re-initialized. We developed an implicit high-order-accurate HWENO-based PDE-reinitialization, which is efficient (stable under any CFL number, with adaptive pseudo-time-step control, permitting fast convergence to the pseudo-time-steady state), and the 5^{th}-order-accurate in space, with a very compact stencil. The re-initialization procedure is formulated as a system of the conservation laws for the normal vector (hyperbolic components), coupled with the level set equation (containing elliptic components). The Lagrangian field (markers) is connected/mapped/anchored to the Eulerian field (signed distance functions) using the high-order-accurate Least-Squares interpolation procedure. Importantly, we do not operator-split the Lagrangian step (marker motion) and the Eulerian steps (fluid dynamics and interface topology defined by re-distancing), but instead fully-couple them non-linearly using Newtons iteration method. For time integration, we employ implicit Runge-Kutta schemes, which are L-stable. Since operator-splitting errors are completely avoided, we can demonstrate high-order-accurate (shown up to the 5^{th}-order) solutions of the non-linear fully-coupled interface tracking problems. Each linear step of the Newton iterations is solved with the Jacobian-free version of the Krylov (GMRES) algorithm. We discuss several GMRES preconditioning techniques, including the classical ILU-based and the Physics-(PDE)-based (PB) preconditionings. In the PB preconditioning, we extract an elliptic component of the re-distancing, forming a Poisson equation, to be effectively solved with the algebraic multigrid method. The efficacies of each preconditioning technique are compared in terms of both the ability to effectively cluster eigenvalues (*Eigenscopy*) of the Jacobian matrices and the number of Krylov steps needed to converge GMRES. As the demonstration test-cases, we use *solid-body translation/rotation* (Zalesak disk, Crusiform problems), *moderate and severe interface deformation, tearing and stretching* (fluid body in 2D and 3D single- and multiple vortices) and 2D/3D curvature-driven flows; showing high-order spatiotemporal convergence for each test-case