
Sue Dollar
Equivalent SPD systems for saddle-point problems

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Consider symmetric saddle-point problems of the form

$$\begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}. \quad (1)$$

The solution to (1) also satisfies the symmetric system

$$\begin{aligned} & \left[\sigma \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} + \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} \begin{pmatrix} D & F^T \\ F & E \end{pmatrix} \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix} \\ &= \begin{pmatrix} \sigma b + A(Db + F^T d) + B^T(Fb + Ed) \\ \sigma d + B(Db + F^T d) - C(Fb - Ed) \end{pmatrix}, \end{aligned} \quad (2)$$

for given real σ , arbitrary symmetric matrices D and E , and arbitrary matrix F .

We show that many popular conjugate gradient-based methods for solving (1) can be reformulated as applying the (preconditioned) conjugate gradient method to (3) for some σ , D , E and F . We also provide conditions for guaranteeing that (3) is positive definite. Using these conditions we propose new conjugate gradient-based methods for solving (1).