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Daniil Svyatskiy  
**An Adaptive Hierarchy of Discretizations for Two-Phase  
Flows in Porous Media**

Applied Mathematics and Plasma Physics  
Theoretical Division  
Mail Stop B284  
Los Alamos National Laboratory  
Los Alamos  
NM 87545  
U S A  
dasvyat@lanl.gov  
Konstantin Lipnikov  
*Los Alamos National Laboratory*  
David Moulton  
*Los Alamos National Laboratory*

We present a multiscale method which builds recursively a problem-dependent multilevel hierarchy of models for flow in heterogeneous porous media. Each model preserves important physical properties, such as local mass conservation. In contrast with classical two-level methods that achieve a total coarsening factor of approximately 10, in each coordinate direction, the multilevel hierarchical approach facilitates large total coarsening factors of 100 or more. Maintenance of the hierarchy of models incurs only a modest computational overhead due to the efficiency of recursive coarsening and adaptive update strategies. The method supports full diffusion tensors on unstructured polyhedral meshes and accommodates general coarsening strategies. Due to its algebraic nature, the method can be adapted to different types of discretization methods, such as the Mixed Finite Element, Finite Volume and Mimetic Finite Differences methods.

To efficiently maintain fine-scale accuracy in the multiscale solution, the method incorporates two adaptive strategies. First, the hierarchy of models is updated locally when the relative permeability, which depends on the water saturation, changes significantly. Second, an efficient error indicator which controls the temporal updates of the flux coarsening parameters. This new strategy concentrates updates around critical times when the invading fluid (water) first enters key features of the reservoir.

To discretize this system in time we use the IMPES time discretization scheme (IMplicit Pressure and EXplicit Saturation). The saturation is updated using the Darcy velocities provided by the pressure solver. Numerical simulations for permeability fields with long correlation lengths, which are challenging problems for multiscale methods, show that even with large coarsening factors, such as

256 in the vicinity of wells, in each coordinate direction, the multiscale solution remains within 5 of the finescale solution.