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Multilevel solution of diffusion operators with cut cell discretizations

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Diffusion operators $\mathcal{L} = \nabla \cdot \beta \nabla$ with discontinuous coefficients $\beta : \mathbb{R}^d \to \mathbb{R}$ are encountered in numerous applications, such as heat conduction, neutron transport, and subsurface flow. These discontinuities represent changes in material properties (thermal conductivity, interaction cross sections, and permeability). Accurate discretization of these operators is challenging when the discontinuities in β are not aligned with the computational mesh. Mesh alignment can be achieved by constraining mesh generation to conform to the geometry of the different materials when using unstructured or boundary-fitted Cartesian grids. However for complicated geometries this may be intractable. An alternative approach is to use a background Cartesian grid and to represent the geometry by intersecting it against the grid. This gives rise to a set of irregular *cut cells* where Cartesian grid cells intersect the geometry.

While originally developed to handle irregular physical boundaries, cut cells can also be used to resolve material interfaces that are interior to the domain. This approach replaces the difficult global grid generation problem with more tractable local intersection problems. When combined with local mesh refinement, high resolution representation of complex geometry can readily be obtained. While this essentially solves the geometry problem, satisfactory discretization of diffusion operators has remained elusive. Second-order finite difference schemes that account for the location and orientation of the interface in a cut cell have been developed, but these approaches are not conservative. Finite volume schemes, such as the ghost fluid method developed in the context of interface tracking using level sets, have also been developed. However analysis of the convergence of these methods is complicated by the lack of smoothness of the solution in the vicinity of the jumps in β.

Because local mesh refinement is necessary for accurate and efficient representation of complex geometry with cut cells, multilevel solvers are essential. We discuss issues related to discretization, grid convergence, and solver efficiency for diffusion operators on cut cell grids.