
Homer F. Walker
**Approximate finite-differences in matrix-free
Newton-Krylov methods**

Mathematical Sciences Department
Worcester Polytechnic Institute
100 Institute Road
Worcester
MA 01609-2280
`walker@wpi.edu`
Peter N. Brown
Rebecca D. Wasyk
Woodward, Carol S.

Newton–Krylov methods are often implemented in “matrix-free” form, in which the Jacobian-vector products required by the Krylov solver are approximated by finite differences. We consider using approximate function values in these finite differences. We first formulate a finite-difference Arnoldi process that uses approximate function values and give backward-error results for it. We then outline a Newton–Krylov method that uses an implementation of the GMRES or Arnoldi method based on this process and develop a local convergence analysis for it, giving sufficient conditions on the approximate function values for desirable local convergence properties to hold. We conclude with numerical experiments involving particular function-value approximations suitable for nonlinear diffusion problems. For this case, conditions are given for meeting the convergence assumptions for both lagging and linearizing the nonlinearity in the function evaluation.