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The Combinatorial Multigrid Solver

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The study of systems on graph Laplacians -the discrete analog of the inhomogeneous Poisson equation- has been a topic of interest for both the scientific computing (SC) and the theoretical computer science (TCS) communities. While the SC community has developed multigrid methods for this problems since the 70s, the TCS community has started studying the problem much more recently, in the middle 90s, through the introduction of Combinatorial Preconditioners, by P. Vaidya. The intense study of Combinatorial Preconditioners has recently lead to algorithms with strictly provable asymptotic properties: the Spielman-Teng solver, with a complexity of $O(m \text{ polylog}(m))$ for general Laplacians with m non-zeros, and our $O(n)$ solver for planar Laplacians.

In this talk we present the Combinatorial Multigrid Solver (CMG). The CMG solver is a multigrid solver derived through the construction of combinatorial preconditioners that are based on the combinatorial geometry of the underlying graph. In that way the CMG solver combines the "black-box/theoretical guarantees" quality of the combinatorial preconditioners approach, with the speed and the parallelism potential of multigrid. To validate our claim, we present experiments with very large 3D Laplacians derived from medical scans.