Marko Huhtanen

APPROXIMATE FACTORS FOR THE INVERSE

Institute of Mathematics
Helsinki University of Technology
Box 1100
FIN-02015
Finland
marko.huhtanen@tkk.fi

Let $\mathcal{W}$ and $\mathcal{V}_1$ be sparse matrix subspaces of $\mathbb{C}^{n\times n}$ containing invertible elements such that those of $\mathcal{V}_1$ are readily invertible. To precondition a large linear system involving a sparse nonsingular matrix $A \in \mathbb{C}^{n\times n}$, in this talk we consider

$$AW \approx V_1 \tag{1}$$

with non-zero matrices $W \in \mathcal{W}$ and $V_1 \in \mathcal{V}_1$ both regarded as variables. The attainability of the possible equality can be verified by inspecting the nullspace of

$$W \longmapsto (I - P_1)AW, \text{ with } W \in \mathcal{W}, \tag{2}$$

where P_1 is the orthogonal projection onto $\mathcal{V}_1$ [1].

Corresponding to the smallest singular values of (2), we have $(I - P_1)AW \approx 0$ if and only if $AW \approx V_1 = P_1AW$. This gives rise to the criterion

$$\min_{W\in\mathcal{W}, ||W||_F=1} ||(I - P_1)AW||_F$$

for a starting point to generate approximate factors W and $V_1 = P_1AW$. Then

$$\frac{\big|\big|AWV_1^{-1} - I\big|\big|}{\big|\big|V_1^{-1}\big|\big|} \leq ||AW - V_1|| \leq \big|\big|AWV_1^{-1} - I\big|\big|\, ||V_1||$$

in the 2-norm, whenever V_1 is invertible. Consequently, the maximum gap between these two approximation problems is determined the condition number of V_1. In the special case $\mathcal{V}_1 = \mathbb{C}I$ the equalities hold in general. This corresponds to the criterion

$$\min_{W\in\mathcal{W}} ||AW - I||_F$$

which constitutes a starting point for constructing sparse approximate inverses.

Bibliography

[1] M. HUHTANEN, *Factoring matrices into the product of two matrices*, BIT, 47 (2007), pp. 793–808.