Multiscale Modeling for Stochastic Forest Dynamics

Maud Comboul

PhD candidate, comboul@usc.edu

University of Southern California, Los Angeles CA

Abstract

This paper investigates a multiscale approach to forest dynamics. An individual-based forest model is coarse-grained to produce an approximated process capable of describing the dynamics at a mesoscopic length scale while preserving fluctuations from tree interaction. From a birth-death-growth stochastic process on a lattice system, the coarse-graining method derives an approximating random process evolving on a coarser grid, thereby reducing the simulation time.

1 Introduction

Forest dynamics are inherently multiscale. At the level of individual trees, the seed dispersal and the competition for light and nutrients are the main drivers of the dynamics. However, natural and anthropogenic disturbances such as fires, which are swift to reshape forests but also can entirely change the species composition of forests, act at mesoscopic length scales. Finally, climate and the available nutrients in the soil are driving factors at macroscopic scales. While individual-based forest models successfully reproduce real forest behaviors (see Comas and Mateu [2007], Särkkä [2006], Särkkä and Renshaw [2006]), they are computationally intensive and prohibit long-term predictions on large spatial domains. The purpose of this study is to coarse-grain the microscopic process driven by the tree interactions in an effort to design an efficient forest model capable of describing the multiscale phenomena as well as interacting with disturbance processes such as fires, tornadoes and logging. The microsopic model is a spatio-temporal marked point process evolving on the two dimensional torus (see Särkkä and Renshaw [2006]). The points emerge and die according to a stochastic birth-death process; their associated marks, which represent tree growth, evolve following a deterministic ODE. When a mark becomes negative, it means that its associated point has died from competition with the neighboring points. The derived coarse-grained model is also a stochastic birth-death process; yet instead of tracking the exact tree locations, it keeps track of the tree-count per patch (see Katsoulakis et al. [2003b], Katsoulakis et al. [2003a]). The approximated model derived from this approach significantly retains the microscopic fluctuations while improving computational tractability.

2 Forest Dynamics

There are many different processes involved in the evolution of forests. At the tree level, the seed dispersal process defines the birth and position of a tree. The neighborhood dynamics is driven by the competition for light and nutrients in the soil. The competition process determines which trees (shade-tolerant or shade-intolerant species) will pursue their growth and which ones will die from oppression. Anthropogenic and natural disturbances such as fires, tornadoes, diseases, insects and logging greatly impact the structure and the species composition of forests. Finally, climate provides the light, moisture and temperature conditions defining the forest tree species composition.

The following terms define the different forest scales (Frelich [2002]):

- Neighborhood: a small grove of trees 10 to 20 meters across.
- Stand: 1 to 100 ha
- Landscape: a collection of stands, usually larger than 1000 ha

In the past 40 years, significant research tackled forest dynamics modeling and more generally complex ecological modeling. The most widely used model, derived from individual tree dynamics, is the gap model (JABOWA, Botkin et al. [1972] and FORET, Shugart and West [1979]). The JABOWA-FORET model class describes forest growth on a coarse grid, representing patches of trees. While the exact locations of trees are absent from the formulation, the patches evolve from individual tree dynamics, which include unstructured random components (random birth, random death and deterministic growth). Patch properties are horizontally homogeneous; yet the vertical structure is dependent on each tree composing the patch: taller trees shade smaller ones in a unidirectional way. The patches evolve independently of each other because the early models are distance independent (JABOWA-FORET). Later models included interaction between adjacent patches (ZELIG, Urban et al. [1991]), however the absence of horizontal

structure within patches rendered the task less effective. The lack of horizontal interactions in the gap forest models influenced later models to refine the grid scale in order to explicitly include the exact tree locations into the formulation (SORTIE, Pacala et al. [1993]). In this model, grid points are either occupied or not by individual trees and the interactions occur over influence zones defined according to the tree size and type. The model formulation in terms of stochastic point process theory, where the trees form a system of dependent random variables (Gavrikov and Stoyan [1995], Stoyan and Stoyan [1998] and Stoyan and Penttinen [2000]) offers well-defined statistical properties that improve the analysis and the model validation procedures. The explicit integration of unobserved intensity processes or interaction potentials generate models that are more realistic; for example the models are able to reproduce the aggregation behavior observed in some natural forests. More recently, models described with marked point processes (Särkkä and Renshaw [2006], Comas and Mateu [2007] and Comas and Mateu [2008]) provide a new layer of complexity. The marks evolve continuously or discretely driven by neighborhood potentials, thereby adding interactions with processes representing tree growth, tree specie or both. A common issue to all the gap and (marked) point process models remains the high computational cost. Individual trees are tracked independently as well as their associated growth parameters and influence zone. This drawback prohibits long term predictions over landscapes scales. The LANDIS model Mladenoff [2004], was developed during the 1990s in an effort to study forests at a landscape scale. LANDIS runs on a mesoscale grid, where each cell represents a forest stand containing tree species age classes. While providing the necessary tools to study the forest interactions with natural and anthropogenic disturbances, it lacks the mechanistic complexity of the individual tree models.

3 Coarse-Graining

Coarse-graining approaches mainly intervene in molecular dynamics to bridge time and length scales. The purpose of those methods is to replace computationally intensive particle system simulations with less complex ones. There are three general methodologies (Ismail [2005]). Substitution methods map the detailed particle model to a simpler one, either by averaging or through dynamic mean field approach by considering the density distribution of the particles as solution of partial differential equations. Those methods are problematic if one wants to restore the particle configuration because of the absence of reverse map. Multiscale methods compute the necessary parameters using the scale specific processes and exchange the information as needed. However, atomistic simulations cannot generally be avoided and the system behavior at intermediary scales remains unexplored. Finally, multiresolution methods parameterize the level of coarse-graining to be performed, thereby offering flexibility by allowing a progressive approach as well as the use of several resolutions within one simulation. The multiresolution methods widely use the wavelet transform approach (Ismail et al. [2002], Ismail [2005]). Though recently, Katsoulakis et al. [2003b], Katsoulakis et al. [2003a], and Katsoulakis and Vlachos [2003] have developed a novel coarse-graining approach for the efficient Monte Carlo simulation of particle systems on lattices. From a stochastic Ising-type microscopic model of particles on a lattice, they derive an approximating coarse-grained birth-death Markov process. Having fewer observables, the coarse random process is far more suitable for computationally extensive simulations. The novelty resides in the fact that the approximated coarse-grained process significantly retains the microscopic interactions between neighboring particles while allowing the description of the system in mesoscopic or macroscopic length scales. The authors have built an essentially theoretical yet extremely useful framework for the efficient simulation of particle systems at any length scale. The method was applied in the paper by Khouider et al. [2003] to the simulation of tropical convection in climate models.

4 Coarse-Grained Forest Model

4.1 Overview

Our understanding of the effect of varying disturbance regimes and their interactions on forest successional trajectories is limited. However, we know that neighborhood effects, such as seed dispersal, light availability, and nutrient feedbacks to the soil, all play a role in tree dynamics (Frelich [2002]). These coupled characteristics are defining properties of a complex adaptive system; *i.e.* the ability to self-organize because of local nonlinear interactions and to adjust to external perturbations. Computer simulations are extremely useful for the study of evolving complex systems especially when the accessible observations are scarce. Yet, the adopted strategy will decide the nature of the model predictions. Unlike the JABOWA-FORET-SORTIE model class, which considers each individual tree interactions, the reduced complexity of stand models, such as LANDIS, allow long-term landscape size predictions; nevertheless, these models heavily trade off important mechanistic properties. Inspired by the multiscale methods often employed in molecular dynamics simulations, the design of a coarse-grained forest model approximated from a detailed description of the tree interactions could produce an effective approach.

Regarding the fine scale forest model, the Markov marked point process (MPP) description presents several advantages. Not only does it offer expandable mechanistic properties through the possibility of adding marks that feedback into the local interactions, but it also has well-defined statistical properties such as Gibbs distributions, which can be computed from the energy of the system. Moreover, the MPP model formulation described in Särkkä and Renshaw [2006] in terms of continuous space-time stochastic process demonstrates broadening aptitudes to more realistic simulations (Comas and Mateu [2007]).

The coarse-graining technique, developed in Katsoulakis et al. [2003b], Katsoulakis et al. [2003a], and Katsoulakis and Vlachos [2003], if successfully adapted to the theory of MPPs, is best suited to our needs. Unlike other methods, which purpose is to preserve only the slow fluctuations, this method derives a noise model that preserves the microscopic fluctuations due to the particle interactions, while rescaling the model by increasing the size of the simulation element. This technique also allows keeping track of the error introduced by the approximation process as a function of the applied coarsening level.

This strategy should let us build forest models at arbitrary mesoscales while preserving the tree level fluctuations. The complexity reduction due to the grid coarsening, significantly reduces the required simulation time, thereby permitting long-term predictions over large forested regions. The next section covers the mathematics associated with the MPP approach to forest dynamics modeling and the coarse-graining theory. Finally I will cover results as well as research goals and I will suggest a timeframe to accomplish the project.

4.2 Mathematical Framework

The microscopic system consists of a stochastic marked point process on a lattice used in Särkkä and Renshaw [2006]. From known dynamics at the scale of individual trees, we derive an approximating coarse-grained random process described in terms of patches of trees. This technique is inspired by the work by Katsoulakis et al. [2003b], Katsoulakis et al. [2003a], and Katsoulakis and Vlachos [2003].

The physical domain is the two-dimensional torus $T_2 := [0,1]^2$. It is first divided into m^2 square coarse cells $D_k, k = 1, ..., m^2$. Let $L_c := \frac{1}{m}\mathbb{Z}^2 \cap T_2$ be the corresponding coarse-grained lattice. Each coarse cell is subdivided into q^2 microcells for a total of $N = (mq)^2$ cells. Let $L := \frac{1}{mq}\mathbb{Z}^2 \cap T_2$ then we have $L_c \subset L$.

4.2.1 Individual-based microscopic process

Let $\Sigma = \{0,1\}^L$ be the configuration space, a point pattern σ at lattice site x is written $\sigma = \{\sigma(x) : x \in L\}$ where $\sigma(x) = 1$ if there is a tree at lattice point x, otherwise $\sigma(x) = 0$.
For a configuration σ we can write the total energy of the system H in terms of h the interparticle potential function and α the external field (taken to be zero for the moment):

$$H(\sigma) = \sum_{x \in L} \sum_{y \neq x; y \in L} h(m_x, m_y; \|x-y\|)\sigma(x)\sigma(y) + \sum \alpha\sigma(x) \tag{1}$$

h is proportional to the overlapping area between two trees while reflecting asymmetric interactions due to the relative sizes of two competing trees (for $b > 0$):

$$h(m_x, m_y; \|x-y\|) = -b \quad \frac{Area(D(x, rm_x) \cap D(y, rm_y))}{\pi r^2 m_x^2} \tag{2}$$

Where $D(x,r)$ is the disk of radius r centered at x.
The size of each tree (or diameter at breast height) $m_x(t)$ evolves in time driven by a linear growth function and lessen by the "force" exerted by all the surrounding trees (Särkkä and Renshaw [2006]).

$$\frac{dm_x}{dt} = \lambda(1 - \frac{m_x(t)}{S}) + \sum_{y \neq x} h(x,y) \tag{3}$$

For a configuration σ, a tree arrives at an empty lattice node x during time interval $[t, t+\Delta t]$ with probability $\alpha_a \Delta t$ and a tree at lattice node y dies either randomly with probability $\alpha_d \Delta t$ or because of competition with neighboring trees when its mark $m_y(t+\Delta t) \leqslant 0$. We can define the underlying stochastic process $\{\sigma_t\}_{t \geq 0}$ as a continuous time Markov process with generator $\mathcal{L}_N f(\sigma) = \sum_{x \in L} c(x,\sigma)[f(\sigma^x)]$, where f is a test function and where the birth/death rate of occurrence $c(\sigma, x)$ is the following:

$$\begin{aligned} c(x,\sigma) &= \alpha_a(1 - \sigma(x)) + \alpha_d \sigma(x) \exp[-U(x)] \\ U(x) &= \sum_{y \neq x, y \in L} h(x,y)\sigma(y) + \alpha \end{aligned} \tag{4}$$

The detailed balance condition is imposed on the dynamic to make sure it leaves the Gibbs measure invariant:

$$c(x,\sigma)\exp(-\beta H(\sigma)) = c(x,\sigma^x)\exp(-\beta H(\sigma^x)) \tag{5}$$

where σ^x is the configuration after a birth/death at site x.

4.2.2 Coarse-grained process

The coarse-graining of the microscopic process previously described is derived from the work by Katsoulakis et al. [2003b], Katsoulakis et al. [2003a], and Katsoulakis and Vlachos [2003]. $\{\eta_t\}_{t \geq 0}$ is the coarse-grained process defined on the coarse lattice L_c. For each coarse cell $D_k \in L_c$, $\eta_t(k)$ tracks the number of trees in D_k: $\eta_t(k) = \sum_{y \in D_k} \sigma_t(y)$. The coarse process η_t takes its values in $\{0, 1, ..., q\}$ instead of taking binary values since each coarse cell contains q micro lattice points. We also define the average of the process $\bar{\eta}_t = \frac{1}{q}\eta_t(k)$
A coarse-grained interaction potential from a coarse cells D_l onto a distinct coarse cell D_k is defined be including all microscopic interactions from the trees $y \in D_l$ on the trees $x \in D_k$:

$$\begin{aligned}\phi(k,l) &= \sum_{x\in D_k}\sum_{y\in D_l} h(x,y)\\ \bar{\phi}(k,l) &= \frac{1}{\eta(l)\eta(k)}\phi(k,l)\end{aligned} \tag{6}$$

Similarly, we define the coarse-grained interaction potential within a coarse cell D_k:

$$\begin{aligned}\phi(k) &= \sum_{x\in D_k}\sum_{y\in D_k; y\neq x} h(x,y)\\ \bar{\phi}(k) &= \frac{1}{\eta(k)(\eta(k)-1)}\phi(k)\end{aligned} \tag{7}$$

We may now express the Hamiltonian of the system in terms of the previously defined coarse-grained process:

$$\begin{aligned}\bar{H}(\eta) = &\sum_{k\in L_c}\sum_{l\in L_c k\neq l}\bar{\phi}(k,l)\eta(k)\eta(l)\\ &+\sum_{k\in L_c}\bar{\phi}(k)\eta(k)(\eta(k)-1)\\ &+\sum_{k\in L_c}\alpha\eta(k)\end{aligned} \tag{8}$$

Now, let $\mathcal{H}_{m,q} = \{0,1,...,q\}^{L_c}$ be the new configuration space, we can define a birth-death type generator $\mathcal{L}_c$ for the Markov process η_t:

$$\mathcal{L}_c g(\eta) = \sum_{k\in L_c} c_a(k,\eta)[g(\eta+\gamma_k)-g(\eta)] + c_d(k,\eta)[g(\eta-\gamma_k)-g(\eta)] \tag{9}$$

Where $\gamma_k \in \mathcal{H}_{m,q}$ is the configuration with only one tree at site $k \in L_c$. The birth rate of a single tree in coarse cell D_k is $c_a(k,\eta) = \alpha_a[q-\eta(k)]$ and the death rate is $c_d(k,\eta) = \alpha_d\eta(k)\exp[-\bar{U}(k)]$.

$$\bar{U}(k) = \sum_{l\in L_c; l\neq k}\left[\bar{\phi}(k,l)+\bar{\phi}(l,k)\right] + \eta(l) + \bar{\phi}(k)(\eta(k)-1) + \alpha \tag{10}$$

Likewise we construct the mark associated with a coarse cell $\bar{m}_k(t)$ by averaging over all the individual tree marks within that cell.

$$\frac{d\bar{m}_k}{dt} = \lambda(\eta(t) - \frac{\bar{m}_k(t)}{S}) + \phi(k) + \sum_{l\in L_c; l\neq k}\phi(k,l) \tag{11}$$

For individual trees, a mark reaching zero implies the death of a tree. However here, the mark for coarse cell k reflects the growth, as an average of $\eta(k)$ trees. A tree dies in coarse cell k and $\eta(k)$ is decreased by 1 when $\frac{d\bar{m}_k}{dt} \leqslant -\frac{\bar{m}_k(t)}{\eta(k)}$.

4.2.3 Coarse-grained process dynamic properties

The coarse-grained process was found to satisfy the detailed balance condition and leaves the Gibbs measure $\mu_{m,q,\beta}(d\eta) = \frac{1}{Z_{m,q,\beta}}\exp(-\beta\bar{H}(\eta))P_{m,q}(d\eta)$ on $\mathcal{H}_{m,q}$ invariant, which is consistent with the microscopic process it approximates (Katsoulakis et al. [2003b]).

4.3 Preliminary Results

The individual-based model is run, starting with the empty torus, until the process reaches stationarity. The obtained point pattern and the associated marks, serve as the initial configuration from which the average tree interactions within coarse cells and between adjacent coarse cells are computed. After counting the trees within each patch and computing the average growth function, the coarse model is run forward in time. The preliminary results obtained with the coarse model are consistent with the micro model predictions in terms of tree count. Our next objective is to accurately quantify the error generated from the approximated model.

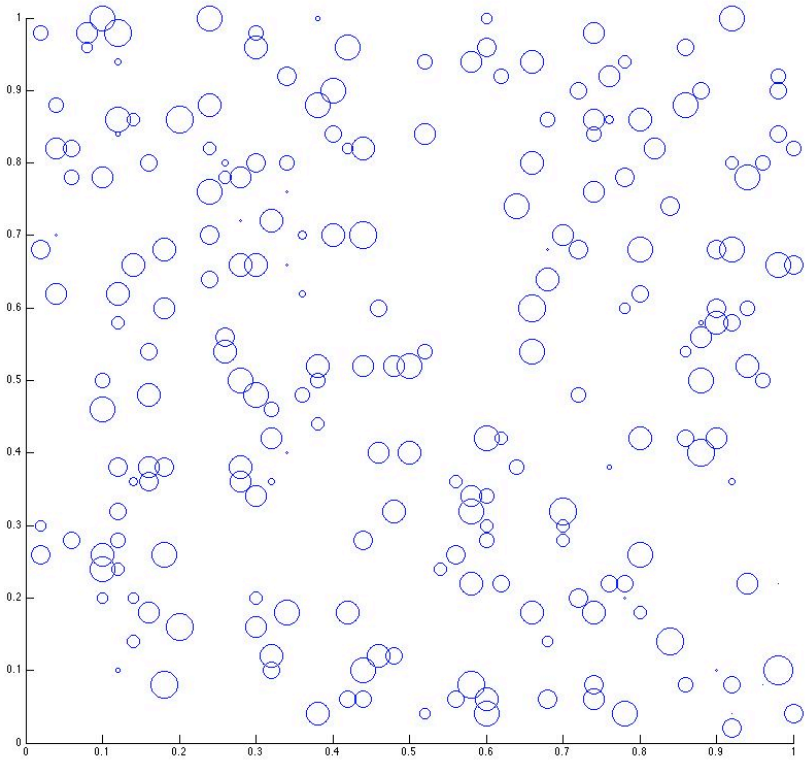

Figure 1: Initial individual-based configuration generated with the stochastic birth-death-growth process

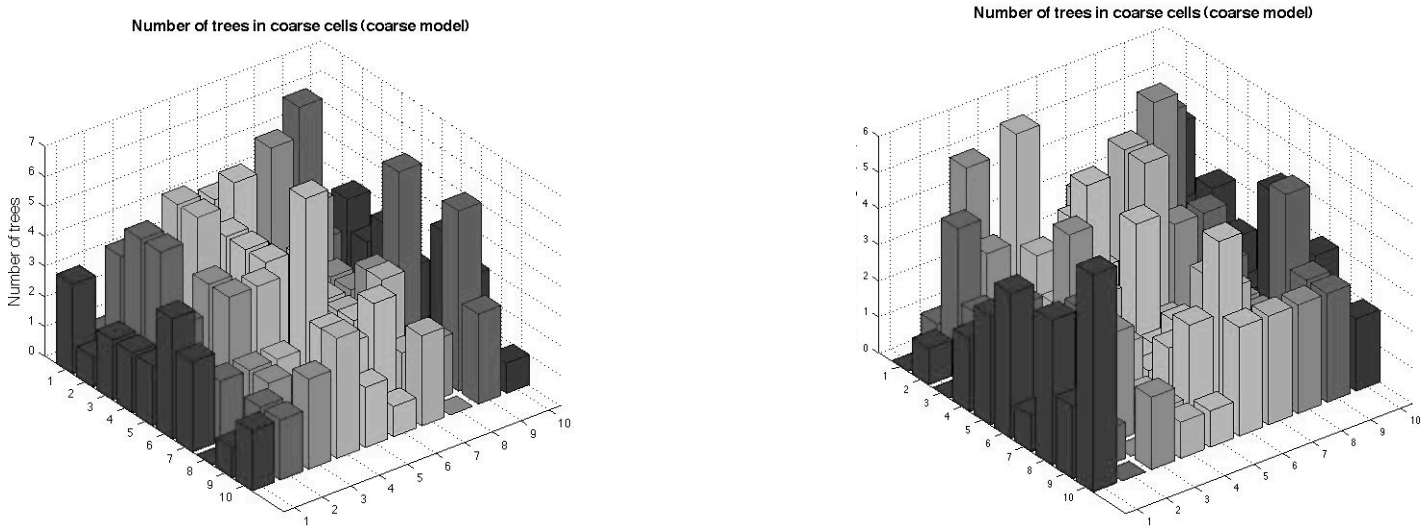

Figure 2: Left figure: coarse-grained model prediction disregarding tree interactions. Right figure: coarse-grained model prediction using the interaction averages from the fine scale process

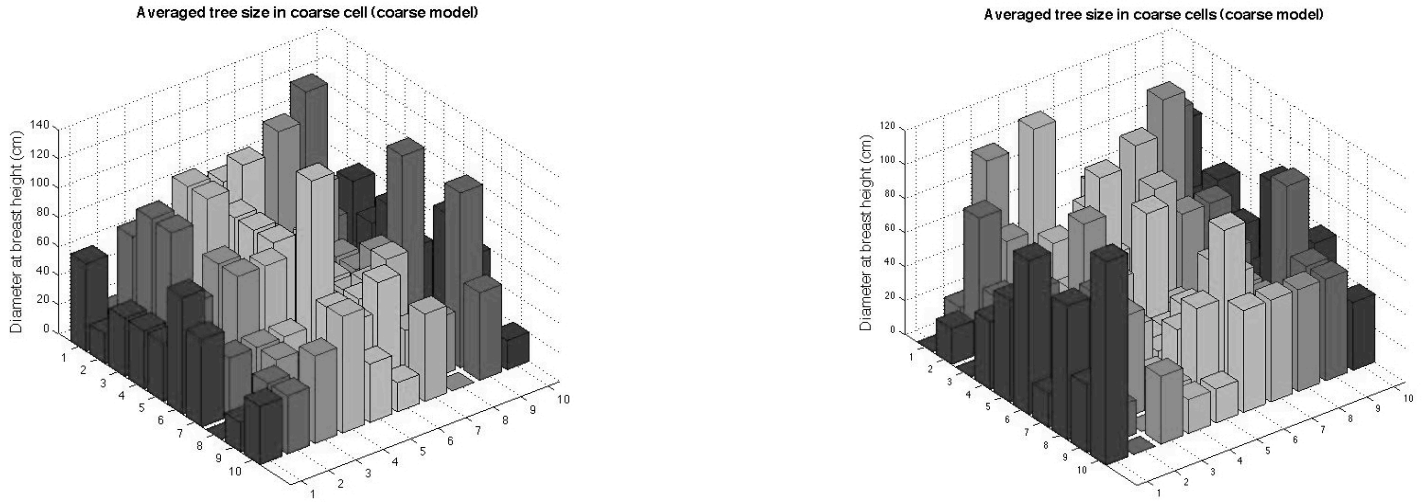

Figure 3: Left figure: coarse-grained model prediction disregarding tree interactions. Right figure: coarse-grained model prediction using the interaction averages from the initial configuration

4.4 Future Work

Having defined a general methodology to describe the dynamics of forest at different length scales, the remaining work mainly resides in the analysis of the simulations and the model validation.

The coarse-graining procedure requires careful handling; the modification of a model defined at a specific scale to describe the process at a different scale may perturb the delicate balance between the input and the model error contributions. The quantification of the error generated from the approximation process should indicate what level of coarse-graining it is reasonable to achieve. As a first validation step, it is important to perform a thorough uncertainty analysis in order to test the sensitivity of the model against input parameter variations and increasing coarsening ranges. In addition, it is fundamental to verify that the fluctuations generated in the original micro model and the ones creates in the approximated coarse model coincide. I expect this first comparison stage to require about two months of coding and simulation time.

The second step consists in defining the potential and the limits of the model. Mainly, refining the model by introducing complexity in the forest composition such as multiple tree species. Furthermore, the choice of the interaction potential between trees greatly influences the forest distribution. Therefore, it is worth testing the degree of detail that can be included in the potential function and more generally in the fine scale model, such that it is reflected in the coarse model outcome.

References

M. F. Acevedo, D. L. Urban, and H. H. Shugart. Models of forest dynamics based on roles of tree species. *Ecological Modelling*, 87, 1996.

O. E. Barndorff-Nielsen, D. R. Cox, and C. Klüppelberg. *Complex Stochastic Systems*. Chapman & Hall/CRC, 2001.

D. B. Botkin, J. F. Janak, and J. R. Wallis. Some ecological consequences of a computer model of forest growth. *The Journal of Ecology*, 60(3), 1972.

F. Bousquet and C. LePage. Multi-agent simulations and ecosystem management: a review. *Ecological Modelling*, 176(3-4), 2004.

D. C. Bragg, D. W. Roberts, and T. R. Crow. A hierarchical approach for simulating northern forest dynamics. *Ecological Modelling*, 173(1), 2004.

H. Bugmann. A review of forest gap models. *Climatic Change*, 51, 2001.

R. T. Busing. A spatial model of forest dynamics. *Vegetatio*, 92, 1991.

C. Comas and J. Mateu. Modelling forest dynamics: A perspective from point process methods. *Biometrical Journal*, 49(2), 2007.

C. Comas and J. Mateu. Space-time dependence dynamics for birth-death point processes. *Statistics and Probability Letters*, 78, 2008.

R. Erban, I. G. Kevrekidis, and H. G. Othmer. An equation-free computational approach for extracting population-level behavior from individual-based models of biological dispersal. *Physica. D*, 215(1), 2006.

L. E. Frelich. *Forest Dynamics and Disturbance Regimes: Studies from Temperate Evergreen-Deciduous Forests*. Cambridge University Press, 2002.

V. Gavrikov and D. Stoyan. The use of marked point processes in ecological and environmental forest studies. *Environmental and Ecological Statistics*, 2, 1995.

H. S. He and D. J. Mladenoff. Spatially explicit and stochastic simulation of forest landscape fire disturbance and succession. *Ecology*, 80(1), 1999.

G. B. M. Heuvelink. Uncertainty analysis in environmental modelling under a change of spatial scale. *Nutrient Cycling in Agroecosystems*, 50, 1998.

A. E. Ismail. *Multiresolution methods for materials modeling via coarse-graining*. PhD thesis, Massachusetts Institute of Technology, 2005.

A. E. Ismail, G. C. Rutledge, and G. Stephanopoulos. Multiresolution analysis in statistical mechanics. i. using wavelets to calculate thermodynamic properties. *The Journal of chemical physics*, 118(10), 2002.

M. Katsoulakis, A. J. Majda, and D. G. Vlachos. Coarse-grained stochastic processes for microscopic lattice systems. *PNAS*, 100(3), 2003a.

M. A. Katsoulakis and P. Plechac ans A. Sopasakis. Error analysis of coarse-graining for stochastic lattice dynamics. *SIAM journal on numerical analysis*, 44(6), 2006.

M. A. Katsoulakis and J. Trashorras. Information loss in coarse-graining of stochastic particle dynamics. *Journal of Statistical Physics*, 122(1), 2006.

M. A. Katsoulakis and D. G. Vlachos. Coarse-grained stochastic processes and kinetic monte carlo simulators for the diffusion of interacting particles. *The Journal of chemical physics*, 119(18), 2003.

M. A. Katsoulakis, A. J. Majda, and D. G. Vlachos. Coarse-grained stochastic processes and monte carlo simulations in lattice systems. *Journal of Computational Physics*, 186(1), 2003b.

B. Khouider, A. J. Majda, and M. A. Katsoulakis. Coarse-grained stochastic models for tropical convection and climate. *PNAS*, 100, 2003.

J. Mateu, J. L. Usó, and F. Montes. The spatial pattern of a forest ecosystem. *Ecological Modelling*, 108, 1998.

D. J. Mladenoff. Landis and forest landscape models. *Ecological Modelling*, 180, 2004.

M. A. Moritz. Spatiotemporal analysis of controls on shrubland fire regimes: age dependency and fire hazard. *Ecology*, 84(2), 2003.

G. Nemhauser, L. Wolsey, and M. Fisher. An analysis of the approximations for maximizing submodular set functions. *Mathematical Porgramming*, 14, 1978.

S. W. Pacala, C. D. Canham, and J. A. Silander. Forest models defined by field measurements: I. the design of a northeastern forest simulator. *Canadian journal of forest research*, 1993.

A. C. Risch, C. Heiri, and H. Bugmann. Simulating structural forest patterns with a forest gap model: a model evaluation. *Ecological Modelling*, 181(2-3), 2005.

A. Särkkä. Modeling growth of trees by space-time growth-interaction processes. In *International Workshop on Spatio-Temporal Modelling (METMA3)*, 2006.

A. Särkkä and E. Renshaw. The analysis of marked point patterns evolving through space and time. *Computational Statistics & Data Analysis*, 51, 2006.

H. H. Shugart and D. C. West. Size and pattern of simulated forest stands. *Forest Science*, 25(1), 1979.

D. Stoyan and A. Penttinen. Recent applications of point process methods in forestry statistics. *Statistical Science*, 15(1), 2000.

D. Stoyan and H. Stoyan. Non-homogeneous gibbs process models for forestry - a case study. *Biometrical Journal*, 40(5), 1998.

B. R. Sturtevant, E. J. Gustason, and H. S. He. Modeling disturbance and succession in forest landscapes using landis: introduction. *Ecological Modelling*, 180(1), 2004.

D. Tilman. Competition and biodiversity in spatially structures habitats. *Ecology*, 75(1), 1994.

D. L. Urban, G. B. Bonan, T. M. Smith, and H. H. Shugart. Spatial applications of gap spatial applications of gap models. *Forest Ecology and Management*, 42(1-2), 1991.

J. Wu and J. L. David. A spatially explicit hierarchical approach to modeling complex ecological systems: theory and applications. *Ecological Modelling*, 153(1-2), 2002.

J. Yang, H. S. He, and E. J. Gustason. A hierarchical fire frequency model to simulate temporal patterns of fire regimes in landis. *Ecological Modelling*, 180(1), 2004.