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Preconditioning the Jacobian system of the extreme type–II Ginzburg–Landau problem

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The nonlinear Schrödinger equation is used in many areas of science and technology and describes, for example, the propagation of solutions in fiber optics or Bose-Einstein condensates in ultra cold traps. The Ginzburg–Landau equations is a prototype of such an equation that is used to model the state and the magnetic field inside superconducting nanodevices. To understand the dynamics of these systems, it is critical to have an efficient solver such that the solution space of can be efficiently explored by, for example, numerical continuation.

For an open, bounded domain $\Omega \subseteq \mathbb{R}^n$ with a piecewise smooth boundary $\partial\Omega$, the Ginzburg–Landau equations for extreme type–II superconductors read

$$\begin{cases} 0 = \left(-\mathrm{i}\nabla - \mathbf{A}\right)^2 \psi - \psi\left(1 - |\psi|^2\right) \quad \text{on } \Omega \\ 0 = \mathbf{n} \cdot (-\mathrm{i}\nabla - \mathbf{A})\psi \quad \text{on } \partial\Omega \end{cases} \tag{1}$$

The unknown $\psi \in H^2_{\mathbb{C}}(\Omega)$ is commonly referred to as *order parameter*. As opposed to the general case, where the magnetic vector potential $\mathbf{A} \in H^2_{\mathbb{R}^n}(\Omega)$ is an unknown of the system, $\mathbf{A}$ is given here by an external magnetic field $\mathbf{H}_0$ via

$$\begin{cases} \nabla \times (\nabla \times \mathbf{A}) = 0, \\ \lim\limits_{\|x\| \to \infty} \nabla \times \mathbf{A} = \mathbf{H}_0; \end{cases}$$

the magnetic field fully penetrates the whole domain and is not altered by the supercurrent. The physical observables are the Cooper-pair density $\rho_{\mathrm{C}} = |\psi|^2$ and the induced magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$.

Extreme type–II superconductors are common in the domain of high-temperature superconductors, for example.

The trivial solution, $\psi = 0$, is the normal non-superconducting state which is the lowest energy state for fields above the critical magnetic field strength. For weak magnetic fields, however, there are non-zero solutions that have a lower energy. These are the famous vortex solutions where the magnetic fields penetrate the sample.

This talk will be concerned with the numerical solution of (1). Applying Newton's method, the focus will be on to solve linear equation systems with the Jacobian operator of (1),

$$\begin{cases} \mathcal{J}(\psi)\varphi = \left((-\mathrm{i}\nabla - \mathbf{A})^2 - 1 + 2|\psi|^2\right)\varphi + \psi^2\overline{\varphi}, \\ 0 = \mathbf{n}\cdot(-\mathrm{i}\nabla - \mathbf{A})\varphi \quad \text{on } \partial\Omega. \end{cases} \tag{2}$$

Properties of $\mathcal{J}(\psi)$ will be outlined that restrict the use of classical solvers. For preconditioning those linear solves, special focus will be on the first part of the operator

$$\begin{cases} \mathcal{K}\varphi = (-\mathrm{i}\nabla - \mathbf{A})^2\varphi, \\ 0 = \mathbf{n}\cdot(-\mathrm{i}\nabla - \mathbf{A})\varphi \quad \text{on } \partial\Omega, \end{cases} \tag{3}$$

and, connected with that, multigrid strategies. Appropriate discretizations will introduced briefly.

Eventually, a solver for $\mathcal{J}(\psi)$ will be constructed which allows for a constant number of linear solver iterations independent of the problem size. Numerical evidence will be presented along with scalability studies to provide insight on the performance of the solver in high-performance computing environments.