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**Local Fourier analysis of interpolation operators for
problems with certain complex stencils – preliminary
results**

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Local Fourier analysis (LFA) (see e.g. [1, 6]) is known to be a valuable tool for the development of geometric multigrid methods for PDEs allowing an effective tuning of multigrid components. The main concept of LFA is the idea of keeping local stencils fixed and treating the (local) problems as if they were part of the associated infinitely large constant coefficient problems. This allows for an optimal choice of smoothers as well as restriction operators.

The infinitely large problem with constant coefficients corresponds to a (l -level) Toeplitz operator. The Toeplitz operators are completely described by their generating symbol, a (l -variate) 2π -periodic function. For second order elliptic problems the generating symbol has a unique zero of order two at the origin. The multigrid methods for Toeplitz matrices and circulant matrices that have been developed in the last years (c.f. [2, 3, 4, 5]), work well for these problems, and they do not depend on the location of the zero. In fact, the zero of the generating symbol just influences the choice of the interpolation operator in the multigrid method.

Combining these developments we are able to provide local interpolation operators for matrices with non-constant stencils with complex entries, consider e.g. the stencil

$$\frac{1}{h^2} \begin{bmatrix} & -e^{2\pi i \varphi_{x,y+\mu}} & \\ -e^{-2\pi i \varphi_{x-\mu,y}} & 4 & -e^{2\pi i \varphi_{x+\mu,y}} \\ & -e^{-2\pi i \varphi_{x,y-\mu}} & \end{bmatrix}$$

where $\varphi_{x-\mu,y}, \varphi_{x+\mu,y}, \varphi_{x,y-\mu}, \varphi_{x,y+\mu} \in [0, 1]$ are some random parameters. This can be considered as a non-shifted variant of the 2d Gauge Laplace matrix that arises in a simplified model of lattice quantum chromodynamics. The idea is similar as in LFA, namely take the local stencil as a constant stencil in an infinitely large system. Using results of Serra-Capizzano and Tablino-Possio in [5] we are able to provide interpolation operators for these matrices, yielding in our local definition of the interpolation.

In this talk we will give a short overview over the used results, introduce the

concept in larger detail and present some numerical results for the two-grid case.

Bibliography

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