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Using Refined Operators to Define A Cell-Structure-Preserving Multigrid Method for the Diffusion Equation

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We consider the standard cell-centered discretization of the diffusion equation on a rectangle with discontinuous diffusion coefficient. Multigrid methods that preserve this cell-based structure are desirable in some applications, such as those that employ local grid refinement. Previously we considered an extension of black box multigrid with a coarsening factor of three, as this approach naturally preserves the cell-based structure. However, a coarsening factor of two is more common in grid refinement algorithms, and in this case the standard application of black box multigrid ignores the cell-based structure and coarsens the structured dual grid. Moreover, using a naive generalization of the interpolation operator that preserves the cell-based structure in conjunction with variational coarsening leads to a twenty-five point coarse-grid stencil.

In this work we develop a variational coarsening algorithm that coarsens by a factor of two, generates nine-point coarse-grid stencils, and preserves the cell-based structure. We first consider the zero-removal case and extend the difference operator on the cell-centers of the finest grid to be a difference operator on the underlying grid consisting of cell-centers, cell-vertices and cell-edges. For this extended operator we derive an operator-induced interpolation, which is used to coarsen variationally to a difference operator on the grid of cell-vertices. This second operator is coarsened in the standard black box multigrid fashion to the grid of coarse-grid cell centers. The resulting operator has a nine-point stencil, and the resulting interpolation operator from fine-grid cell-centers to coarse-grid cell-centers has the same stencil as the transpose of bilinear interpolation. Next we consider non-zero-removal and show how to extend the above method to handle this case. We consider anisotropic coefficient problems and show how to modify the above method to handle this case. We show that the method also works for grids with dimensions that are not powers of two. We present some comparisons of the new method with classic BoxMG.