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Equivalent SPD systems for saddle-point problems

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Consider symmetric saddle-point problems of the form

$$\begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}. \tag{1}$$

The solution to (1) also satisfies the symmetric system

$$\left[\sigma \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} + \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} \begin{pmatrix} D & F^T \\ F & E \end{pmatrix} \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix}\right] \begin{pmatrix} x \\ y \end{pmatrix} \tag{2}$$
$$= \begin{pmatrix} \sigma b + A(Db + F^T d) + B^T(Fb + Ed) \\ \sigma d + B(Db + F^T d) - C(Fb - Ed) \end{pmatrix},$$

for given real σ, arbitrary symmetric matrices D and E, and arbitrary matrix F.

We show that many popular conjugate gradient-based methods for solving (1) can be reformulated as applying the (preconditioned) conjugate gradient method to (2) for some σ, D, E and F. We also provide conditions for guaranteeing that (2) is positive definite. Using these conditions we propose new conjugate gradient-based methods for solving (1) and give numerical results for problems from optimization and fluid dynamics.