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A Fast DP-FETI like Domain Decomposition Algorithm for the solution of Large Electromagnetic Problems

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This paper introduces an efficient domain decomposition algorithm of a class of time-harmonic Maxwell problems in $\mathbf{R^3}$. The present domain decomposition method is a non-overlapping one; it utilizes a set of Lagrange multipliers on the inter-domain interfaces. In addition, the method allows for non-conforming/non-matching triangulations across the interfaces. The proposed algorithm resembles the well known Dual-Primal Finite Element Tearing and Interconnecting (DP-FETI); since both methods eliminate the primal variables, and solve the dense Lagrange multiplier block for the dual unknowns. To achieve convergence of the outer iteration loop, the Robin transmission condition is used to communicate information across interfaces. Using the FETI like algorithm, the present method solves, in the preprocessing step, for the Robin primal subdomain problems multiple times, by exciting one dual unknown each time. This step generates an iteration matrix that is used to update the dual unknowns in the outer-loop iteration. The present method becomes extremely efficient for problems with geometric repetitions, such as antenna arrays, photonic and electromagnetic band gap structures (PBG/EBGs), frequency selective surfaces (FSSs) etc.

The analysis of electrically large electromagnetic (EM) problems with spatial repetitions are of vital importance in practical industrial and engineering applications. Of particular interest is the wave radiation from large finite arrays, since such radiators are the cornerstone of every modern RADAR, satellite communication system or mobile phone base station. Yet another important class of problems that excibits spatial repetitions optics with the photonic crystals and photonic band gap structures. Using "traditional" PDE or even fast IE methods, most of the above-mentioned problems can be quite challenging or even impossible to solve, without using mainframe and parallel computer architectures. In this paper, we propose a domain decomposition methodology based on the Finite Element (FE) approximation that enables the solution of such large scale problems in single a PC.

Despite their great popularity in elliptic differential equations, domain decomposition (DD) methods have found limited application on Maxwell problems. Among the first to consider the non-overlapping alternating Schwarz DD algorithm for the solution of Maxwell problems was Després in [?]. During the years, a number of interface conditions have been proposed to ensure and speed up the convergence of the non-overlapping alternating Schwarz algorithm in both harmonic and Maxwell regime [?],[?].

In this paper a dual primal domain decomposition formulation is developed for the three dimensional Maxwell problem. The method is based on three core ingredients: first an optimized Robin type transmission condition, second a mortar type formulation, and thirdly an efficient FETI solution algorithm. The optimized Robin type transmission condition employed herein is a 3-D vector wave extension of the optimal conditions proposed in [?]. The non-conforming mortar formulation is implemented to relax the mesh generation, and avoid periodic meshes constrains in each domain. The formulation is similar to these described in [?] for elliptic problems. In this work, the primal unknowns are eliminated in the prepossessing step, without the use of the Schur complement. Unlike FETI [?],[?], the dual problem is solved with a computational cheap Gauss-Seidel solver. The Gauss-Seidel iteration matrix is constructed in the preprocessing step, by eliminating of the primal variables.

Numerical results on complex large scale real life industrial problems are used to illustrate the efficiency of the method.

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