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Abstract

Optimized Schwarz preconditioning is applied to a spectral element method for the modified Helmholtz equation and pseudo-Laplacian arising in incompressible flow solvers. The preconditioning is performed on an element-by-element basis. The method enables one to use non-overlapping elements, yielding an effective algorithm in terms of communication between elements and implementation. Two approaches are tested. The first consists of constructing a P_1 finite element problem on each overlapping element. In the second, the preconditioner is applied directly on a non-overlapping spectral element. Numerical results demonstrate an improvement in the iteration count over the classical Schwarz algorithm.

Introduction

The classical Schwarz algorithm uses Dirichlet transmission conditions between subdomains. By introducing a more general Robin boundary condition, it is possible to optimize the convergence characteristics of the original algorithm (Charton et al. 1991; Chevalier et al. 1998; Gander et al. 2002; Gander 2003). In this work, a study of the model equations $u - \Delta u = f$ and pseudo-Laplacian arising in incompressible flow solvers is performed. As suggested by the work of Fischer et al. (2000), the preconditioning is either implemented via a P_1 finite element formulation of the original problem build on the spectral element grid, or directly by solving a smaller spectral element problem without overlap on each spectral element. Although traditional Schwarz preconditioning combined with a coarse grid solver is quite efficient, the need for even more powerful preconditioning techniques stems from atmospheric modeling. Recently (see Thomas and Loft 2002; St-Cyr and Thomas 2004), a semi-implicit SEM was combined with OIFS time stepping, enabling time steps on the order of 20 times the advective CFL condition (Xiu and Karniadakis 2001). This directly reflects as a significant increase in the number of conjugate gradient iterations required to perform the semi-implicit step.

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