Tim Boonen

A multigrid method for the 3D magnetostatic Maxwell equations with solenoidal smoothing

Celestijnenlaan 200 A
B-3001 Leuven
BELGIUM
tim.boonen@cs.kuleuven.ac.be
Stefan Vandewalle

We consider magnetostatic problems formulated as the curlcurl-equation and discretized using first order edge elements. The corresponding system matrix has a very large kernel. Components in this kernel disturb the effectiveness of multigrid when classical pointwise smoothers are used.
The hybrid smoother of R. Hiptmair offers a solution to this problem. This smoother performs additional Gauss-Seidel smoothing in the nodal-based gradient space, hereby damping the kernel-components. Gradient space is accessible as the solution space of the Galerkin product matrix $G^T AG$, with G being the discrete primal gradient matrix with respect to the topology at hand. If the multigrid hierarchy is constructed such that the kernels are nested, then application of the hybrid smoother on all levels gives good results. R. Hiptmair used this hybrid smoother in a geometric multigrid algorithm [1], S. Reitzinger and J. Schöberl integrated it in an algebraic multigrid algorithm [2].
In this talk, a smoother will be presented that only operates on the orthogonal complement of the kernel, leaving the kernel itself untouched. This is achieved by Gauss-Seidel smoothing on facet-based solenoidal space. Solenoidal space is accessible as the solution space of the Galerkin product matrix $\tilde{R}^T A\tilde{R}$, with $\tilde{R}$ being the dual curl matrix with respect to the topology of the problem at hand. This smoother can be applied succesfully on all levels of a multigrid hierarchy, if the orthogonal complements of the kernels are nested.

REFERENCES:

1. R. Hiptmair, *Multigrid method for Maxwell's equations*, SIAM J.Numer.Anal. 36(1999), no.1, 204-255.

2. S. Reitzinger, J. Schöberl, *An algebraic multigrid method for finite element discretizations with edge elements*, Numer.Lin.Alg., 9(2002), 223-238.