Merico E. Argentati

A priori error bounds for eigenvalues approximated by the Ritz values

Department of Mathematical Sciences
University of Colorado at Denver and Health Sciences Center
Denver
P O Box 173364
Campus Box 170
Denver
CO 80217-3364
`rargenta@math.cudenver.edu`
Andrew V. Knyazev

The Rayleigh-Ritz method finds the stationary values of the Rayleigh quotient, called Ritz values, on a given trial subspace as optimal, in some sense, approximations to eigenvalues of a Hermitian operator A. When a trial subspace is invariant with respect to A, the Ritz values are some of the eigenvalues of A. Given two finite dimensional subspaces X and Y of the same dimension, such that X is an invariant subspace of A, the absolute changes in the Ritz values of A with respect to X compared to the Ritz values with respect to Y represent the absolute eigenvalue approximation error. We estimate the error in terms of the principal angles between X and Y. There are several known results of this kind, e.g., for the largest (or the smallest) eigenvalues of A, the maximal error is bounded by a constant times the sine squared of the largest principal angle between X and Y. The constant is the difference between the largest and the smallest eigenvalues of A, called the spread of the spectrum of A.

We prove that the absolute eigenvalue error is majorized by a constant times the squares of the sines of the principal angles between the subspaces X and Y, where the constant is proportional to the spread of the spectrum of A, e.g., for Ritz values that are the largest or smallest contiguous set of eigenvalues of A, we show that the proportionality factor is simply one. Our majorization results imply a very general set of inequalities, and some of the known error bounds follow as special cases. Majorization results of this kind are not apparently known in the literature and can be used, e.g., to derive novel convergence rate estimates of the block Lanczos method.