Khosro, KS Shahbazi

# A Multigrid Solver for High-Order Discontinuous Galerkin Discretizations of the Compressible Navier-Stokes Equations

Department of Mechanical Engineering
University of Wyoming
1000 E University Ave
Laramie
WY 82071-3295 USA
kshahbaz@uwyo.edu
Dimitri Mavriplis

We present a multigrid solver for the system arising from high-order discontinuous Galerkin (DG) discretizations of the compressible Navier-Stokes equations. We use the interior penalty method (D. Arnold, SIAM J. Numer. Anal. 19 (1982), pp. 742-760) for the discretization of the diffusion term and the local Lax-Fredriches fluxes for the convective term. In the interior penalty method, the penalty parameter is chosen based on an explicit expression derived in (K. Shahbazi, J. Comput. Phys. 205 (2005), pp. 401-407). A Hierarchical basis consisting of high-order polynomials is used for spatial discretizations.

The coarse grid approximations and smoothing schemes of our multigrid solver are chosen as follows. The coarse grid operators are direct discretizations of the governing equations at lower approximation orders. For smoothing schemes, we consider both block Jacobi and block Gauss-Seidle iterations, where block corresponds to the matrix arising from the restriction of DG discretizations to a single element. Since we use hierarchical basis, the restriction operator from a high order to a low order vectors is simply an identity matrix with zero columns.

We first verify the performance of the proposed multigrid solver for the two-dimensional Poisson equation on a square domain for different approximation orders and different mesh sizes. We choose the coarsest grid to be the dicretization at approximation order p=0. This allows fast solution of the the coarsest grid system using the geometric multigrid. Mesh- and order-independent convergence rates are achieved if the number of smoothing iterations are chosen large enough.

We then examine the behavior of the scheme in simulating viscous flow over NACA0012 airfoil at zero degree incidence, and with a freestream Mach number of 0.5, and a Reynolds number of 5000. Unlike the Poisson case, adopting p=0 for the coarsest grid, does not yield a robust multigrid solver. Except for the p=1 approximation where fast convergence is obtained, consistent improvements in convergence rates are not observed at higher orders. We attribute this to the

inconsistent discretization of the diffusion operator at p=0. Thus, instead of the p=0, we propose to use the p=1 as the coarsest level. The p=1 (coarsest grid) system itself is efficiently solved using the proposed multigrid scheme. Numerical results verifies the robustness and fast convergence of this scheme.