Elisabeth Ullmann

Iterative solvers for Stochastic Galerkin discretizations of PDEs with random data

Institut fuer Numerische Mathematik und Optimierung
TU Bergakademie Freiberg
09596 Freiberg
Germany
ullmann@math.tu-freiberg.de

We study efficient iterative solvers for Galerkin equations associated with finite element discretizations of second-order elliptic partial differential equations (PDEs) with random coefficient functions. Such systems arise, for example, from discretized stochastic diffusion problems. The Galerkin matrix is a sum of Kronecker products of pairs of matrices associated with the physical and stochastic discretization, respectively. Due to the coupling of (standard) finite element discretizations in the physical space and global polynomial chaos approximations on a probability space, the number of unknowns is huge. Moreover, depending on the models employed for the random coefficient function, the Galerkin matrix can be block-dense, and the cost of a matrix-vector product is non-trivial.

We review a recently proposed Kronecker product preconditioner for the Galerkin matrix which - in contrast to the popular mean-based preconditioner - makes use of the entire information contained in the Galerkin matrix. Furthermore, we extend the idea of Kronecker product preconditioning to the discretized mixed formulation of the stochastic diffusion equation. We demonstrate numerically the improved robustness of Kronecker product preconditioners compared to the mean-based approach with respect to key statistical parameters of lognormal diffusion coefficients.

Finally we discuss perspectives for multilevel techniques for the Galerkin equations. In contrast to previous works, where many researchers have applied multilevel methods exclusively to the deterministic finite element spaces, we focus on multilevel decompositions for the stochastic variational space and combined deterministic/stochastic multilevel approaches.