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**Multigrid preconditioning of linear systems for interior  
point methods applied to a class of box-constrained  
optimal control problems**

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In this work we construct and analyze multigrid preconditioners for operators of the form  $\mathcal{D}_\lambda + \mathcal{K}_h^* \mathcal{K}_h$ , where  $\mathcal{D}_\lambda$  is the multiplication with a relatively “smooth” discrete function  $\lambda > 0$  and  $\mathcal{K}_h$  is a discretization of a compact linear operator  $\mathcal{K}$ . These systems arise when applying interior point methods to the distributed optimal control problem  $\min_u \frac{1}{2}(\|\mathcal{K}u - f\|^2 + \beta\|u\|^2)$  with box constraints  $\underline{u} \leq u \leq \bar{u}$  on the control  $u$ . The presented preconditioning technique is related to the one developed by Drăgănescu and Dupont in [1] for the associated unconstrained problem, and is intended for large-scale problems. As in [1], the quality of the resulting preconditioners is shown to increase as the resolution  $h \downarrow 0$  at a rate that is optimal with respect to  $h$  if the meshes are uniform, but decreases as the smoothness of  $\lambda$  declines. We test this algorithm first on a Tikhnov-regularized backward parabolic equation with  $[0, 1]$  constraints and then on the elliptic-constrained optimization problem

$$\begin{aligned} & \text{minimize} && \frac{1}{2}\|y - f\|^2 + \frac{\beta}{2}\|u\|^2 \\ & \text{subj. to} && \Delta y = u, \ y \in H_0^1(\Omega), \ \underline{u} \leq u \leq \bar{u} \text{ a.e.} \end{aligned}$$

In both cases it is shown that the number of linear iterations per optimization step, as well as the total number of fine-scale matrix-vector multiplications is decreasing with increasing resolution, thus showing the method to be potentially very efficient for truly large-scale problems.

This is joint work with Cosmin Petra from the Argonne National Laboratory.

# Bibliography

- [1] Andrei Drăgănescu and Todd F. Dupont. *Optimal order multilevel preconditioners for regularized ill-posed problems*, Math. Comp., 77 (2008), pp. 2001–2038.