
Andrei Draganescu
A multilevel algorithm for bound-constrained inverse problems

Department of Mathematics and Statistics
University of Maryland
Baltimore County
1000 Hilltop Circle
Baltimore
MD 21250
`draga@math.umbc.edu`

In this work we present some recent results on multilevel (ML) algorithms for a class of inverse problems (IPs) with explicit non-negativity constraints imposed on the control. The IPs under scrutiny are formulated as regularized least-squares minimization problems

$$\text{find } \min_{u \in X} \frac{1}{2} \|Ku - f\|^2 + \frac{\beta}{2} \|u\|^2, \quad u \geq 0, \quad (\dagger)$$

where $K : X \rightarrow Y$ is a compact operator between two Hilbert spaces X and Y , $f \in Y$, and $\beta > 0$ is a regularization parameter. If the non-negativity constraints on the control u were absent from the IP (\dagger) , then the solution would be given by the linear system

$$(\beta I + K^*K)u = K^*f. \quad (\ddagger)$$

It is shown in [1] that (\ddagger) can be solved efficiently by using specially designed ML preconditioners. More precisely, it is shown that the quality of the ML preconditioning increases with resolution $h \downarrow 0$, which results in a number of preconditioned conjugate gradient iterations that decreases with $h \downarrow 0$.

For a large number of applications, the explicit presence of bound-constraints in the IP-formulation can be critical. For example, in the inverse contamination problem [2], where the control represents the initial concentration of an air pollutant, non-negativity constraints not only render a physically meaningful solution, but are essential for the correct recovery of spatially localized solutions. The question addressed in the presented research is whether the qualitative behavior established for the unconstrained problem (\ddagger) can be reproduced in the case of the constrained IP (\dagger) . The problem (\dagger) is solved via the semi-smooth Newton method described in [3] as an outer iteration with an ML-preconditioned CG algorithm being used for solving the linear systems arising at each outer iteration.

Bibliography

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