Baptiste Poirriez
Domain decomposition methods applied to flow simulation in 3D Discrete Fracture Networks

IRISA
Campus de Beaulieu
35042 Rennes Cedex
France
baptiste.poirriez@irisa.fr
Geraldine Pichot
Jocelyne Erhel

Numerical simulations in fractured media are an essential tool for studying hydraulic properties. Discrete Fracture Networks are composed of many multiscale plane fractures intersecting each other, leading to complex geometries. We assume that the rock surrounding the fractures is impervious and aim at simulating the flow in the fractures. Governing equations are Darcy's law and mass continuity, with continuity conditions at the intersections. Mesh generation is rather difficult in this context, because of the geometry, and requires a specific method [2]. We apply a Mixed Hybrid Finite element method and get a large sparse symmetric positive definite (spd) linear system to solve. Direct methods are efficient but become time- and memory- consuming for large systems. Algebraic multigrid method combined with Preconditioned Conjugate Gradient (PCG) converges in few iterations but is also time-consuming. The linear system has a specific structure since the problem is elliptic, but the geometry is neither 2D nor 3D. Domain decomposition methods are attractive because there is a natural geometrical decomposition. In this paper, we explore a preconditioned Schur complement approach.

A first step is to define a partition of the domain. A subdomain is here a set of fractures, and the interface is the set of intersections with fractures of other subdomains. With n subdomains, the matrix A has a block-structure with non singular diagonal blocks $A_{ii}, i = 1, \ldots, n+1$ and nonzero blocks $A_{i,n+1}, i = 1, \ldots, n$. We define the local Schur complement matrix $S_{ii} = A_{n+1,n+1}^{(i)} - A_{i,n+1}^T A_{ii}^{-1} A_{i,n+1}$, the Schur matrix $S = \sum_{i=1}^{n} S_{ii}$ and the associated right-hand side b. Since the matrix S is spd, we apply PCG to solve the linear system $Sx = b$.

We use the classical Neumann-Neumann (NN) preconditioner [1, 6]. Since the partition generates floating subdomains, some local Schur complements S_{ii} are rank-deficient. In order to ensure a kernel of dimension 1, we build a partition such that a subdomain is composed of connected fractures. Then we apply an artificial Dirichlet boundary condition at one edge of the interface to get

a non singular matrix $\tilde{S}_{ii}$. The NN preconditioner is then defined by $M^{-1} = \frac{1}{n}\sum_{i=1}^{n}\tilde{S}_{ii}^{-1}$. To gain in efficiency, we use the Cholesky factorization of A_{ii} to complete the Cholesky factorization of $\tilde{S}_{ii}$, by adapting algorithms aiming at reducing fill-in.

We run numerical experiments with two fracture networks, containing respectively 32 and 128 fractures. These networks are randomly generated, with some large fractures, by using software MP-FRAC-D3 [2]; the domain is partitioned into connected subdomains using the software SCOTCH [4]. The Schur method is implemented in Matlab. In Tables 1 and 2, we give convergence results for CG and PCG(NN) applied to the Schur system. Clearly, NN preconditioner reduces drastically the number of iterations Niter for a small number of subdomains Ndom but is not efficient with many subdomains, as expected.

We use coarse-grid and deflation preconditioners to overcome this difficulty [5, 3]. However, currently, these preconditioners are not efficient. This might be due to the specific structure of the linear system. We further investigate this approach.

Ndom	Niter using CG	Niter using PCG(NN)
2	108	18
4	122	52
8	126	80
16	129	108

Table 1: Number of iterations, solving the Schur system by CG and PCG with Neumann-Neumann preconditioner. The network has 32 fractures.

Ndom	Niter using CG	Niter using PCG(NN)
2	153	37
4	164	75
8	166	96
16	165	149
32	165	211
56	165	275
128	165	373

Table 2: Number of iterations, solving the Schur system by CG and PCG with Neumann-Neumann preconditioner. The network has 128 fractures.

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