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A Large Scale Nonlinear Finite Element Solver Algorithm with Optimal Speed and Robustness

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The speaker will present a new theory, the related algorithmic and programming architecture for solving nonlinear boundary value problems with *optimal speed and robustness* by using large numbers of finite elements.

The new methodology will achieve the desired optimality over a singularly large spectrum of nonlinear finite element models on bounded domains of R^n with $n = 2$ and 3. For example, the algorithm will cover the Galerkin formulation of well-posed nonlinear elliptic systems whose principle part is Lipschitz continuous and strongly monotone in a Sobolev space and certain problems that lack unique solutions such as stationary Navier-Stokes equations. Large variations of stiffness will also be permitted in both magnitude and frequency.

The merit of the new algorithm is not only its speed and scope, but also its mathematically rigorous theory, the elegance in algorithmic design, and simplicity in implementation. The approach will be based on the proven success of the speaker's long time effort starting from early 1990s, recently reported in the article[1] titled "Foundation of nonlinear finite element solvers, Part I", which successfully establishes the corresponding result for second order *quasi-linear* elliptic systems with non-negative lower order terms. Central to the algorithm, the speaker will reformulate a finite element model by generalized Wiener-Hopf equations. This will make element-wise conditioning an inexpensive process, whereby reducing the solution procedure to the straightforward Banach contraction mapping principle: given f, y_0 and m, compute

$$y_{k+1} = (I - R^*TR)y_k + R^*f, \quad k = 0, 1, 2 \ldots m.$$

Here the operator $I - R^*TR$ is strictly contractive with the contraction constant independent of the number of unknowns in the system; T is a scaled direct sum of the local stiffness operators; R and R^* are linear operators conjugate to each other, and I is the identity mapping. Computing Ry and R^*y^* for arbitrary y and y^* is equivalent to solving a linear system defined by a fixed class of sparse M-matrices and their close variants, which can be accomplished by algebraic multi-grid method (AMG) in linear computational count. For a large

[1]This article is submitted to Advances in Computational Mathematics in August, 2005 (95 pages).

class of practical problems, they can also be accomplished by a variety of other linear solver techniques, showing the robustness of the algorithm. From the numerical point of view, R and R^* are optimal conditioners of T in terms of cost, efficiency and robustness. They depend only on a discrete function space modulo the kernel of an appropriate linear analog of T. Throughout the algorithmic design and analysis, non-traditional tools such as topological spaces and discrete measures will be systematically deployed for representing and handling the data structure. This is another novelty of the speaker's approach from the standard methodology.

The speaker's new approach is related neither to Newton-Krylov method and its variants, nor to FAS. At the philosophical level, it is a natural extension of AMG to its fully nonlinear analog without using FAS. It can also be viewed as an extreme exercise of the *finite element tearing and inter-connection* (FETI) philosophy coupled with a novel treatment of degrees of freedom.