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Multilevel methods for solving the Helmholtz equation in unbounded domains

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The incremental unknowns (IU) method has been employed for nearly two decades as a tool of computational fluid dynamics while providing features and benefits similar to those of the classical multigrid method. In particular, several research groups proved that Poisson's equation with Dirichlet boundary conditions can be solved roughly as efficiently using the IU approach as by the classical multigrid [1, 2]. A recent study [5] conducted by the authors proved that, for solving the Helmholtz equation, one can develop a multilevel computational scheme based on the IUs, which is efficient in high and low frequency regimes. We have shown on a sample 2D acoustic scattering problem that for the best performance the number of IU levels used in the preconditioner should be designed for the coarsest grid to have roughly two points per linear wavelength. A recently proposed novel class of methods relying on incomplete factorization of shifted Laplacian operator provides attractive results, but requires significant storage [4]. Memory considerations are especially important when we need to solve 3D problems. To that end, the IU approach appears to be especially advantageous thanks to its low memory requirements. Even if not as efficient as for 2D problems, studies of IU-based techniques for 3D problems proved a promising behavior of the method for solve elliptic problems [3, 7].

A new multilevel preconditioner for the Helmholtz equation in 2D using two types of incremental unknowns has been developed [6]. The transition between the two occurs when the mesh size reaches a predetermined fraction of the wavelength - roughly one quarter wavelength. Conventional IUs based on bilinear interpolation are employed for the fine meshes like in the previous paper [5] by the authors. For coarse mesh sizes novel IUs are designed using a Helmholtz/wave equation-based interpolation. The interpolation coefficients for the coarse meshes with dimension higher than one are derived numerically for stencils resembling integral representations for interior points. In two dimensions the IUs are located on crosses surrounded by square contours. The proposed approach has been proved effective in reducing the condition numbers and accelerating convergence for coarse grids with mesh sizes exceeding the wavelength. Our research has been aimed at devising novel computational schemes that facilitate analysis of large scattering problems. The effort so far has been directed at the development of two-dimensional multilevel preconditioner for high and low frequency cases. The above fast approach has been incorporated into an existing iterative solver. On one hand, one can consider our technique as being related to the multigrid method, in the sense of using stencils constructed based on the ideas stemming from a boundary element method. On the other hand, the idea is not too far from a domain decomposition technique.

Recently, an extension of this method for solving the Helmholtz equation in 3D is considered. While, the first type of IU consists of the adaptation to an unbounded domain problem of the formulas which have been introduced in [3]. The second type is designed using a Helmholtz/wave equation-based interpolation similar to that introduced in [6]. In three dimensions, the IUs are located at the intersections between pairs of planes (three pairs) parallel to the axes, and enclosed in a surface of the surrounding cube. An optimal strategy to combine the two types of IUs is expected to emerge depending on the wavenumber and the size of the mesh.

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