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Numerical simulation of glacial rebound using preconditioned iterative solution methods

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We consider the problem to compute the stress (σ) and displacement ($\mathbf{u}$) fields in a (visco)elastic inhomogeneous layered media, in response to a surface load. The underlying physical phenomenon, which is modeled, is glacial advance and recession, and the post-glacial rebound caused by the latter, which reflects the viscoelastic properties of the mantle.

The material incremental momentum equation for quasi-static infinitesimal perturbations of a stratified, compressible fluid Earth, initially in hydrostatic equilibrium, subject to gravitational forces but neglecting internal forces (cf. [5]) is

$$\nabla \cdot \sigma + \nabla(\mathbf{u} \cdot \nabla \mathbf{p}^{(\mathbf{0})}) + \rho^{(\Delta)}\mathbf{g}^{(\mathbf{0})} + \rho^{(\mathbf{0})}\nabla \mathbf{g}^{(\Delta)} = \mathbf{0}.$$

The first term describes the force from spatial gradients in stress. If a large elastic solid is put in a gravitational field, it becomes gravitationally pre-stressed with pressure $p^{(0)}$. This pressure can be regarded as an initial condition imposed on the problem and does not cause deformations. The second term represents the advection of this pre-stress and describes how it is carried by the moving material. The last two terms describe perturbations of the gravitational force and gravitational acceleration due to changes of density.

In the present study, an incompressible non-selfgravitating (flat) Earth model is used, which implies constant gravity field and constant density, so that these two terms vanish. The equation is further simplified assuming that the advection term describes the advection in the direction of the gravity field only.

Incorporating the above simplifications, we obtain the following form of the governing equilibrium equation

$$\nabla \cdot \sigma + \rho^{(0)} g^{(0)} \nabla(u_d) = \mathbf{0} \quad \mathbf{x} \in \Omega \subset R^d, d = 2, 3 \tag{1}$$

with suitable boundary conditions.

In its full complexity, the model includes viscoelastic constitutive relations. In this work we discuss a purely elastic material behavior only, as is analyzed in [5], for instance.

Problem (1) is discretized using stable mixed finite element pairs or a suitable stabilized formulation, which lead to a system of linear equations with a non-symmetric two-by-two block matrix of a saddle point form.

Results from numerical experiments solving the so-arising algebraic system with preconditioned iterative solution methods are presented. Several preconditioning strategies are tested, based on the techniques and experience described in [4], [3], [1], [2] and other authors. The performance of the tested preconditioned iterative solution methods is compared with that of a commercial FEM package solver.

Bibliography

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