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Time-implicit approach using spatially non-uniform time-step adaptation driven by a-posteriori adjoint error estimates

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We present a space-time finite-volume formulation for the unsteady Euler equations, which allows the use of spatially non-uniform time-step sizes. The space and time dimensions are treated in a unified manner by integrating the equations over control volumes spanning both space and time. The space-time control volumes for the two-dimensional example problems presented in this work are prismatic in nature and are constructed by connecting the vertices of triangular spatial elements at two different time-steps. The triangular spatial elements that form the top and bottom bounding surfaces of the prismatic space-time element are termed temporal faces and have normal vector components that are zero in the spatial dimensions. The space-time element is bounded on the sides by space-time faces and can have either zero or non-zero temporal normal vector components. The normal vectors of the space-time faces have non-zero temporal components in the presence of spatial mesh motion and have zero temporal components when spatial mesh motion is absent. The space-time finite-volume formulation inherently accounts for the effect of dynamically deforming computational meshes. It is shown that discretizing the Euler equations in space-time finite-volume form is identical to discretizing the Euler equations written in the Arbitrary-Lagrangian-Eulerian form.

The primary goal of using the space-time framework is to maintain solution accuracy while reducing the number of unknowns in the overall solution process and potentially lower computational expense. While the formulation presented is capable of simultaneously handling non conformal meshes in both space and time, the scope of this work is limited to conformal spatial meshes with non conformal temporal meshes. At any slice in time, the number of spatial elements remains the same, but across any slice in space, the number of time-steps is allowed to vary. In traditional terms, this translates to non-uniform temporal advancement of spatial elements in an unsteady problem.

The solution process involves first dividing the time domain of interest into slabs of some predetermined temporal thickness that is sufficient to capture the essential nonlinearities in the problem. Then, an a-posteriori error indicator is

employed to identify spatial elements within each temporal slab that have to be advanced with more time-steps of smaller size. In contrast to traditional implicit time-stepping methods where the solution for all spatial elements at a time-step is unknown and solved for implicitly, the implicit system in the space-time framework assumes an unknown solution for all space-time elements within each temporal slab. Compared to advancing on a time-step by time-step basis, the space-time framework is advanced in time on a slab-by-slab basis. A single implicit system of equations within a slab has to be solved for before proceeding to the next temporal slab. It should be noted that the proposed solution method is distinctly different from adapting the time-step size on-the-fly during the nonlinear solution process for each spatial element. While there have been attempts to solve the governing equations in space-time integral form [1-3], advancing spatial elements non-uniformly in time has mostly been limited to explicit time-integration schemes.

For the work here, both local error-based and goal-based a-posteriori error indicators are investigated for use in the space-time framework. The local error-based indicator is constructed by re-evaluating the time derivative term in the governing equations based on a higher order discretization but using the obtained solution. Goal-based error estimates are achieved by solving the adjoint equations in the space-time framework and weighting the non-zero implicit residual with it. Previous work on adapting the time-step size in different parts of the time domain but uniformly for all spatially elements has shown savings in computational expense and total degrees-of-freedom [4]. The proposed method attempts to take this one step further by adapting not only the time-step size in the time domain but additionally adapt the time-step size locally for each spatial element.

Two unsteady problems, one where an isentropic vortex is convected through a rectangular domain and one of a pitching NACA64A010 airfoil in transonic conditions are presented to demonstrate the algorithm. In the vortex convection problem, the local temporal error indicator is used to identify space-time elements which require higher resolution in the time dimension, thus marking them for temporal refinement. In the case of the transonic pitching airfoil, the adjoint-weighted residual method targeting the lift is used as the error indicator. The results indicate that significant reduction in the overall degrees-of-freedom required to solve an unsteady problem can be achieved using the proposed algorithm. Modest improvements in computational expense for specific problems are also observed.

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