
Fei Xue
**Convergence Analysis of Iterative Solvers in Inexact
Rayleigh Quotient Iteration.**

petrinet@math.umd.edu
Howard C. Elman

We present a detailed convergence analysis of preconditioned MINRES for approximately solving the linear systems that arise when Rayleigh Quotient Iteration is used to compute the lowest eigenpair of a symmetric positive definite matrix. We provide insight into the "slow start" of MINRES iteration in both a qualitative and quantitative way, and show that the convergence of MINRES mainly depends on how quickly the unique negative eigenvalue of the preconditioned shifted coefficient matrix is approximated by its corresponding harmonic Ritz value. By exploring when the negative Ritz value appears in MINRES iteration, we obtain a better understanding of the limitation of preconditioned MINRES in this context and the virtue of a new type of preconditioner with "tuning". Finally we show that tuning based on a rank-2 modification can be applied with little additional cost to guarantee positive definiteness of the tuned preconditioner.