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Goal-oriented *hp*-adaptive Discontinuous Galerkin Methods for the Compressible Euler Equations on Unstructured Meshes

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High-order accurate Discontinuous Galerkin (DG) methods [?, ?, ?, ?] have become a popular approach for solving hyperbolic conservation systems over the last decade. The use of adaptive solution strategies via the finite element method has also become a more accepted technique to guarantee and improve numerical accuracy, particularly for cases with complex geometries. The main idea of the adaptive strategies used in this work is to start the computation with a relatively coarse mesh associated with a uniformly low order of discretization on the computation domain. Thereafter, a local or cellwise error indicator arising from an objective functional of interest with engineering properties, such as Lift or Drag, is estimated using an adjoint-based sensitivity analysis technique. This approach results in a spatial error distribution, which is then used to drive local mesh subdivision (h-refinement) or local modification of discretization orders (p-enrichment), in order to improve the accuracy of the objective function and the approximation. One of the main benefits of the given method from a finite element point of view is that the number of unknowns and thus the computational expense are optimized by avoiding excessive resolution in areas of little influence on the quantity of interest.

The *a posteriori* error estimates employed in this work to predict the error distribution involve the inner product of the finite element residuals with the adjoint solution variables. In order to approximate the adjoint solution, we make use of the discrete adjoint strategy [?] where the governing equations are first discretized by the Discontinuous Galerkin method and then linearized, thus reproducing the exact sensitivity derivatives of the original discretization of the governing equations. This approach requires one adjoint solution at each adaptive step, which is equivalent to the cost of one flow solution, since the coefficient matrix of the linear system of equations constructed for the adjoint problem corresponds to the transpose of the full-Jacobian matrix of the flow equations. The solution of this system is relatively straight-forward since many of the entries in the full Jacobian matrix are already computed for the implicit element Jacobi scheme used to solve the flow equations. Moreover, a linearized element Gauss-Seidel smoother and an *hp*-multigrid approach [**?**, **?**] are exclusively used to accelerate the convergence of both the flow and adjoint solvers.

The adaptive mesh approach used in this paper consists of a purely *h*-refinement technique, locally subdividing one triangle into four self-similar children triangles, while keeping the discretization order fixed, as well as a purely *p*-enrichment technique, locally increasing the order of interpolation polynomials while keeping the underlying triangulation fixed, and a combined scheme known as *hp*-adaptive refinement. However, the principal obstacle towards effective use of *hp*-adaptation lies in making a decision between *h*- and *p*-refinement. In this work, the choice is made by utilizing a smoothness indicator to isolate regions with smooth solution behavior where local *p*-enrichment performs more effectively, from areas with singularities or discontinuities where local *h*-refinement is more suitable.

In this paper, we show the convergence properties for both the flow (primal) and the discrete adjoint (dual) problems using *hp*-multigrid solver driven by the element Gauss-Seidel smoother. Then, the overall efficiency and accuracy of the purely *h*-refinement and the purely *p*-enrichment approaches are compared with those of the scheme with uniform mesh or order refinement, respectively. Next, the performance of the *hp*-adaptive approach is compared with the purely *h*- and *p*-refinement. Finally, a case with strong shocks or discontinuities is employed to demonstrate the shock-capturing properties of both *h*- and combined *hp*-adaptive strategies. Future work will focus on extending this approach to unsteady problems.

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