Simulation Process outline

Step 1) Discretize parameter space $\theta \in (0,1)$
$\Rightarrow$ select a sequence of possible values

$$0 \quad \theta^{(1)} \quad \theta^{(2)} \quad \theta^{(3)} \quad \cdots \quad \theta^{(M)} \quad 1$$

Step 2) Construct the posterior for each $\theta^{(1)}, \dots, \theta^{(M)}$

$$\Rightarrow \quad p(\theta^{(m)} \mid y) \propto p(y \mid \theta^{(m)})\, \pi(\theta^{(m)})$$

$\rightarrow$ note this is the same as the general posterior, but using individual discritized θs rather than a smooth curve

Step 3) Sample $\underbrace{\theta_{(1)}}_{}, \dots, \theta_{(m)}$ from $\theta^{(1)}, \dots, \theta^{(M)}$ — m^{th} random draw; $\rightarrow m^{th}$ discritized parameter value
with probability (weight) $\propto p(y \mid \theta^{(m)})\, \pi(\theta^{(m)})$

Derivation

$\rightarrow$ Posterior predictive distribution

$$\rightarrow \quad p(\tilde{y} \mid \underset{\sim}{y}) = ??$$

$$= \int_0^1 \underbrace{p(\tilde{y} \mid \theta)}_{} \, p(\theta \mid \underset{\sim}{y})\, d\theta$$

$\hookrightarrow$ data dist from model ($n = 5$ now)

$$= \int_0^1 \binom{5}{\tilde{y}} \theta^{\tilde{y}} (1-\theta)^{5-\tilde{y}} \frac{\Gamma(102)}{\Gamma(51)\Gamma(51)} \theta^{50} (1-\theta)^{50}\, d\theta$$

$$= \frac{\Gamma(102)}{\Gamma(51)\Gamma(51)} \left(\frac{5!}{\tilde{y}!(5-\tilde{y})!} \right) \underbrace{\int_0^1 \theta^{(51+\tilde{y})-1} (1-\theta)^{(56-\tilde{y})-1}\, d\theta}_{\left[\frac{\Gamma(\tilde{y}+51)\Gamma(56-\tilde{y})}{\Gamma(103)} \right]}$$

$$= \frac{101!\,(50+\tilde{y})!\,(55-\tilde{y})!\,5!}{50!\,50!\,106!\,\tilde{y}!\,(5-\tilde{y})!}$$