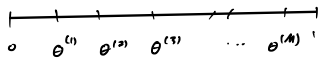


Simulation Process outline

Step 1) Discretize parameter space $\theta \in (0,1)$
 \Rightarrow select a sequence of possible values



Step 2) Construct the posterior for each $\theta^{(1)}, \dots, \theta^{(M)}$

$$\Rightarrow p(\theta^{(m)} | y) \propto p(y | \theta^{(m)}) \pi(\theta^{(m)})$$

\rightarrow note that is the same as the general posterior,
 but using individual discretized θ s rather than a smooth curve

Step 3) Sample $\theta_{(1)}, \dots, \theta_{(m)}$ from $\theta^{(1)}, \dots, \theta^{(M)}$
 \rightarrow m^{th} random draw $\rightarrow m^{\text{th}}$ discretized parameter value
 with probability $\propto p(y | \theta^{(m)}) \pi(\theta^{(m)})$
 (weight)

Derivation

\rightarrow Posterior predictive distribution

$$\rightarrow p(\tilde{y} | \underline{y}) = ?$$

$$\begin{aligned}
 &= \int_0^1 \underbrace{p(\tilde{y} | \theta)}_{\text{data dist from model } (n = s \text{ now})} p(\theta | \underline{y}) d\theta \\
 &= \int_0^1 \left(\frac{s}{\tilde{y}}\right) \theta^{\tilde{y}-1} (1-\theta)^{s-\tilde{y}} \frac{\Gamma(102)}{\Gamma(51)\Gamma(51)} \theta^{50} (1-\theta)^{50} d\theta \\
 &= \frac{\Gamma(102)}{\Gamma(51)\Gamma(51)} \left(\frac{s!}{\tilde{y}!(s-\tilde{y})!}\right) \int_0^1 \theta^{(51+\tilde{y})-1} (1-\theta)^{(50-\tilde{y})-1} d\theta \\
 &= \left[\frac{\Gamma(51+\tilde{y})\Gamma(50-\tilde{y})}{\Gamma(103)} \right] \\
 &= \frac{101! (50+\tilde{y})! (50-\tilde{y})! s!}{50! 50! 102! \tilde{y}! (s-\tilde{y})!}
 \end{aligned}$$