Implementation of Gibbs Sampling within Bayesian Inference and its Applications in Actuarial Science: The app

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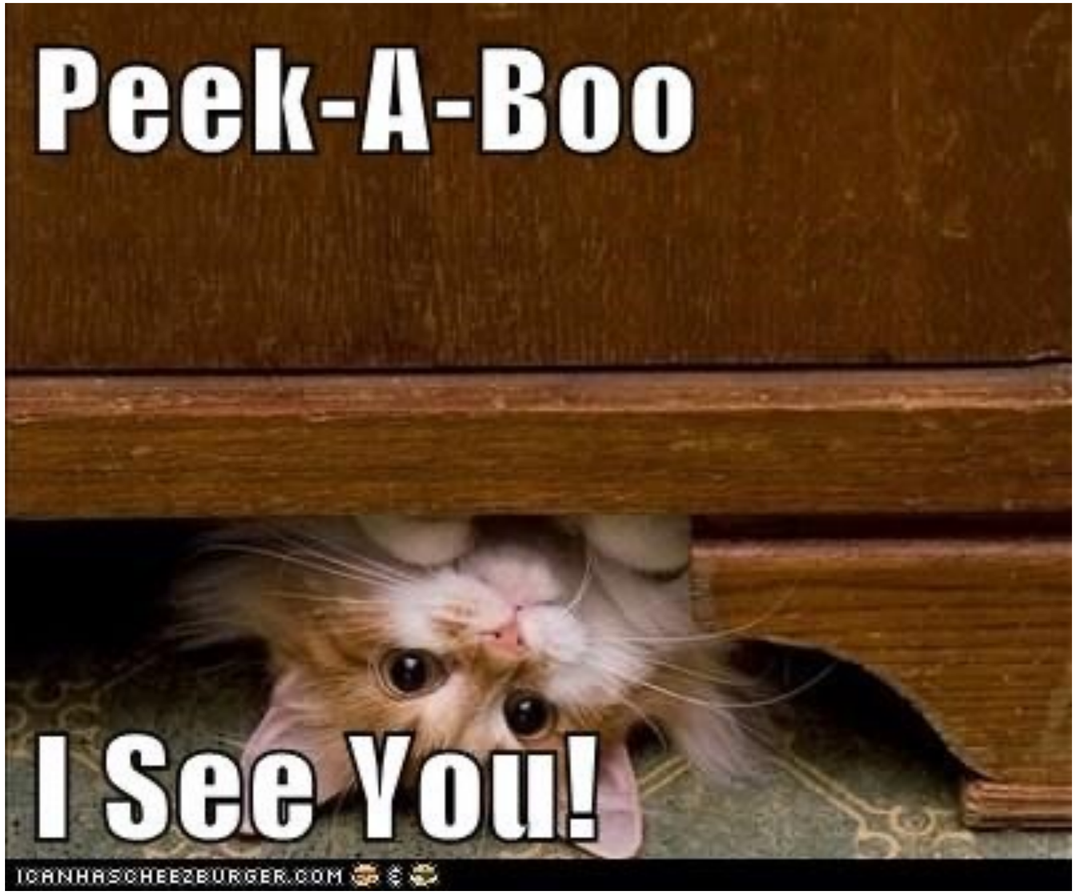

A Bayesian game of peek-a-boo!

Bayesian statistics: The introduction

- Two methodologies in statistics: Classical and Bayesian
- The difference: what quantities are allowed to be random

- Bayesian methods apply probability distributions to model parameters to represent uncertainty about them
- These then get updated with current data to produce posterior probability distributions
- In doing so, parameters become more informed

The situation – A classic example

- An insect lays a large number of eggs, each surviving with probability p. On average, how many eggs will survive?
  - Let X = # of survivors and Y = # of eggs laid
  - X|Y ~ Binomial (Y, P)
  - Y ~ Poisson(λ)
- How can we figure this out?????
- For this situation, we can easily find the distribution of X and figure out E(X) and V(X)

- However, with more complex models, f(X) often does not have a closed form solution……… So what do we do???

A solution

- When something cannot be solved analytically, a useful alternative is simulation
- Essentially, we get an answer our original problem by obtaining a finite sample from the desired distribution and then apply discrete formulas to these samples
- For example, a mean can be estimated with

$$\int x f(x)\, dx \approx \frac{1}{n} \sum x_i$$

Methods we will investigate

- Markov Chain Monte Carlo (MCMC) methods arrive at the distribution of interest by generating dependent sample values from specific distributions
- One class of MCMC methods is called Gibbs sampling, which is essentially one way to construct the specific distributions
- This is what will be demonstrated with the app!

Demonstration

- X = size of a claim
- X| λ ~ Exponential(λ)
- λ ~ Gamma(α, β)

- Goal is to figure out distribution of X

- Note -> In this setup, it can be shown that X ~ Pareto(α, β)

Application

- X = # of policies that file a claim
- N = # of policies in a portfolio
- P = Probability of filing a claim
- X | N, P ~ Binomial(N,P)
- N ~ Poisson(λ)
- P ~ Beta(α, β)

- Goal is to figure out distribution of X, the number of claims filed for an arbitrary portfolio

- In this setup, there is NO closed form solution for f(X)