

Hardest Unit ☹️

Unit 9 – Inferences from Two Samples All Days
Your Bad Planning Professor Colton



Unit 9 - Outline

Unit 9 – Inferences from Two Samples

Intro

Hypothesis Testing Overview for Two Samples

- Review all Steps

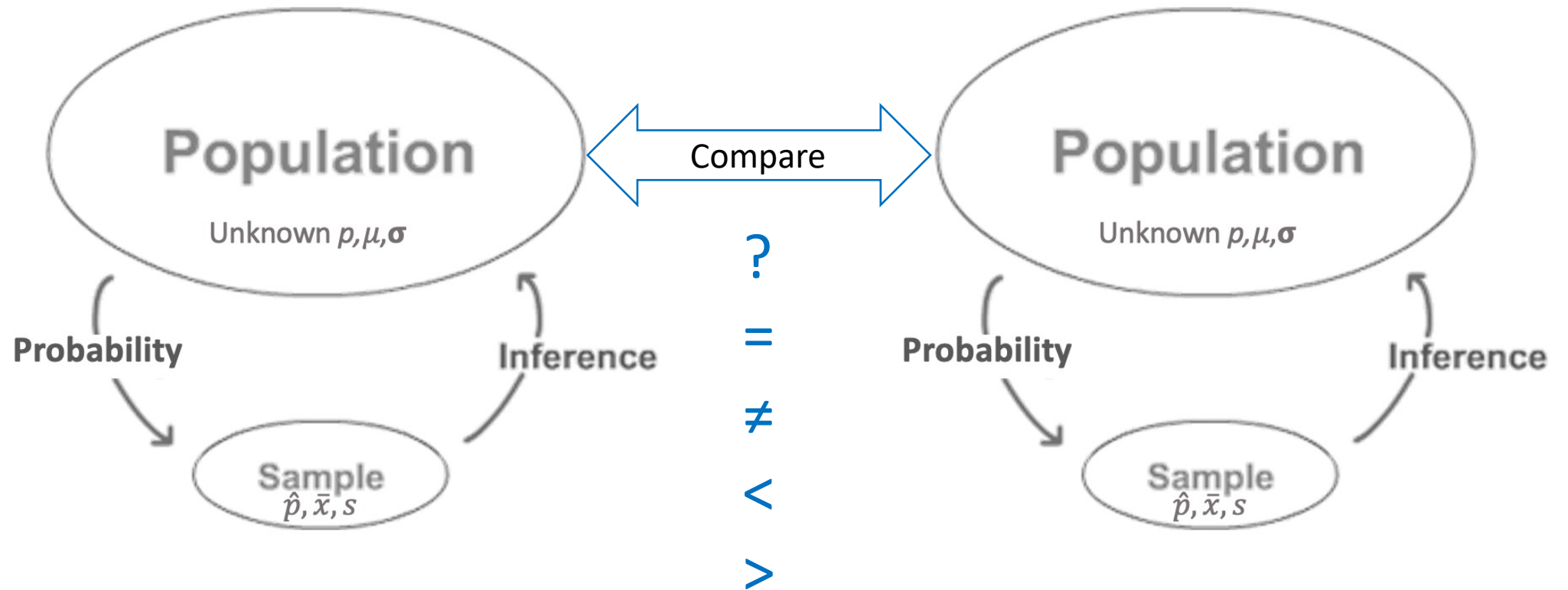
Hypothesis Testing of Two Population Proportions

- Traditional and P-value Methods
- Confidence Intervals
-

Hypothesis Testing of Two Population Means (Independent Samples)

- Independent vs Dependent (Matched vs Paired)
- Means of Independent Samples with sigma known, traditional and pvalue methods
- Means of Independent Samples with sigma unknown, p-value method
- Confidence Intervals

Inference! Our Third Look



Full Problem

Here is an entire TWO sample hypothesis problem worked out perfectly to show us where we are going!

- Then we will break it down piece by piece again!

Setup: Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot? We collected data from random samples of students from CSCC and found that 31 out of 114 males and 63 out of 176 females prefer Starbucks. Test an appropriate hypothesis with $\alpha = 0.05$.

Solution

Hypotheses:

Let p_1 = true proportion of males who prefer Starbucks

Let p_2 = true proportion of females who prefer Starbucks

$$H_0: p_1 - p_2 = 0$$

$$H_A: p_1 - p_2 \neq 0$$

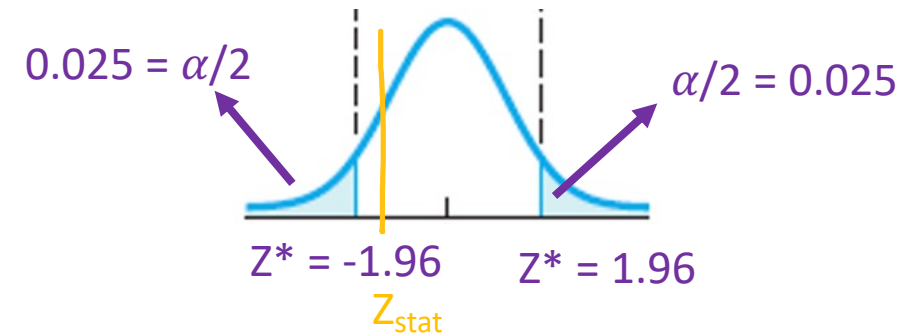
Set $\alpha = 0.05$

Check Assumptions:

- *Randomization: Random sample of males and females were taken*
- *Independence: Males and females are independent groups*
- *Large enough samples:*
 - Males \rightarrow 31 successes and 83 failures, both > 5
 - Females \rightarrow 63 successes and 113 failures, both > 5
- *All conditions are met, appropriate to continue with test!*

Rejection Region:

$$Z^* = \text{invNorm}(\text{area} = 0.05/2, \mu = 0, \sigma = 1) = -1.96$$



Test Statistic:

$$TS = Z_{\text{stat}} = 2\text{-PropZTest}(x_1 = 31, n_1 = 114, x_2 = 63, n_2 = 176, p_1 \neq p_2) = -1.529$$

$$|Z_{\text{stat}}| = 1.529 < 1.96 = |Z^*| \rightarrow \text{Fail to reject } H_0$$

Conclusion and Interpretation:

Because the absolute value our Test Statistic $Z_{\text{stat}} = 1.529$ is less than the absolute value of the Critical Value $Z^* = 1.96$ (5% significance level), we fail to reject the Null hypothesis. We do NOT have sufficient evidence to conclude that the true proportion of male college students who prefer Starbucks is different than that of females.

Hypothesis Test Steps – Reminder

1. **State** the Hypotheses
 - Define parameter + context.
2. **Check** Assumptions.
3. **Determine** and **Sketch** Rejection Region based of Significance Level
4. **Compute** value of Test Statistic / P-value.
5. **Conclude** and **Interpret**
 - State whether you reject H_0 or fail to reject H_0 AND WHY!
 - Interpret your results in the context of the problem

The Hypothesis Statements – Two Samples

1. State the Hypotheses

- **Define parameter + context.**

Define Parameters

- Now we have TWO populations and TWO **parameters**!
- These parameters describe the same quantity (ex: 'true proportion who prefer Starbucks') BUT for different groups (ex: males vs females)!
- So we have to CLEARLY define both of them!
 - Using subscripts of 1 and 2 will be helpful because that is the notation we will use for the calculations and calculator!
 - Order matters and will be important when making our conclusions!

Full Example

Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot? Test an appropriate hypothesis.

Parameters:

Let p_1 = true proportion of males who prefer Starbucks

Let p_2 = true proportion of females who prefer Starbucks

The Hypothesis Statements – Two Samples

1. State the Hypotheses

- Define parameter + context.

Null Hypothesis H_0

- Now we are comparing some quantity (the same quantity) between TWO populations! So we have TWO parameters!
 - We are not necessarily interested in the specific values of these parameters like we were when testing ONE sample (ex: $H_0: p = p_0$)
 - Rather we want to learn about the relationship between the two of them, p_1 ?? p_2
- We start by assuming both parameters are **equivalent**! So their difference is ZERO!
- This can be written in two ways!

Option 1

- Directly equating the two parameters:

$$H_0: p_1 = p_2$$

$$H_0: \mu_1 = \mu_2$$

Full Example

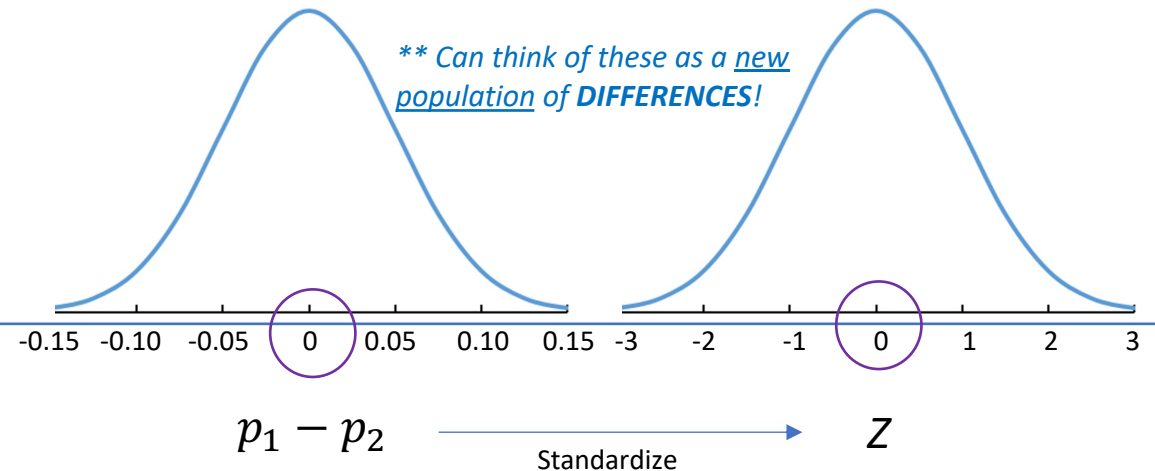
Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot?

Hypotheses:

Let p_1 = true proportion of males who prefer Starbucks

Let p_2 = true proportion of females who prefer Starbucks

$$H_0: p_1 - p_2 = 0$$



Option 2

- Rewrite as a **difference**!

$$H_0: p_1 - p_2 = 0$$

$$H_0: \mu_1 - \mu_2 = 0$$

- Writing our Null hypothesis in this way helps us visualize the Normal curves we use for the CV and TS
- This is an equivalent way to represent it, just a change in perspective to the DIFFERENCE (a single value)

The Hypothesis Statements – Two Samples

1. State the Hypotheses

- Define parameter + context.

Alternative Hypothesis H_A

- Here is where we state our research interest.
- Again, we are interested in the relationship between our two parameters and how we think they are different
 - Our test on the single value of the **difference** may be left-tailed ($<$), right-tailed ($>$), or two-tailed (\neq).
- Alternative can also be written in two ways:

Option 1

$$H_A: p_1 \neq p_2 \rightarrow p_1 - p_2 \neq 0$$

$$H_A: p_1 > p_2 \rightarrow p_1 - p_2 > 0$$

$$H_A: p_1 < p_2 \rightarrow p_1 - p_2 < 0$$

(same for μ_1 and μ_2)

Option 2

Full Example

Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot?

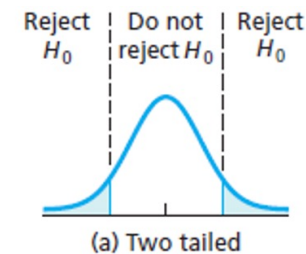
Hypotheses:

Let p_1 = true proportion of males who prefer Starbucks

Let p_2 = true proportion of females who prefer Starbucks

$$H_0: p_1 - p_2 = 0$$

$$H_A: p_1 - p_2 \neq 0$$



Type of Test:

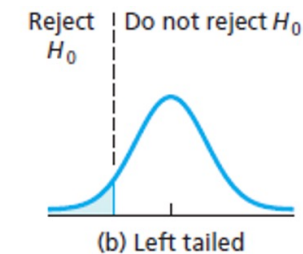
(a) Two tailed

Sign in H_A :

\neq

Rejection Region:

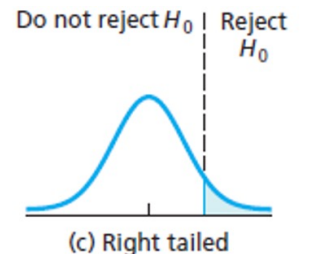
Both sides



(b) Left tailed

$<$

Left side



(c) Right tailed

$>$

Right side

LCQ – Hypotheses

Problem: (1) Define the parameters of interest and (2) State the Null and Alternative for the following scenarios:

a) A company has two separate teams (A and B) who complete accounting reports. From a random sample of recent reports from each team, they want to know if these teams perform differently in terms of proportion of reports completed on time.

b) A company randomly selects employees to complete yearly surveys on whether or not they enrolled in at least one wellness class at the company's site. They want to know if a greater percentage took a wellness class **this year compared to last year**.

LCQ – Hypotheses

Problem: (1) Define the parameters of interest and (2) State the Null and Alternative for the following scenarios:

a) A company has two separate teams (A and B) who complete accounting reports. From a random sample of recent reports from each team, they want to know if these teams perform differently in terms of proportion of reports completed on time.

Let p_1 = the true proportion of completed accounting reports from team A

Let p_2 = the true proportion of completed accounting reports from team B

$H_0: p_1 - p_2 = 0$ OR $H_0: p_1 = p_2$

$H_A: p_1 - p_2 \neq 0$ OR $H_A: p_1 \neq p_2$

Perfect! Both versions are correct, there are just different benefits to each!

The difference version (first way) helps us when looking at the result of our TS, but our calculator uses the direct comparison (second way) in the menus

b) A company randomly selects employees to complete yearly surveys on whether or not they enrolled in at least one wellness class at the company's site. They want to know if a greater percentage took a wellness class **this year compared to last year**.

*Let p_1 be the true proportion of employees **enrolled in at least** one wellness class **THIS YEAR** → Correct! THIS YEAR is our first group, that is the first population*

*Let p_2 be the true proportion of employees **that did not enrolled in** at least one wellness class → INCORRECT! Because this is now representing a different quantity than in p_1 (enrolled vs did NOT enroll); the only thing that should change is the GROUP!*

*Let p_2 be the true proportion of employees **enrolled in at least** one wellness class **LAST YEAR** → Good! LAST YEAR is our second group, now our parameter is for the same quantity but a different POPULATION*

$H_0: p_1 - p_2 = 0$

$H_A: p_1 - p_2 > 0$

Very good!

What if switch the order of our parameters? Now let:

p_1 = LAST year

p_2 = THIS year

$H_0: p_1 - p_2 = 0 \rightarrow$ same

$H_A: p_1 - p_2 < 0 \rightarrow$ different! We still want this year to be greater, so now p_2 is the larger value making the difference less than zero! Order is important

Assumptions

2. Check Assumptions.

- For the most part this step is the same as we have seen before!
 - Random Sample and Large enough sample
 - How we check the Large enough sample assumption depends on the type of test (type of data)
- We just have to do it for both samples now!
- Although now we also have to think about the connection between our two samples
 - Will go over these again when looking at Proportions Tests and Means Tests

Full Example

Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot?

- They random samples of students from CSCC and found that 31 out of 114 males and 63 out of 176 females prefer Starbucks

Check Assumptions:

- *Randomization: Random sample of males and females was taken*
- *Independence: Males and females are independent groups*
- *Large enough samples:*
 - *Males → 31 successes and 83 failures, both > 5*
 - *Females → 63 successes and 113 failures, both > 5*
- *All conditions are met, appropriate to continue with test!*

* will go through these with the proportions slide

Rejection Region and TS – Two Samples

3. Determine and Sketch Rejection Region based of Significance Level

Rejection Region (RR)

- This is the EXACT same as we have seen in all the other types of tests!
 - Which is because we are framing our tests from the perspective of a difference!

4. Compute value of Test Statistic / P-value.

Test Statistic (TS) and P-Value

- SAME logic as with one sample, just different calculations behind the scenes
- Will go over the specifics on each respective Test's slides

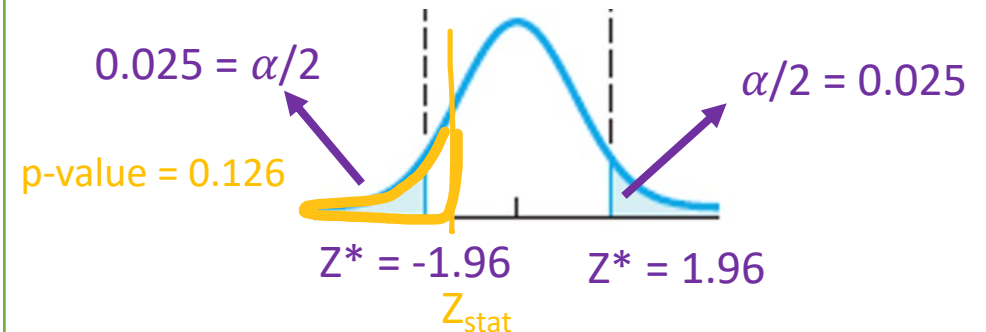
Full Example

Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot?

Rejection Region:

Set $\alpha = 0.05$ (which was set at beginning)

$Z^* = \text{invNorm}(\text{area} = 0.05/2, \mu = 0, \sigma = 1) = -1.96$



Test Statistic:

$TS = Z_{stat} = 2\text{-PropZTest}(x_1 = 31, n_1 = 114, x_2 = 63, n_2 = 176, p_1 \neq p_2) = -1.529$

$|Z_{stat}| = 1.529 < 1.96 = |Z^*| \rightarrow \text{Fail to reject } H_0$

P-Value:

$p\text{-value} = 2\text{-PropZTest}(x_1 = 31, n_1 = 114, x_2 = 63, n_2 = 176, p_1 \neq p_2) = 0.126$

Conclude and Interpret – Two Samples

5. Conclude and Interpret

- State whether you reject H_0 or fail to reject H_0 AND WHY!
- Interpret your results in the context of the problem

This has the SAME structure that we had with ONE sample Hypothesis Tests

- We just need to talk about BOTH parameters for the Alternative!

First Part – Decision and Reasoning

- Because (comparison of TS and CV; OR p-value and α) we (**REJECT** or **FAIL TO REJECT**) the Null Hypothesis.

Second Part – Interpretation

- There (**IS** or **IS NOT**) sufficient evidence to conclude (**THE ALTERNATIVE HYPOTHESIS + CONTEXT**).
- NOTE about wording!

- When thinking about the alternative as a difference (very literally), our wording could be something similar to "There is / is not sufficient to conclude the true difference in proportion of males who prefer Starbucks and of females is not equal to (or less / greater than) zero."
- This doesn't read very well and it's not how we would talk about the results in conversation. So try and **reword** it as a direct comparison of the two population parameters like it is in the example (not in terms of the difference and zero)!

Full Example

Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot?

Conclusion and Interpretation:

Because

- *the absolute value of our Test Statistic $|Z_{stat}| = 1.529$ is less than the Critical Value $Z^* = 1.69$ (5% significance level)*
OR
- *our p-value = 0.126 is greater than the significance level 0.01*

We do have sufficient evidence to conclude that the true proportion of male college students who prefer Starbucks is different than that of females.

** Give MORE INFORMATION IF POSSIBLE

- If REJECT a two-tailed test, say which of the parameters is
- Just look at the sign of the TS and the order of our Hypothesis
- Ex) If conclude $H_A: p_1 - p_2 \neq 0$ and $TS = 2.3 \rightarrow p_1 > p_2$, so say

Hypothesis Tests for Proportions – Two Samples!

- Everything above applies, now we are just going to apply it specifically to a Two Sample Proportions Test!

Proportions Assumptions- Two Samples

2. Check Assumptions.

- Some of the same ideas with some new ones as well!

1) Random samples (both of them)

2) Independence

- Because we have two samples now, we need each group to be **independent** (unrelated, no connection).
- So they results from one group should NOT have any effect / impact on the results of the second group!

3) Large Enough Samples (both of them)

- Because we don't have a Null proportion value like before, we can't check $np_0 \geq 5$ AND $n(1 - p_0) = nq_0 \geq 5$
- So all we have to do is make sure each sample has at least 5 success and 5 failures
 - Sample 1: $n_1\hat{p}_1 \geq 5$, $n_1(1 - \hat{p}_1) = n\hat{q}_1 \geq 5$ AND Sample 2: $n_2\hat{p}_2 \geq 5$, $n_2(1 - \hat{p}_2) = n\hat{q}_2 \geq 5$
 - So we are actually looking at the sample data here (which was a big no no when we were doing one sample...)

One Sample Test Conditions

✓ Randomization Condition

Need to have a random sample

✓ Large Enough Sample Condition

$np_0 \geq 5$ AND $n(1 - p_0) = nq_0 \geq 5$ OR
EXPECT AT LEAST 5 successes and 5 failures

New conditions



TWO Sample Test Conditions

✓ Randomization Condition

Need to have two random samples

✓ Independence Condition

Need to independent samples

✓ Large Enough Sample Condition

AT LEAST 5 successes and 5 failures in EACH collected sample

Full Example

Is there a difference in the proportion of male and female college students who prefer Starbucks as their favorite coffee spot?

- They random samples of students from CSCC and found that 31 out of 114 males and 63 out of 176 females prefer Starbucks

Check Assumptions:

- *Randomization: Random sample of males and females was taken*
- *Independence: Males and females are independent groups*
- *Large enough samples:*
 - Males → 31 successes and 83 failures, both > 5
 - Females → 63 successes and 113 failures, both > 5
- *All conditions are met, appropriate to continue with test!*

LCQ – Assumptions

Problem: Check the conditions for a Hypothesis Test of the two population proportions for the following scenarios:

- a) A company has two separate teams (A and B) who complete accounting reports. From a random sample of 50 recent reports from each team, 40% of Team A's were on time and 36% of Team B's were on time. They want to know if these teams perform differently in terms of proportion of reports completed on time.

- b) A company randomly selects employees to complete yearly surveys on whether or not they enrolled in at least one wellness class at the company's site. Last year's survey showed 81 out of 100 employees had taken a wellness class and this year 102 out of 140 had. They want to know if a greater percentage took a wellness class this year compared to last year.

LCQ – Assumptions

Problem: Check the conditions for a Hypothesis Test of the two population proportions for the following scenarios:

a) A company has two separate teams (A and B) who complete accounting reports. From a random sample of 50 recent reports from each team, 40% of Team A's were on time and 36% of Team B's were on time. They want to know if these teams perform differently in terms of proportion of reports completed on time.

1) Random condition: 'Random sample of 50 reports from each team' Yes! → Have random sample from both groups

2) Independence condition: Separate teams so independent, Yes!! → No reason to think these separate teams impact each other

3) Large enough samples condition: Yes! → Using sample proportions to check these below

● *Team A: $50(.4) = 20 > 5$ and $50(.6) = 30 > 5$*

● *Team B: $50(.36) = 18 > 5$ and $50(.64) = 32 > 5$*

ALL conditions are met! Okay to continue with test!

b) A company randomly selects employees to complete yearly surveys on whether or not they enrolled in at least one wellness class at the company's site. Last year's survey showed 81 out of 100 employees had taken a wellness class and this year 102 out of 140 had. They want to know if a greater percentage took a wellness class this year compared to last year.

1) Random condition: Company randomly selects individuals to complete the surveys each year, Yes!

2) Independence condition: Surveys from different years, Yes! → No reason to think one wellness class enrollment depends on which year it was. And we are not selecting same people because of randomness

3) Large enough samples condition: Yes! → Can directly use the given sample results (no need to get the proportions first) to check these below

● *This year: 102 successes and 38 failures, both > 5*

● *Last year: 81 successes and 19 failures, both > 5*

ALL conditions are met! Okay to continue with test!

Using Calc - Test Statistic and P-Value for Proportions

<https://www.statisticshowto.com/probability-and-statistics/statistics-definitions/prop-proportion/>
<https://www.six-sigma-material.com/Prop-Test.html>

4. Compute value of Test Statistic / P-value.

Setup

A NatGeo Poll interviewed 1200 hiking enthusiasts and 1100 climbers. They asked "Are you more afraid of spiders or snakes???" 768 of the 1200 hikers and 662 of the 1100 climbers, responded "Ewww, snakes...." Is there enough evidence to conclude the proportion of hikers who are more afraid of snakes is different than that of climbers?

Use $\alpha = 0.1$

GOAL: Conduct a Hypothesis Test!

1. 2-PropZTest

- a) x_1 = number of successes in sample 1
- b) n_1 = sample size 1
- c) x_2 = number of successes in sample 2
- d) n_2 = sample size 2
- e) p_1 : Alternative hypothesis with $p_2 \rightarrow$ ** NOTE this is not in terms of the difference

Calculate or Draw

Formula for Z_{stat} by hand:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1 - \hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

\hat{p}_1 = sample proportion from population 1

\hat{p}_2 = sample proportion from population 2

\hat{p} = pooled sample proportion

n_1 = sample size of group 1

n_2 = sample size of group 2

Combined ("pooled")
sample proportion (no subscript) $= \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$

Using Calc - Test Statistic and P-Value for Proportions

<https://www.statisticshowto.com/probability-and-statistics/statistics-definitions/standard-normal-distribution/>
<https://www.six-sigma-material.com/Probability-Distribution-Tables/Standard-Normal-Distribution-Table.html>

4. Compute value of Test Statistic / P-value.

Setup

A NatGeo Poll interviewed 1200 hiking enthusiasts and 1100 climbers. They asked "Are you more afraid of spiders or snakes???" 768 of the 1200 hikers and 662 of the 1100 climbers, responded "Ewww, snakes...." Is there enough evidence to conclude the proportion of hikers who are more afraid of snakes is different than that of climbers?

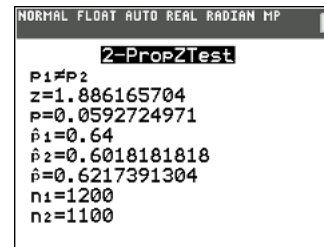
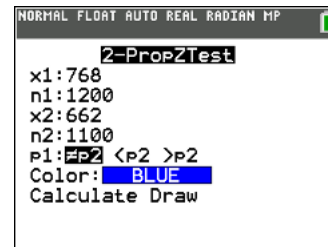
Use $\alpha = 0.1$

GOAL: Conduct a Hypothesis Test!

1. 2-PropZTest

- a) x_1 = number of successes in sample 1
- b) n_1 = sample size 1
- c) x_2 = number of successes in sample 2
- d) n_2 = sample size 2
- e) p_1 : Alternative hypothesis with $p_2 \rightarrow$ ** NOTE this is not in terms of the difference

Calculate or Draw



(p_1 = hikers and p_2 = climbers)

$H_0: p_1 - p_2 = 0$

$H_A: p_1 - p_2 \neq 0$

Calculate Output

$p_1 \neq p_2$ Alternative hypothesis

$z = Z_{\text{stat}}$

$p = p\text{-value}$

\hat{p}_1 = sample proportion 1

\hat{p}_2 = sample proportion 2

\hat{p} = pooled sample proportion

n_1 = sample size 1

n_2 = sample size 2

Formula for Z_{stat} by hand:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1 - \hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

\hat{p}_1 = sample proportion from population 1

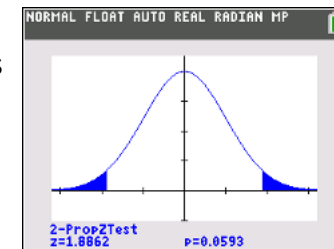
\hat{p}_2 = sample proportion from population 2

\hat{p} = pooled sample proportion

n_1 = sample size of group 1

n_2 = sample size of group 2

Combined ("pooled")
sample proportion (no subscript) = $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$



Draw Output

Plot (and displays values) of $p = p\text{-value}$ and $z = Z_{\text{stat}}$ on the standard normal curve

LCQ – Conclusions and Interpretations

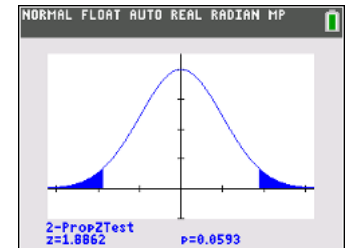
5. Conclude and Interpret

- State whether you reject H_0 or fail to reject H_0 AND WHY!
- Interpret your results in the context of the problem

Problem: Write the conclusions and interpretations for the previous scenarios using our results.

Setup: A NatGeo Poll interviewed 1200 hiking enthusiasts and 1100 climbers. They asked “Are you more afraid of spiders or snakes???” 768 of the 1200 hikers and 662 of the 1100 climbers, responded “Ewww, snakes....” Is there enough evidence to conclude the proportion of hikers who are more afraid of snakes is different than that of climbers? Use $\alpha = 0.1$

Solution:



LCQ – Conclusions and Interpretations

5. Conclude and Interpret

- State whether you reject H_0 or fail to reject H_0 AND WHY!
- Interpret your results in the context of the problem

Problem: Write the conclusions and interpretations for the previous scenarios using our results.

Setup: A NatGeo Poll interviewed 1200 hiking enthusiasts and 1100 climbers. They asked “Are you more afraid of spiders or snakes???” 768 of the 1200 hikers and 662 of the 1100 climbers, responded “Ewww, snakes....” Is there enough evidence to conclude the proportion of hikers who are more afraid of snakes is different than that of climbers? Use $\alpha = 0.1$

Solution: *Let p_1 = the true proportion of hikers who are more afraid of snakes than spiders*
Need these... Let p_2 = the true proportion of climbers who are more afraid of snakes than spiders

$$H_0: p_1 - p_2 = 0$$

$$H_A: p_1 - p_2 \neq 0$$

$$\alpha = 0.1$$

P-Value

$$P\text{-value} = 2\text{-PropZTest}(x_1 = 768, n_1 = 1200, x_2 = 662, n_2 = 1100, p_1 \neq p_2) = 0.0593$$

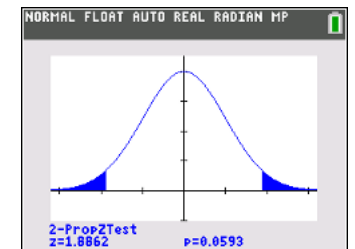
$$p\text{-value} = 0.0593 < 0.10 = \alpha \rightarrow \text{Reject } H_0!$$

Conclusion and Interpretation

First part → *Because our $p\text{-value} = 0.0593$ is less than the significance level 0.10, we reject the Null hypothesis.*

Second part → *There IS sufficient evidence to conclude that the true proportion of hikers who are more afraid of snakes than spiders is different than that of climbers.*

More info → *Further we can say that hikers' proportion is actually greater ($Z_{\text{stat}} = 1.89$).*



Hypothesis Tests for Means – INDEPENDENT Samples (and Known σ)

- All of the previous Hypothesis tests overview applies, now we are just going to apply it specifically to a Two Sample Means Test!
- And going back to the One Sample Means Test, we still have to determine if the population standard deviation is known or unknown.
 - This tells us if we are doing a Z distribution based Test or a T distribution based Test!
- Now we have to also think about the relationship between our two population data sources → This section is for **independent** samples!
 - And we will start with KNOWN population standard deviations σ_1 and σ_2

LCQ: Independent vs Dependent Samples

How to think about samples

- Independent samples → Groups are unrelated, no connection, no relationship
- Dependent samples → Groups have some relationship between one another, can link the two; PAIRS

Problem: Determine if the following scenarios are independent or dependent samples.

- 1) Comparing the blood pressure of STAT 1450 students before the final exam and after completing the final exam.
- 2) Seeing if the height of Faculty is shorter than the undergraduate population.
- 3) Looking to see if there is a difference in the price of the same Video Game Consoles at Target or Walmart.
- 4) A study is conducted to see what effect a new drug has on dexterity. A random sample of 30 students is chosen. They are given a series of tasks to perform and a score reflecting their performance. A dose of the drug is given to the 30 students and they again perform similar tasks and are scored again.

LCQ: Independent vs Dependent Samples

How to think about samples

- Independent samples → Groups are unrelated, no connection, no relationship
- Dependent samples → Groups have some relationship between one another, can link the two; PAIRS

Problem: Determine if the following scenarios are independent or dependent samples.

1) Comparing the blood pressure of STAT 1450 students before the final exam and after completing the final exam.

Dependent! → There is a relationship between the blood pressure before the final and after the completion of the final. Connection is measuring the SAME student twice

2) Seeing if the height of Faculty is shorter than the undergraduate population.

Independent → There is no direct connection (or inherent relationship) between faculty and undergrads

3) Looking to see if there is a difference in the price of the same Video Game Consoles at Target or Walmart.

Independent??? Two different stores

Dependent! → No relationship between Target and Walmart, BUT we are looking at the SAME console at the two different stores (groups). So there is a relationship with the consoles (think pairs of X-boxes, one at Walmart and one at Target; same for a PS4)

4) A study is conducted to see what effect a new drug has on dexterity. A random sample of 30 students is chosen. They are given a series of tasks to perform and a score reflecting their performance. A dose of the drug is given to the 30 students and they again perform similar tasks and are scored again.

Dependent → SAME students before and after drug. So there is a relationship between the two groups

The Hypothesis Statements for Two Samples - Review

1. State the Hypotheses

- **Define parameter + context.**

Define Parameters

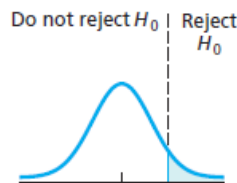
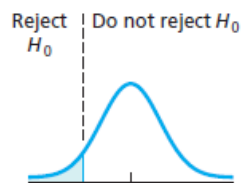
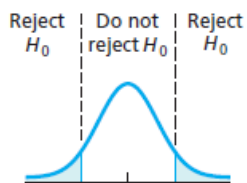
- Now we have TWO populations and TWO **parameters**!
- These describe the same quantity, just for different groups!
 - Quantitative (numeric) → population means μ_1 and μ_2

Null Hypothesis H_0

- We want to learn about the relationship between the two of them, μ_1 ?? μ_2
- So start by assuming both parameters are **equivalent**! So their difference is ZERO!

Alternative Hypothesis H_A

- This is where we state how we think these means are different
- Our test on the single value of the **difference** may be left-tailed (<), right-tailed (>), or two-tailed (\neq).



Type of Test: (a) Two tailed

Sign in H_A : \neq

Critical Values: $-Z^*$ $+Z^*$

(b) Left tailed

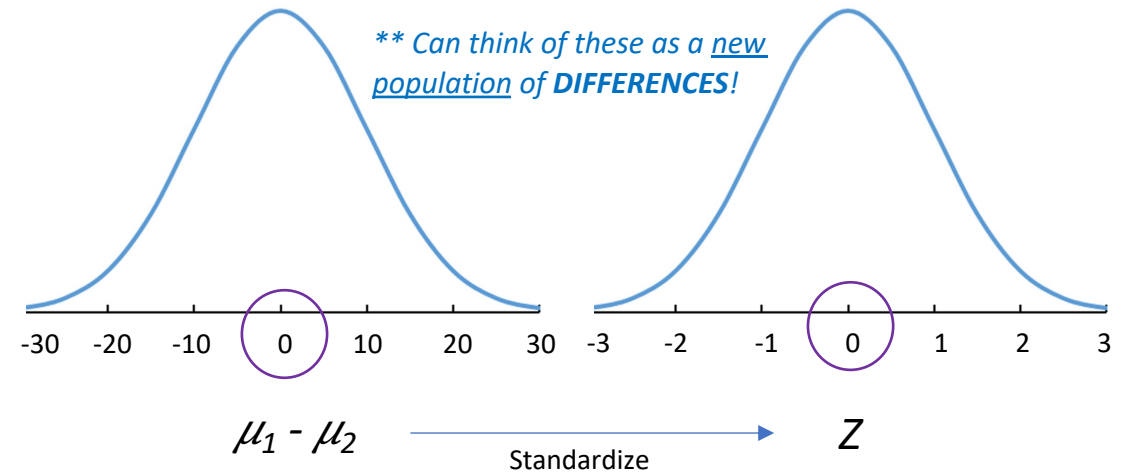
$<$

$-Z^*$

(c) Right tailed

$>$

Z^*



How to write hypotheses

- These can be written in two ways

Difference

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

$$H_A: \mu_1 - \mu_2 < 0$$

$$H_A: \mu_1 - \mu_2 > 0$$

OR

Direct Comparison

$$H_0: \mu_1 = \mu_2$$

$$H_A: \mu_1 \neq \mu_2$$

$$H_A: \mu_1 < \mu_2$$

$$H_A: \mu_1 > \mu_2$$

LCQ – Two Sample Means Hypotheses

Problem: (1) Define the parameters of interest and (2) State the Null and Alternative for the following scenarios:

a) A researcher random sampled 15 infants and 20 toddlers to measure their body temperatures. Let's assume that body temperatures for all persons are normally distributed with some known standard deviation. Is there sufficient evidence to conclude that the mean body temperatures for infants and toddlers differ?

b) A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore and random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. Is there sufficient evidence that the Baltimore housing market is more expensive?

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a) A researcher randomly sampled 15 infants and 20 toddlers to measure their body temperatures. Let's assume that body temperatures for all persons are normally distributed with some known standard deviation. Is there sufficient evidence to conclude that the mean body temperatures for infants and toddlers differ?

Define Parameters

Let μ_1 = The TRUE mean body temperature of infants

Let μ_2 = The POPULATION mean body temperature of toddlers

Hypotheses

$H_0: \mu_1 = \mu_2$ OR $H_0: \mu_1 - \mu_2 = 0 \rightarrow$ Both CORRECT!

$H_A: \mu_1 \neq \mu_2$ OR $H_A: \mu_1 - \mu_2 \neq 0 \rightarrow$ CORRECT! We just want in to be different, not only interested in less than or greater than

b) A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore and random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. Is there sufficient evidence that the Baltimore housing market is more expensive?

Define Parameters

Let P_1 = The average price of homes sold in DC \rightarrow NOPE! We are talking about MEANS, should NOT be any population proportions (p) in our problems

Let P_2 = The average price of homes sold in Baltimore. \rightarrow BE CAREFUL with your notation or shorthand, even if you meant P_2 as 'parameter 2', I would interpret this as a proportion (which is WRONG)

Let μ_1 = The TRUE average price of homes sold in DC \rightarrow Yes!

Let μ_2 = The POPULATION average price of homes sold in Baltimore \rightarrow 'true' and 'population' are synonymous here, both indicate the population parameter

Hypotheses

$H_0: \mu_1 = \mu_2$ OR $H_0: \mu_1 - \mu_2 = 0 \rightarrow$ Both CORRECT!

$H_A: \mu_1 > \mu_2$ OR $H_A: \mu_1 - \mu_2 > 0 \rightarrow$ INCORRECT! Because we want Baltimore to be more expensive, so μ_2 should be LARGER

$H_A: \mu_1 < \mu_2$ OR $H_A: \mu_1 - \mu_2 < 0 \rightarrow$ Now CORRECT!

Mean Assumptions - Independent Samples

2. Check Assumptions.

- EXACT same Assumptions as for Means Test with one sample, we just have to do it for both!
 - (Remember we need to know / be given the population standard deviations σ_1 and σ_2 for now)
- We have to add one more though!
 - Must have an assumption about the connection between our two samples

Independence

- This is a REALLY important assumption now, because we do a different test based on whether this assumption is met or not!
- Right now we are looking at an INDEPENDENT samples test!
 - We need each group to be **independent** (unrelated, no connection).
 - So they results from one group should NOT have any effect / impact on the results of the second group!

One Sample Test Conditions

- ✓ Randomization Condition
Need to have a random sample
- ✓ Large Enough Sample Condition
Normal population OR
 $n \geq 30$

New conditions



INDEPENDENT Samples Test Conditions

- ✓ Randomization Condition
Need to have two random samples
- ✓ Independence Condition
Need to independent samples
- ✓ Large Enough Sample Condition
Normal populations OR
 $n_1 \geq 30$ AND $n_2 \geq 30$

LCQ – Assumptions

Problem: Check the conditions for a Hypothesis Test of the two population proportions for the following scenarios:

a) A researcher randomly sampled 15 infants and 20 toddlers to measure their body temperatures. Let's assume that body temperatures for all persons are normally distributed with some known standard deviation. Is there sufficient evidence to conclude that the mean body temperatures for infants and toddlers differ?

b) A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore a random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. Is there sufficient evidence that the Baltimore housing market is more expensive?

LCQ – Assumptions

Problem: Check the conditions for a Hypothesis Test of the two population proportions for the following scenarios:

a) A researcher randomly sampled 15 infants and 20 toddlers to measure their body temperatures. Let's assume that body temperatures for all persons are normally distributed with some known standard deviation. Is there sufficient evidence to conclude that the mean body temperatures for infants and toddlers differ?

1) Researcher randomly sampled infants and toddlers → Yes!

2) Independent groups, no relationship between infants and toddlers → Good! This is our explanation of why the samples are independent! No mention of measuring the same child once as an infant and years later as a toddler (so reasonable to assume independent!)

3) Large enough samples is met because we are assuming body temperatures for everyone are Normal! → Yes! We have Normal populations for BOTH groups, so no need to look at the sample sizes

b) A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore a random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. Is there sufficient evidence that the Baltimore housing market is more expensive?

1) Random samples were taken as stated in problem → Yes!

2) Independent groups → NOT ENOUGH! IF this is ALL you wrote, NOT full credit!! Need to EXPLAIN why there are independent groups for this specific problem

Independent groups (DC vs Baltimore), there is no relation between DC houses and Baltimore houses → Now this is BETTER!

3) Large enough samples → WHY???? How do you know this??? I don't know that you know if this is all that you write on the Test

Large enough sample, $n \geq 30$ → STILL NOT FULL credit! BE SPECIFIC! (do NOT be general and just write $n \geq 30$); and our cutoff is 30 (not 5, which is the check for $np \geq 5$ for proportions)

Because $n_1 = 32 \geq 30$ and $n_2 = 45 \geq 30$ → YES!! And we had to look at the sample sizes because we don't have information about these populations already being normally distributed

Using Calc - Test Statistic and P-Value for Ind Means and Known σ

4. Compute value of Test Statistic / P-value.

Setup

A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore a random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. Is there sufficient evidence that the Baltimore housing market is more expensive? Use $\alpha = 0.1$

GOAL: Conduct a Hypothesis Test!

Formula for Z_{stat} by hand:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Under the Null hypothesis, the quantity $\mu_1 - \mu_2 = 0$ and everything else is known.

1. 2-SampZTest

- a) Input = Stats
- b) σ_1 = population 1 SD
- c) σ_2 = population 2 SD
- d) \bar{x}_1 = sample 1 mean
- e) n_1 = sample size 1
- f) \bar{x}_2 = sample 2 mean
- g) n_2 = sample size 2
- h) μ_1 : Alternative hypothesis with $\mu_2 \rightarrow$ ** NOTE this is not in terms of the difference

Calculate or Draw

Using Calc - Test Statistic and P-Value for Ind Means and Known σ

4. Compute value of Test Statistic / P-value.

Setup

A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore a random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. Is there sufficient evidence that the Baltimore housing market is more expensive? Use $\alpha = 0.1$

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- g) n_2 = sample size 2
- h) μ_1 : Alternative hypothesis with $\mu_2 \rightarrow$ **** NOTE this is not in terms of the difference**

Calculate or Draw

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS [2] OR [8] TO SELECT AN OPTION

2-SampZTest
Inpt: Data Stats
σ1: 190000
σ2: 190000
x̄1: 443705
n1: 32
x̄2: 450000
n2: 45
μ1: ≠ μ2 > μ2
↓Color: BLUE
```

```
NORMAL FLOAT AUTO REAL RADIAN MP

2-SampZTest
μ1 < μ2
z = -0.1432775072
p = 0.443035484
x̄1 = 443705
x̄2 = 450000
n1 = 32
n2 = 45
```

$(\mu_1 = \text{DC and } \mu_2 = \text{Baltimore})$

$H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 < 0$

Calculate Output

$\mu_1 \neq \mu_2$ Alternative hypothesis

$z = Z_{\text{stat}}$

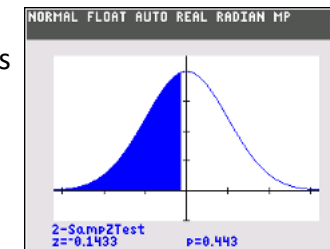
$p = \text{p-value}$

\bar{x}_1 = sample 1 mean

\bar{x}_2 = sample 2 mean

n_1 = sample 1 size

n_2 = sample 2 size



Draw Output

Plot (and displays values) of $p = \text{p-value}$ and $z = Z_{\text{stat}}$ on the standard normal curve

LCQ – Conclusions and Interpretations

5. Conclude and Interpret

- State whether you reject H_0 or fail to reject H_0 AND WHY!
- Interpret your results in the context of the problem

Problem: Write the conclusions and interpretations for the previous scenarios using our results.

Setup: A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore and random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. Is there sufficient evidence that the Baltimore housing market is more expensive? Use $\alpha = 0.1$

Solution:

```
NORMAL FLOAT AUTO REAL Radian MP
2-SampZTest
μ1<μ2
z=-0.1432775072
p=0.443035484
x̄1=443705
x̄2=450000
n1=32
n2=45
```

LCQ – Conclusions and Interpretations

5. Conclude and Interpret

- State whether you reject H_0 or fail to reject H_0 AND WHY!
- Interpret your results in the context of the problem

Problem: Write the conclusions and interpretations for the previous scenarios using our results.

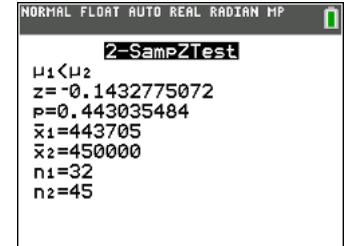
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Solution: Let μ_1 = the true mean sale price of foreclosed homes in DC
Need these... Let μ_2 = the true mean sale price of foreclosed homes in Baltimore

$$H_0: \mu_1 - \mu_2 = 0 \quad \alpha = 0.1$$

$$H_A: \mu_1 - \mu_2 < 0$$

P-Value
 $P\text{-value} = 2\text{-SampZTest}(\sigma_1 = 190000, \sigma_2 = 190000, \bar{x}_1 = 443705, n_1 = 32, \bar{x}_2 = 450000, n_2 = 45, \mu_1 < \mu_2) = 0.443$
 $p\text{-value} = 0.443 < 0.10 = \alpha \rightarrow \text{Fail to reject } H_0$



There is two parts that we need 1) Conclusion and 2) Interpretation

Conclusion and Interpretation

We fail to reject the null hypothesis because the p-value is greater than the significance level \rightarrow This is the correct decision, BUT if this is all you write, you are MISSING the entire INTERPRETATION part; and should be MORE SPECIFIC!! What are the the p-value and significance level???

This would be better for the CONCLUSION (only) \rightarrow We fail to reject the null hypothesis because the p-value = 0.443 is greater than the significance level 0.1 \rightarrow SHOW ME YOU KNOW WHAT YOU'RE DOING!

Can also word the CONCLUSION part like this, which is correct as long as we have all the needed info! \rightarrow Because the p-value 0.443 is greater than $\alpha = 0.1$, we fail to reject the null Hypothesis

Now here is the Interpretation part, which needs to be right after our CORRECT conclusion from above

We do not have sufficient evidence to conclude that the prices in Baltimore are greater than DC \rightarrow Almost there! Correctly said that there is NOT sufficient evidence and talked about the Alternative Hypothesis very good and good context. But MISSING the PARAMETER TRUE MEAN

There is NOT sufficient evidence to claim that the homes in Baltimore are more expensive than homes sold in DC \rightarrow Same thing, MISSING TRUE AVERAGE

There is NOT sufficient evidence to conclude that the true mean sale price of foreclosed homes in Baltimore is more than that of DC \rightarrow NOW this is CORRECT!

We do not have sufficient evidence to conclude that the true average price of Baltimore homes are greater than DC homes \rightarrow This would also be CORRECT!

Hypothesis Tests for Means – INDEPENDENT Samples (and Unknown σ)

- Now we will go over UNKNOWN population standard deviations!
 - This is the scenario when ONLY SAMPLE standard deviations are given
- All of the previous Two Sample overview applies and the Means Hypotheses, Conditions and Interpretations are the same
 - Our test is just based on the T distribution now!
- This is still for **independent** samples!

Using Calc - Test Statistic and P-Value for Ind Means and Unknown σ

4. Compute value of Test Statistic / P-value.

Setup

A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 with standard deviation \$150,000 and in Baltimore a random sample of 45 foreclosed homes sold for \$450,000 with standard deviation \$130,000. Is there sufficient evidence that the Baltimore housing market is more expensive? Use $\alpha = 0.1$

GOAL: Conduct a Hypothesis Test!

1. 2-SampTTest

- a) Input = Stats
- b) \bar{x}_1 = sample 1 mean
- c) Sx_1 = sample 1 SD
- d) n_1 = sample size 1
- e) \bar{x}_2 = sample 2 mean
- f) Sx_2 = sample 2 SD
- g) n_2 = sample size 2
- h) μ_1 : Alternative hypothesis with $\mu_2 \rightarrow$ **** NOTE this is not in terms of the difference**
- i) Pooled: No \rightarrow **** ALWAYS keep as NO for this class**

Calculate or Draw

*** Pooled Standard Deviation

- We have the option to use a weighted average of the two sample standard deviations
- This is appropriate if the two values are similar and results in a slightly better test
- **But we are going to keep it simple and ALWAYS NOT pool the SDs**

Formula for t_{stat} by hand:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Under the Null hypothesis, the quantity $\mu_1 - \mu_2 = 0$ and everything else is known.

*** Degrees of Freedom

- There is a difficult fancy way to determine the DF for two sample T-Tests (which our calc does) that we aren't going to do
- **So only going to be making conclusions using the p-value method**

Using Calc - Test Statistic and P-Value for Ind Means and Unknown σ

4. Compute value of Test Statistic / P-value.

Setup

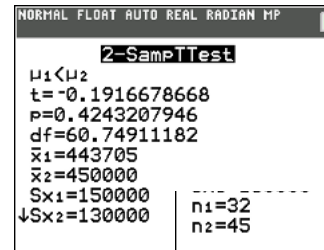
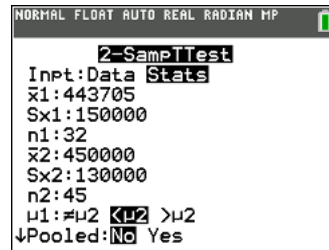
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GOAL: Conduct a Hypothesis Test!

1. 2-SampTTest

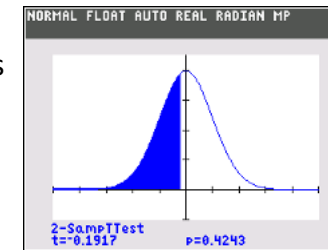
- Input = Stats
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- Sx_1 = sample 1 SD
- n_1 = sample size 1
- \bar{x}_2 = sample 2 mean
- Sx_2 = sample 2 SD
- n_2 = sample size 2
- μ_1 : Alternative hypothesis with $\mu_2 \rightarrow$ **** NOTE this is not in terms of the difference**
- Pooled: No \rightarrow **** ALWAYS keep as NO for this class**

Calculate or Draw



Calculate Output

$\mu_1 \neq \mu_2$ Alternative hypothesis
 $t = t_{\text{stat}}$
 $p = \text{p-value}$
 $df = \text{pooled degrees of freedom}$
 \bar{x}_1 = sample 1 mean
 \bar{x}_2 = sample 2 mean
 Sx_1 = sample 2 SD
 Sx_2 = sample 2 SD
 n_1 = sample size 1
 n_2 = sample size 2



Draw Output

Plot (and displays values) of $p = \text{p-value}$ and $t = t_{\text{stat}}$ on the standard normal curve

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- We have the option to use a weighted average of the two sample standard deviations
- This is appropriate if the two values are similar and results in a slightly better test
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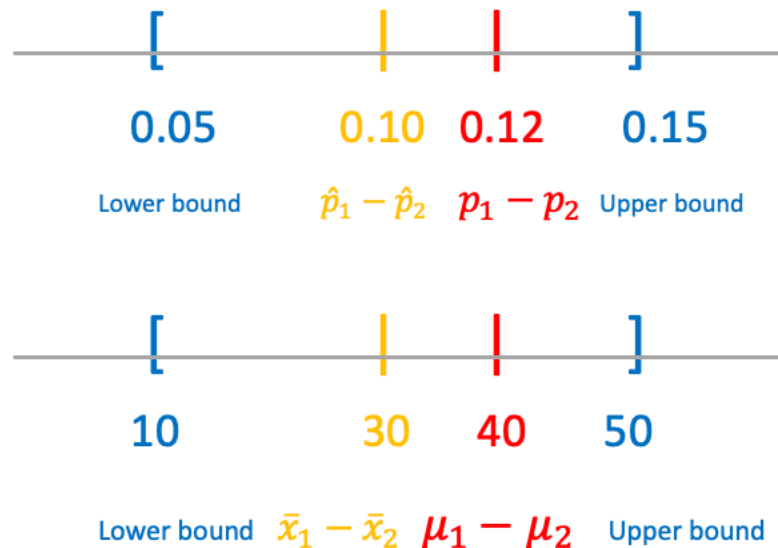
Under the Null hypothesis, the quantity $\mu_1 - \mu_2 = 0$ and everything else is known.

*** Degrees of Freedom

- There is a difficult fancy way to determine the DF for two sample T-Tests (which our calc does) that we aren't going to do
- So only going to be making conclusions using the p-value method**

Confidence Intervals – Two Samples!

- Everything we learned about Confidence Intervals (the different pieces, interpretation, etc.) still applies!
- Now we are just trying to estimate the DIFFERENCE between the two parameters!



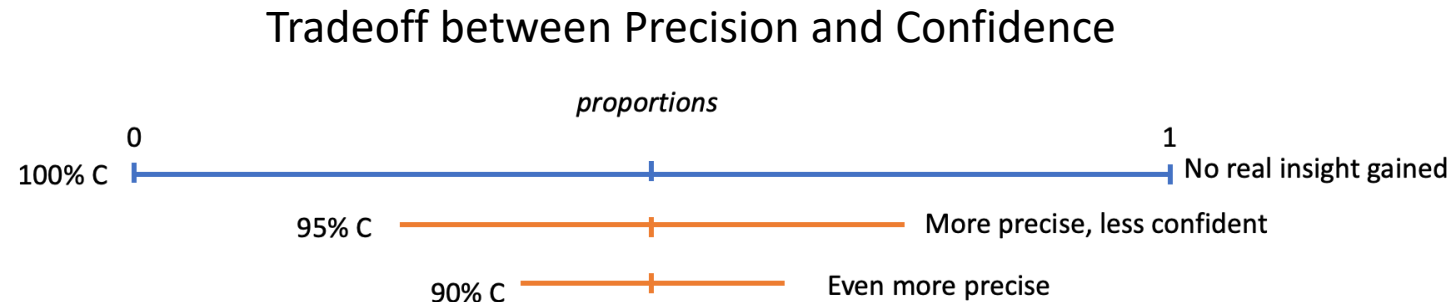
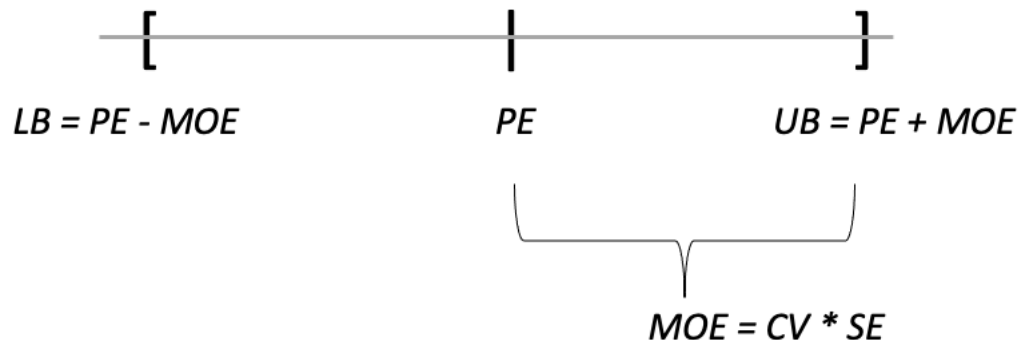
Structure of a Two Sample Confidence Interval

SAME structure as the One Sample Confidence Intervals we learned previously

C.I. = Point Estimate \pm Margin of Error

But Now with Two Samples

- Point Estimate is your best guess of the **DIFFERENCE**; at the center of the interval.
- Margin of Error (MOE) = Critical Value (CV) * Standard Error (SE).
 - CV are the exact same!
 - SE formulas are slightly different because we have an additional sample
- Same relationships with MOE: Smaller MOE, more precise your **estimate** of the difference is.
 - The more confident, the wider your interval is (if everything else stays the same)



Final Confidence Interval for $p_1 - p_2$

2 Proportion Z Interval

Recall: Our point estimate is the sample proportion $\hat{p}_1 = \frac{x_1}{n_1}$, which represents the number of success divided by the sample size, same for the second sample

C.I. = Point Estimate \pm Margin of Error

$$= (\hat{p}_1 - \hat{p}_2) \pm Z^* \sigma_{\hat{p}_1 - \hat{p}_2}$$

$$= (\hat{p}_1 - \hat{p}_2) \pm Z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

TWO Sample Interval Conditions

- ✓ Randomization Condition
Need to have two random samples
- ✓ Independence Condition
Need to independent samples
- ✓ Large Enough Sample Condition
AT LEAST 5 successes and 5 failures in EACH collected sample

Using Calc

GOAL: Find the Two Sample Confidence Interval for Difference in Proportions!

Setup

A NatGeo Poll interviewed 1200 hiking enthusiasts and 1100 climbers. They asked “Are you more afraid of spiders or snakes???” 768 of the 1200 hikers and 662 of the 1100 climbers, responded “Ewww, snakes...” **Calculate** a 95% Confidence Interval for the difference in proportions.

*** Have to state which parameter is 1 and which is 2*

2-PropZInt

- a) x_1 = # of successes (people that said yes) in sample 1
- b) n_1 = sample size
- c) x_2 = # of successes in sample 2
- d) n_2 = sample size 2
- e) C-Level = Confidence level (as a decimal or whole number, both work)

Final Confidence Interval for $p_1 - p_2$

2 Proportion Z Interval

Recall: Our point estimate is the sample proportion $\hat{p}_1 = \frac{x_1}{n_1}$, which represents the number of success divided by the sample size, same for the second sample

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$$= (\hat{p}_1 - \hat{p}_2) \pm Z^* \sigma_{\hat{p}_1 - \hat{p}_2}$$

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- b) n_1 = sample size
- c) x_2 = # of successes in sample 2
- d) n_2 = sample size 2
- e) C-Level = Confidence level (as a decimal or whole number, both work)

$p_1 \rightarrow$ hikers
 $p_2 \rightarrow$ climbers

```
NORMAL FLOAT AUTO REAL RADIAN MP
2-PropZInt
x1:768
n1:1200
x2:662
n2:1100
C-Level:95
Calculate
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
2-PropZInt
(-0.0015,0.07786)
p1=0.64
p2=0.6018181818
n1=1200
n2=1100
```

Final Confidence Interval for $\mu_1 - \mu_2$ INDEPENDENT Samples

2 Sample Z Interval – **KNOWN** σ s

C.I. = Point Estimate \pm Margin of Error

$$= (\bar{x}_1 - \bar{x}_2) \pm Z^* \sigma_{\bar{x}_1 - \bar{x}_2}$$

$$= (\bar{x}_1 - \bar{x}_2) \pm Z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Using Calc

GOAL: Find the Two Sample Confidence Interval for Difference in Means!!

2-SampZInt

- a) Input = Stats
- b) σ_1 = population 1 standard deviation
- c) σ_2 = population 2 standard deviation
- d) \bar{x}_1 = sample mean 1
- e) n_1 = sample size
- f) \bar{x}_2 = sample mean 2
- g) n_2 = sample size
- h) C-Level = Confidence level (as a decimal or whole number, both work)

* Same Critical Value as with a 2 Proportion Z Interval

INDEPENDENT Samples Interval Conditions

- ✓ Randomization Condition
Need to have two random samples
- ✓ Independence Condition
Need to independent samples
- ✓ Large Enough Sample Condition
Normal populations OR
 $n_1 \geq 30$ AND $n_2 \geq 30$

Setup

A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 and in Baltimore a random sample of 45 foreclosed homes sold for \$450,000. Real estate experts say the standard deviation for sales across the nation is \$190,000. **Calculate** a 85% Confidence Interval for the difference in means.

*** Have to state which parameter is 1 and which is 2*

Final Confidence Interval for $\mu_1 - \mu_2$ INDEPENDENT Samples

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Using Calc

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*** Have to state which parameter is 1 and which is 2*

$\mu_1 \rightarrow$ DC

$\mu_2 \rightarrow$ Baltimore

```
NORMAL FLOAT AUTO REAL RADIAN MP
2-SampZInt
Inpt:Data Stats
σ1:190000
σ2:190000
x̄1:443705
n1:32
x̄2:450000
n2:45
C-Level:85
Calculate
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
2-SampZInt
(-69542.56952)
x̄1=443705
x̄2=450000
n1=32
n2=45
```

Final Confidence Interval for $\mu_1 - \mu_2$ INDEPENDENT Samples

2 Sample t Interval – UNKNOWN σ s

C.I. = Point Estimate \pm Margin of Error

$$= (\bar{x}_1 - \bar{x}_2) \pm t^* \sigma_{\bar{x}_1 - \bar{x}_2}$$
$$= (\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Using Calc

GOAL: Find the Two Sample Confidence Interval for Difference in Means!!

2-SameTInt

- a) Input = Stats
- b) \bar{x}_1 = sample mean 1
- c) s_{x_1} = population 1 standard deviation
- d) n_1 = sample size 1
- e) \bar{x}_2 = sample mean 2
- f) s_{x_2} = population 2 standard deviation
- g) n_2 = sample size 2
- h) Pooled = No
- i) C-Level = Confidence level (as a decimal or whole number, both work)

*** Pooled Standard Deviation

- We have the option to use a weighted average of the two sample standard deviations
- This is appropriate if the two values are similar and results in a slightly more precise interval
- **But we are going to keep it simple and ALWAYS NOT pool the SDs**

* Same Critical Value is based on t distribution now

*** Degrees of Freedom

- There is a difficult fancy way to determine the DF for two sample, (which our calc does) that we wouldn't do by hand

INDEPENDENT Samples Interval Conditions

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Need to have two random samples
- ✓ Independence Condition
Need to independent samples
- ✓ Large Enough Sample Condition
Normal populations OR
 $n_1 \geq 30$ AND $n_2 \geq 30$

Setup

A prospective home buyer is trying to decide which city to move to. In Washington D.C, a random sample of 32 foreclosed homes sold for an average of \$443,705 with standard deviation \$150,000 and in Baltimore and random sample of 45 foreclosed homes sold for \$450,000 with standard deviation \$130,000. **Calculate** a 85% Confidence Interval for the difference in means.

*** Have to state which parameter is 1 and which is 2*

Final Confidence Interval for $\mu_1 - \mu_2$ INDEPENDENT Samples

2 Sample t Interval – UNKNOWN σ s

C.I. = Point Estimate \pm Margin of Error

$$= (\bar{x}_1 - \bar{x}_2) \pm t^* \sigma_{\bar{x}_1 - \bar{x}_2}$$
$$= (\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Using Calc

GOAL: Find the Two Sample Confidence Interval for Difference in Means!!

2-SameTInt

- a) Input = Stats
- b) \bar{x}_1 = sample mean 1
- c) Sx_1 = population 1 standard deviation
- d) n_1 = sample size 1
- e) \bar{x}_2 = sample mean 2
- f) Sx_2 = population 2 standard deviation
- g) n_2 = sample size 2
- h) Pooled = No
- i) C-Level = Confidence level (as a decimal or whole number, both work)

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$\mu_1 \rightarrow$ DC

$\mu_2 \rightarrow$ Baltimore

```
NORMAL FLOAT AUTO REAL RADIAN MP
2-SampTInt
Inpt:Data Stats
x1:443705
Sx1:150000
n1:32
x2:450000
Sx2:130000
n2:45
C-Level:85
Pooled:No Yes
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
2-SampTInt
(-45530,32940)
df=31.33141118
x1=443705
x2=450000
Sx1=150000
Sx2=130000
n1=32
n2=45
```

Hypothesis Tests and Confidence Intervals

What each type of inference tells us

- For One Sample
 - Hypothesis tests tell us how a parameter compares to a specific value (greater than, less than, or not equal to)
 - Confidence intervals give us a range of plausible values
- For Two Sample
 - Hypothesis Tests tell us if there is a difference between the two parameters
 - Confidence Intervals give us a range of plausible values for this difference

Both of these inference methods can be used together!

Example

- A hypothesis test on a random sample of 200 American adults found that greater than 50% of them have tried marijuana
 - *This conclusion just tells us that the true proportion is **somewhere above 50%***
- A confidence interval could be constructed to find how much more than 50% of American adults have tried marijuana
 - *Lets say $CI = (0.52, 0.60)$, then we have a specific estimate as to where the true proportion actually is, from 52% to 60%. This tells us **exactly where we think the parameter is and how much greater than 50% it is!** MORE INFORMATION*

CI for Testing

- Confidence Intervals actually give us enough information to say whether or not we would reject a corresponding Hypothesis Test!

LCQ – Confidence Intervals for Testing

Problem: Based on the parameters below, make a conclusion whether we would reject or fail to reject the Hypothesis Test below based on each of the following Confidence Intervals.

Let μ_1 = population mean height of football teams in meters

Let μ_2 = population mean height of soccer teams in meters

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

a) 85% CI for $\mu_1 - \mu_2 \rightarrow (-0.3, -0.05)$

b) 90% CI for $\mu_1 - \mu_2 \rightarrow (-0.1, 0.2)$

c) 90% CI for $\mu_1 - \mu_2 \rightarrow (0.03, 0.36)$

LCQ – Confidence Intervals for Testing

Problem: Based on the parameters below, **make** a conclusion whether we would reject or fail to reject the Hypothesis Test below based on each of the following Confidence Intervals and **explain** why.

Let μ_1 = population mean height of football teams in meters

Let μ_2 = population mean height of soccer teams in meters

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

(-, -)

a) 85% CI for $\mu_1 - \mu_2 \rightarrow (-0.3, -0.05)$

REJECT → because the entire interval is below zero! Here's the long description of why we would reject:

- A confidence interval gives a range of plausible values for the parameter we are estimating, in this case it is the difference between μ_1 and μ_2
- Under the Null, we are assuming this difference is zero (so they are equivalent)
- Well our entire CI is below this null difference of zero, which means it is NOT a plausible value based on our results!
- So this would be enough evidence to show that these two parameters are different
- And also we know that the entire interval is negative. So based on the order of subtraction in the hypotheses, this indicates that μ_2 is LARGER! Average heights of soccer teams are larger based on this interval.
- This conclusion would MATCH the conclusion if we actually did the Hypothesis Test for this. And we would get a negative Test Statistic, also indicating that μ_2 is bigger

(-, +)

b) 90% CI for $\mu_1 - \mu_2 \rightarrow (-0.1, 0.2)$

FAIL TO REJECT → because the entire interval is above zero!

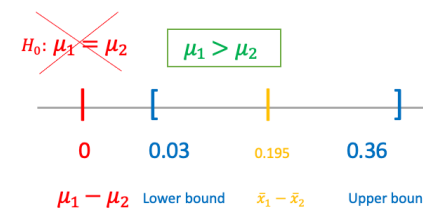
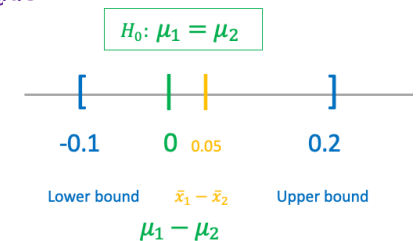
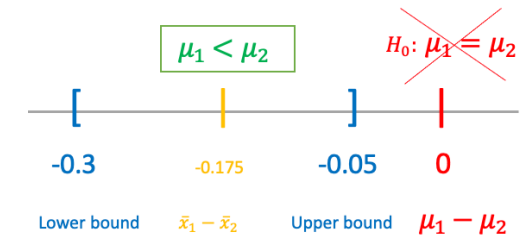
- Now our interval starts negative, crosses zero and end positive (so zero is contained in the interval)
- This says that zero is a plausible value for the difference of these two parameters, which means it's possible that they are equivalent!
- We can NOT say that one is ALWAYS larger than the other, so we would NOT be able to conclude they are significantly different → so would fail to reject the Null

(+, +)

c) 95% CI for $\mu_1 - \mu_2 \rightarrow (0.03, 0.36)$

REJECT → because the entire interval is above zero!

- Our interval suggest that zero is not a possible value for the difference of these two parameters, so it's not possible that they are equivalent based on our results
- That is exactly what rejecting the null hypothesis tells us!
- And because the interval is positive, this time μ_2 is LARGER (subtracting a smaller number from a larger number results in a positive difference) so the average heights of football teams are greater than soccer teams



Two Sample Confidence Interval Interpretations

General Structure

- Here was the structure for ONE sample CI:
 - I am % confident that the true/population parameter + context is between (lower bound) and (upper bound).
- It is not as structured when discussing **TWO parameters**, but we still have the same key parts below

3 Pieces

1. **95% Confident: This is a Confidence Statement**
 - Tells us what percent off ALL possible samples result in a CI that captures the true proportion.
2. **Parameters + Context: We are talking about TWO parameters.**
 - But what parameters??? We ALWAYS need context.
 - Now because we have TWO parameters, we can be specific about which one is greater / less than!
3. **Interval: The range of plausible values for the DIFFERENCE!**
 - Uses the difference of our sample statistics and the MOE based on Two Sample Standard Errors.

** Wording gets a little tricky when the CI of the difference contains zero (negative lower bound and positive upper bound)... but nonetheless same logic

Example

Let p_1 = true proportion of Columbus males who enjoy running

Let p_2 = true proportion of Columbus females who enjoy running

95% CI for $p_1 - p_2 = (0.05, 0.25)$

- We are **95% confident** that the **true proportion of all Columbus males who enjoy running** is **between 0.05 and 0.25 greater than the true proportion of females who enjoy running.**
- Or equivalently but slightly shorter → We are **95% confident** that the **true proportion of all Columbus males who enjoy running** is **between 0.05 and 0.25 greater than that of females.**

LCQ – Two Sample CI Interpretations

Problem: Based on the parameters below, interpret each of the following Confidence Intervals.

Let μ_1 = population mean height of football teams (meters)

Let μ_2 = population mean height of soccer teams (meters)

a) 85% CI for $\mu_1 - \mu_2 \rightarrow (-0.3, -0.05)$

b) 90% CI for $\mu_1 - \mu_2 \rightarrow (-0.1, 0.2)$

c) 95% CI for $\mu_1 - \mu_2 > 0 \rightarrow (0.03, 0.36)$

LCQ – Two Sample CI Interpretations

Problem: Based on the parameters below, interpret each of the following Confidence Intervals.

Let μ_1 = population mean height of football teams (meters)

Let μ_2 = population mean height of soccer teams (meters)

a) 85% CI for $\mu_1 - \mu_2 \rightarrow (-0.3, -0.05) \rightarrow$ *We already talked how this interval indicates $\mu_1 < \mu_2$. So we can phrase our CI interpretation using this knowledge:*

We are 85% confident that the population mean height of football teams is between 0.05 and 0.3 meters less than the population mean height of soccer teams

Or equivalently (rearranging with $\mu_2 > \mu_1$): We are 85% confident that the population mean height of soccer teams is between 0.05 and 0.3 meters greater than the population mean height of football teams

b) 90% CI for $\mu_1 - \mu_2 \rightarrow (-0.1, 0.2) \rightarrow$ *Here we can't say definitively that one mean is greater, so our wording needs to reflect that μ_1 can be less than OR greater than μ_2 (this wording is a little less straightforward than before)*

We are 90% the that true mean height of football teams is between 0.1 meters less than OR 0.2 meters greater than the true mean height of soccer teams

Or equivalently (rearranging with $\mu_2 - \mu_1$): We are 90% confident that the population mean height of soccer teams is between 0.2 meters less than OR 0.1 meters greater than the population mean height of football teams

c) 95% CI for $\mu_1 - \mu_2 > 0 \rightarrow (0.03, 0.36) \rightarrow$ *We know this interval indicates $\mu_1 > \mu_2$. So we can again phrase our CI interpretation in that way:*

We are 95% confident that the true mean height of football teams is between 0.03 and 0.36 meters taller than the true mean height of soccer teams

Or equivalently (rearranging with $\mu_2 < \mu_1$): We are 95% confident that the population mean height of soccer players is between 0.03 and 0.36 meters shorter than the population mean height of football players

LCQ 2 – Two Sample CI Interpretations

Problem: Based on the parameters below, interpret each of the following Confidence Intervals. Then determine if you would reject or fail to the corresponding Hypothesis Test.

Let μ_1 = population mean price (\$) at an Italian restaurant

Let μ_2 = population mean price (\$) at a Mexican restaurant

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

a) 85% CI for $\mu_1 - \mu_2 \rightarrow (-30, -3)$

b) 90% CI for $\mu_1 - \mu_2 \rightarrow (-5, 20)$

c) 95% CI for $\mu_1 - \mu_2 > 0 \rightarrow (3, 10)$

LCQ 2 – Two Sample CI Interpretations


Problem: Based on the parameters below, interpret each of the following Confidence Intervals. Then determine if you would reject or fail to the corresponding Hypothesis Test.

Let μ_1 = population mean price (\$) at an Italian restaurant

Let μ_2 = population mean price (\$) at a Mexican restaurant

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

a) 85% CI for $\mu_1 - \mu_2 \rightarrow (-30, -3)$  **Reject test!** Difference of zero \leftrightarrow equal is NOT plausible based on interval (not captured)

Options

1. *I am 85% confident that the true mean price at the Italian restaurant is less than that of the true mean price at the Mexican restaurant \rightarrow Correct but MISSING the values of the CI! The whole goal of a CI is to find the range of plausible values, so use them in the interpretation!!*
2. *I am 85% confident that the true mean price at the Italian restaurant is less (-30, -3) than that of the true mean price at the Mexican restaurant \rightarrow Correct, but the phrasing for the values should be IMPROVED!! This is NOT how we would try to talk to someone when comparing the prices of two restaurants! Make it flow, how we would naturally speak it*
3. *We are 85% confident that the true mean price of dinner at an Italian restaurant is between \$3 and \$30 less expensive than the true mean price of dinner at a Mexican restaurant \rightarrow PERFECT!!!, saying 'less than' takes care of the negatives and now it reads much better!*
4. *We are 85% confident that the true mean Mexican food price is \$3 to \$30 less expensive than the true mean of Italian food price \rightarrow WRONG!! We have a negative interval for the subtraction, so the second parameter must be larger LARGER (order matters in our interpretation). We could flip it to say it like option 5*
5. *We are 85% confident that the true mean Mexican food price is \$3 to \$30 MORE expensive than the true mean of Italian food price \rightarrow Now this is CORRECT!! Italian less expensive is equivalent to Mexican more expensive!*

LCQ 2 – Two Sample CI Interpretations

Problem: Based on the parameters below, interpret each of the following Confidence Intervals. Then determine if you would reject or fail to the corresponding Hypothesis Test.

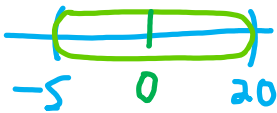
Let μ_1 = population mean price (\$) at an Italian restaurant

Let μ_2 = population mean price (\$) at a Mexican restaurant

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

b) 90% CI for $\mu_1 - \mu_2 \rightarrow (-5, 20)$



Fail to Reject test! Difference of zero \leftrightarrow equal IS plausible based on interval (contained within)

Process

This wording is more tricky than before because we have a negative lower bound and positive upper bound, not strictly less expensive or strictly more expensive.

But we can word our interpretation one bound at a time like so and put the pieces together:

1. Start with μ_1 = Italian \rightarrow *We are 90% confident that the true mean price of an Italian restaurant is between*
2. Now talk about the negative lower bound -5 \rightarrow *(Italian) \$5 less expensive (than Mexican)*
3. Now positive upper bound 20 \rightarrow *\$20 more expensive (than Mexican)*
4. End with μ_2 = Mexican \rightarrow *than the true mean price of a Mexican restaurant*

All together 1 + 2 + 3 + 4 \rightarrow We are 90% confident that the true mean price of an Italian restaurant is between \$5 less expensive and \$20 more expensive than the true mean price at a Mexican restaurant

c) 95% CI for $\mu_1 - \mu_2 > 0 \rightarrow (3, 10)$



Reject test! Difference of zero \leftrightarrow equal is NOT plausible based on interval (NOT contained inside)

Process

We can even do an entirely positive interval like this as well, one bound at a time like so and put the pieces together:

1. Start with μ_1 = Italian \rightarrow *We are 95% confident that the true mean price of an Italian restaurant is between*
2. Now talk about the negative lower bound 3 \rightarrow *(Italian) \$3 more expensive (than Mexican)*
3. Now positive upper bound 10 \rightarrow *\$10 more expensive (than Mexican)*
4. End with μ_2 = Mexican \rightarrow *than the true mean price of a Mexican restaurant*

All together 1 + 2 + 3 + 4 \rightarrow We are 95% confident that the true mean price of an Italian restaurant is between \$3 more expensive and \$10 more expensive than the true mean price at a Mexican restaurant

Can simplify wording a bit cause both are more expensive \rightarrow We are 95% confident that the true mean price of an Italian restaurant is between \$3 and \$10 more expensive than the true mean price at a Mexican restaurant

LCQ 2 – Two Sample CI Interpretations

Problem: Based on the parameters below, interpret each of the following Confidence Intervals.

Let μ_1 = population mean price (\$) at an Italian restaurant

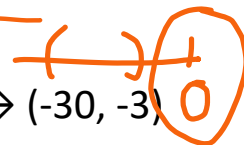
Let μ_2 = population mean price (\$) at a Mexican restaurant

$H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 \neq 0$

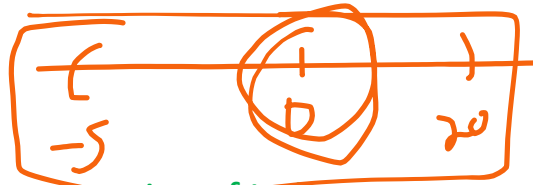
— —
NUT

a) 85% CI for $\mu_1 - \mu_2 \rightarrow (-30, -3)$



1. I am 85% confident that the true mean price at the Italian restaurant is less than that of the true mean price at the Mexican restaurant
2. We are 85% confident that the true mean price of dinner at an Italian restaurant is between \$3 and \$30 less expensive than the true mean price of dinner at a Mexican restaurant
3. We are 85% confident that the true mean Mexican food price is \$3 to \$30 MORE expensive than the true mean of Italian food price

b) 90% CI for $\mu_1 - \mu_2 \rightarrow (-5, 20)$



- We are 90% confident that the true mean price of Italian restaurant is between \$5 less expensive and \$20 more expensive than the true mean price of Mexican
- We are 90% confident that the true mean price of a dinner at an Italian restaurant is between \$5 less and \$20 more than the true mean

Problem Session!!!

Problem 1

Are people waiting longer to marry? In 2007, a random sample of young adults (ages 18-31) showed that 468 of 1872 of those surveyed were married. In 2012, 581 of the 1940 young adults (ages 18-31) randomly surveyed were married. Is there any evidence to suggest the true proportion of young adults who are married has decreased? Use $\alpha = 0.10$.

- a) Define the parameters and state the hypotheses.
- b) Check the conditions to run this test
- c) Carry out the test
- d) Calculate and interpret a 80% confidence interval for the difference in proportions

Problem 1 - Solution

a) Let p_1 = true proportion of married young adults in 2007 and
 p_2 = true proportion of married young adults in 2012

$H_0: p_1 = p_2$ vs. $H_a: p_1 > p_2$

Also acceptable would be $H_0: p_1 - p_2 = 0$ vs. $H_a: p_1 - p_2 > 0$

b) Check the assumptions:

- Whether or not someone is married is a categorical variable.
- It is stated that we have two random samples of young adults (ages 18-31)
- Whether or not one person is married does not affect whether or not others are married, so the groups are independent.
- The number of successes (those married) and failures (those not married) are at least 5 for both samples:
 - Sample 1 has 468 successes and $1872 - 468 = 1404$ failures.
 - Sample 2 has 581 successes and $1940 - 581 = 1359$ failures.

Significance Level: $\alpha = 0.10$

Problem 1 - Solution

Hypothesis test results:

p_1 : proportion of successes for population 1

p_2 : proportion of successes for population 2

$p_1 - p_2$: Difference in proportions

$H_0 : p_1 - p_2 = 0$

$H_A : p_1 - p_2 > 0$

Difference	Count1	Total1	Count2	Total2	Sample Diff.	Std. Err.	Z-Stat	P-value
$p_1 - p_2$	468	1872	581	1940	-0.049484536	0.014469314	-3.4199641	0.9997

c) The z test statistic = -3.420

The P-value = 0.9997

Since the P-value is greater than our significance level of 0.10, we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the true proportion of young adults who are married has decreased.

d) 80% Confidence Interval

I am 80% confident that the true proportion of young adults who were married in 2007 is between 3.1% and 6.8% lower than the true proportion of young adults who were married in 2012.

Note: The result of the test would have been significant at $\alpha = 0.10$ if the statement had called for a left-tailed alternative hypothesis as opposed to right-tailed! That is, if the question posed has been "Is there evidence to suggest the true proportion of young adults who are married has **increased**?"

Problem 3

Is there a difference in the proportion of males and females who participate in Greek life at Miami University? A researcher collected data from a representative sample of students from Miami University and found that 31 out of 114 males and 63 out of 176 females participated in Greek life. If appropriate, Test an appropriate hypothesis with $\alpha = 0.05$.

Problem 3 - Solution

Test to be ran: Two Proportion z-test

Parameters:

- P_1 = Population Proportion of Males that participate in Greek life at Miami
- P_2 = Population Proportion of Females that participate in Greek life at Miami

Hypotheses:

- $H_0: P_1 = P_2$ vs. $H_a: P_1 \neq P_2$
- Or... $H_0: P_1 - P_2 = 0$ vs. $H_a: P_1 - P_2 \neq 0$

Assumptions:

- Random sample of 114 males and 176 females
- Males and females are do not affect each other's probability of joining a fraternity or sorority. (They are independent)
- Each group has 5 successes and 5 failures in their sample
 - Males have 31 successes and 5 failures
 - Females have 63 successes and 113 failures

Significance Level: 5% or 0.05

Problem 3 - Solution

Two sample proportion summary hypothesis test:

p_1 : proportion of successes for population 1

p_2 : proportion of successes for population 2

$p_1 - p_2$: Difference in proportions

$H_0 : p_1 - p_2 = 0$

$H_A : p_1 - p_2 \neq 0$

Test Stat: -1.529

P-value: 0.1263

Hypothesis test results:

Difference	Count1	Total1	Count2	Total2	Sample Diff.	Std. Err.	Z-Stat	P-value
$p_1 - p_2$	31	114	63	176	-0.086024721	0.056270944	-1.5287591	0.1263

Because our p-value of 0.1263 is greater than our significance level of 0.05, we fail to reject the null hypothesis. There is not sufficient evidence that the true population proportion of male Miami students participating in Greek life is different than the true population proportion of female Miami students participating in Greek life.