Second Half!

Unit 6 – Normal Distribution, Sampling Dists and CLT

Your Backpacking Professor Colton

Unit 6 - Outline

<u>Unit 6 – Normal Distribution, Sampling Dists and CLT</u>

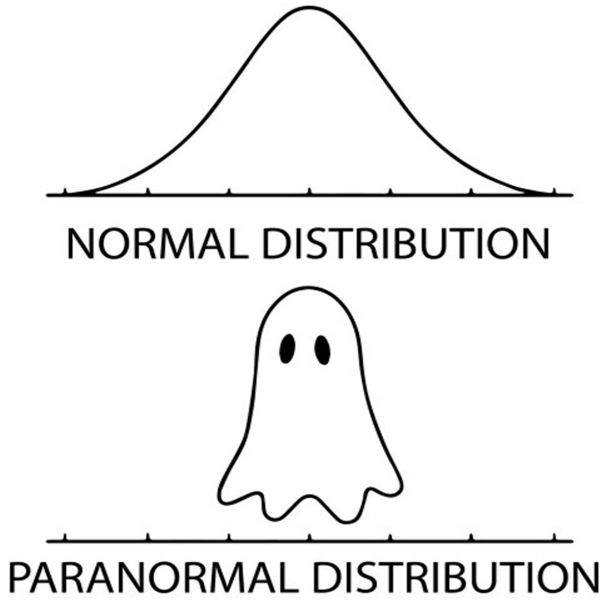
Intro

Density Curves

- Density curve properties

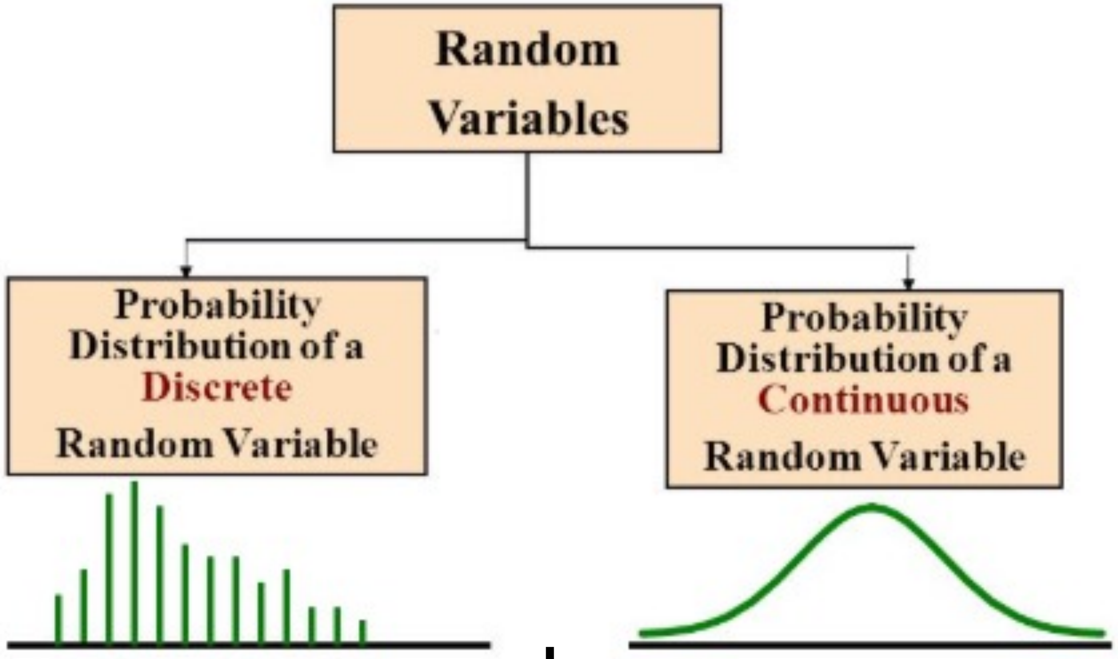
Normal Distribution

- Standard Normal Distributions
- Find z, given area
- Normal Distributions
- Find x, given area



Discrete vs Continuous RVs

X	*P(X)*
Win = 3	0.3
Tie = 1	0.5
Loss = 0	0.2



- Can find probabilities for specific points, e.g. $P(5) = 0.3$, $P(10) = 0.6$
- Can find probabilities of multiple events, such as:
  - $P(5 < X < 10) = P(6) + P(7) + P(8) + P(9)$
  - Just adding up individual probabilities from the probability distribution table.

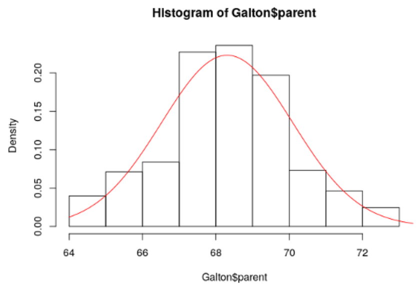
- Can NOT find probabilities for specific points, e.g. $P(5) = 0$, $P(10) = 0$
- Have to find probabilities for intervals:
  - $P(5 \leq X \leq 10)$ or like in the Empirical Rule
  - This is finding the area under the curve between the end points.

https://medium.com/@seema.singh/probability-distributions-discrete-and-continuous-7a94ede66dc0

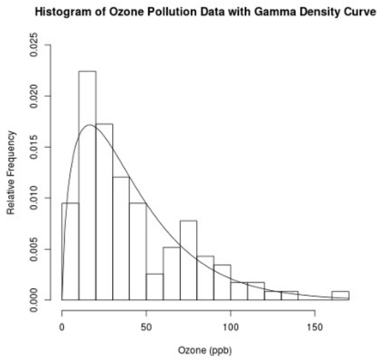
Density Curves

<u>Density Curves</u>

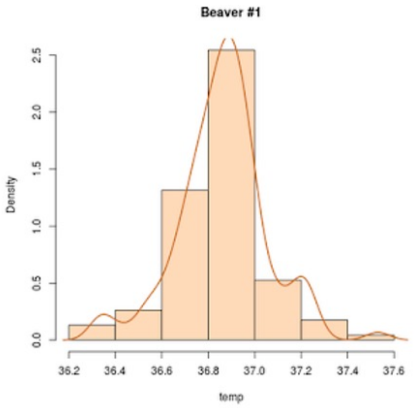
- Describe the overall pattern of the distribution. It's like a "smoothed-out histogram".
- The density curves are an "idealized" picture of the distribution. The proportions obtained from the density curve will not exactly equal the observed proportions.



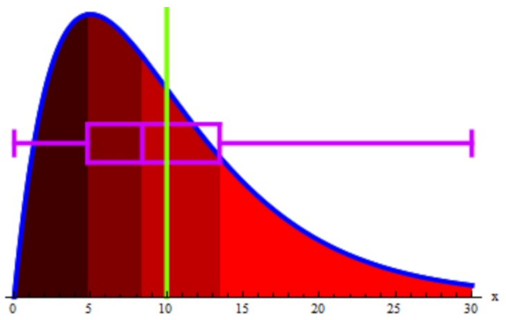
https://homepage.divms.uiowa.edu/~luke/classes/STAT4580/histdens.html



https://chemicalstatistician.wordpress.com/

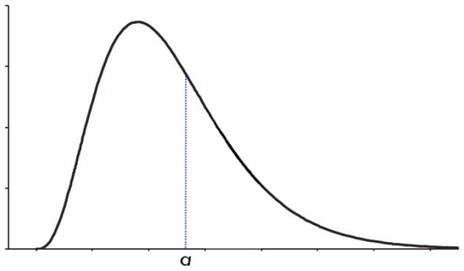


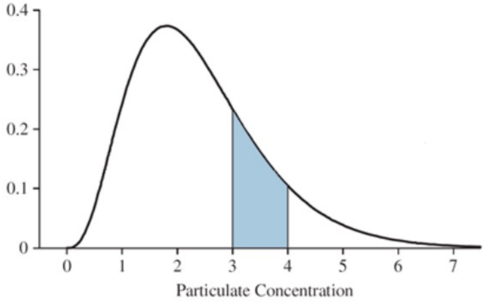
https://www.r-bloggers.com/2012/09/histogram-density-plot-combo-in-r/



- Typically, density curves are used to describe **populations** for <u>continuous random variables</u>

Density Curves Properties





Density Curves Properties

- Remember these are all for continuous distributions (populations):

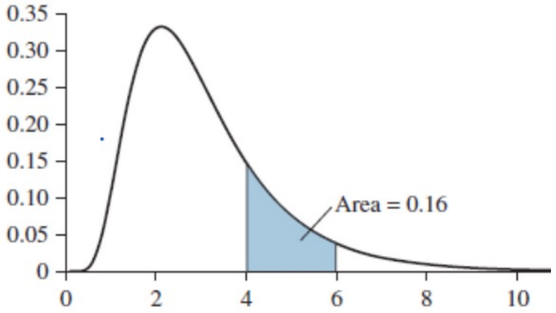
1) The probability of a particular value is 0 → *P(X = a) = 0, for any number a*
   - This is why we have to find probabilities for intervals!
2) The probability between two endpoints is found by calculating the area under the density curve
   - *P(a < X < b) = P(a ≤ X ≤ b), for any number a and b*
3) For any density curve, *the total area under the entire curve equals 1*
   - Analogous to discrete probability distributions and how the sum P(X) = 1, there is a total probability of 100%

X = # of siblings	***P(X)***
0	2/20 = 0.1
1	8/20 = 0.4
2	0.35
3	0.15
Total	**1.00**

Probability Interpretations

- The area under a density curve between any two values *a* and *b* has two interpretations:

1) It is the percent of the population whose values are between *a* and *b*
2) It is the probability that a randomly selected individual will have a value between *a* and *b*



Examples:

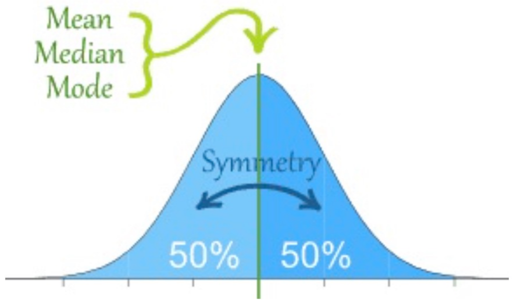
- What proportion of the population is between 4 and 6? P(4 < X < 6) = 0.16
- If a value is chosen at random from this population, what is the probability that it is NOT between 4 and 6?

  Complement!! 1 – 0.16 = 0.84

Normal Distributions

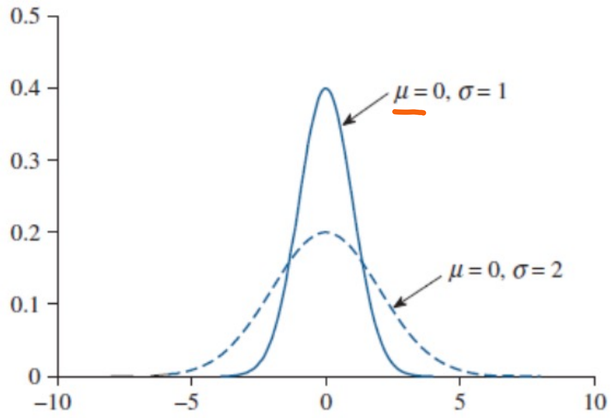
- Probability density curves come in many varieties, depending on the characteristics of the populations they represent.
- Many important statistical procedures can be carried out using only one type of density curve, called a **Normal Curve**
- A population that is represented by a normal curve is said to **Normally Distributed**, or to have (follow ~) a Normal Distribution

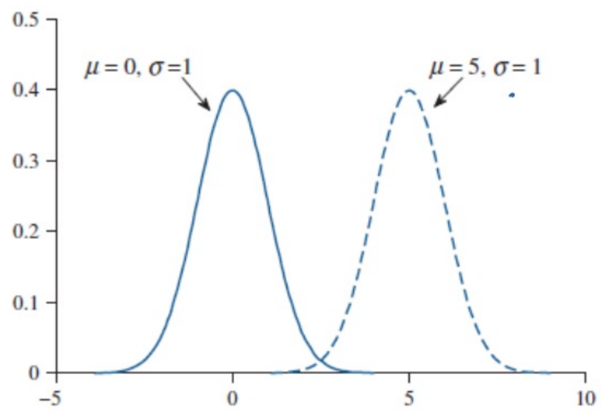
<u>Normal Distribution</u>



- The most special case of a Continuous Probability Distribution, with the following properties:
  - Describes a continuous random variable.
  - It's a symmetric, unimodal, bell-shaped distribution → which implies mean = median = mode
  - Completely described by its **mean** $\boldsymbol{\mu}$ (which determines the location of the peak) and **standard deviation σ** (which measures the spread of the population)

- The notation we will see often is:
  - X ~ Normal(μ , $\boldsymbol{\sigma}$)
  - Example) X ~ Normal(μ = 5 , $\boldsymbol{\sigma}$ = 1)

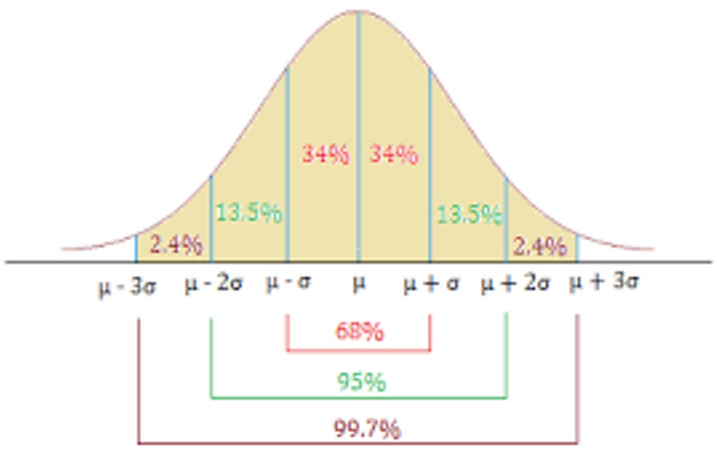




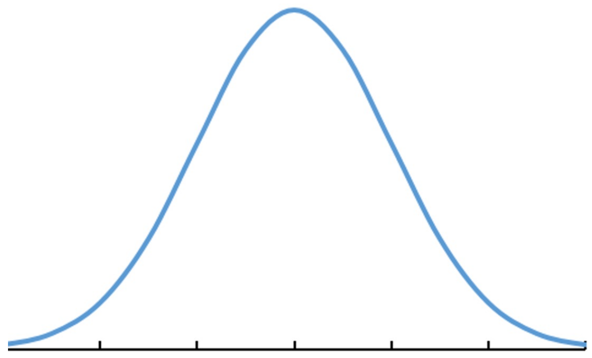
Finding Probabilities with the Normal Curve

<u>Empirical Rule</u>

- We know probabilities associated with intervals that are 1, 2, or 3 standard deviations away from the mean!
  - Could also find probabilities to the left or right of these endpoints.

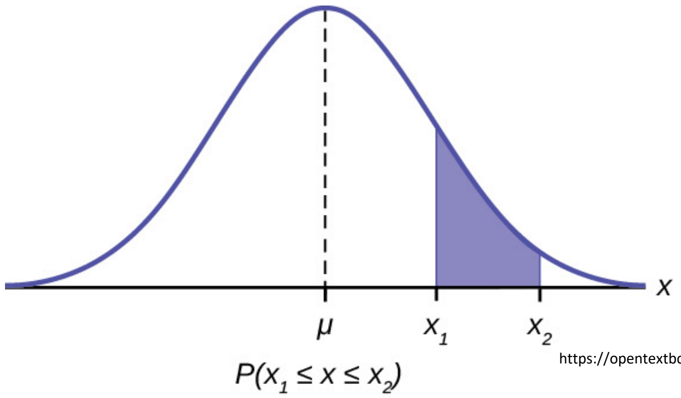


Empirical rule



- But what do we do if we want probabilities for points that don't lie exactly ± 1,2,3 **σ** from μ???

Normal Calc Fun Sess!: Finding Probabilities



https://opentextbc.ca/introbusinessstatopenstax/ch

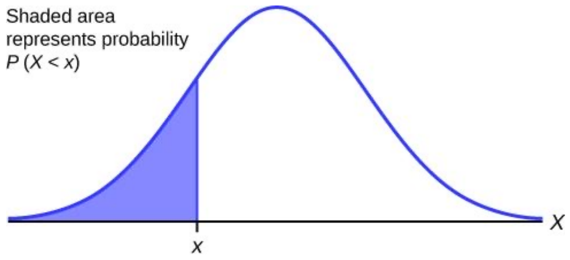
<u>Finding Probabilities</u>

- We are given a value for X (our R.V.), want a percent/probability
- Think about this as being going forward, X → P

<u>GOAL</u>: Use the Normal distribution to calculate probabilities!

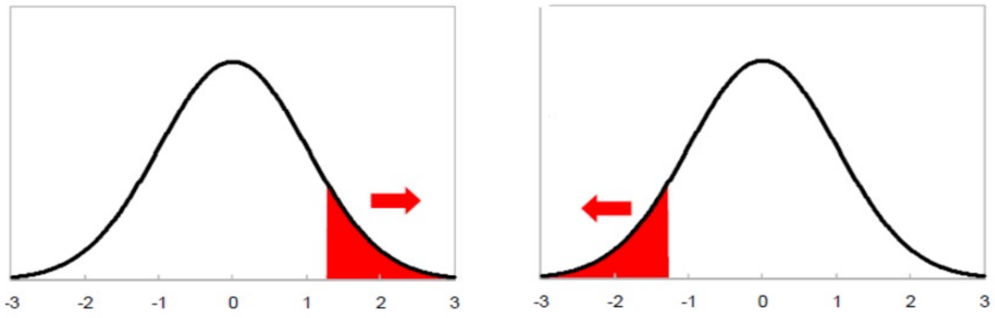
1. Choose correct Dist: 2ND → VARS
    - ALWAYS want **normalcdf()**
2. Enter in information (endpoints and parameters)
    - lower = lower boundary
    - upper = upper boundary
    - μ = Mean
    - **σ** = SD

- If you have TI-83, you would type normalcdf(lower, upper, mean, sd)



https://www.teksguide.org/resource/62-

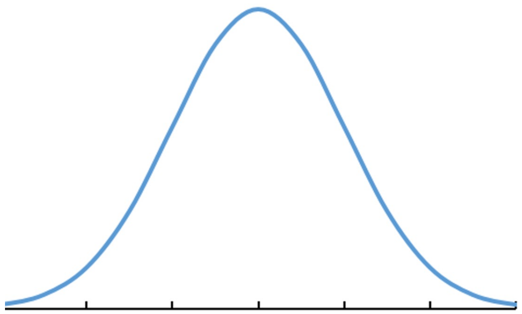
*** <u>Special Cases</u>
If finding a left tailed probability, enter lower = -10000 (some really big negative number)
If finding a right tailed probability, enter upper = 10000 or 10^6 (really big positive number)



https://www.researchgate.net/

Normal Calc Fun Sess!: Finding Probabilities

Setup: Lets say X ~ Normal(μ = 40, $\boldsymbol{\sigma}$ = 5), find the following probabilities.



STEP 1: Draw Normal Curve and Label!!!
STEP 2: Write in terms of a probability statement!!!
STEP 3: Shade the curve
STEP 4: Show work and write answer!

a) What is the probability X is less than 43?

b) What is the probability X is in between 26 and 39?

c) Find the area to the right of 43?

d) What is the probability X greater than or equal to 27?

e) What is the probability X greater than 31 but less than 52? What is the probability X is outside of this interval?

Normal Calc Fun Sess!: Finding Probabilities

Setup: Lets say X ~ Normal(μ = 40, **σ** = 5), find the following probabilities

STEP 1: Draw Normal Curve and Label!!!
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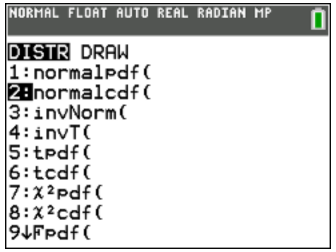


a) What is the probability X is less than 43?

Step 1: Draw and label the curve just like we did with empirical rule problems

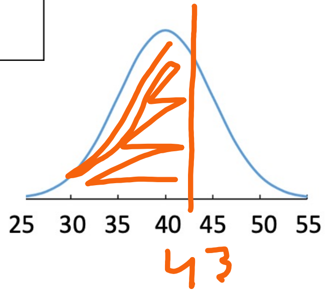
Step 2: Writing as a probability statement means translating the words to notation

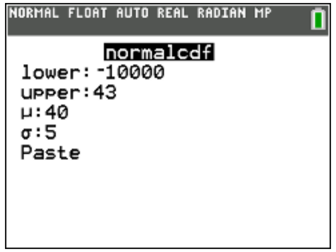
- *(This helps with drawing the picture and showing work for partial credit)*

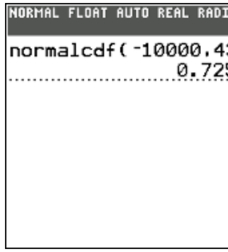
Step 3: Make sure this matches our probability statement (one way to self check your work)

Step 4: Show the calc function and the inputs (REQUIRED WORK)

P(X < 43) = normalcdf(lower = -10000, upper = 43, mean = 40, sd = 5) = 0.7257



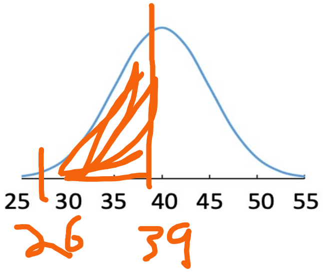


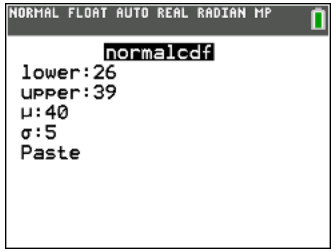


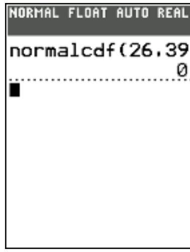
b) What is the probability X is in between 26 and 39?

Unlike in the discrete case, < > vs ≤ ≥ doesn't matter because of the density curve properties (s makes more sense to you)

P(26 < X < 39) = normalcdf(26,39,40,5) = 0.4181







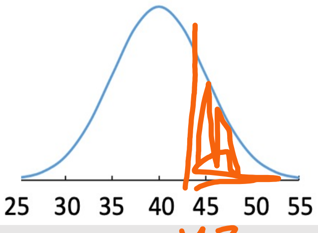
c) Find the area to the right of 43?

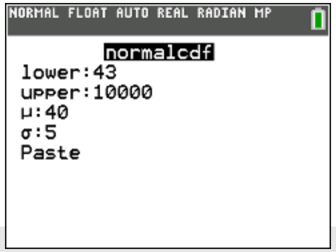
This is an upper tail probability, so there is no upper boundary; just need to put in a large positive number

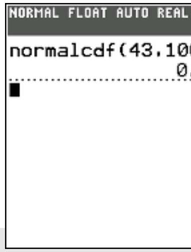
P(X ≥ 43) = normalcdf(lower = 43, upper = 100000, mean = 40, sd = 5) = 0.2743

Notice this area represents the complement of part (a) → So could have answered this without anc normalcdf()

P(X ≥ 43) = 1 - P(X ≤ 43) = 1 - 0.7257 = 0.2743







Normal Calc Fun Sess!: Finding Probabilities

STEP 1: Draw Normal Curve and Label!!!
STEP 2: Write in terms of a probability statement!!!
STEP 3: Shade the curve
STEP 4: Show work and write answer!

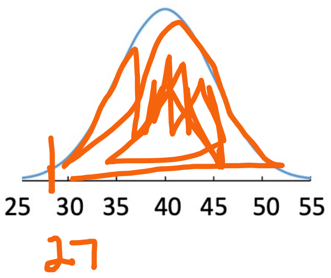
Setup: Lets say X ~ Normal(μ = 40, **σ** = 5), find the following probabilities.

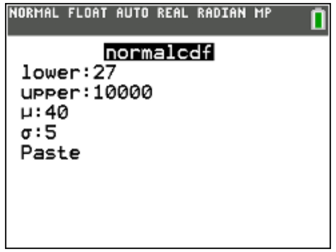
d) What is the probability X greater than or equal to 27?

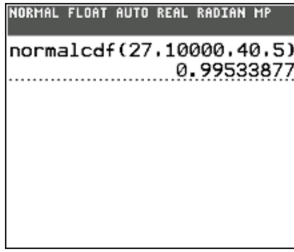
P(X ≥ 27) = normalcdf(lower = 27, upper = 100000, mean = 40, sd = 5) = 0.9953

It would NOT be correct to use upper = 55

- *This technically cuts our region off at 55, but there should be no upper bound*
- *It won't change the answer probably, but want to be correct*







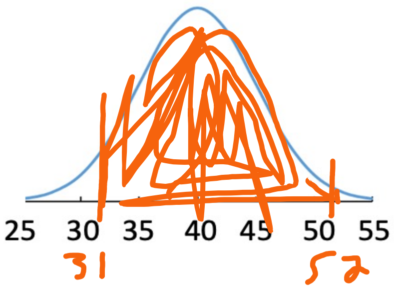
e) What is the probability X greater than 31 but less than 52?

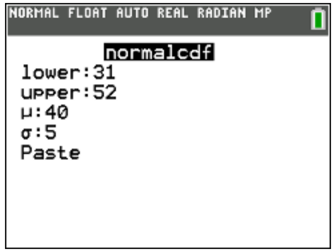
Translating the words directly may lead to P(X > 31 < 52)

- *The idea is correct, but we want to rearrange so that X is in the middle*

P(31 ≤ X ≤ 52) = normalcdf(lower = 31 upper = 52, mean = 40, sd = 5) = 0.9558

- *This will always be the format of P( ) when there is both a lower and upper bound*



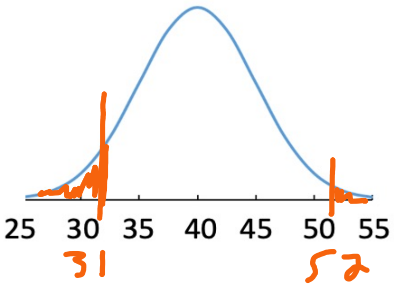


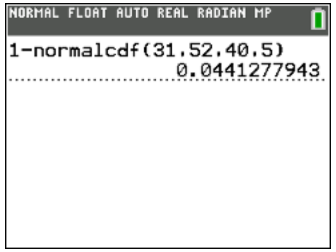
What is the probability X is outside of this interval?

Just use the complement rule whenever finding probabilities OUTSIDE a certain interval!

- *First find the probability for in between the interval (because this is what our calculator can do*
    - *Then do the subtraction from 1 to get the outside region*
- *If you don't do it this way, you would need to do two separate normalcdf() calculations and add the results*

P(X ≤ 31 OR X ≥ 52) = 1 - normalcdf(lower = 31 upper = 52, mean = 40, sd = 5) = 0.0441





Is this event unusual?

Yes because the probability is less than 0.05

Z-Scores and Standard Normal Curve - Review

<u>Z-Score - Conceptual</u>

We can standardize ANY distribution, i.e. turn it into z-scores.

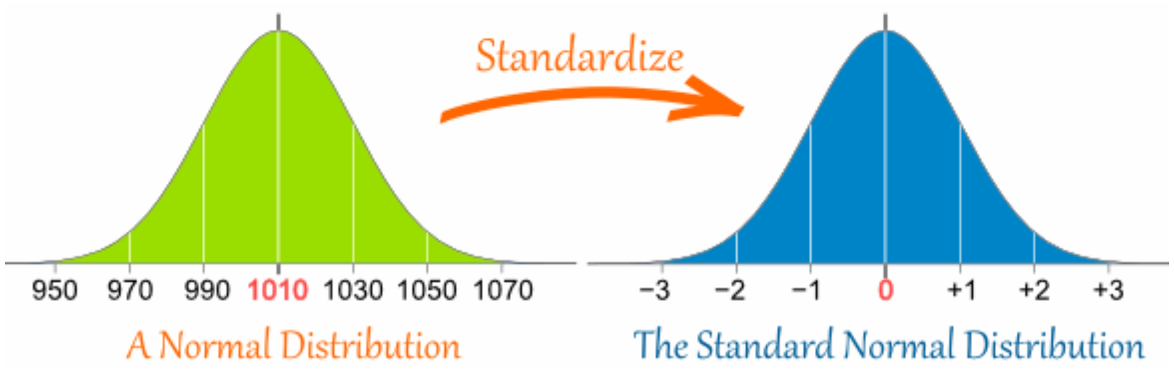
** After standardizing (i.e. converting to Z), our <u>new</u> distribution has mean $\mu_{new} = 0$

if X = μ → $z = \frac{x-\mu}{\sigma} = \frac{\mu - \mu}{\sigma} = 0$

and SD $\sigma_{new} = 1$

(This is why the new Z-scores scale has "steps" equal to 1, which represent 1 SD, 1 Z-score)

But when starting with a Normal distribution, we get a result that is very common!



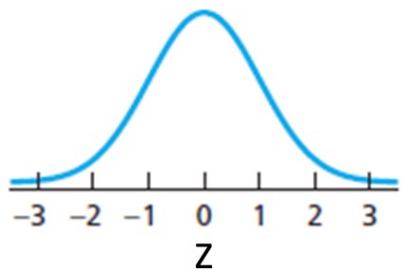
https://www.mathsisfun.com/data/standard-normal-distribution.html

<u>Standard Normal Curve</u>

- Just a specific Normal distribution that has mean mean μ = 0 and standard deviation **σ** = 1

- If a random variable follows a Standard Normal distribution then,

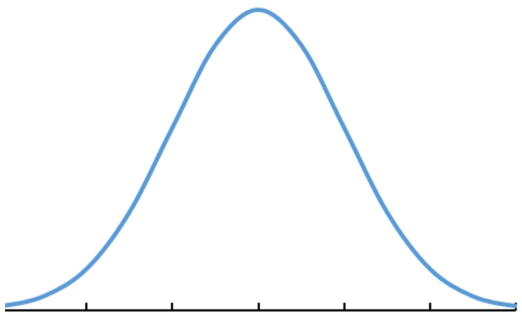
  Z ~ Normal(μ = 0, **σ** = 1)



We can find probabilities for this too!!

Normal Calc Fun Sess!: Finding Probabilities

Same setup: Lets say X ~ Normal(μ = 40, $\boldsymbol{\sigma}$ = 5), find the following probabilities.



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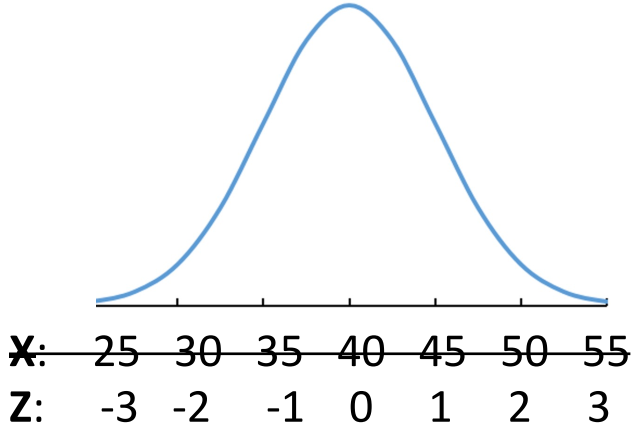
a) What is the probability a standardized score is less than 0.6?

b) What is the probability Z is in between -2.3 and 1.5?

c) What is the probability Z less than zero?

Normal Calc Fun Sess!: Finding Probabilities



Setup: Lets say X ~ Normal(μ = 40, **σ** = 5), find the following probabilities

STEP 1: Draw Normal Curve and Label!!!
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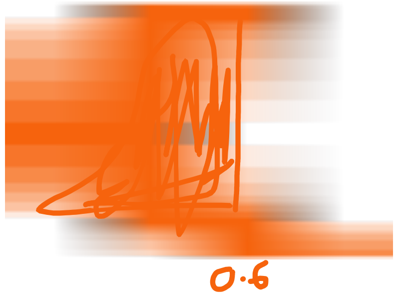
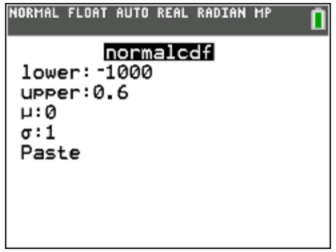
a) What is the probability a standardized score is less than 0.6?

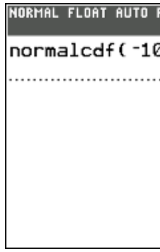
P(~~X~~ Z < 0.6) → *Using the standard normal curve!!!!! NEW $\mu = 0$ and $\sigma = 1$*

- *Really easy to switch to our new distribution, just need to use the new mean and sd*

P(Z < 0.6) = normalcdf(lower = -1000, upper = 0.6, mean = 0, sd = 1) = 0.7257

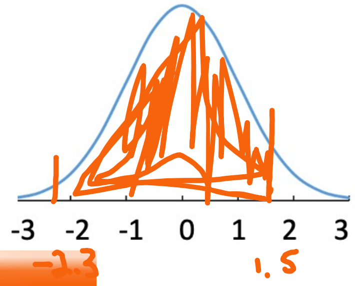


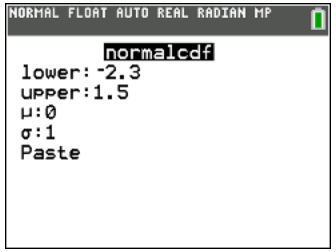


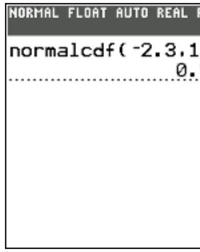


b) What is the probability Z is in between -2.3 and 1.5?

P(-2.3 ≤ Z ≤ 1.5) = normalcdf(lower = -2.3, upper = 1.5, mean = 0, sd = 1) = 0.9224







c) What is the probability Z less than zero?

The mean of the Z curve is zero! Half the data is less than the mean

P(Z ≤ 0) = 0.5 → *no normalcdf() needed*

Finding Percentiles with Normal Distribution

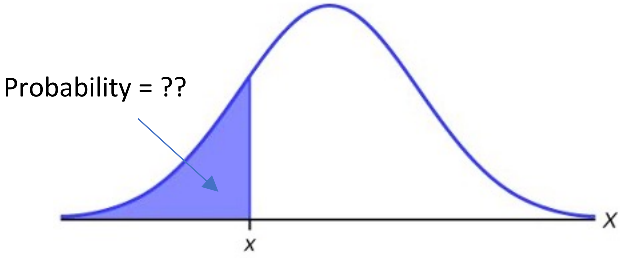
Finding Percentiles

- We are given a probability or percent (0.95 or 95%), want a value for X or a range of X's
- Think of this as working backwards X ← P

Probabilities vs Percentiles

Calculating a **Probability** when…

- We ask for the probability our random variable is below, above, between a value or values.
- Ex. What is the probability of weighing between 10 lbs and 14 lbs?
  - $P(10 \leq X \leq 14) = ??$

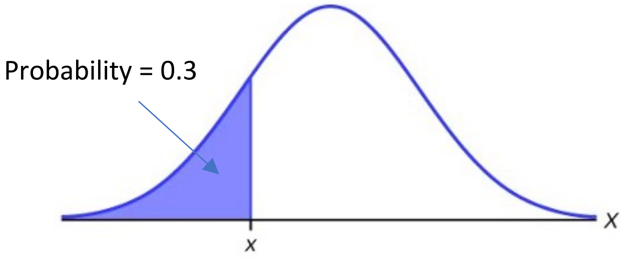


$X = 2, P(X \leq 2) = ??$

Think about this as being going forward, X → P

Calculating a **Percentile** when…

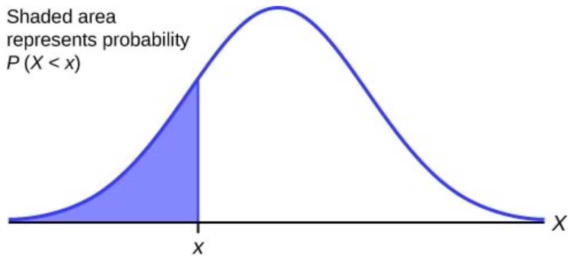
- We ask for the value of the Random Variable that tells us that a certain percentage lies above or below.
- Sometimes can ask for the values of the R.V. that gives us a symmetric interval around the mean that gives us a certain percentage.
- Ex. At what GPA is the 90th percentile?
- Ex. What exam score is needed to be in the top 10%?



$P(X \leq ??) = 0.3, x = ??$

Think about this as being going backwards, P → X

Normal Calc Fun Sess!: Finding Percentiles



https://www.teksguide.org/resource/62-using-normal-distribution

<u>GOAL</u>: Use the Normal distribution to calculate percentiles!

1. Choose invNorm(): 2ND → VARS
2. Enter in information (area and parameters)
   - area = Probability (LEFT, percentile!)
   - μ = Mean
   - **σ** = SD

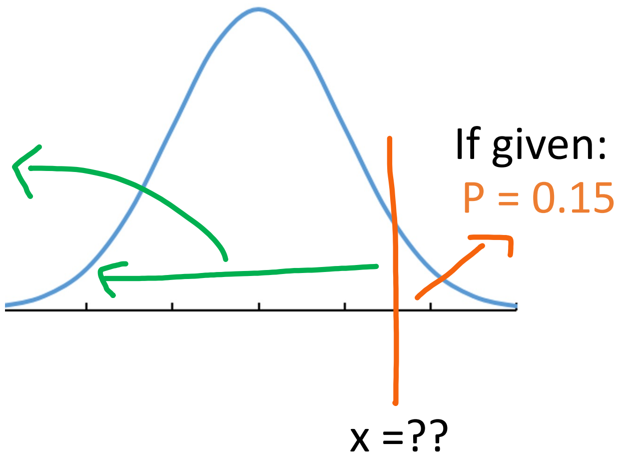
We are entering the probability that corresponds to the left of X (like a Percentile)!

If you have TI-83, you would type invNorm(area,mean,sd)

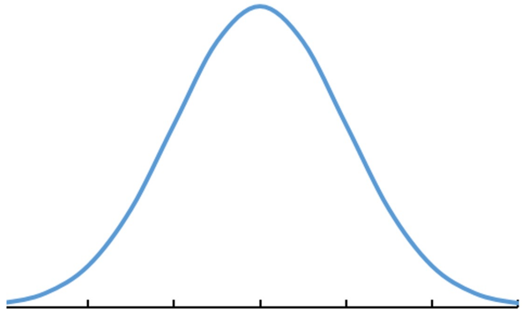
*** If you get a right tail (upper probability), you need to rewrite as probability to the left!

In calc:

P = 1-0.15= 0.85



Normal Calc Fun Sess!: Finding Percentiles



Setup: Lets say X ~ Normal(μ = 10, $\boldsymbol{\sigma}$ = 1.5), find the following values:

STEP 1: Draw Normal Curve and Label!!!
STEP 2: Write in terms of a probability statement!!!
STEP 3: Shade the curve
STEP 4: Show work and write answer!

a) Find the X value that has an area of 0.3 to the left

b) Find the X value that has an area of 0.15 to the right

c) Find the X value with 0.65 probability above

- *P(X   ) need to figure out what else goes in here*
- *Looking for area to left, this means <*
- *But less than what #?? That's what we are trying to figure out, so I put '?*
- *We know that probability statement must equal 0.3, so = 0.3*
- *Then use invNorm() to figure out the ??, which is an x value*

Normal Calc Fun Sess!: Finding Percentiles

Setup: Lets say X ~ Normal(μ = 10, **σ** = 1.5), find the following values:

STEP 1: Draw Normal Curve and Label!!!
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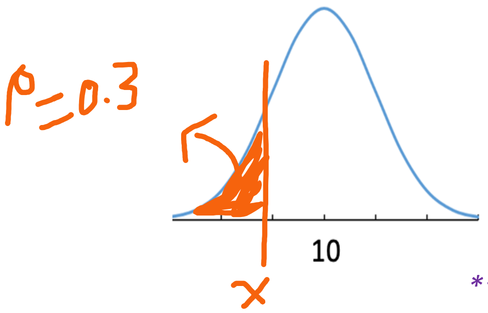
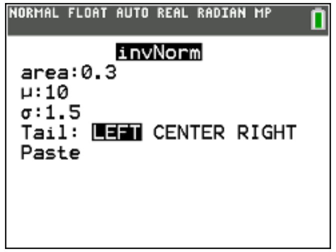
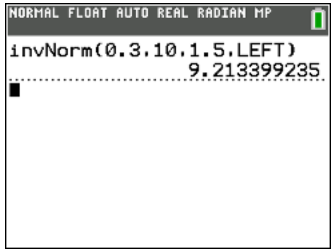






a) Find the X value that has an area of 0.3 to the left

Same steps as before when finding probabilities

P(X ≤ ??) = 0.3 → x = invNorm(area = 0.3, mean = 10, sd = 1.5) = 9.21

→ Can check this by plugging our answer the normalcdf to see if get close to the given probability (some roundoff error)!

*** Remember your calculator might not have that last argument (so just ignore it in these pictures)!*

1 – 0.15 = 0.85

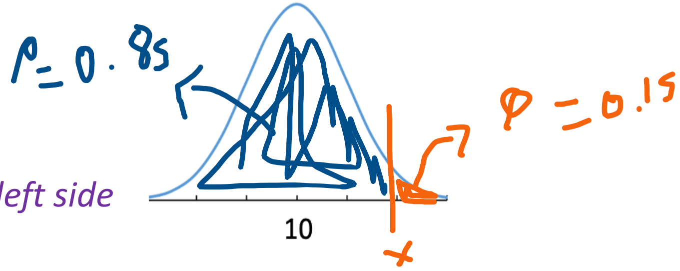
b) Find the X value that has an area of 0.15 to the right

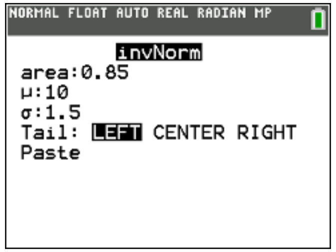
Now we want to the right, which means > in our P(   )

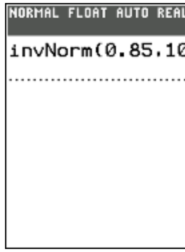
P(X > ??) = 0.15 → need to rewrite as a left-tail probability to enter in our calc!

P(X >??) = 0.15 ⟷ P(X < ??) = 0.85 → so switch direction of inequality and use the left side probability

Now solve for the X value → x = invNorm(area = 0.85, mean = 10, sd = 1.5) = 11.55







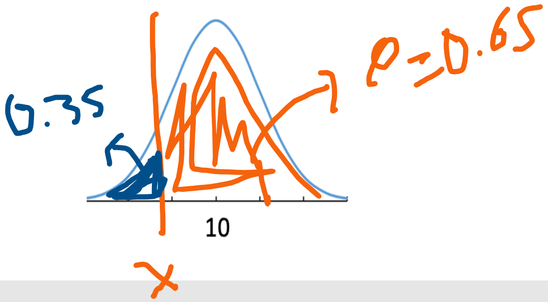
1 – 0.65 = 0.35

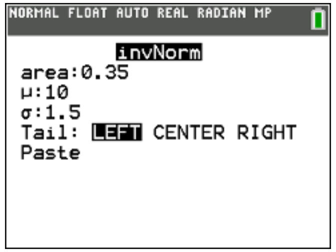
c) Find the X value with 0.65 probability above

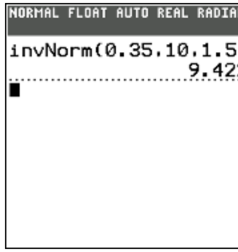
P(X ≥ ??) = 0.65 → need to rewrite as a left-tail probability!

P(X ≥ ??) = 0.65 ⟷ P(X ≤ ??) = 0.35

x = invNorm(area = 0.35, mean = 10, sd = 1.5) = 9.42



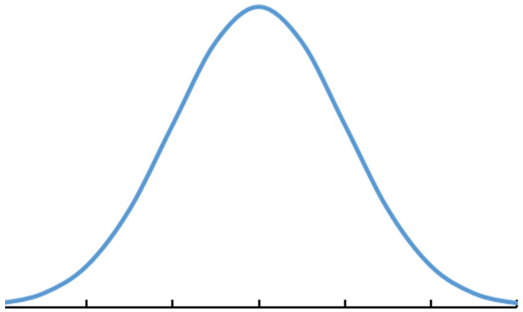




Normal Calc Fun Sess!: Finding Percentiles

STEP 1: Draw Normal Curve and Label!!!
STEP 2: Write in terms of a probability statement!!!
STEP 3: Shade the curve
STEP 4: Show work and write answer!

Setup: Lets say X ~ Normal(μ = 10, **σ** = 1.5), find the following values:



d) Find the X values for the middle 70% of data

e) Find the Z score that represents the 30th percentile

f) Find the Z values that bound the middle 15% of the area under the standard normal curve

Normal Calc Fun Sess!: Finding Percentiles

STEP 1: Draw Normal Curve and Label!!!
STEP 2: Write in terms of a probability statement!!!
STEP 3: Shade the curve
STEP 4: Show work and write answer!

Setup: Lets say X ~ Normal(μ = 10, $\boldsymbol{\sigma}$ = 1.5), find the following values:

d) Find the X values for the middle 70% of data

Need to setup a symmetric interval and find both endpoints!

→ So solve for two X values by writing both as a left-tail probabilities!

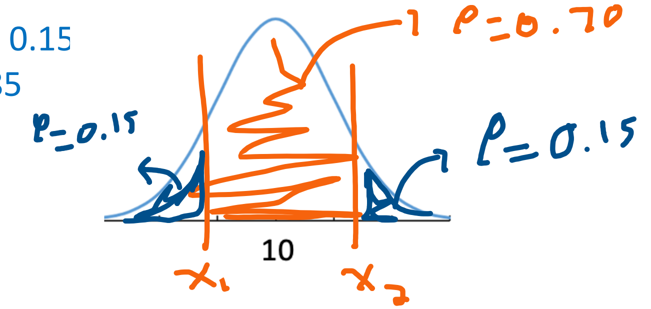
X1: P(X < ??) = 0.15

x1 = invNorm(area = 0.15, mean = 10, sd = 1.5) = 8.45

X2: P(X < ??) = 0.85

x2 = invNorm(area = 0.85, mean = 10, sd = 1.5) = 11.55

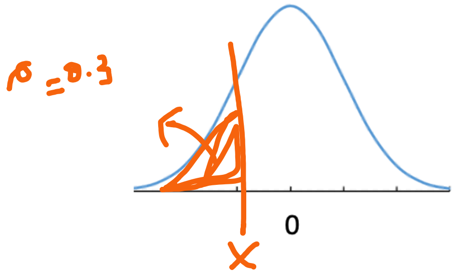
Prob outside: 1 – 0.7 = 0.3
Prob on left (for X1): 0.3 / 2 = 0.15
X2: 0.15 + 0.7 = 1 – 0.15 = 0.85

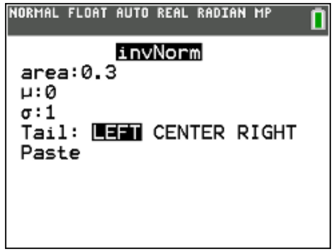


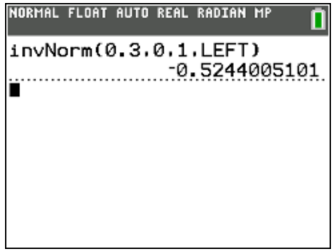
e) Find the Z score that represents the 30th percentile

Using the standard normal curve!!!!! NEW μ = 0 and σ = 1

P(Z ≤ ??) = 0.3) → invNorm(area = 0.3, mean = 0, sd = 1) = -0.52







f) Find the Z values that bound the middle 15% of the area under the standard normal curve

→ Again *need to setup a symmetric interval and find both endpoints! (i.e. write both as a left-tail probabilities!)*

X1: P(X < ??) = 0.425

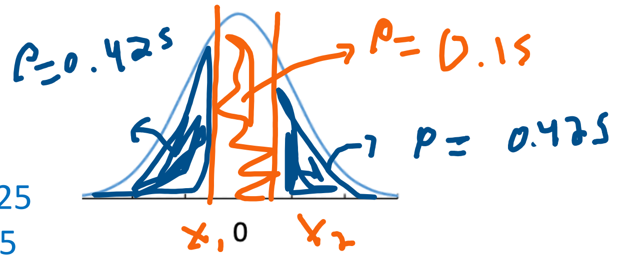
x1 = invNorm(area = 0.425, mean = 10, sd = 1.5) = -0.189

X2: P(X < ??) = 0.575

x2 = nvNorm(area = 0.85, mean = 10, sd = 1.5) = 0.189

→ Because the standard normal curve is centered at zero, symmetric interval endpoints will be the same just the positive and negative version!

Prob outside: 1 – 0.15 = 0.85
Prob on left (for X1): 0.85 / 2 = 0.425
X2: 0.425 + 0.15 = 1 – 0.425 = 0.575



PROBLEM SESSION!!!!!!!!!!!

Finding Probabilities and Percentiles Example

The 3 point shooting percentage (number made/number attempted) for NBA players are Normally Distributed with a mean of 0.33 and standard deviation of 0.05.

What is the probability a randomly selected player makes more than 25% of their 3 point shots?

Finding Probabilities and Percentiles Example

The 3 point shooting percentage (number made/number attempted) for NBA players are Normally Distributed with a mean of 0.33 and standard deviation of 0.05.

What is the probability a randomly selected player makes more than 25% of their 3 point shots?

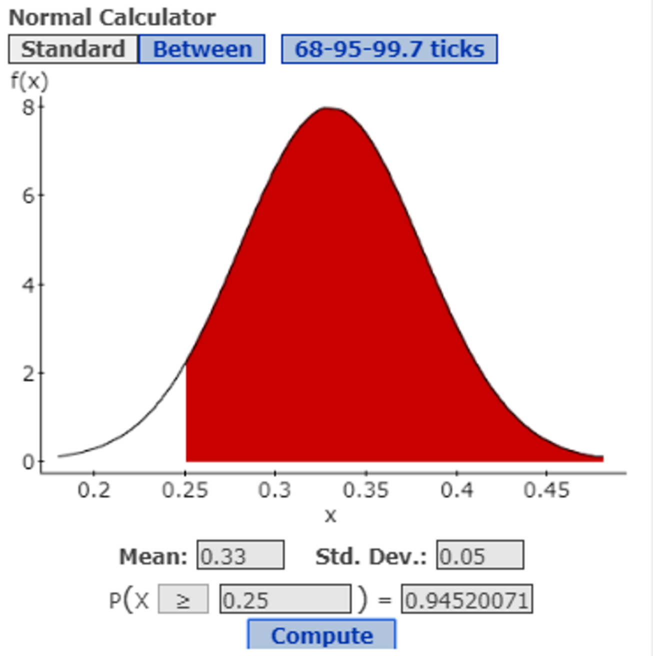
$P(X>=0.25) = ??$

Finding Probabilities and Percentiles Example

The 3 point shooting percentage (number made/number attempted) for NBA players are Normally Distributed with a mean of 0.33 and standard deviation of 0.05.

What is the probability a randomly selected player makes more than 25% of their 3 point shots?

P(X>=0.25) = 0.9452



Finding Probabilities and Percentiles Example

The 3 point shooting percentage (number made/number attempted) for NBA players are Normally Distributed with a mean of 0.33 and standard deviation of 0.05.

What 3 point percentage do the top 1% have?

Finding Probabilities and Percentiles Example

The 3 point shooting percentage (number made/number attempted) for NBA players are Normally Distributed with a mean of 0.33 and standard deviation of 0.05.

What 3 point percentage do the top 1% have?

P(X=>??) = 0.01

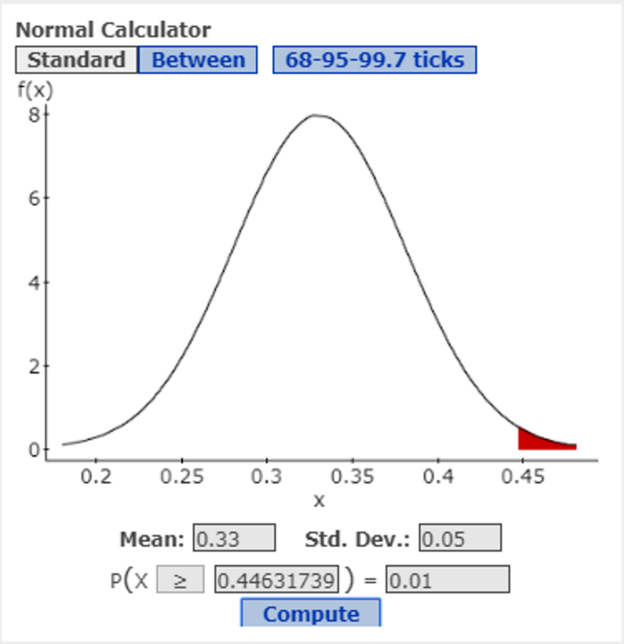
Finding Probabilities and Percentiles Example

The 3 point shooting percentage (number made/number attempted) for NBA players are Normally Distributed with a mean of 0.33 and standard deviation of 0.05.

What 3 point percentage do the top 1% have?

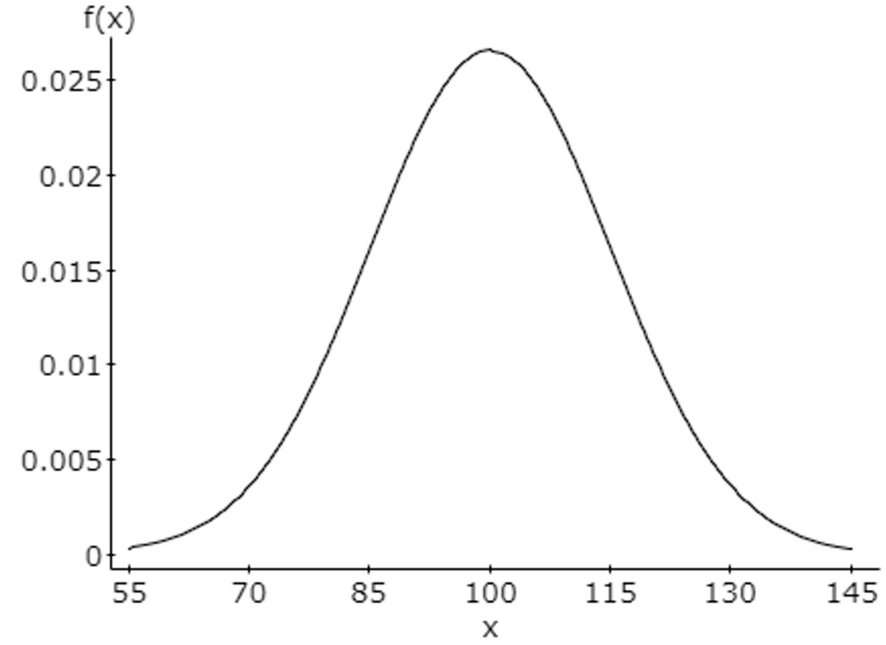
P(X=>0.446) = 0.01

The top 1% make 44.6% of their 3 point shots.



Problem 7

The intelligence quotient (IQ) is a measure of mental ability. IQs are normally distributed with a mean of 100 and a standard deviation of 15.



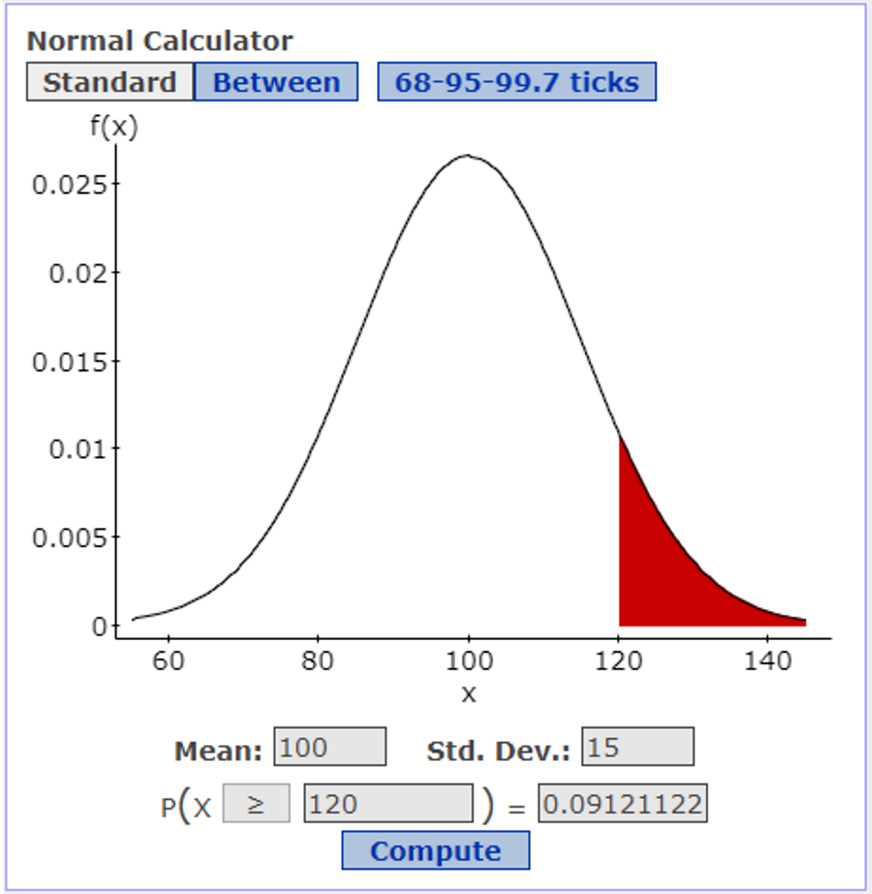
Problem 7a

What is the probability of randomly selecting an individual with an IQ greater than 120?

Problem 7a

What is the probability of randomly selecting an individual with an IQ greater than 120?

P(X>=120) = 0.0912



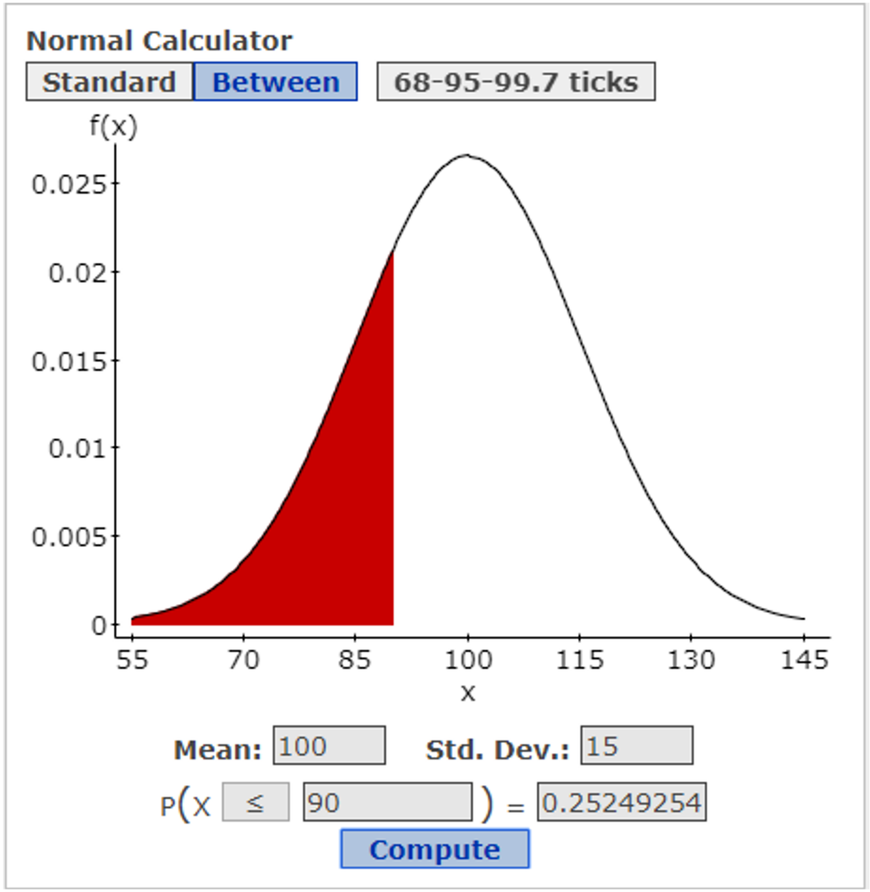
Problem 7b

What is the probability of randomly selecting an individual with an IQ less than 90?

Problem 7b

What is the probability of randomly selecting an individual with an IQ less than 90?

P(X<=90) = 0.2525



Problem 7c

Individuals with mild to moderate intellectual disability have IQs between 40 and 69. What is the probability of randomly selecting an individual with mild to moderate intellectual disability?

Problem 7c

Individuals with mild to moderate intellectual disability have IQs between 40 and 69. What is the probability of randomly selecting an individual with mild to moderate intellectual disability?

P(40<=X<=69) = 0.0194

Problem 7d

Find a symmetric interval about the mean IQ such that 50% of all IQs lie in this interval. What are the endpoints of this interval called?

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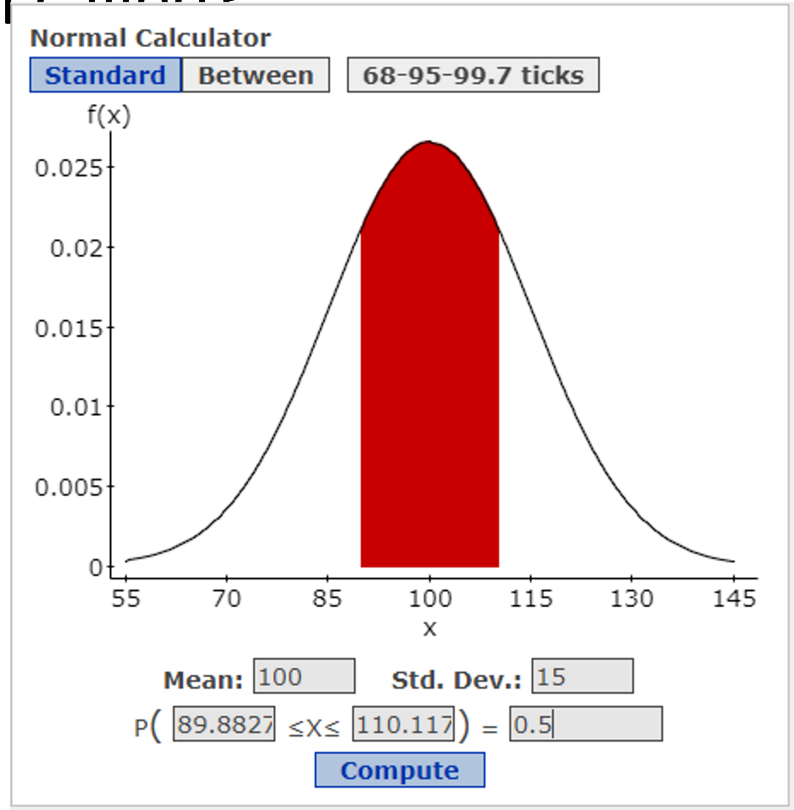
Find a symmetric interval about the mean IQ such that 50% of all IQs lie in this interval. What are the endpoints of this interval called?

The amount data between Q1 and Q3 is 50%.

$P(?<X<?) = 0.5$

Problem 7d

Find a symmetric interval about the mean IQ such that 50% of all IQs lie in this interval. What are the endpoints of this interval called?



The amount data between Q1 and Q3 is 50%.

P(89.88<=X<=110.12) = 0.5

Q1 = 89.88
Q3 = 110.12

Problem 7e

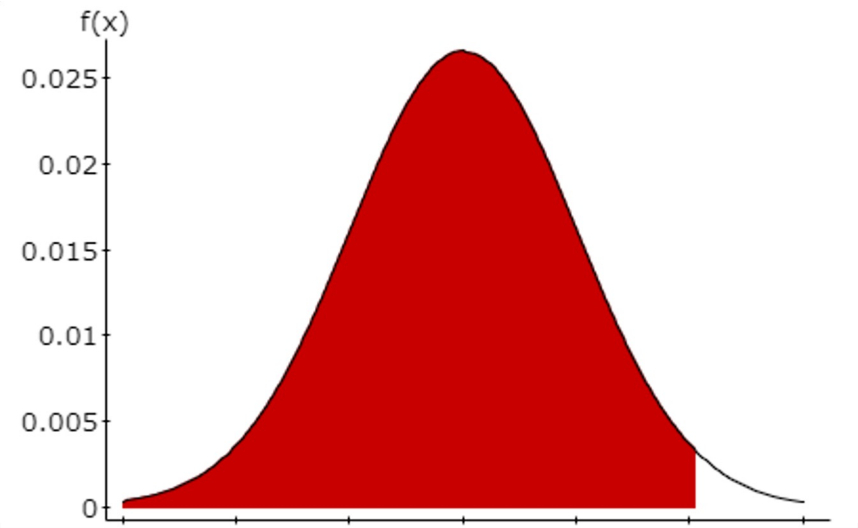
In order to qualify for membership in Mensa (a high IQ society) your IQ must be at (or above) the 98th percentile. What must your IQ be in order to qualify for membership in Mensa? Explain.

Problem 7e

In order to qualify for membership in Mensa (a high IQ society) your IQ must be at (or above) the 98th percentile. What must your IQ be in order to qualify for membership in Mensa? Explain.

To be in the 98th percentile, you need to have an IQ higher than 98% of everyone else. What does that look like?

P(X<=???) = 0.98



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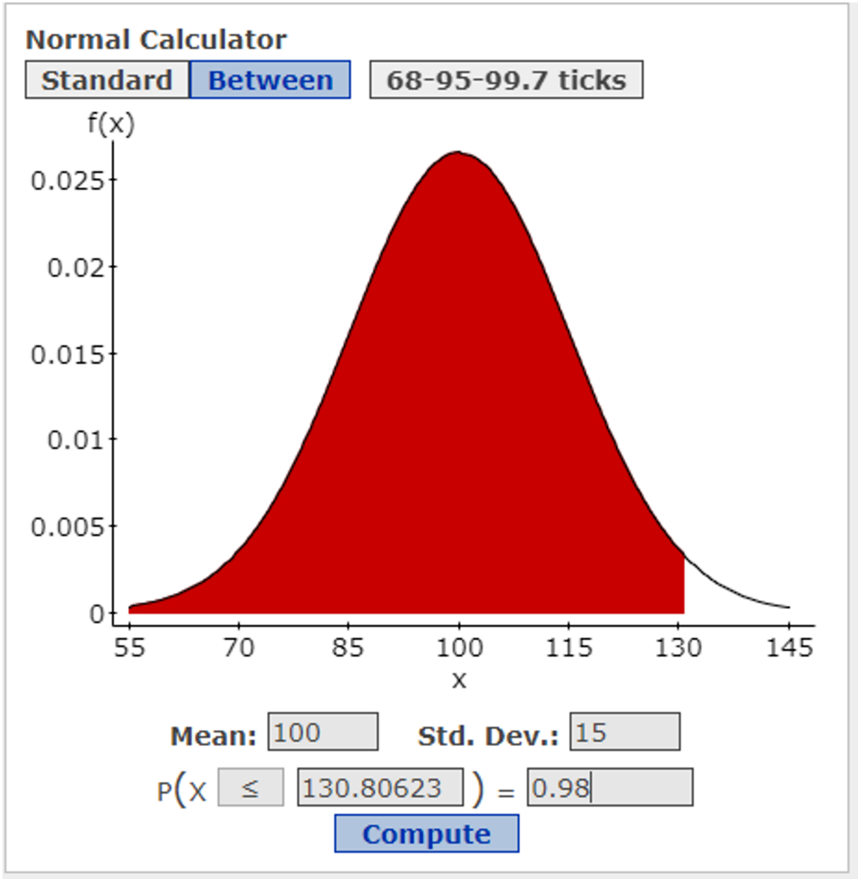


P(X<=130.81) = 0.98

You would need an IQ score of higher than 130.81.

Problem 8

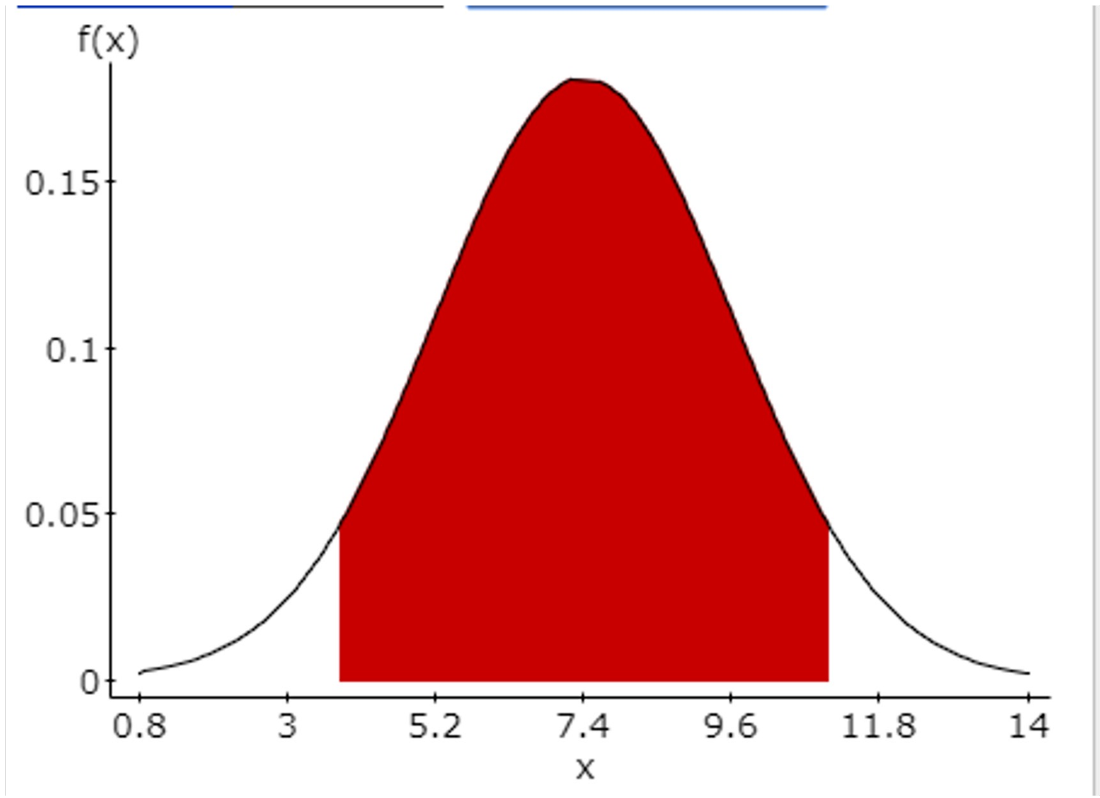
A common complaint of students is back pain due to lugging around laptops and all of the accessories. The weights of laptops are normally distributed with mean 7.4 pounds and standard deviation 2.2 pounds. Find a symmetric interval about the mean weight such that 90% of all laptop weights lie in this interval.

Problem 8

A common complaint of students is back pain due to lugging around laptops and all of the accessories. The weights of laptops are normally distributed with mean 7.4 pounds and standard deviation 2.2 pounds. Find a symmetric interval about the mean weight such that 90% of all laptop weights lie in this interval.

We will start with a picture.

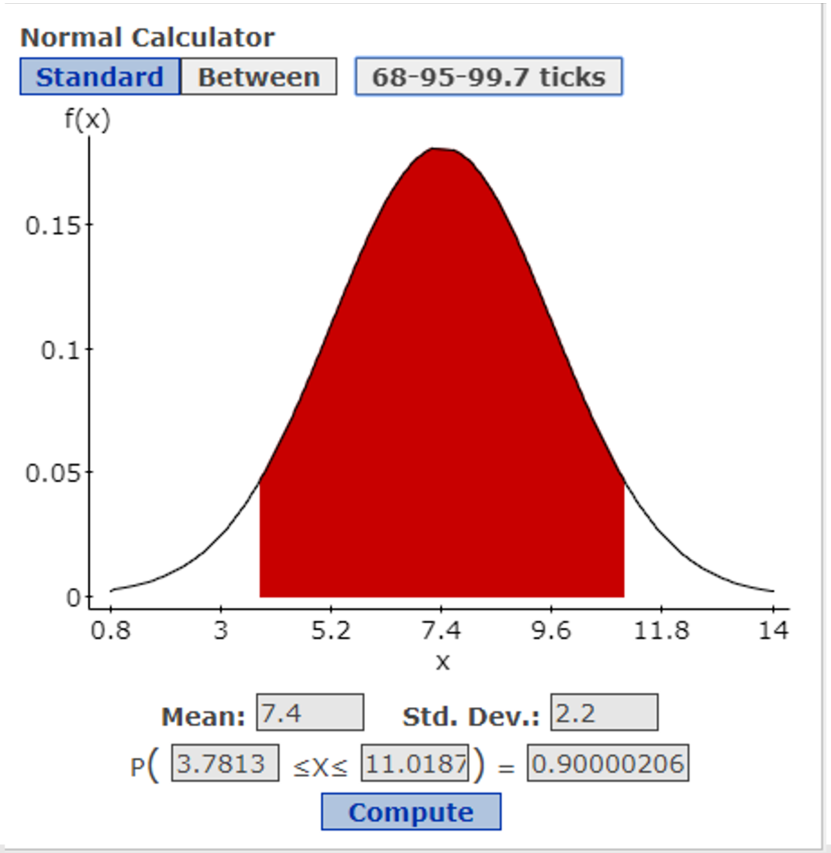
P(??<=X<=??) = 0.9



Problem 8

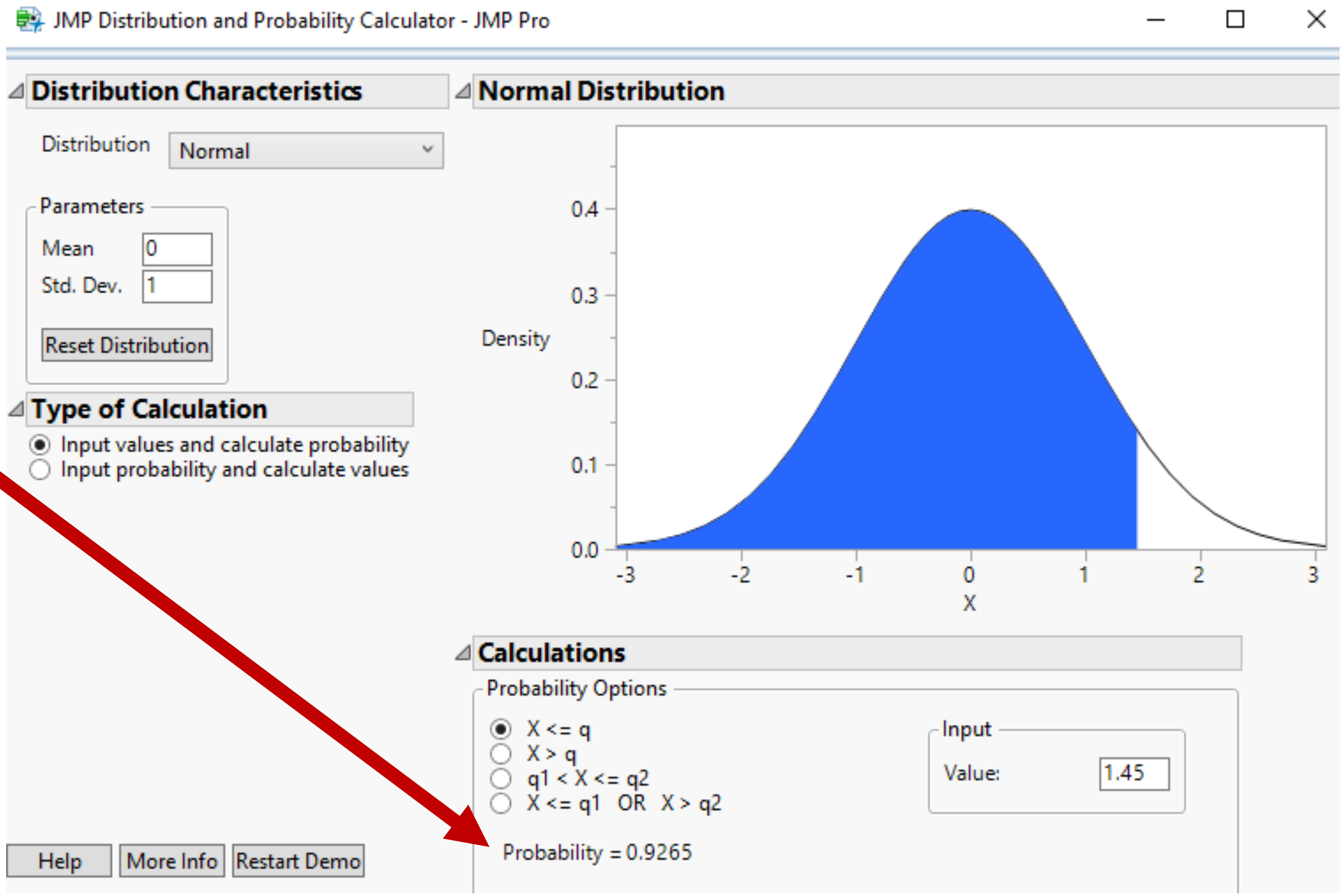
A common complaint of students is back pain due to lugging around laptops and all of the accessories. The weights of laptops are normally distributed with mean 7.4 pounds and standard deviation 2.2 pounds. Find a symmetric interval about the mean weight such that 90% of all laptop weights lie in this interval.

P(3.78<=X<=11.02) = 0.9



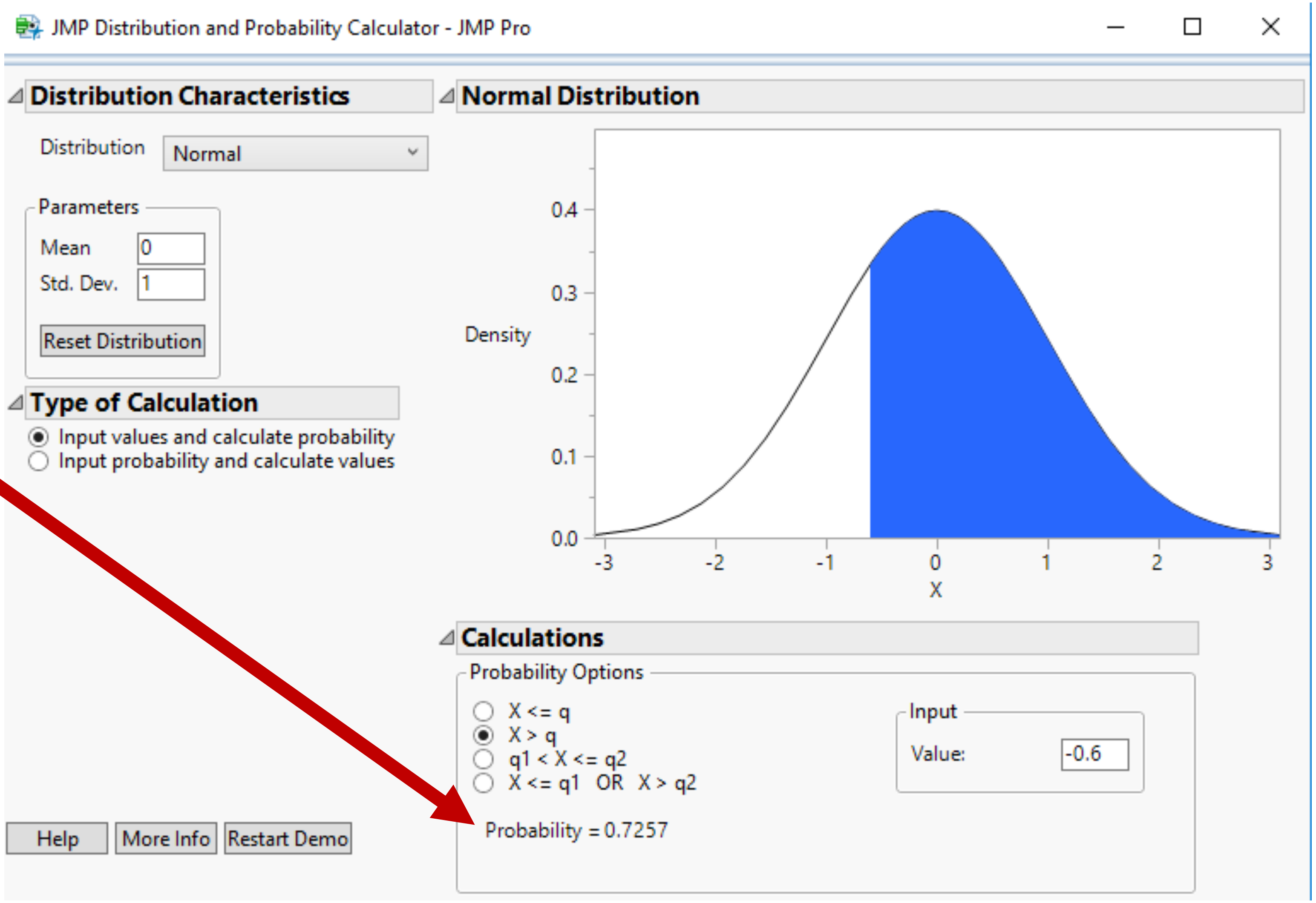
Standard Normal Curve Solution (a)

- P(z ≤ 1.45) = **0.9265**



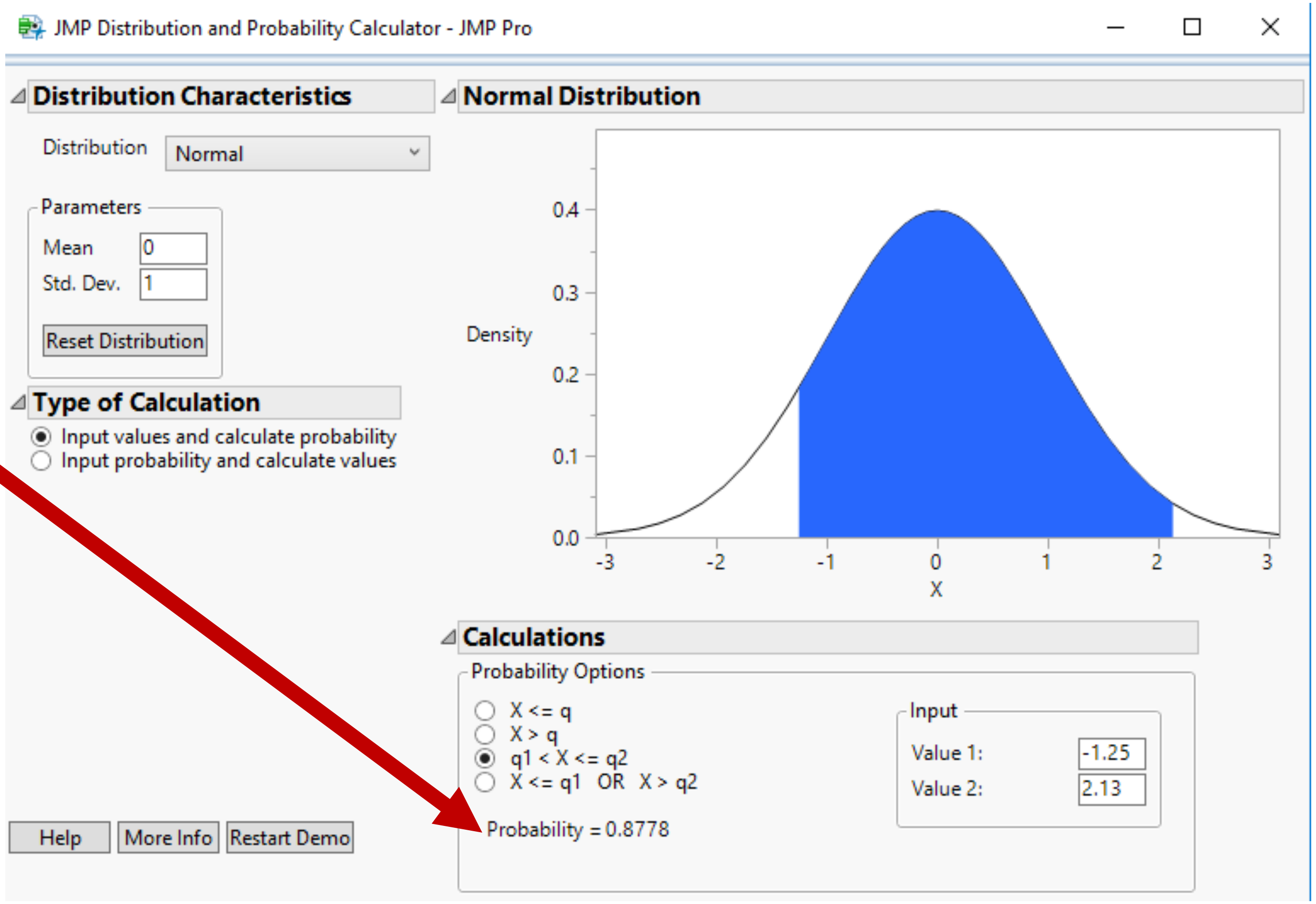
Standard Normal Curve Solution (b)

- P(z $\geq$ -0.6) = **0.7257**



Standard Normal Curve Solution (c)

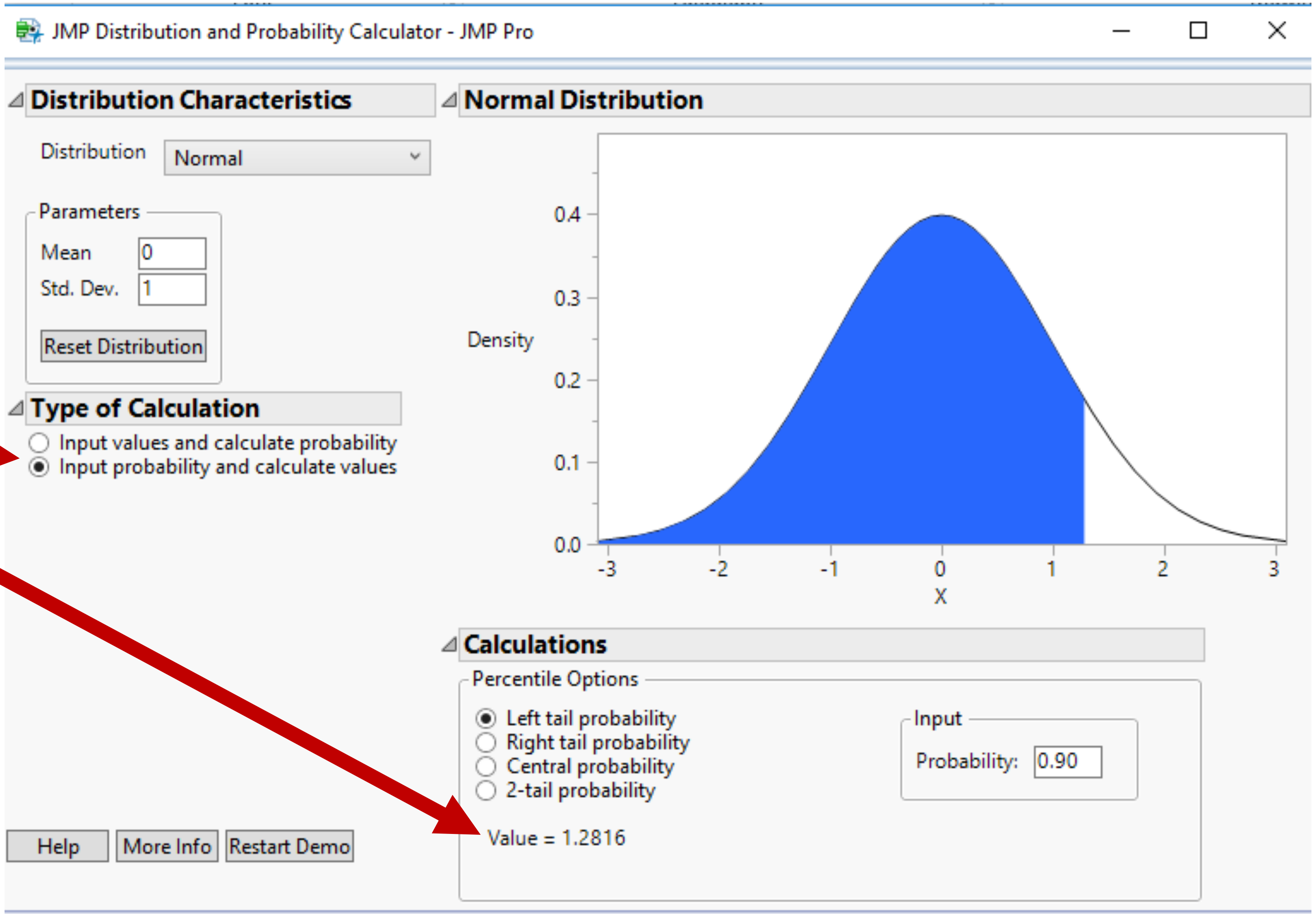
- $P(-1.25 \leq z \leq 2.13)$ = **0.8778**



Standard Normal Curve Solution (d)

- Find the value of the 90th percentile
- The 90th percentile means find *b* such that

  $P(z \leq b) = 0.90$
- The value of the 90th percentile is **1.2816**



Problem #9

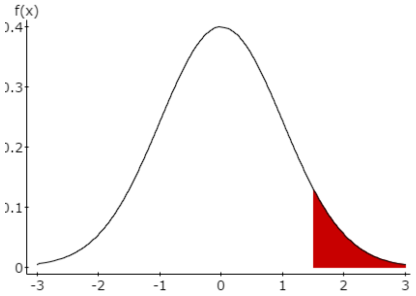
What percent of a standard Normal model is found in each region? Be sure to draw a picture first.

a) z => 1.5
b) z <= 2.25
c) -1 <= z <= 1.15
d) |z| => 0.5

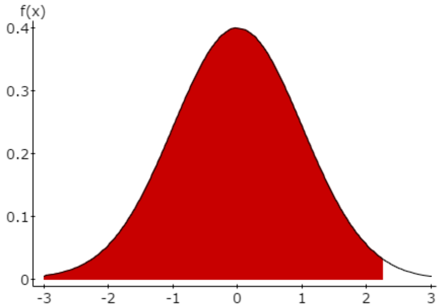
Problem #9 Solution

- Remember the standard normal curve has a mean = 0 and a SD = 1

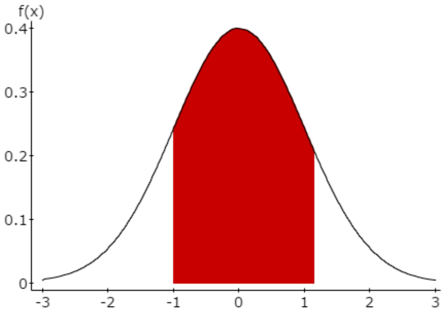
a) P(z >= 1.5) = 0.0668



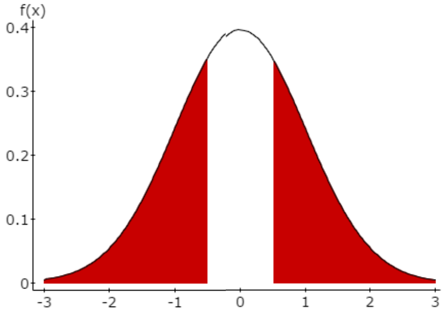
b) P(z <= 2.25) = 0.9878



c) P(-1 <= z <= 1.15) = 0.7163



d) P(|z| => 0.5) = 0.6171



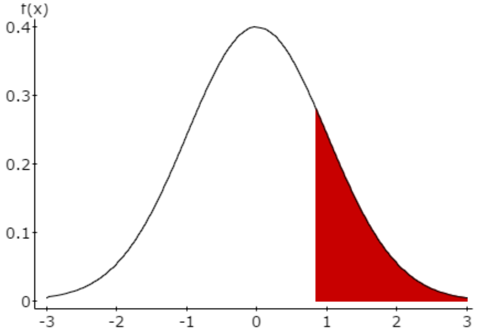
Problem #11

In a standard Normal model, what value(s) of a *z* cut(s) off the region described? Don't forget to draw a picture.

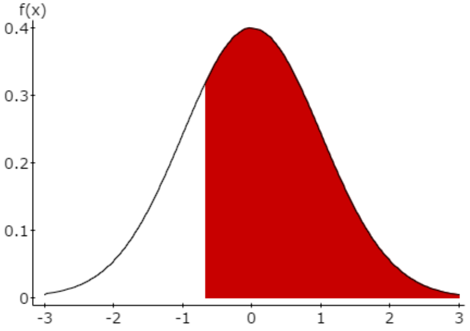
a) the highest 20%
b) the highest 75%
c) the lowest 3%
d) the middle 90%

Problem #11 Solution

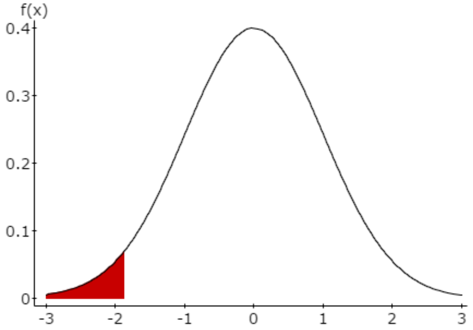
a) $z = 0.8416$



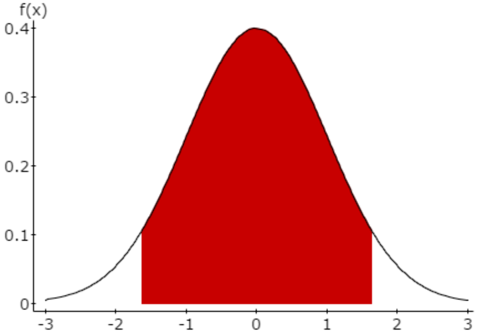
b) $z = -0.6745$



c) $z = -1.8808$



d) $P(-1.645 <= z <= 1.645) = 0.90$



Problem #5

Your company’s Human Resources department administers a test of “Executive Aptitude.” Your company will admit to the executive training program only people who score in the top 3% on the executive aptitude test. They report test grades as *z*-scores, and you got a score of 2.20.

a) With your score of 2.20, did you make the cut?
b) What do you need to assume about test scores to find your answer in part (a)?

Problem #5 Solution

a) **P(z >= 2.20) = 0.0139** which means you scored in the top 1.39%. So, yes, you made the cut.

b) You have to assume that the test scores are normally distributed.

Normal Variables

Find the following probabilities (and percentiles) by using the Normal calculator on JMP. Sketch and label a picture for each problem.

$a)\ \mu = 8, \sigma = 3, x = 17, P(x \geq 17)$

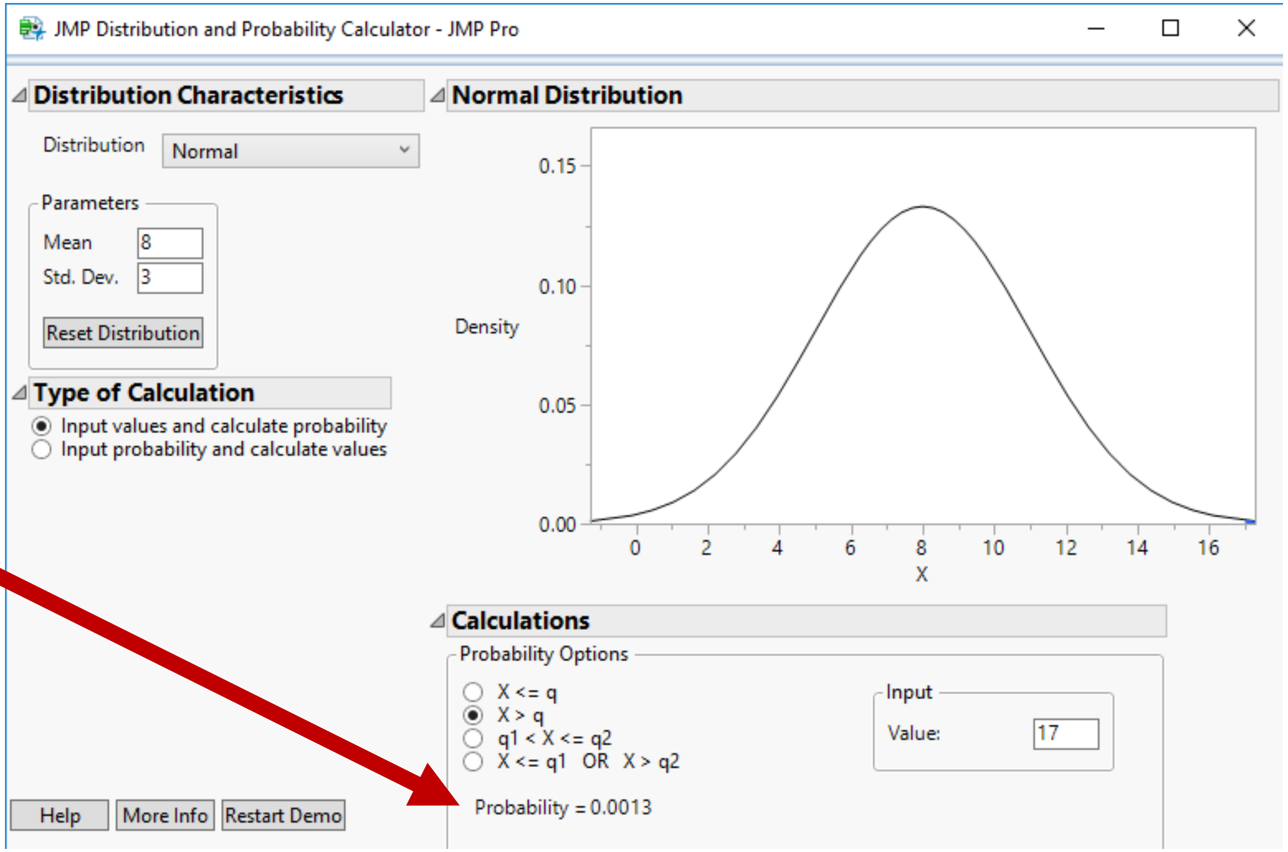
$b)\ \mu = 100, \sigma = 16, x = 80, P(x \leq 80)$

$c)\ \mu = 100, \sigma = 16, x = 80, P(80 \leq x \leq 105)$

$d)\ \mu = 100, \sigma = 16$, find the value of the 99^{th} percentile

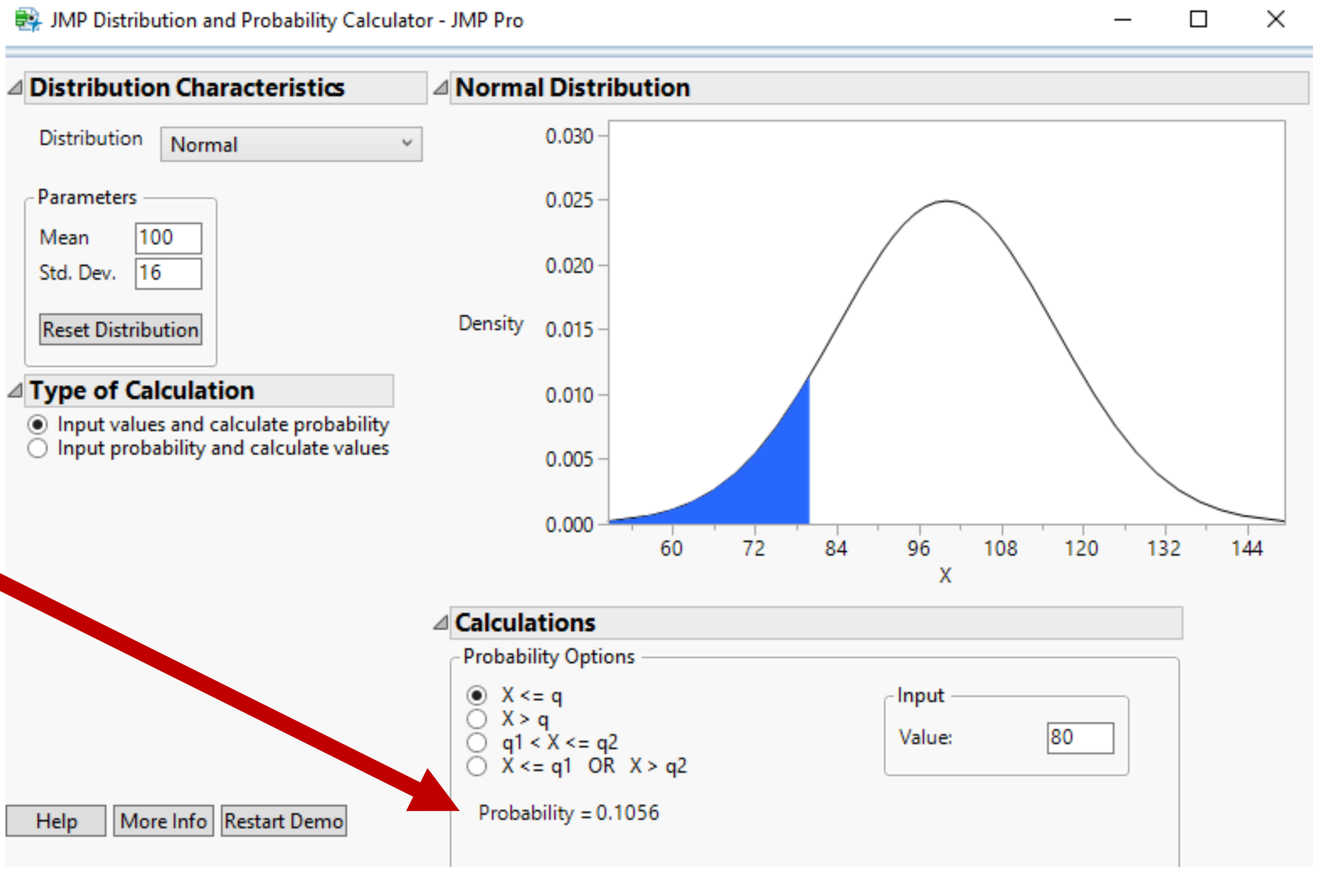
Normal Variables (a) Solution

- $\mu = 8, \sigma = 3, x = 17, P(x \geq 17)$
- $\boldsymbol{P(x \geq 17) = 0.0013}$



Normal Variables (b) Solution

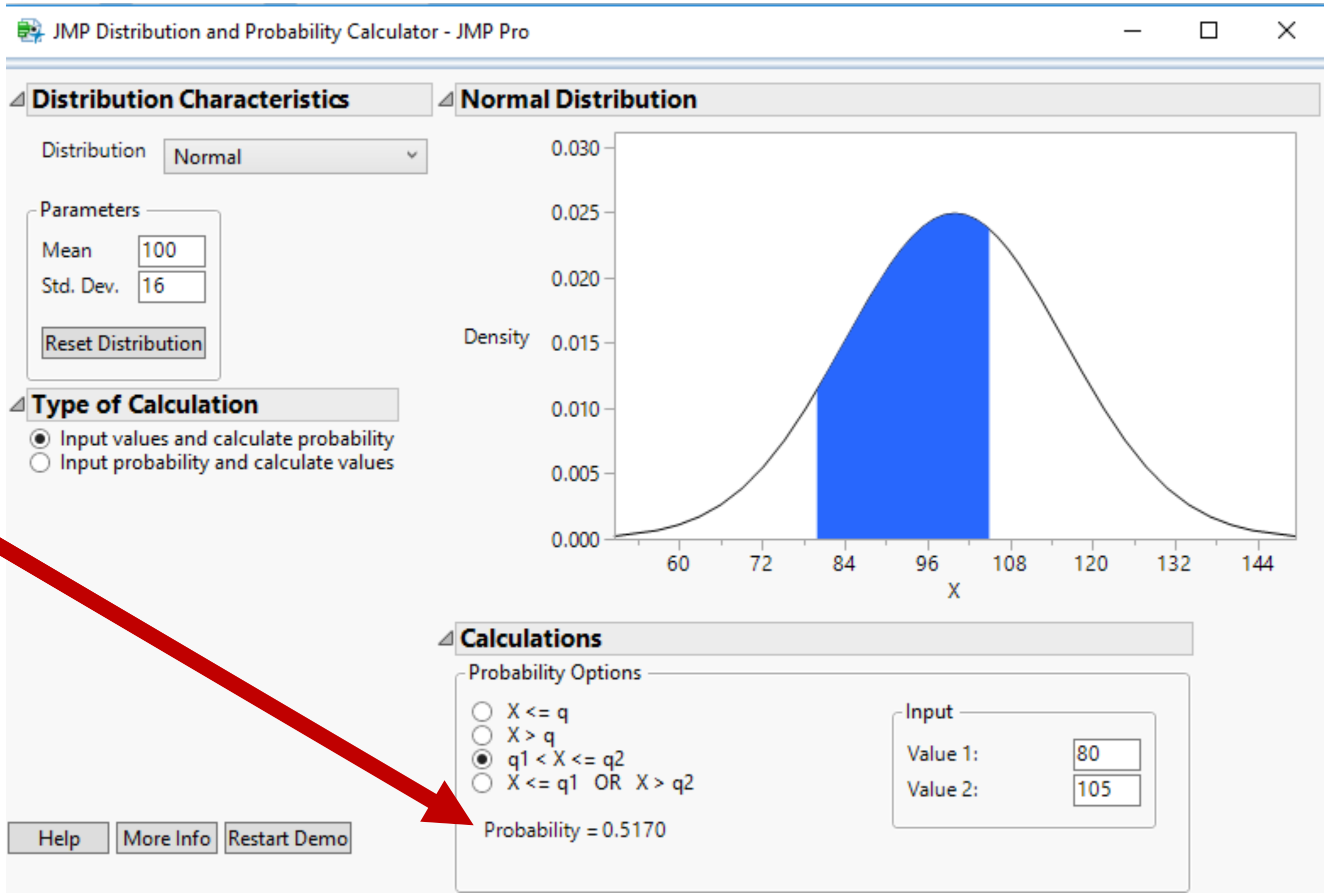
- $\mu = 100, \sigma = 16, x = 80, P(x \leq 80)$
- $\boldsymbol{P(x \leq 80)}$ = **0.1056**



Normal Variables (c) Solution

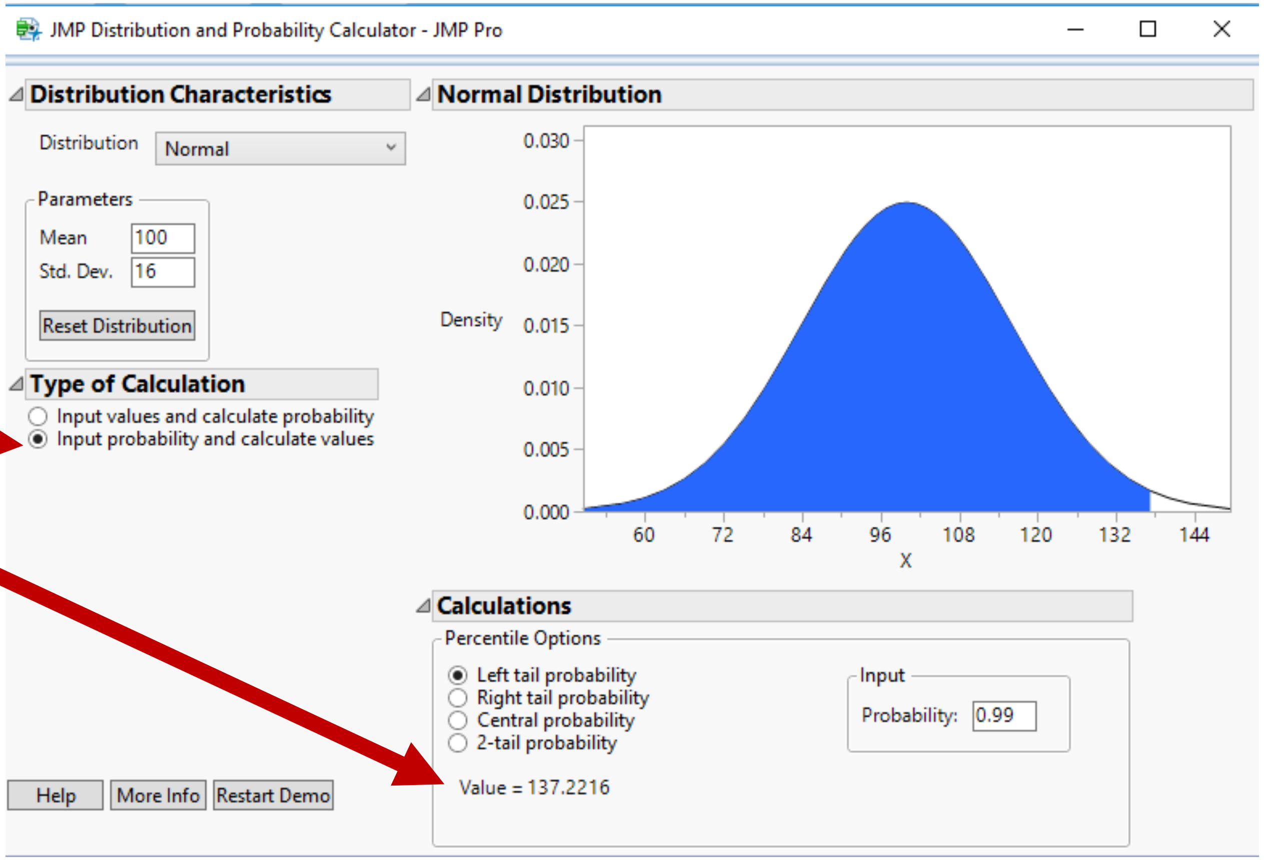
- $\mu = 100, \sigma = 16, x = 80, P(80 \leq x \leq 105)$

- $P(80 \leq x \leq 105)$ = **0.5170**



Normal Variables (d) Solution

- $\mu = 100, \sigma = 16$, find the value of the 99th percentile
- The 99th percentile means find b such that $P(x \leq b) = 0.99$
- The value of the 99th percentile is $\boldsymbol{x = 137.22}$



Problem #7

The Environmental Protection Agency (EPA) fuel economy estimates for automobiles suggest a mean of 24.8 mpg and a standard deviation of 6.2 mpg for highway driving. Assume that a Normal model can be applied.

a) Draw the model for auto fuel economy. Clearly label it, showing what the 68-95-99.7 Rule predicts about miles per gallon.
b) In what interval would you expect the central 68% of autos to be found.
c) About what percent of autos should get more than 31 mpg?
d) About what percent of cars should get between 31 and 37.2 mpg?
e) Describe the gas mileage of the worst 2.5% of all cars.

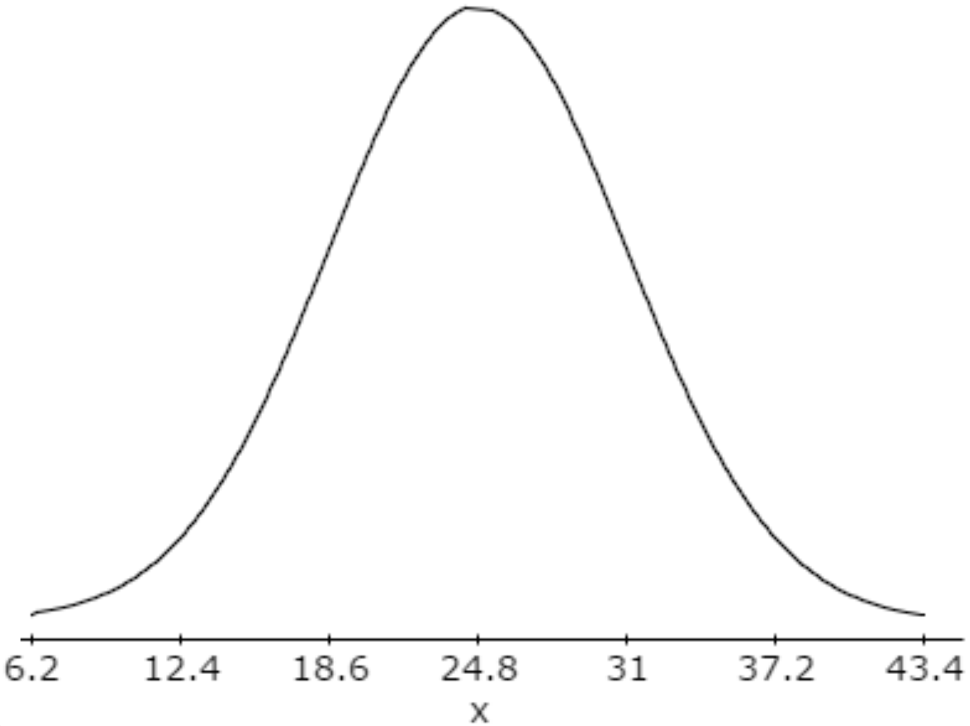
Problem #7 Solution

a) $\mu = 24.8, \sigma = 6.2$

b) 24.8 ± 6.2 = 18.6 mpg to 31 mpg

c) 1 - Normal Distribution[31, 24.8, 6.2] $= 0.1567 \approx 16\%$

d) Normal Distribution[37.2, 24.8, 6.2] - Normal Distribution[31, 24.8, 6.2] $= 0.1359 \approx 13.6\%$



e) Normal Quantile[0.025, 24.8, 6.2] = 12.64, cars that get 12.6 mpg or less are the worst 2.5% of all cars

Problem #31

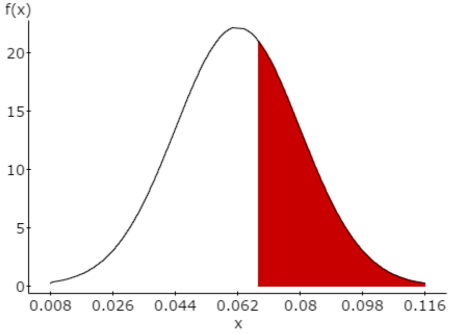
According to the Normal model $N(0.062, 0.018)$ describing mutual fund returns in the 1^{st} quarter of 2013 in Exercise 23, what percent of this group of funds would you expect to have return

a) over 6.8%
b) between 0% and 7.6%
c) more than 1%
d) less than 0%

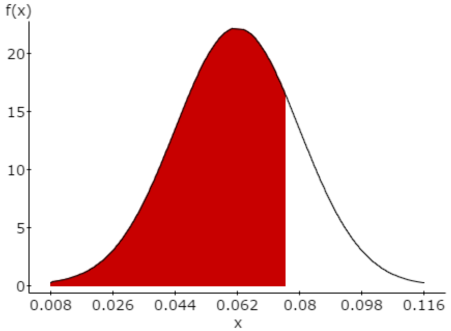
Problem #31 Solution

- $N(0.062, 0.018)$

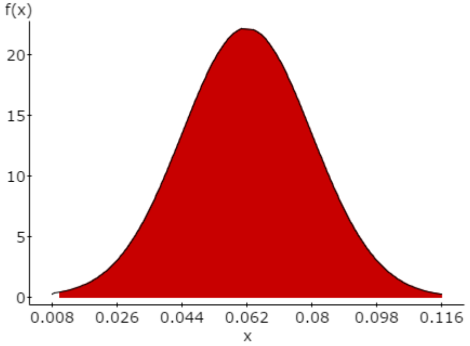
a) P(x => 6.8%) = **0.3694**



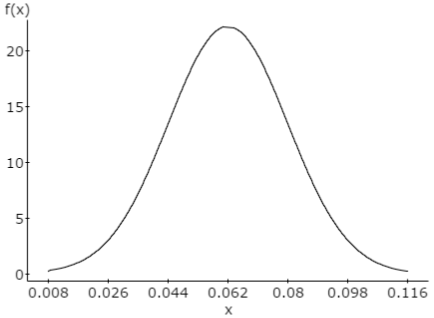
b) P(0% <= x <= 7.6%) = **0.7814**



c) P(x => 1%) = **0.9981**



d) P(x <= 0%) = **0.0003**



Problem #33

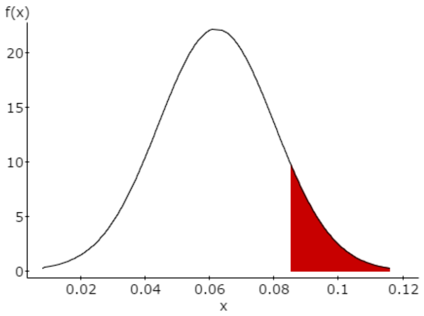
Based on the model *N*(0.062, 0.018) for quarterly returns from Exercise 23, what are the cutoff values for the

a) highest 10% of these funds?
b) lowest 20%?
c) middle 40%?
d) highest 80%?

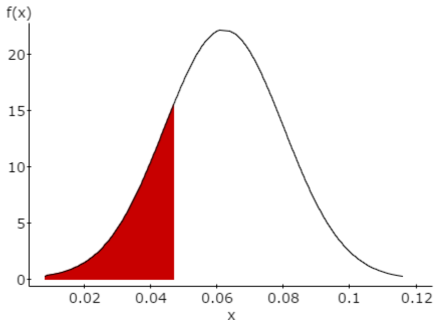
Problem #33 Solution

- $N(0.062, 0.018)$

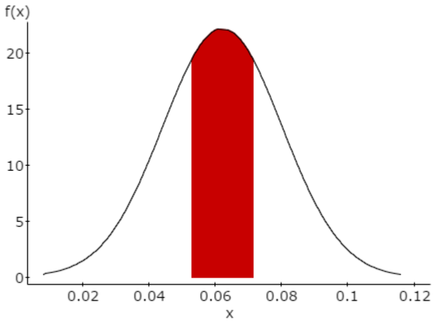
a) Highest 10%, **x = 8.51%**



b) Lowest 20%, **x = 4.69%**



c) Middle 40%, **(5.26%, 7.14%)**



d) Highest 80%, **x = 4.69%**