# Bonus (but necessary) Content

Unit 9 – Inferences from DEPENDENT Samples

Your Extra Professor Colton

# Unit 9 Extra - Outline

Unit 9 – Inferences from Two Samples

Hypothesis Testing of Two Population Means (Dependent Samples)

- Means of Dependent Samples p-value method

# Hypothesis Tests for Means – DEPENDENT Samples

- Much of the logic from Two Sample Hypothesis Tests we have learned still applies!
- Although there are some notable differences in how we conduct a test for dependent samples
- First, let's review what we have learned thus far!

# Independent vs Dependent Samples

- Recall that for Two Sample Hypothesis Tests, we have to know the relationship between our two samples
    - For two sample proportions and the two sample means, both known and unknown population standard deviations, we needed to have INDEPENDENT samples!
    - This was one of the necessary assumptions!
- We had ways to think about our two samples in order to determine if indeed they were independent:

How to think about samples
- Independent samples → Groups are unrelated, no connection, no relationship
- Dependent samples → Groups have some relationship between one another, can link the two; PAIRS

- Let's review the LCQ we had to practice this!

# LCQ: Independent vs Dependent Samples

How to think about samples

- Independent samples → Groups are unrelated, no connection, no relationship
- Dependent samples → Groups have some relationship between one another, can link the two; PAIRS

**Problem**: Determine if the following scenarios are independent or dependent samples.

1) Comparing the blood pressure of STAT 1450 students before the final exam and after completing the final exam.

*Dependent! → There is a relationship between the blood pressure before the final and after the completion of the final. Connection is measuring the SAME student twice*

2) Seeing if the height of Faculty is shorter than the undergraduate population.

*Independent → There is no direct connection (or inherent relationship) between faculty and undergrads*

3) Looking to see if there is a difference in the price of the same Video Game Consoles at Target or Walmart.

~~*Independent??? Two different stores*~~
*Dependent! → No relationship between Target and Walmart, BUT we are looking at the SAME console at the two different stores (groups). So there is a relationship with the consoles (think pairs of X-boxes, one at Walmart and one at Target; same for a PS4)*

4) A study is conducted to see what effect a new drug has on dexterity. A random sample of 30 students is chosen. They are given a series of tasks to perform and a score reflecting their performance. A dose of the drug is given to the 30 students and they again perform similar tasks and are scored again.

*Dependent → SAME students before and after drug. So there is a relationship between the two groups*

# Dependent Samples

Two Types of Dependent Samples

- If samples are dependent, they can be dependent in one of two ways!
- The test that we run is that same for both, but nonetheless it is important to know the structure of our data and how it was obtained!

1) Paired
    - Two values from the SAME subject
    - Examples) Before and After experiments, coordination test for left vs right hand
2) Matched
    - Two values from DIFFERENT subjects connected in some way
    - Examples) Husband and Wife heights, comparing rental car prices from two companies for the same 10 cities

# LCQ: Paired or Matched Samples

**Problem**: Determine if the following scenarios have Paired or Matched samples.

1) Comparing the blood pressure of STAT 1450 students before the final exam and after completing the final exam.

2) Are brothers or sisters smarter? A researcher studied ACT scores of 8 brother and sister pairs.

3) A study is conducted to see what effect a new drug has on dexterity. A random sample of 30 students is chosen. They are given a series of tasks to perform and a score reflecting their performance. A dose of the drug is given to the 30 students and they again perform similar tasks and are scored again.

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# LCQ: Paired or Matched Samples

**Problem**: Determine if the following scenarios have Paired or Matched samples.

1) Comparing the blood pressure of STAT 1450 students before the final exam and after completing the final exam.

*PAIRED → We are measuring the same student twice; repeat measurements from the same person*

2) Are brothers or sisters smarter? A researcher studied ACT scores of 8 brother and sister pairs.

*MATCHED → Dependent sample, connection is from the same family; but not paired because different test-takers*

3) A study is conducted to see what effect a new drug has on dexterity. A random sample of 30 students is chosen. They are given a series of tasks to perform and a score reflecting their performance. A dose of the drug is given to the 30 students and they again perform similar tasks and are scored again.

*PAIRED → Before and after measurements on the SAME student (repeated measures)*

4) Looking to see if there is a difference in the price of the same Video Game Consoles at Target or Walmart.

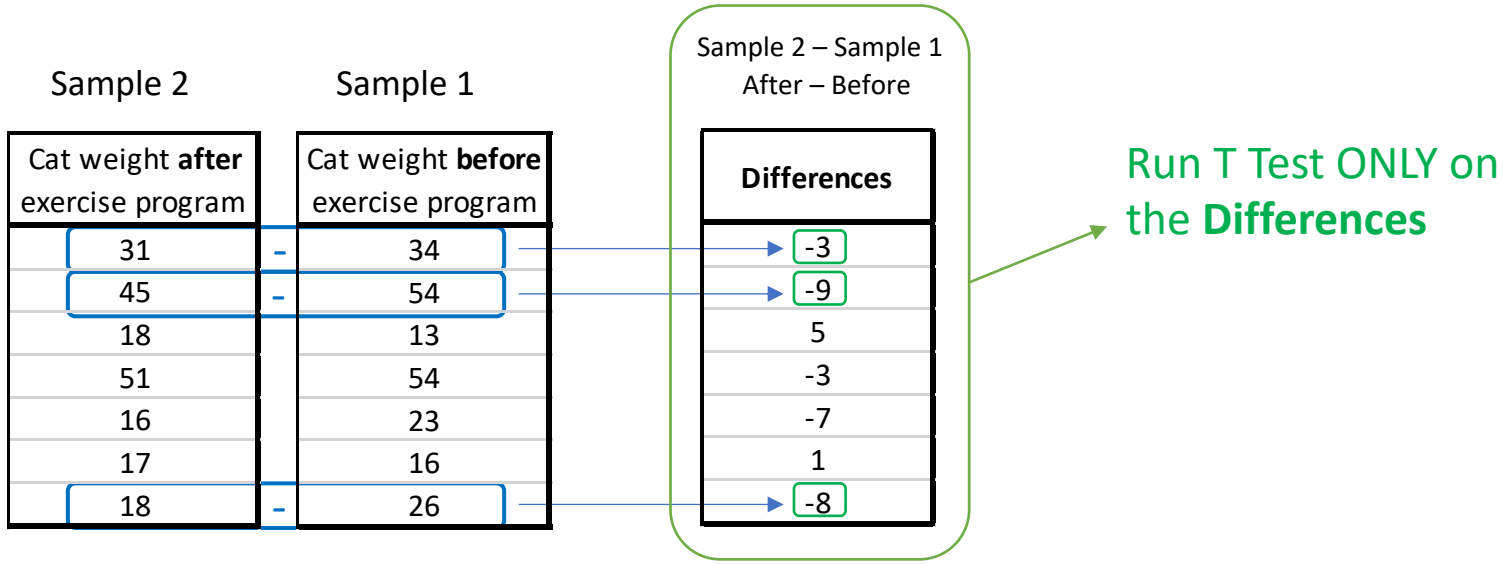
*Matched / Paired? Tough to tell based on context → It will be more obvious when paired*

5) Seeing if the height of Faculty is shorter than the undergraduate population.

*NEITHER → These are INDEPENDENT samples, matched and paired are only for dependent samples*

# Dependent Samples Test (aka Paired T-Test)

- If we had INDEPENDENT samples, we would either run a 2-SampZTest or a 2-SampTTest based on whether or not we knew $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$
- But what do we do if we don't have INDEPENDENT samples and therefore that assumption is NOT MET?? That's what we are going to learn here.
- We will see that we actually end up running a <u>One Sample T Test</u> on the DIFFERENCES between the <u>dependent samples</u>!

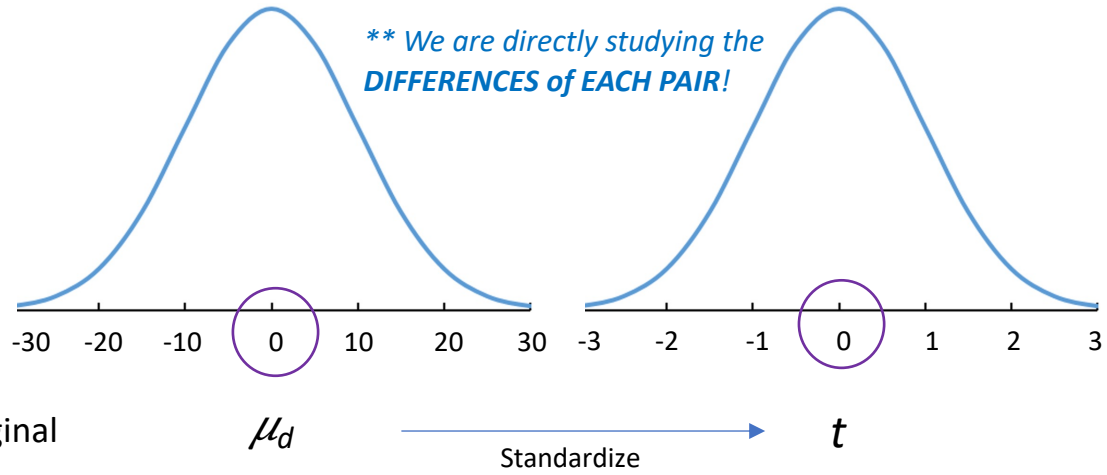



- This might sound similar to how we <u>thought</u> about Two Samples earlier, but the difference is that we are really <u>treating</u> this as ONE sample! We are only going to have ONE sample size, ONE sample mean and ONE sample standard deviation!
- This is because we are actually NOT studying the <u>overall differences between the two population like before</u> (i.e. a separate $\mu_1$ and a separate $\mu_2$). **We are studying the differences between EACH PAIR in the POPULATION!!!**

# The Hypothesis Statements for Dependent Samples

## State the Hypotheses

- **Define parameter + context.**




Define Parameters

- We still have our two **parameters** like before! Population means $\mu_1$ and $\mu_2$
- But now we define a NEW parameter that is the DIFFERENCE between our original two parameters:
  - $\mu_d = \mu_1 - \mu_2$
  - $\mu_d$ represents is the *mean of the differences in each pair of data for the entire population* that the sample is drawn from

*** We still have to define our two parameters in context like before*

Null Hypothesis $H_0$ and Alternative Hypothesis $H_A$

- These are now written using our population mean of the differences, $\mu_d$
- We start by assuming the differences are equal to **ZERO**! Which of course implies the two original parameters are **equivalent**!
- Then we try to show otherwise, the same way we have done!

How to **define** Parameters and **write** Hypotheses

Let $\mu_1$ = true mean + context
Let $\mu_2$ = true mean + context
$\mu_d = \mu_1 - \mu_2$

*** The order of subtraction is IMPORTANT, so we have to be clear when we define $\mu_d$*

$H_0$: $\mu_d = 0$

$H_A$: $\mu_d \neq 0$
$H_A$: $\mu_d < 0$
$H_A$: $\mu_d > 0$

*** The direction here implies which population mean of the pair we believe to be larger*

# LCQ – Dependent Samples Hypotheses

**Problem**: (1) Define the parameters of interest and (2) State the Null and Alternative for the following scenarios:

a) The University is investigating the safety of statistics courses for their students. To do this, they plan a study to compare the blood pressure of STAT 1450 students before the final exam and after completing the final exam. Is there sufficient evidence that the blood pressure has increased from before to after the final?

b) Are brothers or sisters smarter? A researcher studied ACT scores of 8 brother and sister pairs. Is there enough evidence to conclude there is a significant difference in performance on the ACT among brother and sister pairs?

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Define parameters

*$\mu_1$ = true mean blood pressure BEFORE the final exam → We start by defining our two parameters like usual*

*$\mu_2$ = true mean blood pressure AFTER the final exam*

*$\mu_d = \mu_1 - \mu_2$ = BEFORE – **AFTER** → Then we define our new parameter that represents the difference between each pair! We can choose the order of subtraction, but keep in mind that this may affect the direction of the correct Alternative hypothesis*

Write Hypothesis

*$H_0$: $\mu_d = 0$ → Our hypotheses are written in terms of our new parameter; difference equals zero of course implies that there is NO difference between the pairs*

*$H_A$: $\mu_d < 0$ → Based on the context, we want to know if blood pressure has increased! So that means we want to show that the AFTER is larger. Based on our definition of $\mu_d$ we need to have a less than sign indicating we want a negative difference*

b) Are brothers or sisters smarter? A researcher studied ACT scores of 8 brother and sister pairs. Is there enough evidence to conclude there is a significant difference in performance on the ACT among brother and sister pairs?

Define parameters

*$\mu_1$ = true mean ACT score of BROTHERS*

*$\mu_2$ = true mean ACT score of SISTERS*

*$\mu_d = \mu_1 - \mu_2$ = BROTHERS – SISTERS → New parameter special for dependent samples. Again the order is up to us, we could have done SISTER - BROTHER*

Write Hypothesis

*$H_0$: $\mu_d = 0$*

*$H_A$: $\mu_d \neq 0$ → Based on the context, we just want to show a significant difference (no indication of Brothers or Sisters being better). So just need a difference not equal to zero in the Alternative!*

# Using Calc - Test Statistic and P-Value for Paired T-Test

Minor Note → Standard Deviations

- *We will be doing a **T-Test** and using the sample standard deviation of the differe*
- *So we do NOT need to know the original population standard deviations*

## Compute value of Test Statistic / P-value.

**Setup:** The University is investigating the safety of statistics courses for their students. To do this, they plan a study to compare the blood pressure of STAT 1450 students before the final exam and after completing the final exam. 40 randomly selected students are participating in this study.

After the study is completed, they found a mean difference Before – After = -10 mm Hg with standard deviation 30 mm Hg. Is there sufficient evidence that the blood pressure has increased from before to after the final? Use $\alpha = 0.05$

Formula for $t_{stat}$ by hand:

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} \qquad df = n - 1$$

Notation

- $\bar{d}$ = the mean of the differences in each pair of data for the sample
- $\mu_d$ = the mean of the differences in each pair of data for the entire population that the sample is drawn from (assumed to be zero under the Null hypothesis)
- $S_d$ = the standard deviation of the differences in each pair of data for the sample

**GOAL**: Conduct a Hypothesis Test!

Option 1) Summary Stats

T-Test

- Input = Stats
  - a) $\mu_0$ = the Null mean difference of each pair
  - b) $\bar{x} = \bar{d}$ sample mean of differences in each pair
  - c) Sx = $S_d$ sample SD of differences in each pair
  - d) n = sample size of differences
  - e) μ: Alternative hypothesis

Calculate or Draw

*NOTE!! These are the same inputs that we used originally for the One Sample T-Test, just now each quantity has a slightly different meaning*

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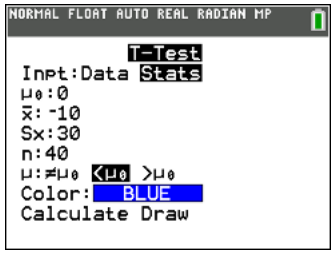
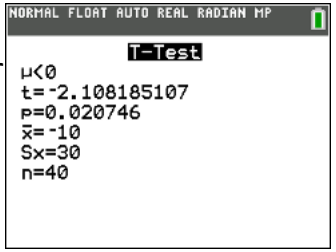
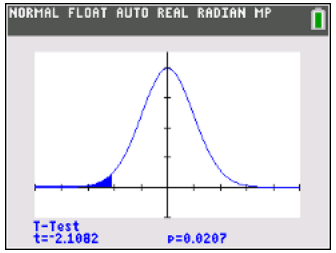



$H_0$: $\mu_d = 0$ Before - After
$H_A$: $\mu_d < 0$




Calculate Output
μ = Alternative hypothesis
t = $t_{stat}$
p = p-value
$\bar{x} = \bar{d}$ sample mean of the differences in each pair
n = sample size of differences




Draw Output
Plot (and displays values) of p = p-value and t = $t_{stat}$ on the t curve with df = n - 1

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## Compute value of Test Statistic / P-value.

**Setup:** Are brothers or sisters smarter? A researcher studied ACT scores of 8 randomly selected brother and sister pairs.

Is there enough evidence to conclude there is a significant difference in performance on the ACT among brother and sister pairs? Use $\alpha = 0.1$

| Brothers' Scores | Sisters' Scores |
|---|---|
| 32 | 25 |
| 24 | 29 |
| 28 | 26 |
| 27 | 34 |
| 33 | 27 |
| 34 | 29 |
| 21 | 25 |
| 22 | 30 |

Formula for $t_{stat}$ by hand:

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} \qquad df = n - 1$$

Notation

- $\bar{d}$ = the mean of the differences in each pair of data for the sample
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**GOAL**: Conduct a Hypothesis Test!

Option 2) Raw Data

1. Enter data
   a) Sample 1 in $L_1$
   b) Sample 2 in $L_2$
   c) Calculate the differences in $L_3 = L_1 - L_2$
   - MATCH the order of subtraction in $\mu_d$
2. T-Test

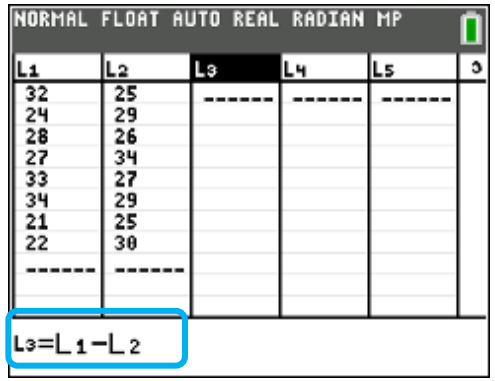
- Input = Data
  a) $\mu_0$ = the Null mean difference of each pair
  b) List = List with differences ($L_3$)
  c) Freq = 1
  d) μ: Alternative hypothesis

Calculate or Draw

# Using Calc - Test Statistic and P-Value for Paired T-Test

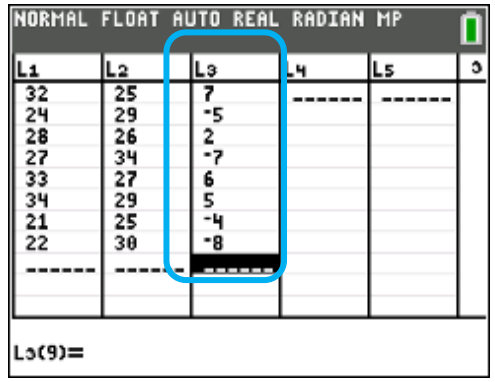
Process Note

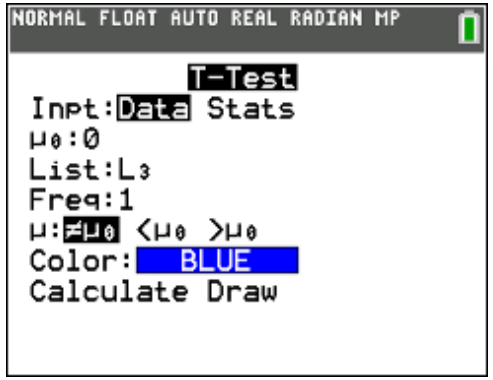
- We could have calculated the differences ourselves and just put them in $L_1$
- But might as well eliminate the chance for a simple algebra mistake and let the calculator do it for us!




Calculate Differences

- Once Samples 1 and 2 are entered, scroll over and up to $L_3$ (click enter)
- Type in the subtraction involving the Lists into the formula bar and click enter

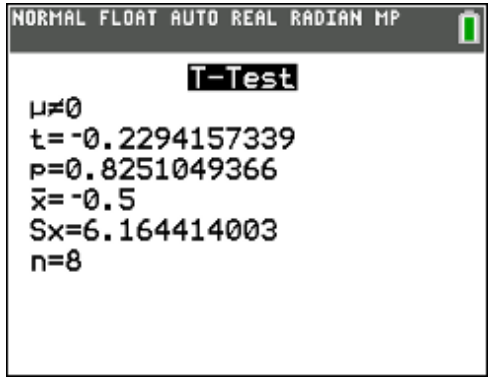



Now run test on list of differences

$H_0: \mu_d = 0$ Brother - Sister

$H_A: \mu_d \neq 0$




Calculate Output

μ = Alternative hypothesis

t = $t_{stat}$

p = p-value

$\bar{x} = \bar{d}$ sample mean of the differences in each pair

n = sample size of differences

# LCQ – Conclusions and Interpretations

**Conclude** and **Interpret**

- State whether you reject $H_0$ or fail to reject $H_0$ AND WHY!
- Interpret your results in the context of the problem

**Problem**: Write the conclusions and interpretations for the previous scenarios using our results.

**A) Setup:** The University is investigating the safety of statistics courses for their students. To do this, they plan a study to compare the blood pressure of STAT 1450 students before the final exam and after completing the final exam. 40 randomly selected students are participating in this study.

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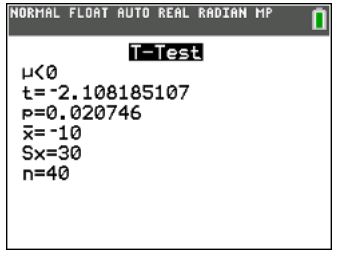


$H_0: \mu_d = 0$ Before - After

$H_A: \mu_d < 0$

**B) Setup:** Are brothers or sisters smarter? A researcher studied ACT scores of 8 randomly selected brother and sister pairs. Assume ACT scores for all students are normally distributed blood pressures are always normally distributed.

Is there enough evidence to conclude there is a significant difference in performance on the ACT among brother and sister pairs? Use $\alpha = 0.1$

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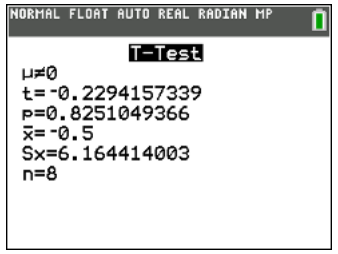


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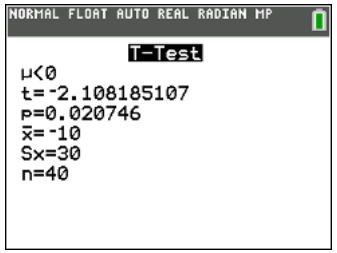


$H_0$: $\mu_d = 0$ Before - **After**
$H_A$: $\mu_d < 0$

*P-Value*

*P-value = T-Test(Input = Stats, $\mu_0 = 0$, $\bar{x} = -10$, Sx = 30, n = 40, $\mu < \mu_0$) = 0.0207*
*p-value = 0.0207 < 0.05 = $\alpha$ → Reject $H_0$!*

Conclusion and Interpretation

*Because our p-value = 0.0207 is less than the significance level = 0.05, we reject the null hypothesis. There is sufficient evidence to conclude that the true mean blood pressure after the final exam is greater than the true mean blood pressure before the final exam*
*→ Explicitly interpreting the alternative would say 'the true mean difference between blood pressure before and after the final is less than zero'*
- *But we know contextually we were testing for After being greater and we rejected! (and also we did Before – After and there was a negative Test Statistic, so After is greater)*

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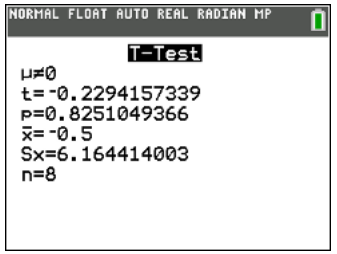


$H_0$: $\mu_d = 0$ Brother - Sister
$H_A$: $\mu_d \neq 0$

*P-Value*
*Entered brother data in L1 and Sister data in L2. Then calculated L3 = L1 – L2*
*P-value = T-Test(Input = Data, List = $L_3$, Fre1 = 1, $\mu \neq \mu_0$) = 0.825*
*p-value = 0.825 > 0.10 = $\alpha$ → Fail to reject $H_0$*

Conclusion and Interpretation

*Because our p-value = 0.825 is greater than the significance level = 0.10, we fail to reject the null hypothesis.*
*There is NOT sufficient evidence to conclude that the true mean ACT scores of brothers is different that the true mean ACT scores of sisters.*
*We could also word the interpretation part like this (equally as correct)→ There is NOT sufficient evidence to conclude there is a significant difference in true mean ACT scores of brothers and sisters*