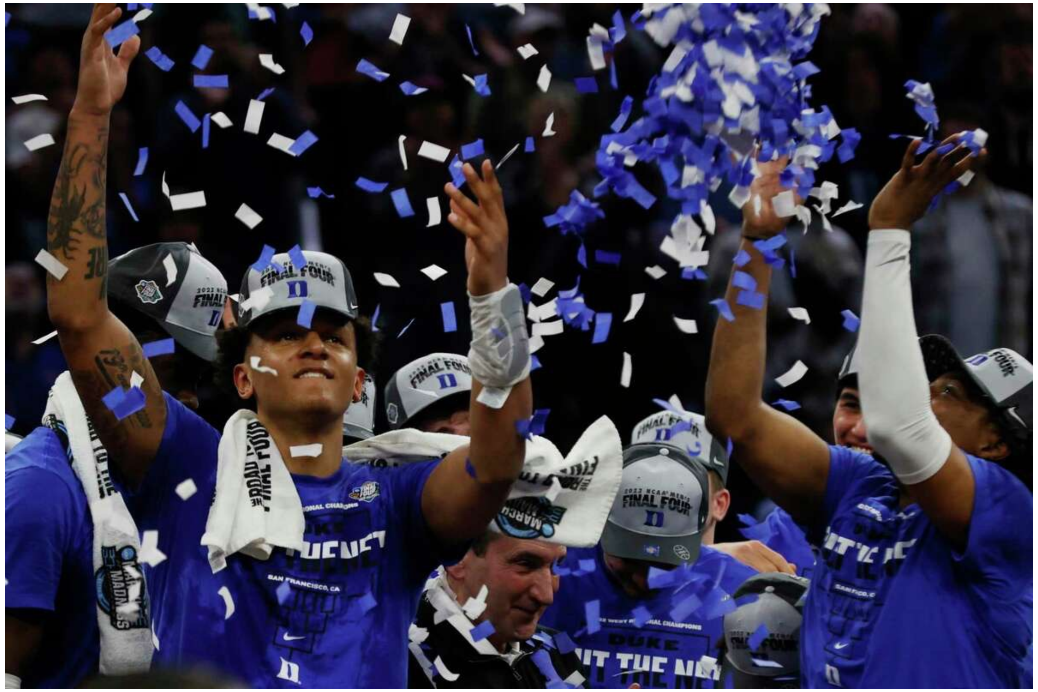
ONLY TWO MORE!!



Unit 4 Bonus – Simulations

Ran out of time Professor Colton

Unit 4 Bonus - Outline

<u>Unit 4 Bonus – Simulation</u>

- Overview of importance and applications
- Goal of a simulation and how to perform one
- Independent events
- Steps of a simulation
- Examples

Simulations

- Simulations are a fantastic tool that researchers in all fields use!!
  - Examples: Simulating the spread of a disease; actuaries use simulations to figure out their expected payout on insurance policies!

- Not always practical to do an experiment (cost, time, etc.), too much (or impossible) math!
  - For example, if I am interested in the number of heads out of 100 tosses. I flip a coin 100 times. That's only one trial! I would need to do this thousands of times to get a good estimate of the final probability!

- <u>Simulations</u> allows us to use computers to mimic what would happen in real life, or what we believe happens in real life.
- Can use computers to simulate or use "devices" that are random.
  - Dice, Coin, Deck of Cards, etc.

- They allow us to perform many, many repetitions all at once AND more easily study complex probability models.
  - Experiment: Toss a coin 10 times. What is the probability of a run of at least three consecutive heads or three consecutive tails???
  - Do you know how to figure this out with equations??

Simulations

- Once we set up a correct probability model, simulation is an effective tool for finding probabilities of complex events!

- We can use random digits to simulate many repetitions quickly.

- The proportion of repetitions on which an event occurs will *eventually be close to its probability*, so simulation can give good estimates of probabilities.

- One last aspect we need to know before we set up some simulations!

Independent Events

Independent Events

- Two events are **independent** if the prior event has NO effect on the subsequent selection.
- For example: Rolling dice, picking cards with replacement, spinning a roulette wheel, and guessing on test questions.
  - These are all independent since your chances of success don't change for each trial.
  - It can also be unrelated stages, such as flipping a coin and then rolling a die.
- Examples of NOT independent events (i.e. dependent):
  - Drawing marbles without replacement or selecting children for a recess sports game.

Probabilities of Independent Events

- If two events are **independent**, the first event does NOT change the probability of the second event!
  - We can just multiply the probabilities!

Independent Events

Example

- If flipping a fair coin three times, what is the probability I flip three heads?
- Probability Model for a single toss:

X	***P(X)***
H	0.5
T	0.5

- Well, probabilities for H or T from each coin toss doesn't depend on the previous toss… So these events are independent!
- This means we can just multiply the probabilities!

P(3 Heads) = P(H) x P(H) x P(H) = ½ x ½ x ½ = 1/8

Steps of a Simulation

1. Set up a probability model that matches the scenario.
   - Think of the probability model for a SINGLE trial!
   - We are only going to work with independent events, so the model will be the same from trial to trial.
2. Decide how you will use random digits to model the scenario.
   - Essentially, figure out how we are going to map random digits to the events in our probability model.
   - We assign numbers to each event to match the corresponding probabilities.
3. Run the simulation several times
   - Generate a sequence of random digits.
4. Collect and summarize results
   - Translate the digits for each trial to a success or failure in terms of the event of interest!
   - Then find the overall probability!

Simulation Example

Setup: Toss a fair coin 10 times. What is the probability of a run of at least three consecutive heads or three consecutive tails?

Simulation

1) Probability Model for a single trial**!**

X	*P(X)*
H	0.5
T	0.5

GOAL:

Outcomes

Mapping

Generated Numbers

2) There are many ways to do this define our random number scheme:

- Option 1
  - Generate random digits from 0-9 (for example: 14151349807190836243)
  - **Let 0-4 represent H and 5-9 represent T**
  - This works (is valid) because the probabilities match!
    - 5 numbers that represent H out of 10 total possible numbers = 5/10 = 0.5! And same for Tails, 5/10 = 50% of Tails
- Option 2 (easier way)
  - Generate random zeros and ones (for example: 00110101011011)
  - **Let 0 = H and 1 = T**
  - Obviously there is a 50% chance for each H and T! So we are good!

Simulation Example Continued

Setup: Toss a fair coin 10 times. What is the probability of a run of at least three consecutive heads or three consecutive tails?

Simulation

3) Actually generate the random numbers!

- One tool we can use is this website: www.random.org/integers/
- We need to figure out how many numbers to generate!
- For this experiment, we are flipping 10 coins. So lets generate 10 numbers (i.e. 10 trials).
  - But we want to do this many times, we will do a small simulation with say 20 experiments!
  - So we need 10 x 20 = 200 random numbers.

4) Determine if each experiment was a success or a failure based on the event of interest, whether there was a set of 3 consecutive zeroes (Heads) or three consecutive ones (Tails).

- If yes, Success! If no, Failure!
- Then calculate the final simulated probability.
  - *P(at least three consecutive H or T) = number of successes / number of experiments*

Simulation Example Results

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10	Success / Failure??
Experiment 1	0	0	1	1	0	1	0	1	1	1	S
Experiment 2	1	1	0	0	0	0	0	0	1	1	S
Experiment 3	0	0	1	0	1	1	0	1	0	0	F
Experiment 4	0	1	1	0	0	0	1	1	1	0	S
Experiment 5	0	0	1	1	0	1	1	0	0	0	S
Experiment 6	1	0	1	0	0	1	0	1	1	0	F
Experiment 7	0	0	0	0	1	1	1	0	1	1	S
Experiment 8	1	1	0	0	0	1	1	1	1	0	S
Experiment 9	0	1	1	0	1	0	1	1	0	0	F
Experiment 10	0	1	0	0	0	1	0	1	1	1	S
Experiment 11	1	0	0	1	0	0	1	0	1	1	F
Experiment 12	0	0	0	1	0	1	0	0	1	0	S
Experiment 13	0	1	0	0	0	1	1	0	0	0	S
Experiment 14	0	1	0	1	1	1	1	1	0	1	S
Experiment 15	0	1	0	1	0	0	1	0	0	1	F
Experiment 16	1	1	1	0	0	0	1	0	1	0	S
Experiment 17	1	1	0	0	0	0	1	1	1	1	S
Experiment 18	1	0	0	0	0	0	0	0	1	1	S
Experiment 19	0	1	1	1	1	1	0	1	0	0	S
Experiment 20	1	0	1	1	1	0	1	1	0	0	S

Total Successes = 15
Total Experiments = 20
Final Probability -> P(at least three consecutive H or T) = 15/20 = 0.75

Simulation Example Conclusion

- Twenty repetitions are <u>not enough to be confident</u> that our <u>estimate is accurate</u>. We can have a computer to do <u>many thousands of repetitions</u>.
- A long simulation (or hard mathematics) finds that the <u>true probability</u> is about 0.826, so even our small simulation didn't do too badly.

LCQ

GOAL:

Outcomes

Mapping

Generated Numbers

Explain how to use random.org to model the following situations.

a) A roll of one six-sided die.

b) A coin flip.

c) Picking a card out of a standard deck of cards. (52 cards are in a standard deck)

d) The number of girls in a family of with 2 children. (Assume sexes are equally likely)
Hint: Is there an equal chance of having 2 girls as it is as to having 1 girl?

LCQ

GOAL:

Outcomes

↕ Mapping

Generated Numbers

Explain how to use random.org to model the following situations.

a) A roll of one six-sided die.

Could we just do 1-6 and let the numbers represent themselves? YES!!!

6 total numbers we can generate from, and each has 1/6 probability which matches that of actually rolling a die

b) A coin flip.

Generate numbers from 0-99 (100 total), let 0-49 (50 total) = H and 50-99 (total) = T

c) Picking a card out of a standard deck of cards. (52 cards are in a standard deck)

Probability model: 1/52 for each card (with replacement)
Randomly generate a number from 1 – 52 (52 total)
Assign each card a unique number: Ace of hearts = 1, 2 of hearts = 2,…. King of hearts = 13, ace of diamonds = 14, …., king of spades = 52

d) The number of girls in a family of with 2 children. (Assume sexes are equally likely)
Hint: Is there an equal chance of having 2 girls as it is as to having 1 girl? *Not equally likely!*

One option:
- *Repeat the strategy of the H/T simulation!*
- *Generate 0/1, 0 = girl and 1 = boy*

1st	2nd	
0	1	= 1 G
1	0	= 1 G
0	1	…
1	1	
1	1	= 2 G
0	1	
1	1	
1	0	…
0	0	= 0 G
0	1	

Another option:
- *Generate numbers 0-3 (4 total)*
- *Assign having no girls 0, have the first child being a girl 1, assign the first child being a boy and the second being a girl a 2, and assign having both children being girls a 3.*
- *P(0 G) = P(2 G) = ¼, P(1 G) = ½*

Both kids	
1	= 1 G
3	= 2 G
1	…
2	= 1 G
1	…
0	= 0 G
0	
2	
1	
1	

PROBLEM SESSION!!!!!!!!!!!

Simulation Example

We are interested in the probability that a student gets at least 7 multiple choice questions correct out of 10 if they randomly guess on each. Each question has 4 answer choices. Run a simulation to answer this question.

321765431326543416343456897613 2...

Simulation Example: Numbering scheme

We are interested in the probability that a student get at least 7 multiple choice questions correct out of 10 if they randomly guess on each. Each question has 4 answer choices. Run a simulation to answer this question.

321765431326543416343456897 6132…

We want to select numbers to represent the percentage/proportion we observe in the real world.

4 choices, we have a 1 in 4 or 25% chance of guess correct.

So we want to select numbers such that 25% of them are representing a correct choice, the other 75% representing the wrong answers.

Let 1 represent a correct answer, 2-4 incorrect. We have four total choices (1, 2, 3, 4) with one fourth of them (1) representing a correct answer. Not the only way to do this, could let 0 and 1 be getting a correct answer and 2-7 be incorrect. I still have one fourth (0 and 1) being the even guess correct out of the total of 8 (0, 1, 2, 3, 4, 5, 6, 7)

What are we interested in? We want to know if we get <u>at least</u> 7 of the 10 correct. So our event is **whether we get at least 7 correct answers or not**.

Simulation Example: An Experiment

We are interested in the probability that a student get at least 7 multiple choice questions correct out of 10 if they randomly guess on each. Each question has 4 answer choices. Run a simulation to answer this question.

321~~8~~76543132654341634345 6~~89~~76132…

Let’s use 0-1 as a correct answer, 2-7 as incorrect. If my sequence of random digits contains numbers that aren’t in my scheme (such as 9). Then we just skip it, it doesn’t count as wrong or right (act like it’s not there). This is kinda what we did back in LU 1 with Table A and randomly selecting people for our samples.

Each number is a trial representing an answer to a mc question. The first 10 numbers are…

32**1**~~8~~76543**1**3

How many 0’s and 1’s do we have? **2**
Is this at least 7? **No. So our response or result of this experiment failure.**

Once we run enough experiments, out of them how many successes do we observe? That number divided by the total number of experiments is your probability you report.

Question 3

You decide to test your luck by playing the Mega Millions lottery. In the Mega Millions game, you select 5 numbers with each value being between 1 and 70. You also get to pick one extra number known as the Mega Ball which you can be between 1 and 25. If you were to use the random numbers listed below as a way to select which numbers to use, which numbers do you use? [Hint: Pick the five numbers first, then pick the Mega Ball number.]

86167677872189877636493271516176436937838817

Question 3

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86 **16** 76 77 87 **21** 89 87 76 **36 49 32** 7151617643693783 88 **17**

5 Numbers picked are 16, 21, 36, 49, 32

Mega Ball is 17

Question 7

A couple is interested in the probability of having exactly 4 girls if they were to have a total of 4 children. Assume that it is equally likely to have a girl and a boy. Use the set of random numbers below to answer this question. Use the digits 00-49 to represent having a girl, and the digits 50-99 to represent having a boy.

Run the simulation by filling in the table based on the random numbers.

Then find the final probability of interest.

Trial	Random #s	Number of Girls	Result	Trial	Random #s	Number of Girls	Result
1	08 28 14 43			**11**	88 32 25 22		
2	63 73 14 42			**12**	01 51 93 31		
3	10 48 09 40			**13**	70 05 84 03		
4	23 70 81 41			**14**	54 50 03 28		
5	72 89 10 05			**15**	16 98 98 12		
6	84 46 25 26			**16**	86 52 85 36		
7	63 88 95 26			**17**	73 00 56 96		
8	92 92 96 46			**18**	53 70 87 60		
9	10 57 15 73			**19**	89 08 56 77		
10	37 41 03 02			**20**	80 85 14 18		

Question 7

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P(exactly 4 girls) = 0.15

Trial	Random #s	Number of Girls	Result	Trial	Random #s	Number of Girls	Result
1	08 28 14 43	4	1 or Yes	11	88 32 25 22	3	0 or No
2	63 73 14 42	2	0 or No	12	01 51 93 31	3	0 or No
3	10 48 09 40	4	1 or Yes	13	70 05 84 03	2	0 or No
4	23 70 81 41	2	0 or No	14	54 50 03 28	2	0 or No
5	72 89 10 05	2	0 or No	15	16 98 98 12	2	0 or No
6	84 46 25 26	3	0 or No	16	86 52 85 36	1	0 or No
7	63 88 95 26	1	0 or No	17	73 00 56 96	1	0 or No
8	92 92 96 46	1	0 or No	18	53 70 87 60	0	0 or No
9	10 57 15 73	2	0 or No	19	89 08 56 77	1	0 or No
10	37 41 03 02	4	1 or Yes	20	80 85 14 18	2	0 or No