4.2.2 → Deriving base rates CONT...

→ Approximated change in average rate differential

→ The second method utilizes the derived beginning of notes equation. The last term in it $\left(\frac{R_c}{R_p}\right)$ (rating factor: R_c → current, R_p → proposed) is commonly known as the off-balance factor, which accounts for the way changes in the rating factors impact the base rate required to achieve the target premium (or rate change) amount.

$$OBF = \frac{\text{Avg Current rating factor}}{\text{Avg Proposed rating factor}}$$

→ So, the equation can be rewritten as

◎ $$\text{Proposed Base Rate} = \frac{\text{Proposed Avg Premium} - \text{Proposed Additive Fee}}{\text{Current Avg Premium} - \text{Current Additive Fee}} \times \text{Current Base rate} \times OBF$$

→ This method is called the approximate change in average rate differential b/c it allows us to approximate the overall change in the average relativity, i.e. the off-balance factor. For instance, assuming we have two rating variables, class & Territory, the overall off-balance factor is approximated as:

$$OBF_{overall} = \frac{\text{Current Avg rating factor}}{\text{Proposed Avg rating factor}} \quad ▢$$

$$\approx \frac{\text{Current Avg Class factor} \times \text{Current Avg Territory factor}}{\text{Proposed Avg Class factor} \times \text{Proposed Avg Territory factor}}$$

$$= \frac{\text{Current Avg Class factor}}{\text{Proposed Avg Class factor}} \times \frac{\text{Current Avg Territory factor}}{\text{Proposed Avg Territory factor}}$$

$$\Rightarrow \; = OBF_{class} \times OBF_{Territory}$$

→ Since non-changing rating variables will have a change factor of 1, we only need to focus on the rating variables whose relativities are changing. This offers some time-saving advantages over using the previous method.

→ When calculating the off-balance factor for each rating variable, there are two mathematically equivalent options:

→ 1) The current weighted average rating factor divided by the proposed weighted average rating factor (as shown in ▢). In this case, use the variable premium at base level or adjusted exposures when weighting.

→ 2) The reciprocal of the weighted average relativity change factor, i.e. the proposed relativity divided by the current relativity for each level.

$$OBF = \frac{1}{\text{Avg Relativity Change Factor}}$$

In this case, use the variable premium (not at base level) when weighting relativity change factors.

→ We will demonstrate the method using the same example as earlier.

→ Example → Calculate the base rate using the approximated change in average rate differential.

→ The only piece we haven't calculated already is the off-balance factor. Since the relativities for both rating variables are changing, we will need to calculate the off-balance factors for each of them.

→ Approach 1 → The current weighted average rating factor divided by the proposed weighted average rating factor

→ For class, we have previously calculated the proposed average rating factor to be 1.2105. Now, using the previously calculated current variable premium at base level, calculate the current average rating factor:

Class	Current Relativity	Current Variable Premium at Base Level
A	1.00	29,000
B	1.20	32,200

$$\text{Current Avg Class factor} = \frac{1.00(29,000) + 1.20(32,200)}{29,000 + 32,200} = 1.1052$$

→ Therefore, the off-balance factor for class is:

$$OBF_{class} = \frac{1.1052}{1.2105} = 0.9131$$

→ For territory, we have previously calculated the proposed average rating factor to be 0.9331. Now, using the previously calculated current variable premium at base level, calculate the current average rating factor:

Territory	Current Relativity	Current Variable Premium at Base Level
X	1.00	39,200
Y	0.90	31,600

$$\text{Current Avg Territory factor} = \frac{1.00(39,200) + 0.90(31,600)}{39,200 + 31,600} = 0.9554$$

→ Therefore, the off-balance factor for Territory is:

$$OBF_{Territory} = \frac{0.9554}{0.9331} = 1.0239$$

→ Approach 2 → The reciprocal of the weighted average relativity change factor

→ For class:

Class	Relativity Change Factor	Current Variable Premium
A	1.00 / 1.00 = 1.0000	200(100 × 1.00 × 1.00 + 50 × 1.00 × 0.90) = 29,000
B	1.40 / 1.20 = 1.1667	200(80 × 1.20 × 1.00 + 90 × 1.20 × 0.90) = 38,640

$$\text{Class Avg relativity change factor} = \frac{1.000(29,000) + 1.1667(38,640)}{29,000 + 38,640} = 1.0952$$

$$\Rightarrow OBF_{class} = \frac{1}{1.0952} = 0.9131 \checkmark$$

→ For territory:

Territory	Relativity Change Factor	Current Variable Premium
X	1.00 / 1.00 = 1.0000	200(100 × 1.00 × 1.00 + 80 × 1.20 × 1.00) = 39,200
Y	0.85 / 0.90 = 0.9444	200(50 × 1.00 × 0.90 + 90 × 1.20 × 0.90) = 28,440

$$\text{Territory Avg Relativity change factor} = \frac{1.000(39,200) + 0.9444(28,440)}{39,200 + 28,440} = 0.9766$$

$$\Rightarrow OBF_{Territory} = \frac{1}{0.9766} = 1.0239 \checkmark$$

→ Both approaches result in the same off-balance factors. Therefore, the approximated overall off-balance factor is

$$OBF \approx 0.9131 \times 1.0239 = 0.9349$$

→ Finally, the proposed base rate is

$$\text{Proposed base rate} = \frac{260.01 - 25}{236.375 - 25} \times 200 \times 0.9349 = \$207.89$$

→ If we were to calculate the base rate using the exact off balance factor instead, we would need the relativity change factors & the current variable premium for each combination of rating variables.

Class	Territory	Combined Current Relativity	Combined Proposed Relativity	Relativity Change Factor	Current Variable Premium
A	X	1.00	1.00	1.00 / 1.00 = 1.0000	200 × 100 × 1.00 = 20,000
A	Y	0.90	0.85	0.85 / 0.90 = 0.9444	200 × 50 × 0.90 = 9,000
B	X	1.20	1.40	1.40 / 1.20 = 1.1667	200 × 80 × 1.20 = 19,200
B	Y	1.08	1.19	1.19 / 1.08 = 1.1019	200 × 90 × 1.08 = 19,440

$$\text{Avg relativity change factor} = \frac{1.0000(20,000) + 0.9444(9,000) + 1.1667(19,200) + 1.1019(19,440)}{20,000 + 9,000 + 19,200 + 19,440} = 1.0692$$

$$\Rightarrow OBF = \frac{1}{1.0692} = 0.9353$$

$$\Rightarrow \text{Proposed base rate} = \frac{260.01 - 25}{236.375 - 25} \times 200 \times 0.9353 = 207.98$$

→ Extension of Exposures

→ In the two methods above, in addition to approximating the components, we also covered deriving the base rate w/o approximation. Now, we'll explain a third method to derive the base rate w/o approximation.

→ Similar to the extension of exposures method from section 4.2, this method involves re-rating all unique combinations of rating variables using the proposed rating factors & a made-up base rate called the seed base rate.

▢ $$\text{Proposed base rate} = \text{Seed Base rate} \times \frac{\text{Proposed Avg Premium} - \text{Proposed Additive Fee}}{\text{Proposed Avg Premium w/ Seed base rate} - \text{Proposed Additive Fee}}$$

→ The seed base rate can be anything other than 0, but setting it at 1 is usually the simplest.

→ Let's redo the same motivating example.

→ Example → Calculate the base rate using the extension of exposures method.

→ To keep things simple, we'll select a seed base rate of 1. We need to calculate the total premium for each combination of levels of the rating variable using the combined proposed relativity & the seed base rate. We're calculating the total premium here, so don't forget to include the expense fee of 25.

Class	Territory	Exposures	Combined Proposed Relativity	Proposed Premium with Seed Base Rate
A	X	100	1.00	100(1 × 1.00 + 25) = 2,600
A	Y	50	0.85	50(1 × 0.85 + 25) = 1,292.50
B	X	80	1.40	80(1 × 1.40 + 25) = 2,112
B	Y	90	1.19	90(1 × 1.19 + 25) = 2,357.10

$$\text{Proposed Avg Premium w/ Seed Base rate} = \frac{2600 + 1292.50 + 2112 + 2,357.10}{100 + 50 + 80 + 90} \Rightarrow 26.13$$

→ Therefore,

$$\text{Proposed base rate} = 1 \times \frac{260.01 - 25}{26.13 - 25} = 207.98 \checkmark$$

→ Summary

→ In total, we have explored deriving the base rate using the following methods:

- → Approximated average rate differential
  - → Exact average rate differential
- → Approximated change in average rate differential
  - → Exact change in average rate differential
- → Extension of exposures

→ It is important to note that as the number of rating variables or the categories w/in each rating variable increases, calculating the exact components needed to derive the base rate becomes increasingly complex & eventually impractical. In such cases, approximations are generally preferred to reduce effort & maintain feasibility.

→ Example → You are given the following information.

Class	Exposures	Premium	Current Relativities	Proposed Relativities
1A	500	$40,000	0.80	0.70
2B	1,000	$100,000	1.00	1.00
3C	100	$15,000	1.50	1.60
Total	1,600	$155,000		

- The current base rate is $100.
- The desired overall change is 5%.

Calculate the proposed base rate.

→ Since the problem does not specify whether or not there are any additive premiums, we should assume all premiums are variable. This should be your default assumption on the exam. There are multiple ways to solve this problem. We'll demonstrate a couple of them.

→ The quickest way is to apply equation ◎. Note that the total relativity change factor is computed as a weighted average of the individual change factors using the premiums (not at base level) as weights.

Class	Exposures	Proposed Relativities	Relativity Change Factors
1A	500	0.70	500 × 1 × 0.70 = 350
2B	1,000	1.00	1,000 × 1 × 1.00 = 1,000
3C	100	1.60	100 × 1 × 1.60 = 160
Total	1,600		$\frac{350 + 1,000 + 160}{1,600} = 0.9438$

$$OBF = \frac{1}{0.9742} = 1.0265$$

→ The proposed average premium divided by the current average premium must equal the rate change factor of 1.05. So the proposed base rate is

$$\text{Proposed base rate} = 1.05 \times 100 \times 1.0265 = \$107.78$$

→ Alternate Solution

→ Another fairly quick solution to the problem is to use the extension of exposures method. We'll assume the seed base rate is 1. The entry in the bottom row of the rightmost column is the proposed average premium per exposure using the seed rate which can be found by summing the proposed premiums using the seed base rate & dividing by the total number of exposures.

Class	Exposures	Proposed Relativities	Relativity Change Factors
1A	500	0.70	500 × 1 × 0.70 = 350
2B	1,000	1.00	1,000 × 1 × 1.00 = 1,000
3C	100	1.60	100 × 1 × 1.60 = 160
Total	1,600		$\frac{350 + 1,000 + 160}{1,600} = 0.9438$

→ We can obtain the current average premium by dividing total premium by total exposures.

$$\text{Current Avg Premium} = \frac{155,000}{1,600} = 96.875$$

→ Then, the proposed average premium must be equal to the current average premium multiplied by the target rate change factor.

$$\text{Proposed Avg Premium} = 96.875 \times 1.05 = 101.7185$$

→ So, using equation ▢, the proposed base rate is

$$\text{Proposed base rate} = 1 \times \frac{101.7185 - 0}{0.9438 - 0} \approx \$107.78$$

→ Assignment

→ Q1) Given the following information:

- Current base rate = $90
- Current average premium per exposure = $110
- Loss and LAE ratio = 75%
- Fixed expense ratio = 16.82%
- Variable expense ratio = 15%
- Target profit provision = 5%
- Rating algorithm = {Base Rate × Factor × (1.0 − Discount)} + Expense Fee
- Selected rate change = indicated rate change

Factor	Current Differential	Proposed Differential
1	0.95	0.95
2	1.00	1.00

Discount	Current Discount	Proposed Discount
Yes	5.0%	10.0%
No	0.0%	0.0%

Factor	Discount	Exposures
1	Yes	100
1	No	200
2	Yes	300
2	No	400

(a) Calculate the indicated rate change.
(b) Calculate the proposed expense fee.
(c) Calculate the proposed base rate using the approximated average rate differential method.
(d) Calculate the proposed base rate using the *exact* average rate differential method.

→

Row	C	D	E	F	G	H	I	J	K	L	M
2		Factor	Current Differential	Proposed Differential							
3		1	0.95	0.95					Indicated rate change =	14.775%	=(E18+E19)/(1-E20-E21)-1
4		2	1	1							
5					=1-E7				Proposed fee =	$23.13	=E17*E19/(1-E20-E21)
6		Discount	Current Discount	Proposed Discount	Current discount factor	Proposed Discount factor					
7		Yes	5%	10%	0.95	0.90			Proposed avg premium =	$126.25	=(1+L3)*E17
8		No	0%	0%	1.00	1.00					
9									Avg Factor factor =	0.985	=(F3*SUM(F11:F12)+F4*SUM(F13:F14))/SUM($F$11:$F$14)
10		Factor	Discount	Exposures							
11		1	Yes	100					Avg Discount factor =	0.960	=(H7*SUM(F11,F13)+H8*SUM(F12,F14))/SUM($F$11:$F$14)
12		1	No	200							
13		2	Yes	300					Approximated proposed avg rating factor =	0.9456	=L9*L11
14		2	No	400							
15									Approximated average rate differential method: Proposed base rate =	$109.06	=($L$7-$L$5)/L13
16	Current base rate =	$90									
17	Current average premium per exposure =	$110							=I19*J19		
18	Loss and LAE ratio =	75%			Factor	Discount	Factor	Discount	Combined	Exposures	
19	Fixed expense ratio =	16.82%			1	Yes	0.95	0.90	0.855	100	
20	Variable expense ratio =	15%			1	No	0.95	1.00	0.950	200	
21	Target profit provision =	5%			2	Yes	1.00	0.90	0.900	300	
22	Rating algorithm =	(Base rate x Factor (1.0 - Discount)) + Expense fee			2	No	1.00	1.00	1.000	400	
23	Selected rate change =	Indicated rate change							Exact avg rating factor =	0.9455	=SUMPRODUCT(K19:K22,L19:L22)/SUM(L19:L22)
25									Exact average rate differential method: Proposed base rate =	$109.07	=($L$7-$L$5)/L23

→ Q2) An insurance company is targeting a 10% rate increase for the book of business described below.

Class	Territory	Exposures
A	X	50
A	Y	80
B	X	120
B	Y	65
C	X	90
C	Y	75

Class	Current Relativity	Proposed Relativity
A	1.00	1.00
B	1.20	1.40
C	1.90	1.75

Territory	Current	Proposed
X	1.00	1.00
Y	0.80	0.80

- All premium is variable.
- The current base rate is $500.
- The current average premium is $630.625.

Calculate the proposed base rate using:

(a) The **approximated** change in average rate differential method.
(b) The extension of exposures method.

Row	T	U	V	W	X	Y	Z	AA	AB	AC	AD
25				=XLOOKUP($T28,$T$36:$T$38,$U$36:$U$38)					=Y28*Z28		
26				Current relativity		Proposed relatives		=$U$45*(PRODUCT(V28:X28)+PRODUCT(V29:X29))		=V28*AB28*$U$48	
27	Class	Territory	Exposures	Class	Territory	Class	Territory	Current variable premium	Combined proposed relativities	Proposed premium with seed base rate	
28	A	X	50	1.00	1.00	1.00	1.00	$57,000	1.00	$50.00	
29	A	Y	80	1.00	0.80	1.00	0.80		0.80	$64.00	
30	B	X	120	1.20	1.00	1.40	1.00	$103,200	1.40	$168.00	
31	B	Y	65	1.20	0.80	1.40	0.80		1.12	$72.80	
32	C	X	90	1.90	1.00	1.75	1.00	$142,500	1.75	$157.50	
33	C	Y	75	1.90	0.80	1.75	0.80		1.40	$105.00	
35	Class	Current Relativity	Proposed Relativity	Relativity change factor							
36	A	1	1	1.000	=V36/U36				Class avg relativity change factor =	1.020	=(W36*AA28+W37*AA30+W38*AA32)/SUM(AA28:AA33)
37	B	1.2	1.4	1.167							
38	C	1.9	1.75	0.921					Class off-balance factor =	0.981	=1/AC36
40	Territory	Current	Proposed						Proposed base rate =	$539.40	=U47/U46*U45*AC38
41	X	1	1								
42	Y	0.8	0.8								
43									Proposed avg premium with seed base rate =	$1.286	=SUM(AC28:AC33)/SUM(V28:V33)
44	Target rate increase =	10%									
45	Current base rate =	$500							Proposed base rate =	$539.40	=U48*(U47/AC43)
46	Current average premium =	$630.625									
47	Proposed average premium =	$693.688	=(1+U44)*U46								
48	Seed base rate =	$1									