6.6.1 → Frequency-Severity Technique #1

→ Assumptions + uses

→ Frequency–Severity technique #1 applies the development technique separately to claim counts + claim severities. As such, its first assumption is closely related to the first key assumption for the development technique.

→ Here are the three key assumptions for this technique:

→ 1) Claim counts + severities recorded to date will continue to develop in a similar fashion in the future.

→ 2) Claim counts have a consistent definition throughout the experience period.

→ For instance, it is not appropriate to combine claimant counts w/ occurrence counts, where all claimants under a single occurrence are recorded as one claim, unless the method of counting is consistent between the two.

→ 3) The mix of claim types is reasonably homogeneous

→ for instance, it would not be reasonable to combine the average values of slip-and-fall claims w/ those from class action lawsuits involving thousands of insured parties, as the resulting average would have little significance.

→ The second + third assumptions are common between all three of the frequency-severity techniques we will discuss.

→ Technique

→ Frequency-Severity #1 technique (FS #1) can be broken into four main steps:

→ 1) Project + select ultimate claim counts

→ 2) Project + select ultimate severity

→ 3) Project ultimate claims as ultimate counts times ultimate severity

→ 4) Compute unpaid claim / IBNR estimate (if needed)

→ Essentially, we're just performing the development technique on claim counts + severities separately to get ultimate claim counts + ultimate severities. Then, for each AY, we project the ultimate claims by multiplying ultimate claim counts by ultimate severity. Finally, we subtract paid claims to date to estimate unpaid claims or subtract reported claims to date to estimate IBNR (broad definition).

→ We'll demonstrate how this method can be used w/ an example.

→ Examples

→ An actuary at BIG Auto Insurance Company gathers the following data:

Accident Half-Year	Cumulative Reported Claims in $000s as of (months)					
	6	12	18	24	30	36
2005–1	511	983	1,214	1,362	1,384	1,384
2005–2	560	983	1,212	1,364	1,385	
2006–1	509	997	1,210	1,359		
2006–2	574	977	1,212			
2007–1	514	985				
2007–2	558					

Accident Half-Year	Cumulative Reported Claim Counts as of (months)					
	6	12	18	24	30	36
2005–1	400	759	935	1,048	1,065	1,065
2005–2	452	763	937	1,052	1,068	
2006–1	405	772	934	1,047		
2006–2	458	760	938			
2007–1	407	762				
2007–2	449					

Assume no further development after 36 months, and that the trend rate for both claim counts and average claim cost is 0%.

Compute the IBNR for each accident half-year using a frequency-severity technique.

→ Step 1: Project + select ultimate claim counts

→ Start by performing the development method on the reported claim counts. Data is aggregated by accident half-years. For example, "2005–1" represents the first 6 months in 2005 (January through June), + "2005–2" represents the last 6 months of 2005 (July through December).

→ The resulting development factors are given below.

Accident Half-Year	Reported Claim Counts Age-to-Age Factors				
	6-12	12-18	18-24	24-30	30-36
2005–1	1.898	1.232	1.121	1.016	1
2005–2	1.688	1.228	1.123	1.015	
2006–1	1.906	1.210	1.121		
2006–2	1.659	1.234			
2007–1	1.872				

→ Notice that there appears to be some seasonality w/in the first development period. The development factors for claims reported during the first half of the year are significantly higher than the development factors for claims reported during the second half of the year. If we go back + take a peek at the reported claim counts, the reported counts at 6 months are slightly higher for accident half years ending in December, confirming our suspicion.

→ This could be due to any number of reasons. Perhaps policyholders are more motivated to quickly report auto claims towards the end of the second half of the year b/c they want to make sure they have a usable vehicle for the winter months but are fine w/ walking / riding a bike during the summer months towards the end of the first half of the year.

→ Whatever the case is, it is clear that the 6-to-12 month development factors are materially different for different halves of the year. B/c of this, taking a straight average over all accident half-years ending would be inappropriate. Instead, we'll average over half-years ending in December since the only accident half-year that has yet to mature 12 months, i.e. 2007-2, ends in December.

$$(6 \text{ to } 12 \text{ factor}) = \frac{1.688 + 1.659}{2} = 1.674$$

→ Development factors do not appear to differ between halves of the year for any other maturity. So we can take a straight average for all other maturities. The results are summarized in the table below.

	Reported Claim Counts Development Factors				
	6-12	12-18	18-24	24-30	30-36
Selected	1.674	1.226	1.122	1.016	1

→ We can use the selected factors to project the ultimate claim counts for each accident half-year.

Accident Half-Year	CDF	Ultimate Counts
2005–1	1	1,065
2005–2	1	1,068
2006–1	1.016	1,063
2006–2	1.139	1,069
2007–1	1.397	1,064
2007–2	2.338	1,050

↳ $= 1.674 \times 1.226 \times 1.122 \times 1.016 \times 1$

→ Step 2: Project + select ultimate severity

→ Although we aren't given the severities, we can easily find them for each historical period by dividing cumulative reported claim amounts by cumulative reported claim counts: losses = claim counts × severity
↳ "claims"
⇒ $\text{Severity} = \frac{\text{Losses}}{\text{Claim counts}}$

Accident Half-Year	Cumulative Reported Severities as of (months)					
	6	12	18	24	30	36
2005–1	1,277.50	1,295.13	1,298.40	1,299.62	1,299.53	1,299.53
2005–2	1,238.94	1,288.34	1,293.49	1,296.58	1,296.82	
2006–1	1,256.79	1,291.45	1,295.50	1,297.99		
2006–2	1,253.28	1,285.53	1,292.11			
2007–1	1,262.90	1,292.65				
2007–2	1,242.76					

→ $= \frac{1,384}{1,065} \times 1000$ ↳ (units change)

→ Next, we apply the development method to the reported severities.

Accident Half-Year	Reported Severity Age-to-Age Factors				
	6-12	12-18	18-24	24-30	30-36
2005–1	1.014	1.003	1.001	1	1
2005–2	1.040	1.004	1.002	1	
2006–1	1.028	1.003	1.002		
2006–2	1.026	1.005			
2007–1	1.024				

→ We don't notice the same type of seasonality in the age-to-age factors for severity. So, we'll take the averages for each maturity as our selected factors.

	Reported Severity Development Factors				
	6-12	12-18	18-24	24-30	30-36
Selected	1.026	1.004	1.002	1	1

→ We can use the selected factors to project ultimate claim severity for each accident half-year.

Accident Half-Year	CDF	Ultimate Severities
2005–1	1	1,299.53
2005–2	1	1,296.82
2006–1	1	1,298.07
2006–2	1.002	1,294.45
2007–1	1.006	1,299.78
2007–2	1.032	1,282.24

→ Step 3: Project ultimate claims

→ multiply the projected ultimate counts by the projected ultimate severities for each accident half-year to get the ultimate claims estimates.

Accident Half-Year	Ultimate Counts	Ultimate Severities	Ultimate Claims
2005–1	1,065	1,299.53	1,384,000
2005–2	1,068	1,296.82	1,385,000
2006–1	1,063	1,298.07	1,380,437
2006–2	1,069	1,294.45	1,383,146
2007–1	1,064	1,299.78	1,383,227
2007–2	1,050	1,282.24	1,345,759

→ Step 4: Compute IBNR estimates

→ Finally, subtract the current reported claim amounts for each accident half-year from the ultimate claims projections to estimate the IBNR for each accident half-year.

Accident Half-Year	Ultimate Claims	Cumulative Reported Claim Amounts	Estimated IBNR
2005–1	1,384,000	1,384,000	**0**
2005–2	1,385,000	1,385,000	**0**
2006–1	1,380,437	1,359,000	**21,437**
2006–2	1,383,146	1,212,000	**171,146**
2007–1	1,383,227	985,000	**398,227**
2007–2	1,345,759	558,000	**787,759**

→ Notes → Though we only observe claim counts develop upward in the example above, note that it is not uncommon for reported claim counts to actually develop downward. This can occur in certain lines of business where a majority of claims are reported quickly, but some end up closing w/o payment. Depending on how they are defined, reported claim counts may or may not include claims closed w/o payment. If they do not, + more previously reported claims are closed w/o pay than new claims are reported, claim counts would decrease.

→ *(1.75 points)* Given the following:

Accident Year	Cumulative Total Closed Claim Counts			
	12 Months	24 Months	36 Months	48 Months
2007	550	700	800	840
2008	700	855	925	
2009	625	800		
2010	675			

Accident Year	Cumulative Claim Counts Closed with No Payment			
	12 Months	24 Months	36 Months	48 Months
2007	30	80	105	120
2008	55	105	130	
2009	35	60		
2010	40			

Accident Year	Projected Ultimate Severity per Claim Closed with Payment
2007	$3,350
2008	$3,400
2009	$3,275
2010	$3,450

- Assume no further closed claim development after 48 months.
- Use an all-year volume-weighted average for all factor selections.

Use the frequency-severity technique to estimate the ultimate claim amount for accident year 2010.

→ We need to estimate the ultimate claim amount for AY 2010 using the FS #1 technique. This approach requires consistency between claim count + severity definitions, so we must consider only claims closed w/ payment.

→ The data provides total closed claim counts + counts closed w/ no payment. To find the counts closed w/ payment, we subtract counts closed w/o payment from the total counts for each AY at each maturity.

→ Step 1: Derive paid claim counts

→ The number of claims closed w/ payment is:

paid counts = Total closed counts − Closed w/ no payment counts

Accident Year	12 mo	24 mo	36 mo	48 mo
2007	520	620	695	720
2008	645	750	795	
2009	590	740		
2010	635			

→ Step 2: Calculate volume-weighted LDFs for paid counts

→ using the formula:

$$LDF_{x-y} = \frac{\Sigma \text{ paid counts at } y}{\Sigma \text{ paid counts at } x}, \quad n_x = n_y \text{ (same \# of AYs in sum)}$$

→ 12–24: $\frac{620 + 750 + 740}{520 + 645 + 590} = 1.202$

→ 24–36: $\frac{695 + 795}{620 + 750} = 1.088$

→ 36–48: $\frac{720}{695} = 1.036$

→ Step 3: Project ultimate paid claim counts for AY 2010

$635 \times \underbrace{1.202 \times 1.088 \times 1.036}_{\text{CDF}} = 860.16$

→ Step 4: Apply severity for AY 2010

ultimate claims = $860.17 \times 3,450 = 2,967,642$

→ Alternative solution

→ Rather than directly developing paid counts, develop total + no-pay claims separately, then subtract to find paid counts. (This is just changing the order of operations: 1) Derive ⇒ Develop vs 2) Develop ⇒ Derive)

→ Step 1: Develop no-pay closed counts

Accident Year	12 mo	24 mo	36 mo	48 mo
2007	30	80	105	120
2008	55	105	130	
2009	35	60		
2010	40			

→ No pay LDFs:

→ 12–24: $\frac{80 + 105 + 60}{30 + 55 + 35} = 2.042$

→ 24–36: $\frac{105 + 130}{80 + 105} = 1.270$

→ 36–48: $\frac{120}{105} = 1.143$

→ AY 2010 projected no-pay counts:

$40 \times \underbrace{2.042 \times 1.270 \times 1.143}_{\text{CDF}} = 118.56$

→ Step 2: Develop total closed counts

Accident Year	12 mo	24 mo	36 mo	48 mo
2007	550	700	800	840
2008	700	855	925	
2009	625	800		
2010	675			

→ Total LDFs:

→ 12–24: $\frac{700 + 855 + 800}{550 + 700 + 625} = 1.256$

→ 24–36: $\frac{800 + 925}{700 + 855} = 1.109$

→ 36–48: $\frac{840}{800} = 1.050$

→ AY 2010 projected total closed counts:

$675 \times \underbrace{1.256 \times 1.109 \times 1.050}_{\text{CDF}} = 987.51$

→ Step 3: Derive paid counts

$987.51 - 118.56 = 868.95$

→ Step 4: Apply severity

$868.95 \times 3450 = 2,997,882$

→ Notes → Notice that this result differs from our earlier calculation (by ≈ 30,000). These methods are not consistent in that they won't produce the same answer, but both approaches are valid. Therefore, either method would receive full credit on the exam.

→ Assignment

→ Q1) Given the following:

Accident year	Cumulative Reported Claim Counts		
	12 Months	24 Months	36 Months
2009	210	312	320
2010	221	340	
2011	212		

Accident year	Cumulative Reported Claims (000s)		
	12 Months	24 Months	36 Months
2009	$1,175	$2,100	$2,375
2010	$1,210	$2,305	
2011	$1,215		

- Assume no reported claim count development or claim development after 36 months.

Use a frequency-severity technique to estimate the ultimate claim amount for accident year 2011.

→

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2		Accident Year	Cumulative Reported Claim Counts				Accident Year	Cumulative Reported Claims (000s)				Accident Year	Severity (000s)		
3			12 Months	24 Months	36 Months			12 Months	24 Months	36 Months			12 Months	24 Months	36 Months
4		2009	210	312	320		2009	$1,175	$2,100	$2,375		2009	$5.60	$6.73	$7.42
5		2010	221	340			2010	$1,210	$2,305			2010	$5.48	$6.78	
6		2011	212				2011	$1,215				2011	$5.73		
7															
8			=D4/C4												
9		Accident Year	Claim Count Age-to-Age Factors									Accident Year	Claims Age-to-Age Factors		
10			12-24	24-36	36-Ult								12-24	24-36	36-Ult
11		2009	1.486	1.026	1.000							2009	1.203	1.103	1.000
12		2010	1.538									2010	1.238		
13		2011													
14															
15		=C10	=AVERAGE(XLOOKUP(B17,$C$10:$E$10,$C$11:$E$13))												
16		Age-to-age	Factor									Age-to-age	Factor		
17		12-24	1.512									12-24	1.221		
18		24-36	1.026									24-36	1.103		
19		36-Ult	1.000									36-Ult	1.000		
20															
21		=LEFT(B17,2)&"-ult"	=PRODUCT(C17:$C$19)												
22		Age-to-ult	CDF									Age-to-ult	CDF		
23		12-ult	1.551									12-ult	1.346		
24		24-ult	1.026									24-ult	1.103		
25		36-ult	1.000									36-ult	1.000		
26															
27			=C6*C23												
28		Est ult reported claim counts =	328.8									Est ult reported claims =	$7.71		
29															
30								=C28*M28*1000							
31							Est ult reported claims amount =	$2,536,095							