2.6.0 → Overview

→ overview → up to this point, we have discussed adjustments for estimating each of the individual components in the fundamental insurance equation below during the ratemaking process.

Premium = Losses + LAE + UW Expenses + Profit

→ In this section, we look at how to incorporate each of these components to determine whether the current rates being charged are appropriate for meeting the profit target at the aggregate level. Specifically, we will cover two general approaches for determining the overall rate indication:

→ 1) Pure Premium Method

→ 2) Loss Ratio Method

2.6.1 → Pure Premium Method

→ Overview → Under the pure premium method, we use pure premium to calculate the indicated average rate per exposure, i.e. the average rate that should be charged as indicated by our analysis. The formula for this method is:

$$\text{Indicated Avg rate} = \frac{\text{Pure Premium including LAE} + \text{Fixed UW expense per exposure}}{1 - \text{Variable UW expense \%} - \text{Target UW Profit \%}}$$

→ This is also referred to as the indicated average premium per exposure.

→ Recall that we derived this formula in section 2.5.2. As a refresher, start w/ the fundamental insurance equation:

→ Premium = Losses + LAE + UW Expenses + Profit

→ let P_I = Indicated premium

L = Losses

E_L = LAE

E_F = Fixed UW expenses

V = Variable UW expenses

Q_T = Target UW Profit %

X = Exposures

→ then we can solve for the indicated premium as:

$$P_I = L + E_L + (E_F + V P_I) + Q_T P_I$$

$$P_I - V P_I - Q_T P_I = L + E_L + E_F$$

$$P_I (1 - V - Q_T) = \downarrow$$

$$P_I = \frac{L + E_L + E_F}{1 - V - Q_T}$$

→ Divide by the # of exposures to find the indicated average premium per exposure:

$$\frac{P_I}{X} = \frac{(L + E_L + E_F)/X}{1 - V - Q_T}$$

$$\bar{P}_I = \frac{\bar{L} + \bar{E}_L + \bar{E}_F}{1 - V - Q_T}$$

→ note that the denominator is the variable permissible loss ratio.

→ Lets try out a simple example

→ Examples

→

You are given the following information:

Accident Year	Trended Ultimate Loss & LAE	Earned Exposures
2015	$129,300	1,525
2016	$146,800	1,810
2017	$152,100	1,730
2018	$157,400	1,845

- Fixed expense per exposure = $15
- Variable expense ratio = 10%
- Target profit ratio = 3%

Calculate the indicated rate per exposure.

→ We can use the loss + LAE + earned exposure data to find pure premium including LAE. We will use a weighted average of the data, although a 4-year straight average would also be valid

Pure Premium including LAE = $\frac{129,300 + 146,800 + 152,100 + 157,400}{1525 + 1810 + 1730 + 1845}$

↓

$= \$84.75$

→ now we can calculate the indicated rate per exposure as:

$$\bar{P}_I = \frac{\bar{L} + \bar{E}_L + \bar{E}_F}{1 - V - Q_T}$$

$$= \frac{84.75 + 15}{1 - 0.1 - 0.03}$$

↓

$= \$114.65$

→

You are given the following information:

Calendar / Accident Year	Reported Loss and LAE	Earned Exposures
2012	$2,258,000	3,600
2013	$2,346,000	3,850
2014	$2,895,000	4,910
2015	$2,921,000	5,600

	Age-to-Age Development Factor
12-24 Months	1.23
24-36 Months	1.16
36-48 Months	1.09
48-60 Months	1.03
60 Months to Ultimate	1.01

- Losses are evaluated as of December 31, 2016
- Annual severity trend = 2.5%
- Annual frequency trend = -1.0%
- Fixed expense = $25 per exposure
- Variable permissible loss ratio = 78%
- All policies are annual
- The future policy period begins July 1, 2017
- The proposed rates will be in effect for 12 months

Determine the indicated rate.

→ First, develop each year's reported loss + LAE. We can find the cumulative development factors by multiplying the age-to-age development factor for a given age by all subsequent factors.

	Cumulative Development Factor
12 Months to Ultimate	1.6179
24 Months to Ultimate	1.3154
36 Months to Ultimate	1.1339
48 Months to Ultimate	1.0403
60 Months to Ultimate	1.0100

→ Then, determine the age of each year's reported loss + LAE, & multiply by the appropriate CDF to determine ultimate losses + LAE.

Calendar / Accident Year	Reported Loss and LAE	Age as of 12/31/2016	CDF	Ultimate Losses and LAE
2012	$2,258,000	60 months	1.0100	$2,280,580
2013	$2,346,000	48 months	1.0403	$2,440,544
2014	$2,895,000	36 months	1.1339	$3,282,719
2015	$2,921,000	24 months	1.3154	$3,842,153

→ Next, we can trend ultimate losses + LAE. The severity trend is 2.5%, & the frequency trend is -1.0%. So, the pure premium trend is

Pure Premium trend = $(1 + \text{sev trend})(1 + \text{freq trend}) - 1$

$= (1 + 0.025)(1 - 0.01) - 1$

↓

$= 1.475\%$

→ Since we are trending losses, the trend period is from the average accident date of each Calendar/accident year (i.e. July 1 of each year) to the future average accident date. Since rates are in effect from 7/1/2017 to 6/30/2018 & policies are annual, losses can occur between 7/1/2017 & 6/30/2019. So, the future average accident date is 7/1/2018.

Calendar / Accident Year	Ultimate Losses and LAE	Trend Period	Trend Factor	Trended Ultimate Losses and LAE
2012	$2,280,580	6 years	$(1.01475)^6 = 1.0918$	$2,490,002
2013	$2,440,544	5 years	$(1.01475)^5 = 1.0760$	$2,625,923
2014	$3,282,719	4 years	$(1.01475)^4 = 1.0603$	$3,480,727
2015	$3,842,153	3 years	$(1.01475)^3 = 1.0449$	$4,014,688

→ Now, we can calculate pure premium including LAE. I will use a 4-year weighted average, although other averages may also be reasonable.

Pure Premium including LAE = $\frac{2,490,000 + \cdots + 4014,688}{3,600 + \cdots + 5,600}$

↓

$= \$702.19$

→ We're given that the fixed expense per exposure is 25, & the variable permissible loss ratio is 78%. Therefore, we can calculate the indicated rate as

Indicated rate = $\frac{702.19 + 25}{0.78}$

↓

$= \$932.30$

→

Given the following information for a Homeowners book of business in State X:

- All policies are annual.
- Proposed effective date is January 1, 2011 and rates will be in effect for twelve months.
- Selected current loss trend is 3%.
- Selected projected loss trend is 2%.

Calendar Accident Year	Earned Exposures	Non-Cat Reported Losses and Paid ALAE	Loss Development Factor to Ultimate	ULAE Factor	Amount of Insurance Years (AIY) to Earned Exposure Exponential Fit
2005	5,400	$2,025,000	1.0000	1.050	$270.00
2006	8,600	$3,440,000	1.0500	1.050	$283.50
2007	9,600	$3,408,000	1.1000	1.050	$297.68
2008	10,000	$5,400,000	1.1500	1.050	$312.56
2009	11,000	$5,500,000	1.2000	1.050	$328.19
2010					$344.60
2011					$361.83
2012					$379.92

- Selected fixed expense using expense data through 2009 = $47 per exposure
- Variable expenses = 14.4% of premium
- Fixed expense trend = 3.0%
- Profit and contingency provision = 9% of premium
- State X experience is fully credible
- Modeled catastrophe pure premium = $35.15
- Arithmetic average of last 20 years' non-modeled cat to AIY ratio = 0.370
- Expected reinsurance recoveries from a reinsurance contract with coverage from June 1, 2011 to June 1, 2012 = $350,000
- Cost of reinsurance = $680,000
- Assume no projected growth in exposures

(a) Calculate the projected ultimate non-catastrophe pure premium.

(b) Calculate the projected non-modeled catastrophe pure premium.

(c) Calculate projected net reinsurance cost per exposure.

(d) Calculate the projected fixed expense per exposure.

(e) Calculate the indicated rate.

→ a) → To determine the projected ultimate non-catastrophe pure premium, we will first need to develop & trend the non-cat losses + ALAE + apply a load for ULAE. We are given that the loss development factors to ultimate, the ULAE factors, & the current & projected loss trends. So, we will just need to determine the trend periods

→ For the current trend step, we trend from the midpoint of each calendar accident year (i.e. July 1 of each year) to the midpoint of the latest time period (i.e. 7/1/2009).

→ For the projected trend step, we trend from the midpoint of the latest time period (i.e 7/1/2009) to the midpoint of the future time period. Losses can occur between 1/1/2011 & 12/31/2012 for annual policies w/ effective dates between 1/1/2011 & 12/31/2011. So, the midpoint of the future time period is 1/1/2012, which means the projected trend period is 2.5 years.

→ Then, calculate the projected ultimate non-cat loss + LAE for each year as

Non-cat Loss & ALAE × LDF to Ult × ULAE factor × Current trend factor × Projected trend factor

→ Divide by exposures to find the projected ultimate non-cat pure Premium.

Calendar Accident Year	Projected Ultimate Non-Cat Loss and LAE	Projected Ultimate Non-Cat Pure Premium
2005	$2{,}025{,}000\,(1.00)\,(1.05)(1.03)^4(1.02)^{2.5} = 2{,}514{,}570$	$\frac{2{,}514{,}570}{5{,}400} = 465.66$
2006	$3{,}440{,}000\,(1.05)\,(1.05)(1.03)^3(1.02)^{2.5} = 4{,}354{,}609$	$\frac{4{,}354{,}609}{8{,}600} = 506.35$
2007	$3{,}408{,}000\,(1.10)\,(1.05)(1.03)^2(1.02)^{2.5} = 4{,}387{,}897$	$\frac{4{,}387{,}897}{9{,}600} = 457.07$
2008	$5{,}400{,}000\,(1.15)\,(1.05)(1.03)^1(1.02)^{2.5} = 7{,}056{,}975$	$\frac{7{,}056{,}975}{10{,}000} = 705.70$
2009	$5{,}500{,}000\,(1.20)\,(1.05)(1.03)^0(1.02)^{2.5} = 7{,}281{,}715$	$\frac{7{,}281{,}715}{11{,}000} = 661.97$

→ Finally, we can select a projected ultimate non-cat pure Premium. Since there are more exposures in recent years compared to older years, I will use a 5 year exposure-weight

Projected ultimate non-cat PP = $\frac{2{,}514{,}570 + \cdots + 7{,}281{,}715}{5{,}400 + \cdots + 11{,}000}$

↓

$= \$593.90$

→ b) → I will assume the non-modeled cat to AIY ratios do not include ULAE, similar to the loss data given in the table. So, first load the average ratio for ULAE.

$1.05\,(0.370) = 0.3885$

→ Then, we need to determine the future average AIY per exposure. Since losses for the future policy period can occur between 1/1/2011 & 12/31/2012, take an average of the AIY per exposure values for 2011 & 2012.

Average AIY per exposure = $\frac{361.82 + 379.92}{2} = \$370.875$

→ Therefore

Projected non-modeled cat PP = $0.3885\,(370.875)$

↓

$= \$144.08$

→ c) → The net cost of reinsurance is the cost of reinsurance less the expected recoveries

Net reinsurance cost = $680{,}000 - 350{,}000 = \$330{,}000$

→ Then divide by the # of exposures to determine the projected net reinsurance costs per exposure. Since we are assuming no projected growth in exposures, use the latest earned exposures of 11,000.

Net reinsurance cost per exposure = $\frac{330{,}000}{11{,}000}$

↓

$= \$30$

→ d) → We're given that the fixed expense trend is 3%, & the selected fixed expense through 2009 is $47 per exposure. So, to project fixed expenses, we need to determine the trend period.

→ We're not told when fixed expenses are incurred. However, since most expenses are incurred at the start of a policy, we will assume this is the case for all fixed expenses in this problem. So, we trend from the average written date in 2009 (i.e. 7/1/2009) to the average written date of the new rates (i.e. 7/1/2011), which means the expense trend period is 2 years.

Projected fixed expense per exposure = $47\,(1.03)^2$

↓

$= \$49.86$

→ e) Assume the non-cat, non-modeled cat, & modeled cat Pure Premiums to find the pure premium including LAE.

Pure Premium including LAE = $593.90 + 144.08 + 35.15$

↓

$= \$773.13$

→ To find the indicated rate, include the pure premium, fixed expense per exposure, & net reinsurance cost per exposure in the numerator. The denominator includes the variable expense provision & profit + contingency provision.

Indicated rate = $\frac{773.13 + 49.86 + 30}{1 - 0.144 - 0.09}$

↓

$= \$1087.46$

→ Assignment

→ Q1)

You are given:

Calendar / Accident Year	Reported Loss and ALAE	Earned Exposures
2003	2,000,000	2,000
2004	2,300,000	2,300
2005	1,900,000	1,900

- Losses are evaluated as of December 31, 2006
- Loss & ALAE Development Factors:

12 months to ultimate	1.400
24 months to ultimate	1.200
36 months to ultimate	1.050
48 months to ultimate	1.000

- Annual severity trend = +5.0% (trend is exponential)
- Annual frequency trend = -3.0% (trend is exponential)
- Commission = 10.0%
- Taxes = 7.0%
- Variable portion of General and Other Acquisition = 6%
- Total fixed expense = $20 per exposure
- Profit load = 2.0%
- All policies are annual
- Filed rates will be in effect for one year
- Proposed effective date for the rate change is October 1, 2007

(a) Determine the developed and trended Loss and ALAE by accident year.
(b) Determine the indicated pure premium. (Use a weighted average by earned exposures.)
(c) Determine the indicated rate.

→ a)

→ b)

→ c)

	B	C	D	E	F	G	H	I	J	K
1						=C3*F3		=(3*(1+I7)^H3		
2	Calendar / Accident Year	Reported Loss and ALAE	Earned Exposures	Age (as of 12/31/2006)	CDF	Developed loss and ALAE	Trend period	Trended ultimate loss and LAE		
3	2003	2,000,000	2,000	48	1.000	2,000,000	5.25	$2,202,041		
4	2004	2,300,000	2,300	36	1.050	2,415,000	4.25	$2,610,667		
5	2005	1,900,000	1,900	24	1.200	2,280,000	3.25	$2,419,960		
6				** Order of operations: Develop losses, then trend						
7	age-to-ultimate	factor				Pure Premium trend	1.85%	=(1+C14)^(1+C15)-1		
8	12 months to ultimate	1.400								
9	24 months to ultimate	1.200					Indicated pure premium and ALAE	$1,166.56	=SUM(I3:I5)/SUM(D3:D5)	
10	36 months to ultimate	1.050								
11	48 months to ultimate	1.000					Indicated rate	$1,582.08	=(I9+C22)/(1-SUM(C18:C21))	
12										
13	Trend	Factor								
14	Frequency	5%								
15	Severity	-3%								
16										
17	Fees	Ratio / amount per exposure								
18	Commision	10%								
19	Taxes	7%								
20	Variable portion of gener	6%								
21	Profit load	2%								
22	Total fixed expense (per	20								

Problem solving tip → Sometimes you just have to read the problem & choose an interpretation that makes sense. If you're worried that your assumption isn't correct, just state what your assumption is & move forward anyway. Unless your assumption is illogical, that's probably enough for full credit