3.3 → Special classification

3.3.2 → Increased limits ratemaking

- → Increased limits ratemaking is the process of determining increased limit factors (ILFs)
  - → why needed? → limited data leads to volatile results. multivariate approaches are better for sparse data, but can produce illogical rating factors (e.g. decrease in factor for a higher limit)
  - ⇒ have specialized approaches

→ Standard ILF approach

→ $Rate_H = ILF_{(H)} \times Rate_B$ → Assumptions
- → 1) All underwriting expenses are variable
- → 2) The variable expense + profit provisions do not vary by limit
- → 3) Frequency + severity are independent
- → 4) Frequency is the same for all limits

→ $\text{Indicated } ILF(H) = \frac{LAS(H)}{LAS(B)}$

→ LAS (H), or the limited average severity at H, is the severity assuming every loss is capped at limit H regardless of the actual policy limit. LAS(B), or the limited severity at B, is the severity assuming every loss is capped at the base limit.

$$\rightarrow LAS(H) = \frac{(\text{total losses of claims} \le H) + (\#\text{ claims} > H) \times H}{\text{Total } \#\text{ claims}}$$

→ Censored losses → for $H_2 > H_1$

$$\rightarrow LAS(H_2) = LAS(H_1) + P(X > H_1) \times LAS[(H_2 - H_1) \text{ xs } H_1]$$

where

$$\rightarrow P(X > H_1) = \frac{\#\text{ of claims} > H_1}{\#\text{ of policies w/ limits} > H_1}$$

&

$$\rightarrow LAS[(H_2 - H_1) \text{ xs } H_1] = \frac{\text{losses in layer between } H_1 + H_2}{\#\text{ of claims} > H_1}$$

→ Noticing that the numerator of $P(X > H_1)$ equals the denominator of $LAS[(H_2 - H_1) \text{ xs } H_1]$ allows us to take a shortcut:

$$LAS(H_2) = LAS(H_1) + \frac{\text{losses in the layer between } H_1 + H_2}{\#\text{ of claims in policies w/ limits} > H_1}$$

Limit

	Base = H_1 claims	Base = H_1 losses	H_2 claims	H_2 losses	H_3 claims	H_3 losses
$0 < x \le H_1$						
$H_1 < x \le H_2$						
$H_2 < x$						

$$\rightarrow LAS(H_1) = \frac{\overset{\langle \text{raw in that layer} \rangle}{\sum_{H_i, i \in 1,2,3} \left[\sum_{0 < x \le H_1} \text{losses} \right]} + \overset{\langle \text{capped above layers} \rangle}{\sum_{H_i, i \in 2,3} H_1 \times (\#\text{ claims } H_1 \le x < H_3)}}{\sum_{All} \text{claims}}$$

$$\rightarrow LAS((H_2 - H_1) \text{ xs } H_1) = \frac{\left(\overset{\langle \text{raw in that layer} \rangle}{\sum_{H_i, i=2,3} \left[\sum_{H_1 < x \le H_2} \text{losses} \right]} + \overset{\langle \text{capped above layers} \rangle}{\sum_{H_3} H_2 \times \#\text{ claims } (H_2 \le x < H_3)} \right) - \overset{\langle H_1 \text{ accounted for} \rangle}{H_1 \times \#\text{ claims used in } (\,)}}{\sum \text{claims used in } (\,)}$$

$$\rightarrow ILF(H_2) = LAS(H_1) + \underline{P(H_1 \le x < H_2)} \times LAS((H_2 - H_1) \text{ xs } H_1)$$

$$\hookrightarrow = \frac{\sum \text{claims used in } (\,)}{\sum_{H_i, i=2,3} \text{claims}}$$

$$\rightarrow LAS((H_3 - H_2) \text{ xs } H_2) = \frac{\overset{\langle \text{raw} \rangle}{\sum_{H_3} \left[\sum_{H_2 \le x \le H_3} \text{losses} \right]} - \overset{\langle H_2 \text{ accounted for} \rangle}{H_2 \times \#\text{ claims used in } (\,)}}{\#\text{ claims used in } (\,)}$$

$$\rightarrow ILF(H_3) = LAS(H_2) + \underline{P(H_2 \le x < H_3)} \times LAS((H_3 - H_2) \text{ xs } H_2)$$

$$\hookrightarrow = \frac{\#\text{ claims used in } (\,)}{\sum_{H_3} \text{claims}}$$

→ Fitted data approach
- → Due to typically limited data on large claims, the standard ILF approach may produce results that are volatile. The random fluctuations in empirical data can be smoothed out by fitting curves to the empirical data. Commonly used distributions are lognormal, pareto, + truncated pareto.
- → Let f(x) denote the continuous distribution of losses of size (x) + H be the limit being priced. The formula for LAS(H) is:

$$\rightarrow LAS(H) = \int_0^H x f(x)\,dx + \int_H^\infty H f(x)\,dx \rightarrow \text{just the expected value of a function of a random variable}$$

$$\Rightarrow \text{let } f(x) = x \text{ \& let } g(x) = \begin{cases} x & x < H \\ H & x \ge H \end{cases} \Rightarrow LAS(H) = E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

- → The first term includes all losses below \$10,000 at their actual loss amounts multiplied by the probability of these claims occurring, while the second term represents the limit, \$10,000 multiplied by the probability that losses exceed the limit.
- → thus, the increased limit factor for limit H is represented as follows:

$$\rightarrow ILF(H) = \frac{\int_0^H x f(x)\,dx + \int_H^\infty H \cdot f(x)\,dx}{\int_0^B x f(x)\,dx + \int_B^\infty B f(x)\,dx}$$

- → The main challenge w/ this approach is identifying a distribution that accurately represents the expected losses.

→ Multivariate approach
- → Techniques like GLMs are effective at handling sparse data, though high, sparsely populated limits still pose challenges
- → GLMs don't assume a uniform frequency across risks, which allows them to account for both loss capping + behavioral differences at different limits, which can occasionally result in illogical ratings

3.3.3 → Deductible pricing

→ overview → there are two types of deductibles: fixed dollar + percentage
- → Deductibles are popular b/c
  - → premium reduction (from insured's perspective)
  - → Eliminates small nuisance claims
  - → Provides incentive for loss control (financial incentive for insureds + prevent losses since they are partially responsible for paying)
  - → Controls catastrophe exposure

→ Deductible relativities, found via the Loss elimination ratio (LER) approach
- → Assuming all expenses are variable + the variable expenses + profit are a constant percentage of premium, the indicated deductible relativity for going from one deductible level, B, to a higher one, D can be calculated as

$$\rightarrow \text{Indicated deductible relativity} = \frac{(L + E_L)_D}{(L + E_L)_B} = \langle \ldots \rangle = 1 - LER(D)$$

where $(L+E_L)_B$ + $(L+E_L)_D$ are the losses + LAE net of the deductible B + D, respectively.

→ the loss elimination ratio → $LER(D) = \frac{(L+E_L)_B - (L+E_L)_D}{(L+E_L)_B}$
- → represents the proportion of losses eliminated by going from one deductible to another. Speak differently, the loss elimination ratio is one minus the proportion of losses that remain after switching deductible.
- → Notes → for policies w/ full coverage, set B = 0. Then the formulas essentially become

$$\rightarrow \text{Indicated deductible relativity} = \frac{\text{losses + LAE assuming deductible D}}{\text{Total ground up losses + LAE}}$$

&

$$\rightarrow LER(D) = \frac{\text{losses + LAE eliminated by Deductible D}}{\text{total ground-up losses + LAE}}$$

→ Two methods to calculate LER:
- → Method 1 → The ratio of losses eliminated by the deductible + the total ground-up losses
- → Method 2 → One minus the ratio of the losses that remain after applying the deductible to the ground-up losses

→ Net reported losses → the portion of reported losses that exceed the deductibles
- → when using net losses (as opposed to ground-up losses), data from policies w/ higher deductibles cannot be used to calculate the LER for lower deductibles.
  - → e.g.) Data from \$500 deductible cannot be used to calculate the LER for a \$250 deductible.
  - But the reverse works!

have: \$500 ded ↛ want: \$250 ded
want: \$500 ded ← have: \$250 ded

→ Fitted data approach
- → Assume f(x) denotes the continuous distribution of losses of size x + D is the size of the deductible. Assuming no deductible in the base level policy, the losses eliminated can be calculated as:

$$\rightarrow \text{Eliminated loss} = \int_0^D x f(x)\,dx + \int_D^\infty D f(x)\,dx \xrightarrow{\text{this is just}} \text{if } g(x) = \begin{cases} x & x < D \\ D & x \ge D \end{cases} \Rightarrow E(g(x))$$

- → Then the LER is the eliminated loss divided by the expected loss:

$$\rightarrow LER(D) = \frac{\text{Eliminated loss}}{\text{Expected loss}}$$

$$\downarrow \;= \frac{\int_0^D x f(x)\,dx + \int_D^\infty D f(x)\,dx}{\int_0^\infty x f(x)\,dx} \Rightarrow = \frac{E(g(x))}{E(x)}$$

→ Practical considerations → the following should be considered when using the LER approach:
- → 1) LER calculations assume claiming behavior will be the same for each deductible + ignore the differences
  - → e.g.) under a \$250 ded, a claim of \$1100 will likely be reported. But for a \$1000 ded, that same loss may not be reported as the payment of \$100 doesn't outweigh the premium surcharge for having a claim
- → 2) LER does not recognize the behavioral differences that arise when insureds are allowed to choose their policy deductible
  - → e.g.) low risk insureds ⇒ often choose high deductibles (believe unlikely to file claims) ⇒ often higher ded policies are more profitable for insurers than lower ded policies
- → univariate + multivariate ARE able to capture these behavioral differences
  - ⇒ Good idea to use multiple methods before selecting final credits
- → Percentage credits → if LER for a \$1000 ded is say 35% ⇒ 35% of the total losses will be eliminated by moving to this ded.
  - → say the policy premium is \$5000, then this should equate to a premium credit of \$5000 × 0.35 = \$1750, but
  - → BUT, this is much larger than the deductible (which is the amount of money the insurer saves per policy (given a claim))
    - ⇒ Insurer can remedy this by:
      - → capping the credit at say \$750
      - → Calculating different set of credits for different policies (e.g. different credits for low, medium, + high valued homes), segregating risks helps minimize the likelihood of such scenarios
      - → use % deductibles, which allows the credit to vary by coverage amount

3.3.4 → Workers Compensation size of risk

→ ⟨... maybe read? ...⟩

3.3.5 → Insurance to value

- → Insurance to value (ITV) describes the relationship between the chosen amount of insurance + the total value or replacement cost of the insured item.
  - → this is what insurers should charge rates on to ensure equitable + adequate rates.
- → Issues that arise when properties (for homeowners insurance) are not insured to their full value of replacement cost
  - → 1) the insurance payout will not adequate to cover a total or near-total loss, leaving the insured unable to fully recover their pre-loss condition
    - → e.g.) If house is \$250K, but only insured for \$200K, a loss > \$200K will be capped at \$200K, which doesn't cover the entire loss
  - → 2) If the insurer bases the rate on the assumption that all homes are insured to their full value, the premium charged will not be sufficient to cover the expected loss on underinsured homes
- → Importance of ITV
  - → when properties are not insured to the same level ⇒ rate inequity
  - → And if all homes are underinsured by the same percentage ⇒ the resulting premium may not be adequate to cover all losses, but it would be equitable
    - → over time, the insurer will raise the base rate so that the aggregate premium covers the expected aggregate losses.
    - → However, in this scenario, fully insured homes end up bearing part of the cost of the underinsured homes.
  - → It is important to note that the issues of inequity + inadequacy of rates only exist if partial losses are possible. If all losses are total losses, the total premium collected will be adequate to cover all losses, + rates will be equitable, even if some homes are underinsured
  - → Typically, most losses are partial rather than total. To ensure rates that are both equitable + adequate, insurers should charge rates based on the ITV level.
- → other ITV initiatives
  - → offer guaranteed replacement cost (GRC), usually w/ a cap
  - → use more sophisticated tools to estimate replacement costs
  - → Implement property inspections, indexation clauses, & education of insureds
- → Coinsurance clause
  - → In addition to adjusting rates based on the ITV level, insurers have another tool at their disposal to achieve greater equity among risks: coinsurance. Coinsurance refers to two or more parties jointly covering a portion of the insured risk.
  - → In property insurance, coinsurance is a policy provision where the insured agrees to maintain coverage at a specific ITV level (usually 80% or 90%). If the insured fails to meet this requirement, they may be subject to a penalty in the event of a claim, where insurance payout is reduced proportionally according to the degree of underinsurance.
  - → Assume a required coinsurance percentage (c) + a policy is underinsured (ie F < cV). To determine the indemnity payment, we first need to calculate the apportionment ratio (a), which is the ratio of the coverage amount (or face value, F) to the coinsurance requirement (cV).

$$\rightarrow a = \min\left(\frac{F}{cV}, 1\right) = \begin{cases} \frac{F}{cV}, & F < cV \\ 1, & F \ge cV \end{cases}$$

  - → Then, the indemnity payment (I) is the (100a)% of the loss (L), capped at face value

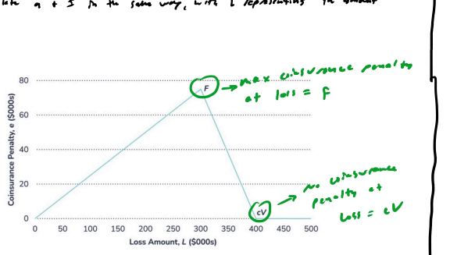
$$\rightarrow I = \min(aL, F) = \begin{cases} aL, & L < cV \\ F, & L \ge cV \end{cases}$$

  - → There will be a coinsurance penalty (e) when the policy is underinsured (ie F < cV) + experiences a partial loss less than the coinsurance requirement (i.e. L < cV)

$$\rightarrow e = \min(L, F) - I = \begin{cases} (1-a)L, & L \le F < cV \\ F - aL, & F \le L < cV \\ 0, & \text{otherwise} \end{cases}$$

  - → Notes → The situation above does not account for deductibles. If the policy includes a deductible, D, we calculate a + I in the same way, with L representing the amount of loss before the deductible. In this case, I is the indemnity payment before the deductible

→ V = true value
→ F = coverage amount, F < V + cV
→ cV = coinsurance requirement
→ c = % underinsured (relative to coinsurance requirement)
→ I = what the insurer pays
→ e = what the insured pays
→ when the loss is greater than the coinsurance requirement, there is no coinsurance penalty



→ Insurance to value → policyholder covers portion of loss greater than the face value
- ⇒ when this is the case, coinsurance penalty ≠ policyholder payment

Review example of underinsured

- → Coinsurance promotes adequate rates ⇒ the insurer will not be exposed to unknown levels of risk + can rate similar risks accordingly
- — — — equitable — — ⇒ share losses w/ underinsureds so ITV insureds do not have excessive rates

3.4 → Credibility

3.4.1 → measuring credibility

- → Necessary criteria for measures of credibility
  - → 1) $0 \le Z \le 1$
  - → 2) As n increases, Z should increase
  - → 3) As n increases, Z should increase at a decreasing rate
- → Three commonly used methods for measuring credibility
  - → method 1 → Classical credibility (aka limited fluctuation credibility)
    - → Estimate = (Z × observed exp.) + (1 − Z) related experience
    - → simplifying assumptions:
      - → Exposures are homogeneous (i.e. each exposure has the same expected # of claims)
      - → claim occurrence follows a Poisson distribution, so E(Y) = V(Y)
      - → there is no variation in loss size (i.e. constant severity)
    - → The expected number of claims needed for full credibility is:

$$E(Y) = \left(\frac{z_{(p+1)/2}}{k}\right)^2 \hookrightarrow \text{will be given } p = 80\% \text{ \& } k = 5\%\text{, for example}$$

    - → If the number of observed claims is equal to or greater than the standard for full credibility, i.e. Y ≥ E(Y), then Z = 1. otherwise, Z is calculated using the square root rule:

$$Z = \sqrt{\frac{Y}{E(Y)}}, \text{ where } Y < E(Y)$$

    - → The number of exposures needed for full credibility is the number of claims needed for full credibility divided by the expected frequency. Then, use it w/ the # of observed exposures in the above square root equation

$$\#\text{ exposures} = \#\text{ claims} \times \text{Freq}$$

$$\hookrightarrow = \frac{\#\text{ exposures}}{\#\text{ claims}}$$

    - → Classical credibility has three main advantages:
      - → 1) It is the most commonly used credibility method
      - → 2) The data required is readily available
      - → 3) The calculations are very straightforward

→ using exposure-based credibility as opposed to claims-based would be beneficial if the definition of a claim has not been consistent over time

    - → The main disadvantages of classical credibility include the following:
      - → 1) The simplifying assumptions used in the derivation may not be true in practice
      - → 2) It does not take into account the quality of the related experience used as a complement of credibility
  - → Method 2 → Bühlmann credibility (aka least squares credibility)
    - → calculates the credibility-weighted estimate as follows:

$$\text{Estimate} = Z \times \text{observed Experience} + (1 - Z) \times \text{Prior mean}$$

    - → under Bühlmann credibility, Z is defined as follows:

$$Z = \frac{N}{N + K}$$

where
- → N = number of observations
- → K = the expected value of process variance (EVPV) divided by the variance of hypothetical means (VHM)

    - → Unlike classical credibility, Z calculated under the Bühlmann method will never equal 1.
    - → Bühlmann credibility is used w/in the insurance industry + is generally accepted. The major challenge w/ this approach is the determination of EVPV + VHM. Similar to classical probability, Bühlmann credibility relies on a lot of simplifying assumptions that must be assessed to determine if this approach is appropriate for the situation.
  - → Method 3 → Bayesian analysis
    - → Bayesian analysis, as the name implies, is based on the Bayes Theorem. This method has no specific calculation of Z. Instead, it adjusts the prior estimate to reflect new information in a probabilistic manner. Bayesian analysis is not commonly used due to its complexity.
    - → It is noteworthy that Bühlmann, or least squares credibility, corresponds to the weighted least squares line related to the Bayesian estimate. In specific mathematical cases, the Bayesian estimate is identical to the least squares credibility estimate.