3.6.2 → Loss ratio method

→ overview → under the loss ratio method, we use the loss ratio to calculate the indicated average rate change from the current rate level. The formula for this method is:

$$\boxed{\text{Indicated avg rate change} = \frac{\text{Loss + LAE ratio} + \text{Fixed UW Expense \%}}{1 - \text{Variable UW Expense \%} - \text{Target UW Profit \%}} - 1}$$

→ notes → the indicated average rate & indicated average rate change are not associated w/ a specific rating variable. They are the overall average rate or overall average rate change across all rating variables. note that this is implied, so the word "overall" is dropped from the terms.

→ we can derive the formula for the loss ratio method by starting w/ the indicated premium formula that we derived in the previous subsection.

$$P_I = \frac{L + E_L + E_F}{1 - V - Q_T}$$

→ let P_C represent the premium at the current rate level & F represent the fixed expense ratio. Then, we can find the indicated change factor as

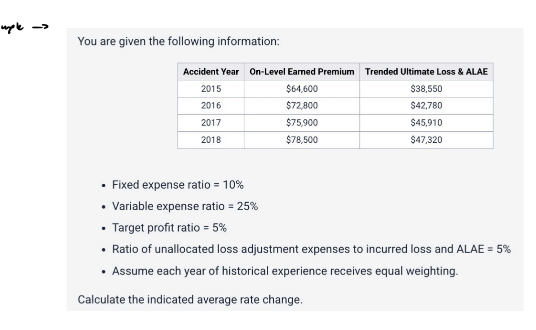
$$\text{Indicated change factor} = \frac{P_I}{P_C} = \frac{\frac{L + E_L}{P_C} + \frac{E_F}{P_C}}{1 - V - Q_T} = \boxed{\frac{\text{Loss + LAE ratio} + F}{1 - V - Q_T}}$$

→ this is the factor that the current rate needs to be multiplied by to find the indicated rate. Then, we can find the indicated average rate change under the LR method as

$$\boxed{\text{Indicated avg rate change} = \text{Indicated change factor} - 1 = \frac{\text{Loss + LAE ratio} + F}{1 - V - Q_T} - 1}$$

→ lets practice using this method in the following example.

→ Example →



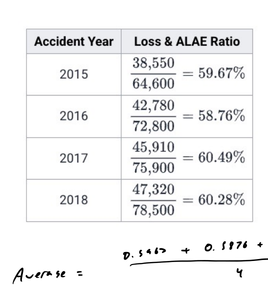
You are given the following information:

Accident Year	On-Level Earned Premium	Trended Ultimate Loss & ALAE
2015	$64,600	$38,550
2016	$72,800	$42,780
2017	$75,900	$45,910
2018	$78,500	$47,320

- Fixed expense ratio = 10%
- Variable expense ratio = 25%
- Target profit ratio = 5%
- Ratio of unallocated loss adjustment expenses to incurred loss and ALAE = 5%
- Assume each year of historical experience receives equal weighting.

Calculate the indicated average rate change.

→ First, calculate the loss ratios for each year, & find their average.



Accident Year	Loss & ALAE Ratio
2015	$\frac{38,550}{64,600} = 59.67\%$
2016	$\frac{42,780}{72,800} = 58.76\%$
2017	$\frac{45,910}{75,900} = 60.49\%$
2018	$\frac{47,320}{78,500} = 60.28\%$

$$\text{Average} = \frac{0.5967 + 0.5876 + 0.6049 + 0.6028}{4} = 59.80\%$$

order doesn't matter
⇒ can select loss from all years + ALAE + THEN load ULAE
OR
can load ULAE to all years & THEN select loss & LAE ratio

→ we're given that the ratio of unallocated loss adjustment expense to incurred loss + ALAE ratio is 5%. So, load the loss + LAE ratio for ULAE.

$$\text{Loss + LAE ratio} = 1.05(0.5980) = 62.79\%$$

→ Finally,

$$\text{Indicated avg rate change} = \frac{\text{Loss + LAE ratio} + F}{1 - V - Q_T} = \frac{0.6279 + 0.10}{1 - 0.25 - 0.05} - 1 = 3.99\%$$

→ Comparison of methods

→ There are two main differences between the pure premium method (PP method) & the LR method (LR method).

→1) → The underlying loss measure used in the calculations is different. The PP method utilizes pure premium & the LR method utilizes the loss ratio. This means that the premium at CRL is needed for the LR method, but not the PP method. In addition, clearly defined exposures are needed for the PP method, but not the LR method.

→ B/c of this, the PP method is typically used when premium at the CRL is not available or difficult to calculate accurately. For instance, if there are significant changes to rating variables for an insurance product w/ a complex rating algorithm, this can make it difficult to calculate on-level premiums.

→ on the other hand, the LR method is preferred when exposure data is not available or exposures are not clearly defined for an insurance product. For example, commercial general liability policies can have several sub-lines to cover a wide variety of risks, but these sub-lines may have different exposure bases, which can make it difficult to establish a consistent exposure measure.

→ 2) → The two methods result in different outputs. The PP method results in an indicated rate, while the LR method results in an indicated rate change.

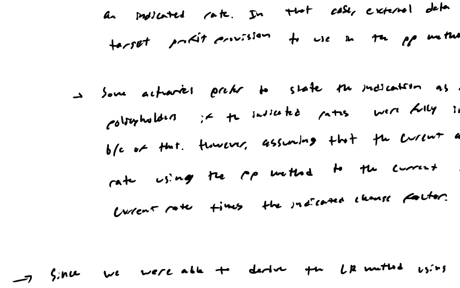
→ Since the LR method involves calculating the percent change from current rates, this method cannot be used for a new company that wants to determine an indicated rate. In that case, external data or judgement can be used to estimate expected PP + expense provisions + to select a target profit provision to use in the PP method.

→ Some actuaries prefer to state the indication as a percent change to existing rates, as this gives an approximation of the average impact on existing policyholders if the indicated rates were fully implemented (assuming no changes in policyholder retention). So, the LR method may be preferred b/c of that. However, assuming that the current average rate is available, the indicated rate change can still be found by comparing the indicated rate using the PP method to the current rate. Likewise, the indicated rate to charge can be found from the LR method by multiplying the current rate times the indicated change factor.

→ Since we were able to derive the LR method using the result from the PP method, these methods should be equivalent. However, this equivalency depends on whether consistent data + assumptions are used for both methods. For instance, if the premium at CRL is calculated using the parallelogram method, rather than the extension of exposures method (which is a more accurate approach), then the two methods might produce different results.

→ Examples

→



Given the following data for private passenger auto bodily injury basic limits, answer the questions below. Show all work.

- Policies are annual.
- Proposed Effective Date = July 1, 2005
- Rates are in effect for one year.
- Current Rate = $225

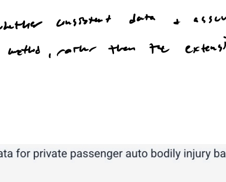
Calendar Accident Year	Earned Exposures	Loss & ALAE as of December 31, 2004
2002	450	$52,000
2003	500	$54,000
2004	530	$40,000

- Age-to-age loss development factors
  12-24 months = 1.50
  24-36 months = 1.15
  36-48 months = 1.05
  48-ultimate = 1.06
- Frequency trend = 2%
- Severity trend = 5%
- Target loss ratio = 65%

(a) Calculate the indicated statewide rate level change using the loss ratio method.

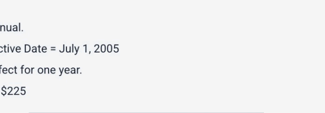
(b) Using your results from part (a) above, illustrate the equivalency of the loss ratio method and the pure premium method.

→ a) → We can first develop each year's loss + ALAE. Use the age-to-age loss development factors + find the cumulative development factors.



	Cumulative Development Factor
12 Months to Ultimate	1.9199
24 Months to Ultimate	1.2800
36 Months to Ultimate	1.1130
48 Months to Ultimate	1.0600

→ Then, determine the age of each year's loss + ALAE, + multiply by the appropriate CDF to determine ultimate losses + ALAE. Since we are not given any information on ULAE, assume that ultimate losses + LAE are equal to ultimate losses + ALAE (i.e. ULAE ratio = 0%).

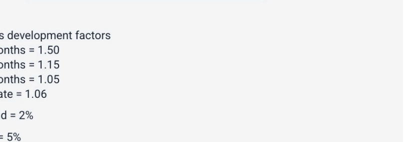


Calendar Accident Year	Loss and ALAE	Age as of 12/31/2004	CDF	Ultimate Losses and LAE
2002	$52,000	36 months	1.1130	$57,876
2003	$54,000	24 months	1.2800	$69,117
2004	$40,000	12 months	1.9199	$76,797

→ Next, we can trend ultimate losses + LAE. The severity trend is 5%, + the frequency trend is 2%. So the pure premium trend is

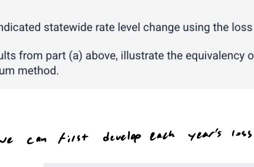
$$\text{PP trend} = (1+0.05)(1+0.02) - 1 = 7.10\%$$

→ Since we are trending losses, the trend period is from the average accident date of each calendar/accident year (i.e. July 1 of each year) to the future average accident date (i.e. 7/1/2006).



Calendar Accident Year	Ultimate Losses and LAE	Trend Period	Trend Factor	Trended Ultimate Losses and LAE
2002	$57,876	4 years	$(1.071)^4 = 1.3157$	$76,148
2003	$69,117	3 years	$(1.071)^3 = 1.2285$	$84,909
2004	$76,797	2 years	$(1.071)^2 = 1.1470$	$88,089

→ We're not given any premium trend or a way to calculate premium trend, so we can ignore that step. To find premium at the current rate level for a year, multiply earned exposures by the current rate.



Calendar Accident Year	On-Level Earned Premium
2002	$225\,(450) = \$101,250$
2003	$225\,(500) = \$112,500$
2004	$225\,(530) = \$119,250$

→ Now, we have the information needed to calculate the loss + LAE ratio. I will use a 3-year weighted average.

$$\text{Loss + LAE ratio} = \frac{76,148 + 84,909 + 88,089}{101,250 + 112,500 + 119,250} = 74.82\%$$

→ We are given that the target loss ratio (↳ i.e. PLR) is 65%. Therefore

$$\text{Indicated rate change} = \frac{0.7482}{0.65} - 1 = 15.11\% \Rightarrow \text{new rate} = \text{CRL} \times \text{Indicated change factor} = 225\,(1.1511) \approx \$258.99$$

→ b) → Use a 3-year weighted average to determine the PP including LAE.

$$\text{PP including LAE} = \frac{76,148 + 84,909 + 88,089}{450 + 500 + 530} = \$168.34$$

→ The target loss ratio is 65%, which means the indicated rate is

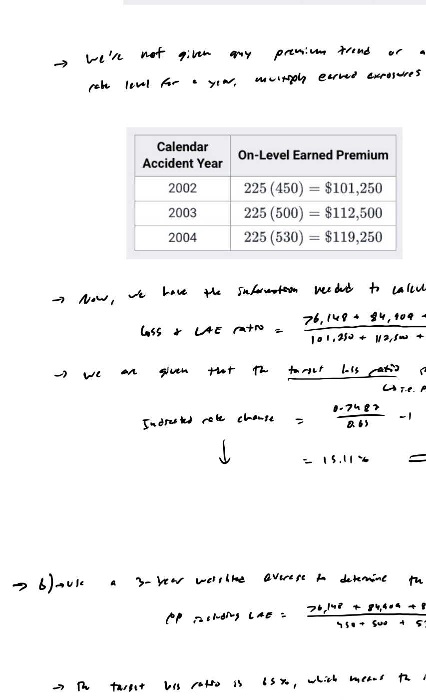
$$\text{Indicated rate} = \frac{168.34}{0.65} = \$258.99$$

→ Therefore,

$$\text{Indicated rate change} = \frac{258.99}{225} - 1 = 15.11\%$$

→ So, the LR method + the PP method result in the same indicated rate change.

→



You are given the following information:

- Policies are annual.
- Proposed effective date = October 1, 2018.
- Rates are in effect for one year.
- Policies are assumed to be written uniformly throughout the year.
- Loss trend = 3%.
- Premium trend = 0%.

Accident Year	Estimated Ultimate Losses (in thousands)
2015	$970
2016	$967
2017	$956

Calendar Year	Earned Premium (in thousands)
2015	$1,122
2016	$1,155
2017	$1,281

Effective Date	Overall Average Rate Change
Initial	-
7/1/2015	+3%
1/1/2017	+5%

- Assume each historical loss ratio receives equal weighting.
- All expenses are variable.
- Permissible loss ratio = 80%.

Calculate the indicated average rate change.

→ Using the LR method, the indicated average rate change is:

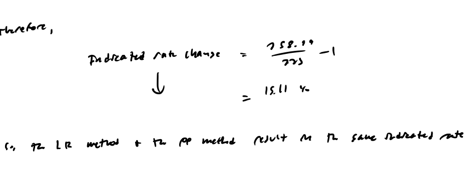
$$\text{Indicated avg rate change} = \frac{\text{Loss + LAE ratio} + F}{1 - V - Q_T}$$

→ We are given that all expenses are variable, which means $F = 0$. In addition, the PLR is 80%, so $1 - V - Q_T = 0.80$. Therefore,

$$\text{Indicated avg rate change} = \frac{\text{Loss + LAE ratio}}{0.80} - 1$$

→ To calculate the projected loss + LAE ratio, we need to trend ultimate losses + bring premiums to CRL.

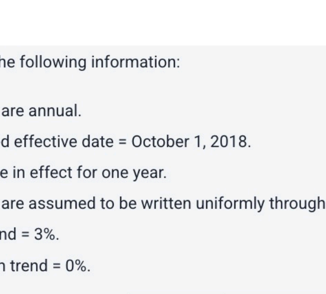
→ We'll start by calculating the trended ultimate losses for each AY.



Accident Year	Trend From Date	Trend To Date	Trend Period	Trended Ultimate Losses (in thousands)
2015	7/1/2015	10/1/2019	4.25	$1,100
2016	7/1/2016	10/1/2019	3.25	$1,065
2017	7/1/2017	10/1/2019	2.25	$1,022

→ Next, calculate the EPs at CRL using the parallelogram method

→ Draw a diagram to illustrate rate changes, + calculate the cumulative rate level index for each rate group.



Rate Group	Effective Date	Cumulative Rate Level Index
A	Initial	1.0000
B	7/1/2015	1.0300
C	1/1/2017	1.0815

→ Compute the portion of the premium for each CY that corresponds to each rate group. Then, calculate the average cumulative rate level indexes.

Calendar Year	Portion			Average Cumulative Rate Level Index
	A	B	C	
2015	0.875	0.125	-	1.00375
2016	0.125	0.875	-	1.02625
2017	-	0.500	0.500	1.05575

→ Calculate the on-level factors + use them to calculate the EPs at the CRL. The values are tabulated below.

Calendar Year	On-Level Factor	Premium at Current Rate Level (in thousands)
2015	1.0775	$1,209
2016	1.0538	$1,217
2017	1.0244	$1,312

→ We can now calculate the LR for each year.

Year	Trended Ultimate Losses (in thousands)	Premium at Current Rate Level (in thousands)	Loss Ratio
2015	$1,100	$1,209	0.9098
2016	$1,065	$1,217	0.8746
2017	$1,022	$1,312	0.7786

→ Then, find the projected loss ratio for the forecast period as the average of the LRs.

$$\text{Loss + LAE ratio} = \frac{0.9098 + 0.8746 + 0.7786}{3} = 0.8543$$

→ Finally, we can calculate the indicated average rate change as

$$\text{Indicated avg rate change} = \frac{0.8543}{0.80} - 1 = 6.79\%$$

→

A company is reviewing the rate level adequacy. Given the following information for a book of business:

- All policies are annual.
- Current rates have been in effect for three years.
- New rates will be in effect for 18 months beginning on July 1, 2013.
- Annual premium trend = -1%.
- Annual loss trend = +3%.
- Loss adjustment expense provision = 2.5% of loss.
- Historical expense ratios:
  - Fixed = 6%.
  - Variable = 30%.
- Underwriting profit and contingencies provision = 5%.
- Ultimate losses are estimated using the reported development technique.
- On January 1, 2014, the company will reduce agency commissions by 3% of premium.

Calendar Year Ending	Earned Premium ($000s)
December 31, 2011	$2,163
December 31, 2012	$2,120

Accident Year	Reported Losses ($000s)				
	12 months	24 months	36 months	48 months	60 months
2008	$780	$928	$1,030	$1,083	$1,094
2009	$765	$921	$1,004	$1,053	
2010	$760	$920	$1,012		
2011	$805	$966			
2012	$890				

Calculate the indicated rate change.

→ We can calculate the indicated rate change using the LR method.

$$\text{Indicated Avg rate change} = \frac{\text{loss + LAE ratio} + F}{1 - V - Q_T} - 1$$

→ There are no changes to fixed expenses or the UW profit + contingencies provision. So, we know that $F = 0.06$ + $Q_T = 0.05$.

→ The historical variable expense ratio is 30% of premium. However, agency commissions will be reduced by 3% of premium on 1/1/2014. New rates will be in effect from 7/1/2013 to 12/31/2014. So, assuming policies are written uniformly over time, 2/3 of the policies will be at the new lower commission levels. Therefore, the VER should be reduced by

$$\tfrac{2}{3}(0.03) = 2\% \Rightarrow V = 0.30 - 0.02 = 28\%$$

→ Substitute the values found so far into the formula for the indicated average rate change.

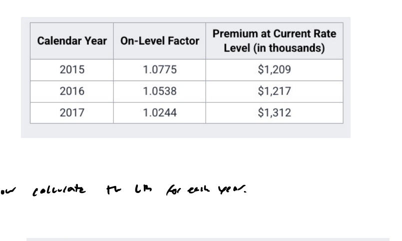
$$\text{Indicated avg rate change} = \frac{\text{loss + LAE ratio} + 0.06}{1 - 0.28 - 0.05} - 1 = \frac{\text{loss + LAE ratio} + 0.06}{0.67} - 1$$

→ To find the loss + LAE ratio, we will need to adjust premiums + losses to the level expected in the future policy period.

→ For premiums, we are given that current rates have been in effect for 3 years, so there is no need to on-level premiums. However, we will need to apply the premium trend of −1%.

→ The average earned date of each CY is the midpoint of that year. Since policies will be written at the new rates between 7/1/2013 + 12/31/2014, premium will be earned on those policies between 7/1/2013 + 12/31/2015. So, the average earned date of the future policy period is 10/1/2014.

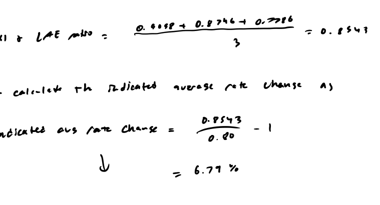
(see note below)



Calendar Year	Earned Premium ($000s)	Trend Period ✱	Trend Factor	Trended Earned Premium ($000s)
2011	$2,163	3.25 years	$(1 - 0.01)^{3.25} = 0.9679$	$2,093
2012	$2,120	2.25 years	$(1 - 0.01)^{2.25} = 0.9776$	$2,073

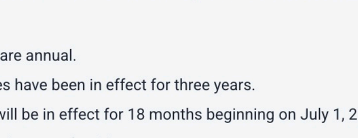
→ For losses, we will need to develop + trend losses for 2011 + 2012, + apply a load for LAE. We won't need to adjust other loss data since we only have 2 years of premium data.

→ To develop the losses, we will first need to find the age-to-age development factors based on all of the data available.



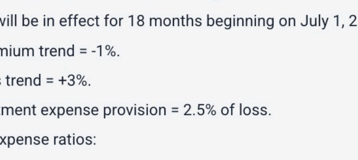
Accident Year	12-24 Months	24-36 Months	36-48 Months	48-60 Months
2008	1.1897	1.1099	1.0515	1.0102
2009	1.2039	1.0901	1.0488	
2010	1.2105	1.1000		
2011	1.2000			

→ From the above, there don't seem to be any patterns in the age-to-age factors. Therefore, I will select a straight average of the factors. Note that other selections may also be acceptable.



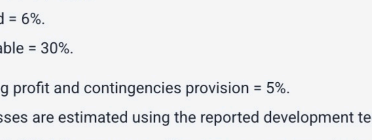
	12-24 Months	24-36 Months	36-48 Months	48-60 Months
Average LDFs	1.2010	1.1000	1.0501	1.0102

→ Then, find the cumulative development factors. Since no tail factor is given, I will assume there is no loss development after 60 months.



	12 Months to Ultimate	24 Months to Ultimate	36 Months to Ultimate	48 Months to Ultimate	60 Months to Ultimate
CDFs	1.4015	1.1669	1.0608	1.0102	1.0000

→ We can develop losses by multiplying by the appropriate CDF. Then, trend losses using the loss trend of 3%. The trend period is from the average accident date of each accident year (i.e. July 1 of each year) to the future average accident date (i.e. 10/1/2014). Finally, apply a load for LAE.



Accident Year	Reported Losses ($000s)	CDF	Trend Factor ✱	LAE Factor	Trended Ultimate Losses and LAE
2011	$966	1.1669	$(1.03)^{3.25} = 1.1008$	1.025	$1,272
2012	$890	1.4015	$(1.03)^{2.25} = 1.0688$	1.025	$1,366

→ I will use a 2-year weighted average to find the loss + LAE ratio.

$$\text{Loss + LAE ratio} = \frac{1,272 + 1,366}{2,093 + 2,073} = 63.33\%$$

→ Therefore,

$$\text{Indicated avg rate change} = \frac{\text{Loss + LAE ratio} + 0.06}{0.67} - 1 = \frac{0.6333 + 0.06}{0.67} - 1 = 3.48\%$$

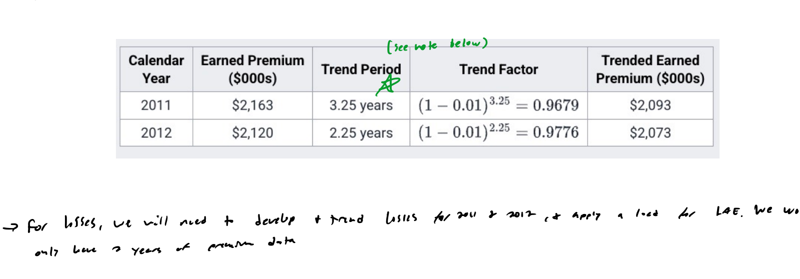
→ Note: Trend periods → If we are using CY/AY (↗ for premium, ↗ for losses) aggregation for trending EP + using earned dates, then the trend periods for premium + loss will match exactly.

→ If we're trending EP using written dates, then the trend periods will have the same total length, but the start + endpoints will differ.

→ Specifically, the written trend for EP will both start & end one half policy period sooner than the earned trend (assuming uniform writings)

→ Assignment

→ Q1 →



Answer the questions below. Show all work.

- Experience period on-level earned premium = $1,500,000
- Experience period trended and developed losses = $870,000
- Experience period earned exposure = 10,000
- Premium-related expenses factor = 18%
- Non-premium-related expenses = $30,000
- Profit and Contingency factor = 2%

(a) Calculate the indicated rate level change using the loss ratio method.

(b) Calculate the indicated rate level change using the pure premium method.

(c) Describe one situation in which it is preferable to use the loss ratio method, and one situation in which it is preferable to use the pure premium method.

→ a) LR method — Indicated rate level change

$$= \frac{\text{Loss + LAE ratio} + F}{1 - V - Q_T} - 1 = \frac{\frac{\text{Developed + trended losses}}{\text{EP @ CRL}} + \frac{\text{Fixed expense \$}}{\text{EP @ CRL}}}{1 - V - Q_T} - 1 = \frac{\frac{870,000}{1,500,000} + \frac{30,000}{1,500,000}}{1 - 0.18 - 0.02} - 1 = \frac{0.58 + 0.02}{0.8} - 1 = 0.75 - 1 = -25\%$$

→ check: $\text{Ind rate change} = \frac{P_I}{P_C}$

$$-0.25 = \frac{P_I}{(\text{EP @ CRL})/X} - 1$$

$$\Rightarrow P_I \overset{\checkmark}{=} \$112.50$$

→ b) PP method → Indicated rate

$$= \frac{(\bar{L} + \bar{E}_L) + \bar{F}}{1 - V - Q_T} = \frac{[(\text{Loss + LAE}) + \text{Fixed expenses}]/X}{1 - V - Q_T} = \frac{(870,000 + 30,000)/10,000}{1 - 0.18 - 0.02} = \frac{90}{0.8} = \$112.50$$

X : exposures

→ Check: $\text{Indicated rate change} = \frac{P_I}{P_C} - 1 = \frac{112.50}{150} - 1 \overset{\checkmark}{=} -0.25$

→ c) → The PP method is typically used when premium at CRL is not readily available or is difficult to calculate accurately. This situation often occurs, for instance when an insurance product w/ a complex rating algorithm undergoes significant changes to its rating variables, complicating the calculation of on-level premiums.

→ On the other hand, the LR method is the preferred approach when exposure data for an insurance product is either unavailable or the exposures are not well-defined. A common example is commercial general liability policies. These policies often cover a wide variety of risks across several sub-lines, each with different exposure bases. This diversity can make it challenging to establish a consistent measure of exposure.