3.1.3 → Univariate classification

→ overview → Once the rating variable & the levels w/in the rating variable have been determined, the next step is determining the amount of rate variation among the different levels. A rating variable needs at least two levels, w/ one designated as the base level. The rates for all non-base levels are expressed relative to the base level.

→ Many approaches can be used to determine rate differentials. This section focuses on traditional univariate methods, which determine the rate differentials by analyzing the historical experience for each level of a rating variable independently. We will look at three approaches:

→ 1) Pure premium approach
→ 2) Loss ratio approach
→ 3) Adjusted pure premium approach

→ These methods produce a set of indicated rate relativities as output. If there are changes to the relativities for any or all levels of the rating variables, the insurer will typically adjust the base rate accordingly to account for the expected increase or decrease in premium. This practice, known as base rate offsetting, will be discussed in a later section.

→ The following example will be used to demonstrate all three traditional univariate methods.

→ Example → (used below)

A classification analysis of homeowners insurance consists of only two rating variables: amount of insurance (AOI) and territory. You are given the following information:

- The exposure, premium, and loss data is summarized as follows:

Amount of Insurance (AOI)	Territory	Exposures	Loss & ALAE	Premium at Current Rate Level
Low	1	10	$303.55	$567.60
Medium	1	110	$4,586.67	$7,804.50
High	1	180	$10,807.86	$17,368.56
Low	2	130	$6,416.59	$12,298.00
Medium	2	120	$8,136.00	$14,190.00
High	2	140	$13,668.48	$22,514.80
Low	3	150	$9,165.85	$18,447.00
Medium	3	120	$10,072.37	$18,447.00
High	3	40	$4,834.74	$8,362.64
TOTAL		1,000	$67,992.11	$120,000.10

- The true underlying loss cost relativities and the current relativities used in the company's rating structure are given in the tables below.

AOI	True Relativity	Charged Relativity
Low	0.728	0.800
Medium	1.000	1.000
High	1.440	1.360

Territory	True Relativity	Charged Relativity
1	0.615	0.600
2	1.000	1.000
3	1.238	1.300

- All underwriting expenses are variable. The variable expense provision (V) is 25% of premium, and the target profit percentage (Q_T) is 5% of premium.
- Territory 2 and Medium AOI are the base levels.

→ Pure premium approach

→ The basic pure premium approach determines the indicated relativities by comparing the expected pure premiums for each of the levels w/in a rating variable.

→ When using the pure premium approach, the need to trend & develop the loss & ALAE depends on the nature of the portfolio.

→ Stable portfolios for short tailed lines may not require adjustments for classification analysis.

→ For portfolios that are growing or shrinking or those w/ changing loss & ALAE distributions, applying aggregate trend & development factors to the loss & ALAE of individual years before summing can improve the multi-year pure premium analysis.

→ Long-tailed lines may have risk classes w/ varying trend & development rates, requiring separate adjustments to individual risks or levels.

→ When classification analysis involves multiple claim types (e.g. workers compensation indemnity & medical), trending & development of losses should be performed on each claim type separately before combining.

→ Data should be adjusted for extraordinary & catastrophic events prior to a classification analysis.

→ The table below shows the indicated relativity for each territory using the pure premium approach.

(1) Territory	(2) Exposures	(3) Loss & ALAE	(4) Indicated Pure Premium	(5) Indicated Relativity	(6) Indicated Relativity to Base
1	300	$15,698.08	$52.33	0.7696	**0.7231**
2	390	$28,221.07	$72.36	1.0643	**1.0000**
3	310	$24,072.96	$77.65	1.1421	**1.0731**
Total	1,000	$67,992.11	$67.99	1.0000	0.9396

(5) = (4)/(4 total); (6) → (5i)/(5_2) OR (4i)/(4_2)

→ The indicated pure premium for each territory is calculated as the loss & ALAE divided by the exposures. Then, the indicated relativity in column (5) can be calculated as the pure premium for each territory divided by the total pure premium. This value represents the indicated relationship between the given territory & the total. The insurer usually selects a base level & expresses the rates for other levels w/ respect to the base. Thus, the individual relativities relative to the base level are needed. In this example, the indicated relativity to base in column (6) is calculated as the indicated relativity for each territory divided by the indicated relativity for territory 2.

→ Note that while the values in column (6) can be calculated directly from the pure premiums in column (4), the indicated relativities in column (6) are useful when the insurer wants to compare the normalized values of current, indicated, & competitors' relativities.

→ Notes → In the given example, it was assumed that the variable expense provision & target provision are the same for all risks. If these provisions vary by the type of risk, the indicated pure premiums for each level can be adjusted by the applicable provisions before determining the indicated relativities.

→ The table below compares the true relativities & the indicated relativities determined using the PP method.

Territory	True Relativity	Pure Premium Indication
1	0.615	0.7231
2	1.000	1.000
3	1.238	1.0731

→ The indicated PP relativity is too high for territory 1 & too low for territory 3. This is b/c the PP method ignores the correlation between the exposures of different rating variables. When calculating the pure premium for each level of a rating variable, it is assumed that exposures are uniformly distributed across all other rating variables. If there is a distributional bias in some or all of the other risk characteristics, the resulting PPs may be materially biased.

→ In our example, Territory 1 has a disproportionately high share of risks w/ a high amount of insurance. These risks have high loss & ALAE, which results in a high indicated PP & a high indicated relativity. Conversely, the indicated PP relativity for territory 3 is lower than the actual relativity due to the disproportionately high share of risks w/ a low amount of insurance.

→ Loss ratio approach

→ The loss ratio approach determines the indicated relativities by comparing the LRs for each of the levels w/in a rating variable. When using this approach, EP should be brought to the current level for each class of risk.

→ The table below shows the indicated relativity for each territory using the LR approach.

(1) Territory	(2) Premium at Current Rate Level	(3) Loss & ALAE	(4) Loss & ALAE Ratio	(5) Indicated Change Factor	(6) Current Relativity	(7) Indicated Relativity	(8) Indicated Relativity to Base
1	$25,740.66	$15,698.08	60.99%	1.0763	0.6000	0.6458	**0.6354**
2	$49,002.80	$28,221.07	57.59%	1.0164	1.0000	1.0164	**1.0000**
3	$45,256.64	$24,072.96	53.19%	0.9388	1.3000	1.2204	**1.2007**
Total	$120,000.10	$67,992.11	56.66%	1.0000			

(4): (3)/(3 total); (5): (4)/(4 total); (6) = factor from prior row; (7) = (5)×(6); (8): (7i)/(7_2)

→ The loss & ALAE ratio in column (4) is calculated as the loss & ALAE divided by the premium at current rate level. Then, the indicated change factor in column (5) is the loss & ALAE ratio for each territory divided by the total loss & ALAE ratio. This value represents how much the territory relativities should be changed so that the loss & ALAE ratios for every territory will be equivalent. The product of the indicated change factor & the current relativity is the indicated relativity, which is shown in column (7). Finally, column (8) shows the indicated relativity for each level relative to the base level.

→ The table below compares the true relativities & indicated relativities determined using both the PP & LR methods.

Territory	True Relativity	Pure Premium Indication	Loss Ratio Indication
1	0.615	0.7231	0.6354
2	1.000	1.000	1.0000
3	1.238	1.0731	1.2007

→ The indicated territorial relativities determined using the LR approach are closer to the true relativities than those calculated using the PP approach. Using the current as the denominator of the loss & ALAE ratio helps to adjust for the distributional bias in the amount of insurance rating variable. The remaining distortion arises from the discrepancies between the true amount of insurance (AOI) relativities & the AOI relativities charged by the insurer. If the AOI relativities charged are the same as the true AOI relativities, then the indicated territorial relativities produced by the LR method will match the true territorial relativities.

→ Besides adjusting for the distributional bias in other rating variables, the indicated relativities determined using the LR approach also compensate for the inequity present in other rating variables. In the example, Territory 1 consists mainly of risks w/ a high amount of insurance, which are underpriced b/c the charged relativity for high AOI is lower than the true relativity. To compensate for this, the indicated relativity for Territory 1 developed using the LR is higher than the true relativity. The drawbacks of this adjustment is that all insureds in territory 1, not just those w/ a high amount of insurance, are charged an additional amount to correct for the inequity in the amount of insurance relativities.

→ Adjusted pure premium approach

→ The LR approach requires premium at CRL for each level of the rating variable, which may not always be available or practical to obtain. In such cases, it is necessary to use the pure premium approach. To mitigate the impact of any distributional bias, the pure premium approach can be performed using exposures adjusted by the exposure-weighted average relativity of all other variables.

→ The exposure-weighted AOI relativity for each territory is calculated as follows:

AOI	Charged AOI Factor	Exposures by Territory		
		1	2	3
Low	0.80	10	130	150
Medium	1.00	110	120	120
High	1.36	180	140	40
Wtd Avg AOI Relativity by Territory		1.2093	1.0626	0.9497

→ For example, the weighted average AOI relativity of territory 1 is

$$\frac{10(0.80) + 110(1.00) + 180(1.36)}{10 + 110 + 180} = 1.2093$$

→ The table below shows the indicated relativity for each territory using the PP approach w/ adjusted exposures.

(1) Territory	(2) Exposures	(3) Wtd Avg AOI Relativity	(4) Adjusted Exposures	(5) Loss & ALAE	(6) Indicated Adjusted Pure Premium	(7) Indicated Relativity	(8) Indicated Relativity to Base
1	300	1.2093	362.8	$15,698.08	$43.27	0.6820	**0.6354**
2	390	1.0626	414.4	$28,221.07	$68.10	1.0733	**1.0000**
3	310	0.9497	294.4	$24,072.96	$81.77	1.2887	**1.2007**
Total	1,000		1,071.6	$67,992.11	$63.45	1.0000	0.9317

(4) = (2)×(3); (6) = (5)/(4); (7) = (6i)/(6 base); (8) → (7i)/(7_2)

→ As the exposures have been adjusted using the current amount of insurance relativities charged, the resulting indicated relativities are the same as those calculated using the LR approach. As w/ the LR approach, the remaining distortions in the indicated relativities come from the variation between the true AOI relativities & the AOI relativities charged by the insurer.

→ Note → Calculating the weighted averages of relativities is not strictly necessary. As you may have noticed, the adjusted exposures equal the numerator in the weighted average calculations.

Territory	Adjusted Exposures
1	10(0.80) + 110(1.00) + 180(1.36) = 362.8
2	130(0.80) + 120(1.00) + 140(1.36) = 414.4
3	150(0.80) + 120(1.00) + 40(1.36) = 294.4

→ Example →

Given the following information:

	As of January 1, 2011	As of July 1, 2011
Base Rate	$200	$250
Good Driver Discount Factor	0.85	0.75
Territory 1 Factor	1.00	1.00
Territory 2 Factor	1.10	1.10

Exposures	Good Driver Discount	
	Yes	No
Territory 1	750	250
Territory 2	600	150

Loss and ALAE	Good Driver Discount	
	Yes	No
Territory 1	$90,000	$40,000
Territory 2	$80,000	$20,000

- The rating algorithm is base rate × good driver discount factor × territory factor.
- Territory 1 and No Good Driver Discount remain the base classification.

Use the loss ratio method to calculate indicated territorial relativities.

→ first, for each territory, calculate the EP @ CRL using the extension of exposures method, ensuring that the most recent available data (as of July 1, 2011) is used. This requires using the exposures from each good driver level, multiplied by the appropriate current rate. The rates are determined by using the base rate, adjusted for the applicable good driver discount factor & territory factor.

→ Terr 1 E = $\sum_{T1}$ (Exposure × rating)

= $\sum_{T1}$ [Exposures × (Base rate × Good driver discount × territory factor)]

= 750 (50 × 0.75 × 1.00) + 250 (250 × 1.00 × 1.00)

= 203,125

Terr 2 EP = $\sum_{T2}$ [Exposure × rating]

= $\sum_{T2}$ [Exposure × (base rate × good driver discount factor × territory factor)]

= 600 (200 × 0.75 × 1.10) + 150 (250 × 1.00 × 1.10)

= 165,000

→ Next, for each territory, calculate the total loss & ALAE by summing across the row for each good driver discount level.

→ Terr 1 loss & ALAE = 90,000 + 40,000 = 130,000
Terr 2 loss & ALAE = 80,000 + 20,000 = 100,000

→ This provides the information needed to calculate the indicated relativities using the loss ratio method:

→ The loss & ALAE ratio is calculated as the loss & ALAE divided by the premium at the CRL
→ The indicated change factor is the loss & ALAE ratio for the territory divided by the overall loss & ALAE ratio
→ Multiply this factor by the current relativities for each territory to determine the indicated relativity
→ Finally, divide by the indicated relativity of Territory 1, which serves as the base rate

(4): (3)/(2); (5): (4i)/(4 total); (6) = (5)×(1); (7): (6i)/(6 base)

Territory	Premium at Current Rate Level	Loss & ALAE	Loss & ALAE Ratio	Indicated Change Factor	Current Relativity	Indicated Relativity	Indicated Relativity to Base
1	$203,125	$130,000	0.6400	1.0243	1.00	1.0243	**1.0000**
2	$165,000	$100,000	0.6061	0.9700	1.10	1.0670	**1.0417**
Total	$368,125	$230,000	0.6248	1.0000			

→ Summary → After identifying risk characteristics to be used as rating variables, companies can apply univariate methods such as pure premium, loss ratio, or adjusted pure premium analysis to calculate the indicated rate differentials for each level of each variable. However, these univariate techniques can introduce distortions, which is why many companies have transitioned to multivariate methods — now more accessible w/ modern technology. The fundamentals of these multivariate techniques will be discussed in the next section. Nevertheless, some companies continue to use univariate approaches, adjusting for known distortions as needed.

→ Assignment

→ Q1) A private passenger auto insurance company uses only two rating variables: territory and marital status. The distribution of exposures is:

Marital Status	Territory		
	1	2	3
Married	123	79	87
Single	74	123	33

The rating factors for each variable are:

Marital Status	Current Relativity
Married	1.00
Single	1.15

Territory	Current Relativity
1	0.60
2	1.00
3	0.90

Losses/LAE for each category during the experience period are:

Territory	Marital Status	Loss & LAE
1	Married	$7,760
1	Single	$5,789
2	Married	$8,307
2	Single	$16,038
3	Married	$8,233
3	Single	$3,873

- No fixed expense adjustment is necessary.
- All policies have the same underwriting expense and target profit.

(a) *(2.5 points)* Using the adjusted pure premium approach and maintaining the same base classes, develop the indicated relativity for policyholders who are single.

(b) *(0.5 point)* Explain why the adjusted pure premium approach is preferable to the pure premium method.

→ a)

Exposures	Territory			Total
Marital Status	1	2	3	
Married	123	79	87	289
Single	74	123	33	230
Total	197	202	120	519

Marital Status	Current Relativity
Married	1
Single	1.15

Territory	Current Relativity
1	0.6
2	1
3	0.9

Territory	Marital Status	Loss & LAE
1	Married	$7,760
1	Single	$5,789
2	Married	$8,307
2	Single	$16,038
3	Married	$8,233
3	Single	$3,873

=SUMIFS($D$19:$D$24,$B$19:$B$24,C$28,$C$19:$C$24,$E29)

Loss & LAE	Territory			Total
Marital Status	1	2	3	
Married	$7,760	$8,307	$8,233	$24,300
Single	$5,789	$16,038	$3,873	$25,700
Total	$13,549	$24,345	$12,106	$50,000

=SUM(C28:E28) =SUM(C28:C29)

=vlookup(<...>)

Territory	Factor	Marital Status: Married	Marital Status: Single
1	0.6	123	74
2	1	79	123
3	0.9	87	33
	Weighted avg territory by marital status	0.800	0.857

=SUMPRODUCT($B$5:$B7,K5:K7)/SUM(K5:K7)

Marital Status	Exposures	Weighted avg territory by marital status	Adjusted exposures	Loss & LAE	Indicated adjusted pure premium	Indicated relativity	Indicated relativity to base
	=vlookup(<...>)		=I13*J13	=vlookup(<...>)	=L13/K13	=M13/$M$15	=N13/$N$13
Married	289	0.800	231.1	$24,300	$105.15	0.900	1.000
Single	230	0.857	197.1	$25,700	$130.39	1.117	1.240
Total	519		428.2	$50,000	$116.77	1.000	1.110

→ b) The adjusted pure premium approach is preferable to the pure premium method b/c it mitigates the impact of any distributional bias, while the pure premium method does not.

→ Q2) You are given the following:

Territory	Current Rates	Earned Premium at CRL	Reported Loss & LAE
A	110	575,000	330,000
B	55	716,667	525,000
C	100	416,667	290,000
D	160	256,667	135,000

Assume Territory A is the base territory.

Using the loss ratio approach, calculate the indicated territory relativities.

→

Territory	Current Rates	Earned Premium at CRL	Reported Loss & LAE	Loss ratio	Indicated change factor	Current relativity	Indicated relativity	Indicated relativity to base
				=S37/R37	=T37/$T$41	=Q37/$Q$37	=U37*V37	=W37/$W$37
A	$110	$575,000	$330,000	57.4%	0.881	1.000	0.881	1.000
B	$55	$716,667	$525,000	73.3%	1.125	0.500	0.562	0.638
C	$100	$416,667	$290,000	69.6%	1.068	0.909	0.971	1.102
D	$160	$256,667	$135,000	52.6%	0.807	1.455	1.174	1.333
Total		$1,965,001	$1,280,000	65.1%	1.000			
		=SUM(R37:R40)	=SUM(S37:S40)					