4.2.3 → Capping rates

→ Overview → After calculating relativities + deriving an appropriate base rate, an insurer may opt to limit the extent to which a rate revision affects the premium paid by any specific class. This practice, known as capping, is employed to mitigate large premium fluctuations that could lead to policyholder dissatisfaction, lapses, or regulatory issues. In essence, capping places a limit on the impact of a rating variable's adjustment to protect certain insured groups from steep increases while still striving to meet the overall target rate change.

→ Capping helps maintain fairness among insureds by ensuring that rate changes changes are not excessive or unfairly discriminatory. While capping may deviate from strictly cost-based pricing, it balances equity considerations with practical market realities + regulatory requirements. Additionally, limiting large rate increases can support customer retention + foster regulatory goodwill, aligning w/ the broader goals of sustainable + equitable ratemaking.

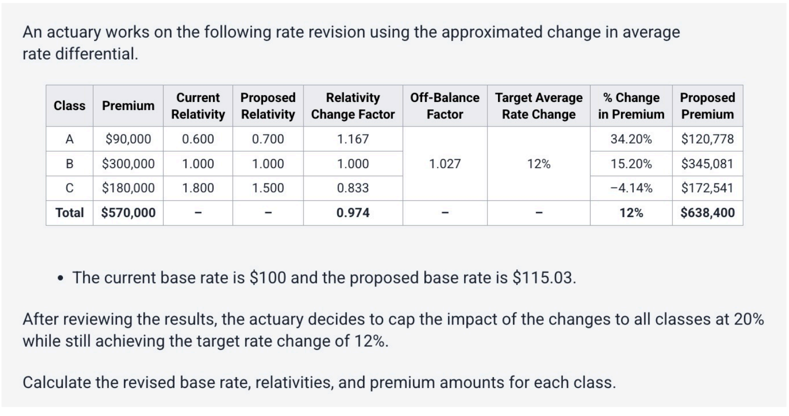
→ Because capping generally reduces the total premium collected from the capped classes, this may lead to a premium shortfall relative to the target rate change. To meet the overall rate objectives, actuaries must adjust the base rate + other class relativities so that the non-capped classes contribute enough to make up for the shortfall. Here, we will explore two methods for handling a premium shortfall:

- → Method 1 → A structured, formula-based adjustment for both capped + non-capped classes.
- → Method 2 → An intuitive, proportional reallocation approach that simplifies adjustments, especially useful for multi-capped portfolios

→ Capping a non-base class

→ To explain this process, let's work through an example rate revision. Assume that all premiums are variable.

→ Example →



An actuary works on the following rate revision using the approximated change in average rate differential.

Class	Premium	Current Relativity	Proposed Relativity	Relativity Change Factor	Off-Balance Factor	Target Average Rate Change	% Change in Premium	Proposed Premium
A	$90,000	0.600	0.700	1.167	1.027	12%	34.20%	$120,778
B	$300,000	1.000	1.000	1.000			15.20%	$345,081
C	$180,000	1.800	1.500	0.833			−4.14%	$172,541
Total	**$570,000**	–	–	**0.974**	–	–	**12%**	**$638,400**

- The current base rate is $100 and the proposed base rate is $115.03.

After reviewing the results, the actuary decides to cap the impact of the changes to all classes at 20% while still achieving the target rate change of 12%.

Calculate the revised base rate, relativities, and premium amounts for each class.

→ Notice that the total % change for class A (34.20%) exceeds the cap (20%). To comply w/ the cap, we need to limit the amount of premium class A pays:

$$\text{Class A capped premium} = \text{Current premium} * (1 + \text{Cap\%}) = \$90{,}000 * (1+0.20) = \$108{,}000$$

→ This results in a shortfall of $120,778 − $108,000 = $12,778, which must be made up for by classes B + C to meet the overall premium target.

→ Method 1: Formulas

→ This method starts by reducing the capped class's relativity in order to comply w/ the cap. We can derive the relativity for capped class using the equation below:

$$\boxed{\text{Initial capped relativity} = \frac{\text{Current relativity} * (1 + \text{cap\%})}{\text{OBF} * (1 + \text{Target rate change})}}$$

→ Using our example, the relativity for class A must be capped at:

$$\text{Class A capped relativity} = \frac{0.600 * (1+0.20)}{1.027 * 1.12} = 0.626$$

→ In order to offset the shortfall in premium caused by the cap, the base rate needs to be increased. The extra amount that each of the non-capped classes need to pay in order to make up for the shortfall is proportional to their proposed premium amounts (prior to the cap). This allows us to preserve the relativities for each of the non-capped classes while still increasing the amount of premium paid to cover the shortfall.

$$\boxed{\text{Shortfall adjustment factor} = 1 + \frac{\text{Premium shortfall}}{\text{Total proposed premium from uncapped classes}}}$$

→ In our example, there are two uncapped classes, class B + class C. Sum their proposed premiums to get the denominator of the adjustment factor.

$$\text{Shortfall adjustment factor} = 1 + \frac{\$12{,}778}{\$345{,}081 + \$172{,}541} = 1.025$$

→ Multiply the proposed base rate by the shortfall adjustment factor to find our final base rate. In our example,

$$\text{Final base rate} = \$115.03 * 1.025 = \$117.87$$

→ Increasing the base rate to cover the shortfall causes the premium amount for the capped classes to increase + exceed the cap once again. Thus, we need to re-adjust the relativity of the capped class by dividing it by the shortfall adjustment factor.

$$\boxed{\text{Final capped relativity} = \frac{\text{Initial capped relativity}}{\text{Shortfall adjustment factor}}}$$

$$\Rightarrow \text{Final class A relativity} = \frac{0.626}{1.025} = 0.611$$

→ As for the final premium amounts, we've already established that the final premium for class A must be $108,000 in order to comply w/ the cap. The relativities for classes B + C are not touched, but the base rate is scaled by the shortfall adjustment factor. Thus the final premium amounts for classes B + C are just their proposed premiums multiplied by the shortfall adjustment factor.

$$\boxed{\text{Final premium} = \text{Proposed premium} * \text{Shortfall adjustment factor}} \quad ▣$$

→ The table below summarizes the results.

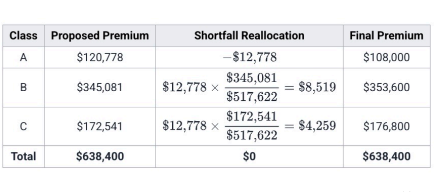
Class	Shortfall	Capped Relativity	Shortfall Adjustment Factor	Final Base Rate	Final Relativity	Final Premium	% Change in Premium
A	$12,778	0.626	1.025	$117.87	0.611	$108,000	20%
B	–	–			1.000	$353,600	17.87%
C	–	–			1.500	$176,800	−1.78%
Total	$12,778	–	–	–	–	$638,400	12%

→ Notes → Notice that the % change in premium is not larger than 20% for any one class + that the overall target rate change is achieved. Additionally, the total final premium amount is the same as the total final premium we found before applying the cap. Verifying that these relationships hold true is a good way to check your work.

→ Method 2: Manually relocate the shortfall

→ The second method relies on intuition. If one class has a shortfall in premium, then the lost premiums must be made up for by the other classes. However, this redistribution doesn't occur randomly; premiums must be reallocated to the other classes in proportion to their proposed premiums, as evidenced by the equations above.

→ Using the same example as before, capping class A at a 20% change results in a shortfall of $12,778. This must be redistributed to classes B + C, which have a total proposed premium of $345,081 + $172,541 = $517,622 between the two of them.

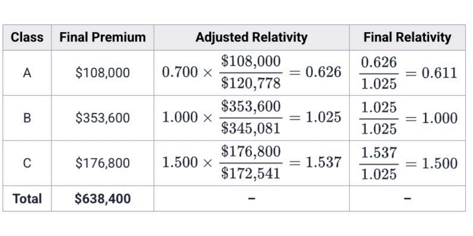


Class	Proposed Premium	Shortfall Reallocation	Final Premium
A	$120,778	$-\$12{,}778$	$108,000
B	$345,081	$\$12{,}778 \times \frac{\$345{,}081}{\$517{,}622} = \$8{,}519$	$353,600
C	$172,541	$\$12{,}778 \times \frac{\$172{,}541}{\$517{,}622} = \$4{,}259$	$176,800
Total	**$638,400**	**$0**	**$638,400**

→ Reallocating the shortfall in this way effectively alters each of the relativities, not the base rate. We can derive the adjusted relativities by comparing the final premium amounts to the proposed premium amounts.

$$\boxed{\text{Adjusted relativity} = \text{Proposed relativity} * \frac{\text{Final premium}}{\text{Proposed premium}}}$$

→ The relativities produced in this way are valid. Combined w/ the original proposed base rate, they result in the correct final premium amounts. However, if we want to preserve the original base class, the adjusted relativities will need to be rebased. Divide each of the adjusted relativities by the adjusted relativity for the base class to obtain the final relativities.



Class	Final Premium	Adjusted Relativity	Final Relativity
A	$108,000	$0.700 \times \frac{\$108{,}000}{\$120{,}778} = 0.626$	$\frac{0.626}{1.025} = 0.611$
B	$353,600	$1.000 \times \frac{\$353{,}600}{\$345{,}081} = 1.025$	$\frac{1.025}{1.025} = 1.000$
C	$176,800	$1.500 \times \frac{\$176{,}800}{\$172{,}541} = 1.537$	$\frac{1.537}{1.025} = 1.500$
Total	$638,400	–	–

→ Rebasing also impacts the base rate. In order to offset the change in the relativities, we multiply the proposed base rate by the adjusted relativity for the base class.

$$\text{Final base rate} = \$115.03 * 1.025 = \$117.87$$

→ As long as we preserve the original base class, this method will produce the same final base rate, relativities, + premium amounts as the first method.

→ Notes → Note that the source material presents only the first method, not the second method. However the second method is intuitive + widely accepted.

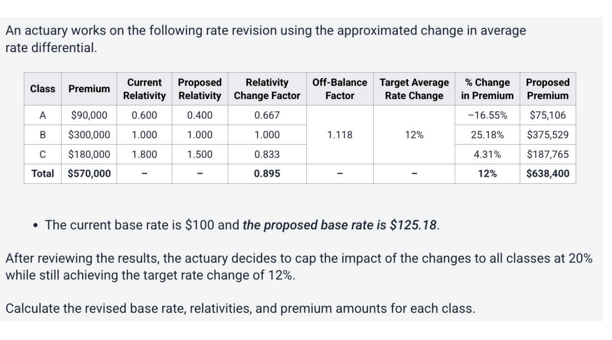
→ To summarize:

- → Method 1: Formula based approach
  - → 1) Approach
    - → Method 1 adjusts the capped class's relativity to meet the cap + then applies a calculated shortfall adjustment factor to ensure the total premium aligns w/ the target rate change.
    - → It is structured + systematic, typically involving two main recalculations.
  - → 2) Key steps
    - → Initial adjustment → Reduce the capped class's relativity to meet the cap
    - → Shortfall adjustment factor → Calculate a factor to adjust the base rate, applied proportionally to uncapped classes' premiums to meet the target rate.
    - → Final adjustments → Recalculate the capped class's relativity by dividing it by the shortfall adjustment factor to ensure it precisely meets the cap
  - → 3) Benefits
    - → Method 1 provides a precise, formula-driven solution, ideal for formal calculations
    - → It is effective for structured, standardized scenarios, where accuracy in achieving the cap + target rate is prioritized
  - → 4) Drawbacks
    - → Method 1 can feel complex + less intuitive due to its step-by-step recalculations, especially in large or multi-capped portfolios
- → Method 2: Manual reallocation of shortfall
  - → 1) Approach
    - → Method 2 reallocates the premium shortfall from the capped class proportionally across uncapped classes based on their proposed premiums
    - → It finalizes premiums first, then adjusts the base rate + relativities to align w/ these allocated premiums
  - → 2) Key steps
    - → Initial reallocation → Calculate the shortfall from the capped class + redistribute it proportionally to uncapped classes
    - → Final adjustments → Adjust the base rate + rebased relativities to maintain consistency, setting the base class relativity to 1.00
  - → 3) Benefits
    - → Method 2 is practical + intuitive, avoiding the need for recalculations + providing a streamlined way to achieve the target rate
    - → Especially useful in real-world situations w/ multiple capped classes, as it requires only one reallocation + rebasing step
  - → 4) Drawbacks
    - → May feel less structured than Method 1 due to the reallocation process
    - → Requires a final re-basing step for consistency, setting the base class relativity to 1.00
- → Summary comparison
  - → Method 1 is a structured, formula-driven approach that achieves precise results but involves step-by-step adjustments
  - → Method 2 is intuitive, reallocating shortfalls in a single step + rebasing to maintain consistency around the base class

→ Capping the base class

→ Applying a cap to the base class inherently lowers the base rate when compared to the uncapped proposed base rate. As a result, there are a few changes to the first approach to handling the premium shortfall. The second approach is essentially unchanged. Nonetheless, we will demonstrate both approaches using a modified version of the example above where the base class is capped.

→ Example →



An actuary works on the following rate revision using the approximated change in average rate differential.

Class	Premium	Current Relativity	Proposed Relativity	Relativity Change Factor	Off-Balance Factor	Target Average Rate Change	% Change in Premium	Proposed Premium
A	$90,000	0.600	0.400	0.667	1.118	12%	−16.55%	$75,106
B	$300,000	1.000	1.000	1.000			25.18%	$375,529
C	$180,000	1.800	1.500	0.833			4.31%	$187,765
Total	**$570,000**	–	–	**0.895**	–	–	**12%**	**$638,400**

- The current base rate is $100 and *the proposed base rate is $125.18*.

After reviewing the results, the actuary decides to cap the impact of the changes to all classes at 20% while still achieving the target rate change of 12%.

Calculate the revised base rate, relativities, and premium amounts for each class.

→ This time, the total % change in premium for the base class, i.e. Class B, exceeds the cap (20%).

$$\text{Class B capped premium} = \text{Current premium}\ (1 + \text{Cap\%}) = \$300{,}000\ (1+0.20) = \$360{,}000$$

→ This results in a shortfall of $375,529.41 − $360,000 = $15,529.41 which must be made up for by classes A + C

→ Method 1: Formulas

→ As we alluded to above, applying a cap to the base class effectively lowers the base rate. Computing the capped base rate is as simple as multiplying the current base rate by 1 plus the cap percentage.

→ Another way to do this is by first calculating an adjustment factor as 1 plus the cap percentage over 1 plus the proposed change in premium for the base class.

$$\boxed{\text{Capped Base Adjustment factor} = \frac{1 + \text{cap \%}}{1 + \text{Proposed \% change in Premium}}}$$

→ We can multiply the adjustment factor by the proposed base rate to get the final base rate.

$$\text{Capped Base Adjustment factor} = \frac{1+0.20}{1+0.2518} = 0.959$$

$$\Downarrow$$

$$\text{Final base rate} = 0.959 * \$125.18 = \$120$$

→ This adjustment gives us a base rate that is as close to the proposed base rate as possible w/o exceeding the cap

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→ Since the base rate is decreased by the capped adjustment factor, the non-base relativities should be increased by the same factor to make up for it, i.e. by dividing by it (as factors are multiplicative w/ base rate: base rate × factor = Premium ⟹ if decrease base rate by x%, then $[\text{Base rate}\ (1-x)] * [\text{factor}/(1-x)] \overset{✓}{=} \text{Premium}$) Furthermore, we would calculate the same shortfall adjustment factor. The relativities need to be increased by its factor to make up for the shortfall.

→ The shortfall adjustment factor for this example is:

$$\text{Shortfall adjustment factor} = 1 + \frac{\$15{,}529.41}{\$75{,}105.88 + \$187{,}764.71} = 1.059$$

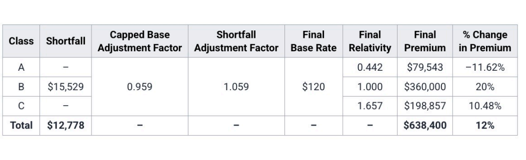
→ Thus, the final relativities for the non-capped classes are

$$\text{Class A final relativity} = 0.400 * \frac{1.059}{0.959} = 0.442$$

$$\text{Class C final relativity} = 1.500 * \frac{1.059}{0.959} = 1.657$$

→ Calculating the final premium amounts works the same way as before. The premium for the capped class will equal the current premium by 1 plus the cap percentage, + the final premiums for the uncapped classes can be found by using equation ▣.

→ These adjustments are summarized in the table below.



Class	Shortfall	Capped Base Adjustment Factor	Shortfall Adjustment Factor	Final Base Rate	Final Relativity	Final Premium	% Change in Premium
A	–	0.959	1.059	$120	0.442	$79,543	−11.62%
B	$15,529				1.000	$360,000	20%
C	–				1.657	$198,857	10.48%
Total	$12,778	–	–	–	–	$638,400	12%

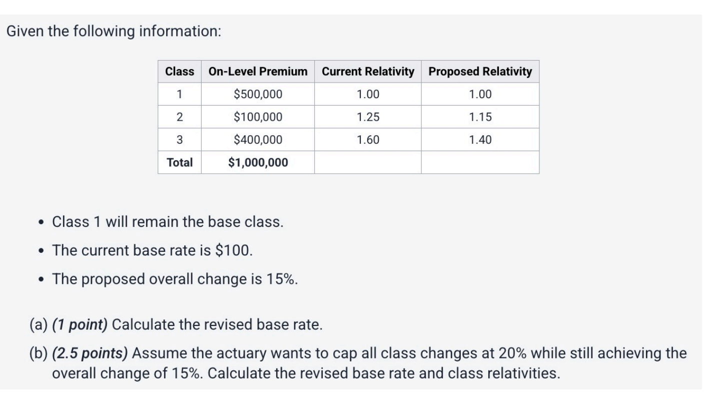
→ Method 2: Manually reallocate the shortfall

→ Method 2 is largely the same when the base class is capped. Determine the shortfall in premium, reallocate that shortfall to the uncapped classes proportional to their proposed premiums to obtain the final premiums for each class, + then derive the base rate/relativities using these premiums.

→ The results of using this method on our second example are depicted below.

Class	Proposed Premium	Shortfall Reallocation	Final Premium	Adjusted Relativity	Final Relativity	Final Base Rate
A	$75,106	$4,437	$79,543	0.424	0.442	$120
B	$375,529	−$15,529	$360,000	0.959	1.000	
C	$187,765	$11,092	$198,8574	1.589	1.657	
Total	$638,400	$0	$638,400	–	–	–

→ Example →



Given the following information:

Class	On-Level Premium	Current Relativity	Proposed Relativity
1	$500,000	1.00	1.00
2	$100,000	1.25	1.15
3	$400,000	1.60	1.40
Total	$1,000,000		

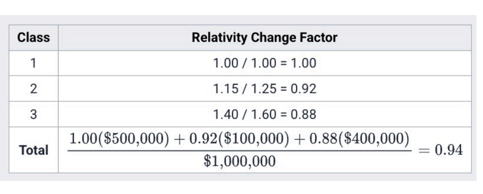
- Class 1 will remain the base class.
- The current base rate is $100.
- The proposed overall change is 15%.

(a) *(1 point)* Calculate the revised base rate.

(b) *(2.5 points)* Assume the actuary wants to cap all class changes at 20% while still achieving the overall change of 15%. Calculate the revised base rate and class relativities.

→ a) → First, compute the weighted average relativity change factor:

- → For each class, divide the proposed relativity by the current relativity
- → Use premium as weights.
- → This results in a weighted average of 0.94



Class	Relativity Change Factor
1	1.00 / 1.00 = 1.00
2	1.15 / 1.25 = 0.92
3	1.40 / 1.60 = 0.88
Total	$\frac{1.00(\$500{,}000) + 0.92(\$100{,}000) + 0.88(\$400{,}000)}{\$1{,}000{,}000} = 0.94$

→ Then, the off-balance factor is the reciprocal of the weighted average relativity change factor:

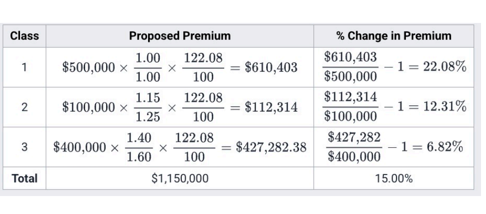
$$\text{OBF} = \frac{1}{0.94}$$

→ We are given the current base is $100, + that the target rate change percentage is 15%. Therefore,

$$\text{Proposed base rate} = \$100 * 15\% * \frac{1}{0.94} = \$122.08$$

→ b) → Regardless of what method we choose, we'll need to start by determining the proposed premium + % changes in premium for each class. We can find the proposed premiums by adjusting the current premiums for the changes in the base rate + relativities. In the previous part, we calculated both the relativity change factor + the proposed base rate. With these in hand, we have all the necessary information to calculate the proposed premium for each class.

$$\text{Proposed premium} = \text{Current premium} * \text{Relativity change factor} * \text{Base rate change factor}$$



Class	Proposed Premium	% Change in Premium
1	$\$500{,}000 \times \frac{1.00}{1.00} \times \frac{122.08}{100} = \$610{,}403$	$\frac{\$610{,}403}{\$500{,}000} - 1 = 22.08\%$
2	$\$100{,}000 \times \frac{1.15}{1.25} \times \frac{122.08}{100} = \$112{,}314$	$\frac{\$112{,}314}{\$100{,}000} - 1 = 12.31\%$
3	$\$400{,}000 \times \frac{1.40}{1.60} \times \frac{122.08}{100} = \$427{,}282.38$	$\frac{\$427{,}282}{\$400{,}000} - 1 = 6.82\%$
Total	$1,150,000	15.00%

→ Class 1, the base class, has a change in premium that exceeds the cap of 20%. You can use either of the methods discussed above to derive the final base rate + relativities, but we'll demonstrate the second method below.

→ Start by calculating the premium shortfalls for class 1.

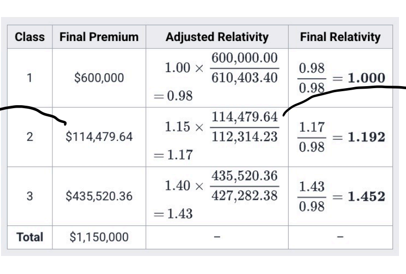
$$\text{Shortfall} = \text{Proposed premium} - \text{Capped premium} = 500{,}000\ (22.08\% - 20.00\%) = 10{,}403.40$$

→ Next, allocate this shortfall to classes 2 + 3 based on the proposed premium:

$$\text{Class 2 Reallocation} = \frac{112{,}314.23}{112{,}314.23 + 427{,}282.38} = 20.81\%$$

$$\text{Class 3 reallocation} = \frac{427{,}282.38}{112{,}314.23 + 427{,}282.38} = 79.19\%$$

→ Add the shortfall allocation to the proposed premium to determine the final premium. Note that the total remains at 1.15 million. Then, calculate the adjusted relativities based on the proposed relativities. To find the final relativity, divide each adjusted relativity by the base class, Class 1, so that class 1 relativity is standardized to 1.



Class	Final Premium	Adjusted Relativity	Final Relativity
1	$600,000	$1.00 \times \frac{600{,}000.00}{610{,}403.40} = 0.98$	$\frac{0.98}{0.98} = \mathbf{1.000}$
2	$114,479.64	$1.15 \times \frac{114{,}479.64}{112{,}314.23} = 1.17$	$\frac{1.17}{0.98} = \mathbf{1.192}$
3	$435,520.36	$1.40 \times \frac{435{,}520.36}{427{,}282.38} = 1.43$	$\frac{1.43}{0.98} = \mathbf{1.452}$
Total	$1,150,000	–	–

= Proposed premium + (0.2081 * 10,403.40)

— — — + (0.7919 * — — —)

$$= \frac{\text{Final premium}}{\text{Proposed premium}}$$

→ Remember that the only thing that impacts the proposed base rate when using this method is rebasing the relativities at the very end. To get the final base rate, multiply the proposed base rate by the adjusted relativity for Class 1:

$$\text{Final base rate} = \$122.08 * 0.98 = \$120$$

→ It's no surprise that the base rate increased by 20%, as we already knew it was capped at that level.

→ Assignment

→ Q1)

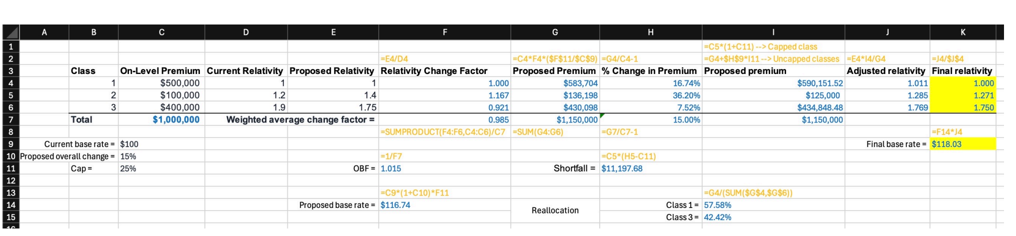
Given the following information:

Class	On-Level Premium	Current Relativity	Proposed Relativity
1	$500,000	1.00	1.00
2	$100,000	1.20	1.40
3	$400,000	1.90	1.75
Total	$1,000,000		

- Class 1 will remain the base class.
- The current base rate is $100.
- The proposed overall change is 15%.

Assume the actuary wants to cap all class changes at 25% while still achieving the overall change of 15%. Calculate the revised base rate and class relativities.

→



	A	B	C	D	E	F	G	H	I	J	K
1									=C5*(1+C11) --> Capped class		
2						=E4/D4	=C4*F4*($F$11/$C$9)	=G4/C4-1	=G4+$H$9*I11 --> Uncapped classes	=E4*I4/G4	=J4/$J$4
3		Class	On-Level Premium	Current Relativity	Proposed Relativity	Relativity Change Factor	Proposed Premium	% Change in Premium	Proposed premium	Adjusted relativity	Final relativity
4		1	$500,000	1	1	1.000	$583,704	16.74%	$590,151.52	1.011	1.000
5		2	$100,000	1.2	1.4	1.167	$136,198	36.20%	$125,000	1.285	1.271
6		3	$400,000	1.9	1.75	0.921	$430,098	7.52%	$434,848.48	1.769	1.750
7		Total	$1,000,000		Weighted average change factor =	0.985	$1,150,000	15.00%	$1,150,000		
8						=SUMPRODUCT(F4:F6,C4:C6)/C7	=SUM(G4:G6)	=G7/C7-1			=F14*J4
9		Current base rate =	$100							Final base rate =	$118.03
10		Proposed overall change =	15%			=1/F7		=C5*(H5-C11)			
11		Cap =	25%		OBF =	1.015	Shortfall =	$11,197.68			
12											
13						=C9*(1+C10)*F11			=G4/(SUM($G$4,$G$6))		
14					Proposed base rate =	$116.74	Reallocation	Class 1 =	57.58%		
15								Class 3 =	42.42%		