3.3.3 → Deductible pricing

→ overview → Initially, many insurance policies provided full coverage, meaning the insurer covered the entire loss amount. Over time, insurers introduced deductible clauses, making the insured responsible for covering losses up to a specified deductible amount, while the insurer pays any amount above the deductible, up to the policy limits.

→ There are two basic types of deductibles:

→ 1) Flat dollar deductibles → Stated as a dollar amount below which losses are not covered by the policy (e.g. $250). Flat dollar deductibles can vary widely, from relatively small amounts (e.g. $100 or $250) on personal lines to significantly larger amounts (e.g. $100,000 or more) on large commercial policies.

→ 2) Percentage deductibles → Stated as a percentage of the coverage amount. For example, a 5% deductible on a $500,000 home insurance is the same as a flat dollar deductible of $25,000. Percentage deductibles are commonly used in property policies, particularly for perils like earthquakes or hurricanes that can lead to catastrophic losses.

→ Deductibles are popular among both insureds + insurers for several reasons:

→ Premium reduction → Deductibles reduce insurance premiums as the insured covers a portion of the losses

→ Eliminates small nuisance claims → Deductibles minimize the occurrence of small nuisance claims, which are often more expensive to investigate + handle than the actual claim itself. This allows insurers to manage their expenses better + ultimately keep rates lower than they would be otherwise.

→ Provides incentive for loss control → Deductibles provide a financial incentive for the insured to prevent losses since they bear the initial loss.

→ Controls catastrophic exposure → Deductibles reduce an insurer's exposure to catastrophic losses + lower the risk of insolvency.

→ Besides the univariate + multivariate methods, deductible relativities can be determined using the loss-elimination ratio (LER) approach.

→ Loss elimination ratio (LER) approach

→ Assuming all expenses are variable + the variable expenses + profit are a constant percentage of premium, the indicated deductible relativity for going from one deductible level, B, to a higher one, D can be calculated as

$$\text{Indicated deductible relativity} = \frac{(L+E_L)_D}{(L+E_L)_B}$$

where $(L+E_L)_B$ + $(L+E_L)_D$ are the losses + LAE net of the deductible B + D, respectively.

→ This equation can be rewritten as

$$\text{Indicated deductible relativity} = \frac{(L+E_L)_B - (L+E_L)_B + (L+E_L)_D}{(L+E_L)_B}$$

$$= 1 - \frac{(L+E_L)_B - (L+E_L)_D}{(L+E_L)_B}$$

$$= 1 - LER(D)$$

→ The loss elimination ratio

$$LER(D) = \frac{(L+E_L)_B - (L+E_L)_D}{(L+E_L)_B}$$

represents the proportion of losses eliminated by going from one deductible to another. Speaking differently, the loss elimination ratio is one minus the proportion of losses that remain after switching deductible.

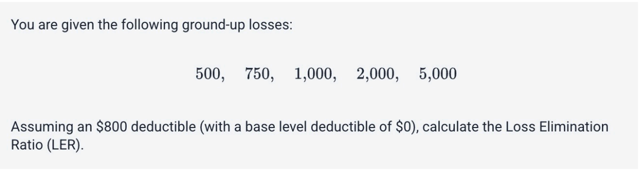
→ Notes → for policies w/ full coverage, set B = 0. Then the formulas essentially become

$$\text{Indicated deductible relativity} = \frac{\text{Losses + LAE assuming deductible D}}{\text{Total ground up losses + LAE}}$$

+

$$LER(D) = \frac{\text{Losses + LAE eliminated by Deductible D}}{\text{total ground-up losses + LAE}}$$

→ If the ground-up loss of each claim is known, the calculation of the LER is straightforward. To illustrate, consider the following simple example:

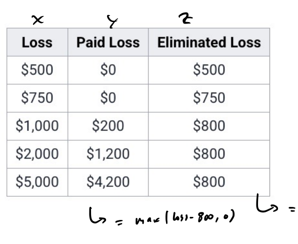
→ 

You are given the following ground-up losses:

500, 750, 1,000, 2,000, 5,000

Assuming an $800 deductible (with a base level deductible of $0), calculate the Loss Elimination Ratio (LER).

→ The ground-up loss, paid loss, + eliminated loss are summarized in the following table:



x Loss	y Paid Loss	z Eliminated Loss
$500	$0	$500
$750	$0	$750
$1,000	$200	$800
$2,000	$1,200	$800
$5,000	$4,200	$800

↳ = max(loss − 800, 0)   ↳ = loss − paid loss

→ There are two methods to calculate the LER:

→ Method 1 → The ratio of the losses eliminated by the deductible to the total ground up losses.

$$LER(\$800) = \frac{\text{Total eliminated loss}}{\text{Total loss}} = \frac{\text{Total eliminated loss}}{\text{Total loss}} = \frac{\Sigma z_i}{\Sigma x}$$

$$= \frac{500+750+800+800+800}{500+750+1000+2000+5000} = \frac{\$3650}{\$9250} = 0.395$$

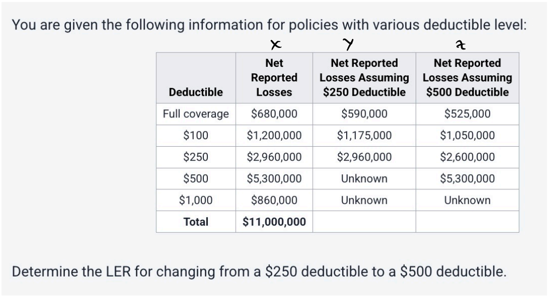
→ Method 2 → one minus the ratio of the losses that remain after applying the deductible to the ground-up losses.

$$LER(\$800) = 1 - \frac{\text{Total paid loss}}{\text{Total loss}} = 1 - \frac{\Sigma y_i}{\Sigma x_i}$$

$$= 1 - \frac{0+0+200+1200+4200}{500+750+1000+2000+5000} = 1 - \frac{\$5600}{\$9250} = 0.395$$

→ The previous calculations were possible b/c the ground-up losses were available. However, companies may not have ground-up losses for all claims, as insureds often do not report losses below their policy deductible. In such cases, the data may only include net losses (losses above the deductible). When using net losses, data from policies w/ higher deductibles cannot be used to calculate the loss elimination ratio for lower deductibles. For instance, data from $500 deductible policies cannot be used to calculate ratios for a $250 deductible. However, data from policies w/ lower deductibles can be used. Actuaries often aggregate data from all policies w/ lower deductibles to increase data volume for analysis.

→ The example below shows the calculation for the credit to move from a $250 to a $500 deductible.



You are given the following information for policies with various deductible level:

Deductible	x Net Reported Losses	y Net Reported Losses Assuming $250 Deductible	z Net Reported Losses Assuming $500 Deductible
Full coverage	$680,000	$590,000	$525,000
$100	$1,200,000	$1,175,000	$1,050,000
$250	$2,960,000	$2,960,000	$2,600,000
$500	$5,300,000	Unknown	$5,300,000
$1,000	$860,000	Unknown	Unknown
Total	**$11,000,000**		

Determine the LER for changing from a $250 deductible to a $500 deductible.

→ In the table above, we are given net reported losses, which are the portion of reported losses that exceed the deductibles. The goal is to calculate the loss elimination ratio when changing from a $250 deductible to a $500 deductible, so only policies w/ deductibles of $250 or less can be used.

→ The LER is calculated as:

$$LER(\$500) = \frac{\text{losses eliminated by going from Deductible} = \$250 \text{ to Deductible} = \$500}{\text{net reported losses assuming Deductible } \$250}$$

$$= \frac{\text{Net reported losses assuming ded } \$250 - \text{net reported losses assuming ded. } \$500}{\text{net reported losses assuming Deductible } \$250}$$

$$= \frac{(590{,}000 + 1{,}175{,}000 + 2{,}960{,}000) - (525{,}000 + 1{,}050{,}000 + 2{,}600{,}000)}{590{,}000 + 1{,}175{,}000 + 2{,}960{,}000}$$

$$= \frac{\$4{,}725{,}000 - \$4{,}175{,}000}{\$4{,}725{,}000} = 0.1164$$

→ Fitted data Approach

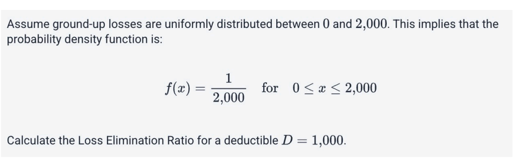
→ Assume f(x) denotes the continuous distribution of losses of size x + D is the size of the deductible. Assuming no deductible in the base level policy, the losses eliminated can be calculated as:

$$\boxed{\text{Eliminated loss} = \int_0^D x f(x)\,dx + \int_D^\infty D f(x)\,dx} \xrightarrow{\text{this is just}} \text{if } g(x) = \begin{cases} x & x<D \\ D & x \geq D \end{cases} \Rightarrow E(g(x))$$

→ Then the LER is the eliminated loss divided by the expected loss:

$$LER(D) = \frac{\text{Eliminated loss}}{\text{Expected loss}} = \frac{\int_0^D x f(x)\,dx + \int_D^\infty D f(x)\,dx}{\int_0^\infty x f(x)\,dx} \Rightarrow = \frac{E(g(x))}{E(x)}$$

→ Example

→ 

Assume ground-up losses are uniformly distributed between 0 and 2,000. This implies that the probability density function is:

$$f(x) = \frac{1}{2{,}000} \quad \text{for} \quad 0 \leq x \leq 2{,}000$$

Calculate the Loss Elimination Ratio for a deductible $D = 1{,}000$.

→ Step 1 → Calculate the expected loss (ground-up)

→ The expected loss, or mean of a uniform distribution from 0 to 2000 is simply the midpoint:

$$\underset{E(x)}{\text{Expected loss}} = \frac{0+2000}{2} = 1000$$

→ Step 2 → Calculate the losses eliminated by deductible D

→ The loss eliminated by the deductible D = 1000 can be split into two parts

→ 1) Losses eliminated below D → This represents the portion of losses fully absorbed by the deductible. It's calculated as the integral of x·f(x) from 0 to D:

$$\int_0^{1000} x \frac{1}{2000}\,dx = \frac{1}{4000} x^2 \Big|_0^{1000} = 250$$

→ 2) Fixed deductible for losses above D → This represents the portion of losses partially absorbed by the deductible. It's calculated as the deductible amount times the probability that a loss exceeds D:

$$\int_{1000}^{2000} D \frac{1}{2000}\,dx = \underset{\hookrightarrow = 1000}{D} \frac{1}{2000}\left[2000 - 1000\right] = 500$$

→ So the total losses eliminated by the deductible D = 1000 is → 250 + 500 = 750

→ Step 3 → Calculate the LER

→ Finally, calculate the LER using the formula:

$$LER(D) = \frac{\text{Eliminated loss}}{\text{Expected loss}} = \frac{750}{1000} = 0.75$$

→ The loss elimination ratio for a deductible of 1000 is 75%, meaning that 75% of the total losses are expected to be eliminated by the deductible.

→ Practical considerations

→ The following should be considered when using the LER approach:

→ 1) LER calculations assume claiming behavior will be the same for each deductible + ignore the difference in claiming behavior for different deductibles.

→ For example, an insured may report a claim of $1,100 under a $250 deductible policy but is less likely to report the same claim under a $1000 deductible policy. This is because the risk of an increase in premium for a claim surcharge outweighs the small net payment of $100.

→ 2) LER also does not recognize the behavioral differences that arise when insureds are allowed to choose their policy deductible.

→ Usually, lower-risk insureds will choose higher deductibles since they believe it is unlikely for them to have a claim. As a result, policies w/ higher deductibles are often more profitable than policies w/ lower deductibles.

→ Univariate & multivariate methods are able to capture the behavioral differences mentioned above. Thus, it may be prudent to determine deductible rate relativities using various approaches before selecting the final set to use.

→ Another consideration is the average percentage credit determined using LER approach. This credit is applied to all policies w/ a specific deductible amount. For example, in an earlier example w/ ground-up losses, we calculated the LER for a $800 deductible as 39.5%. This means that if the total policy premium is $5000, the credit from full coverage to a $800 deductible is $5000 × 39.5% = $1,975. This premium savings is significantly more than the deductible amount, which disproportionately benefits the insured. The insurer can handle this situation in a few ways, such as:

→ 1) Implementing a cap on the amount of dollar credit from the deductible.

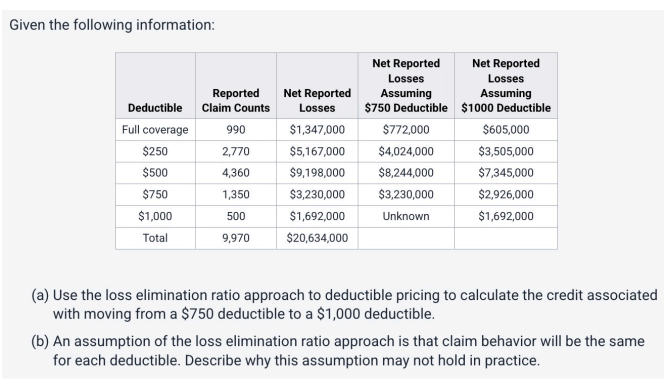
→ For example, the maximum deductible credit for switching from full coverage to a $250 deductible might be capped at $200.

→ 2) Calculating a different set of credits for different policies.

→ For example, a homeowners insurer may offer different deductible credits for low-, medium-, + high-valued homes. Segregating risks in this way helps minimize the likelihood of such reversals.

→ 3) Using percentage-based deductibles

→ This approach links the deductible amount to the insurance coverage amount, increasing the deductible as the coverage amount rises

→ Example → 

Given the following information:

Deductible	Reported Claim Counts	Net Reported Losses	Net Reported Losses Assuming $750 Deductible	Net Reported Losses Assuming $1000 Deductible
Full coverage	990	$1,347,000	$772,000	$605,000
$250	2,770	$5,167,000	$4,024,000	$3,505,000
$500	4,360	$9,198,000	$8,244,000	$7,345,000
$750	1,350	$3,230,000	$3,230,000	$2,926,000
$1,000	500	$1,692,000	Unknown	$1,692,000
Total	9,970	$20,634,000		

(a) Use the loss elimination ratio approach to deductible pricing to calculate the credit associated with moving from a $750 deductible to a $1,000 deductible.

(b) An assumption of the loss elimination ratio approach is that claim behavior will be the same for each deductible. Describe why this assumption may not hold in practice.

→ a) → only policies w/ a deductible of $750 or less can be used to calculate the loss elimination ratio. For ease of readability, we will use the net reported losses in 000's.

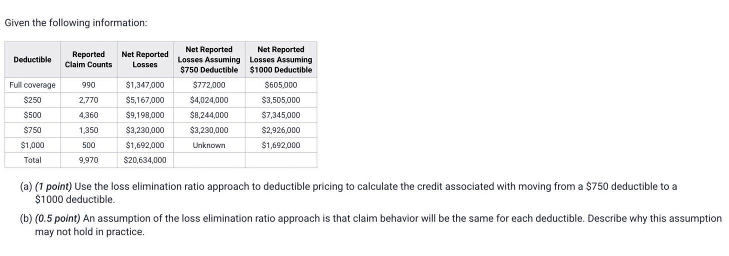
→ The LER is the ratio of losses eliminated by increasing the deductible from $750 to $1000, relative to the net reported losses under the $750 deductible. Remember to only use the first 4 rows in the table; do not use the row for the $1000 deductible.

$$\rightarrow LER = \frac{(772-605) + (4024-3505) + (8244-7345) + (3230-2926)}{772+4024+8244+3230}$$

$$= 11.61\%$$

→ b) In practice, claim behavior might vary based on the selected amount of deductible due to different financial incentives. For example, it is more likely for an insured to file a claim of $1050 under a $250 deductible policy than under a $1000 deductible policy. W/ a $250 deductible, the insured will receive a payment of $800, so it is worth the effort to report the claim. If the deductible is $1000, the net payment of $50 is small, + reporting the claim will increase the risk of an increase in premium from a claim surcharge.

→ Assignment

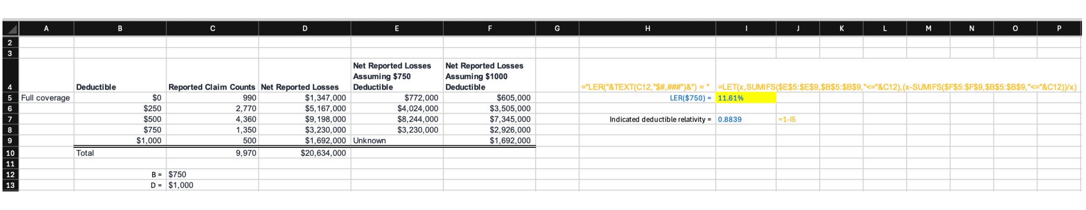
→ Q1) 

Given the following information:

Deductible	Reported Claim Counts	Net Reported Losses	Net Reported Losses Assuming $750 Deductible	Net Reported Losses Assuming $1000 Deductible
Full coverage	990	$1,347,000	$772,000	$605,000
$250	2,770	$5,167,000	$4,024,000	$3,505,000
$500	4,360	$9,198,000	$8,244,000	$7,345,000
$750	1,350	$3,230,000	$3,230,000	$2,926,000
$1,000	500	$1,692,000	Unknown	$1,692,000
Total	9,970	$20,634,000		

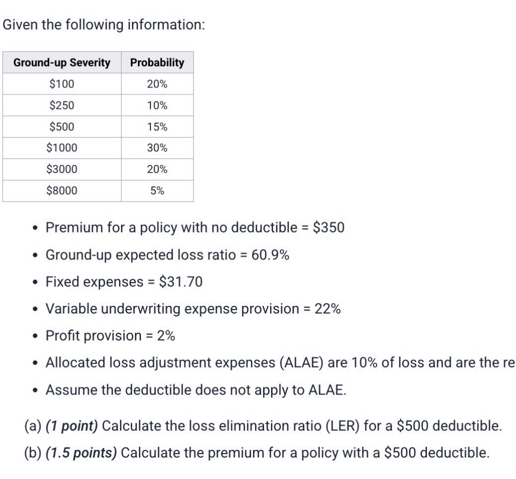
(a) *(1 point)* Use the loss elimination ratio approach to deductible pricing to calculate the credit associated with moving from a $750 deductible to a $1000 deductible.

(b) *(0.5 point)* An assumption of the loss elimination ratio approach is that claim behavior will be the same for each deductible. Describe why this assumption may not hold in practice.

→ a) 

Row	A	B	C	D	E	F	G	H	I	J
4		Deductible	Reported Claim Counts	Net Reported Losses	Net Reported Losses Assuming $750 Deductible	Net Reported Losses Assuming $1000 Deductible		="LER("&TEXT(C12,"$#,###")&") = "	=LET(x,SUMIFS($E$5:$E$9,$B$5:$B$9,"<="&C12),(x-SUMIFS($F$5:$F$9,$B$5:$B$9,"<="&C12))/x)	
5	Full coverage	$0	990	$1,347,000	$772,000	$605,000		LER($750) =	11.61%	
6		$250	2,770	$5,167,000	$4,024,000	$3,505,000				
7		$500	4,360	$9,198,000	$8,244,000	$7,345,000		Indicated deductible relativity =	0.8839	=1-I5
8		$750	1,350	$3,230,000	$3,230,000	$2,926,000				
9		$1,000	500	$1,692,000	Unknown	$1,692,000				
10		Total	9,970	$20,634,000						
12		B =	$750							
13		D =	$1,000							

→ b) < see notes for example >

→ Q2) 

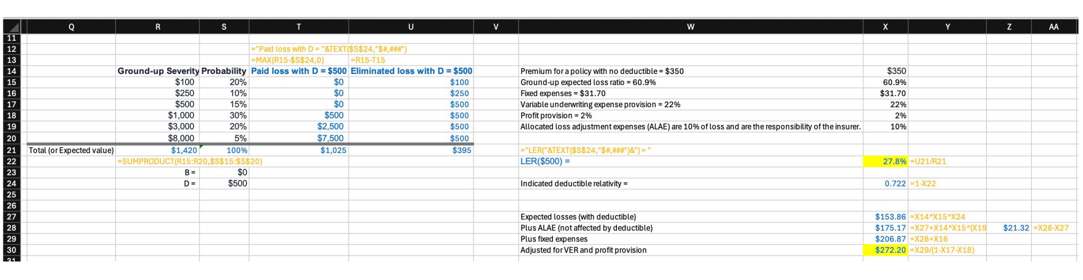
Given the following information:

Ground-up Severity	Probability
$100	20%
$250	10%
$500	15%
$1000	30%
$3000	20%
$8000	5%

- Premium for a policy with no deductible = $350
- Ground-up expected loss ratio = 60.9%
- Fixed expenses = $31.70
- Variable underwriting expense provision = 22%
- Profit provision = 2%
- Allocated loss adjustment expenses (ALAE) are 10% of loss and are the responsibility of the insurer.
- Assume the deductible does not apply to ALAE.

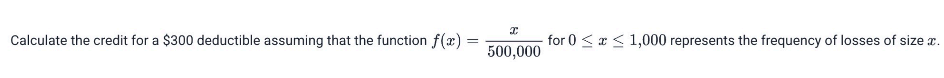
(a) *(1 point)* Calculate the loss elimination ratio (LER) for a $500 deductible.

(b) *(1.5 points)* Calculate the premium for a policy with a $500 deductible.



→ a) → b)

Row	Q	R	S	T	U	W	X	Y	Z	AA
12				="Paid loss with D = "&TEXT($S$24,"$#,###")						
13				=MAX(R15-$S$24,0)	=R15-T15					
14		Ground-up Severity	Probability	Paid loss with D = $500	Eliminated loss with D = $500	Premium for a policy with no deductible = $350	$350			
15		$100	20%	$0	$100	Ground-up expected loss ratio = 60.9%	60.9%			
16		$250	10%	$0	$250	Fixed expenses = $31.70	$31.70			
17		$500	15%	$0	$500	Variable underwriting expense provision = 22%	22%			
18		$1,000	30%	$500	$500	Profit provision = 2%	2%			
19		$3,000	20%	$2,500	$500	Allocated loss adjustment expenses (ALAE) are 10% of loss and are the responsibility of the insurer.	10%			
20		$8,000	5%	$7,500	$500					
21	Total (or Expected value)	$1,420	100%	$1,025	$395	="LER("&TEXT($S$24,"$#,###")&") = "				
22		=SUMPRODUCT(R15:R20,$S$15:$S$20)				LER($500) =	27.8%	=U21/R21		
23		B =	$0							
24		D =	$500			Indicated deductible relativity =	0.722	=1-X22		
27						Expected losses (with deductible)	$153.86	=X14*X15*X24		
28						Plus ALAE (not affected by deductible)	$175.17	=X27+X14*X15*(X19	$21.32	=X28-X27
29						Plus fixed expenses	$206.87	=X28+X16		
30						Adjusted for VER and profit provision	$272.20	=X29/(1-X17-X18)		

→ Q3) 

Calculate the credit for a $300 deductible assuming that the function $f(x) = \frac{x}{500{,}000}$ for $0 \leq x \leq 1{,}000$ represents the frequency of losses of size x.

$$\rightarrow \text{Let } g(x) = \begin{cases} x & x<d \\ D & x \geq d \end{cases} \Rightarrow LER = \frac{E(g(x))}{E(x)} = \frac{291}{666.67} = 0.4365$$

↳ eliminated losses

$$\rightarrow E(x) = \int_0^{1000} x \frac{x}{500000}\,dx = \int_0^{1000} \frac{x^2}{500000}\,dx = \frac{1}{3} x^3 \left(\frac{1}{500{,}000}\right)\Big|_0^{1000} = 666.67$$

$$E(g(x)) = \int_0^D x \frac{x}{500{,}000}\,dx + \int_D^{1000} D \frac{x}{500{,}000}\,dx$$

$$= \frac{1}{3} x^3 \left(\frac{1}{500{,}000}\right)\Big|_0^D + \frac{1}{2} D x^2 \left(\frac{1}{500{,}000}\right)\Big|_D^{1000}$$

$$= \frac{1}{3} D^3 \left(\frac{1}{500{,}000}\right) + \frac{1}{2} D \left(\frac{1}{500{,}000}\right)\left[1000^2 - D^2\right]$$

$$= \frac{1}{500{,}000} (D)\left[\frac{1}{3} D^2 + \frac{1}{2}\left(1000^2 - D^2\right)\right]$$

< let D = 300 >

$$= 291$$

OR

$$\text{let } h(x) = \begin{cases} 0 & x<D \\ x-D & x \geq D \end{cases} \Rightarrow \text{Indicated deductible relativity} = 1 - LER$$

↳ paid losses

$$\Rightarrow LER = 1 - \text{Indicated Deductible relativity} = 1 - \frac{E(h(x))}{E(x)} = 1 - \frac{375.67}{666.67} = 0.4365$$