

6.6.7 → Frequency-severity technique (FS)

→ Assumptions & uses

- Similar to Technique #1, frequency-severity technique also relies on the development method. However, technique #2 incorporates trend analysis for exposure, frequency, & severity. Given its reliance on the development method, the same reasoning for assumptions is straightforward:
- #1) Claim counts & severities reported to date will continue to develop in a similar fashion in the future.
- #2) Claim counts have a consistent distribution throughout the experience period.
- #3) The ratio of claim years is reasonably homogeneous.
- This technique is especially useful when estimating the ultimate loss or IBNR for AIs that are relatively immature, especially if the CDFs for the early years are highly leveraged. Although it fails in obtaining CDFs using the development method, they are often (though not always) provided during the exam. \rightarrow lots of variation (for CDFs, the same fits worse)

→ Technique

→ FS #2 technique can be broken into five steps:

- 1) Project ultimate claim counts
- 2) Compute claim counts & exposures, & select ultimate frequency
- 3) Project & select ultimate severity
- 4) Project ultimate claims
- 5) Compute unpaid claim / IBNR estimate (if needed)

→ Essentially, we select a frequency/severity as an average historical ultimate frequency/severity trended to the year we're interested. Then we calculate an ultimate claim estimate for that year as

$$\text{Ay ult claim} = \frac{\text{Projected claim counts} \times \text{Selected severity}}{\text{Ay exposures} \times \text{Selected frequency}}$$

→ We will demonstrate this w/ an example similar to a past loss exam question.

→ Examples

You are given the following information for a worker's compensation line of business.

Accident Year	Reported as of December 31, 2010	Payroll (\$000)
2007	800	\$4,200
2008	790	\$4,000
2009	650	\$3,100
2010	340	\$620

As of Month	Cumulative Development Factor
12	2.5
24	1.3
36	1.1
48	1.0

- The annual severity trend is +3%.
- The annual claim count trend is -1%.
- The annual payroll trend is +5%.

Estimate the AY2010 IBNR using a frequency-severity technique.

→ Step 1: Project ultimate claim counts

- Start by developing the reported claim counts to estimate. Use the CDFs provided to project each day's reported claim counts to ultimate.
- Since our historical trends to project 48-month ultimate losses, trend each ultimate claim count using the given trend (-1%) to Ay 2010 level.
- To obtain the projected claim counts, multiply the reported claim counts by the claim count CDF & the trend factor.

Accident Year	Reported Claim Counts	Claim Count CDF	Claim Count Trend Factor	Projected Claim Counts
2007	800	1.0	$(1 - 0.01)^3$	776
2008	790	1.1	$(1 - 0.01)^2$	852
2009	650	1.3	$(1 - 0.01)$	837

→ Since 2010 data, just like the method (i.e. don't know the same year's losses higher to project ultimate claims for)

→ Step 2: Select ultimate frequency

- We have not been explicitly provided w/ the company's definition of exposure. However, we can assume that the unit of exposure is year of payroll since that's the most common exposure base for workers' compensation, & the provided payroll data is in hundreds of dollars.
- Rank the exposures for each day to same level. Then divide the projected claim counts by the provided exposures for each day to get the projected frequencies. For example:

$$\text{Ay 2010 Projected Payroll} = \frac{776}{(100,000 \times (1 + 0.05)^3)} = 0.671\%$$

Accident Year	Payroll (\$000)	Payroll Trend Factor	Trended Payroll	Projected Frequency
2007	\$100,000	$(1 + 0.05)^3$	\$115,763	0.671%
2008	\$115,000	$(1 + 0.05)^2$	\$126,788	0.672%
2009	\$120,000	$(1 + 0.05)$	\$126,000	0.664%

→ Same, figure 2010

- The projected frequencies seem stable. Thus, the selected frequency is the simple average of the projected frequencies across days.

$$\text{Selected frequency} = \frac{0.671\% + 0.672\% + 0.664\%}{3} = 0.669\%$$

→ Step 3: Project & select ultimate severity

- Develop the reported severities to estimate, & trend to Ay 2010 level using the 3-year severity trend.

Accident Year	Reported Severity	CDF	Severity Trend Factor	Projected Severity
2007	\$4,200	1.0	$(1 + 0.03)^3$	\$4,589
2008	\$4,000	1.2	$(1 + 0.03)^2$	\$5,092
2009	\$3,100	1.4	$(1 + 0.03)$	\$4,470

- The projected severities seem stable. Thus, the selected severity is the simple average of the projected severities across AYs.

$$\text{Selected severity} = \frac{4,589 + 5,092 + 4,470}{3} = 4,712$$

→ Step 4: Project ultimate claims

- Using the selected frequency, we can project the ultimate claim counts for Ay 2010. Multiply Ay 2010 trended payroll by the selected frequency + set projected ultimate claim counts for Ay 2010.

$$\text{Ay 2010 Projected Claim Counts} = \text{Ay 2010 Exposures} \times \text{Selected Frequency}$$

$$= 100,000 \times 0.669\% =$$

$$= 669$$

- This means that the company expects 669 claims to arise from the insured payroll of \$10 million.

- Multiply the projected claim counts by the selected severity to find the Ay 2010 ultimate claims.

$$\text{Ay 2010 Ultimate Claims} = \text{Ay Projected Claim Counts} \times \text{Selected Severity}$$

$$= 669 \times 4,712$$

$$= 3,101,088$$

→ Step 5: Compute IBNR estimate

- Finally, estimate the losses for Ay 2010 by subtracting cumulative reported claims from projected ultimate claims. We are not given Ay 2010 cumulative reported claims directly, but we can plug cumulative reported claim counts & severity.

- Multiply the cumulative reported claim counts by the cumulative reported severity for Ay 2010 to get reported claims.

$$\text{Ay 2010 Reported Claims} = \text{Ay 2010 Cumulative Reported Claims} \times \text{Ay 2010 Reported Severity}$$

$$= 6,791,413 \times 0.669 = 4,500,000$$

$$= 2,291,413$$

- Then, the difference between the ultimate claims & the reported claims.

$$\text{IBNR est} = \text{Ay 2010 Ultimate Claims} - \text{Ay 2010 Reported Claims}$$

$$= 3,101,088 - 2,291,413$$

$$= 809,675$$

→ Assignment

→ Q1)

You are given the following information about a particular line of business:

Accident Year	Reported Claim Counts Excluding Claims Closed With No Payment
2006	200
2007	250
2008	300

- The 48-to-ultimate development factor for claim counts is 1.010.
- The 48-to-ultimate development factor for reported severity is 1.025.
- The selected annual frequency trend is +2.0% for 2006 to 2009.
- The selected annual severity trend is -1.5% for 2006 to 2009.
- Volume-weighted averages are used to calculate development factors.
- Exposures have been constant and there is no exposure trend.

- (a) (3.25 points) Use the frequency-severity technique to calculate the expected ultimate claim cost estimate for accident year 2009.

- (b) (0.75 points) State the three key assumptions underlying the frequency-severity technique.

→ Step 1: Identify the appropriate FS technique

- We're asked to estimate the ultimate claim cost for Ay 2009, but no data is provided for 2009. This is a strong indicator that the question is permitting us to use FS #2 technique, but does not specify which FS technique to use.

- This is type of wording you should expect to see on the exam. You'll have to determine which technique the exam wants you to use based on the information given in the problem. For example, Technique #1 requires trend rates & age-weighted factor, whereas Technique #2 relies on cumulative development factors.

- Second, notice that the calculation for projected claim counts under Technique #2 bears a strong resemblance to the expected claim counts.

$$\text{Ay Projected Claim Counts} = \text{Ay Exposures} \times \text{Selected Frequency}$$

$$\downarrow \qquad \qquad \qquad \Rightarrow \text{Ay Exposures} \times \frac{\text{Projected Claim Counts}}{\text{Projected Exposures}}$$

- In both cases, we create a ratio of some quantity (claim counts or claim amounts) to exposure. Then we multiply that ratio by Ay exposures to project claim counts (or FS #2 technique) or ultimate claims (Technique #1). Thus, the way we calculate the project for this technique is very similar to normalizing.

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- Therefore, we can conclude that FS #2 is the appropriate technique to use here.

- Now, we need to calculate the projected claim counts for Ay 2009.