

### 3.4 Summary

 15m

This section aimed to offer a general overview of credibility procedures applied in ratemaking. After selecting the method for calculating the credibility measure, it is essential for the actuary to thoroughly understand and accurately document any simplifying assumptions used.

The section also discussed desirable qualities for selecting related experience, known as the complement of credibility. Additionally, it presented various methods for determining the complement of credibility for both first-dollar and excess ratemaking, evaluating each method against specified criteria.

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### Measuring Credibility

The amount of credibility ( $Z$ ) should meet the following criteria:

1.  $0 \leq Z \leq 1$ .
2. As  $n$  increases,  $Z$  should increase.
3. As  $n$  increases,  $Z$  should increase at a decreasing rate.

### CLASSICAL CREDIBILITY

The formula for the credibility-weighted estimate is:

$$\text{Estimate} = Z \times \text{Observed Experience} + (1 - Z) \times \text{Related Experience}$$

Assuming exposures are homogeneous, claim occurrence follows a Poisson distribution, and a constant severity, the expected number of claims needed for full credibility is:

$$E(Y) = \left( \frac{z_{(p+1)/2}}{k} \right)^2$$

If the number of observed claims does not meet the full credibility standard,  $Z$  is calculated using the square root rule:

$$Z = \sqrt{\frac{Y}{E(Y)}} \text{ where } Y < E(Y)$$

The number of exposures needed for full credibility is the number of claims needed for full credibility divided by the expected frequency.

Advantages of classical credibility:

1. Most commonly used.
2. Data is readily available.
3. Calculations are straightforward.

Disadvantages of classical credibility:

1. Simplifying assumptions used may not be true in practice.
2. Does not consider the quality of complement of credibility.

## BÜHLMANN CREDIBILITY

The formula for the credibility-weighted estimate is:

$$\text{Estimate} = Z \times \text{Observed Experience} + (1 - Z) \times \text{Prior Mean}$$

Formula for  $Z$ :

$$Z = \frac{N}{N + K}$$

where:

- $N$  = Number of observations
- $K$  = The expected value of process variance (EVPV) divided by the variance of hypothetical means (VHM)

Bühlmann credibility is commonly used within the insurance industry but the determination of EVPV and VHM can be challenging.

## BAYESIAN ANALYSIS

Bayesian analysis has no specific calculation of  $Z$  but adjusts the prior estimate to reflect new information using Bayes Theorem. Bayesian analysis is not commonly used due to its complexity.

## Complements of Credibility

## FIRST DOLLAR RATEMAKING

Formula for the complement of credibility using the rate change from the larger group applied to present rates method:

$$C = \frac{\text{Current Loss Cost of Subject Experience}}{\text{Larger Group Current Average Loss Cost}} \times \frac{\text{Larger Group Indicated Loss Cost}}{\text{Larger Group Current Average Loss Cost}}$$

Formula for the complement of credibility using the trended present rates method:

$$C = \text{Present Rate} \times \text{Loss Trend Factor} \times \frac{\text{Prior Indicated Loss Cost}}{\text{Loss Cost Implemented with Last Review}}$$

$$C = \frac{\text{Loss Trend Factor}}{\text{Premium Trend Factor}} \times \frac{\text{Prior Indicated Rate Change Factor}}{\text{Prior Implemented Rate Change Factor}}$$

Steps to calculate the complement of credibility using Harwayne's method, assuming the complement is for Class 1 in State A:

1. Calculate the weighted average pure premium for State A.
2. Calculate the weighted average pure premium for other states based on State A's exposure distribution by class.
3. Calculate the adjustment factors.
4. Apply the adjustment factors to the Class 1 pure premium in other states.
5. Calculate the complement of credibility.

Method \ Desirable Quality	Loss Costs of a Larger Group that Includes the Group being Rated	Loss Costs of a Larger Related Group	Rate Change from Larger Group Applied to Present Rates	Harwayne's Method	Trended Present Rates	Competitor Rates
Accurate	Yes	May be inaccurate	Accurate over the long run	Yes	Depends on accuracy of historical loss costs	May be inaccurate
Unbiased	Likely biased	Generally biased	Unbiased	Unbiased	Unbiased	May be biased
Independent	Independent if subject is excluded or does not dominate the larger group	Yes	Depends on size of subject relative to the larger group	Mostly independent	Depends on data used	Yes
Available	Yes	Yes	Yes	Yes	Yes	No
Easy to Compute	Yes	Yes	Yes	No	Yes	Yes

Method \ Desirable Quality	Loss Costs of a Larger Group that Includes the Group being Rated	Loss Costs of a Larger Related Group	Rate Change from Larger Group Applied to Present Rates	Harwayne's Method	Trended Present Rates	Competit Rates
Logical	Yes, if all risks in the group have something in common	Yes, if the group is chosen reasonably	Yes, if the group is chosen reasonably	Hard to explain due to computational complexity	Yes	Yes

## EXCESS RATEMAKING

Formulas for the complement of credibility for the layer  $L$  excess of  $A$  under different methods:

### 1. Increased limits analysis

$$C = L_A \times \left( \frac{\text{ILF}_{A+L} - \text{ILF}_A}{\text{ILF}_A} \right)$$

### 2. Lower limits analysis

$$C = L_d \times \left( \frac{\text{ILF}_{A+L} - \text{ILF}_A}{\text{ILF}_d} \right)$$

### 3. Limits analysis

$$C = \text{Expected Loss Ratio} \times \sum_{d>A} \text{Premium}_d \times \frac{(\text{ILF}_{\min(d, A+L)}) - \text{ILF}_A}{\text{ILF}_d}$$

### 4. Fitted curves

$$C = \text{Total Limits Losses} \times \frac{\int_A^{A+L} (x - A) \cdot f(x) dx + \int_{A+L}^{\infty} L \cdot f(x) dx}{\int_{-\infty}^{\infty} x \cdot f(x) dx}$$

Method \ Desirable Quality	Increased Limits Analysis	Lower Limits Analysis	Limits Analysis	Fitted Curves
Accurate	Inaccurate if there is a difference in loss severity distribution	More accurate than increased limits analysis	Same level as increased limits and lower limits analyses	Most accurate if fitted curve replicates distribution of actual data

<b>Method \ Desirable Quality</b>	<b>Increased Limits Analysis</b>	<b>Lower Limits Analysis</b>	<b>Limits Analysis</b>	<b>Fitted Curves</b>
<b>Unbiased</b>	Biased if there is a difference in loss severity distribution	More biased than increased limits analysis	Same level as increased limits and lower limits analyses	Least biased if fitted curve replicates distribution of actual data
<b>Independent</b>	Generally independent	Generally independent	Generally independent	Less independent
<b>Available</b>	Practical if data is available	Not as available	Not as available	Not as available
<b>Easy to Compute</b>	Yes	Yes	Time-consuming but straightforward	Computationally complex
<b>Logical</b>	Controversial	Controversial	Controversial	Most logically related to losses in the excess layer