4.2.2 → Deriving base rates

→ Deriving base rates: no change to relativities

→ After determining the target average premium, rating algorithm, proposed relativities, & additive premium, the final step is to derive a base rate that results in the target average premium. If the proposed relativities are the same as the current ones, this isn't too difficult.

→ We can derive an equation for the proposed base rate by relating it to the proposed average premium. To keep the formulas in this section a bit more compact, we'll assume that any additive premium is an expense fee. Each equation we derive relies on the following relationship:

$$\boxed{\text{Avg Premium} = \text{Base rate} * \text{Avg rating factor} + \text{Additive fee}}$$

→ The most obvious way to derive a formula for the proposed base rate would be by directly rearranging the relationship from above:

$$\boxed{\text{Proposed base rate} = \frac{\text{Proposed avg Premium} - \text{Proposed additive fee}}{\text{Proposed avg rating factor}}}$$

→ Using this formula would be perfectly valid; however it would require calculating the proposed weighted average rating factor which can take a while. We'll come back to this equation later, but for now, let's find a quicker method.

→ For the sake of readability, we will use some variables: we will denote the average premium, base rate, weighted average rating factor, & expense fee w/ $\bar{P}, B, \bar{R}, A$, respectively. Furthermore, we will use the subscripts P to indicate "Proposed" & C to indicate "Current". Now we can relate the formulas above for the proposed & current average premiums:

$$\bar{P}_P = B_P * \bar{R}_P + A_P$$
$$\bar{P}_C = B_C * \bar{R}_C + A_C$$

→ If we take a ratio of the proposed variable premium to the current variable premium, we discover another relationship:

$$\frac{B_P * \bar{R}_P}{B_C * \bar{R}_C} = \frac{\bar{P}_P - A_P}{\bar{P}_C - A_C}$$

$$B_P * \bar{R}_P = \frac{\bar{P}_P - A_P}{\bar{P}_C - A_C} * (B_C * \bar{R}_C)$$

$$* \quad B_P = \frac{\bar{P}_P - A_P}{\bar{P}_C - A_C} * B_C * \frac{\bar{R}_C}{\bar{R}_P}$$

→ Since we're assuming relativities aren't changing in this scenario, the proposed average rating factor is equal to the current average rating factor, i.e. $\bar{R}_P = \bar{R}_C$. Thus, they cancel out & leave us w/

$$B_P = \frac{\bar{P}_P - A_P}{\bar{P}_C - A_C} * B_C$$

$$\therefore \text{Proposed base rate} = \frac{\text{Proposed variable premium}}{\text{Current variable premium}} * \text{Current base rate}$$

$$= \left(1 + \frac{\text{\% Change in}}{\text{variable premium}}\right) * \text{Current base rate}$$

→ Additionally, this implies that if there are no fixed fees & the premium is entirely variable, we can simply find the proposed base rate as the current base rate multiplied by the rate change factor.

→ Remember though, the equation above only applies when relativities are not changing.

→ Example

→ You are given:

Class	Exposures	Relativity	On-Level Earned Premium
1	250	1.00	$48,750
2	200	1.10	$42,600
3	300	1.25	$72,000

- There is a fixed expense fee of 15 per exposure.

The company wishes to implement a rate change such that the new average premium amount is 250. There are no changes made to the rating factors, but the fixed expense fee increases to 30.

Calculate the proposed base rate.

→ We are not given the current base rate or current average premium, so we will need to solve for both before using the equation above.

→ The quickest way to find the current base rate is by using the information given about the base class (class 1) since it has a relativity of 1. Recall from the last section that the premium for a single exposure can be expressed as

$$\text{Total premium} = \underbrace{\text{Base rate} * \text{Relativity}}_{\text{variable premium}} + \text{Additive premium}$$

→ So for class 1,

$$48{,}750 = 250 \times (\text{Current base rate} * 1.00 + 15)$$

$$\Rightarrow \text{Current base rate} = \frac{48{,}750}{250} - 15 = 180$$

→ To get the current average premium, divide the total premium for all classes by the total number of exposures

$$\text{Current Avg Premium} = \frac{48{,}750 + 42{,}600 + 72{,}000}{250 + 200 + 300} = \$217.80$$

→ Thus,

$$\text{Proposed Base rate} = \frac{250 - 30}{217.80 - 15} \times 180 = 195.27$$

→ Deriving base rates: changing relativities

→ If the rate review results in one or more rating variables changing relativities, there are three viable methods for deriving the proposed base rate:

→ Approximated average rate differential

→ Approximated change in average rate differential

→ Extension of exposures

→ Each method has its own advantages, disadvantages, & applications. We will discuss each of these techniques below.

→ Approximated average rate differential

→ To begin, let's bring back the original equation:

$$\text{Proposed base rate} = \frac{\text{Proposed avg Premium} - \text{Proposed additive fee}}{\text{Proposed avg rating factor}}$$

→ The method is called approximated average rate differential because it allows us to approximate the proposed average rating factor (a.k.a. proposed rate differential) component in the equation. Specifically, we will approximate the average rating factor as the product of the weighted average rating factors (rate differentials) for each individual rating variable. For instance, if there are two rating factors, class & territory, the average rating factor would be approximated as:

$$\boxed{\text{Avg rating factor} \approx \text{Avg Class factor} * \text{Avg Territory factor}}$$

→ When calculating the weighted average rating factors for each rating variable, there are a few different options for the weights. The most accurate option would be to weigh by the current variable premium at base level. Variable premium at base level is the total multiplicative premium a level would be charged if it were re-rated as the base level. In other words, it is the variable premium if the level had a relativity of 1.

→ This is equivalent to weighing relativities by the adjusted exposures (in the pure premium method) that was used to calculate the proposed relativities.

→ The last option is to weigh relativities for each rating variable by unadjusted exposures. This is generally less accurate than either of the two other options, so avoid this option when using the approximated average rate differential.

→ That said, when calculating the exact average rating factor, weighing by exposures is perfectly acceptable. This is because in order to calculate the exact average rating factor, we would need to find the total relativity for each combination of rating factor levels, thus essentially means we're combining the rating factors into one big rating factor.

→ We will shortly show an example that is similar to an exam problem that uses both methods; however, they are included here to help you understand the differences, making the comparison clearer or more intuitive.

→ Example →

An insurance company is targeting a 10% rate increase for the book of business described below.

Class	Territory	Exposures
A	X	100
A	Y	50
B	X	80
B	Y	90

Class	Current	Proposed
A	1.00	1.00
B	1.20	1.40

Territory	Current	Proposed
X	1.00	1.00
Y	0.90	0.85

Assume that the rating algorithm is in the form:

$$\text{Premium} = \text{Base Rate} \times \text{Relativity} + \text{Expense Fee}$$

- The current and proposed expense fees are both 25.
- The current base rate is 200 and the current average rate is 236.375.

Calculate the base rate using the approximated average rate differential.

→ To derive the proposed base rate, we need three components: the proposed average premium, the proposed additive fee, & the proposed weighted average rating factor. The fee is given to us as 25, & the proposed average premium can be found by multiplying the current average premium by 1 plus the target rate change:

$$\text{Proposed Avg Premium} = \text{Current Avg premium} \times (1 + \text{target rate change})$$
$$= 236.375 \times (1 + 0.10)$$
$$= 260.01$$

→ The third component is the proposed average rating factor. The first question to answer is how to get this: we need to calculate the proposed weighted average rating factors for both Class & Territory. We could use the exposures as weights, but it would be more accurate to weigh using the variable premium at base level for each level.

→ When calculating the variable premium at base level for a specific level of a rating variable, we set it as though that level is the base level (for that rating variable). For example, in Class B, there are 80 exposures from Territory X & 90 from Territory Y. So, the current variable premium at base level for Class B is 80 times Territory X's current relativity, plus 90 times Territory Y's current relativity, all multiplied by the current base rate of 200. Class B's relativity is omitted since we treat it as the base level, i.e. w/ a relativity of 1.

Class	Current Variable Premium at Base Level
A	200(100 × 1.00 × 1.00 + 50 × 1.00 × 0.90) = 29,000
B	200(80 × 1.00 × 1.00 + 90 × 1.00 × 0.90) = 32,200

Territory	Current Variable Premium at Base Level
X	200(100 × 1.00 × 1.00 + 80 × 1.20 × 1.00) = 39,200
Y	200(50 × 1.00 × 1.00 + 90 × 1.20 × 1.00) = 31,600

→ Next, compute the weighted average rating factors for both rating variables.

→ The proposed average class factor is:

$$\text{Proposed avg class factor} = \text{Wtd Avg of proposed class relativities}$$
$$= \frac{29{,}000}{29{,}000 + 32{,}200}(1.00) + \frac{32{,}200}{29{,}000 + 32{,}200}(1.40)$$
$$= 1.2105$$

→ Note that we are using the base-level current variable premium for Class A & Class B as the weights.

→ The proposed average territory factor is:

→ Proposed average territory factor is:

$$\text{Proposed avg territory factor} = \text{Wtd Avg of Proposed territory relativities}$$
$$= \frac{39{,}200}{39{,}200 + 31{,}600}(1.00) + \frac{31{,}600}{39{,}200 + 31{,}600}(0.85)$$
$$= 0.9331$$

→ Note that we are using the base-level variable premiums for Territories X & Y as the weights.

→ Then, combine them to get the approximated proposed average rating factor:

$$\text{Proposed Avg rating factor} \approx \text{Proposed avg class factor} * \text{Proposed avg territory factor}$$
$$= 1.2105 * 0.9331$$
$$= 1.1294$$

→ Finally, apply the beginning of the notes equation to estimate the proposed base rate that results in a 10% rate increase.

$$\text{Proposed base rate} = \frac{\text{Proposed Avg Premium} - \text{Proposed additive fee}}{\text{Proposed avg rating factor}}$$
$$= \frac{260.01 - 25}{1.1294}$$
$$= 208.08$$

→ If we were to calculate the base rate using the exact proposed average rate differential instead, we would need to find the proposed relativities for each possible combination of levels of the two rating factors. The combined proposed relativities would be the product of the individual proposed relativities for the two levels.

Class	Territory	Class Relativity	Territory Relativity	Combined Proposed Relativity
A	X	1.00	1.00	1.00 × 1.00 = 1.00
A	Y	1.00	0.85	1.00 × 0.85 = 0.85
B	X	1.40	1.00	1.40 × 1.00 = 1.40
B	Y	1.40	0.85	1.40 × 0.85 = 1.19

→ Once the combined relativities are established, compute the exposure-weighted average to find the overall proposed average rating factor:

Combined Proposed Relativity	Exposures
1.00	100
0.85	50
1.40	80
1.19	90

$$\text{Proposed Avg rating factor} = \frac{\Sigma(\text{Combined proposed relativity} \times \text{Exposures})}{\Sigma \text{Exposures}}$$
$$= \frac{1.00(100) + 0.85(50) + 1.40(80) + 1.19(90)}{100 + 50 + 80 + 90}$$
$$= 1.13$$

→ This average rating factor feeds directly into the formula to solve for the required base rate:

$$\text{Proposed Base rate} = \frac{\text{Proposed Avg Premium} - \text{Proposed Additive fee}}{\text{Proposed Avg rating factor}}$$
$$= \frac{260.01 - 25}{1.13}$$
$$= 207.98$$

→ Thus, using the exact distribution of exposures & proposed relativities, the base rate that achieves the target average premium is \$207.98

→ Note → You may be wondering why we would approximate the average rating factor when deriving the exact one seems rather easy. Though the two methods take a similar amount of time for the example above, this will not always be the case. As the number of rating variables increases, computing the exact average rating factor becomes much more !

→ For instance, consider a scenario where we have 4 rating variables, each w/ 3 categories. To compute the exact average rating factor, we would need the relativities & exposure counts for all $3^4 = 81$ combinations of rating factor levels. In contrast, the approximation would only require us to calculate the current variable premium at base level for $4 * 3 = 12$ combinations.