3.2.0 → Overview

→ Overview → In the preceding section, we examined univariate approaches to classification ratemaking. We recognize that the primary shortcoming of these univariate approaches lies in their inability to accurately capture the effect of other rating variables.

→ To achieve more precise outcomes, many companies have transitioned to alternative methods. This section will review two such methodologies: iteratively standardized one-way approaches, such as minimum bias procedures, & multivariate approaches. We will explore why multivariate approaches became popular & what benefits they offer. The discussion of multivariate approaches will focus on one particular method: generalized linear models (GLMs).

→ At the end of this section, we will briefly touch upon how data mining techniques & external data sources can complement multivariate analysis in the context of classification ratemaking.

3.2.1 → Minimum Bias Procedures

→ Overview → Minimum bias procedures are iteratively standardized univariate approaches that were popular during the latter half of the 20th century. Each procedure involves the following steps:

→ 1) Select a rating structure, such as additive, multiplicative, or combined

→ 2) Choose a bias function, which measures the difference between the procedure's observed loss statistics & indicated loss statistics. Examples of bias functions are balance principle, least squares, χ^2, & maximum likelihood functions.

→ 3) Weighting both sides of the bias function by the exposures in each cell to adjust for uneven mix of business.

→ 4) "Minimizing the bias", which is accomplished by equating the sum of indicated pure premium & the sum of the weighted observed loss costs for every level of every rating variable.

→ The only minimum bias procedure covered in the syllabus uses a multiplicative rating structure w/ the balance principle. Let's see how this minimum bias procedure works using the same data from section 3.1.3.

→ Example →

Amount of Insurance (AOI)	Territory	Exposures	Loss & ALAE	Premium at Current Rate Level
Low	1	10	$303.55	$567.60
Medium	1	110	$4,586.67	$7,804.50
High	1	180	$10,807.86	$17,368.56
Low	2	130	$6,416.59	$12,298.00
Medium	2	120	$8,136.00	$14,190.00
High	2	140	$13,668.48	$22,514.80
Low	3	150	$9,165.85	$18,447.00
Medium	3	120	$10,072.37	$18,447.00
High	3	40	$4,834.74	$8,362.64
TOTAL		1,000	$67,992.11	$120,000.10

<long data>

The current base rate is $118.25. The base levels are Territory 2 and Medium AOI.

→ First, organize the exposures & loss data as follows:

→ Exposures →

AOI \ Territory	1	2	3	Total
Low	10	130	150	290
Medium	110	120	120	350
High	180	140	40	360
Total	300	390	310	1,000

<wide data>

→ Loss & ALAE →

AOI \ Territory	1	2	3	Total
Low	$303.55	$6,416.59	$9,165.85	$15,885.99
Medium	$4,586.67	$8,136.00	$10,072.37	$22,795.04
High	$10,807.86	$13,668.48	$4,834.74	$29,311.08
Total	$15,698.08	$28,221.07	$24,072.96	$67,992.11

→ Let t_1, t_2, & t_3 denote the relativities for territories 1, 2, & 3 & a_L, a_M, & a_H denote the relativities for low, medium, & high AOI. Equate the exposure-weighted loss costs, which is the observed loss & ALAE, & the indicated loss costs, which is the total premium calculated using the indicated relativities, for each level of each rating variable.

$$\begin{aligned}
\text{Territory 1:}\quad & 15{,}698.08 = 118.25\,(10 \times a_L \times t_1 + 110 \times a_M \times t_1 + 180 \times a_H \times t_1)\\
\text{Territory 2:}\quad & 28{,}221.07 = 118.25\,(130 \times a_L \times t_2 + 120 \times a_M \times t_2 + 140 \times a_H \times t_2)\\
\text{Territory 3:}\quad & 24{,}072.96 = 118.25\,(150 \times a_L \times t_3 + 120 \times a_M \times t_3 + 40 \times a_H \times t_3)\\
\text{Low AOI:}\quad & 15{,}885.99 = 118.25\,(10 \times a_L \times t_1 + 130 \times a_L \times t_2 + 150 \times a_L \times t_3)\\
\text{Medium AOI:}\quad & 22{,}795.04 = 118.25\,(110 \times a_M \times t_1 + 120 \times a_M \times t_2 + 120 \times a_M \times t_3)\\
\text{High AOI:}\quad & 29{,}311.08 = 118.25\,(180 \times a_H \times t_1 + 140 \times a_H \times t_2 + 40 \times a_H \times t_3)
\end{aligned}$$

→ There is no closed-form solution to these equations since they are linearly dependent. To solve for the relativities of one rating variable, start w/ the seed relativities for the other rating variables. A logical choice for the seed relativities is the univariate pure premium relativities. We will use the indicated territory relativities calculated using the adjusted pure premium, which are:

$$t_1 = 0.6354$$

$$t_2 = 1.0000$$

$$t_3 = 1.2007$$

→ Substitute these values into the AOI equations to solve for the AOI relativities.

$$15{,}885.99 = 118.25\,(10 \times a_L \times 0.6354 + 130 \times a_L \times 1.0000 + 150 \times a_L \times 1.2007)$$
$$a_L = 0.4245$$

$$22{,}795.04 = 118.25\,(110 \times a_M \times 0.6354 + 120 \times a_M \times 1.0000 + 120 \times a_M \times 1.2007)$$
$$a_M = 0.5772$$

$$29{,}311.08 = 118.25\,(180 \times a_H \times 0.6354 + 140 \times a_H \times 1.0000 + 40 \times a_H \times 1.2007)$$
$$a_H = 0.8197$$

→ Next, substitute the AOI relativities into the territory equations to obtain the new territory relativities.

$$15{,}698.08 = 118.25\,(10 \times 0.4245 \times t_1 + 110 \times 0.5772 \times t_1 + 180 \times 0.8197 \times t_1)$$
$$t_1 = 0.6167$$

$$28{,}221.07 = 118.25\,(130 \times 0.4245 \times t_2 + 120 \times 0.5772 \times t_2 + 140 \times 0.8197 \times t_2)$$
$$t_2 = 0.9977$$

$$24{,}072.96 = 118.25\,(150 \times 0.4245 \times t_3 + 120 \times 0.5772 \times t_3 + 40 \times 0.8197 \times t_3)$$
$$t_3 = 1.2284$$

→ Thus, after one complete iteration, the relativities are as follows:

$$\begin{array}{lll} a_L = 0.4245 & a_M = 0.5772 & a_H = 0.8197 \\ t_1 = 0.6167 & t_2 = 0.9977 & t_3 = 1.2284 \end{array}$$

→ The process of discarding the previous relativities & solving for new ones is repeated until there is no material change in any of the relativities

- Iteration 2

$$\begin{array}{lll} a_L = 0.4197 & a_M = 0.5755 & a_H = 0.8268 \\ t_1 = 0.6137 & t_2 = 0.9970 & t_3 = 1.2332 \end{array}$$

- Iteration 3

$$\begin{array}{lll} a_L = 0.4189 & a_M = 0.5752 & a_H = 0.8280 \\ t_1 = 0.6132 & t_2 = 0.9969 & t_3 = 1.2340 \end{array}$$

- ...

→ The relativities should converge to the following values:

$$\begin{array}{lll} a_L = 0.4187 & a_M = 0.5751 & a_H = 0.8282 \\ t_1 = 0.6131 & t_2 = 0.9969 & t_3 = 1.2342 \end{array}$$

→ It is common to rebase these relativities so that the base levels have a relativity of 1. Using medium AOI & territory 2 as the base levels, the relativities to base for every level of both rating variables are:

$$\begin{array}{lll} a_L = 0.728 & a_M = 1.000 & a_H = 1.440 \\ t_1 = 0.615 & t_2 = 1.000 & t_3 = 1.238 \end{array}$$

→ These values match the true relativities that were given in section 3.1.3.

AOI	True Relativity		Territory	True Relativity
Low	0.728		1	0.615
Medium	1.000		2	1.000
High	1.440		3	1.238

→ Industry remarks → In reality, this approach is impractical, given that typical rating plans incorporate at least 10 variables.

→ Sequential analysis

→ The approach described above involves multiple rounds of univariate analysis on rating variables. Each iteration adjusts based on the exposure weight & the indications derived from the previous variable in the sequence. Implementing calculations w/ several rating variables generally requires at least some level of spreadsheet programming.

→ Another technique related to the minimum bias procedure is sequential analysis. Sequential analysis is the only classification ratemaking method permitted for voluntary personal automobile insurance in California. It involves the following steps:

→ 1) Conduct a standard univariate analysis on the first variable to determine the indicated relativities

→ 2) Calculate the indicated relativities for the second variable using the adjusted pure premium approach. This involves adjusting the exposures based on the indicated relativities of the first variable obtained in step 1

→ 3) Repeat the adjusted pure premium approach for all remaining variables, each time adjusting the exposures based on the analysis of all prior variables

→ 4) Conclude the process after completing a single pass through the sequence of chosen rating variables

→ In contrast to minimum bias procedures, which iterate until convergence is achieved, sequential analysis is not iterative. The main drawback of sequential analysis is that it does not have a closed-form solution, resulting in varying outcomes based on the order of rating variables in the sequence.

→ Notes → You should have a basic understanding of minimum bias procedures & sequential analysis, knowing their pros & cons. For minimum bias procedures, exam questions will likely ask you to perform only one iteration due to their complexity.

→ Assignment

→ Q1) An insurance company uses two variables for pricing its auto insurance product - Gender and Territory.

The ratemaking actuary has decided to use the following data, together with a multiplicative minimum bias model and the balance principle, to develop new classification rates.

Exposure Units

	Y1 (Male)	Y2 (Female)
X1 (Territory 1)	2,000	3,000
X2 (Territory 2)	2,500	4,000
X3 (Territory 3)	1,000	2,000

Observed Loss Cost Relativities → equivalent to pure premium

	Y1 (Male)	Y2 (Female)
X1 (Territory 1)	4.0	6.0
X2 (Territory 2)	2.0	5.0
X3 (Territory 3)	3.0	1.0

The initial values for territorial relativities are:

X1 = 3.0
X2 = 2.0
X3 = 1.5

Calculate the relativities for both Gender and Territory after one complete iteration.

→ Territory 1 → $26{,}000 = 2000\,x_1 y_1 + 3000\,x_1 y_2$

---- 2 → $25{,}000 = 2500\,x_2 y_1 + 4000\,x_2 y_2$

---- 3 → $5000 = 1000\,x_3 y_1 + 2000\,x_3 y_2$

→ Male → $16000 = 2000\,x_1 y_1 + 2500\,x_2 y_1 + 1000\,x_3 y_1$

Female → $40{,}000 = 3000\,x_1 y_2 + 4000\,x_2 y_2 + 2000\,x_3 y_2$

(0) Seed values → Territory: $x_1 = 3,\ x_2 = 2,\ x_3 = 1.5$

(1) Iteration ⇒ Male → $16{,}000 = 2000(3)\,y_1 + 2500(2)\,y_1 + 1000(1.5)\,y_1$

⇒ $y_1 = 1.28$

Female → $40{,}000 = 3000(3)\,y_2 + 4000(2)\,y_2 + 2000(1.5)\,y_2$

⇒ $y_2 =$

⇒ Territory 1 → $26{,}000 = 2000\,x_1(1.28) + 3000\,x_1(2)$

⇒ $x_1 = 3.037$

---- 2 → $25{,}000 = 2500\,x_2(1.28) + 4000\,x_2(2)$

⇒ $x_2 = 2.232$

---- 3 → $5{,}000 = 1000\,x_3(1.28) + 2000\,x_3(2)$

⇒ $x_3 = 0.9470$

→ $x = (3.037, 2.232, 0.947)$
$y = (1.28, 2.00)$