Insurance Basics

The Fundamental Insurance Equation

Basic Insurance Terms

- Premium: The amount the insured pays for insurance coverage.
  - Written premium: The total premium of all policies issued during a specified period.
  - Earned premium: The portion of the written premium for which coverage has already been provided as of a specific point in time.
  - Unearned premium: The portion of the written premium for which coverage has not yet been provided as of a specific point in time.
  - In-force premium: The full-term premium for policies that are still active at a specific point in time.
- Exposure: The basic unit of risk that underlies the insurance premium.
  - Written exposures: The total exposures of all policies issued during a specified period.
  - Earned exposures: The portion of the written exposures for which coverage has already been provided as of a specific point in time.
  - Unearned exposures: The portion of the written exposures for which coverage has not yet been provided as of a specific point in time.
  - In-force exposures: The number of insured units that are exposed to loss at a specific point in time.
- Claim: A demand for indemnification.
  - Open claim: Claim that has not been settled.
  - Closed claim: Claim that has been settled.
- Claimant: The person making the claim.
- Date of loss/accident date/occurrence date: The date when the event that resulted in the loss occurred.
- Report date: The date the claim is reported.
- Unreported/incurred but not reported (IBNR) claims: Claims that are not known to the insurer.
- Loss: The amount of compensation paid or payable to the claimant.
  - Paid losses: The amount that has been paid to claimants.
  - Case reserve/case outstanding: An estimate of the amount required to ultimately settle any reported claim, excluding any payments already made.
  - Reported loss/case incurred loss: The sum of paid losses and the current case outstanding for a claim.
  - Ultimate loss: The amount needed to close and settle all claims for a defined group of policies.
- Incurred but not reported (IBNR) reserve: The estimated amount required to ultimately settle all unreported claims.
- Incurred but not enough reported (IBNER) reserve: The difference between the estimated amount required to ultimately settle the reported claims and the aggregate reported losses at a given valuation date.
- Loss adjustment expenses (LAE): Expenses incurred in the process of settling claims.
  - Allocated loss adjustment expenses (ALAE): LAE that are directly attributable to a specific claim.
  - Unallocated loss adjustment expenses: LAE that cannot be attributed to the processing of a specific claim.
  - Defense and cost containment (DCC): Expenses related to defense, litigation, and cost containment services.
  - Adjusting and other (A&O): LAE not included in DCC.
- Underwriting expenses/operational and administrative expenses: Expenses incurred in the acquisition and servicing of policies.
  - Commissions and brokerage: The amounts paid to insurance agents or brokers as compensation for generating business.
  - Other acquisition costs: Expenses paid to acquire business other than commission and brokerage.
  - Taxes, licenses, and fees: All taxes, licensing fees, and miscellaneous fees paid by the insurer, excluding federal income taxes.
  - General expenses: The remaining expenses associated with the insurer's operation and any other miscellaneous costs.
- Underwriting profit: The profit generated by an insurance company through its course of business.

Fundamental Insurance Equation

$$\text{Premium} = \text{Losses} + \text{LAE} + \text{UW Expenses} + \text{UW Profit}$$

Key points to consider when performing ratemaking based on the fundamental insurance equation:

1. Ratemaking is prospective.
2. Balance should be attained at the aggregate & individual levels.

Basic Insurance Ratios

$$\text{Frequency} = \frac{\text{Number of Claims}}{\text{Number of Exposures}}$$

$$\text{Severity} = \frac{\text{Losses}}{\text{Number of Claims}}$$

$$\text{Pure Premium} = \frac{\text{Losses}}{\text{Number of Exposures}} = \text{Frequency} \times \text{Severity}$$

$$\text{Average Premium} = \frac{\text{Premium}}{\text{Number of Exposures}}$$

$$\text{Loss Ratio} = \frac{\text{Losses}}{\text{Premium}} = \frac{\text{Pure Premium}}{\text{Average Premium}}$$

$$\text{LAE Ratio} = \frac{\text{Loss Adjustment Expenses}}{\text{Losses}}$$

$$\text{UW Expense Ratio} = \frac{\text{UW Expenses}}{\text{Premium}}$$

$$\text{Operating Expense Ratio} = \text{UW Expense Ratio} + \frac{\text{LAE}}{\text{Earned Premium}}$$

$$\begin{aligned}\text{Combined Ratio} &= \text{Loss Ratio} + \frac{\text{LAE}}{\text{Earned Premium}} + \frac{\text{UW Expenses}}{\text{Written Premium}} \\ &= \text{Loss Ratio}(1 + \text{LAE Ratio}) + \frac{\text{UW Expenses}}{\text{Written Premium}}\end{aligned}$$

$$\text{Retention Ratio} = \frac{\text{Number of Policies Renewed}}{\text{Number of Potential Renewal Policies}}$$

$$\text{Close Ratio} = \frac{\text{Number of Accepted Quotes}}{\text{Number of Quotes}}$$

Policies, Coverages, and Claims

Policy Types and Coverages

Common types of personal insurance: homeowners insurance, auto insurance, renters insurance, personal articles floater, and personal umbrella insurance.

Common types of commercial insurance: commercial property insurance, commercial auto insurance, workers compensation insurance, general liability insurance, professional liability insurance, and cyber insurance.

Claims

The claim process includes the following steps:
1. Reporting a claim
2. Reviewing a claim
3. Performing claim transactions
4. Recovering payments from:
   - Deductibles
   - Salvage
   - Subrogation
   - Reinsurance

Exposures

Criteria of a good exposure base:
1. Proportional to expected loss
2. Practical
3. Historical precedence

Understanding Insurance Data

Compiling Data

Internal data includes:
1. Risk information
2. Accounting information

External data includes:
1. Data call/statistical plans
2. Other aggregated insurance industry data
3. Competitor rate filings/manuals
4. Third-party data unrelated to insurance

Data considerations:
1. Limited data
2. Multiple currencies
3. Large claims
4. Terminology differences

Areas of focus of a data review:
1. Consistency with financial statement data
2. Consistency with prior data
3. Reasonableness of the data
4. Clarity and accuracy of data definitions

Aggregating Data

General goals of data aggregation:
1. Accurately match losses and premium
2. Use the most recent data available
3. Minimize the cost associated with gathering and retrieving data

Calendar year aggregation groups data according to calendar year.
- Advantages:
  - Data is readily available
  - No future development
  - Data is easily accessible
- Disadvantages
  - Mismatch between premium and losses
  - Inability to capture major developments

Accident year aggregation groups losses according to accident data.
- Advantages:
  - Easy to achieve and understand
  - Better match of premium and losses than calendar year aggregation
  - Useful for identifying the impact of major claim events or changes due to economic or regulatory forces
- Disadvantages
  - Requires estimation of future development
  - Less accurate matching of premium and losses compared to policy/underwriting year aggregation

Policy/Underwriting year aggregation groups data according to the year in which the policies were written.
- Advantages:
  - Best match between losses and premium
  - Useful for identifying the impact of underwriting or pricing changes
- Disadvantages
  - Longer development time
  - Difficult to understand and isolate the impact of a significant event

Report year aggregation groups data according to the report date.
- Advantage:
  - Provides more stable data than accident year aggregation
- Disadvantage:
  - Development on incurred but not reported claims is excluded

Measuring Exposures and Premium

The calendar/calendar-accident year and policy year methods can be used to aggregate:
1. Written exposures/premium
2. Earned exposures/premium
3. Unearned exposures/premium

$$\begin{matrix}\text{EOY Unearned}\\\text{Exposure}\end{matrix} = \begin{matrix}\text{Written}\\\text{Exposure}\end{matrix} - \begin{matrix}\text{Earned}\\\text{Exposure}\end{matrix} + \begin{matrix}\text{BOY Unearned}\\\text{Exposure}\end{matrix}$$

$$\begin{matrix}\text{EOY Unearned}\\\text{Premium}\end{matrix} = \begin{matrix}\text{Written}\\\text{Premium}\end{matrix} - \begin{matrix}\text{Earned}\\\text{Premium}\end{matrix} + \begin{matrix}\text{BOY Unearned}\\\text{Premium}\end{matrix}$$

In force exposures and premiums are measured as of a specific time. The in-force premium is based on the full-term premium.

Factors to consider when measuring exposures/premium:
1. Mid-term adjustments (cancellations, premium amendments)
2. Varying policy terms
3. Earning patterns

When summarizing data assume that all policies were written at the mid-point of the period.

Measuring Losses

Aggregation of losses is based on:
1. Choice of relevant statistics
   - Paid losses
   - Reported losses
2. Data aggregation method
   - Calendar year
   - Accident year
   - Policy year
   - Report year
3. Period of time
   - Accounting period
   - Valuation date

The Ratemaking Process

Ratemaking Principles, Considerations, and Adjustments

Ratemaking Principles

1. A rate is an estimate of the expected value of future costs.
2. A rate provides for all costs associated with the transfer of risk.
3. A rate provides for the costs associated with an individual risk transfer.
4. A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer.

Shock Losses

Individual losses that occur infrequently and that are excessively large.

Ways to adjust for shock losses:

- Excluding shock losses entirely
- Capping losses at the basic limit
- Capping losses at a different large loss threshold, e.g., a percentile of the loss distribution or a percent of the insured value

To determine a shock loss provision,

1. Sum the shock losses (or excess losses).
2. Sum the non-shock losses (or non-excess losses).
3. Calculate the ratio of shock losses to non-shock losses (or excess losses to non-excess losses).
4. Calculate the shock loss loading factor (or excess loss loading factor) as 1 plus the ratio found above.

Multiply the historical non-shock losses (or non-excess losses) by the factor above to incorporate the long-term average expected shock loss (or excess loss).

Catastrophe Data

Losses from unusually severe disasters, both natural and man made, that result in a large number of claims.

- Non-modeled catastrophe analysis is used for events that occur somewhat regularly over a period of many years
- Modeled catastrophe analysis is used for events that occur very infrequently but that result in high-severity claims

To determine a non-modeled cat loss provision,

1. follow the same process used for the shock loss provision, or
2. develop a pure premium or loss ratio for the non-modeled cat exposure.

Modeled cat losses can be estimated using a stochastic model and then added to the non-modeled cat loss provision to get a total cat loss provision.

Reinsurance

Reinsurance is insurance purchased by a primary insurance company to transfer some of its financial risk.

There are two main types:

1. Proportional: the same proportion of premium and losses are ceded to the reinsurer.
2. Non-proportional: the primary insurer cedes a portion of premium to the reinsurer, and the reinsurer assumes some predefined portion of the losses.

Proportional reinsurance doesn't need to be explicitly included in a ratemaking analysis, but non-proportional reinsurance should be. To adjust for non-proportional reinsurance,

1. reduce projected losses for any expected recoveries and reduce total premium by cost of reinsurance, or
2. include the cost of reinsurance minus the expected recoveries as an expense item.

Rate and Benefit Changes

Examining Effects of Rate and Benefit Changes

Premium can be affected by rate changes, while losses can be affected by benefit or coverage changes. These changes include those mandated by law.

Direct effects from rate and benefit changes are direct and obvious impacts on premium or losses, while indirect effects are changes in claimant behavior that impact premium or losses.

Effects on Premium

The direct effects of rate changes on premium can be calculated by comparing the historical premiums for all in-force policies to the premium that would be charged using the current rates.

Effects on Losses

The ideal approach for calculating the direct effects of benefit/coverage changes on losses is restating individual claims. This involves adjusting historical claims data to reflect how each claim would appear under the new benefit levels.

Alternatives:

- Analyze the average restatement effect for groups of claims
- Simulating losses under new conditions

Adjusting Historical Premium

Premiums can be adjusted to the current rate level, or made on-level, using one of the following approaches:
1. Extension of exposures method
2. Parallelogram method, a.k.a. geometric method

Extension of Exposures Method

Restate the historical premiums to the current rate level by rerating each policy.

The extension of exposures method is the most accurate current rate level method provided the necessary detailed data is readily available, but it also has the following limitations:
- The knowledge of the relevant rating characteristics for each policy in the historical period may not be readily accessible.
- It involves extensive calculations, requiring substantial computing power.
- Incorporating changes in schedule rating guidelines for commercial products can be difficult as schedule rating allows underwriters to apply subjective debits and credits.

Parallelogram Method

The steps for the parallelogram method are:
1. Determine the timing and amount of the rate changes.
2. Calculate the cumulative rate level index for each rate level group.
3. For each time period, calculate the portion of premium that corresponds to each rate level group.
4. Calculate the weighted average cumulative rate level index for each time period.
5. Calculate the on-level factor for each time period as:
$$\text{On} - \text{Level Factor} = \frac{\text{Current Cumulative Rate Level Index}}{\text{Avg Rate Level Index for Historical Period}}$$
6. Apply the on-level factor to the earned premium for the appropriate time period.

The parallelogram method is less accurate than the extension of exposures method. It has the following disadvantages:
- It assumes policies are written uniformly throughout the time period.
- The method is typically applied at the aggregate level using overall average changes, even though the effects may vary by class.

If policies are not written uniformly throughout the time period, the parallelogram method can be adjusted by:
1. Using shorter periods of time than a year, such as months or quarters.
2. Calculating the actual distribution of policy writings. That can be used to determine more accurate weightings for calculating the historical average rate level.

Adjusting Historical Losses

The steps of the parallelogram method can be applied when adjusting losses for benefit or coverage changes. Instead of an on-level factor, calculate the benefit change loss adjustment factor as
$$\text{Adjustment Factor} = \frac{\text{Current Loss Level}}{\text{Avg Loss Level of Historical Period}}$$

Development

Development Triangles for Diagnostics

Development triangles can be examined to understand and identify changes in the data used for ratemaking and reserving. Common examples of development triangles used for diagnostics include:
- Paid losses divided by reported losses
- Total closed claim counts divided by reported claim counts
- Claim counts on closed with payment divided by total closed claim counts
- Claim counts on closed without payment divided by total closed claim counts

Developing Premiums

Premiums typically need to be developed when an incomplete year of data is used or for lines of business that utilize premium audits. This is commonly done by studying historical patterns of premium development.

Developing Losses

One method for developing losses is the chain-ladder method (a.k.a. the development technique). The steps for this method are
1. Compile claims data.
2. Calculate age-to-age development factors.
3. Calculate averages of the age-to-age factors for each age (e.g., arithmetic average, medial average, geometric average, volume-weighted average).
4. Select claim development factors.
5. Select tail factor.
6. Calculate cumulative claim development factors.
7. Project ultimate claims.

Trending

Considerations for Ratemaking Data

There are several considerations to make when selecting data to use for trending:
- Trending should be done after adjusting for anomalies and rate and benefit changes.
- Monthly, quarterly, or yearly data can be used, along with different levels of granularity.
- Exposures that are impacted by time-related influences, like inflation, may need trended separately.
- For premiums, trending should be done on average premium per exposure. Either written or earned premium can be used, as written premium is a leading indicator of changes in earned premium.
- Loss trends can be estimated by analyzing pure premium directly or by analyzing frequency and severity separately. Frequency and severity are often analyzed separately because they can change for different reasons and in different directions.
- Exposures, premiums, and losses should be trended consistently when determining the rate level indication.

Estimating a Trend

The choice of method depends on the stability of the data and the type of trend being measured.

Approaches for estimating a trend include:
1. Simple average of percent changes
2. Linear and exponential trend fitting

For premium trends, changes in average premium are relatively stable, so simple average often suffices. Loss trends are more volatile, so simple averages may be less reliable, and curve fitting methods are used.

The pure premium trend can be found as:

$$\left(1+\begin{matrix}\text{Pure Premium}\\ \text{Trend}\end{matrix}\right)=\left(1+\begin{matrix}\text{Frequency}\\ \text{Trend}\end{matrix}\right)\left(1+\begin{matrix}\text{Severity}\\ \text{Trend}\end{matrix}\right)$$

Seasonality

To smooth out the seasonality in the data, it is common to use 12-month rolling data.

Limits

When trending loss data that is subject to limits, the effect of the severity trend on basic limits losses and excess losses should be considered. For a positive severity trend,

$$\begin{matrix}\text{Basic}\\ \text{Limit Trend}\end{matrix}\le\begin{matrix}\text{Total}\\ \text{Limit Trend}\end{matrix}\le\begin{matrix}\text{Excess}\\ \text{Limit Trend}\end{matrix}$$

The relationship above is reversed for a negative severity trend.

Determining Trend Periods

The experience period is the time period in which the historical data occurs, while the forecast period is the future time period to which historical data is projected.

The trend period is the length of time from the midpoint of the experience period to the midpoint of the forecast period.

The experience and forecast periods depend on
- the type of data (e.g., earned or written premium or losses)
- the length of the policy term (e.g., 6 or 12 months)
- the type of data aggregation (e.g., calendar/accident year or policy year)
- the length of time rates are expected to be in effect

One-Step Trending

The one-step trending approach involves applying a single trend to historical data to adjust it to the forecast period.
- Written premium is trended from the average written date in each historical period to the average written date at the new rate level.
- If the trend is estimated from earned premium data, earned premium is trended from the average earned date in each historical period to the average earned date at the new rate level.
- If the trend is estimated from written premium data, earned premium is trended from the average written date for a time period of earned premium to the average written date at the new rate level.
- Losses are trended from the average accident date in each historical period to the average accident date at the new rate level.

Two-Step Trending

The two-step trending approach is used when the future trend rate is expected to differ from the historical trend rate.

Step 1 is the current trend step, which adjusts each year's historical data to the average level of the most recent time period in the trend data. This step can be performed in two ways:
1. Adjust historical data by applying an annual trend factor
2. Adjust historical data to align with the most recent known average

Step 2 is the projected trend step, which is to adjust the data from the average date of the latest time period to the average date of the future policy period.

$$\begin{matrix}\text{Total}\\ \text{Trend Factor}\end{matrix}=\begin{matrix}\text{Current}\\ \text{Trend Factor}\end{matrix}\times\begin{matrix}\text{Projected}\\ \text{Trend Factor}\end{matrix}$$

Overlap Fallacy

The overlap fallacy states that developing and trending data will double-count the effect of inflation and other possible changes. This is not true, as development brings immature values to their expected ultimate values, while trending adjusts the ultimate values from the historical experience period to the period the rates will be in effect.

Expenses and Profit

Loss Adjustment Expenses

ALAE

Allocated loss adjustment expenses (ALAE) can be incorporated in ratemaking using one of the following methods:
- Include ALAE with losses.
- Study development and trend patterns for ALAE separately.

ULAE

Unallocated loss adjustment expenses (ULAE) can be incorporated in ratemaking by assuming that ULAE tracks with loss and ALAE dollars consistently over time. The steps for this approach are:
1. Calculate the ratios of calendar year paid ULAE to calendar year paid loss plus ALAE over several years.
2. Select a ratio to use.
3. Apply the ratio to each year's reported loss plus ALAE.

Other methods for incorporating ULAE are:
- Using count-based allocation methods.
- Studying how claim adjusters spend their time.

Underwriting Expenses

The historical expense ratios for each category of underwriting expense typically use the following data:

Expense	Data Used	Divided By
General Expense	Countrywide	Earned Premium (or Exposure/Policy Counts)
Other Acquisition	Countrywide	Written Premium (or Exposure/Policy Counts)
Commissions and Brokerage	Countrywide/ Statewide	Written Premium (or Exposure/Policy Counts)
Taxes, Licenses, and Fees	Statewide	Written Premium (or Exposure/Policy Counts)

All Variable Expense Method
All underwriting expenses are treated as variable to premium, i.e., as a constant percentage of premium.
1. Derive the expense ratio for each year and category.
2. Select a ratio for each expense type.
3. Sum the selections for each expense category to find the total underwriting expense provision.

If any expenses are constant or almost constant for each risk, this method will undercharge risks with below-average premium and overcharge risks with above-average premium.

Premium-Based Projection Method
Fixed and variable expenses are handled separately, although they are both related to premium data.
1. Derive the expense ratio for each year and category.
2. Select a ratio for each expense type.
3. Divide each selected expense ratio into fixed and variable ratios.
4. Sum the fixed expense ratio selections by expense category to find the total fixed expense provision. Repeat for variable expenses.

If needed, find the fixed expense per exposure as
$$\begin{array}{c}\text{Fixed Expense}\\ \text{Per Exposure}\end{array} = \text{Fixed Expense Ratio} \times \begin{array}{c}\text{Projected}\\ \text{Avg Premium}\end{array}$$
This method will result in distorted fixed expense ratios if historical and projected premium levels differ materially.

Exposure/Policy-Based Projection Method
Variable expenses are related to premium data, while fixed expenses are related to exposure or policy count data.
1. Split each expense into fixed and variable components.
2. Derive the expense ratio for each year and category. Divide by exposures (or policy counts) for fixed expenses and premium for variable expenses. Trend the fixed expense ratios if needed.
3. Select a ratio for each expense type.
4. Sum the fixed expense ratio selections for each expense category to find the fixed expense provision. Repeat for variable expenses.

If needed, find the fixed expense per exposure as
$$\begin{array}{c}\text{Projected Fixed}\\ \text{Expense Ratio}\end{array} = \frac{\text{Avg Projected Fixed Expense Per Exposure}}{\text{Projected Avg Premium}}$$
This method could be enhanced by:
- improving the method for splitting expenses into fixed and variable portions
- collecting data at a finer level (other than countrywide)
- updating the allocation of fixed expenses that vary by certain characteristics (such as new vs. renewal business)
- incorporating the impact of economies of scale on expenses

Underwriting Profit Provision
$$\text{Total Profit} = \text{Investment Income} + \text{UW Profit}$$
Investment income is income earned from capital funds or policyholder-supplied funds (i.e., unearned premium reserves and loss reserves).

Underwriting profit is the sum of profits generated from individual policies. It is typically set to achieve the target total rate of return after investment income has been considered.

The variable permissible loss ratio (VPLR) is the percentage of each premium dollar that is intended to pay for projected losses and LAE and projected fixed expenses.
$$\text{VPLR} = 1 - V - Q_T$$
The total permissible loss ratio (PLR) is the percentage of each premium dollar that is intended to pay for projected losses and LAE.
$$\text{PLR} = 1 - F - V - Q_T$$

Overall Rate Level Indications

Pure Premium Method
Pure premium is used to calculate the indicated average rate per exposure.
$$\text{Indicated Avg Rate } = \frac{\bar{L} + \bar{E}_L + \bar{E}_F}{1 - V - Q_T}$$

Loss Ratio Method
The loss ratio is used to calculate the indicated average rate change from the current level.
$$\text{Indicated Avg Rate Change} = \frac{\text{Loss and LAE Ratio} + F}{1 - V - Q_T} - 1$$

Comparison of Methods
These methods will produce equivalent results as long as consistent data and assumptions are used for both. The main differences are:
1. The loss measures are different. The pure premium method uses pure premium, while the loss ratio method uses the loss ratio.
2. The two methods result in different outputs. The pure premium method results in an indicated rate, while the loss ratio method results in an indicated rate change.

Risk Classification

Traditional Risk Classification

Importance of Charging Equitable Rates

Process of adverse selection:

1. Insurer fails to implement a rating variable that other competitors are using to segment risks.
2. Insurer experiences a distributional shift toward higher-risk insureds.
3. The rate charged is inadequate, and the insurer becomes unprofitable.
4. Insurer raises the average rate.
5. Steps 2 to 4 will repeat until the insurer improves its rate segmentation, becomes insolvent, or focuses on higher-risk insureds.

Skimming the cream: Using a risk characteristic to identify, attract, and select the lower-risk insureds that exist in the insured population.

Criteria for Evaluating Rating Variables

Statistical criteria:

1. Statistical significance
2. Homogeneity
3. Credibility

Operational criteria:

1. Objective
2. Inexpensive to administer
3. Verifiable

Social criteria:

1. Affordability
2. Causality
3. Controllability
4. Privacy concerns

Legal criteria: Compliance with applicable laws and regulations.

Univariate Classification

1. **Pure premium approach**: Calculate the indicated relativities to base as the pure premium of each level divided by the pure premium of the base level.
2. **Loss ratio approach**: Calculate the indicated relativities as the product of the indicated change factor and the current relativity, where the indicated change factor is the loss & ALAE ratio of each level divided by the total loss & ALAE ratio. The indicated relativity to base is the indicated relativity for each level divided by the indicated relativity for the base level.
3. **Adjusted pure premium approach**: Adjust exposures by the weighted average current relativity from other rating variables. Then, calculate the indicated relativities to base as the adjusted pure premium of each level divided by the adjusted pure premium of the base level.

The output of the unadjusted pure premium approach is distorted by any distributional bias in other rating variables.

Univariate Classification with Credibility

1. **Pure premium approach**:
   i. Re-base the current (or competitor's) relativities to total.
   ii. Re-base the indicated relativities to total.
   iii. Calculate the credibility-weighted average of the relativities from steps (i) and (ii).
2. **Adjusted pure premium approach**: Same as pure premium method, but use *adjusted exposures* as weights.
3. **Loss ratio approach**: Same as pure premium method, but use *premium at base level* as weights.
   - If the complement is no change, re-basing can be skipped. Additionally, instead of relativities, we can also calculate the credibility-weighted average between the indicated change factors and 1.

Multivariate Classification

Minimum Bias Procedures

Minimum bias procedures are iteratively standardized univariate approaches that account for an uneven mix of business. The process of a minimum bias procedure that uses a multiplicative rating structure with balance principle is summarized below.

1. Equate the exposure-weighted loss costs to the indicated loss costs for each level of each rating variable.
2. Calculate the seed relativities for all but the first variable.
3. Using the seed relativities, solve for the relativities of the first variable.
4. Discard the seed relativities for the second variable and use the result from Step 3 to calculate the relativities for the second variable.
5. Repeat the process until convergence is achieved.
6. Re-base the relativities.

Multivariate Classification

Factors leading to the use of multivariate statistical techniques in classification ratemaking:

1. Enhancement in computing power
2. Improved granularity and accessibility of data
3. Competitive pressure

Benefits of multivariate methods:

1. They automatically adjust for exposure correlations between rating variables.
2. They attempt to capture only systematic effects (signals) and ignore unsystematic effects (noise).
3. They generate model diagnostics.
4. They allow consideration of response correlation.

Generalized Linear Models

GLM analysis is typically performed on loss cost data instead of loss ratios because:

1. Modeling loss ratios requires on-level premiums at the granular level.
2. There may be an a priori expectation of frequency and severity patterns.
3. Loss ratio models become obsolete when rates and rating structures are changed.
4. There is no standard distribution for modeling loss ratios.

GLMs achieve all the benefits of the multivariate methods and have the following additional benefits:

1. Transparent
2. Parameter estimates and statistical diagnostics
3. Iterations can be tracked
4. Model output is a series of multipliers

GLM diagnostics:

1. Standard errors: Measure the variability associated with a parameter
2. Measures of deviance: Measure the extent to which the fitted values deviate from the actual observations
3. Practical diagnostic: Run the GLM for individual years across a multi-year dataset to assess the consistency of results over time

Practical considerations:

- Ensuring data adequacy to avoid GIGO
- Identify when unusual outcomes dictate further exploratory analysis
- Evaluating model results from both statistical and business perspectives
- Developing appropriate methods to communicate model results based on the company's ratemaking objectives

Supplemental external information:

- Geo-demographics
- Weather
- Property characteristics
- Information about insured individuals or businesses

Data Mining Techniques

1. Factor analysis
2. Cluster Analysis
3. Classification and Regression Trees (CART)
4. Multivariate Adaptive Regression Spline (MARS)
5. Neural network

Special Classification

Territorial Ratemaking

Challenges with territorial ratemaking:

1. High correlation of territory with other rating variables.
2. Limited data within each territory.

Steps of territorial ratemaking:

1. Establishing territorial boundaries
   a. Determine the basic geographic unit
   b. Estimate the geographic risk associated with each unit
2. Determining rate relativities for the territories

Types of basic spatial smoothing techniques:

1. Distance-based
   - Combines data from a primary geographic unit with data from neighboring units, using weights determined by distance from the primary unit. Weights decrease as the distance from the primary unit increases.
   - Advantages: Easy to understand and implement
   - Disadvantages: Assumes distance has the same impact on urban and rural risks and fails to consider natural or artificial boundaries.
   - Application: Weather-related perils
2. Adjacency-based
   - Combines data from a primary geographic unit with data from rings of adjacent units, with closer adjacent rings having more weight.
   - Advantages: Accounts for the differences between urban and rural risks and consider physical boundaries.
   - Application: Perils driven by socio-demographic characteristics

Types of clustering techniques:

1. Quantile methods: Clusters are created based on equal numbers of observations or weights.
2. Similarity methods: Clusters are created based on the closeness of the estimators.
   - Average linkage similarity method: Joins clusters with lower variances.
   - Centroid similarity method: More responsive to outliers.
   - Ward's clustering method: Produces clusters with equal numbers of observations.

A contiguity constraint must be added if contiguous boundaries are desired.

Increased Limits Ratemaking

Reasons for growing importance in increased limits ratemaking:
1. Growth in personal wealth leads individuals to seek higher insurance coverage.
2. Inflationary trends have a greater impact on increased limits losses than on basic limits losses.
3. Surge in lawsuits and larger jury awards impact increased limits losses disproportionately.

Issues with using standard ratemaking approaches for increased limits ratemaking:
1. Limited data causes volatile results
2. GLMs may produce counterintuitive results

Standard ILF Approach

$$\text{Indicated ILF}(H) = \frac{\text{LAS}(H)}{\text{LAS}(B)}$$

where

$$\text{LAS}(x) = \frac{(\text{Total Losses of Claims} < x) + (\text{\# of Claims} \geq x) \times x}{\text{Total \# of Claims}}$$

When losses are censored, policies with a limit smaller than the limit being priced cannot be used. For $x < y$,

$$\text{LAS}(y) = \text{LAS}(x) + \frac{\text{Losses in the Layer between } x \text{ and } y}{\text{\# of Claims in Policies with Limits} > x}$$

Historical losses used in the calculation of the ILFs should be trended and developed to ultimate.

Deductible Pricing

Types of deductibles:
1. Flat dollar deductibles
2. Percentage deductibles

Reasons why deductibles are popular:
1. Deductibles reduce insurance premiums.
2. Deductibles minimize the occurrence of small nuisance claims.
3. Deductibles provide a financial incentive for the insured to prevent losses.
4. Deductibles reduce an insurer's exposure to catastrophic losses and lower the risk of insolvency.

Loss Elimination Ratio (LER) Approach

$$\text{Indicated Deductible Relativity} = \frac{(L + E_L)_D}{(L + E_L)_B} = 1 - \text{LER}(D)$$

where

$$\text{LER}(D) = \frac{(L + E_L)_B - (L + E_L)_D}{(L + E_L)_B}$$

Only policies with deductibles less than or equal to the deductible being priced can be used to determine loss elimination ratios.

Considerations when using the LER approach:
1. LER ignores the difference in claiming behavior for different deductible levels.
2. LER fails to recognize the behavioral differences that arise when insureds are allowed to choose their policy deductible.

When the premium savings implied by the deductible relativity is greater than the deductible amount, the insurer can:
1. Implement a cap on the amount of dollar credit from the deductible.
2. Calculate a different set of credits for different policies.
3. Use percentage deductibles.

Workers Compensation Size of Risk

Approaches used to adjust for the inequity of rates from using the All Variable Expense Method:
1. Apply the variable expense provision to the first $\$X$ of standard premium only.
2. Charge an expense constant to all risks.
3. Apply a premium discount to policies with premium exceeding a specified amount.

Theories explaining why smaller workers compensation risks typically have worse loss experience compared to larger risks:
1. Small companies often have less advanced safety programs.
2. Small companies may lack programs designed to help injured employees return to work.
3. Small insureds may have less motivation to prevent or manage injuries compared to large insureds.

Insurers can add a loss constant to the premium of small insureds to equalize the final expected loss ratios across all insured sizes.

Insurance to Value (ITV)

The relationship between the selected insurance level and the overall value or replacement cost of the insured item.

Issues when properties are not fully insured:
1. The insurance payout will not be adequate to fully cover a total or near-total loss.
2. If the insurer assumes all homes are insured to full value when determining premium, the underinsured risks would be undercharged, leading to inequitable rates.

The issues of inequity and inadequacy of rates only exist if partial losses are possible, which is a common case in real life. Insurers have taken the following actions to ensure insurance to full value:
1. Vary rates based on the ITV level.
2. Implement a coinsurance clause, which reduces the insurance payout proportionally by the level of underinsurance.

$$\text{Apportionment Ratio}, a = \min\left(\frac{F}{cV}, 1\right) = \begin{cases} \frac{F}{cV}, & F < cV \\ 1, & F \geq cV \end{cases}$$

$$\text{Indemnity Payment}, I = \min(aL, F) = \begin{cases} aL, & L < cV \\ F, & L \geq cV \end{cases}$$

$$\text{Coinsurance Penalty}, e = \min(L, F) - I = \begin{cases} (1-a)L, & L < F < cV \\ F - aL, & F \leq L < cV \\ 0, & \text{otherwise} \end{cases}$$

Other ITV initiatives:
1. Offer guaranteed replacement cost (GRC), usually with a cap.
2. Use more sophisticated tools to estimate replacement costs.
3. Implement property inspections, indexation clauses, and education of insureds.

Credibility

Measuring Credibility

The amount of credibility (Z) should meet the following criteria

1. $0 \leq Z \leq 1$.
2. As n increases, Z should increase.
3. As n increases, Z should increase at a decreasing rate.

Classical Credibility

$$\text{Estimate} = Z \times \text{Observed Experience} + (1 - Z) \times \text{Related Experience}$$

Assuming exposures are homogeneous, claim occurrence follows a Poisson distribution, and a constant severity, the expected number of claims needed for full credibility is:

$$E(Y) = \left(\frac{z_{(p+1)/2}}{k}\right)^2$$

If the number of observed claims does not meet the full credibility standard, Z is calculated using the square root rule:

$$Z = \sqrt{\frac{Y}{E(Y)}} \text{ where } Y < E(Y)$$

Bühlmann Credibility

$$\text{Estimate} = Z \times \text{Observed Experience} + (1 - Z) \times \text{Prior Mean}$$

where

$$Z = \frac{N}{N + K}$$

and

- N = Number of observations
- K = The expected value of process variance (EVPV) divided by the variance of hypothetical means (VHM)

Bühlmann credibility is commonly used within the insurance industry but the determination of EVPV and VHM can be challenging.

Bayesian Analysis

Bayesian analysis has no specific calculation of Z but adjusts the prior estimate to reflect new information using Bayes Theorem. Bayesian analysis is not commonly used due to its complexity.

Complements of Credibility

First Dollar Ratemaking

Rate change from the larger group applied to present rates:

$$C = \begin{array}{c}\text{Current Loss Cost}\\ \text{of Subject Experience}\end{array} \times \frac{\text{Larger Group Indicated Loss Cost}}{\text{Larger Group Current Average Loss Cost}}$$

Trended present rates:

$$C = \begin{array}{c}\text{Present}\\ \text{Rate}\end{array} \times \begin{array}{c}\text{Loss Trend}\\ \text{Factor}\end{array} \times \frac{\text{Prior Indicated Loss Cost}}{\text{Prior Implemented Loss Cost}}$$

$$C = \frac{\text{Loss Trend Factor}}{\text{Premium Trend Factor}} \times \frac{\text{Prior Indicated Rate Change Factor}}{\text{Prior Implemented Rate Change Factor}}$$

Harwayne's Method (Complement is for Class 1 in State A):

1. Calculate the weighted average pure premium for State A.
2. Calculate the weighted average pure premium for other states based on State A's exposure distribution by class.
3. Calculate the adjustment factors.
4. Apply the adjustment factors to the Class 1 pure premium in other states.
5. Calculate the complement of credibility.

Excess Ratemaking

Complement of credibility for the layer L excess of A:

1. Increased limits analysis

$$C = L_A \times \left(\frac{\text{ILF}_{A+L} - \text{ILF}_A}{\text{ILF}_A}\right)$$

2. Lower limits analysis

$$C = L_d \times \left(\frac{\text{ILF}_{A+L} - \text{ILF}_A}{\text{ILF}_d}\right)$$

3. Limits analysis

$$C = \text{LR} \times \sum_{d>A} P_d \times \frac{\text{ILF}_{\min(d,A+L)} - \text{ILF}_A}{ILF_d}$$

4. Fitted Curves

$$C = \text{Total Limits Losses} \times \frac{\int_A^{A+L} (x - A) \cdot f(x)\,\text{d}x + \int_{A+L}^{\infty} L \cdot f(x)\,\text{d}x}{\int_{-\infty}^{\infty} x \cdot f(x)\,\text{d}x}$$

Complements for First Dollar Ratemaking

Method / Desirable Quality	Loss Costs of a Larger Group that Includes the Group being Rated	Loss Costs Of a Larger Related Group	Rate Change from Larger Group Applied to Present Rates	Harwayne's Method	Trended Present Rates	Competitor's Rates
Accurate	Yes	May be inaccurate	Accurate over the long run	Yes	Depends on accuracy of historical loss costs	May be inaccurate
Unbiased	Likely biased	Generally biased	Unbiased	Unbiased	Unbiased	May be biased
Independent	Independent if subject is excluded Or does not dominate the larger group	Yes	Depends on size of subject relative to the larger group	Mostly independent	Depends on data used	Yes
Available	Yes	Yes	Yes	Yes	Yes	No
Easy to Compute	Yes	Yes	Yes	No	Yes	Yes
Logical	Yes, if all risks in the group have something in common	Yes, if the group is chosen reasonably	Yes, if the group is chosen reasonably	Hard to explain due to computational complexity	Yes	Yes

Complements for Excess Ratemaking

Method / Desirable Quality	Increased Limit Analysis	Lower Limits Analysis	Limits Analysis	Fitted Curves
Accurate	Inaccurate if there is a difference in loss severity distribution	More accurate than increased limits analysis	Same level as increased limits and lower limits analyses	Most accurate if fitted curve replicates distribution of actual data
Unbiased	Biased if there is a difference in loss severity distribution	More biased than increased limits analysis	Same level as increased limits and lower limits analyses	Least biased if fitted curve replicates distribution of actual data
Independent	Generally independent	Generally independent	Generally independent	Less independent
Available	Practical if data is available	Not as available	Not as available	Not as available
Easy to Compute	Yes	Yes	Time-consuming but straightforward	Computationally complex
Logical	Controversial	Controversial	Controversial	Most logically related to losses in the excess layer

Implementation

Constraints and Considerations

Regulatory Constraints

Potential U.S. regulatory constraints:

- Restricting the use of certain actuarially indicated rates
- Limiting the amount a rate can change, either the overall average rate or the rate for a single group/individual
- Requiring insurance companies to provide customers with advanced notice of a rate change
- Prohibiting the use of certain rating variables, such as credit scores in personal lines
- Prescribing specific ratemaking techniques, such as multivariate analysis

Options companies have to combat constraints:

- Take legal action to challenge the regulation
- Revise their underwriting guidelines to avoid writing policies at an inadequate rate level
- Change their marketing scheme to try to minimize the number of new policyholders whose rates are believed to be inadequate
- If a rating variable is banned, they could use a different but related variable

Operational Constraints

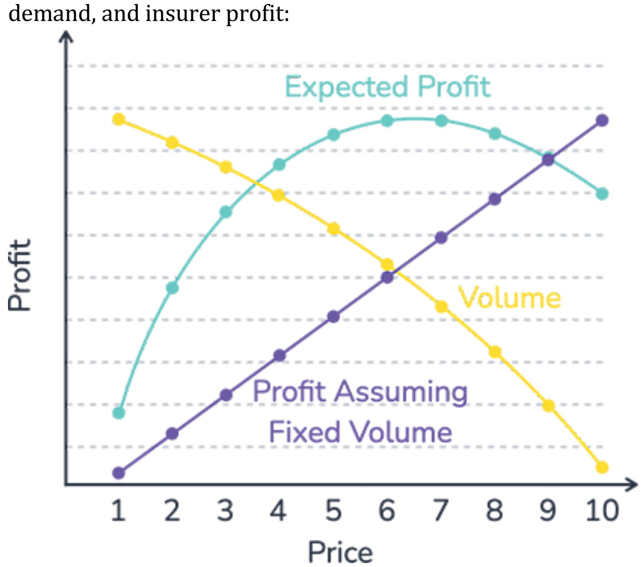
Potential challenges when implementing a new rate or rating variable:

- Systems limitations
- Resource constraints
- Limited access to information (e.g. policyholder data)

A **cost-benefit analysis** can be used to evaluate a company's potential options and select an appropriate course of action. In general, this involves evaluating the costs and the benefits of taking a certain action over a specified **time horizon.**

Marketing Considerations

Relationship between insurance price, the volume of policyholder demand, and insurer profit:



Companies tend to classify insureds as either new business or renewal business and analyze them separately. Some factors that affect a policyholder's likelihood to renew are:

- Prices of competing products
- Total cost of the product
- Rate changes
- Characteristics of the insured/sensitivity to price
- Customer satisfaction and brand loyalty

Traditional Techniques for Incorporating Marketing Considerations

Competitive Comparisons:

$$\begin{array}{c}\text{\% Competitive}\\\text{Position}\end{array} = \frac{\text{Competitor Premium}}{\text{Company Premium}} - 1$$

$$\begin{array}{c}\text{\$ Competitive}\\\text{Position}\end{array} = \begin{array}{c}\text{Competitor}\\\text{Premium}\end{array} - \begin{array}{c}\text{Company}\\\text{Premium}\end{array}$$

$$\text{\% Win} = \frac{\begin{array}{c}\text{\# Risks Meeting Criteria}\\\text{(e. g. , Premium Lower than Competitor)}\end{array}}{\text{Total \# of risks}}$$

$$\text{Rank} = \begin{array}{c}\text{Rank of Company Premium}\\\text{Compared to Several Competitors}\end{array}$$

Other Techniques and Metrics:

- **Close ratio** – measures the rate at which prospective insureds accept a new quote.
$$\text{Close Ratio} = \frac{\text{\# of Accepted Quotes}}{\text{Total \# of Quotes}}$$
- **Retention ratio –** measures the rate at which existing policyholders renew their policies upon expiration.
$$\text{Retention Ratio} = \frac{\text{\# of Policies Renewed}}{\text{Total \# of Potenial Renewals}}$$
- **Growth –** attracting new policyholders while retaining existing ones.
$$\text{\% Policy Growth} = \frac{\text{New Policies Written} - \text{Lost Policies}}{\text{Policies at Start of Period}}$$
- **Distributional analysis** – studying the distributions of new and renewal business by customer segment and how those distributions change over time.
- **Dislocation analysis** – studying the way different rate changes impact policyholders' propensity to renew their policy.

Systematic Techniques for Incorporating Marketing Considerations

- **Lifetime value analysis** attempts to understand the profitability of an insured over a longer period of time by acknowledging that not all insureds will renew and that certain actions can influence some insureds to stay.
- **Optimized pricing** utilizes multivariate statistical modeling techniques to produce better estimates for policyholder retention, policyholder conversion, and customer demand.

Underwriting Cycles

- **Soft market** – Prices are low and growth is high
- **Hard market** – Prices are high and growth is low

Finalizing Rates

Handling an Imbalanced Fundamental Insurance Equation

Non-pricing solutions:

- Reduce expenses (UW expenses, LAE, or both)
- Reduce the expected loss (e.g., coverage modifications, loss control programs, etc.)

Pricing solutions:

- Change the target UW profit %
- Implement a rate change

Steps for implementing a rate change:

1. Select an overall average premium.
2. Construct a rating algorithm.
3. Select relativities for each rating variable.
4. Calculate any applicable additive components.
5. Derive the base rate required to reach the selected overall average premium.

Expense Fees and Other Additives

- **Variable premium** – accounts for differences in risk characteristics and is the product of the base rate and any relativities.
- **Additive premium** – a flat premium amount for each policyholder regardless of risk characteristics.
- **Expense fee** – the fixed expense per exposure adjusted for variable expenses and the target UW profit.

$$\text{Expense Fee per Exposure} = \frac{\text{Fixed Expense per Exposure}}{\text{Permissible Loss Ratio}} = \frac{\text{Fixed Expense per Exposure}}{1 - V - Q_t}$$

Other additive components to premium would be treated in the same way an expense fee would.

$$\text{Additive Premium per Exposure} = \frac{\text{Additive Amount per Exposure}}{1 - V - Q_t}$$

Deriving Base Rates: No Change to Relativities

If there are no changes to relativities and all premium is variable:

$$\begin{array}{c}\text{Proposed}\\\text{Base Rate}\end{array} = \frac{\text{Proposed Variable Premium}}{\text{Current Variable Premium}} \times \begin{array}{c}\text{Current}\\\text{Base Rate}\end{array} = \left(1 + \begin{array}{c}\text{\% Change in}\\\text{Variable Premium}\end{array}\right) \times \text{Current Base Rate}$$

Deriving Base Rates: Changing Relativities

If the rate review results in one or more rating variables changing relativities, there are three viable methods for deriving the proposed base rate.

Approximated Average Rate Differential

$$\begin{array}{c}\text{Proposed}\\\text{Base Rate}\end{array} = \frac{\text{Proposed Avg Premium} - \text{Proposed Additive Fee}}{\text{Proposed Avg Rating Factor}}$$

Approximate the average rating factor as the product of the weighted average rating factors for each individual rating variable.

Options for weighting:

1. Current variable premium at base (most accurate)
2. Adjusted exposures (equivalent to weighting by variable premium at base)
3. Unadjusted exposures (least accurate)

Approximated Change in Average Rate Differential

$$\begin{array}{c}\text{Proposed}\\\text{Base Rate}\end{array} = \frac{\begin{array}{c}\text{Proposed}\\\text{Avg Premium}\end{array} - \begin{array}{c}\text{Proposed}\\\text{Additive Fee}\end{array}}{\begin{array}{c}\text{Current}\\\text{Avg Premium}\end{array} - \begin{array}{c}\text{Current}\\\text{Additive Fee}\end{array}} \times \begin{array}{c}\text{Current}\\\text{Base Rate}\end{array} \times \text{OBF}$$

Only requires you to assess rating variables that are changing relativities. Approximate the overall off-balance factor as the product of the off-balance factors for each individual rating variable.

Options for calculating an off-balance factor:

1. Weight rating factors by current variable premium at base or adjusted exposures
$$\text{OBF} = \frac{\text{Avg Current Rating Factor}}{\text{Avg Proposed Rating Factor}}$$
2. Weight relativity change factors by current variable premium (not at base)
$$\text{OBF} = \frac{1}{\text{Avg Relativity Change Factor}}$$

Extension of Exposures

Re-rate all unique combinations of rating variables using the proposed rating factors and a seed base rate.

$$\begin{array}{c}\text{Proposed}\\\text{Base Rate}\end{array} = \begin{array}{c}\text{Seed}\\\text{Base Rate}\end{array} \times \frac{\begin{array}{c}\text{Proposed}\\\text{Avg Premium}\end{array} - \begin{array}{c}\text{Proposed}\\\text{Additive Fee}\end{array}}{\begin{array}{c}\text{Proposed Avg Premium}\\\text{with Seed Base Rate}\end{array} - \begin{array}{c}\text{Proposed}\\\text{Additive Fee}\end{array}}$$

Capping Rates

- **Capping** – limiting the impact to premium a rate revision can have on a single class.
- **Shortfall** – the difference between the unlimited proposed premium and the proposed premium limited at the cap.

Capping a Non-Base Class:

Method 1: Formulas

1. Reduce the capped class's relativity in order to comply with the cap.

$$\text{Initial Capped Relativity} = \frac{\text{Current Relativity} \times (1 + \text{Cap \%})}{\text{OBF} \times (1 + \text{Target Rate Change \%})}$$

2. Calculate the shortfall adjustment factor.

$$\begin{matrix}\text{Shortfall} \\ \text{Adjustment Factor}\end{matrix} = 1 + \frac{\text{Premium Shortfall}}{\text{Total Prop. Prem from Uncapped Classes}}$$

3. Calculate the final base rate.

$$\text{Final Base Rate} = \text{Proposed Base Rate} \times \begin{matrix}\text{Shortfall} \\ \text{Adjustment Factor}\end{matrix}$$

4. Re-adjust the relativity for the capped class.

$$\text{Final Capped Relativity} = \frac{\text{Initial Capped Relativity}}{\text{Shortfall Adjustment Factor}}$$

5. Compute the final premium amounts for the uncapped classes (if needed).

$$\text{Final Premium} = \text{Proposed Premium} \times \begin{matrix}\text{Shortfall} \\ \text{Adjustment Factor}\end{matrix}$$

Method 2: Manually Reallocate the Shortfall

1. Compute the premium shortfall as a result of capping.
2. Reallocate the premium shortfall to other classes in proportion to their proposed premiums.
3. Compute the adjusted relativities for each class.

$$\begin{matrix}\text{Adjusted} \\ \text{Relativity}\end{matrix} = \text{Proposed Relativity} \times \frac{\text{Final Premium}}{\text{Proposed Premium}}$$

4. Rebase to derive the final relativities and base rate.

Capping the Base Class

Capping the base class inherently lowers the base rate, reducing the premium for all classes. This changes Method 1 slightly but does not change Method 2.

Method 1: Formulas

1. Compute the capped base adjustment factor.

$$\begin{matrix}\text{Capped Base} \\ \text{Adjustment Factor}\end{matrix} = \frac{1 + \text{Cap \%}}{1 + \text{Proposed \% Change in Premium}}$$

2. Calculate the final base rate.

$$\text{Final Base Rate} = \text{Proposed Base Rate} \times \begin{matrix}\text{Capped Base} \\ \text{Adjustment Factor}\end{matrix}$$

3. Calculate the shortfall adjustment factor

$$\begin{matrix}\text{Shortfall} \\ \text{Adjustment Factor}\end{matrix} = 1 + \frac{\text{Premium Shortfall}}{\text{Total Prop. Prem from Uncapped Classes}}$$

4. Calculate the final relativities for the non-capped classes.

$$\begin{matrix}\text{Final} \\ \text{Relativity}\end{matrix} = \begin{matrix}\text{Proposed} \\ \text{Relativity}\end{matrix} \times \frac{\text{Shortfall Adjustment Factor}}{\text{Capped Base Adjustment Factor}}$$

5. Compute the final premium amounts for the uncapped classes (if needed).

$$\text{Final Premium} = \text{Proposed Premium} \times \begin{matrix}\text{Shortfall} \\ \text{Adjustment Factor}\end{matrix}$$

Method 2: Manually Reallocate the Shortfall

Same as capping a non-base class.

Other Considerations

Minimum Premiums

Similar to an expense fee, a minimum premium ensures that each individual risk pays enough premium to cover expected expenses and losses.

$$\text{Effect of Minimum Premium} = \frac{\text{Total Premium with Minimum}}{\text{Total Premium without Minimum}} - 1$$

When adding a minimum premium to a policy, the base rate should be multiplied by the following factor to offset the increase in the average premium.

$$\text{Minimum Premium Offset Factor} = \frac{1}{1 + \text{Effect}}$$

Premium Transition Rules

The insurer would select a maximum and minimum allowable change to premium that a policyholder could observe over a single renewal. If a rate review indicates that a policyholder should receive an increase/decrease larger than the selected minimum/maximum allowable change, the portion exceeding the minimum/maximum allowable change is deferred until the next renewal.

When implementing such a rule, the company needs to carefully consider the minimum/maximum premium change amounts, the period of time needed to fully transition to the new rate, and the effect on the total average premium.

Ratemaking Using External Rates

Challenges when using external rates:

- The company must gain access to a competitor's rating manual or a rating bureau filing which is usually not public information.
- The company's expenses and expected loss costs may differ.

Potential adjustments:

- The company can adjust its fixed expense fee to reflect anticipated differences in fixed expenses.
- The company can adjust for anticipated differences in the variable PLR by multiplying the base rate and expense fee by a factor of:

$$\frac{\text{External Variable Permissible Loss Ratio}}{\text{Company Variable Permissible Loss Ratio}}$$

- If the company anticipates that its expected loss cost to be different, it may increase/decrease the base rate to account for the percent difference.
- The company can make judgmental adjustments to rating factors/base rates to account for anticipated differences in loss costs.

Communication and Monitoring

It is important for an actuary to communicate the expected effect of the rate change to relevant stakeholders, such as management or key regulators. The actual effects of the rate change should be monitored so that the expected effects of a rate change can be compared to its actual effects.

Other Ratemaking Topics

Claims-Made Ratemaking

Claims-Made Coverage

- **Report lag** - the time between when a claim occurs and when it is reported to the insurer.
- **Coverage trigger** - the event that must occur for a claim to be covered under the insurance policy.
  - **Occurrence-based policy** - the coverage trigger is the occurrence of a claim.
  - **Claims-made policy** - the coverage trigger is the reporting of a claim.

Under a claims-made policy, claims must occur after **the retroactive date** and be reported during the policy term to be covered.

A claims-made policy reaches **maturity** when renewals no longer significantly extend coverage, ensuring nearly all claims reported during the period are covered.

Report Year Aggregation

Denote the number of claims coming from report year x with report lag r as $L(x, r)$

Report Year	Report Lag (Years)			
	0	1	2	3
2018	$L(2018, 0)$	$L(2018, 1)$	$L(2018, 2)$	$L(2018, 3)$
2019	$L(2019, 0)$	$L(2019, 1)$	$L(2019, 2)$	$L(2019, 3)$
2020	$L(2020, 0)$	$L(2020, 1)$	$L(2020, 2)$	$L(2020, 3)$
2021	$L(2021, 0)$	$L(2021, 1)$	$L(2021, 2)$	$L(2021, 3)$
2022	$L(2022, 0)$	$L(2022, 1)$	$L(2022, 2)$	$L(2022, 3)$

where Report Year $=$ Accident Year $+$ Report Lag in Years.

5 Principles of Claims–Made Policies

1. A claims-made policy should always cost less than an occurrence policy as long as claim costs are increasing.
2. If there is a sudden, unpredictable change in the underlying trends, the claims-made policy priced based on the prior trend will be closer to the correct price than an occurrence policy based on the prior trend.
3. If there is a sudden, unexpected shift in the reporting pattern, the cost of a mature claims-made policy will be affected relatively little, if at all, relative to the occurrence policy.
4. Claims-made policies incur no liability for (pure) IBNR, so the risk of reserve inadequacy is greatly reduced.
5. The investment income earned from claims-made policies is substantially less than under-occurrence policies.

Switching Between Policy Types

Switching from occurrence to claims-made

- If the retroactive date is the same as the expiration date for the occurrence policy, there will be no gaps or overlaps in coverage.
- If the retroactive date is before the expiration of the occurrence policy, there will be an overlap in coverage. In this case, the retroactive date should be coordinated with the expiring occurrence policy.

Switching from claims-made to occurrence

- Creates a gap in coverage. The insured is susceptible to claims that occurred before the switch but are not reported until afterward.
- **Tail coverage** (a.k.a. extended reporting endorsement) can be purchased in order to make sure the insured is fully protected.

Retirement

- If an insured retires after having a claims-made policy, they are susceptible to claims that occurred during their final year of employment but were not reported during the policy year.
- Tail coverage can be purchased in order to make sure the insured is fully protected.

Commercial Lines Ratemaking

Commercial risks tend to be very large and highly differentiated, making it difficult to classify them into homogenous groups. Instead, actuaries lean on techniques for rating individual risks in order to accurately price commercial lines policies. These techniques fall into two categories:

1. techniques that modify the manual rate
2. techniques that compute a premium for a specific large commercial risk

Experience Rating

Experience rating uses a particular insured's historical claim experience to predict their future experience. The adjustment made to the future manual premium is called the **experience modification** and is calculated as a credibility-weighted average of an **experience component** and an **expected component**.

ISO Commercial General Liability Experience Rating Plan

$$\text{CD} = \frac{(\text{AER} - \text{EER})}{\text{EER}} \times Z$$

where

$$\text{AER} = \frac{\begin{matrix}\text{Reported BLL \&} \\ \text{ALAE Limited at MSL}\end{matrix} + \begin{matrix}\text{Expected Unreported BLL} \\ \text{\& ALAE Limited at MSL}\end{matrix}}{\text{Company Subject BLL \& ALAE}}$$

and

$$\text{EER} = \frac{\text{Expected Ultimate BLL \& ALAE Limited at MSL}}{\text{Company Subject BLL \& ALAE}}$$

- CD is the percentage credit or debit (i.e., the mod)
- AER is the **actual experience ratio** (i.e., the experience component)
- EER is the **expected experience ratio** (i.e., the experience component)
- Z is the credibility

Expected Unreported BLL & ALAE Limited at MSL

$$= \overbrace{\begin{matrix}\text{Expected Ultimate BLL} \\ \text{ALAE Limited at MSL}\end{matrix}}^{\text{EER} \times \begin{matrix}\text{Company Subject} \\ \text{BLL and ALAE}\end{matrix}} \times \text{\% Unreported}$$

NCCI Worker's Compensation Experience Rating Plan

$$M = \frac{\left[Z_p \times A_p + \left(1 - Z_p\right) \times E_p\right] + \left[Z_e \times A_e + \left(1 - Z_e\right) \times E_e\right]}{E}$$

or

$$M = \frac{A_p + w \times A_e + (1 - w) \times E_e + B}{E + B}$$

where

- M is the experience modification factor
- A_p is the **actual primary loss**
- A_e is the **actual excess loss**
- E_p is the **expected primary loss**
- E_e is the **expected excess loss**
- $E = E_p + E_e$
- Z_p is the **primary credibility**
- Z_e is the **excess credibility**
- B is the **ballast value**, such that $Z_p = \frac{E}{E+B}$
- $w = \frac{Z_e}{Z_p}$ is the weighting value

<u>Schedule Rating</u>

Schedule rating allows underwriters to adjust the manual rate for perceived differences in risk by applying a series of credits and debits to the manual rate. The characteristics used in a schedule rating plan can be either objective and quantifiable or subjective.

Be careful not to "double-count" the effects of a risk characteristic when using a schedule rating plan. Factors that are already reflected in a company's loss experience should not also be given a schedule credit or debit.

<u>Composite Rating</u>

Composite rating allows an insurer to track multiple exposure bases using one **composite exposure base**. If a risk is sufficiently large, it can be made a **loss-rated risk**, meaning the composite rate can be based entirely on the individual entity's prior claims experience.

ISO Composite Rating Plan for Loss-Rated Risks

1. Trend and develop reported losses and ALAE to the future policy period.
2. Select a composite exposure base and measure it for each year during the experience period.
3. Trend exposures to the future policy period and sum to get total trended composite exposure.
4. Compute the adjusted premium.
$$\text{Adjusted Premium} = \frac{\text{Trended Ultimate Loss \& ALAE}}{\text{Expected Loss \& ALAE Ratio}}$$
5. Calculate the composite rate as the ratio of the adjusted premium to the total trended composite exposure.
$$\text{Composite Rate} = \frac{\text{Adjusted Premium}}{\text{Total Trended Composite Exposure}}$$

<u>Retrospective Rating</u>

Under a retrospective rating plan, the insurer uses the insured's actual experience **during the policy period** to determine their premium for that period. An initial premium amount is charged at the beginning of the policy period and adjustments to the premium are billed during the period.

Worker's Compensation Retrospective Rating Plan

$$\frac{\text{Retro}}{\text{Premium}} = \left[\frac{\text{Basic}}{\text{Premium}} + \frac{\text{Converted}}{\text{Losses}}\right] \times \frac{\text{Tax}}{\text{Multiplier}}$$

where

$$\text{Converted Losses} = \text{Limited Reported Losses} \times \text{LCF}$$

and

$$\begin{matrix}\text{Basic} \\ \text{Premium}\end{matrix} = \left[\begin{matrix}\text{Expense} \\ \text{Allowance}\end{matrix} - \text{ELR} \times (\text{LCF} - 1) + \begin{matrix}\text{Net Insurance} \\ \text{Charge}\end{matrix}\right] \times \begin{matrix}\text{Standard} \\ \text{Premium}\end{matrix}$$

- LCF is the **loss conversion factor**.
- ELR is the **expected loss ratio**.
- **Standard premium** is the manual premium multiplied by any experience modification factors and any additional premium charges.
- The final retrospective premium is constrained to being between a minimum and a maximum premium amount.

<u>Large Deductible Policies</u>

- **Claims handling** – Either the insured or the insurer could be tasked with handling claims that fall entirely below the deductible.
  - If the **insurer** assumes responsibility, they incur additional expenses from handling these claims.
  - If the **insured** assumes responsibility, they will have less incentive to keep losses down, increasing the expected loss cost.
- **Application of the deductible** – The deductible can apply to losses only or to losses and ALAE.
- **Deductible processing** – The insurer is responsible for paying all claims in full and seeks reimbursement for amounts that do not exceed the deductible, incurring credit risk.
- **Risk margin** – Losses that exceed the large deductible tend to be more volatile, necessitating a higher target profit margin.

$$\begin{matrix}\text{Large Deductible} \\ \text{Premium}\end{matrix} = \frac{\begin{matrix}\text{Excess} \\ \text{Loss}\end{matrix} + \begin{matrix}\text{Expenses} \\ \text{Variable to Loss}\end{matrix} + \begin{matrix}\text{Fixed} \\ \text{Expense}\end{matrix}}{1 - \begin{matrix}\text{Variable} \\ \text{Expense \%}\end{matrix} - \begin{matrix}\text{Target} \\ \text{Profit \%}\end{matrix}}$$

Basic Reserving Techniques

Introduction to Reserving Principles

Components of Unpaid Claims

IBNR is made of the following 4 components:

1. An amount for claims incurred but not reported (**pure IBNR**).
2. A provision for claims reported but not recorded.
3. A provision for future development on known claims, a.k.a. **incurred but not enough reported (IBNER)**.
4. An estimate for reopened claims.

Key Relationships and Dates

$$\begin{matrix}\text{Unpaid} \\ \text{Claims}\end{matrix} = \begin{matrix}\text{Case} \\ \text{Outstanding}\end{matrix} + \begin{matrix}\text{IBNR} \\ \text{(broad definition)}\end{matrix}$$

$$\begin{matrix}\text{Unpaid} \\ \text{Claims}\end{matrix} = \begin{matrix}\text{Ultimate} \\ \text{Claims}\end{matrix} - \begin{matrix}\text{Paid} \\ \text{Claims}\end{matrix}$$

$$\begin{matrix}\text{Ultimate} \\ \text{Claims}\end{matrix} = \underbrace{\begin{matrix}\text{Paid} \\ \text{Claims}\end{matrix} + \begin{matrix}\text{Case} \\ \text{Outstanding}\end{matrix}}_{\text{Reported claims}} + \text{IBNR}$$

- **Accounting date**: This date separates paid versus unpaid claim amounts.
- **Valuation date**: The date through which transactions are included in the estimate. This date can be before, on, or after the accounting date.
- **Review date**: The cutoff date for information known to the actuary.

Importance of Accurately Estimating Unpaid Claims

- **Internal Management**: Estimating unpaid claims is essential for decision-making in areas such as pricing, underwriting, and finance. Inaccurate estimates of unpaid claims can lead to poor decisions that can negatively impact an insurer's financial strength.
- **Investors**: Accurate unpaid claims estimates are crucial for decision-making, and inaccurate estimates could lead to misleading financial metrics.
- **Insurance Regulators**: Inaccurate unpaid claims estimates can result in misrepresenting an insurer's actual financial position.

Regulatory Compliance

New York insurance law mandates that insurers maintain reserves in an estimated aggregate amount to pay for all losses or claims incurred on or before the settlement date.

Currently, countries around the world require an annual **Statement of Actuarial Opinion** signed by a qualified actuary commonly referred to as the **Appointed Actuary**, containing the actuary's opinion on the reasonableness of the reserve for carried loss and LAE.

Homogeneity and Credibility of Data

Actuaries can improve the accuracy of estimating unpaid claims by grouping similar types of claims together based on characteristics such as:

- Consistent coverage
- Claim count volume
- Reporting patterns
- Ability to develop appropriate case estimates
- Settlement patterns
- Likelihood of reopening claims
- Average settlement value

An actuary must balance the trade-off between the granularity of the analysis and the sufficiency/credibility of the data.

Chain Ladder Method (a.k.a. Development Method)

Steps of the Development Technique

1. *Compile Claims Data*
   Rows represent accident years, columns represent development years, and diagonals represent calendar years.

2. *Calculate Age-to-Age Factors*
   Divide each cell by the previous entry for that accident year.

3. *Calculate Averages*
   Calculate an average development factor for each maturity. Common choices are the arithmetic, geometric, medial, and volume-weighted averages.

4. *Select Claim Development Factors*
   Select an average development factor for each maturity. Consider the following when making your selection:
   - Smooth progression of factors
   - Stability
   - Credibility of experience
   - Changes in patterns
   - Applicability of the historical experience
   - Shock losses/CAT losses
   - Actuarial judgement

5. *Select Tail Factor*
   Three approaches for evaluating a tail factor:
   - Industry benchmark development factors
   - Fit an exponential curve
   - Utilizing reported-to-paid ratios at the latest observed paid development period

6. *Calculate Cumulative Claim Development Factors*
   Multiply claims development factors to obtain cumulative development factors (CDFs) to ultimate.

7. *Project Ultimate Claims*
   Multiply the most recent accident year observed losses by their corresponding CDFs to project ultimate losses.

Key Assumptions:

1. Claims recorded to date will continue to develop in a similar fashion in the future.
2. For accident years that are not fully developed, claims observed thus far give us relevant information on claims that have yet to be observed.
3. Throughout the policy period:
   - there is consistent claims processing (claim settlement rates and case outstanding adequacy).
   - the mix of claim types is stable.
   - policy limits and deductibles (if any) are stable.
   - reinsurance retention limits (if any) are stable.

Uses for the Development Method

The development method works well:

1. when there is a large amount of credible historical claims data available.
2. for high-frequency, low-severity lines with stable and timely reporting of claims.
3. when the presence or absence of large claims does not greatly distort the data.

It is valid in other scenarios also, as long as the key assumptions of the method are met.

Analyzing the Impact of Changes in Insurer's Environment

<table>
<tr><th rowspan="2">Description</th><th colspan="2">Impact on Development Method</th></tr>
<tr><th>Paid</th><th>Reported</th></tr>
<tr><td>Increase in exposures</td><td colspan="2">If exposures distribution is stable year over year, claims increase but development factors are unaffected.

If exposures distribution change, causing the average accident date to shift, claims projection may be distorted.</td></tr>
<tr><td>Increase in claim ratios</td><td colspan="2">Claims increase, but development factors are unaffected</td></tr>
<tr><td>Speedup in claim settlement rates</td><td>Overestimates development factors and ultimate claims</td><td>No effect</td></tr>
<tr><td>Increase in case outstanding adequacy</td><td>No effect</td><td>Overestimates development factors and ultimate claims</td></tr>
<tr><td>Change in product mix</td><td colspan="2">Development factors change to reflect the new mix</td></tr>
</table>

Expected Claims Method

Using earned premiums, the ultimate claims for a given accident year can be expressed as

$$\text{Ultimate Claims} = \underbrace{\frac{\text{Expected Claims}}{\text{Earned Premium}}}_{\text{Expected Loss Ratio}} \times \text{Earned Premium}$$

and using earned exposures as

$$\text{Ultimate Claims} = \underbrace{\frac{\text{Expected Claims}}{\text{Earned Exposures}}}_{\text{Expected Pure Premium}} \times \text{Earned Exposures}$$

In both equations, the ratio is also referred to as an expected loss ratio (ECR).

<u>Determining the ECR</u>

1. Adjust the historical data
2. Calculate the average claim ratios
3. Select the ECR

<u>Key Assumptions</u>

An actuary can more accurately estimate total unpaid claims using a prior estimate rather than relying solely on the claims experience observed to date. Claims paid to date for a given AY provide no useful information on future claim payments for that AY.

<u>Uses for the Expected Claims Method</u>

The expected claims method works well:

1. When an insurer enters a new line of business or territory.
2. When operational or environmental changes make historical data unreliable.
3. When development factors for early maturities are highly leveraged.
4. When data is limited or unavailable.

<u>Analyzing the Impact of Changes in Insurer's Environment</u>

The EC method implicitly ignores environmental changes, making it a suitable choice in unstable environments.

Description	Impact on Expected Claims Method
Increase in claim ratios	The EC method reflects changes from the years used to determine the ECR; if not captured, ultimate claims may be understated.
Increase in case outstanding adequacy	No effect
Change in product mix	Can be distorted when different lines of business have significantly different ECRs.

Bornhuetter-Ferguson & Benktander Methods

<u>Bornhuetter-Ferguson (BF) Method</u>

The BF method is a blend of the development technique together with the expected claims technique.

To determine the reserve using the BF technique:

1. Estimate ultimate losses using the expected claims method.
2. Calculate the cumulative loss development factors (CDF) using the development method.
3. Estimate ultimate losses using either reported claims
$$\begin{aligned}\begin{matrix}\text{Ultimate}\\\text{Claims}\end{matrix} &= \begin{matrix}\text{Actual}\\\text{Reported Claims}\end{matrix} + \begin{matrix}\text{Expected}\\\text{Claims}\end{matrix} \times \%\text{ Unreported}\\ &= \begin{matrix}\text{Actual}\\\text{Reported Claims}\end{matrix} + \begin{matrix}\text{Expected}\\\text{Unreported Claims}\end{matrix}\end{aligned}$$
or paid claims
$$\begin{aligned}\begin{matrix}\text{Ultimate}\\\text{Claims}\end{matrix} &= \begin{matrix}\text{Actual}\\\text{Paid Claims}\end{matrix} + \begin{matrix}\text{Expected}\\\text{Claims}\end{matrix} \times \%\text{ Unpaid}\\ &= \begin{matrix}\text{Actual}\\\text{Paid Claims}\end{matrix} + \begin{matrix}\text{Expected}\\\text{Unpaid Claims}\end{matrix}\end{aligned}$$
4. To estimate the IBNR, subtract actual reported losses; to estimate the unpaid claims, subtract actual paid claims.

The BF method can also be viewed as a **credibility-weighted average** between the development estimate and the expected claim ratio estimate.

$$\begin{matrix}\text{Ultimate}\\\text{Claims}\end{matrix} = \begin{matrix}\text{Development}\\\text{Ultimate Claims}\end{matrix} \times \frac{1}{\text{CDF}} + \begin{matrix}\text{Expected}\\\text{Claims}\end{matrix} \times \left(1 - \frac{1}{\text{CDF}}\right)$$

Advantages of the BF method:

- Works well for long tails and for limited data
- Withstands early random fluctuations in AYs
- More accurate than the reported development technique when faced with large and unusual claims

Benktander Method

The Benktander method combines the BF technique and the development technique in a credibility-weighted average. Additional key facts are:

- Essentially an iterative version of the BF technique that will eventually converge to the development method's ultimate claims estimate.
- Places more weight on the development technique, making it more responsive to the changes in the claims data than the BF method, but results in more stable estimates than the development technique.

Analyzing the Impact of Changes in Insurer's Environment

When changes occur in the book of business, the BF and Benktander methods, as weighted averages of the development and expected claims methods, adopt the traits of those methods.

Description	Impact on BF & Benktander Methods	
	Paid	Reported
Increase in claim ratios	Underestimates ultimate claims if the increase is not captured by the ECR, though not by as much as the EC method would.	
Increase in case outstanding adequacy	No effect	Overestimates ultimate claims by less than the development method but more than the EC method.
Change in product mix	Can be distorted when the changing lines of business have varying development patterns or ECRs.	

Cape Cod Method

Commonly utilized by reinsurers and is applicable to all lines of insurance, including both short-tail and long-tail lines. It operates on the assumption that unreported claims will develop based on expected claims.

The Cape Cod method differs from the Bornhuetter-Ferguson method in how the expected claim ratio is derived:

$$\begin{aligned}\text{Cape Cod ECR} &= \frac{\text{Total Reported Claims to Date}}{\text{Total Used-up Premium}}\\ &= \frac{\sum \text{Reported Claims}}{\sum\left(\begin{matrix}\text{On} - \text{Level}\\ \text{Earned Premium}\end{matrix} \times \%\ \text{Reported}\right)}\end{aligned}$$

Additional key facts:

- Only reported claims are used to estimate ultimate losses.
- Data from the year in which the estimate is made is included.
- Claims ratios must be adjusted to the same level and year of estimation, as needed.

Advantages/Disadvantages

- An advantage of the Cape Cod method, when compared to the development technique, is its resilience against early random fluctuations in the development of an accident year or time interval.
- Disadvantages include the use of on-level premiums and the requirement of having sufficient and credible enough reported claims data, which results in the Cape Cod method not being as suitable as the BF method when data is thin or volatile.

Analyzing the Impact of Changes in Insurer's Environment

Description	Impact on the Cape Cod Method
Increase in claim ratios	Underestimates ultimate claims, though not as much as the BF method would since the Cape Cod ECR is based on reported claims through the valuation date.
Increase in case outstanding adequacy	Overestimates ultimate claims by less than the development method but more than the BF method.
Change in product mix	Can be distorted when the changing lines of business have varying development patterns.

Frequency-Severity Techniques

- Pros:
  - Produce more stable estimates than the development method, especially for earlier maturities.
  - Give actuaries greater insight into the claims process.
  - Allow inflation to be incorporated in unpaid claims estimates.
- Cons:
  - Incorrect assumptions about inflation result in inaccuracy.
  - Require detailed data.

<u>Frequency-Severity Technique #1</u>

Apply the development technique separately to claim counts and claim severities.

Key Assumptions

1. Claim counts and severities recorded to date will continue to develop in a similar fashion in the future.
2. Claim counts have a consistent definition throughout the experience period.
3. The mix of claim types is reasonably homogenous.

Technique

1. Project and select ultimate claim counts.
2. Project and select ultimate severity.
3. Project ultimate claims as ultimate counts times ultimate severity.
4. Compute unpaid claim/IBNR estimate (if needed).

<u>Frequency-Severity Technique #2</u>

Utilize the development method while incorporating trend analysis for exposure, frequency, and severity. Especially useful when estimating the ultimate loss or IBNR for accident years that are relatively immature or have highly leveraged CDFs.

Key Assumptions

1. Claim counts and severities recorded to date will continue to develop in a similar fashion in the future.
2. Claim counts have a consistent definition throughout the experience period.
3. The mix of claim types is reasonably homogenous.

Technique

1. Project ultimate claim counts.
2. Compare claim counts to exposures and select ultimate frequency.
3. Project and select ultimate severity.
4. Project ultimate claims.

$$\text{AY Ult. Claims} = \overbrace{\text{Projected Claim Counts}}^{\text{AY Exposures} \times \text{Selected Frequency}} \times \text{Selected Severity}$$

5. Compute unpaid claim/IBNR estimate (if needed).

<u>Disposal Rate Method</u>

The **disposal rate** for a particular accident year at maturity t is the percentage of ultimate claim counts that have been closed at that maturity.

$$\text{AY Disposal Rate}_t = \frac{\text{AY Cumulative Closed Claim Counts}_t}{\text{AY Ultimate Closed Claim Counts}}$$

Key Assumptions

1. Claim counts and severities recorded to date will continue to develop in a similar fashion in the future.
2. Claim counts have a consistent definition throughout the experience period.
3. The mix of claim types is reasonably homogenous.
4. There are no significant partial claim payments made.

Technique

1. Project and select ultimate claim counts by accident year.
2. Create disposal rate triangle and select disposal rates by maturity.
3. Project claim counts using the selected disposal rates.

$$\begin{matrix}\text{Closed Claim Counts} \\ \text{Between } t_1 \text{ and } t_2\end{matrix} = \frac{\text{Open Claim Counts}}{1 - \text{Latest Selected DR}} \cdot \left(\begin{matrix}\text{Selected} \\ \text{DR}_{t_2}\end{matrix} - \begin{matrix}\text{Selected} \\ \text{DR}_{t_1}\end{matrix}\right)$$

4. Analyze and select incremental severities by maturity.
5. Calculate incremental severities by maturity age and accident year.
6. Project unpaid claims by multiplying claim counts by severities.
7. Compute ultimate claim/IBNR estimate (if needed).

<u>Tail Severities</u>

The severities in the oldest maturity ages are often highly unstable, especially in the long-tailed lines of business. This is mainly due to thin data. In this case, we can consider combining the severities of multiple maturity ages and select a **tail severity**.

Considerations when Selecting a Tail Severity

- Combine data over the ages where the results become erratic.
- Consider the influence on the total projections of selecting a particular age.
- Consider the percentage of claims expected to be closed beyond the selected maturity.

Case Outstanding Techniques

Case Outstanding Technique #1

Develop case outstanding and use it to project incremental paid claims.

- Pros:
  - Useful for short-tailed lines where most or all claims are reported during the first development year
  - Pairs well with claims-made coverages
- Cons:
  - Case outstanding ratios are difficult to interpret
  - Large claims during the experience period can distort case outstanding and unpaid claims estimates

Technique

1. Calculate and select the remaining-in-case ratios (a.k.a. ratio of case outstanding to previous case outstanding)

$$\text{Remaining}-\text{in}-\text{Case Ratio}=\frac{\text{Current Case Outstanding}}{\text{Prior Case Outstanding}}$$

2. Project case outstanding

$$\begin{array}{c}\text{Projected}\\\text{Case Outstanding}\end{array}=\begin{array}{c}\text{Selected}\\\text{Remaining}-\text{in}-\text{Case Ratio}\end{array}\times\begin{array}{c}\text{Prior Case}\\\text{Outstanding}\end{array}$$

3. Calculate and select the paid-on-case ratios (a.k.a. ratio of incremental paid claims to previous case outstanding)

$$\text{Paid}-\text{on}-\text{Case Ratio}=\frac{\text{Incremental Paid Claims}}{\text{Prior Case Outstanding}}$$

4. Project incremental payments

$$\begin{array}{c}\text{Projected Incremental}\\\text{Paid Claims}\end{array}=\begin{array}{c}\text{Selected}\\\text{Paid}-\text{on}-\text{Case Ratio}\end{array}\times\begin{array}{c}\text{Prior}\\\text{Case Outstanding}\end{array}$$

Case Outstanding Technique #2

Project future unpaid claims by combining case outstanding using industry-based development factors.

- Pros:
  - Allows us to project unpaid claims when case outstanding is the only available internal information
  - Commonly used by self-insurers or after mergers with them
- Cons:
  - Relies on the insurer's ability to obtain industry CDFs
  - Assumes that the obtained industry CDFs are representative of the future claim development within the company
  - Large claims during the experience period can distort case outstanding and unpaid claims estimates

Key Assumptions

1. Claims recorded to date will develop in a similar fashion in the future to the industry benchmark.
2. Case outstanding give us relevant information on claims that have yet to be observed.
3. Throughout the policy period:
   - there is consistent claims processing
   - the mix of claim types, policy limits (if any), and reinsurance retention limits (if any) are stable.

Technique

Combine industry-based paid and reported CDFs to create a singular case outstanding development factor.

$$\begin{array}{c}\text{Case OS}\\\text{Development Factor}\end{array}=1+\frac{(\text{Reported CDF}-1)(\text{Paid CDF})}{\text{Paid CDF}-\text{Reported CDF}}$$

Multiply by AY case outstanding to project unpaid claims.

Adjusting and Evaluating Reserving Techniques

Berquist-Sherman Paid Claim Development Adjustment

(Focuses on Settlement Rate Adjustment)

- Restates paid claims data by interpolating paid amounts between the historical disposal rate and the most recent disposal rate for each maturity.
- Assumes that disposal rates are roughly proportional to the total percentage of ultimate claims paid for each maturity

<u>Mechanics of the Adjustment</u>

1. Analyze the historical disposal rates. If the disposal rates changed significantly over the year, a B-S paid claims development adjustment is warranted.
2. (Assuming adjustment is appropriate) Restate the cumulative paid claims.
   1. Select the disposal rates by maturity.
   2. Interpolate between paid claims of consecutive maturities.
      - If historical disposal rate < selected disposal rate, then adjust paid claims upward.
      - If historical disposal rate > selected disposal rate, then adjust paid claims downward.
3. Perform the paid development method.

<u>Paid B-S Interpolation Formulas</u>

If historical disposal rate < selected disposal rate:

$$\begin{matrix}\text{Adjusted}\\ \text{Paid}_t\end{matrix} = \text{Paid}_t + (\text{Paid}_{t+1} - \text{Paid}_t)\left(\frac{\begin{matrix}\text{Selected}\\ \text{DR}_t\end{matrix} - \begin{matrix}\text{Historical}\\ \text{DR}_t\end{matrix}}{\begin{matrix}\text{Historical}\\ \text{DR}_{t+1}\end{matrix} - \begin{matrix}\text{Historical}\\ \text{DR}_t\end{matrix}}\right)$$

If historical disposal rate > selected disposal rate:

$$\begin{matrix}\text{Adjusted}\\ \text{Paid}_t\end{matrix} = \text{Paid}_t - (\text{Paid}_t - \text{Paid}_{t-1})\left(\frac{\begin{matrix}\text{Historical}\\ \text{DR}_t\end{matrix} - \begin{matrix}\text{Selected}\\ \text{DR}_t\end{matrix}}{\begin{matrix}\text{Historical}\\ \text{DR}_t\end{matrix} - \begin{matrix}\text{Historical}\\ \text{DR}_{t-1}\end{matrix}}\right)$$

<u>Alternative: Two-point Exponential Fits</u>

After determining that an adjustment is needed:

1. Compute adjusted cumulative closed claim counts by multiplying the estimated ultimates for each accident year by the selected disposal rates for each maturity.
2. Apply the equation $Y = ae^{bx}$ to get adjusted closed claim amounts, where
   - Y is the adjusted closed claim amount
   - X is the adjusted closed claim count
   - a and b are regression parameters (typically given)
     - If the historical claim count is below the adjusted claim count, then adjust paid claims upward, i.e., use a and b at the next maturity.
     - If the historical claim count is above the adjusted claim count, then adjust paid claims downward, i.e., use a and b at the current maturity.

Berquist-Sherman Reported Claim Development Adjustment

(Focuses on Case Outstanding Adequacy Adjustment)

- Restates reported claims data at a common level of case outstanding adequacy.
- Assumes that annual changes in the average case outstanding at each maturity are due to changes in case outstanding adequacy or trends in claim severity.

<u>Mechanics of the Adjustment</u>

1. Evaluate the data.
   1. Analyze the % change in claim severities.
   2. Analyze the % change in average case outstanding.
   3. If the changes deviate significantly, a B-S reported claims development adjustment is warranted.
2. (Assuming adjustment is appropriate) Restate the cumulative reported claims.
   1. Select a severity trend. (If a severity trend is given by the question, we can use it directly.)
   2. Adjust the average case outstanding by trending the most recent data backward.
   3. Restate the cumulative reported claims using the adjusted average case outstanding.
3. Perform the reported development method.

<u>Reported B-S Formulas</u>

$$\begin{matrix}\text{Average}\\ \text{Case OS}\end{matrix} = \frac{\text{Case OS}}{\text{Open Claim Counts}} = \frac{\text{Reported Claims} - \text{Paid Claims}}{\text{Reported Counts} - \text{Closed Counts}}$$

$$\begin{matrix}\text{Adj. Reported}\\ \text{Claims}\end{matrix} = \begin{matrix}\text{Adj. Average}\\ \text{Case OS}\end{matrix} \times \left(\begin{matrix}\text{Reported}\\ \text{Counts}\end{matrix} - \begin{matrix}\text{Closed}\\ \text{Counts}\end{matrix}\right) + \begin{matrix}\text{Paid}\\ \text{Claims}\end{matrix}$$

Changes in Settlement Rate and Case Outstanding Adequacy

1. Perform the B-S paid adjustment to obtain the adjusted cumulative paid claims as normal.
2. Perform the B-S reported adjustment to obtain the adjusted average case outstanding (not the adjusted reported claims).
3. Adjust the closed claim count for each period using the selected disposal rates from Step 1.
4. Restate the adjusted reported claims as the product of the adjusted average case outstanding and the adjusted open claim counts, plus the adjusted paid claims.

$$\begin{matrix}\text{Adj. Reported}\\ \text{Claims}\end{matrix} = \begin{matrix}\text{Adj. Average}\\ \text{Case OS}\end{matrix} \times \left(\begin{matrix}\text{Reported}\\ \text{Counts}\end{matrix} - \begin{matrix}\text{Adj. Closed}\\ \text{Counts}\end{matrix}\right) + \begin{matrix}\text{Adj. Paid}\\ \text{Claims}\end{matrix}$$

Evaluating Reserving Techniques

Components of an Unpaid Claims Analysis

Berquist and Sherman note that actuaries should include the following elements in an unpaid claims analysis whenever possible:

- Projections of reported claims
- Projections of paid claims
- Projections of ultimate reported claim counts and severities
- Estimates of the number and average amount of outstanding claims
- Claim ratio estimates

In addition, they recommended that actuaries include concepts from credibility, regression analysis, and data smoothing in unpaid claims analyses in order to enhance the stability of any estimates.

Rather than performing detailed rate reviews each period, smaller companies tend to "roll forward" the results from the previous detailed analysis and approximate the changes between the current date and the date of the last analysis in order to get an up-to-date picture of their position.

Expected Emergence

- Expected Reported Claims Between t_1 and t_2

$$\overbrace{\begin{pmatrix}\text{Estimated} \\ \text{Ult. Claims}\end{pmatrix} - \begin{pmatrix}\text{Latest} \\ \text{Rept. Claims}\end{pmatrix}}^{\text{Estimated IBNR}} \times \overbrace{\frac{\%\text{ Rept}_{t_2} - \%\text{ Rept}_{t_1}}{1 - \text{Latest }\%\text{ Rept}}}^{\%\text{ Expected to Emerge}}$$

or

$$\left(\begin{matrix}\text{Estimated} \\ \text{Ult. Claims}\end{matrix} - \begin{matrix}\text{Latest} \\ \text{Rept. Claims}\end{matrix}\right) \times \frac{\dfrac{1}{\text{Rept CDF}_{t_2}} - \dfrac{1}{\text{Rept CDF}_{t_1}}}{1 - \dfrac{1}{\text{Latest Rept CDF}}}$$

- Expected Paid Claims Between t_1 and t_2

$$\overbrace{\left(\begin{matrix}\text{Estimated} \\ \text{Ult. Claims}\end{matrix} - \begin{matrix}\text{Latest} \\ \text{Paid Claims}\end{matrix}\right)}^{\text{Estimated Unpaid Claims}} \times \overbrace{\frac{\%\text{ Paid}_{t_2} - \%\text{ Paid}_{t_1}}{1 - \text{Latest }\%\text{ Paid}}}^{\%\text{ Expected to be Paid}}$$

or

$$\left(\begin{matrix}\text{Estimated} \\ \text{Ult. Claims}\end{matrix} - \begin{matrix}\text{Latest} \\ \text{Paid Claims}\end{matrix}\right) \times \frac{\dfrac{1}{\text{Paid CDF}_{t_2}} - \dfrac{1}{\text{Paid CDF}_{t_1}}}{1 - \dfrac{1}{\text{Latest Paid CDF}}}$$

Options for Handling a Discrepancy in IBNR

1. **Leave the IBNR reserve as is**: For example, this would be appropriate if we believed that the excess reported claims were due to random fluctuation, such as an unusually large loss, but that future development would return to expected levels.
2. **Increase the IBNR reserve**: For example, this would be appropriate if we believed that the excess reported claims were due to an increasing claims ratio.
3. **Decrease the IBNR reserve**: For example, this would be appropriate if we believed that the excess reported claims were due to an increase in the report rate.

Interpolating Between Dates

- When claims are uniformly reported or paid between maturities, we can linearly interpolate between the two neighboring quantities.
  - Fairly accurate when the difference between maturities is small.
- In practice, claims tend to develop faster at the start of a period and slower toward the end.

Other Reserving Topics

Recoveries

Salvage and Subrogation

- **Salvage** refers to the proceeds from selling damaged property that the insurer collects.
- **Subrogation** refers to the insurer's right to recover claim payments from a third party responsible for damages and/or injuries.

Data Considerations

Insurers vary in how they process and track S&S:

- Some insurers track case estimates and payments separately for various recovery types (i.e., salvage, subrogation, deductibles, and collateral sources).
- Others consolidate claims data across all recovery types; often, they only record payments without estimating any case outstanding.
- Some consider recoveries as negative claim payments and avoid maintaining distinct recovery data.

Estimating S&S Recoveries

To estimate unpaid S&S or unreported S&S, use either of the following:

$$\text{Unpaid S\&S} = \text{Ultimate S\&S} - \text{Received S\&S}$$

$$\text{Unreported S\&S} = \text{Ultimate S\&S} - \text{Reported S\&S}$$

The development technique is a common approach to estimating ultimate salvage and subrogation. Another common approach is the ratio method, which makes use of the ratio of S&S to gross claims and then uses those ratios in conjunction with ultimate claims to estimate ultimate S&S.

The ratio method offers two distinct advantages over the development technique:

- Less leverage in development factors
- Refined selection of ultimate S&S ratios

One disadvantage of the ratio method is that an error in selecting ultimate losses will likely lead to an error in the S&S estimate also.

Reinsurance

In a reinsurance contract, a primary insurer cedes some or all of its risk to a reinsurance company. There are two types of reinsurance:

- **Quota share** reinsurance is similar to the application of coinsurance. Both parties share a percentage of the total risk.
- **Excess of loss** reinsurance covers all claim amounts above the primary insurer's retention. This is similar to a deductible provision. There are several types:
  - Per-risk excess of loss reinsurance covers claims for a single policy.
  - Per-occurrence excess of loss reinsurance covers claims resulting from one occurrence or event that impacts multiple policies.
  - Aggregate excess of loss reinsurance covers aggregate losses within a specific policy period.

Considerations for Data Analysis

To ensure a thorough analysis, consistent assumptions, and proper validation, actuaries should consider the following throughout the estimation process:

1. Comparing net and gross data
2. Quota share analysis
3. Excess of loss analysis
4. Consistent assumptions
5. Net claim development patterns

Unpaid ALAE

ALAE Data Considerations

Actuaries must comprehend industry-specific data variations and distinguish between ALAE and claims data to accurately estimate unpaid ALAE, as combining them can lead to inaccurate estimates due to differing development patterns.

Estimating Unpaid ALAE

There are two methods for ALAE estimation: the development technique and the ratio technique. The development technique is used to estimate paid ALAE, and if case outstanding data is available, it can also be used for reported ALAE estimation.

Additionally, an additive ratio approach, known as additive development, serves as an alternative to the multiplicative ratio approach for ALAE calculation, particularly when ratios are small in the early stages to enhance stability.

The ratio method offers several advantages:

1. Acknowledges the correlation between ALAE and claims.
2. Results in less leveraged development factors compared to paid ALAE dollars.
3. Provides flexibility for actuarial judgment when selecting ultimate ALAE ratios from recent years within the experience period.

Disadvantages include:

1. Potential for claims with no payment but significant ALAE.
2. Errors in estimating ultimate claims can lead to errors in the ultimate ALAE estimate.

Unpaid ULAE: Dollar-Based Techniques

Dollar-based techniques relate ULAE dollars to claim dollars. ULAE expenditures are assumed to follow the general timing of claims (reported or paid) and are proportional to their size.

Classic (Traditional) Approach

Mechanics

1. Calculate the historical ratio of paid ULAE-to-paid claims, W_C, for each calendar year.
$$W_C = \frac{\text{CY Paid ULAE}}{\text{CY Paid Claims}}$$
2. Analyze the historical ratios for any trends or patterns.
3. Choose an ULAE-to-claims ratio, W_C^*.
4. Estimate the unpaid ULAE.
$$\begin{matrix}\text{Unpaid}\\\text{ULAE}\end{matrix} = W_C^* \times \left[\begin{matrix}\text{Pure}\\\text{IBNR}\end{matrix} + 0.5\left(\begin{matrix}\text{Case}\\\text{Outstanding}\end{matrix} + \text{IBNER}\right)\right]$$

Key Assumptions

- The ratio of paid ULAE-to-paid claims is stable enough such that it can reasonably estimate the ultimate ULAE-to-ultimate claims relationship.
- The future activity and cost associated with managing claims that have not been reported yet are directly proportional to pure IBNR, and those that have been reported but not yet closed are directly proportional to case outstanding and IBNER.
- 50% of ULAE costs occur when a claim is opened and 50% when a claim is closed.

Mango-Allen Refinement

The Mango-Allen refinement is nearly identical to the classical technique except that instead of using the actual calendar period claims, expected paid claims are used:
$$W_M = \frac{\text{CY Paid ULAE}}{\text{CY Expected Paid Claims}}$$

Kittel Refinement

In this approach, the ratio, W_K, is defined as:
$$W_K = \frac{\text{CY Paid ULAE}}{0.5(\text{CY Paid Claims} + \text{CY Incurred Claims})}$$
where
$$\begin{aligned}&0.5(\text{CY Paid Claims} + \text{CY Incurred Claims})\\&\quad = \text{Claims Opened and Paid}\\&\quad\quad + 0.5(\text{Payments on Prior Outstanding})\\&\quad\quad + 0.5(\text{Claims Opened Remaining Open})\end{aligned}$$

Note that in this approach, the term "incurred claims" includes both reported and IBNR claims. Thus, it also equals the paid claims plus the change in **total** reserves, including both case outstanding and IBNR.

Key Assumptions

- ULAE is sustained as claims are reported even if no claim payments are made.
- ULAE payments for a specific calendar year are related to both the reporting and payment of claims.

Generalized Approach

In the generalized approach, three weights, U_1, U_2, and U_3, are defined as follows:

- U_1 is the percent of ultimate ULAE spent in opening claims
- U_2 is the percent of ultimate ULAE spent in maintaining claims
- U_3 is the percent of ultimate ULAE spent in closing claims
- $U_1 + U_2 + U_3 = 100\%$

This method estimates the ULAE-to-claims ratio, W, for each calendar year as the paid ULAE, M, divided by the claims basis, B:
$$W = \frac{M}{B}$$
where $B = (U_1 \times R) + (U_2 \times P) + (U_3 \times C)$, and R, P, and C are defined as:

- R - ultimate cost of claims reported during the calendar year
- P - paid claims during the calendar year
- C - the ultimate cost of claims closed during the calendar year

Unpaid ULAE can then be estimated using any of the following approaches:

Approach	Unpaid ULAE
Bornhuetter-Ferguson	$W^* \times (L - B)$
Expected Claims	$(W^* \times L) - M$
Development	$M \times \left(\frac{L}{B} - 1\right)$

Key Assumptions

- ULAE expenses are directly tied to the dollar amount of claims being handled.
- ULAE amounts spent when opening claims are proportional to the ultimate cost of those claims when reported.
- ULAE amounts spent during the maintenance of claims correspond to the payments made during their handling.
- ULAE expenses for closing claims are proportionate to the final cost of the claims being closed.

Simplified Generalized Approach

This approach estimates the claim basis, $B_{estimate}$, by replacing R with L and letting $U_3 = 0$:
$$B_{estimate} = (U_1 \times L) + (U_2 \times P)$$
W is defined as:
$$W = \frac{M}{B_{estimate}}$$

Then, the following formula is used to estimate the unpaid ULAE:
$$\begin{aligned}\begin{matrix}\text{Unpaid}\\\text{ULAE}\end{matrix} &= W_C^* \times \left[\text{U}_1 \times \begin{matrix}\text{Pure}\\\text{IBNR}\end{matrix} + \text{U}_2 \times \left(\begin{matrix}\text{Case}\\\text{Outstanding}\end{matrix} + \text{IBNER} + \begin{matrix}\text{Pure}\\\text{IBNR}\end{matrix}\right)\right]\\&= W_C^* \times \left[\begin{matrix}\text{Pure}\\\text{IBNR}\end{matrix} + \text{U}_2 \times \left(\begin{matrix}\text{Case}\\\text{Outstanding}\end{matrix} + \text{IBNER}\right)\right]\end{aligned}$$

The classical and Kittel approaches are just specific cases of the generalized approach.

Unpaid ULAE: Count Based Techniques

Count-based techniques assume that the cost of each claim's transaction is the same regardless of claim size and that a claim which is open longer will cost more.

- The Brian technique projects future transaction volumes and assumes similar ULAE costs for the five transaction types (initiating, managing, disbursing, concluding, and reopening claims) based on historical ULAE-to-transaction volume ratios.
- The Wendy Johnson technique follows a similar approach to Brian's technique but instead focuses on two key transactions: reporting and maintenance.
- The Mango-Allen Claim Staffing Technique works by estimating the future staff count needed to handle claims and then multiplying by the future ULAE per claim staff member.
- Kay Kellogg Rahardjo introduces a specialized pricing method for claims-handling services, particularly aimed at Third-Party Claims Administrators (TPAs). She also points that as claim duration prolongs, the cost of managing the claim over its remaining lifespan also increases.
- Spalla advocates utilizing advanced claim information systems to track employees' time on individual claims. Spalla's method enables the calculation of average ULAE per transaction type, including overhead costs, and provides relative cost comparisons.

Generalized Approach with Claim Counts

$$W = \frac{M}{b}$$

where the claim count basis, b, can be calculated as:

$$b = (v_1 \times r) + (v_2 \times o) + (v_3 \times c)$$

This relies on the following inputs:

- r = reported claim counts
- o = open claim counts
- c = closed claim counts
- v_1 = relative cost of handling the reporting of a claim
- v_2 = relative cost of managing an open claim
- v_3 = relative cost of closing a claim

Select w^*, or a series of w_i^* terms, which can be different for each calendar year to take inflation into account. Then, the unpaid ULAE can be estimated as:

$$\text{Unpaid ULAE} = \sum_i w_i^* \times [(v_1 \times r_i) + (v_2 \times o_i) + (v_3 \times c_i)]$$

where:

- r_i is the number of claims reports in calendar year i
- o_i is the number of open claims at the end of calendar year i
- c_i is the number of claims to be closed during calendar year i

Unpaid ULAE: Triangle-Based Techniques

Triangle-based techniques can also be used to estimate unpaid ULAE.

- Triangle-Based Development Techniques. In this method, actuaries allocate paid ULAE in previous calendar years to accident years. It will be inaccurate if the allocations are incorrect or altered over time.
- Slifka's Method (Time-and-Motion Study Approach). The Slifka method recommends employing a time-and-motion study to understand how resources are allocated between managing the current accident year's claims and the prior accident year's claims.
- Paid ULAE Triangles Based on Time and Motion Studies. This creates a ULAE triangle, and traditional development techniques can then be applied to estimate ultimate ULAE and indicated unpaid ULAE.