3.4.1 → Measuring Credibility

→ Necessary criteria for measures of credibility

→ The initial step in applying credibility is to assess the reliability of the actuarial estimate derived from observed experience. For homogeneous risks, the credibility assigned to the observed experience, often represented by Z, should satisfy the following three criteria:

→1) $0 \le Z \le 1$
→2) As n increases, Z should increase
→3) As n increases, Z should increase at a decreasing rate

→ Methods for measuring credibility

→ Three commonly used methods for determining credibility are classical credibility, Bühlmann credibility, & Bayesian analysis. Since these methods have been covered extensively in previous exams, this subsection is meant to be an overview.

→ Method 1 → Classical credibility

→ Classical credibility, also known as limited fluctuation credibility, calculates the credibility-weighted estimate using the following formula:

$$\boxed{\text{Estimate} = (Z \times \text{observed exp.}) + (1-Z)\ \text{related experience}}$$

→ Let Y denote the total # of claims & S represent the total amount of losses. The observed experience is considered fully credible when there is a high probability, P, that it will not differ from the expected experience by more than some arbitrary threshold (k). Mathematically, that means:

$$\boxed{P[(1-k)E(S) \le S \le (1+k)E(S)] = P}$$

→ Using the central limit theorem, the equation can be achieved when:

$$\boxed{\sqrt{\frac{(1+k)E(S) - E(S)}{V(S)}} = z_{(P+1)/2}}$$

→ Furthermore, if we make the following simplifying assumptions about the observed experience:

→ Exposures are homogeneous (i.e. each exposure has the same expected # of claims)
→ Claim occurrence follows a Poisson distribution, so $E(Y) = V(Y)$
→ There is no variation in loss size (i.e. constant severity)

then the expected number of claims needed for full credibility is:

$$\boxed{E(Y) = \left(\frac{z_{(P+1)/2}}{k}\right)^2}$$

→ If the number of observed claims is equal to or greater than the standard for full credibility, i.e. $Y \ge E(Y)$, then $Z = 1$. Otherwise, Z is calculated using the square root rule:

$$\boxed{Z = \sqrt{\frac{Y}{E(Y)}}, \text{ where } Y < E(Y)}$$

→ The number of exposures needed for full credibility is the number of claims needed for full credibility divided by the expected frequency. Then, use it w/ the # of observed exposures in the above square root equation.

→ Example →

Given the following information:

- The full credibility standard is set so that the observed value is within 5% of the true value 95% of the time.
- It is assumed that exposures are homogeneous, claim occurrence follows a Poisson distribution, and there is no variation in claim costs.
- The observed pure premium based on 500 claims is $100.
- The pure premium of the related experience is $150.
- $z_{0.95} = 1.645$, $z_{0.975} = 1.960$, and $z_{0.995} = 2.575$.

Calculate the credibility-weighted pure premium estimate.

→ First, determine the number of claims required for full credibility. We are given $p = 0.95$ & $k = 0.05$

$$E(Y) = \left(\frac{z_{(P+1)/2}}{k}\right)^2 = \left(\frac{z_{0.975}}{0.05}\right)^2 = \left(\frac{1.960}{0.05}\right)^2 = 1536.64$$

→ Since the # of claims, 500, is less than 1536.64, calculate Z using the square root formula.

$$Z = \sqrt{\frac{500}{1537}} \approx 0.57$$

→ Then, the credibility-weighted pure premium estimate is:

$$0.57 \times \$100 + (1-0.57) \times \$150 = \$121.48$$

→ More info on classical credibility

→ Classical credibility has three main advantages:

→1) It is the most commonly used credibility method
→2) The data required is readily available
→3) The calculations are very straightforward

→ The main disadvantages of classical credibility include the following:

→1) The simplifying assumptions used in the derivation may not be true in practice
→2) It does not take into account the quality of the related experience used as a complement of credibility

→ Method 2 → Bühlmann credibility

→ Bühlmann credibility is also called least squares credibility. It calculates the credibility-weighted estimate as follows:

Estimate = $Z \times$ observed Experience + $(1-Z) \times$ prior mean

→ under Bühlmann credibility, Z is defined as follows:

$$Z = \frac{n}{n+k}$$

where

→ n = number of observations
→ k = the expected value of process variance (EVPV) divided by the variance of hypothetical means (VHM)

→ Unlike classical credibility, Z calculated under the Bühlmann method will never equal 1.

→ Bühlmann credibility is used w/in the insurance industry & is generally accepted. The major challenge w/ this approach is the determination of EVPV + VHM. Similar to classical probability, Bühlmann credibility relies on a set of simplifying assumptions that must be assessed to determine if this approach is appropriate for the situation.

→ Method 3 → Bayesian analysis

→ Bayesian analysis, as the name implies, is based on the Bayes Theorem. This method has no specific calculation of Z. Instead, it adjusts the prior estimate to reflect new information in a probabilistic manner. Bayesian analysis is not commonly used due to its complexity.

→ It is noteworthy that Bühlmann, or least squares credibility, corresponds to the weighted least squares line related to the Bayesian estimate. In specific mathematical cases, the Bayesian estimate is identical to the least squares credibility estimate.

→ Assignment

→ Q1) We discussed three criteria for credibility.

(a) *(0.75 points)* State the criteria.

(b) *(0.75 points)* Assess whether $F(x) = x^2, 0 \le x \le 1$, meets each of the criteria in part (a) above. Show all work.

→ a) The amount of credibility, Z, should meet the following criteria:

→1) $0 \le Z \le 1$
→2) As the risk size, n, increases, Z should increase. Mathematically, that is $\frac{dZ}{dn} \ge 0$
→3) As the risk size increases, Z should increase at a decreasing rate. Mathematically, that is $\frac{d\left(\frac{Z}{n}\right)}{dn} > 0$?

→ b) 1) $Z = F(x) = x^2$ ranges between 0 & 1 for $0 \le x \le 1$ ⇒ ✓

2) $\frac{dF(x)}{dx} = 2x > 0, \ 0 \le x \le 1$ ⇒ ✓

3) $\frac{d}{dx}\left(\frac{x^2}{x}\right) = 1 > 0$ ⇒ X