3.3.2 → Increased limit ratemaking

→ Overview → Insurance policies w/ third-party liability coverage usually offer different amounts of insurance coverage, known as limits of insurance. The basic limit has a relativity of 1 & is typically the lowest level of coverage offered. Higher limits are referred to as increased limits.

→ Increased limits ratemaking, which is the process of determining indicated increased limit factors (ILFs), is growing in importance for several reasons.

→ As personal wealth rises, individuals have more assets to protect, creating a need for higher levels of insurance coverage.

→ General inflation raises costs, & the resulting trends have a more pronounced effect on increased limits losses compared to basic limits losses.

→ The frequency of lawsuits & the size of jury awards have grown significantly due to social inflation, which, like general inflation, disproportionately affects increased limits losses.

→ Insurance lines where increased limits are especially crucial include private passenger & commercial auto liability, umbrella policies, & any commercial products offering liability coverage, such as contractors or professional liability

→ There are two types of policy limits offered.

→ 1) Single limits → These restrict the total amount the insurer will pay for a single claim. For instance, if an umbrella policy has a $1M limit, it will cover up to that amount for any single claim.

→ 2) Compound limits → These apply two or more limits to the covered losses & can be further categorized as follows:

→ Split limit → includes separate limits for each claimant & each occurrence. For example, a split limit of $10,000/$20,000 in personal automobile insurance means the policy will pay each injured party up to $10,000, w/ a total cap of $20,000 for all injured parties in an accident.

→ Occurrence/aggregate limit → These set caps for individual occurrences & for all incidents in a policy period. For example, a professional liability policy w/ a limit of $500,000/$1,500,000 will pay no more than $500,000 for any single occurrence & no more than $1,500,000 for all occurrences incurred w/in the policy period

→ In this subsection, we will focus on determining the ILFs for single limits only. Compound & split limits are more complex b/c both limits need to be taken into account; therefore, they will not be discussed here.

→ The challenge w/ using standard ratemaking approaches to determine ILFs is that limited data can lead to volatile results. Multivariate approaches like GLM are more effective at handling sparse data than univariate methods but may produce counterintuitive results, such as a decrease in expected losses when the limit increases. Consequently, special techniques have been developed for increased limits ratemaking.

→ Standard ILF approach

→ The rate at the increased limit H is expressed as the ILF for limit H times the rate for the basic limit,

$$\boxed{Rate_H = ILF_{(H)} \times Rate_B}$$

→ The following assumptions will be used in the calculation of the ILF.

→ 1) All underwriting expenses are variable

→ 2) The variable expense & profit provisions do not vary by limit

→ 3) Frequency & severity are independent

→ 4) Frequency is the same for all limits

→ Note that the assumptions above may not be realistic. For example, the claim experience for higher limits can be very volatile, which makes them hard to price & risky for the insurer. As a result, the insurer may require a higher profit provision for higher limits. However, the assumptions above will help to simplify the calculations. Using all the assumptions above, we can derive the following formula for the indicated ILF for a given limit H:

$$\rightarrow \text{Indicated } ILF(H) = \frac{Rate_H}{Rate_B}$$

$$= \frac{\left[\frac{L+E_L}{1.0-V-Q_T}\right]_H}{\left[\frac{L+E_L}{1.0-V-Q_T}\right]_B} \quad \text{< from assumption 1 > variable UW expenses}$$

$$= \frac{[L+E_L]_H}{[L+E_L]_B} \quad \text{< from assumption 2 > VER + profit don't vary by limit}$$

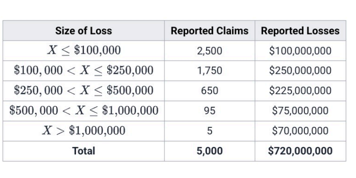
$$= \frac{Frequency_H \times Severity_H}{Frequency_B \times Severity_B} \quad \text{< from assumption 3 > Freq} \perp\!\!\!\perp \text{Sev}$$

$$= \frac{Severity_H}{Severity_B} \quad \text{< from assumption 4 > } Freq_i = Freq_j$$

$$\boxed{= \frac{LAS(H)}{LAS(B)}}$$

→ LAS(H), or the limited average severity at H, is the severity assuming every loss is capped at limit H regardless of the actual policy limit. LAS(B), or the limited severity at B, is the severity assuming every loss is capped at the basic limit.

→ We illustrate the calculation of ILF using the following uncensored data on 5,000 claims.



Size of Loss	Reported Claims	Reported Losses
$X \le \$100{,}000$	2,500	$100,000,000
$\$100{,}000 < X \le \$250{,}000$	1,750	$250,000,000
$\$250{,}000 < X \le \$500{,}000$	650	$225,000,000
$\$500{,}000 < X \le \$1{,}000{,}000$	95	$75,000,000
$X > \$1{,}000{,}000$	5	$70,000,000
Total	**5,000**	**$720,000,000**

→ The limited average severity at $100,000 is calculated by capping every claim at $100,000 & dividing by the total # of claims. For the 2500 claims w/ a size of loss less than $100,000, the losses are uncapped. The remaining 2,500 claims have losses greater than $100,000, so their losses are capped at $100,000. Thus, LAS($100,000) is equal to:

$$\rightarrow LAS(\$100{,}000) = \frac{\overset{\text{reported losses (uncapped)}}{100{,}000{,}000} + (1750+650+95+5)\times 100{,}000}{5000}$$

→ Similarly, the limited average severity at $250,000 can be calculated as follows:

$$LAS(\$250{,}000) = \frac{\$100{,}000{,}000 + \$250{,}000{,}000 + (650+95+5)\times 250{,}000}{5000} = \$107{,}500$$

→ The indicated limit factor (ILF) for $250,000 is:

$$\rightarrow ILF(\$250K) = \frac{LAS(\$250K)}{LAS(\$100K)} = \frac{\$107{,}500}{\$70{,}000} = 1.536$$

→ Using the same approach, we can calculate the ILF for $500,000:

$$\rightarrow LAS(\$500K) = \frac{\$100{,}000{,}000 + \$250{,}000{,}000 + \$225{,}000{,}000 + (95+5)\times\$500{,}000}{5000} = \$125{,}000$$

$$\rightarrow ILF(\$500K) = \frac{LAS(\$500K)}{LAS(\$100K)} = \frac{125{,}000}{70{,}000} = 1.786$$

→ And the indicated ILF for $1,000,000:

$$LAS(\$1M) = \frac{\$100{,}000{,}000 + \$250{,}000{,}000 + \$225{,}000{,}000 + \$75{,}000{,}000 + 5\times\$1{,}000{,}000}{5000} = \$131{,}000$$

$$ILF(\$1M) = \frac{LAS(\$1M)}{LAS(\$100K)} = \frac{\$131{,}000}{\$70{,}000} = 1.871$$

→ The calculation for the limited average severities is fairly straightforward, & can be summarized by the formula below:

$$\boxed{LAS(H) = \frac{(\text{total losses of claims} \le H) + (\#\text{ claims} > H)\times H}{\text{Total \# claims}}}$$

→ Censored losses

→ The losses in the example above are uncensored, meaning the data reflects the full loss amounts w/o applying policy limits. However, the data available to the actuary is typically censored at the policy limit, so the actuary does not know the complete loss amounts

→ For instance, take a policy w/ $100,000 limit & a claim cost of $200,000. The claims database may only record the insurer's payment, which is capped at $100,000. As a result, the actuary analyzing the data from the claims database would not be aware of the full loss amount.

→ To see how to calculate the ILFs given censored losses, let's consider the data below.

Size of Loss	$100,000 Limit		$250,000 Limit		$500,000 Limit	
	Claims	Losses	Claims	Losses	Claims	Losses
$X \le \$100{,}000$	2,181	$167,234,578	578	$29,584,913	730	$36,987,408
$\$100{,}000 < X \le \$250{,}000$			656	$121,508,473	582	$91,264,808
$\$250{,}000 < X \le \$500{,}000$					273	$95,558,703
$\$500{,}000 < X \le \$1{,}000{,}000$						
Total	2,181	$167,234,578	1,234	$151,093,386	1,585	$223,810,919

→ For LAS($100,000), the experience from all policies can be used since the lowest limit is $100,000. Losses that are less than $100,000 will not be censored, while losses greater than $100,000 will be capped at $100,000.

$$LAS(\$100K) = \frac{\$167{,}234{,}578 + \$29{,}584{,}913 + \$36{,}987{,}408 + (656+582+273)\times\$100{,}000}{5{,}000}$$

$$= \frac{\$384{,}906{,}899}{5{,}000}$$

$$= \$76{,}981.38$$

→ When calculating LAS($250,000), the policies w/ a limit of $100,000 cannot be used, since there is no way of knowing what the losses would be if these policies had a limit of $250,000. Instead, LAS($250,000) is calculated as the sum of LAS($100,000) & the limited average severity for the layer between $100K & $250K. The latter is known as the limited average severity at $150,000 excess of $100,000 or LAS($150,000 xs $100,000).

→ LAS($100,000) has already been calculated above. The limited average severity for the layer between $100,000 & $250,000 includes the following:

→ For the 656 claims from policies w/ a limit of $250,000 & a size of loss between $100,000 & $250,000, the first $100,000 has been included in LAS($100,000). Thus, these claims contribute a total of $121,508,473 − 656 × $100,000 = $55,908,473.

→ For the 582 claims from policies w/ a $500,000 limit & a size of loss between $100,000 & $250,000, the first $100,000 has been included in LAS($100,000). Thus, these claims contribute a total of $91,264,808 − 582 × $100,000 = $33,064,808.

→ For the 273 claims from policies w/ a $500,000 limit & a size of loss between $250,000 & $500,000, the first $100,000 of each claim has been included in LAS($100,000). We are only interested in the limited average severity for the layer between $100,000 & $250,000, so each claim will be capped at $250,000. Thus, these claims contribute a total of 273 × ($250,000 − $100,000) = $40,950,000

→ The total losses in the layer between $100,000 & $250,000 is the sum of the three, which is:

$$\$55{,}908{,}473 + \$33{,}064{,}808 + \$40{,}950{,}000 = \$129{,}923{,}281$$

→ To calculate the limited average severity, divide this by the total number of claims that contributed to the sum, which is 656 + 582 + 273 = 1,511

$$LAS(\$150{,}000 \text{ xs } \$100{,}000) = \frac{\$129{,}923{,}281}{1511} = \$85{,}984.96$$

→ We cannot just add $85,984.96 to LAS($100,000) b/c the amount is based on the claims w/ losses that exceed $100,000. Thus, we need to multiply it by the probability that losses exceed $100,000, which is

$$P(X \ge \$100K) = \frac{656+582+273}{1234+1585} = \frac{1511}{2819} = 0.536$$

→ Thus, the amount that we will add to LAS($100,000) to calculate LAS($250,000) is:

$$\$85{,}984.96 \times 0.536 \approx \$46{,}088.43$$

→ This is equivalent to dividing the losses in that layer by the total # of claims with a limit greater than $100,000:

$$\frac{\$129{,}923{,}281}{2{,}819} \approx \$46{,}088.43$$

→ With this, we can calculate the increase limit factor for $250,000 as follows:

$$ILF(\$250K) = \frac{LAS(\$250K)}{LAS(\$100K)} = \frac{\$123{,}069.81}{\$76{,}981.38} = 1.60$$

→ The same technique can be applied to calculate ILF($500K). Only losses from policies w/ a limit of $500,000 & above can be used to calculate the limited average severity for the layer between $250,000 & $500,000. The contribution from this layer of loss is equal to:

$$\frac{\$95{,}558{,}703 - 273 \times \$250{,}000}{1585} = 17{,}229.47$$

→ Adding this to LAS($250,000) gives LAS($500,000), which is used to determine ILF($500,000).

$$LAS(\$500K) = \$123{,}069.81 + \$17{,}229.47 = \$140{,}299.28$$

$$ILF(\$500K) = \frac{LAS(\$500K)}{LAS(\$100K)} = \frac{\$140{,}299.28}{\$76{,}981.38} = 1.82$$

→ To summarize the process above, for $H_2 > H_1$,

$$LAS(H_2) = LAS(H_1) + P(X>H_1) \times LAS[(H_2-H_1) \text{ xs } H_1]$$

where

$$\rightarrow P(X>H_1) = \frac{\#\text{ of claims} > H_1}{\#\text{ of policies w/ limits} > H_1} \quad \&$$

$$\rightarrow LAS[(H_2-H_1) \text{ xs } H_1] = \frac{\text{losses in layer between } H_1 \,\&\, H_2}{\#\text{ of claims} > H_1}$$

→ Noticing that the numerator of $P(X>H_1)$ equals the denominator of $LAS[(H_2-H_1) \text{ xs } H_1]$ allows us to take a shortcut:

$$LAS(H_2) = LAS(H_1) + \frac{\text{losses in the layer between } H_1 \,\&\, H_2}{\#\text{ of claims in policies w/ limits} > H_1}$$

→ Other Considerations

→ There are several factors to consider when using this approach to calculate increased limit factors:

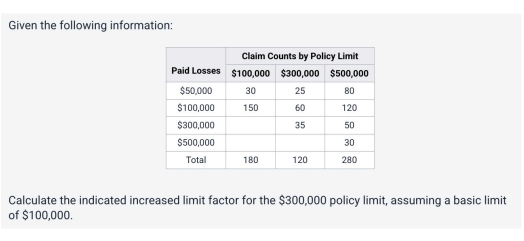
→ 1) As the increased limits ratemaking analysis outlined above are intended to produce rate relativities for future policies, historical losses used in calculating ILFs should be adjusted to reflect any anticipated trends. This adjustment is especially relevant for increased limit losses, as the magnitude of a possible trend is greater on increased limit losses than on basic limit losses.

→ 2) If the empirical data is recent, some claims may still be unsettled. Larger claims often develop differently from small ones, which can impact the calculation of increased limit factors. Ideally, all claims should reach ultimate development before applying these techniques.

→ 3) In the case w/ deductibles, losses are truncated from below. While it is possible to add back deductible amounts for known claims, it is impossible to determine how many claims were fully eliminated by the deductible.

→ Example

→



Given the following information:

Paid Losses	Claim Counts by Policy Limit		
	$100,000	$300,000	$500,000
$50,000	30	25	80
$100,000	150	60	120
$300,000		35	50
$500,000			30
Total	180	120	280

Calculate the indicated increased limit factor for the $300,000 policy limit, assuming a basic limit of $100,000.

→ First, calculate the limited average severity at $100,000.

→ We can use all the data because the lowest policy limit is $100,000

→ For claims w/ losses of $50,000, simply use $50,000, as it is below the $100,000 limit.

→ For all other claims, cap the loss at $100,000

→ Finally, divide by the total # of claims

$$LAS(\$100K) = \frac{(30+25+80)\times\$50{,}000 + (150+60+120+35+50+30)\times\$100{,}000}{180+120+280}$$

$$= \frac{\$51{,}250{,}000}{580}$$

$$= \$88{,}362$$

→ Next, calculate the contribution from the layer between $100,000 & $300,000.

→ For this layer, we can only use losses from policies w/ limits of $300,000 or greater.

→ Policies w/ a limit of $100,000 would never contribute to this layer.

→ The first $100,000 of each loss has already been accounted for in the LAS at $100,000, so claims w/ paid losses of $50,000 & $100,000 do not contribute to this layer

→ For claims w/ paid losses greater than $100,000, subtract $100,000 from each loss amount, & cap the remaining losses at $300,000.

→ The resulting total losses are $23,000,000. Divide this total by the total # of claims from policies w/ limits of $300,000 or $500,000, so that we don't need to account for the probability of claims exceeding $100,000

→ The LAS for this layer is:

$$\frac{35\times(\$300{,}000 - \$100{,}000) + (50+30)\times(\$300{,}000 - \$100{,}000)}{120+180}$$

$$= \frac{23{,}000{,}000}{400}$$

$$= \$57{,}500$$

→ Add that to LAS($100K) & calculate LAS($300K) as:

$$LAS(\$300K) = \$88{,}362 + \$57{,}500 = \$145{,}862$$

→ Finally, calculate the ILF for the $300,000 policy limit as:

$$ILF(\$300K) = \frac{LAS(\$300K)}{LAS(\$100K)} = \frac{\$145{,}862}{\$88{,}362} = 1.651$$

→ Fitted data approach

→ Due to typically limited data on large claims, the standard ILF approach may produce results that are volatile. The random fluctuations in empirical data can be smoothed out by fitting curves to the empirical data. Commonly used distributions are lognormal, Pareto, & truncated Pareto.

→ Let f(x) denote the continuous distribution of losses of size (x) & H be the limit being priced. The formula for LAS(H) is:

$$\boxed{LAS(H) = \int_0^H x f(x)\,dx + \int_H^\infty H \cdot f(x)\,dx} \longrightarrow \text{just the expected value of a function of a random variable}$$

$$\Rightarrow \text{let } f(x) = x \;\&\; \text{let } g(x) = \begin{cases} x & x < H \\ H & x \ge H \end{cases} \Rightarrow LAS(H) = E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

→ For example, the LAS at a limit of $10,000 can be calculated as follows:

$$LAS(\$10{,}000) = \int_0^{10000} x f(x)\,dx + 10{,}000 \int_{10{,}000}^{\infty} f(x)\,dx$$

→ The first term includes all losses below $10,000 at their actual loss amounts multiplied by the probability of those claims occurring, while the second term represents the limit, $10,000 multiplied by the probability that losses exceed the limit.

→ Thus, the increased limit factor for limit H is represented as follows:

$$\boxed{ILF(H) = \frac{\int_0^H x f(x)\,dx + \int_H^\infty H \cdot f(x)\,dx}{\int_0^B x f(x)\,dx + \int_B^\infty B f(x)\,dx}}$$

→ The main challenge w/ this approach is identifying a distribution that accurately represents the expected losses.

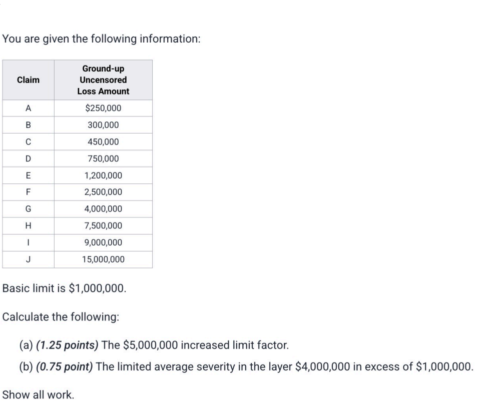
→ Multivariate Approach

→ Actuaries often analyze increased limit factors using a multivariate framework. Techniques such as GLMs are more effective at handling sparse data, though high, sparsely populated limits still pose challenges.

→ Unlike univariate methods that rely on limited average severities, GLMs do not assume a uniform frequency across risks. This allows GLMs to account for both loss capping & behavioral differences at different limits, which can occasionally yield counterintuitive results (e.g. expected losses decreasing as limits rise). Consequently, actuaries may use both methods to help determine appropriate increased limit factors.

→ Assignment

→ Q1)



You are given the following information:

Claim	Ground-up Uncensored Loss Amount
A	$250,000
B	300,000
C	450,000
D	750,000
E	1,200,000
F	2,500,000
G	4,000,000
H	7,500,000
I	9,000,000
J	15,000,000

Basic limit is $1,000,000.

Calculate the following:

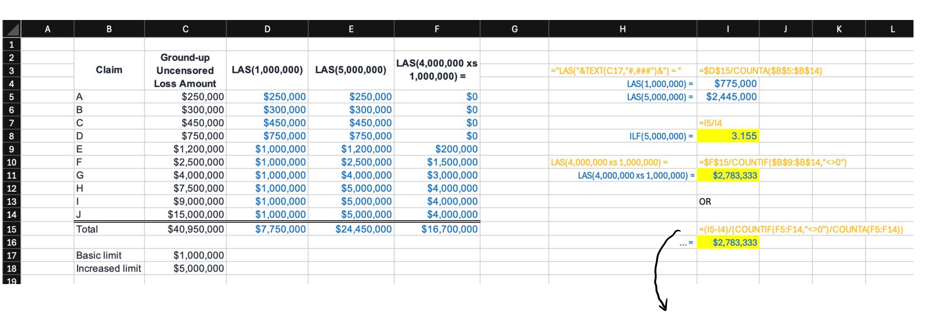
(a) *(1.25 points)* The $5,000,000 increased limit factor.

(b) *(0.75 point)* The limited average severity in the layer $4,000,000 in excess of $1,000,000.

Show all work.

→ a)

→ b)

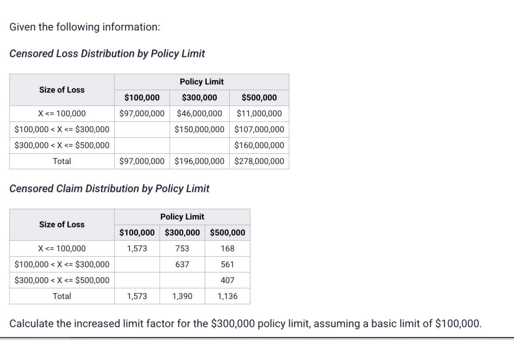


	A	B	C	D	E	F	G	H	I	J	K	L
1												
2												
3		Claim	Ground-up Uncensored Loss Amount	LAS(1,000,000)	LAS(5,000,000)	LAS(4,000,000 xs 1,000,000) =		="LAS("&TEXT(C17,"#,###")&") = "	=$D$15/COUNTA($B$5:$B$14)			
4								LAS(1,000,000) =	$775,000			
5		A	$250,000	$250,000	$250,000	$0		LAS(5,000,000) =	$2,445,000			
6		B	$300,000	$300,000	$300,000	$0						
7		C	$450,000	$450,000	$450,000	$0			=I5/I4			
8		D	$750,000	$750,000	$750,000	$0		ILF(5,000,000) =	3.155			
9		E	$1,200,000	$1,000,000	$1,200,000	$200,000						
10		F	$2,500,000	$1,000,000	$2,500,000	$1,500,000		LAS(4,000,000 xs 1,000,000) =	=$F$15/COUNTIF($B$9:$B$14,"<>0")			
11		G	$4,000,000	$1,000,000	$4,000,000	$3,000,000		LAS(4,000,000 xs 1,000,000) =	$2,783,333			
12		H	$7,500,000	$1,000,000	$5,000,000	$4,000,000						
13		I	$9,000,000	$1,000,000	$5,000,000	$4,000,000			OR			
14		J	$15,000,000	$1,000,000	$5,000,000	$4,000,000						
15		Total	$40,950,000	$7,750,000	$24,450,000	$16,700,000			=(I5-I4)/(COUNTIF(F5:F14,"<>0")/COUNTA(F5:F14))			
16								... =	$2,783,333			
17		Basic limit	$1,000,000									
18		Increased limit	$5,000,000									
19												

$$LAS(H_2) = LAS(H_1) + P(X>H_1) \times LAS[(H_2-H_1) \text{ xs } H_1]$$

$$\Rightarrow LAS[(H_2-H_1) \text{ xs } H_1] = \frac{LAS(H_2) - LAS(H_1)}{P(X>H_1)}$$

→ Q2)



Given the following information:

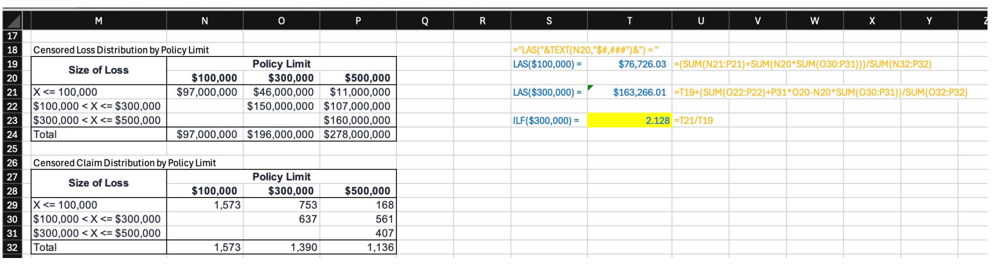
Censored Loss Distribution by Policy Limit

Size of Loss	Policy Limit		
	$100,000	$300,000	$500,000
X <= 100,000	$97,000,000	$46,000,000	$11,000,000
$100,000 < X <= $300,000		$150,000,000	$107,000,000
$300,000 < X <= $500,000			$160,000,000
Total	$97,000,000	$196,000,000	$278,000,000

Censored Claim Distribution by Policy Limit

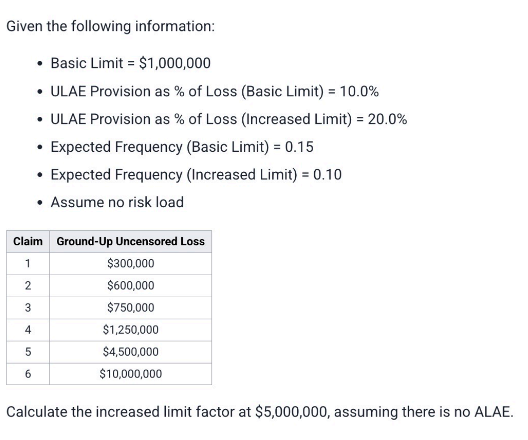
Size of Loss	Policy Limit		
	$100,000	$300,000	$500,000
X <= 100,000	1,573	753	168
$100,000 < X <= $300,000		637	561
$300,000 < X <= $500,000			407
Total	1,573	1,390	1,136

Calculate the increased limit factor for the $300,000 policy limit, assuming a basic limit of $100,000.



	M	N	O	P	Q	R	S	T	U	V	W	X	Y
17													
18	Censored Loss Distribution by Policy Limit						="LAS("&TEXT(N20,"$#,###")&") = "						
19	Size of Loss	Policy Limit					LAS($100,000) =	$76,726.03	=(SUM(N21:P21)+SUM(N20*SUM(O30:P31)))/SUM(N32:P32)				
20		$100,000	$300,000	$500,000									
21	X <= 100,000	$97,000,000	$46,000,000	$11,000,000			LAS($300,000) =	$163,266.01	=T19+(SUM(O22:P22)+P31*O20-N20*SUM(O30:P31))/SUM(O32:P32)				
22	$100,000 < X <= $300,000		$150,000,000	$107,000,000									
23	$300,000 < X <= $500,000			$160,000,000			ILF($300,000) =	2.128	=T21/T19				
24	Total	$97,000,000	$196,000,000	$278,000,000									
25													
26	Censored Claim Distribution by Policy Limit												
27	Size of Loss	Policy Limit											
28		$100,000	$300,000	$500,000									
29	X <= 100,000	1,573	753	168									
30	$100,000 < X <= $300,000		637	561									
31	$300,000 < X <= $500,000			407									
32	Total	1,573	1,390	1,136									

→ Q3)

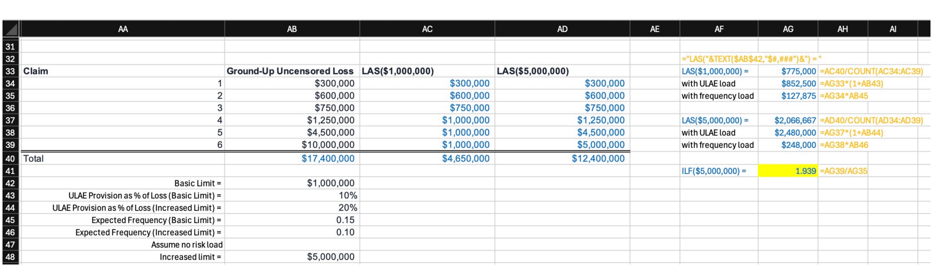


Given the following information:

- Basic Limit = $1,000,000
- ULAE Provision as % of Loss (Basic Limit) = 10.0%
- ULAE Provision as % of Loss (Increased Limit) = 20.0%
- Expected Frequency (Basic Limit) = 0.15
- Expected Frequency (Increased Limit) = 0.10
- Assume no risk load

Claim	Ground-Up Uncensored Loss
1	$300,000
2	$600,000
3	$750,000
4	$1,250,000
5	$4,500,000
6	$10,000,000

Calculate the increased limit factor at $5,000,000, assuming there is no ALAE.



	AA	AB	AC	AD	AE	AF	AG	AH	AI
31									
32						="LAS("&TEXT($AB$42,"$#,###")&") = "			
33	Claim	Ground-Up Uncensored Loss	LAS($1,000,000)	LAS($5,000,000)		LAS($1,000,000) =	$775,000	=AC40/COUNT(AC34:AC39)	
34	1	$300,000	$300,000	$300,000		with ULAE load	$852,500	=AG33*(1+AB43)	
35	2	$600,000	$600,000	$600,000		with frequency load	$127,875	=AG34*AB45	
36	3	$750,000	$750,000	$750,000					
37	4	$1,250,000	$1,000,000	$1,250,000		LAS($5,000,000) =	$2,066,667	=AD40/COUNT(AD34:AD39)	
38	5	$4,500,000	$1,000,000	$4,500,000		with ULAE load	$2,480,000	=AG37*(1+AB44)	
39	6	$10,000,000	$1,000,000	$5,000,000		with frequency load	$248,000	=AG38*AB46	
40	Total	$17,400,000	$4,650,000	$12,400,000					
41						ILF($5,000,000) =	1.939	=AG39/AG35	
42	Basic Limit =	$1,000,000							
43	ULAE Provision as % of Loss (Basic Limit) =	10%							
44	ULAE Provision as % of Loss (Increased Limit) =	20%							
45	Expected Frequency (Basic Limit) =	0.15							
46	Expected Frequency (Increased Limit) =	0.10							
47	Assume no risk load								
48	Increased limit =	$5,000,000							