2.3.2 → Developing Premiums

→ Overview → when performing a ratemaking analysis, the ultimate amount of premium for the experience period may not be known. In that case, premiums will need to be developed to their ultimate value before being used in an analysis. Two common cases where premiums typically need to be developed are:

→ 1) when an incomplete year of data is used

→ 2) for lines of business that utilize premium audits

→ Incomplete data

→ Recall that data used for ratemaking should balance both responsiveness + stability. So, for a PY analysis, where it takes longer for the PY to close, it may be beneficial to use a recent, but incomplete year of data

→ While the initial premium amounts will be known for all policies once they are written, those premium amounts may change up until the policies expire due to changes in or cancellations of the policies. So, if the premium data for active policies is used, that data should be developed to ultimate. This is commonly done by studying historical patterns of premium development, specifically to estimate the effect of mid-term changes or cancellations on PY premium.

→ Premium audits

→ For lines of business that have premium audits, the insured will typically be charged an estimated premium at the beginning of the policy period based on their estimated exposure. Then, once their actual exposure is known at the end of the period, their premium will be adjusted.

→ For example, premium for workers compensation will be estimated based on payroll. A payroll audit will usually be done about 3 to 6 months after the policy expires to determine the final premium amount.

→ For ratemaking, premiums will often be developed by studying historical patterns. These patterns can depend on several factors, such as:

→ Types of plans available from the insurer or allowed by the jurisdiction

→ Internal company policies, e.g. auditing procedures, marketing strategy, or accounting policy

→ The stability of the historical relationship between the estimated premium + the final audited premium

→ If CY data is used, premium development generally won't be necessary, as the data is final at the end of the CY. If audit patterns are changing though, an actuary may still choose to adjust CY premiums.

→ On the other hand, AY + PY data may still be developing at the end of the year. So, premium development factors are needed in that case to adjust premiums to their ultimate values after audits are performed.

→ Consider the following example to see how premiums can be developed to ultimate.

→ Example →

You are given the following information:

- Initial estimates of policy year premium are $2 million per month for the first 6 months of 2021 and $2.3 million per month for the last 6 months of 2021.
- The final audit occurs three months after policy expiration.
- On average, audits result in 10% additional premium.
- All policies are annual and written uniformly over the year.

Calculate the policy year 2021 premium development factor from

1. 12 to 24 months.
2. 24 to 36 months.

→ 1) → Policies are annual + are written uniformly between 1/1/2021 + 12/31/2021. So they also expire uniformly between 1/1/2022 + 12/31/2022. Final audits occur 3 months after expiration, which means audits will occur uniformly between 4/1/2022 + 3/31/2023.

→ At 12 months (as of 12/31/2021), no audits will have occurred. So we will have WP of

$$\underbrace{2{,}000{,}000}_{\text{premium/month}}\ \underbrace{(6)}_{6\text{ months}} + 2{,}300{,}000\ (6) = 25{,}800{,}000$$

→ At 24 months (as of 12/31/2022), audits will have been completed on policies issued during the first 9 months of the year. Policies written during those 9 months will have their premium increased by 10%. Thus, we will have WP of

$$[2{,}000{,}000\ (6) + 2{,}300{,}000\ (3)]\ 1.1 + 2{,}300{,}000\ (3) = 27{,}690{,}000$$

⇒ Therefore, the 12 to 24 month premium development factor is

$$\frac{27{,}690{,}000}{25{,}800{,}000} = 1.0733$$

→ 2) → At 36 months (as of 12/31/2023), all audits will have been completed, which means all policies will have had their premium increased by 10%. So, WP at 36 months is

$$(2{,}000{,}000\ (6) + 2{,}300{,}000\ (6))\ (1.1) = 28{,}380{,}000$$

⇒ the 24 to 36 month premium development factor is

$$\frac{28{,}380{,}000}{27{,}690{,}000} = 1.0249$$

→ Assignment

→ Q1)

You are given the following information:

- Initial estimates of policy year premium are $3 million per month for the first 6 months of 2020 and $3.5 million per month for the last 6 months of 2020.
- The final audit occurs six months after policy expiration.
- On average, audits result in 30% additional premium.
- All policies are annual and written uniformly over the year.

Calculate the policy year 2020 premium development factor from

(a) 12 to 24 months.
(b) 24 to 36 months.

a) → @ 12 months (12/31/2020) → No audits have been done

⇒ WP = 3,000,000 (6) + 3,500,000 (6) = 39,000,000

→ @ 24 months (12/31/2021) → (need 18 months to do an audit)

⇒ audits done on only 6 months of policies

⇒ WP = [3,000,000(6)] 1.3 + 3,500,000 (6) = 44,400,000

⇒ 12 to 24 month development factor $= \frac{44{,}400{,}000}{39{,}000{,}000} = 1.13846$

b) → @ 36 months → all audits have been done

⇒ WP = [3,000,000 (6) + 3,500,000 (6)] 1.3 = 50,700,000

⇒ 24 to 36 month development factor $= \frac{50{,}700{,}000}{44{,}400{,}000} = 1.14189$

2.3.3 → Developing Losses

→ Overview → As previously mentioned, there are several techniques that can be used to develop losses to their ultimate values. In this subsection, we'll introduce one approach that utilizes development triangles. Additional approaches will be covered in Section 6.

→ Chain ladder method

→ The chain ladder method, also called the development technique, works by assuming historical patterns predict future development patterns. While this method also applies to a variety of claims data, data should be homogeneous. Homogeneity increases the predictive value of the data, + thus increases its credibility.

→ To see how this method works, lets revisit a prior example.

→ Example → ABC insurance has a line of business which covers automobile damage. B/c automobile damage claims settle quickly, losses are assumed to be fully paid after 48 months

→ On 1/1/23, ABC insurance asks us to estimate ultimate losses on this LOB.

→ Step 1 → Compile claims data

Here is the table of claims data we compiled previously.

Accident Year	Paid Claims by Age ($US Millions)			
	12 Months	24 Months	36 Months	48 Months
2019	102	135	161	160
2020	100	121	152	
2021	108	150		
2022	110			

→ While we are given that losses are fully paid after 48 months for this example, one consideration that should typically go into this step is how many development periods need to be studied. If possible, an actuary should analyze development until it ceases (i.e. until the values remain the same from one age to the next). The number of development periods needed can vary by LOB, jurisdiction, + data type (paid claims vs reported claims)

→ For instance, short-tailed LOBs, such as automobile physical damage + homeowners insurance, typically have claims that are settled relatively quickly. In that case, the ultimate amount of claims will be known fairly quickly. However, for long-tailed lines, such as commercial general liability, workers compensation, or personal umbrella, it may be years before the ultimate amount is known.

→ Another consideration is what level of granularity to use for the development triangles. The analysis could be done by LOB, coverages, or types of losses. The analysis could also be done on a national, regional, or state level + at monthly, quarterly, or annual intervals. When choosing the level of granularity, the focus should be on balancing the homogeneity + credibility of the data.

→ In general, when estimating ultimate claims, it is important that an actuary has a thorough understanding of the LOB being analyzed.

→ Step 2 → Calculate age-to-age factors

→ Development methods typically assume that losses develop in a multiplicative way from one age to the next, instead of an additive way. So, after aggregating data, the next step is to find the ratios of paid claim amounts at successive maturities within each AY. These ratios are called link ratios or age-to-age development factors.

Using the paid claims data we've found, the age-to-age development factors are bolded below:

Accident Year	Age-to-Age Development Factors		
	12-24 Months	24-36 Months	36-48 Months
2019	135/102 = **1.324**	161/135 = **1.193**	160/161 = **0.994**
2020	121/100 = **1.210**	152/121 = **1.256**	
2021	150/108 = **1.389**		

→ Using this table of values, how should we develop the paid claim amounts for AY 2020, AY 2021, + AY 2022?

→ For AY 2020, we already know the paid claim amount after 36 months of development. We only have one 36-48 months development factor, which equals 0.994. So, to develop AY 2020 paid claims to 48 months (when it will be fully paid), we can multiply the value at 36 months of development by the 36-48 month development factor. Thus, the estimated ultimate paid claims amount for AY 2020 is:

152 * 0.994 = 151.056

→ NOTE → Keep in mind that cumulative paid claims will eventually include all losses paid on closed claims. So, by developing paid claims, we estimate the ultimate losses.

→ However for AY 2021, we have two different 24-36 month factors. So, to develop these losses to 36 months, we could choose 1.193 or 1.256, or some combination of both

→ Similarly, for developing AY 2022 losses, we also have 3 different 12-24 month factors. So, how do we choose which factors to use?

→ Step 3 → Calculate averages

→ In practice, the most common approaches are to first find the arithmetic average, the medial average, the geometric average, + the volume-weighted average of the age-to-age factors.

→ Average → finds the arithmetic average of all development factors for a given age

→ Medial average → Like the arithmetic average, except the highest + lowest values are excluded (for less than 3 values, simply take the arithmetic average).

→ Geometric average → finds the geometric average by taking the n^{th} root of the product of n factors

→ Volume weighted average → Also called the ratio of total losses, finds the sum of the cumulative losses in a given development year divided by the sum of the corresponding cumulative losses from the previous development year. The numerator + denominator must have the same # of entries. For example, the development factor for 12-24 month is:

$$\frac{135 + 121 + 150}{102 + 100 + 108} = 1.310$$

Here is a summary of our results:

Accident Year	Age-to-Age Development Factors		
	12-24 Months	24-36 Months	36-48 Months
2019	1.324	1.193	0.994
2020	1.210	1.256	
2021	1.389		
Arithmetic Average	1.307	1.224	0.994
Medial Average	1.324	1.224	0.994
Geometric Average	1.305	1.224	0.994
Volume-Weighted Average	1.310	1.223	0.994

→ Step 4 → Select claim development factors

→ After calculating averages of the age-to-age factors, we need to determine which factors to use. There isn't necessarily a correct choice, but it is important to think critically when selecting factors + be able to provide reasoning for the selection made. Here are some key points to keep in mind when reviewing the claim development experience:

→ Smooth progression → Ideally, there should be a smooth decrease across development years.

→ Stability → within each column, a low variance in development factors signifies stability.

→ Credibility of experience → volume + homogeneity determine credibility. If credibility is lacking, then industry benchmark factors may need to be used instead as a last resort

→ Change in patterns → Reviewing age-to-age factors can help identify patterns that reflect internal + external changes

→ Applicability of the historical period → Actuaries should use qualitative information to determine how applicable historical development will reflect future claim development

→ Shock losses / Cat losses → Sometimes we might observe a single very large claim that distorts the development factors for a given year/age

→ Generally, development factors should be selected based on what is expected to happen in the future. Consider a few examples:

→ Stability within column → If the factors are relatively consistent across the periods being studied, the all-year arithmetic averages may be appropriate. If the pattern seems to be changing over time, it may be preferable to rely on more recent years using a two or three year average

→ Volatility in the business → A LOB may be too volatile to rely only on the past two or three years. In that case, a weighted average can be used, with more weight given to recent years

→ Varying development factors → If development factors vary widely between AYs or if there is an anomaly in certain AYs, the medial average can be used to remove the highest + lowest

→ Step 5 → Select tail factor

→ If historical data triangle doesn't reach ultimate, then a tail factor can be used. The tail factor accounts for any development after the most recent period included in the triangle. Two possible approaches for selecting a tail factor are:

→ Fitting curves to historical patterns + extrapolating the development beyond the historical data patterns

→ Performing special studies that include more years of data

→ For our example, we assume that losses are fully paid after 48 months. Since we have data at 48 months, we won't need a tail factor in this case. If we assumed that claims were still developing after 48 months instead, we would need to select a tail factor that accounts for development between 48 months + whenever claims reach ultimate levels.

→ Step 6 → Calculate cumulative claim development factors (CDF)

→ After selecting development tail factors, the next step is to calculate cumulative claim development factors, also called age-to-ultimate development factors, or claim development factors to ultimate, for each maturity. Each CDF projects the remaining growth through maturity.

→ The CDF for a given maturity is found by multiplying the development factor for that maturity by the development factors for all subsequent maturities + by any tail factor

→ For instance, assume we selected the arithmetic average factors for the example above. The 36 month CDF is just

CDF at 36 months = 0.994

→ The 24-month CDF is the product of the 24-36 month + 36-48 month development factors

CDF at 24 months = (1.224) (0.994) = 1.217

→ The 12-month CDF is the product of all three development factors.

CDF at 12 months = (1.307) (1.224) (0.994) = 1.591

→ Step 7 → Project ultimate claims

→ Once the age-to-ultimate factors are calculated, we can estimate ultimate claims by multiplying the cumulative claims paid to date by the appropriate age-to-ultimate factor. In the development triangle, the latest diagonal gives us the current cumulative paid claims. Therefore, we can calculate the estimated claims as

Accident Year	Cumulative Paid Claims	CDF	Ultimate Claims
2019	160	1.000	160 (1.000) = 160.00
2020	152	0.994	152 (0.994) = 151.06
2021	150	1.217	150 (1.217) = 182.52
2022	110	1.591	110 (1.591) = 175.00

$$\text{Total Ultimate Claims} = 160.00 + 151.06 + 182.52 + 175.00 = \mathbf{668.58}$$

→ Summary → Chain ladder method steps

1) Compile claims data
2) Calculate age-to-age factors
3) Calculate averages
4) Select claim development factors
5) Select tail factor
6) Calculate cumulative claim development factors
7) Project ultimate claims

→ Example →

You are given the following accident year data as of December 31, 2021:

Accident Year	Incremental Reported Claims by Age ($US Millions)				
	12 Months	24 Months	36 Months	48 Months	60 Months
2017	231	82	51	32	14
2018	218	77	46	30	
2019	257	85	54		
2020	290	96			
2021	310				

- Accident year 60-to-ultimate development factor = 1.02.
- Age-to-age development factors are selected using a volume-weighted average.

Estimate the ultimate reported claims for accident year 2021 using the chain ladder method.

→

Accident Year	Incremental Reported Claims by Age ($US Millions)				
	12 Months	24 Months	36 Months	48 Months	60 Months
2017	231	82	51	32	14
2018	218	77	46	30	
2019	257	85	54		
2020	290	96			
2021	310				

Accident Year	Cumulative Reported Claims by Age ($US Millions)				
	12 Months	24 Months	36 Months	48 Months	60 Months
2017	231	313	364	396	410
2018	218	295	341	371	
2019	257	342	396		
2020	290	386			

Accident Year	Age-to-age factors (months)				
	12-24	24-36	36-48	48-60	
2017	1.355	1.163	1.088	1.035	
2018	1.353	1.156	1.088		
2019	1.331	1.158			(actually not needed cause volume-weighted method)
2020	1.331				

Age-to-age	Volume-weighted factor	
12-24	1.341	=SUM(D13:D16)/SUM(C13:C16)
24-36	1.159	
36-48	1.088	
48-60	1.035	
60-Ult (tail)	1.02	(given)

Age-Ult	Factor	
12-Ult	1.786	=PRODUCT(C19:C23)
24-Ult	1.332	
36-Ult	1.149	
48-Ult	1.056	
60-Ult	1.020	

Months	AY 2021 reported claims	Projected ultimate reported claims	
12 months	310	553.7	=C40*C33
24 months	96	127.8	
36 months	54	62.0	
48 months	30	31.7	
60 months	14	14.3	
Total	504	789.5	=SUM(D40:D44)

→ Assignment

→ Q1 →

Given the following data, calculate the projected ultimate accident year 2011 losses.

Accident Year	As of December 31, 2012	
	Paid Losses	Case Reserves
2009	$16,000	$3,000
2010	$12,000	$4,000
2011	$10,000	$7,000
2012	$5,000	$8,000

- Projected ultimate accident year 2010 losses = $18,000
- 12-24 reported link ratio = 1.20
- 24-36 reported link ratio = 1.15

→

Accident Year	As of December 31, 2012 Paid Losses	Case Reserves	Incurred losses	
2009	$16,000	$3,000	$19,000	=SUM(J51:K51)
2010	$12,000	$4,000	$16,000	
2011	$10,000	$7,000	$17,000	
2012	$5,000	$8,000	$13,000	
			-> This is the latest diagonal because of 'As of 12/12/2012'	

Age-to-age	Factor (link ratio)	
12-24	1.20	(given)
24-36	1.15	(given)
36-48	X	

< using age-to-ult factors (CDFs) >

Estimated ultimate losses AY 2010 @ 36 months (as of 12/12/2012)
$18,000 =L52*PRODUCT(J59)
(given) (solve for X ==> 1.125

AY 2011 incurred losses $17,000 (given)
Estimated ultimate losses AY 2011 @ 24 months (as of 12/12/2012) $21,993.75 =L53*PRODUCT(J58:J59)

→ Q2 →

You are given the following information:

Earned Exposures	Accident Year	Cumulative Paid Claims ($000s) Age of Development (Months)				
		12	24	36	48	60
50,000	2010	500	550	660	660	660
50,000	2011	465	545	645	645	
50,000	2012	500	540	675		
50,000	2013	500	545			
50,000	2014	510				

Assume claims are fully developed after 60 months.

(a) Describe an unusual pattern in the data and a potential cause for this pattern.

(b) Using the paid claim development technique and considering the unusual pattern identified in part (a), estimate the unpaid claims as of December 31, 2014.

Show all work and explain how you address the unusual data pattern.

→ a) In AY 2011, the paid claims at 12 months appear lower compared to other AYs. The pattern seems unusual b/c the # of exposures did not change. Since claims appear to follow a similar pattern to other AYs at 24 months, we can reason that this pattern is attributable to a slowdown in claims settlement during the first 12 months, after which claims for that period were processed + caught up with.

→ b)

Earned Exposures	Accident Year	Cumulative Paid Claims ($000s) Age of Development (Months)				
		12	24	36	48	60
50,000	2010	500	550	660	660	660
50,000	2011	465	545	645	645	
50,000	2012	500	540	675		
50,000	2013	500	545			
50,000	2014	510				

Earned Exposures	Accident Year	Cumulative Paid Claims ($000s) Age to Age (Months)			
		12-24	24-36	36-48	48-60
50,000	2010	1.100	1.200	1.000	1.000
50,000	2011	1.172	1.183	1.000	
50,000	2012	1.080	1.250		
50,000	2013	1.090			

Age-to-age	Medial average factor
12-24	1.095
24-36	1.200
36-48	1.000
48-60	1.000

Age-to-ult	Factor	
12-24	1.314	=PRODUCT(R92:R95)
24-36	1.200	
36-48	1.000	
48-60	1.000	

Months	AY 2014 paid claims	Estimated ult paid claims		
12 months	510	670.1	=S81*R98	
24 months	545	654.0		
36 months	675	675.0		
48 months	645	645.0		
60 months	660	660.0		
Total	3035	3304.1	==> Unpaid:	269.14 =S109-R109

→ Q3 →

Given the following data evaluated as of December 31, 2015:

Accident Year	Company Data: Cumulative Paid Claims ($000) as of (months)			
	12	24	36	48
2012	850	950	1,950	2,450
2013	700	2,200	3,300	
2014	900	1,600		
2015	1,000			

Accident Year	Industry Data: Paid Claims Age-to-Age Factors		
	12	24	36
2012	1.97	1.24	1.10
2013	2.03	1.27	
2014	2.03		

- There is no development after 48 months.

(a) Calculate the company's accident year 2015 ultimate claims using the paid claim development technique and the company's historical paid claim activity.

(b) Calculate the company's accident year 2015 ultimate claims using the paid claim development technique and the industry claim development factors.

(c) Recommend and briefly justify an estimate of the company's accident year 2015 ultimate claims.

→ a)

Accident Year	Company Data: Cumulative Paid Claims ($000) as of (months)			
	12	24	36	48
2012	850	950	1,950	2,450
2013	700	2,200	3,300	
2014	900	1,600		
2015	1,000			

Accident Year	Age-age factors (months)		
	12-24	24-36	36-48
2012	1.118	2.053	1.256
2013	3.143	1.500	
2014	1.778		

Age-to-age	Factor (simple average method)	
12-24	2.013	=AVERAGE(Z126:Z128) --> simple average is good for stability
24-36	1.776	--> Another option is to just use the most recent year's factors (i.e. the latest diagonal)
36-48	1.256	

Age-to-ult	CDF	
12-ult	4.492	=PRODUCT(Z131:Z133)
24-ult	2.232	
36-ult	1.256	

Months	AY 2014 Paid Claims	Estimated ult paid claims	
12	1000	4492.0	=Z141*Z136
24	1600	3570.9	
36	3300	4146.2	
48	2450	2450.0	
Total	8350	14659.0	

→ b)

Accident Year	Industry Data: Paid Claims Age-to-Age Factors		
	12-24	24-36	36-48
2012	1.97	1.24	1.1
2013	2.03	1.27	
2014	2.03		

Age-to-age	Factor (simple average method)	
12-24	2.010	=AVERAGE(Z153:Z155)
24-36	1.255	
36-48	1.100	

Age-to-ult	CDF	
12-ult	2.775	=PRODUCT(Z158:Z160)
24-ult	1.381	
36-ult	1.100	

Months	AY 2014 Paid Claims	Estimated ult paid claims	
12	1000	2774.8	=Z168*Z163
24	1600	2208.8	
36	3300	3630.0	
48	2450	2450.0	
Total	8350	11063.6	

→ c)

Valid sample answers would include the following:

- I would recommend using $2,774,805 based on industry data. The company data is too volatile to be given any credibility. The large fluctuations in LDFs could be due to large losses and thus don't give good indications of what is to come.
- I would use an industry CDF estimate of $2,774,805 because the company age-to-age factors (ATAs) are very volatile, and at early maturity (@ 12 months), you are subject to a highly leveraged CDF.
- Since the ATA factors for company data are really volatile and seem biased by large losses, I would recommend using the estimate in part (b) only based on the payment pattern of the industry, which is more stable.
- Recommend average of these two estimates because it takes the company development patterns into consideration but supplements with industry data to help smooth out potential noise in company development.

⇒ Note on selecting factors

You can select anything that seems reasonable, whether that's an average, a particular age-to-age factor, or anything else. Use your actuarial judgement when selecting factors. And remember, you don't have to make the same type of selection at each maturity either. If the age-to-age factors look unstable at 12 months but fairly stable for other maturities, it might be reasonable to select the medial average at 12 months and the straight average for other maturities.

On the exam in general, there are no strict rules for what factor to select as long as you make a reasonable choice. The default selection would be the straight average, but if there's a reason to not do that, make a different selection and explain your reasoning for doing so.