6.6.3 → Disposal rate method

→ Assumptions + uses

→ The final frequency-severity technique we will explore is often referred to as the **disposal rate method**. The disposal rate for a particular AY & development year is determined by dividing the cumulative closed claim counts by the ultimate closed claim counts. In other words, it is the percentage of ultimate claims that have been closed.

$$\text{AY Disposal Rate}_t = \frac{\text{AY Cumulative Closed Claim Counts}_t}{\text{AY Ultimate Closed Claim Counts}}$$

where subscript t represents the maturity (12 months, 24 months, etc.)

→ Like the other FS techniques we've discussed, the disposal rate method also utilizes the development technique. Therefore, the same three key assumptions apply. In addition, the disposal rate method also makes a fourth assumption:

- →1) Claim counts + severities recorded to date will continue to develop in a similar fashion in the future.
- →2) Claim counts have a consistent definition throughout the experience period
- →3) The mix of claim types is reasonably homogeneous
- →4) There are no significant partial claim payments made.

→ To clarify, the fourth assumption essentially means that the disposal rate method assumes that exactly one payment is made on each claim. So, although the reported amount for an individual claim may develop, it is not paid until it is fully developed. Once it is fully developed, a single payment is made for the full amount of the claim, + the claim closes.

→ Technique

→ The disposal rate method can be broken into seven steps:

- →1) Project + select ultimate claim counts by AY
- →2) Create disposal rate triangle + select disposal rates by maturity
- →3) Project claim counts using the selected disposal rates
- →4) Analyze + select incremental severities by maturity
- →5) Calculate incremental severities by maturity age + AY
- →6) Project unpaid claims by multiplying claim counts by severities
- →7) Compute ultimate claim / IBNR estimate (if needed)

→ We'll show this technique in action using an example.

→ Examples

→ You are given:

Accident Year	Cumulative Closed Claim Counts				
	12 Months	24 Months	36 Months	48 Months	Ultimate
2020	225	345	445	500	500
2021	230	350	450		520
2022	230	365			532
2023	240				555

Accident Year	Incremental Paid Claims			
	0-12 Months	12-24 Months	24-36 Months	36-48 Months
2,020	225,000	480,000	600,000	550,000
2021	232,300	483,600	595,000	
2022	236,900	546,750		
2023	254,400			

↳ solving for these #s

- Severity trend = 2%

Assume no partial claim payments and no development past 48 months.

Create an incremental paid severities triangle. Then, estimate AY2023 unpaid claims using the disposal rate method.

→ a) Creating an incremental paid severities triangle

→ The incremental paid severities are calculated by dividing the incremental paid claims by the incremental closed claim counts. Given incremental paid claims, we can first determine the incremental closed claim counts based on the cumulative closed claim counts provided, + then calculate the incremental paid severities.

Accident Year	Incremental Closed Claim Counts			
	0-12 Months	12-24 Months	24-36 Months	36-48 Months
2020	225	120	100	55
2021	230	120	100	
2022	230	135		
2023	240			

e.g.) $\frac{55{,}000}{55} = 10{,}000$

Accident Year	Incremental Paid Severities = Incremental Paid Claims / Incremental Closed Claim Counts			
	0-12 Months	12-24 Months	24-36 Months	36-48 Months
2020	1,000	4,000	6,000	10,000
2021	1,010	4,030	5,950	
2022	1,030	4,050		
2023	1,060			

→ b) Estimating unpaid claims using the disposal rate method

→ Step 1: Project + select ultimate claim counts

→ Since the ultimate claim counts are provided to us, we can skip this step. This is fairly typical on the exam. If ultimate counts were not given, we would most likely also be given reported claim counts that could be used to estimate ultimate claim counts.

→ Step 2: Create disposal rate triangle + select disposal rates

→ To create the disposal rate triangle, divide each cell of the cumulative closed claim counts triangle by its respective ultimate loss.

Accident Year	Historical Disposal Rates			
	12 Months	24 Months	36 Months	48 Months
2020	$225/500 = 0.450$	$345/500 = 0.690$	$445/500 = 0.890$	$500/500 = 1.000$
2021	$230/520 = 0.442$	$350/520 = 0.673$	$450/520 = 0.865$	
2022	$230/532 = 0.432$	$365/532 = 0.686$		
2023	$240/555 = 0.432$			

→ The disposal rates appear to be fairly stable, so select an average disposal rate for each development period.

Selected Disposal Rates			
12 Months	24 Months	36 Months	48 Months
0.439	0.683	0.878	1.000

→ Note that attempting to perform the development method on the historical disposal rates would be a mistake. The historical disposal rates are already a reflection of historical development patterns. In general, don't develop disposal rates.

→ Step 3: Project claim counts using selected disposal rates

→ W/ the selected disposal rates in hand, we can use them to project closed claim counts for future periods. We also want to preserve the ultimate estimates we used to calculate the disposal rates, meaning the projected claim counts for a given AY at month 48 must equal the ultimate claim counts we are given for that year.

→ To preserve the estimated ultimate claim counts, we can estimate the closed claim counts between two maturities w/ linear interpolation. Let t_1 + t_2 represent two different maturities where $t_1 < t_2$. Also, "DR" is used to abbreviate "Disposal Rate" in the formulas below. For a given AY:

→ Method 1

$$\text{Closed Claim Counts Between } t_1 + t_2 = \frac{\text{open claim counts}}{1 - \text{Latest Selected DR}} \cdot (\text{Selected } DR_{t_2} - \text{Selected } DR_{t_1})$$

where open claim counts = Ultimate claim counts − Closed claim counts

→ Let's see how this formula works in action. AY2023 is currently at 12 months. It is estimated to have 555 − 240 = 315 open claim counts, + the selected disposal rate at 12 months is 0.439. Thus, the claims count closed

→ between 12 + 24 months is $\frac{315}{1-0.439}(0.683-0.439) = 137$,

→ between 24 + 36 months is $\frac{315}{1-0.439}(0.878-0.683) = 109$,

→ between 36 + 48 months is $\frac{315}{1-0.439}(1-0.878) = 69$

} = round (0 decimals, 0)

→ Logic: ult claims − closed claims = open claims
555 − 240 = 315

⇒ Based on selected DR, 240 is 43.9% of the ult claims

⇒ So there is (1 − DR) = 1 − 0.439 = 0.561, 56.1% of the remaining claims to close over the following development periods, BUT this must = 315

⇒ 315 = (1 − 0.439) × "adjusted" ultimate

⇒ $\frac{315}{(1-0.439)}$ = "adjusted ultimate"
↳ accounting for variation in selected DR

⇒ Then we divvy out the "adjusted ultimate" incrementally

→ Method 2

→ In words: The open claims are allocated proportionally to the incremental disposal rates for the remaining maturity periods.

→ For example, 315 = open claim count (same as before), + there are 3 remaining maturity periods w/ incremental disposal rates as follows:

- → 12−24 months: 0.683 − 0.439 = 0.244 = A
- → 24−36 months: 0.878 − 0.683 = 0.195 = B
- → 36−48 months: 1 − 0.878 = 0.122 = C

→ Then the corresponding proportions of those 315 open claims that we allocate are:

- → 12−24 months: $\frac{A}{A+B+C}$ = ... = 43.48%
- → 24−36 months: $\frac{B}{A+B+C}$ = ... = 34.71%
- → 36−48 months: $\frac{C}{A+B+C}$ = ... = 21.81%

$$\text{open claims} * \frac{\Delta\, DR_{\text{remaining}}}{1 - \text{Selected DR}}$$
↳ for each t_1, t_2

→ And so the projected claim counts are:

- → 12−24 months: 315 × 43.48% = 137
- → 24−36 months: 315 × 34.71% = 109
- → 36−48 months: 315 × 21.81% = 69

→ We can verify if the projected incremental claim counts plus the latest claim counts sum to the estimated ultimate claim counts:

AY 2023 ultimate closed claim counts = 240 + 137 + 109 + 69 = 555 ✓

→ Our projections agree w/ the given estimated ultimate, which is what we were hoping to see. If they did not match, that would indicate that we most likely made a mistake in projecting the claim counts.

→ Step 4: Analyze + select incremental severities

→ Our eventual goal is to project AY 2023 unpaid claims. So we'll need to trend the historical incremental severities to AY2023 level using the given severity trend. We can skip over the first column of the development triangle since AY 2023 has already developed for 12 months.

Accident Year	Incremental Severity Trended to AY2023		
	12-24 Months	24-36 Months	36-48 Months
2020	$4{,}000(1.02)^3 = 4{,}244.83$	$6{,}000(1.02)^3 = 6{,}367.24$	$10{,}000(1.02)^3 = 10{,}612.08$
2021	$4{,}030(1.02)^2 = 4{,}192.81$	$5{,}950(1.02)^2 = 6{,}190.38$	
2022	$4{,}050(1.02) = 4{,}131$		

* This just has to be trended to the same year, doesn't matter which year for when we are making a selection
→ just pick latest year so don't have to do that anything
⇒ Then we get same answer at end b/c we then re-trend to whatever year we are estimating for (if needed)

⇒ We trend w/ ( )^3 each time b/c we are trending to incremental severities for each maturity to AY 2023, not CY 2023. So the trend/inflation levels should also correspond to the AY, not the CY.

→ Note that the 12 to 24-month severity for AY 2023 would correspond to claims that occur during 2023 but are made from 1/1/2024 through 12/31/2024.

→ Think about it this way. AY 2020 claims paid between 12 + 24 months would be at CY 2021 levels of inflation. That means they are at the same trend/inflation level as AY 2021 claims paid between 0 + 12 months. However, we don't want to compare claim payments from two different maturity periods to each other. What we want is to compare claims from the same maturity period over three different AYs. So get AY 2020 claims paid between 12 + 24 months at CY 2023 level of inflation to the same trend/inflation level as AY 2023 claims paid between 12 + 24 months (which would be at CY 2024 level of inflation), we need to trend them one year.

Maybe think about the corresponding CYs to the AYs when trending, + that is what we want to be equivalent

→ Notice that trended incremental severities appear to be decreasing year-over-year. As such, we will forgo taking an average of historical severities + simply select the most recent severities (ie. severities on the rightmost diagonal).

Selected Incremental Severities Trended to AY2023		
12-24 Months	24-36 Months	36-48 Months
4,131	6,190.38	10,612.08

→ Step 5: Calculate incremental severities by maturity age + AY

→ For this step, we would trend the selected incremental severities for each maturity to each AY we need to estimate unpaid claims for. In this case, we are only interested in projecting unpaid claims for a single AY, so we can skip over this step.

→ Step 6: Project unpaid claims

→ For each maturity, multiply the selected incremental closed claim counts by the selected incremental severity to get the projected incremental paid claims. Then, sum over all maturities to get the AY 2023 unpaid claims estimate.

AY 2023 unpaid claims = 137 (4,131) + 109 (6,190.38) + 69 (10,612.08) = 1,871,741.39

→ You are given the following information:

Accident Year	Claim Counts Reported				
	0-12 Months	12-24 Months	24-36 Months	36-48 Months	Ultimate
1998	30	40	30	0	100
1999	33	44	33		110
2000	34	46			115
2001	36				120

Accident Year	Open Claim Counts			
	12 Months	24 Months	36 Months	48 Months
1998	10	20	20	0
1999	12	24	24	
2000	10	21		
2001	11			

Accident Year	Incremental Average Closed Claim Severity			
	0-12 Months	12-24 Months	24-36 Months	36-48 Months
1998	1,500	3,000	4,500	7,500
1999	1,575	3,015	4,725	
2000	1,653	3,308		
2001	1,736			

} → trend 2 years
} → trend 1 year
↳ @ CY 2001 levels already
just going to use the historical data, + only projecting the 24−36 + 36−48 maturities

Assume an annual inflation of 5.0%

Estimate the ultimate loss for accident year 2000 using the disposal rate method.

→ Frequency

→ To create the disposal rate triangle, we need cumulative closed claim counts, which we can obtain by subtracting open claim counts at each period from the cumulative reported counts for that AY.

Accident Year	12 Months	24 Months	36 Months	48 Months
1998	30 − 10 = 20	70 − 20 = 50	100 − 20 = 80	100 − 0 = 100
1999	33 − 12 = 21	77 − 24 = 53	110 − 24 = 86	–
2000	34 − 10 = 24	80 − 21 = 59	–	–
2001	36 − 11 = 25	–	–	–

→ Divide each cell by the provided ultimate claim counts to get the disposal rates.

Accident Year	12 Months	24 Months	36 Months	48 Months
1998	20 / 100 = 0.200	50 / 100 = 0.500	80 / 100 = 0.800	100 / 100 = 1.000
1999	21 / 110 = 0.191	53 / 110 = 0.482	86 / 110 = 0.782	–
2000	24 / 115 = 0.209	59 / 115 = 0.513	–	–
2001	25 / 120 = 0.208	–	–	–

→ Disposal rates seem to be fairly consistent year-over-year, so select the simple averages.

Maturity	Selected Disposal Rate
12 Months	(0.200 + 0.191 + 0.209 + 0.208) / 4 = **0.202**
24 Months	(0.500 + 0.482 + 0.513) / 3 = **0.498**
36 Months	(0.800 + 0.782) / 2 = **0.791**
48 Months	**1.000** (Fully settled by then)

→ Now, apply the linear interpolation to project incremental closed claim counts for the remaining maturities for AY 2000.

AY 2000 closed claim counts between 24 + 36 months = $\frac{115-59}{1-0.498}(0.791-0.498) = 33$

AY 2000 closed claim counts between 36 + 48 months = $\frac{115-59}{1-0.498}(1-0.791) = 23$

→ Notice that the projected closed counts plus the historical closed count add to the given AY 2000 ultimate counts.

59 + 33 + 23 = 115

→ Severity

→ Next, trend the given incremental severities to AY 2000 level using the given 5% inflation trend. Since we are only missing incremental claim severities for 24−36 months + for 36−48 months, we can skip over the first two columns of the table.

Accident Year	Incremental Average Closed Claim Severity Trended to AY2000	
	24-36 Months	36-48 Months
1998	$4{,}500(1+0.05)^2 = 4{,}961.25$	$7{,}500(1+0.05)^2 = 8{,}268.75$
1999	$4{,}725(1+0.05) = 4{,}961.25$	
Selected	4,961.25	8,268.75

→ Aggregate

→ Multiply the incremental severity by the corresponding incremental closed counts to get the total incremental claims for that maturity. Then add them up to estimate the ultimate claims for AY 2000.

→ Historical (AY 2000):

→ 0−12: 24 × 1,653 = 39,672

→ 12−24: 35 × 3,308 = 115,780

→ Projected:

→ 24−36: 33 × 4,961.25 = 163,721.25

→ 36−48: 23 × 8,268.75 = 190,181.25

→ Total:

AY 2000 ult closed claims = 39,672 + 115,780 + 163,721.25 + 190,181.25 = 510,472.71

→ Note that we should multiply the historical severities by the historical counts.

→ Notes → For those who may be wondering, b/c we were not given a separate trend for claim severity, the inflation trend of 5% per year implies that the severity trend is also 5%. If inflation is 5% per year, then claim costs are increasing at an average 5% per year. Severity is a component of claim costs, so we can assume severity is trending at 5% per year.

→ Assignment

→ Q4

You are given the following information about a particular line of business:

Accident Year	Incremental Loss and ALAE Payments on Closed Claims ($000s)			
	Age of Development in Months			
	12	24	36	48
2004	400.0	300.0	180.0	75.0
2005	504.0	378.0	226.8	
2006	662.0	496.0		
2007	833.0			

Accident Year	Incremental Number of Claims Closed				
	Age of Development in Months				Ultimate Counts
	12	24	36	48	
2004	500	300	150	50	1,000
2005	600	360	180		1,200
2006	750	450			1,500
2007	900				1,800

The annual severity trend is 5.0%.

Use the frequency-severity disposal rate technique to calculate the AY2007 unpaid claims as of December 31, 2007.

→