3.1.4 → Univariate classification w/ credibility

→ overview → Univariate classification is a cornerstone of traditional insurance ratemaking, used to determine rate differentials by analyzing historical experience for each level of a rating variable independently. Building on the previous section, we introduce how credibility adjustments into these univariate techniques.

→ Credibility is vital in enhancing the reliability of the estimates derived from univariate analyses. By blending class-specific data w/ broader, more stable datasets, we can mitigate the impact of random variations & ensure more accurate rate indications. While the detailed principles & calculations of credibility will be discussed in a later section, this section focuses on its practical application w/in the context of univariate classification.

→ we will revisit the motivating example, add a credibility requirement to it, & demonstrate how to adjust the three primary univariate methods to incorporate credibility.

→ Example →

A classification analysis of homeowners insurance consists of only two rating variables: amount of insurance (AOI) and territory. You are given the following information:

- The exposure, premium, and loss data is summarized as follows:

Amount of Insurance (AOI)	Territory	Exposures	Loss & ALAE	Premium at Current Rate Level
Low	1	10	$303.55	$567.60
Medium	1	110	$4,586.67	$7,804.50
High	1	180	$10,807.86	$17,368.56
Low	2	130	$6,416.59	$12,298.00
Medium	2	120	$8,136.00	$14,190.00
High	2	140	$13,668.48	$22,514.80
Low	3	150	$9,165.85	$18,447.00
Medium	3	120	$10,072.37	$18,447.00
High	3	40	$4,834.74	$8,362.64
TOTAL		1,000	$67,992.11	$120,000.10

- The true underlying loss cost relativities and the current relativities used in the company's rating structure are given in the tables below.

AOI	True Relativity	Charged Relativity
Low	0.728	0.800
Medium	1.000	1.000
High	1.440	1.360

Territory	True Relativity	Charged Relativity
1	0.615	0.600
2	1.000	1.000
3	1.238	1.300

- All underwriting expenses are variable. The variable expense provision (V) is 25% of premium, and the target profit percentage (Q_T) is 5% of premium.
- Territory 2 and Medium AOI are the base levels.
- The credibility for each level is calculated as $\sqrt{\frac{n}{500}}$, limited at 1, where n is the number of exposures.
- The complement of credibility is no change.

→ for comparison purposes, lets continue to derive the Territory indicated relativities. We will start by calculating the credibility as instructed.

Territory	Exposures	Credibility
1	300	$\min\left(1, \sqrt{\frac{300}{500}}\right) = 0.7746$
2	390	$\min\left(1, \sqrt{\frac{390}{500}}\right) = 0.8832$
3	310	$\min\left(1, \sqrt{\frac{310}{500}}\right) = 0.7874$

→ In general, the credibility estimate is the credibility-weighted average between the subject experience & a credibility complement. This can be expressed as:

$$\boxed{\text{Credibility Estimate} = \text{Credibility} * \text{Experience} + (1 - \text{Credibility}) * \text{complement}}$$

→ In the context of univariate classification, the subject experience is represented by the indicated relativities, & the credibility complements are typically the current relativities.

→ Pure Premium approach

→ when taking the average between two sets of relativities, its crucial that both sets have the same basis. Thus, the first step is to re-base the relativities to be relative to the total. To do this, we calculate the average of the relativities, weighted by the exposures.

$$\frac{300(0.60) + 390(1.00) + 310(1.30)}{1000} = 0.9730$$

→ we then divide the current relativity for each level by this average to re-base them to the total, as shown in column (4).

→ The indicated relativities in column (5) are taken from the previous section. Note that we should use the relativities before they are released to the base level.

→ The credibility factors in column (6) were calculated earlier.

→ With these elements, we can now calculate the credibility-weighted indicated relativities, as shown in column (7). Finally, we rebase the relativities to territory 2 in column (8).

(1) Territory	(2) Exposures	(3) Current Relativity	(4) Normalized Current Relativity	(5) Indicated Relativity to Total	(6) Credibility	(7) Cred-Wtd Indicated Relativity	(8) Cred-Wtd Indicated Relativity to Base
1	300	0.600	0.6166	0.7696	0.7746	0.7351	**0.6935**
2	390	1.000	1.0277	1.0643	0.8832	1.0600	**1.0000**
3	310	1.300	1.3361	1.1421	0.7874	1.1833	**1.1164**
Total	1,000	0.9730	1.0000	1.0000			

- (3)total = [(3) × (2)] ÷ (2)total
- (4) = (3) ÷ (3)total
- (7) = (6) × (5) + [1 - (6)] × (4)
- (8) = (7) ÷ (7)base

→ Adjusted Pure Premium approach

→ Let's now proceed to the adjusted pure premium approaches. The calculations are identical to the pure premium approach, except we will weight the current relativities by the adjusted exposures, which we calculated in the previous section

(1) Territory	(2) Adjusted Exposures	(3) Current Relativity	(4) Normalized Current Relativity	(5) Indicated Relativity	(6) Credibility	(7) Cred-Wtd Indicated Relativity	(8) Cred-Wtd Indicated Relativity to Base
1	362.8	0.600	0.6336	0.6820	0.7746	0.6710	**0.6264**
2	414.4	1.000	1.0560	1.0733	0.8832	1.0713	**1.0000**
3	294.4	1.300	1.3728	1.2887	0.7874	1.3066	**1.2197**
Total	1,071.6	0.9470	1.0000	1.0000			

- (3)total = [(3) × (2)] ÷ (2)total
- (4) = (3) ÷ (3)total
- (7) = (6) × (5) + [1 - (6)] × (4)
- (8) = (7) ÷ (7)base

→ Loss ratio approach

→ Recall that in the LR approach, the indicated relativities (before being re-based to the base level) are calculated by multiplying the current relativities by the indicated change factors. As a result, this set of indicated relativities is already relative to the current relativity for the base level. In other words, the two sets of relativities are already on the same basis.

→ In the table below, column (3) lists the indicated relativities (before re-basing) from the previous section. The credibility factors in column (4) were calculated earlier. We then calculate the credibility-weighted average between the current & indicated relativities in column (5). Finally, we rebase the relativities to Territory 2 in column (6).

(1) Territory	(2) Current Relativity	(3) Indicated Relativity	(4) Credibility	(5) Cred-Wtd Indicated Relativity	(6) Cred-Wtd Indicated Relativity to Base
1	0.600	0.6458	0.7746	0.6355	**0.6264**
2	1.000	1.0164	0.8832	1.0145	**1.0000**
3	1.300	1.2204	0.7874	1.2373	**1.2197**

- (5) = (4) × (3) + [1 - (4)] × (2)
- (6) = (5) ÷ (5)base

→ An equivalent approach is to calculate the credibility-weighted average between the indicated change factors & 1 (i.e. no change), & then apply the current relativities as summarized in the table below.

(1) Territory	(2) Current Relativity	(3) Indicated Change Factor	(4) Credibility	(5) Cred-Wtd Indicated Change Factor	(6) Cred-Wtd Indicated Relativity	(7) Cred-Wtd Indicated Relativity to Base
1	0.600	1.0763	0.7746	1.0591	0.6355	**0.6264**
2	1.000	1.0164	0.8832	1.0145	1.0145	**1.0000**
3	1.300	0.9388	0.7874	0.9518	1.2373	**1.2197**

- (5) = (4) × (3) + [1 - (4)] × 1.0000
- (6) = (5) × (2)
- (7) = (6) ÷ (6)base

→ Unsurprisingly, the loss ratio method & the adjusted pure premium method yield the same relativities.

→ Note → the second approach (credibility-weighting the change factors) is typically more convenient as it allows us to bypass calculating the indicated relativities.

→ The shortcuts above are also acceptable when the complement is no change, which is often the case in exam questions. If other complements are used, we would need to re-base both sets of relativities to be relative to the total, like in the pure premium approaches. Furthermore, we often lack the exposures data when employing the LR approach. Instead, we can weight the relativities by the premium at base level, which is the premium divided by the current relativities.

→ If we were to rebase the relativities in the LR approach, here is what it would look like:

(1) Territory	(2) Premium at Current Rate Level	(3) Premium at CRL at Base	(4) Current Relativity	(5) Normalized Current Relativity	(6) Indicated Relativity	(7) Normalized Indicated Relativity	(8) Credibility	(9) Cred-Wtd Indicated Relativity	(10) Cred-Wt Indicate Relativit to Base
1	$25,740.66	$42,901.10	0.600	0.6336	0.6458	0.6820	0.7746	0.6710	**0.6264**
2	$49,002.80	$49,002.80	1.000	1.0560	1.0164	1.0733	0.8832	1.0713	**1.0000**
3	$45,256.64	$34,812.80	1.300	1.3728	1.2204	1.2887	0.7874	1.3066	**1.2197**
Total	$120,000.10	$126,716.70	0.9470	1.0000	0.9470	1.0000			

- (3) = (2) ÷ (4)
- (4)total = [(4) × (3)] ÷ (3)total
- (5) = (4) ÷ (4)total
- (6)total = [(6) × (3)] ÷ (3)total
- (7) = (6) ÷ (6)total
- (9) = (8) × (7) + [1 - (8)] × (5)
- (10) = (9) ÷ (9)base

→ Note → notice that we get the same average relativities using the adjusted exposures:

$$\frac{362.8(0.60) + 414.4(1.00) + 294.4(1.30)}{1071.6} = 0.9470$$

or the premium at base level:

$$\frac{\$42,901.10(0.60) + \$49,002.80(1.00) + \$34,812.80(1.300)}{\$126,716.70} = 0.9470$$

as weights. Keep this in mind, as it will come in handy in section 4.2.

→ Examples →

Given the following information:

	As of January 1, 2011	As of July 1, 2011
Base Rate	$200	$250
Good Driver Discount Factor	0.85	0.75
Territory 1 Factor	1.00	1.00
Territory 2 Factor	1.10	1.10

Exposures	Good Driver Discount	
	Yes	No
Territory 1	750	250
Territory 2	600	150

Loss and ALAE	Good Driver Discount	
	Yes	No
Territory 1	$90,000	$40,000
Territory 2	$80,000	$20,000

use these to calculate Premium @ CRL

- The rating algorithm is base rate × good driver discount factor × territory factor.
- Territory 1 and No Good Driver Discount remain the base classification.
- The credibility for each level is calculated as $\frac{n}{n+500}$, where n is the number of exposures.

Using the loss ratio method and the following credibility complements, calculate the credibility-weighted indicated territorial relativities.

(a) The complement of credibility is no change.

(b) A competitor's rating relativities are used as the credibility complement.

Territory 1	1.00
Territory 2	1.20

→ a) → Since the complement is no change, we can either weight the indicated change factors or the indicated relativities. We will demonstrate the former approach.

→ start by calculating premiums, losses, loss ratios & the indicated change factors for each territory:

Territory	Premium at Current Rate Level	Loss & ALAE	Loss & ALAE Ratio	Indicated Change Factor
1	$203,125	$130,000	64.00%	1.0243
2	$165,000	$100,000	60.61%	0.9700

→ Then, calculate the credibility for each territory:

$$\text{Credibility}_{T1} = \frac{750 + 250}{(750 + 250) + 500} = 0.667$$

$$\text{Credibility}_{T2} = \frac{600 + 150}{(600 + 150) + 500} = 0.600$$

→ Next, calculate the credibility-weighted averages of the indicated change factors, & then the indicated relativities. Finally, re-base the indicated relativities so that Territory 1 has a final relativity of 1.

Territory	Credibility-Weighted Indicated Change Factor	Credibility-Weighted Indicated Relativity	Re-based Indicated Relativity
1	$0.6667(1.0243) + (1 - 0.6667)(1) = 1.0162$	$1.0162(1.00) = 1.0162$	$\frac{1.0162}{1.0162} = \mathbf{1.0000}$
2	$0.6000(0.9700) + (1 - 0.6000)(1) = 0.9820$	$0.9820(1.10) = 1.0802$	$\frac{1.0802}{1.0162} = \mathbf{1.0630}$

→ b) → Since the complement is something else, we must rebase both sets of relativities. We can calculate the indicated relativities using the specified change factors & current territory factors.

Territory	Competitor's Relativities	Indicated Relativities
1	1.00	$1.0243(1.00) = 1.0243$
2	1.20	$0.9700(1.10) = 1.0670$

→ To rebase both sets of relativities, we need the average relativities. Begin by calculating the premiums at base level.

→ Territory 1 → $\frac{\$203,125}{1.00} = \$203,125$

→ Territory 2 → $\frac{\$165,000}{1.10} = \$150,000$

→ use these premiums as weights to calculate the average competitor relativity & the average indicated relativity.

→ The average competitor relativity is

$$\frac{\$203,125(1.00) + \$150,000(1.20)}{\$203,125 + \$150,000}$$

→ The average indicated relativity is

$$\frac{\$203,125(1.0243) + \$150,000(1.067)}{\$203,125 + \$150,000} = 1.0425$$

→ Divide by the average relativities to obtain the relativities to the total:

Territory	Competitor's Relativities	Indicated Relativities	Competitor's Relativities to Total	Indicated Relativities to Total
1	1.00	1.0243	$\frac{1.00}{1.0850} = 0.9217$	$\frac{1.0243}{1.0425} = 0.9826$
2	1.20	1.0670	$\frac{1.20}{1.0850} = 1.1060$	$\frac{1.0670}{1.0425} = 1.0236$
Total	1.0850	1.0425		

→ Now, we are ready to calculate the credibility-weighted relativities. Multiply the credibility by the indicated relativity & '1 minus the credibility' by the competitor's relativity. Finally, divide by the credibility-weighted indicated relativity for Territory 1, which serves as the base class.

Territory	Credibility-Weighted Indicated Relativity	Re-based Indicated Relativity
1	$0.6667(0.9826) + (1 - 0.6667)(0.9217) = 0.9623$	$\frac{0.9623}{0.9623} = \mathbf{1.0000}$
2	$0.6000(1.0236) + (1 - 0.6000)(1.1060) = 1.0565$	$\frac{1.0565}{0.9623} = \mathbf{1.0979}$

→ Note → Equivalently, we could also calculate the average relativities using the adjusted exposures as weights.

Good Driver Discount	Charged Good Driver Discount Factor	Territory	
		1	2
Yes	0.75	750	600
No	1.00	250	150

Territory	Weighted Average Good Driver Discount Factor	Exposures	Adjusted Exposures
1	$\frac{750(0.75) + 250(1.00)}{750 + 250} = 0.8125$	$750 + 250 = 1{,}000$	$1{,}000(0.8125) = 812.5$
2	$\frac{600(0.75) + 150(1.00)}{600 + 150} = 0.8000$	$600 + 150 = 750$	$750(0.8000) = 600$

→ The average competitor's relativity is

$$\frac{812.50(1.00) + 600(1.20)}{812.50 + 600} = 1.0850$$

& the average indicated relativity is

$$\frac{812.50(1.0243) + 600(1.0670)}{812.50 + 600} = 1.0425$$

→ Given the following information:

Territory	Historical Earned Exposures	Current Territorial Relativity	Average Relativity for Other Factors*	Reported Losses	Reported Claim Count
1	4,000	0.60	1.30	$420,000	600
2	16,000	1.00	1.05	$1,250,000	1,320
3	3,750	0.52	1.20	$360,000	390

*Weighted-average rate relativity for all factors except territory.

- Territory 2 will remain the base territory.
- Full credibility standard is 1,082 claims.
- Complement of credibility is no change.

Calculate the credibility-weighted indicated territorial relativities.

→ Since we're provided the exposure & loss data, we will use one of the pure premium methods. Specifically, since the average relativities are given, we are expected to use the adjusted pure premium method.

→ First, calculate the indicated relativities relative to the total

→ Compute the adjusted exposures for each territory by multiplying the earned exposures by the average relativity for the other factors. This accounts for differences within the territories due to other factors.

→ Divide the reported losses by the adjusted exposures to obtain the adjusted pure premium for each territory, as well as for the total of all territories

→ Compute the indicated relativity for each territory by dividing the territory's indicated adjusted pure premium by the total indicated adjusted pure premium

Territory	Adjusted Exposures	Indicated Adjusted Pure Premium	Indicated Relativity
1	$4{,}000(1.30) = 5{,}200$	$\frac{\$420{,}000}{5{,}200} = \$80.77$	$\frac{\$80.77}{\$76.60} = 1.0544$
2	$16{,}000(1.05) = 16{,}800$	$\frac{\$1{,}250{,}000}{16{,}800} = \$74.40$	$\frac{\$74.40}{\$76.60} = 0.9713$
3	$3{,}750(1.20) = 4{,}500$	$\frac{\$360{,}000}{4{,}500} = \$80.77$	$\frac{\$80.00}{\$76.60} = 1.0443$
Total	26,500	$\frac{\$2{,}030{,}000}{26{,}500} = \$76.60$	1.0000

→ Next, rebase the current relativities so they are relative to the total. The average current relativity is:

$$\frac{5200(0.60) + 16,800(1.00) + 4,500(0.52)}{26,500} = 0.84$$

→ Divide the current relativities by this average to obtain the normalized current relativity, which now has a weighted average of 1 when weighted by the adjusted exposures.

Territory	Current Relativity	Normalized Current Relativity
1	0.60	$\frac{0.60}{0.84} = 0.7143$
2	1.00	$\frac{1.00}{0.84} = 1.1905$
3	0.52	$\frac{0.52}{0.84} = 0.6190$
Total	0.84	1.0000

→ Now that both sets of relativities are relative to the total, we can calculate the credibility-weighted averages. We need the credibility factors for each territory.

→ Territory 1: $\sqrt{\frac{600}{1082}} = 0.7447$

→ Territory 2: $\sqrt{\frac{1320}{1082}} > 1$ (capped at 1)

→ Territory 3: $\sqrt{\frac{390}{1082}} = 0.6004$

→ We can now calculate the credibility-weighted relativities & rebase them so that territory 2 has a relativity of 1.

→ To obtain the credibility-weighted indicated relativity, add the product of the credibility & the indicated relativity to the product of '1 minus the credibility' & the normalized current relativity.

→ Finally, divide by the credibility-weighted indicated relativity for Territory 2, the base territory. This gives the re-based indicated relativity by territory.

Territory	Cred-Wtd Indicated Relativity	Re-based Indicated Relativity
1	$0.7447(1.0544) + (1 - 0.7447)(0.7143) = 0.9675$	$\frac{0.9675}{0.9713} = \mathbf{0.9961}$
2	$1.0000(0.9713) + (1 - 1.0000)(1.1905) = 0.9713$	$\frac{0.9713}{0.9713} = \mathbf{1.0000}$
3	$0.6004(1.0443) + (1 - 0.6004)(0.6190) = 0.8744$	$\frac{0.8744}{0.9713} = \mathbf{0.9002}$

→ Assignment

→ Q1)

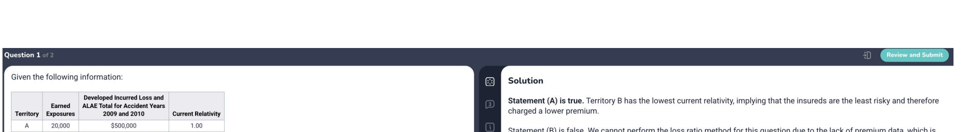
(3 points) You are given the following information:

Territory	Premium	Incurred Losses & ALAE	Claim Count	Current Relativity
1	$520,000	$420,000	600	0.60
2	$1,680,000	$1,250,000	1,320	1.00
3	$450,000	$360,000	390	0.52
Total	$2,650,000	$2,030,000	2,310	

- Full credibility standard is 1,082 claims and partial credibility is calculated using the square root rule.
- The complement of credibility is no change.

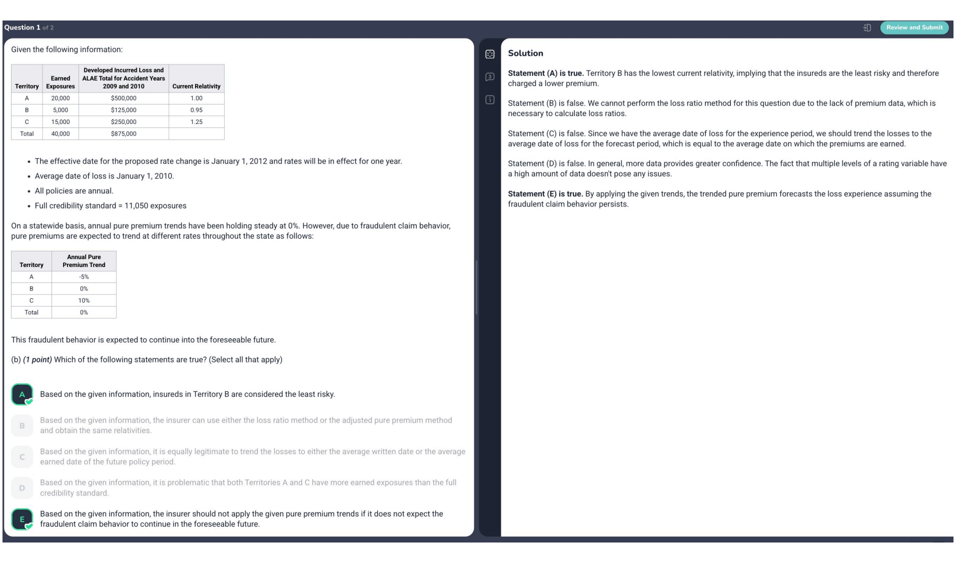
Calculate the indicated territorial relativities using this most recent experience. Assume that Territory 2 remains the base territory.

→



	B	C	D	E	F	G	H	I	J	K	L
1											
2						=D4/C4	=G4/$G$7	=MIN(1,SQRT(E4/$C$9))	=I4*H4+(1-I4)*$C$10	=F4*J4	=K4/$K$5
3	Territory	Premium	Incurred losses & ALAE	Claim Count	Current Relativity	Loss ratio	Indicated change factor	Credibility	Credibility-weighted indicated change factor	Credibility-weighted indicated relativity	Re-based
4	1	$520,000	$420,000	600	0.6	80.8%	1.054	0.745	1.040	0.624	0.643
5	2	$1,680,000	$1,250,000	1,320	1	74.4%	0.971	1.000	0.971	0.971	1.000
6	3	$450,000	$360,000	390	0.52	80.0%	1.044	0.600	1.027	0.534	0.550
7	Total	$2,650,000	$2,030,000	2,310		76.6%	1.000				
8											
9	Credibility standard	1,082									
10	Complement of credibility	1.00									

→ Q2)



Given the following information:

Territory	Earned Exposures	Developed Incurred Loss and ALAE Total for Accident Years 2009 and 2010	Current Relativity
A	20,000	$500,000	1.00
B	5,000	$125,000	0.95
C	15,000	$250,000	1.25
Total	40,000	$875,000	

- The effective date for the proposed rate change is January 1, 2012 and rates will be in effect for one year.
- Average date of loss is January 1, 2010.
- All policies are annual.
- Full credibility standard = 11,050 exposures

On a statewide basis, annual pure premium trends have been holding steady at 0%. However, due to fraudulent claim behavior, pure premiums are expected to trend at different rates throughout the state as follows:

Territory	Annual Pure Premium Trend
A	-5%
B	0%
C	10%
Total	0%

This fraudulent behavior is expected to continue into the foreseeable future.

(b) *(1 point)* Which of the following statements are true? (Select all that apply)

☑ A Based on the given information, insureds in Territory B are considered the least risky.

☐ B Based on the given information, the insurer can use either the loss ratio method or the adjusted pure premium method and obtain the same relativities.

☐ C Based on the given information, it is equally legitimate to trend the losses to either the average written date or the average earned date of the future policy period.

☐ D Based on the given information, it is problematic that both Territories A and C have more earned exposures than the full credibility standard.

☑ E Based on the given information, the insurer should not apply the given pure premium trends if it does not expect the fraudulent claim behavior to continue in the foreseeable future.

Solution

Statement (A) is true. Territory B has the lowest current relativity, implying that the insureds are the least risky and therefore charged a lower premium.

Statement (B) is false. We cannot perform the loss ratio method for this question due to the lack of premium data, which is necessary to calculate loss ratios.

Statement (C) is false. Since we have the average date of loss for the experience period, we should trend the losses to the average date of loss for the forecast period, which is equal to the average date on which the premiums are earned.

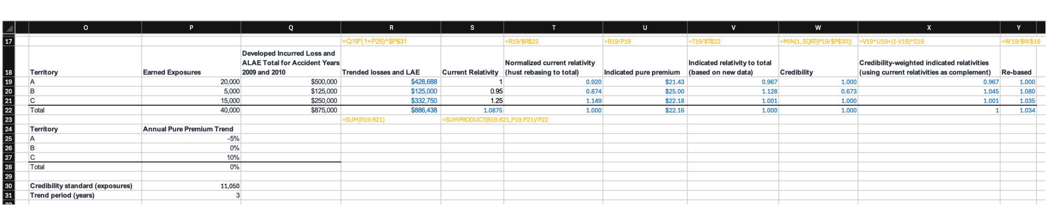
Statement (D) is false. In general, more data provides greater confidence. The fact that multiple levels of a rating variable have a high amount of data doesn't pose any issues.

Statement (E) is true. By applying the given trends, the trended pure premium forecasts the loss experience assuming the fraudulent claim behavior persists.

→ Q3)

(using setup above)

(a) *(2.75 points)* Assuming Territory A is the base territory, calculate the credibility-weighted indicated relativities to the base territory.



	O	P	Q	R	S	T	U	V	W	X	Y
17				=Q19*(1+P25)^$P$31		=R19/$R$22	=R19/P19	=T19/$T$22	=MIN(1,SQRT(P19/$P$30))	=V19*U19+(1-V19)*S19	=W19/$W$19
18	Territory	Earned Exposures	Developed Incurred Loss and ALAE Total for Accident Years 2009 and 2010	Trended losses and LAE	Current Relativity	Normalized current relativity (hust rebasing to total)	Indicated pure premium	Indicated relativity to total (based on new data)	Credibility	Credibility-weighted indicated relativities (using current relativities as complement)	Re-based
19	A	20,000	$500,000	$428,688	1	0.920	$21.43	0.967	1.000	0.967	1.000
20	B	5,000	$125,000	$125,000	0.95	0.874	$25.00	1.128	0.673	1.045	1.080
21	C	15,000	$250,000	$332,750	1.25	1.149	$22.18	1.001	1.000	1.001	1.035
22	Total	40,000	$875,000	$886,438	1.0875	1.000	$22.16	1.000	1.000	1	1.034
23				=SUM(R19:R21)	=SUMPRODUCT(R19:R21,P19:P21)/P22						
24	Territory	Annual Pure Premium Trend									
25	A	-5%									
26	B	0%									
27	C	10%									
28	Total	0%									
29											
30	Credibility standard (exposures)	11,050									
31	Trend period (years)	3									