6.4.1 → Bornhuetter-Ferguson Method

→ Overview → The development method multiplies actual claims by a cumulative development factor, but it can result in unreliable projections if development patterns are changing. On the other hand, the expected claims (EC) technique provides more stability but at the expense of ignoring historical results.

→ The Bornhuetter-Ferguson (BF) method is a blend of these methods. It balances these approaches by splitting the ultimate claims into actual reported (or paid claims) & the expected reported claims (or unpaid claims), giving more weight to actual claims over time. While it can be used for the number of claims & w/ ALAE, the BF approach is most often applied to reported claims + paid claims.

→ We project ultimate claims using either of the following:

$$\text{Ultimate claims} = \text{Actual reported claims} + \underbrace{\text{Expected claims} \times \%\text{ unreported}}_{\text{Expected unreported claims}}$$

$$\text{Ultimate claims} = \text{Actual paid claims} + \underbrace{\text{Expected claims} \times \%\text{ Unpaid}}_{\text{Expected unpaid claims}}$$

where

$$\%\text{ unreported} = 1 - \frac{1}{\text{Reported Age-to-Ult Factor}}$$

$$\%\text{ unpaid} = 1 - \frac{1}{\text{paid Age-to-Ult Factor}}$$

→ To be clear, the expected claims is the ultimate claims estimate w/ the EC method, & the factors are calculated using the development method. Since actual reported claims & actual paid claims are known, the focus of the technique is on estimating expected unreported claims or expected unpaid claims.

→ If we estimate ultimate claims w/ the first of the above equations, it is called the reported Bornhuetter-Ferguson method.
→ — — — — — — — — — — second — — — — — — — — — — — — paid Bornhuetter-Ferguson method.

→ After projecting the ultimate claims, we subtract the actual reported claims to estimate IBNR, or subtract the actual paid claims to estimate the unpaid claims:

$$\text{IBNR Estimate} = \text{Ultimate claims} - \text{Actual reported claims}$$
$$= \text{Expected claims} \times \%\text{ unreported}$$

$$\text{Unpaid claims Estimate} = \text{Ultimate claims} - \text{Actual paid claims}$$
$$= \text{Expected claims} \times \%\text{ unpaid}$$

→ Assumptions + uses

→ As a hybrid between the EC + development methods, the BF method inherits some assumptions from each of them:

→ 1) Like the EC method, the BF method assumes that the unreported (or unpaid) claims will develop based on the expected claims, which means the claims reported (or paid) to date provide no information for further development.

→ 2) Thus, it assumes that future claims, both reported & paid, develop in the same pattern as in the development method.

→ The BF method can be applied to all lines of insurance, both short-tail & long-tail. Yet, it is favorable for:

→ Long-tail → The BF method is frequently employed for long-tail lines of insurance, especially during the initial stages of development when claim development factors are influential.

→ Limited Data → It is also well suited for situations w/ limited or volatile data, such as when insurers venture into new lines of business or territories w/ insufficient historical claims development experience. In these cases, actuaries may rely on benchmarks from similar lines or industry-wide experience to establish development patterns & expected claims ratios.

→ One advantage of the BF technique is its ability to withstand early random fluctuations in AYs, leading to more accurate projections. Unlike the development technique, which can be overly conservative when faced w/ large + unusual claims for a specific year, the BF method remains relatively unaffected by such scenarios.

→ Applying the BF technique

→ To determine the reserve using the BF technique:
→ 1) Estimate ultimate losses using the expected claims method
→ 2) Calculate the cumulative loss development factors (CDF) using the development method
→ 3) Estimate ultimate losses using one of the top two formulas above
→ 4) To estimate IBNR, subtract actual reported losses; to estimate unpaid claims, subtract actual paid claims

→ To better explain the mechanics of the BF technique, consider a simple example similar to a past CAS exam question.

→ Example →

Given the following as of December 31, 2022:

Earned premium ($000s)	$5,000
Expected claim ratio	80%
Paid claims at 12 months ($000s)	$2,000
Paid age-to-ultimate factor at 12 months	1.6

Using the Bornhuetter-Ferguson method, calculate the total unpaid claims estimate for AY2022.

→ Step 1: Estimate ultimate losses using EC method

→ The expected claims ratio is 80% & the EP is $5,000,000. Thus, the AY2022 expected claims is:

AY2022 Expected claims = 5,000,000 × 0.8 = 4,000,000

→ Step 2: Calculate CDF

→ Since this value has already been provided (1.6), we can move onto the next step

→ Step 3: Estimate ultimate claims

→ Since we have the paid claims development factor, we will apply the paid BF method:

$$\text{AY2022 ultimate claims} = 2{,}000{,}000 + 4{,}000{,}000\left(1 - \frac{1}{1.6}\right) = 3{,}500{,}000$$

→ Step 4: Estimate total unpaid claims

→ Subtract the actual paid claims to calculate the unpaid claims estimate

AY2022 Total unpaid claims = 3,500,000 − 2,000,000 = 1,500,000

→ Credibility weighted average

→ The BF method can be expressed as the weighted average of the development + expected claims approaches. For example (from the top two above formulas):

$$\text{Ultimate claims} = \text{Actual paid claims} + \text{Expected claims}\left(1 - \frac{1}{CDF}\right)$$
$$= \text{Actual paid claims}\left[\frac{CDF}{CDF}\right] + \text{Expected claims}\left(1 - \frac{1}{CDF}\right)$$
$$= \text{Actual paid claims} \times CDF\left(\frac{1}{CDF}\right) + \cdots$$
$$= \text{Development ultimate claims}\left(\frac{1}{CDF}\right) + \text{Expected claims}\left(1 - \frac{1}{CDF}\right)$$

→ This means that in the example above, we could also have estimated ultimate claims as:

$$\text{ultimate claims} = 3{,}200{,}000\left(\frac{1}{1.6}\right) + 4{,}000{,}000\left(1 - \frac{1}{1.6}\right) = 3{,}500{,}000$$

where 3,200,000 is the ultimate claims estimated using the development method.

→ The above equation implies that as claims approach maturity, the age-to-ultimate factor decreases & the weight assigned to the development method increases. Practically speaking, this makes sense b/c the closer the claims are to maturity, the more data is available on those claims. Therefore, higher weight is given to ultimate claims calculated using actual claims experience.

→ Also, observe that the result of the above equation, the BF method can also be interpreted as a credibility-weighted average, where $Z = \frac{1}{CDF}$ serves as the credibility factor.

$$\text{Ultimate claims} = \text{Development ultimate claims} \times Z + \text{Expected claims} \times (1 - Z)$$

→ One issue that arises is when a CDF is less than 1. This in turn causes the credibility factor to be greater than 1, & thus, in theory invalidates the BF method.

→ In practice, actuaries still use the BF method w/ these CDFs. One way of addressing this issue is to limit the CDFs to a minimum of 1.00. The second approach, which is more common, is to still calculate the BF projected ultimate claims but then rely on another technique to select the ultimate claims for that year.

→ Example →

You are given the following information:

Earned Premium	Accident Year	Cumulative Reported Claims as of (months) 12	24	36	48
$1,000	1999	$250	$500	$750	$825
1,000	2000	200	350	490	
1,500	2001	300	450		
1,800	2002	400			

- The expected loss ratio is 75%.
- Loss development factors should be calculated using a simple arithmetic average.
- The tail factor is 1.05 for development from 48 months to ultimate.

Using the reported Bornhuetter-Ferguson method, estimate the total IBNR as of December 31, 2002.

→ Step 1: Estimate ultimate losses using EC method

→ For each AY, multiply the EP by the expected loss ratio of 75% to determine the expected loss for that year.

Accident Year	Expected Loss
1999	1,000(75%)=750
2000	1,000(75%)=750
2001	1,500(75%)=1,250
2002	1,800(75%)=1,350

→ Step 2: Calculate CDF

→ First, we'll calculate the age-to-age development factors using the given table of cumulative reported claims. To do this, divide each number in the cell directly to its left. Additionally, we can include the provided 48-to-ultimate factor as LDF for completeness.

Accident Year	Age-to-Age Development Factors 12-24	24-36	36-48	48-Ult.
1999	500 / 250 = 2.00	750 / 500 = 1.50	825 / 750 = 1.10	1.05
2000	350 / 200 = 1.75	490 / 350 = 1.40		
2001	450 / 300 = 1.50			

→ Next, compute the arithmetic average of each age-to-age development factor, as the question specifically instructs us to use this method.

→ Arithmetic average across all values:
→ 12–24: $\frac{2.00 + 1.75 + 1.50}{3} = 1.75$
→ 24–36: $\frac{1.50 + 1.40}{2} = 1.45$
→ 36–48: $\frac{1.10}{1} = 1.10$
→ 48–ult: given = 1.05

→ From this, we can calculate the age-to-ult (CDF) factors, for each year, we now have the cumulative reported claim development factors.

Development Age	CDF
12–Ult.	1.75 × 1.45 × 1.10 × 1.05 = **2.931**
24–Ult.	1.45 × 1.10 × 1.05 = **1.675**
36–Ult.	1.10 × 1.05 = **1.155**
48–Ult.	**1.050**

→ Apply the appropriate factor to each AY. For example, we use 1.05 for the AY 1999, as it has already reached 48 months as of 12/31/2002.

Accident Year	CDF
1999	1.050
2000	1.155
2001	1.675
2002	2.931

→ Step 3: Estimate ultimate claims

→ The actual reported claims as of 12/31/2002 plus the expected loss multiplied by (1 minus 1 over the CDF), provides the BF estimated ultimate losses.

Accident Year	Actual Reported Claims	Expected Loss	CDF	BF Ultimate Loss
1999	825	750	1.050	860.71
2000	490	750	1.155	590.65
2001	450	1,125	1.675	903.26
2002	400	1,350	2.931	1,289.38

→ Step 4: Estimate total unpaid claims

→ For each AY, start w/ the estimated ultimate loss we just calculated & subtract the actual reported claims.

AY1999 IBNR = 860.71 − 825 = 35.71
AY2000 IBNR = 590.65 − 490 = 100.65
AY2001 IBNR = 903.26 − 450 = 453.26
AY2002 IBNR = 1289.38 − 400 = 889.38

→ Sum the IBNRs for each AY to obtain the total IBNR, which is 1,479.

→ Assignment

→ Q1) You are given the following information:

Accident Year	Earned Premium ($000)	Case Incurred Losses ($000), Valued as of December 31, 2001	Expected Loss Ratio
1998	200	100	80%
1999	1,000	1,000	80%
2000	1,500	900	80%
2001	1,500	600	80%

Selected age-to-age incurred loss development factors:

12 - 24 months	1.250
24 - 36 months	1.100
36 - 48 months	1.050
48 - 60 months	1.080

There is no further development after 60 months.

Calculate the IBNR reserve as of December 31, 2001 using the Bornhuetter-Ferguson method.

→

Accident Year	Age (as of 12/31/2001)	Earned Premium ($000)	Case Incurred Losses ($000), Valued as of December 31, 2001	ExpectedLoss Ratio	Est ult losses (EC method)	Dev ult claims	Est ult losses (BF)	Est IBNR
1998	48	200	100	80%	160	108	111.85	11.85
1999	36	1,000	1,000	80%	800	1134	1094.03	94.03
2000	24	1,500	900	80%	1200	1122.66	1138.00	238.00
2001	12	1,500	600	80%	1200	935.55	1030.40	430.40

Total est losses = 774,782.70

Age-to-age	Dev Factor
12 - 24 months	1.25
24 - 36 months	1.1
36 - 48 months	1.05
48 - 60 months	1.08

Lookup	Age-to-ult	Dev factor
12	12-ult	1.559
24	24-ult	1.247
36	36-ult	1.134
48	48-ult	1.080
60	60-ult	1

→ Q2)

Accident Year	Earned Premium	Case Incurred Losses (Age of Development in Months) 12	24	36	48
2001	$19,000	$4,850	$9,700	$14,100	$16,200
2002	20,000	5,150	10,300	14,900	
2003	21,000	5,400	10,800		
2004	22,000	7,200			

- Expected Loss Ratio = 0.90
- Assume no development past 48 months.
- Loss development factors should be calculated using a simple average.

Using the Bornhuetter-Ferguson method, calculate the indicated IBNR for accident year 2004 as of December 31, 2004.

→

Accident Year	Age	Earned Premium	Est ult losses (EC method)	Agg losses	CDF	Dev losses	Est ult loss (BF)	Est IBNR
2001	48	$19,000	$17,100	$16,200	1.000	$16,200.00	$16,200.00	$0.00
2002	36	$20,000	$18,000	$14,900	1.149	$17,117.51	$17,223.33	$2,323.33
2003	24	$21,000	$18,900	$10,800	1.666	$17,993.47	$18,333.99	$7,533.99
2004	12	$22,000	$19,800	$7,200	3.332	$23,990.53	$21,057.86	$13,857.86

Expected loss ratio = 90%

Accident Year	12-24	24-36	36-48	48-Ult
2001	2.00	1.45	1.15	1.00
2002	2.00	1.45		
2003	2.00			

Age-to-age	Selected factor
12-24	2.00
24-36	1.45
36-48	1.15
48-Ult	1.00

Age-to-ult	CDF
12 12-ult	3.332
24 24-ult	1.666
36 36-ult	1.149
48 48-ult	1.000

→ Q3) Given the following for policy year 2006 for a line of business:

Premium	$1,600,000
Expected loss reported at 24 months	68%
Expected loss reported at 36 months	82%
Reported loss as of December 31, 2007	$800,000
Bornhuetter-Ferguson estimate of ultimate loss	$1,133,000

Calculate the expected loss ratio that was used in the Bornhuetter-Ferguson estimate of ultimate loss for policy year 2006.

~~AY2006 est ult loss (BF) = Actual ult losses $\left(\frac{1}{CDF}\right)$ + Est ult losses $\left(1 - \frac{1}{CDF}\right)$~~ Know other formulas too!

→ Since development is at 24 months, apply the BF formula:

ultimate claims = Actual reported claims + Expected claims × % unreported (↳ premium × ECR)

1,133,000 = 800,000 + [1,600,000 × ECR] × (1 − 0.68)

⇒ ECR = 65%

6.4.2 → Benktander Method

→ Overview → When we discussed the BF method, we highlighted its stability as a significant advantage. This is b/c it places more emphasis on the expected claims in the early development years & less on the emerging actual claims.

→ However, this means that the estimate of unpaid claims for the most recent AYs, using the BF method is heavily influenced by the actuary's judgement, which may not always be beneficial.

→ The Benktander method, introduced in 1976, combines the BF technique & the development technique in a credibility-weighted average. Specifically, while the BF technique depends on the EC method in estimating the expected claims, the Benktander method utilizes the projected ultimate claims from an initial BF projection:

$$\text{Benktander ultimate claims} = \text{Development ultimate claims} \times \left(\frac{1}{CDF}\right) + \text{BF ultimate claims}\left(1 - \frac{1}{CDF}\right)$$

→ Thus, the Benktander method is essentially an iterative version of the BF technique, w/ the main difference being the derivation of expected claims. After sufficient iterations, the Benktander projection of ultimate claims will converge to the projected ultimate claims generated by the development technique.

→ Note that, compared to the BF method, the Benktander method places more weight on the changes in the claims data. Yet, it still provides more stable estimates than the development technique.

→ Example →

You are given the following information for an accident year:

Earned Premium	Reported Loss as of 12 months	Expected Loss Ratio
$10,000,000	$4,000,000	80%

The expected reporting pattern is as follows:

Age (in months)	Percent Reported
12	60%
24	80%
36	100%

Calculate the ultimate loss estimate using the Benktander method at age 12 months.

→ Start by calculating the BF ultimate loss estimate. It is reported loss(es), plus the expected losses multiplied by 1 minus the reported as of 12 months

BF ult claims = 4,000,000 + (10,000,000 × 0.80)×(1 − 0.6) = 7,200,000

→ Then, calculate the ultimate loss estimate again using the BF formula, but this time, use 7,200,000 for the expected losses.

Benktander ult claims = 4,000,000 + 7,200,000 × (1 − 0.6) = 6,880,000

→ Hopefully, you can see now that the Benktander method simply involves applying the BF method twice. There isn't much new to learn.

→ Assignment

→ Q1) Given the following information:

Estimate of Ultimate Claims Using Expected Claims Method	$9,000,000
Paid Claims	$8,000,000
Paid Age-to-Ultimate Factor	1.250

a. (1 point). Calculate the unpaid loss estimate using the Benktander method.
b. (0.5 point). Briefly describe an advantage the Benktander method generally has over the Bornhuetter-Ferguson method.

→ a) BF ult claims = Actual paid claims + Expected paid (% unpaid) claims

$= 8,000,000 + 9,000,000\left(1 - \frac{1}{1.250}\right)$

$= 9,800,000$

Benktander ult claims = Actual paid claims + BF ult (% unpaid) claims

$= 8,000,000 + 9,800,000\left(1 - \frac{1}{1.25}\right)$

$= 9,960,000$

⇒ Unpaid claims = Benktander ult claims − Paid claims

$= 9,960,000 - 8,000,000$

$= 1,960,000$

→ b) The Benktander method responds more quickly to new experience compared to the BF method

→ Q2) (2 points) You are given the following information:

Accident Year	Earned Premium	Expected Loss Ratio	Paid Age to Ultimate Factor	Paid Losses as of December 31, 2003
2002	$25,000	65%	2.30	$7,200
2003	15,000	75%	4.00	3,375

Use the Benktander method to estimate the ultimate losses for accident years 2002 and 2003. Show all work.

→

Accident Year	Earned Premium	Expected Loss Ratio	Paid Age to Ultimate Factor	Paid Losses as of 12/31/2003	BF ult losses	Benktander ult losses
2002	$25,000	65%	2.3	$7,200	$16,384.78	$16,460.96
2003	$15,000	75%	4.0	$3,375	$11,812.50	$12,234.38

=$F4+(G4*(1-1/$E4))