1.2.1 → Policy limits

→ Policy limits

→ def → max amount the insurer will pay for a single loss

→ $X \wedge u = \begin{cases} X, & X < u \\ u, & X \geq u \end{cases}$

↓ limited loss variable

capped at u ⇒ pay whichever is lower, X or u

→ Limited expected value → $E(X \wedge u) = \int_0^u x f(x)\,dx + \underbrace{\int_u^\infty u f(x)\,dx}_{= u S(u)}$

→ Extended to k^{th} moment → $E[(X \wedge u)^k] = \int_0^u x^k f(x)\,dx + u^k S(u)$

→ Alternative calculation ⟶

→ $E[X \wedge u] = \int_0^u S(x)\,dx$

$E[(X \wedge u)^k] = \int_0^u k x^{k-1} S(x)\,dx$

$E(g(X)) = \int_{\underset{\uparrow}{0}}^\infty g'(x) S(x)\,dx$

Always zero, regardless of domain of RV

X → must be non-negative

$g(0) = 0$

→ Mixtures → Even w/ these, formulas for $E[(X \wedge u)^k]$ work by taking the weighted average of the raw moments for each component

$\underset{\substack{\downarrow \\ \text{mixture}}}{E(X \wedge u)} = \sum_{i=1}^n w_i E[X_i \wedge u]$

→ Assignment

→ Q1) $f(x) = \dfrac{375000}{x^4}, \; x > 50$

$V(X \wedge 500) = E[(X \wedge 500)^2] - (E(X \wedge 500))^2$

$= \left[\int_{50}^{500} x^2 f(x)\,dx + \int_{500}^\infty 500^2 f(x)\,dx\right] - \left[\int_{50}^{500} x f(x)\,dx + \int_{500}^\infty 500 f(x)\,dx\right]^2$

$= (\ldots)$

$= (6750 + 250) - (74.25 + 0.5)^2$

$= 1412.4375$

OR

$X \sim$ SP Pareto$(\alpha = 3, \theta = 50)$

$$E[(x \wedge 500)^2] = \frac{\alpha\theta^k}{\alpha-k} - \frac{k\theta^\alpha}{(\alpha-k)x^{\alpha-k}}$$

$$\downarrow \quad = \frac{3(50^2)}{3-2} - \frac{2(50^3)}{(3-2)500^{3-2}}$$

$$= 7000$$

$$E(x \wedge 500) = \frac{3(50)}{3-1} - \frac{50^3}{(3-1)500^{3-1}}$$

$$\downarrow \quad = 74.75$$

$$V(X \wedge 500) = 7000 - 74.75^2$$

$$\downarrow \quad = 1412.4375$$

1.2.2 → Deductibles

→ def → Deductible is the amount the <u>policyholder</u> is responsible for paying before the insurer will pay anything on the claim

→ Two types → ordinary & franchise deductibles

→ ordinary deductibles

→ from the <u>policyholder perspective</u>, deductibles act as a cap/limit

$$\boxed{X \wedge d = \begin{cases} x & x < d \\ d & x \geq d \end{cases}}$$

→ The insurer then pays the remaining amount

$$\boxed{(x-d)_+ = \begin{cases} 0 & x \leq d \\ x-d & x > d \end{cases}}$$

→ full loss amount = sum of parts

$$(X \wedge d) + (x-d)_+ = X$$

$\downarrow$ $= \min(x,d)$ $\quad$ $\downarrow$ $\max(x-d,0)$

$\Rightarrow$ $E[X \wedge d] + E((x-d)_+) = E(x)$

$\Rightarrow$ Expected insurance payment can be calculated as

★ $E[(x-d)_+] = E(x) - E(x \wedge d)$

→ recommended formula b/c pieces are easy to find / given in table

→ ★ Does not hold for higher moments

$$E[(x-d)_+^2] \neq E(x^2) - E[(x \wedge d)^2]$$

$\Rightarrow$ need to use integration

$$E[(x-d)_+] = \int_0^d \overset{\text{Below}}{0\,\cancel{f(x)dx}} \qquad \int_d^\infty \overset{\text{above}}{(x-d)f(x)\,dx}$$

$$\downarrow \quad = 0 + \swarrow$$

$$=$$

→ can be extended to the k^{th} moment too

$$E[(x-d)_+^k] = \int_d^\infty (x-d)^k f(x)\,dx$$

using survival method $= \int_d^\infty S(x)\,dx$

k^{th} moment → $E[(x-d)_+^k] = \int_d^\infty k(x-d)^{k-1} S(x)\,dx$

→ loss Elimination ratio (LER) → measures how much the insurer saves by imposing an ordinary deductible

$$\Rightarrow \boxed{LER = \frac{E(x \wedge d)}{E(x)}}$$

→ Payment per loss vs per payment

→ ex) losses: 2 3 7 9 14

Policy has an ordinary deductible of 5

→ Expected payment per loss $= \frac{\text{Total payment}}{\#\text{ losses}} = \frac{0+0+2+4+9}{5} = 3$

→ Expected payment per payment $= \frac{\text{Total payment}}{\#\text{ payments}} = \frac{2+4+9}{3} = 5$

$$\Rightarrow \underbrace{E[Y^P]}_{\text{payment per payment RV}} \geq \underbrace{E[Y^L]}_{\text{payment per loss RV}}$$

→ Formulas

→ $E[Y^L] = E[(x-d)_+]$

→ $E[Y^P] = E[Y^L \mid Y^L > 0]$

$\downarrow \quad = E[X-d \mid x > d]$

→ $f(x \mid x > d) = \frac{f(x)}{1-F(d)} = \frac{f(x)}{S(d)}, \quad x > d$

$$\Rightarrow \quad E(Y^P) = E[X-d \mid X>d]$$

$$= \int_d^{\infty} (x-d)\, f(x \mid X>d)\, dx$$

$$= \int_d^{\infty} (x-d)\, \frac{f(x)}{S(d)}\, dx$$

$$= \frac{1}{S(d)} \int_d^{\infty} (x-d)\, f(x)\, dx$$

$$= \frac{E[(X-d)_+]}{S(d)}$$

$$= \frac{E[Y^L]}{S(d)}$$

→ Easier way to switch between expected payment per payment & expected payment per loss

$$\boxed{E(Y^P) = \frac{E(Y^L)}{S(d)} \iff E(Y^L) = E(Y^P)\, S(d)}$$

→ works for higher order moments too $E[(Y^P)^k] = E[(Y^L)^k]\, S(d) \iff \langle \dots \rangle$

→ Y^L is the general notation for payment per loss RV & is not ALWAYS $(x-d)_+$ (only when there is an ordinary deductible & no other coverage modifications)

→ with no deductible, $Y^P = Y^L$ (all losses are payments)

→ Example → losses $X \sim$ Exponential ($\lambda = 1/500$) w/ ordinary deductible of 100

→ define variables → $Y^L = (X-100)_+$

$Y^P = X - 100 \mid X > 100$

→ Goal → $V(Y^P) = E[(Y^P)^2] - (E(Y^P))^2 = ??$

$$\rightarrow E(Y^P) = \frac{E(Y^L)}{S(d)}$$

$$= \frac{E(X) - E(X \wedge d)}{e^{-d\lambda}} \quad \hookrightarrow d = 100 \qquad E(X \wedge d) \rightarrow = \theta(1 - e^{-x/\theta})$$

$$= \frac{\frac{1}{1/500} - 90.63}{0.8187}$$

$$= 500$$

$$\rightarrow E[(Y^L)^2] = \int_d^{\infty} ?(x-d)\, f(x)\, dx$$

$$= \int_d^\infty 2(x-100)\, e^{-x/500}\, dx$$

$$= (\cdots)$$

$$= 404{,}365.3265$$

$$E[(Y^P)^2] = \frac{E[(Y^L)^2]}{S(d)}$$

$$= 500{,}000$$

$$\Rightarrow V(Y^P) = 500{,}000 - 500^2$$

$$= 250{,}000$$

OR

$Y^P = X - d \mid X > d \sim \mathrm{Exp}(1/500)$ by memoryless property

$$\Rightarrow V(Y^P) = V(\mathrm{Exp}(1/500))$$

$$= 250{,}000$$

→ def → $X \wedge d$ = limited loss variable

$X - d \mid X > d$ = excess loss variable

$e(d)$ = mean excess loss function

$= E[X - d \mid X > d]$

→ Shortcuts for mean excess loss functions

loss	Mean	Excess loss	Mean Excess loss
$\mathrm{Exp}(\theta)$	θ	$\mathrm{Exp}(\theta)$	θ
uniform(a, b)	$\frac{a+b}{2}$	uniform$(0, b-d)$	$\frac{b-d}{2}$
pareto(α, θ)	$\frac{\theta}{\alpha-1}$	pareto$(\alpha, \theta+d)$	$\frac{\theta+d}{\alpha-1}$
S-P Pareto(α, θ)	$\frac{\alpha\theta}{\alpha-1}$	pareto(α, θ)	$\frac{d}{\alpha-1}$
Beta$(1, b, \theta)$	$\frac{\theta}{1+b}$	Beta$(1, b, \theta-d)$	$\frac{\theta-d}{1+b}$

→ Franchise deductibles

→ def → pays full amount of losses over the deductible (in contrast to ordinary deductible which only pays amount in excess of the deductible)

$$\boxed{Y^L = \begin{cases} 0 & X \le d \\ X & X > d \end{cases}}$$

→ Expected value → $E(Y^L) = \int_d^\infty x f(x)\,dx$

$$= \int_d^\infty x f(x)\,dx - \int_d^\infty d\, f(x)\,dx + \int_d^\infty d\, f(x)\,dx$$

$$= \int_d^\infty (x-d) f(x)\,dx + d\int_d^\infty f(x)\,dx$$

$$= E[(X-d)_+] + d\,S(d)$$

$$\Rightarrow \quad E[Y^P] = \frac{E(Y^L)}{S(d)}$$

$$= \frac{E[(X-d)_+] + d\,S(d)}{S(d)}$$

$$= e(d) + d$$

→ Example → $X \sim \text{Pareto}(\alpha = 2.5, \theta = 100)$

→ $E(Y^P) = e(d) + d$

↳ $X-d \mid X>d \sim \text{Pareto}(\alpha, \theta+d)$

$\Rightarrow e(d) = E[X-d \mid X>d]$

$= \frac{\theta+d}{\alpha-1}$

$$= \frac{100+50}{2.5-1} + 50$$

$$= 150$$

→ Impact of deductibles on claim frequency

→ Recall # losses ≠ # payments necessarily when there is a deductible d

→ If # of losses N is modelled by an $(a,b,0)$ class distribution, the # of payments N' will follow the same distribution, but w/ modified parameters

Dist	N	N'
Poisson	λ	$v\lambda$
Binomial	n, p	n, vp
NB	r, β	$r, v\beta$

where v is the prob of a loss being greater than the deductible $P(X>d)$ = prob of + payment

→ Assignment

→ Q1) $X \sim Exp(\theta)$

$e(\theta) = E(X-d \mid X>d)$

$X-d \mid X>d \sim Exp(\theta)$

$\Rightarrow e(\theta) = \theta$

→ Q2) → $X \sim Exp(\theta = 2000)$

$d = 500$

$d' = 1000$

→ $|E(X-d) - E(X-d')| = ??$

→ $E(X-d) = \int_{500}^{\infty} (X-500) \frac{1}{2000} e^{-x/2000} \, dx$

$\downarrow$ $= \langle \dots \rangle$

$= 1552.602$

$E(X-d') = \int_{1000}^{\infty} (X-1000) \frac{1}{2000} e^{-x/2000} \, dx$

$\downarrow$ $= \langle \dots \rangle$

$= 1213.061$

$\Rightarrow |E(X-d) - E(X-d')| = 344.54$

OR

$$\rightarrow E[\overset{= Y^L}{(X-d)_+}] = E(X) - \underset{= \theta(1-e^{-d/\theta})}{E(X \wedge d)}$$

$\downarrow$ $= 2000 - 2000(1 - e^{-500/2000})$ $= 442.398$

✓ $= 1552.602$

→ $E[(X-d')_+] = \langle \dots \rangle$

$\downarrow$ ✓ $= 1213.061$

1.2.3 → Coinsurance

→ def → Coinsurance is the portion of the loss the insurer is responsible for

→ Suppose loss X → insurer pays αX, $0 < \alpha < 1$ + policyholder pays $(1-\alpha)X$

→ of course $E(\alpha X) = \alpha E(X)$

→ Coinsurance & deductible → Coinsurance comes into play *After* the deductible is met

$$\boxed{\begin{aligned} &\rightarrow Y^L = \begin{cases} 0 & x \le d \\ \alpha(x-d) & x > d \end{cases} \\ &\rightarrow E(Y^L) = \alpha\left[E(X) - E(X \wedge d)\right] \end{aligned}}$$

→ If coinsurance is applied *before* the deductible, then instead of applying the entire loss X against the deductible, the insurer will only apply the coinsured portion of the loss, αX.

The policyholder will still be responsible for the remaining portion $(1-\alpha)X$ + amount needed to meet deductible

$$\Rightarrow \boxed{\begin{aligned} Y^L &: \begin{cases} 0 & \alpha X \le d \\ \alpha X - d & \alpha X > d \end{cases} \\ &\downarrow \\ &= \begin{cases} 0 & x \le \frac{d}{\alpha} \\ \alpha\left(x - \frac{d}{\alpha}\right) & x > \frac{d}{\alpha} \end{cases} \end{aligned}}$$

→ ex) if $\alpha = 0.8$, $x = 3$, $d = 2$

After ⇒ payment $= 0.8(3-2) = 0.8$

Before ⇒ payment $= 0.8(3) - 2 = 0.4$

⇒ to solve for Expected value, make the following adjustment

$$E(Y^L) = \alpha\left[E(X) - E\left(X \wedge \frac{d}{\alpha}\right)\right]$$

→ General formula for all coverage modifications

$$E(Y^L) = \alpha\left[\underset{*}{E(X \wedge m)} - E(X \wedge d)\right]$$

X = loss RV

u = policy limit (∞ if no limit)

d = deductible (0 if none)

α = coinsurance (1 if none)

m = maximum covered loss

$$\boxed{\downarrow = \frac{u}{\alpha} + d}$$

$$\rightarrow \boxed{Y^L = \begin{cases} 0 & X \le d \\ \alpha(X-d) & d < X < m \\ m & X \ge m \end{cases}}$$

→ if only policy limit ⇒ $m = u$

if — — — — & deductible ⇒ $m = u + d$

if — — — — — — — — & coinsurance ⇒ $m = \frac{u}{\alpha} + d$

→ $E[X \wedge \infty] = \int_0^\infty S(x)\,dx = E(X)$

$E(X \wedge 0) = \int_0^0 S(x)\,dx = 0$

$\Rightarrow E[(X-0)_+] = E(X) - E(X \wedge 0) = E(X)$

$\rightarrow$ examples $\rightarrow$ losses $X \sim \text{Pareto}(\alpha = 2, \theta = 1000)$

$d = 200$ & $u = 2000$

$\rightarrow E(Y^L) = E(X \wedge m) - E(X \wedge d)$

$m = u + d = 2000 + 200 = 2200$

$E(X \wedge d) = \frac{\theta}{\alpha - 1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha - 1}\right], \alpha \neq 1$

$= E(X \wedge 2200) - E(X \wedge 200)$

$= \frac{1000}{2-1}\left[1 - \left(\frac{1000}{2200+1000}\right)^{2-1}\right] - (\ldots)$

$= 687.50 - 166.67$

$= 520.83$

$\rightarrow$ losses $X \sim \text{Gamma}(\alpha = 2, \theta = 100)$

$u = 250, c = ??$

$E(cX) = E(X \wedge u) \longrightarrow \int_0^u x f(x)\,dx + u S(u) = 200c$

OR

$\alpha\theta\,\Gamma(\alpha+1, u/\theta) + u[1 - \Gamma(\alpha, u/\theta)]$

$2(100)\,\Gamma\left(3, \frac{250}{100}\right) + 250\left[1 - \Gamma\left(2, \frac{250}{100}\right)\right]$

$\Rightarrow N \sim \text{Poisson}(\lambda = 2.5)$

$\Rightarrow \Gamma(3, 2.5) = 1 - P(N < 3)$

$= 1 - P(N = 0, 1, 2)$

$= 1 - \left[\frac{e^{-2.5}\,2.5^0}{0!} + \cdots + \frac{e^{-2.5}\,2.5^2}{2!}\right]$

$= 0.4562$

$\Gamma(2, 2.5) = (\ldots) = 0.7127$

$\underbrace{2(100)}_{} = \alpha\theta = \overset{c\,E(X)}{\overbrace{d}}$

$= 2(100)(0.4562) + 250(1 - 0.7127)$

$200c \approx 163$

$\Rightarrow c \approx 0.815$

OR

$\Gamma(\alpha, \frac{u}{\theta}) = \text{GAMMA.DIST}(x, \alpha, \beta = \theta, \text{TRUE})$

$\hookrightarrow = (\ldots)(250, 3, 100)$

$\downarrow$

chose x so that $x = \frac{u}{\theta}$

$\Rightarrow X \sim \text{Pareto}(\alpha = 3, \theta = 400)$

$B = \begin{cases} 0.15(400 - X) & X < 500 \\ 0 & X \geq 500 \end{cases}$ ★ Be careful defining RV

$= 0.15(500 - X)_+$

switched compared to usual

$\Rightarrow E(B) = E[0.15(500 - X)_+]$

$$= 0.15\, E[(500-x)_+]$$

$$= \left[E(500) - \underbrace{E(500 \wedge X)}\right]$$

$\hookrightarrow = \min(50, X) = \min(x, 500)$

$= E(X \wedge 500)$

$$= \left[500 - \frac{400}{3-1}\left(1-\left(\frac{400}{500+400}\right)^{3-1}\right)\right]$$

$$= 50.93$$

→ Assignment

→ Q1 → loss $x \sim Exp(\theta = 600)$

$d = 200,\ \alpha = 0.80,\ u = 800$

$$\to Y^L = \begin{cases} 0 & x \le d \\ \alpha(x-d) & d < x < m \\ m & x \ge m \end{cases}$$

$\hookrightarrow = \frac{u}{\alpha} + d$

$$= \begin{cases} 0 & x \le 200 \\ 0.8(x-200) & 200 < x < 1200 \\ 1200 & x \ge 1200 \end{cases}$$

$$\to E(Y^L) = \alpha\left[E(x \wedge m) - E(x \wedge d)\right] \xrightarrow{} = \theta(1-e^{-x/\theta})$$

$$= 0.8\left[E(x \wedge 1200) - E(x \wedge 200)\right]$$

$$= 0.8\left[600(1-e^{-1200/600}) - 600(1-e^{-200/600})\right]$$

$$E(Y^P) = E(Y^L/x>d) = \frac{278.974}{S(d)}$$

$\hookrightarrow = e^{-200/600}$

$$= 389.3397$$

OR

Square deductible && memoryless property → $X-d/X>d = X$

$$E(Y^P) = E(Y^L) = \alpha\, E[x \wedge 1000]$$

$$= 0.8(600)(1-e^{-1000/600})$$

$$= 389.3397$$

1.2.4 → Inflation

→ Inflation & its impacts on coverage modifications

$$E[(1+r)X \wedge d] = (1+r)\, E\left[X \wedge \frac{d}{1+r}\right]$$

→ Example → losses $X \sim$ S.P pareto $(\alpha = 2, \theta = 100)$

$r = 0.10$, $u = 220$

$$E[(1+0.10)X \wedge 220] = 1.1\, E\left[X \wedge \frac{220}{1+0.1}\right] \xrightarrow{} \frac{\alpha\theta^k}{\alpha-k} - \frac{k\theta^\alpha}{(\alpha-k)x^{\alpha-k}}$$

$$= 1.1\left[\frac{2\cdot 100}{2-1} - \frac{(1)\,100^2}{(2-1)\,200^{2-1}}\right]$$

$$\downarrow \quad = 165$$

<u>OR</u>

$Y = (1+r)X$

$\downarrow = 1.1X \sim$ S-P Pareto $(\alpha = 2,\ \theta = 1.1(100))$ $\quad \theta \downarrow = 110$

$$\Rightarrow E[Y \wedge 220] = \frac{2(110)}{2-1} - \frac{(1)\,100^2}{(2-1)\,220^{2-1}}$$

$$\downarrow \quad \overset{\checkmark}{=} 165$$

<u>OR</u>

$$E[1.1X \wedge 220] = \int_0^\infty \min(1.1x, 220)\, f(x)\, dx$$

$$= \int_0^\infty 1.1 \min\left(x, \frac{220}{1.1}\right) f(x)\, dx$$

$$= 1.1 \int_0^{200} \min(x, 200)\, f(x)\, dx$$

$$\downarrow \quad = 1.1\, E[X \wedge 200]$$

↪ losses $X_{16} \sim$ Uniform $[0, 1000)$

$r = 0.2$, $d = 300$

2016 → $E(X) = 500$

$$E(X \wedge 300) = \int_0^{300} S(x)\, dx$$

$$= \int_0^{300} \left(1 - \frac{x}{1000}\right) dx$$

$= \langle \dots \rangle$

$\downarrow = 255$

2017 → $E(1.2X) = 600$

$$E((1+0.2)X \wedge 300) = (1+0.2)\, E\left[X \wedge \frac{300}{1+0.2}\right]$$

$$= 1.2 \int_0^{250} S(x)\, dx$$

$= \langle \dots \rangle$

$\downarrow = 262.5$

$$LER_{16} - LER_{17} = \frac{255}{500} - \frac{262.5}{600}$$

↪ $LER = \dfrac{E(X \wedge d)}{E(X)}$

$$= 0.51 - 0.4375$$

$$\downarrow = 0.0725$$

→ updating all coverage modification formula for inflation

$$E(Y^L) = \alpha(1+r)\left[E\left(X \wedge \frac{m}{1+r}\right) - E\left(X \wedge \frac{d}{1+r}\right)\right]$$

$$E\left[(Y^P)^k\right] = \frac{E\left[(Y^L)^k\right]}{S_X\left(\frac{d}{1+r}\right)}$$

$$\Updownarrow$$

$$E\left[(Y^L)^k\right] = E\left[(Y^P)^k\right] S_X\left(\frac{d}{1+r}\right)$$

↳ or use $X^* = (1+r)X$ + $\theta^* = (1+r)\theta$

+ $E(Y^L) = \alpha\left[E(X^* \wedge m) - E(X^* \wedge d)\right]$

↳ use w/ scale distributions

→ Example → $S(x) = 1 - \frac{x}{200}$, $0 \leq x \leq 200$

⇒ $X \sim$ uniform $(0, 200)$

$d = 20$, $m = 100$ ⇒ $u = 100 - 20 = 80$

$$V(Y^P) = E\left[(Y^P)^2\right] - E(Y^P)$$

$$= \frac{E\left[(Y^L)^2\right]}{S(d)} - \frac{E(Y^L)}{S(d)}$$

★ Solve for Y^L + convert

$$= \frac{4053.333}{0.9} - \frac{56}{0.9}$$

$$= 632.0988$$

$$\boxed{\to E\left[(Y^L)^k\right] = \int_d^m (x-d)^k f(x)\,dx + u\,S(m)}$$

$$\to Y^L: \begin{cases} 0 & x < 20 \\ x-20 & 20 < x < 100 \\ 80 & x \geq 100 \end{cases}$$

$$\to E\left[(Y^L)^2\right] = \int_{20}^{100} (x-20)^2 \frac{1}{200}\,dx + 80\,S(100)$$

$$= (\ldots)$$

$$= 4053.333$$

$$\to E(Y^L) = \int_{20}^{100} (x-20) \frac{1}{200}\,dx + 80\,S(100)$$

$$= (\ldots)$$

$$= 56$$

→ Assignment

→ Q1) losses $X \sim$ Pareto $(\alpha = 2, \theta = 1600)$

$u = 2457$, $r = 0.05$

$$\%\Delta = \frac{E[(1+r)X \wedge 2457] - E(X \wedge 2457)}{E(X \wedge 2457)} = \frac{997.766 - 968.991}{968.991} = 0.029695$$

$$\to E[X \wedge 2457] = \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{u+\theta}\right)^{\alpha-1}\right]$$

$$= \frac{1600}{2-1}\left[1 - \left(\frac{1600}{2457+1600}\right)^{2-1}\right]$$

$$= 968.991$$

$\rightarrow E[\underset{\sim}{1.05X} \wedge 2457] =$ < same as above except w/ 1680 for 1600 >

$\sim \text{Pareto}(\alpha = 2, \theta = 1.05(1600)) = 997.766$
$d = 1680$

$\rightarrow$ Q2 $\rightarrow$ losses $X \sim \text{Pareto}(\alpha = 4, \theta = 1500)$, $r = 0.20$

$E[((1+r)X - 100)_+] = E[(X^* - 100)_+]$

$\hookrightarrow \sim \text{Pareto}(\alpha = 4, \theta = 1.2(1500))$

$= E(X^*) - E[X^* \wedge 100]$

$= \dfrac{\theta^*}{\alpha - 1} - \dfrac{\theta^*}{\alpha - 1}\left[1 - \left(\dfrac{\theta^*}{d + \theta}\right)^{\alpha - 1}\right]$

$= \dfrac{1800}{4-1} - \dfrac{1800}{4-1}\left[1 - \left(\dfrac{1800}{100 + 1800}\right)^{4-1}\right]$

$= 510.1618$