CA – Quiz 1

Probability

The annual number of claims for a policy follows a Poisson distribution with mean Λ. The Λ parameter varies by policy according to a gamma distribution with $\alpha = 2$ and Θ.The Θ parameter varies by school district according to a beta distribution with $a = b = 3$ and $\theta = 8$.

Calculate the standard deviation of the annual number of claims by a randomly selected policy in a randomly selected school district.

(round to the nearest 0.01)

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The loss amounts in year 2022 follow a lognormal distribution with parameters $\mu = 2$ and $\sigma^2 = 3$.

Inflation increases the size of the previous year's loss amounts by 2% each year.

Calculate the variance of loss amounts in year 2024.

(round to the nearest whole number)

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The following table contains the probability density function of five random variables.

Random Variable	Probability Density Function
V	$f_V(v) = \dfrac{1}{120v}\left(\dfrac{v}{\theta}\right)^6 e^{-v/\theta}$
W	$f_W(w) = \dfrac{3\theta^3}{(w+\theta)^4}$
X	$f_X(x) = \dfrac{5\left(\frac{x}{\theta}\right)^5}{x\left(1+\left(\frac{x}{\theta}\right)^5\right)^2}$
Y	$f_Y(y) = \dfrac{6}{y}\left(\dfrac{y}{\theta}\right)^6 e^{-(y/\theta)^6}$
Z	$f_Z(z) = \dfrac{\theta^2}{z^3} e^{-\theta/z}$

You use the number of positive raw or central moments to determine the tail weight of these distributions (existence of moments test).

Determine the random variable whose distribution has the heaviest tail weight.

A V

B W

C X

D Y

E Z

You are given:

- X has density $f(x)$, where $f(x) = \dfrac{500{,}000}{x^3}$, for $x > 500$.
- Y has density $g(y)$, where $g(y) = \dfrac{ye^{-y/500}}{250{,}000}$.

Which of the following are true?

I. X has an increasing mean residual life function.
II. Y has an increasing hazard rate.
III. X has a heavier tail than Y based on the hazard rate test.

A I only

B II only

C III only

D I, II, and III

E The correct answer is not given by (A), (B), (C), or (D).

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An actuary for a medical device manufacturer initially models the failure time for a particular device with an exponential distribution with mean 4 years.

The distribution is replaced with a spliced model whose density function:

- is uniform over [0, 3].
- is proportional to the initial modeled density function after 3 years.
- is continuous.

Calculate the value-at-risk at $p = 0.60$, $\mathrm{VaR}_{0.60}$ (the 60^{th} percentile).

A. Less than 2.0

B. At least 2.0, but less than 3.0

C. At least 3.0, but less than 4.0

D. At least 4.0, but less than 5.0

E. At least 5.0

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Between 9 am and 3 pm Big National Bank employs 2 tellers to service customer transactions. The time it takes Teller X to complete each transaction follows an exponential distribution with a mean of 10 minutes. Transaction times for Teller Y follow an exponential distribution with a mean of 15 minutes. Both Teller X and Teller Y are continuously busy while the bank is open.

On average, every third customer transaction is a deposit and the amount of the deposit follows a Pareto distribution with parameters $\alpha = 3$ and $\theta = 5{,}000$. Each transaction that involves a deposit of at least 7,500 is handled by the branch manager. The time the tellers spend screening deposits, including those passed on to the manager, is also factored into their exponential transaction times, and these are counted as completed transactions.

Calculate the expected total deposits made through the tellers each day.

A Less than 31,000

B At least 31,000, but less than 32,500

C At least 32,500, but less than 35,000

D At least 35,000, but less than 37,500

E At least 37,500

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You are given the following information on a parallel system with four components:

- All components in the system are independent.
- The lifetimes of three components are exponentially distributed with a mean of two years.
- The other component has a lifetime that is exponentially distributed with a mean of three years.
- The system has operated for one year.

What is the probability that the system functions for another one year?

A Less than 0.86

B At least 0.86, but less than 0.88

C At least 0.88, but less than 0.90

D At least 0.90, but less than 0.92

E At least 0.92

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Finish Quiz

You are given the following information regarding a series system with two independent machines, X and Y:

- The hazard rate function, in years, for machine i is denoted by $r_i(t)$.
- $r_X(t) = \ln 1.06, \quad \text{for } t > 0$
- $r_Y(t) = \dfrac{1}{20-t}, \quad \text{for } 0 < t < 20$
- Both machines are currently three years old.

Calculate the probability that the system fails when the machines are between five and nine years old.

A Less than 0.305

B At least 0.305, but less than 0.315

C At least 0.315, but less than 0.325

D At least 0.325, but less than 0.335

E At least 0.335

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A given system is governed by a 5-state Markov chain, with states 0, 1, 2, 3, and 4. These 5 states fall into two classes: {0, 1, 3} and {2, 4}.

Determine which of the following statements regarding this Markov chain and its classes is/are true.

I. The chain is ergodic.
II. At least one of the two classes must be recurrent.
III. If state 2 is positive recurrent, then state 4 must be positive recurrent.

A I only

B II only

C III only

D I, II, and III

E The correct answer is not given by (A), (B), (C), or (D).

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Finish Quiz

You are given:

1. All states in an irreducible Markov chain are recurrent.
2. If all states in a finite Markov chain are recurrent, the Markov chain is irreducible.
3. If a one-dimensional random walk has a transition probability of 0.5 in either direction, all states in the Markov chain are recurrent.

Determine which of the following is correct.

A I is true; II is true; III is true

B I is false; II is false; III is true

C I is true; II is false; III is true

D I is true; II is true; III is false

E I is false; II is true; III is false

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Kevin and Kira are in a history competition:

- In each round, every child still in the contest faces one question. A child is out as soon as he or she misses one question. The contest will last at least 5 rounds.
- For each question, Kevin's probability and Kira's probability of answering that question correctly are each 0.8; their answers are independent.

Calculate the conditional probability that both Kevin and Kira are out by the start of round five, given that at least one of them participates in round 3.

A Less than 0.15

B At least 0.15, but less than 0.18

C At least 0.18, but less than 0.21

D At least 0.21, but less than 0.24

E At least 0.24

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Let e_x denote the curtate expectation of life of (x).

Which of the following statement is true concerning the inequality $e_{x+1} > e_x$?

A The inequality cannot be true.

B The inequality is true if and only if $e_{x+1} > \frac{p_x}{q_{x+1}}$.

C The inequality is true if and only if $e_{x+1} > \frac{p_x}{p_{x+1}q_{x+1}}$.

D The inequality is true if and only if $e_{x+1} > \frac{p_x + 1}{q_x}$.

E The inequality is true if and only if $e_{x+1} > \frac{p_x}{q_x}$.

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You are using the following technique to implement the Rejection Method for simulating a random variable X having density f:

- Step 1: Simulate Y having density g and simulate a random number U.
- Step 2: If $U \leq \dfrac{f(Y)}{2.5 \cdot g(Y)}$, set $X = Y$. Otherwise return to step 1.

Let Z denote the number of iterations required to produce the first value of X.

Calculate the probability that Z is an odd number.

A Less than 0.50

B At least 0.50, but less than 0.55

C At least 0.55, but less than 0.60

D At least 0.60, but less than 0.65

E At least 0.65

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You are given the following information about the distribution of losses:

- Ground-up losses follow a Pareto distribution with $\alpha = 2$ and θ.
- The insurance payment for each loss is subject to a maximum payment of 20,000.
- A random sample of five insurance payments are drawn:

$$800 \quad 1{,}700 \quad 9{,}000 \quad 15{,}000 \quad 20{,}000$$

Estimate θ using the method of moments.

A Less than 15,000

B At least 15,000, but less than 20,000

C At least 20,000, but less than 25,000

D At least 25,000, but less than 30,000

E At least 30,000

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You have been given the following information to compare two hypotheses using the Neyman-Pearson lemma:

- X_i follows an exponential distribution where $f(x) = \dfrac{e^{-x/\lambda}}{\lambda}, x > 0$.
- $H_0 : \lambda = 1$
- $H_1 : \lambda = 2$
- The numerator of the likelihood ratio test will hold the results for H_0 and the denominator will hold the results H_1.
- The test will be based on a random sample of size 100 with $\bar{X} = 1.5$.

Calculate the lower limit of the critical region value for the likelihood ratio test described above so that the significance level will be 5%.

A Less than 1.1

B At least 1.1, but less than 1.2

C At least 1.2, but less than 1.3

D At least 1.3, but less than 1.4

E At least 1.4

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Determine which of the following statements about GLMs are true.

I. The saturated model has the highest possible deviance.

II. Deviance follows a chi-square distribution for all models in the exponential family.

III. Deviance is a useful measure of goodness of fit for all models in the exponential family.

A I only

B II only

C III only

D I, II and III

E The correct answer is not given by (A), (B), (C), or (D).

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You are given the following information for a fitted GLM used to model policy renewals:

Response variable	Renewal outcome (1 or 0)
Response distribution	Bernoulli
Link	Logit

Observation i	Leverage, h_i	Deviance residual, d_i
1	0.034	0.428
2	0.041	0.492
3	0.038	0.553
⋮	⋮	⋮

Consider the following statements:

I. The standardized deviance residual for observation 1 is greater than the standardized deviance residual for observation 2.

II. The standardized deviance residuals should have an approximately standard normal distribution, provided the numbers of observations for each covariate pattern are not too small.

III. Deviance residuals can be plotted in observation order to assess serial correlation.

Determine which of the above statements are true.

A. I only

B. II only

C. III only

D. I, II, and III

You are given a Poisson regression model of the form:

$$\mathrm{E}\left[Y_i\right] = \beta_1 + \beta_2 x_i$$

Maximum likelihood estimates of the beta coefficients are obtained using iterative weighted least squares. W is the weight matrix. X is the design matrix. Z has the beta values from the prior iteration applied to the explanatory variables as well as the correction term.

Using the estimates of the initial iteration, the following matrices are calculated:

$$\mathrm{b}^{(1)} = \begin{bmatrix} 3.60 \\ 0.12 \end{bmatrix}$$

$$\mathrm{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\left(\mathrm{X}^T\mathrm{W}\mathrm{X}\right)^{(1)} = \begin{bmatrix} 2.336 & 13.249 \\ 13.249 & 102.538 \end{bmatrix}$$

$$\left[\left(\mathrm{X}^T\mathrm{W}\mathrm{X}\right)^{(1)}\right]^{-1} = \begin{bmatrix} 1.602 & -0.207 \\ -0.207 & 0.0365 \end{bmatrix}$$

$$\left(\mathrm{X}^T\mathrm{W}\mathrm{z}\right)^{(1)} = \begin{bmatrix} 9.158 \\ 54.671 \end{bmatrix}$$

Calculate the estimate for the first diagonal element of the weight matrix, w_{11}, on the second iteration.

A. Less than 0.22

B. At least 0.22, but less than 0.24

C. At least 0.24, but less than 0.26

You have fit a smoothing spline model to a set of data, using the tuning parameter λ.

Consider the following statements regarding the tuning parameter λ in a smoothing spline model:

I. Larger values of λ result in smoother splines.
II. Larger values of λ result in greater effective degrees of freedom for the model.
III. Larger values of λ result in a more biased model.

Determine which of the above statements are true.

A I only

B II only

C III only

D I, II, and III

E The answer is not given by (A), (B), (C), or (D).

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A modeler creates a cubic spline model with Property Claim Frequency as the response variable and Age of Building Construction as a continuous predictor. The modeler put knots in at Age = $\{10, 20, 30, 50\}$.

The modeler believes that she is overfitting on the ends of the distribution and decides to impose an additional constraint that the curve before the first knot at Age = 10 and after the last knot at Age = 50 will be linear.

Calculate the number of degrees of freedom used by this new model.

(round to the nearest whole number)

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