CA – Quiz 2

Stats

Finish Quiz

We are given a sample from a random variable:

$$1.3 \quad 2.7 \quad 8.2 \quad 5.3 \quad b$$

A rectangular kernel with a bandwidth of b is used to estimate the probability density function of the random variable.

The kernel estimate of the cumulative distribution function at $x = 6.75$ is 0.80 using the rectangular kernel.

Calculate the maximum possible value of b.

Incorrect Answer

14% A Less than 2.5

43% At least 2.5, but less than 3.5

26% At least 3.5, but less than 4.5

13% D At least 4.5, but less than 5.5

5% E At least 5.5

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X_i is the severity of claim i, which has an exponential distribution with mean = θ.

The payment for the claim under an insurance policy is capped at u.

There are $(n+s)$ total claims, with $\{x_1, x_2, \ldots, x_n\}$ claims with payment less than u, and s claims with payment capped at u.

Determine which of the following is the MLE for θ.

9% A $\frac{1}{n}\sum_{i=1}^{n} x_i + \frac{u}{s}$

6% B $\frac{1}{n}\sum_{i=1}^{n} x_i + \frac{nu}{s}$

6% C $\frac{1}{n}\sum_{i=1}^{n} x_i + \frac{u}{n}$

73% D $\frac{1}{n}\sum_{i=1}^{n} x_i + \frac{su}{n}$

6% E None of (A), (B), (C), or (D) are correct.

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Mrs. Actuarial Gardener has used a global positioning system to lay out a perfect 20-meter by 20-meter gardening plot in her back yard.

Her husband, Mr. Actuarial Gardener, decides to estimate the area of the plot. He paces off a single side of the plot and records his estimate of its length. He repeats this experiment an additional 4 times along the same side. Each trial is independent and follows a normal distribution with a mean of 20 meters and a standard deviation of 2 meters. He then averages his results and squares that number to estimate the total area of the plot.

Which of the following is a true statement regarding Mr. Gardener's method of estimating the area?

Incorrect Answer

4% A On average, it will underestimate the true area by at least 1 square meter.

2% B On average, it will underestimate the true area by less than 1 square meter.

35% C On average, it is an unbiased method.

40% ✔ On average, it will overestimate the true area by less than 1 square meter.

18% ✖ On average, it will overestimate the true area by at least 1 square meter.

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Determine which of the following statements is true about an estimator $\hat{\theta}$ and parameter θ.

I. If $\hat{\theta}$ is unbiased and efficient, then $\hat{\theta}$ is the MVUE of θ.

II. If $\hat{\theta}$ is the MVUE of θ, then $\hat{\theta}$ is efficient.

III. The efficiency of $\hat{\theta}$ is the estimator's variance divided by the Rao-Cramér lower bound.

Incorrect Answer

39% — I only

23% — B — II only

5% — C — III only

17% — D — I, II, and III

16% — The correct answer is not given by (A), (B), (C), or (D),

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Let $X_1, X_2, ..., X_n$ be a random sample from an exponential distribution with the following density function:

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0$$

You use the following estimator to estimate λ.

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} X_i}$$

Determine which of the following statements are true. (select all that apply)

65% ✔ $\hat{\lambda}$ is the method of moments estimator of λ.

77% ✔ $\hat{\lambda}$ is the maximum likelihood estimator of λ.

62% ✔ $\hat{\lambda}$ is a sufficient statistic for λ.

50% D $\hat{\lambda}$ is the minimum variance unbiased estimator of λ.

31% E $\hat{\lambda}$ attains the Rao-Cramér lower bound.

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You are given the following information:

- τ is a parameter of a distribution.
- The true value of τ is 2.
- $\hat{\alpha}$ and $\hat{\beta}$ are two uncorrelated estimators for τ.
- $\mathrm{E}\left[\hat{\alpha}\right] = 2.0$
- $\mathrm{Var}\left[\hat{\alpha}\right] = 5.0$
- $\mathrm{E}\left[\hat{\beta}\right] = 3.0$
- $\mathrm{Var}\left[\hat{\beta}\right] = 1.0$

Consider the class of estimators of τ which are of the form: $w\hat{\alpha} + (1 - w)\hat{\beta}$.

Calculate the value of w that results in an estimator with the smallest mean square error.

Incorrect Answer

18% A Less than 0.2

55% ✓ At least 0.2, but less than 0.4

14% C At least 0.4, but less than 0.6

6% ✗ At least 0.6, but less than 0.8

8% E At least 0.8

Finish Quiz

You are given the following information:

- X and Y are random variables with unknown means μ_x and μ_y.
- X and Y have unknown but equal variances.
- Samples from X and Y are taken with sizes 40 and 50, respectively.
- $\frac{\sum X_i}{40} = \bar{X} = 80.1$ and $\frac{\sum Y_i}{50} = \bar{Y} = 78.8$
- $\frac{\sum \left(X_i - \bar{X}\right)^2}{40} = 34.8$ and $\frac{\sum \left(Y_i - \bar{Y}\right)^2}{50} = 25.0$
- $H_0 : \mu_x = \mu_y$
- $H_1 : \mu_x > \mu_y$

Determine the p-value of the hypothesis test.

5% A Less than 0.05

6% B At least 0.05, but less than 0.10

48% ✔ At least 0.10, but less than 0.15

4% D At least 0.15, but less than 0.20

36% E At least 0.20

Finish Quiz

Two six-sided dice (X and Y) are rolled 1,000 times each. The outcomes are in the table below.

Outcome	X	Y
1	94	157
2	244	168
3	135	197
4	158	136
5	128	220
6	241	122

An outcome of 4 or more is considered a success. Let P be the probability of a success.

- $H_0 : P_X = P_Y$
- $H_1 : P_X \neq P_Y$

Calculate the minimum significance level at which the null hypothesis will be rejected.

28% A Less than 0.025

51% At least 0.025, but less than 0.035

7% C At least 0.035, but less than 0.045

5% D At least 0.045, but less than 0.055

8% E At least 0.055

Finish Quiz

Let X be a single observation from the distribution:

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\theta}\right)^2\right)$$

You are testing the null hypothesis $H_0 : \theta = y$ against the alternative hypothesis $H_1 : \theta = 10y$. The null hypothesis is rejected if $X > k$. The probability of a Type I error is 5.0%.

Calculate the probability of a Type II error.

Incorrect Answer

65% ✔ Less than 4.0%

12% B At least 4.0%, but less than 8.0%

14% ✖ At least 8.0%, but less than 12.0%

6% D At least 12.0%, but less than 16.0%

3% E At least 16.0%

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To test whether taking a driver's education course improves driving skills, you set up an experiment. Participants are given a pretest of their driving skills and are tested on their skills again after the course. The data is contained in the table below.

User ID	Pretest	Posttest	Difference
1	62	60	-2
2	93	94	1
3	97	100	3
4	77	80	3
5	69	75	6
6	72	78	6
7	99	100	1
8	55	57	2
9	90	91	1
10	66	63	-3
11	76	83	7
12	51	49	-2
13	74	84	10
14	52	61	9
15	97	99	2
16	54	52	-2
Sum	1,184	1,226	42
Sum of Squares	91,940	98,616	352

Assumes the scores follow a Normal distribution.

Calculate the statistic to test if there is a significant difference in the two scores.

1% A Less than -3

B At least -3, but less than -1

Finish Quiz

You are given the following information about a process that follows the normal distribution:

- The variance is known and $\sigma^2 = 25$.
- $H_0 : \mu = 35$
- $H_1 : \mu = 30$
- The sample size, n, is equal to 16.

Determine the minimum possible Type I error such that the probability of Type II error is no more than 2.5%.

13% A Less than 2%

68% ✔ At least 2%, but less than 3%

10% C At least 3%, but less than 4%

5% D At least 4%, but less than 5%

4% E At least 5%

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Let X be a single observation from a continuous distribution with density function:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda > 0$.

The null hypothesis $H_0 : \lambda = 1$ is tested against the alternative hypothesis $H_1 : \lambda > 1$.

Determine the critical region corresponding to the uniformly most powerful test of significance level 0.05.

Incorrect Answer

36% $X < 0.051$

10% B $X > 0.051$

14% C $X < 2.996$

38% $X > 2.996$

1% E $X > 3.689$

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Let x denote a single observation from a distribution with the density function:

$$f(x;\theta) = \begin{cases} \dfrac{2(x+\theta)}{1+2\theta}, & \text{for } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

where $\theta \geq 0$.

Determine the uniformly most powerful critical region of significance level α for testing the null hypothesis $H_0 : \theta = 0$ against the alternative hypothesis $H_1 : \theta = 1$.

Incorrect Answer

7% A $x \geq \sqrt{\alpha}$

22% B $x \leq \alpha$

15% C $x \leq \sqrt{1-\alpha}$

15% $x \geq \sqrt{1-\alpha}$

40% $x \leq \sqrt{\alpha}$

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Finish Quiz

Let $X_1, \ldots, X_n$ be a random sample from a distribution with density function $f(x) = \dfrac{2\theta^2}{(x+\theta)^3}$, where n is large. You wish to test the hypothesis $H_0 : \theta = 1$ against the hypothesis $H_1 : \theta \neq 1$.

You perform this test, and the null hypothesis is rejected at a 1% significance using the Chi-squared approximation for the likelihood ratio test. Let Λ be the likelihood ratio for this hypothesis test.

Calculate the maximum possible value of Λ.

✖ Incorrect Answer

12% ✖ Less than 0.01

14% B At least 0.01, but less than 0.02

19% C At least 0.02, but less than 0.03

50% ✔ At least 0.03, but less than 0.04

5% E At least 0.04

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You are given the following information about a random sample:

- $X_1, \ldots, X_n$ is a random sample where $n = 100$ and the X_i are distributed $\mathrm{N}(\theta, 4)$.
- $H_0 : \theta = 1$
- $H_1 : \theta \neq 1$
- In this problem, the likelihood ratio test is defined as the likelihood function for the null hypotheses divided by the likelihood function for the alternative hypotheses. The likelihood function in the denominator uses $\hat{\theta}$, an estimate of μ from the random sample, calculated as the maximum likelihood estimate for the normal distribution.
- $\bar{X} = 2$
- The formula for the Normal Distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), \quad \text{for } -\infty < x < \infty$$

Calculate the test statistic that is a function of the likelihood ratio defined above in terms of the Chi-Square distribution and select the range below that contains the p-value for this test.

✖ Incorrect Answer

54% ✔ Less than 0.005

6% B At least 0.005, but less than 0.01

19% ✖ At least 0.01, but less than 0.025

11% D At least 0.025, but less than 0.05

Finish Quiz

You are given the following information about two loss severity distributions fit to a sample of 275 closed claims:

- For the Exponential distribution, the natural logarithm of the likelihood function evaluated at the maximum likelihood estimate is -828.37.
- For the Weibull distribution, the natural logarithm of the likelihood function evaluated at the maximum likelihood estimate is -826.23.
- The Exponential distribution is a subset of the Weibull distribution.
- The null hypothesis is that the exponential distribution provides a better fit than the Weibull distribution.

Calculate the minimum significance level at which one would reject the null hypothesis.

6% A Less than 0.5%

7% B At least 0.5%, but less than 1.0%

10% C At least 1.0%, but less than 2.5%

65% ✔ At least 2.5%, but less than 5.0%

13% E At least 5.0%

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You observe the following distribution for a random sample of 100 claims.

Layer	Number of Claims
Less than 5,000	31
Between 5,000 and 10,000	19
Between 10,000 and 15,000	26
Greater than 15,000	24

You are testing the null hypothesis that claim severity follows a Weibull distribution with $\theta = 10{,}000$ and $\tau = 1.10$, against the alternative hypothesis that claim severity does not follow the distribution specified in the null hypothesis.

Using the chi-square goodness-of-fit test to evaluate the null hypothesis, what is the conclusion?

Incorrect Answer

6% A Do not reject the null hypothesis at the 5.0% significance level

10% Reject the null hypothesis at the 5.0% significance level, but not at the 2.5% significance level

63% Reject the null hypothesis at the 2.5% significance level, but not at the 1.0% significance level

7% D Reject the null hypothesis at the 1.0% significance level, but not at the 0.5% significance level

14% E Reject the null hypothesis at the 0.5% significance level

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Finish Quiz

You are given:

- A coin is tossed 100 times.
- 58 heads are observed.

Calculate the upper bound of the 90% symmetric confidence interval for the true probability of tossing a heads.

5% A Less than 0.62

8% B At least 0.62, but less than 0.64

8% C At least 0.64, but less than 0.66

75% D At least 0.66, but less than 0.68

3% E At least 0.68

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Finish Quiz

You are given:

- A random sample of n losses is observed from a normal distribution with mean μ and variance σ^2.
- A hypothesis test is conducted to investigate the mean of the loss distribution.
- $H_0 : \mu = k$
- $H_1 : \mu \neq k$
- Multiple $100q\%$ confidence intervals for μ are constructed:

q	$100q\%$ Confidence Interval
0.800	$(22.35, 27.65)$
0.900	$(21.55, 28.45)$
0.950	$(20.83, 29.17)$
0.975	$(20.15, 29.85)$
0.990	$(19.31, 30.69)$

Calculate the p-value of the test if $k = 21$.

✖ Incorrect Answer

3% A Less than 1.0%

7% ✖ At least 1.0%, but less than 2.5%

28% C At least 2.5%, but less than 5.0%

56% ✔ At least 5.0%, but less than 10.0%

Finish Quiz

You are given an independent random sample of size 40, $x_1, x_2, \ldots, x_{40}$, from an exponential distribution with mean 150.

Calculate the probability that the second smallest observation (the observation with one observation smaller and 38 observations larger in the dataset) is greater than 20.

Incorrect Answer

13% Less than 0.02

63% At least 0.02, but less than 0.04

5% C At least 0.04, but less than 0.06

14% D At least 0.06, but less than 0.08

4% E At least 0.08

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