$= E[(x-d)_+] + d\,S(d)$

$\Rightarrow E[Y^P] = \dfrac{E(Y^L)}{S(d)}$

$= \dfrac{E[(x-d)_+] + d\,S(d)}{S(d)}$

$= e(d) + d$

→ Example → $X \sim \text{Pareto}(\alpha = 2.5, \theta = 100)$

→ $E(Y^P) = e(d) + d$

$X-d \mid X>d \sim \text{Pareto}(\alpha, \theta + d)$

$\Rightarrow e(d) = E[X-d \mid X>d]$

$= \dfrac{\theta + d}{\alpha - 1}$

$= \dfrac{100+50}{2.5-1} + 50$

$= 150$

→ Impact of deductibles on claim frequency

→ Recall # losses ≠ # payments necessarily when there is a deductible d

→ If # of losses N is modelled by an $(a,b,0)$ class distribution, the # of payments N' will follow the same distribution, but w/ modified parameters

Dist	N	N'
Poisson	λ	$v\lambda$
Binomial	n, p	n, vp
NB	r, β	$r, v\beta$

where v is the prob of a loss being greater than the deductible $P(X>d)$ = prob of + payment

---

→ Assignment

→ Q1) $X \sim \text{Exp}(\theta)$

$e(d) = E(X-d \mid X>d)$

$X-d \mid X>d \sim \text{Exp}(\theta)$

$\Rightarrow e(d) = \theta$

→ Q2) → $X \sim \text{Exp}(\theta = 2000)$

$d = 500$

$d' = 1000$

→ $|E(X-d) - E(X-d')| = ??$

→ $E(X-d) = \int_{500}^{\infty} (x-500)\frac{1}{2000} e^{-x/2000}\,dx$

$= \langle \ldots \rangle$

$= 1557.602$

$E(X-d') = \int_{1000}^{\infty} (x-1000)\frac{1}{2000} e^{-x/2000}\,dx$

$= \langle \ldots \rangle$

$= 1213.061$

$\Rightarrow |E(X-d) - E(X-d')| = 344.54$

OR

→ $E[(X-d)_+] = E(X) - E(X \wedge d)$, with $E[(X-d)_+] = Y^L$ and $E(X \wedge d) = \theta(1-e^{-d/\theta})$

$= 2000 - 2000(1-e^{-500/2000})$

$= 442.399$

$= 1557.602$ ✓

→ $E[(X-d')_+] = \langle \ldots \rangle$

$= 1213.061$ ✓

1.2.3 → Coinsurance

→ def → Coinsurance is the portion of the loss the insurer is responsible for

→ Suppose loss X → insurer pays αX, $0<\alpha<1$ + policyholder pays $(1-\alpha)X$

→ of course $E(\alpha X) = \alpha E(X)$

→ Coinsurance + deductible → Coinsurance comes into play After the deductible is met

$$\boxed{\begin{aligned} &\to Y^L = \begin{cases} 0 & x \le d \\ \alpha(x-d) & x>d \end{cases} \\ &\to E(Y^L) = \alpha[E(X) - E(X\wedge d)] \end{aligned}}$$

→ If coinsurance is applied before the deductible, then instead of applying the entire loss X against the deductible, the insurer will only apply the coinsured portion of the loss, αX. The policyholder will still be responsible for the remaining portion $(1-\alpha)X$ + amount needed to meet deductible

$$\Rightarrow \boxed{\begin{aligned} Y^L &= \begin{cases} 0 & \alpha x \le d \\ \alpha x - d & \alpha x > d \end{cases} \\ &= \begin{cases} 0 & x \le \frac{d}{\alpha} \\ \alpha(x - \frac{d}{\alpha}) & x > \frac{d}{\alpha} \end{cases} \end{aligned}}$$

→ ex) if $\alpha = 0.8$, $x = 3$, $d = 2$

After ⇒ payment $= 0.8(3-2) = 0.8$

Before ⇒ payment $= 0.8(3) - 2 = 0.4$

⇒ to solve for Expected value, make the following adjustment

$E(Y^L) = \alpha[E(X) - E(X \wedge \frac{d}{\alpha})]$

→ General formula for all coverage modifications

$E(Y^L) = \alpha[E(X\wedge m) - E(X\wedge d)]$ ★

X = loss RV

u = policy limit (∞ if no limit)

d = deductible (0 if none)

α = coinsurance (1 if none)

m = maximum covered loss

$$\boxed{\downarrow = \frac{u}{\alpha} + d}$$

$$\to \boxed{Y^L = \begin{cases} 0 & x \le d \\ \alpha(x-d) & d<x<m \\ m & x \ge m \end{cases}}$$

→ if only policy limit ⇒ $m = u$

if — — — + deductible ⇒ $m = u + d$

if — — — — — — + coinsurance ⇒ $m = \frac{u}{\alpha} + d$

→ $E[X \wedge \infty] = \int_0^\infty S(x)\,dx = E(X)$

$E(X\wedge 0) = \int_0^0 S(x)\,dx = 0$

⇒ $E[(X-0)_+] = E(X) - E(X\wedge 0) = E(X)$

→ examples → losses $X \sim \text{Pareto}(\alpha = 2, \theta = 1000)$

$d = 200$ & $u = 2000$

→ $E(Y^L) = E(X\wedge m) - E(X\wedge d)$, with $m = u + d = 2000 + 200 = 2200$ and $E(X\wedge d) = \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right]$, $\alpha \neq 1$

$= E(X\wedge 2200) - E(X\wedge 200)$

$= \frac{1000}{2-1}\left[1 - \left(\frac{1000}{2200+1000}\right)^{2-1}\right] - \langle \ldots \rangle$

$= 687.50 - 166.67$

$= 520.83$

→ losses $X \sim \text{Gamma}(\alpha = 3, \theta = 100)$

$u = 250$, $c = ??$

$E(cX) = E(X\wedge u)$ → $\int_0^u x f(x)\,dx + u\,S(u) = 200c$

OR

$\alpha\theta\,\Gamma(\alpha+1, u/\theta) + u[1 - \Gamma(\alpha, u/\theta)]$

$3(100)\,\Gamma(3, \frac{250}{100}) + 250\left[1 - \Gamma(3, \frac{250}{100})\right]$

$c\,E(X) = \alpha\theta\,c$

$3(100)c = 3(100)(0.4563) + 250(1-0.7127) = 200c$

$200c \approx 163$

⇒ $c \approx 0.815$

→ $N \sim \text{Poisson}(\lambda = 2.5)$

⇒ $\Gamma(3, 2.5) = 1 - P(N<3)$

$= 1 - P(N = 0,1,2)$

$= 1 - \left[\frac{e^{-2.5} 2.5^0}{0!} + \cdots + \frac{e^{-2.5} 2.5^2}{2!}\right]$

$= 0.4563$

$= \langle \ldots \rangle$

$= 0.7127$

OR

$\Gamma(\alpha, \frac{x}{\theta})$: GAMMA.DIST(x, α, β = θ, TRUE)

$= \langle \ldots \rangle (250, 3, 100)$

choose x so that $x = \frac{u}{\theta}$

→ $X \sim \text{Pareto}(\alpha = 3, \theta = 400)$

$B = \begin{cases} 0.15(500 - X) & x<500 \\ 0 & x \ge 500 \end{cases}$ ★ be careful defining RV

$= 0.15(500 - X)_+$ switched compared to usual

⇒ $E(B) = E[0.15(500 - X)_+]$

$= 0.15\,E[(500 - X)_+]$

$= \left[E(500) - E(500 \wedge X)\right]$, where $\min(500, X) = \min(X, 500) = E(X\wedge 500)$

$= \left[500 - \frac{400}{3-1}\left(1 - \left(\frac{400}{500+400}\right)^{3-1}\right)\right]$

$= 50.43$

→ Assignment

→ Q1 → loss $X \sim \text{Exp}(\theta = 600)$

$d = 200$, $\alpha = 0.80$, $u = 800$

→ $Y^L = \begin{cases} 0 & x \le d \\ \alpha(x-d) & d<x<m \\ m & x \ge m \end{cases}$, $m = \frac{u}{\alpha} + d$

$= \begin{cases} 0 & x \le 200 \\ 0.8(x-200) & 200<x<1200 \\ 1200 & x \ge 1200 \end{cases}$

→ $E(Y^L) = \alpha[E(X\wedge m) - E(X\wedge d)]$, with $E(X\wedge d) = \theta(1-e^{-x/\theta})$

$= 0.8[E(X\wedge 1200) - E(X\wedge 200)]$

$= 0.8\left[600(1-e^{-1200/600}) - 600(1-e^{-200/600})\right]$

$E(Y^P) = E(Y^L \mid X>d) = \dfrac{278.974}{S(d)}$, $S(d) = e^{-200/600}$

$= 389.3397$

OR

Ignore deductible b/c memoryless property → $X - d \mid X>d = X$

$E(Y^P) = E(Y^L) = \alpha\,E(X\wedge 1000)$

$= 0.8(600)(1 - e^{-1000/600})$

$= 389.3397$

1.2.4 → Inflation

→ Inflation & its impacts on coverage modifications

$E[(1+r)X \wedge d] = (1+r)\,E\left[X \wedge \frac{d}{1+r}\right]$

→ Example → losses $X \sim$ S-P Pareto $(\alpha = 3, \theta = 100)$

$r = 0.10$, $u = 220$

$E[(1+0.10)X \wedge 220] = 1.1\,E\left[X \wedge \frac{220}{1+0.1}\right]$, with $E(X\wedge x) = \frac{\alpha\theta}{\alpha-1} - \frac{k\theta^\alpha}{(\alpha-1)x^{\alpha-1}}$

$= 1.1\left[\frac{3\cdot 100}{3-1} - \frac{(1)100^3}{(3-1)200^{3-1}}\right]$

$= 165$

OR

$Y = (1+r)X$

$= 1.1X \sim$ S-P Pareto $(\alpha = 3, \theta = 1.1(100) = 110)$

⇒ $E[Y\wedge 220] = \frac{3(110)}{3-1} - \frac{(1)110^3}{(3-1)220^{3-1}}$

$= 165$ ✓

OR

$E[1.1X \wedge 220] = \int_0^\infty \min(1.1x, 220) f(x)\,dx$

$= \int_0^\infty 1.1\min(x, \frac{220}{1.1}) f(x)\,dx$

$= 1.1\int_0^{200} \min(x, 200) f(x)\,dx$

$= 1.1\,E[X\wedge 200]$

→ losses $X_{16} \sim \text{Uniform}(0, 1000)$

$r = 0.2$, $d = 300$

2016 → $E(X) = 500$

$E(X\wedge 300) = \int_0^{300} S(x)\,dx$

$= \int_0^{300} \left(1 - \frac{x}{1000}\right) dx$

$= \langle \ldots \rangle$

$= 255$

2017 → $E(1.2X) = 600$

$E[(1+0.2)X\wedge 300] = (1+0.2)\,E\left[X\wedge \frac{300}{1+0.2}\right]$

$= 1.2\int_0^{250} S(x)\,dx$

$= \langle \ldots \rangle$

$= 262.5$

$LER_{16} - LER_{17} = \dfrac{255}{500} - \dfrac{262.5}{600}$, $LER = \dfrac{E(X\wedge d)}{E(X)}$

$= 0.51 - 0.4375$

$= 0.0725$

→ updating all coverage modification formula for inflation

$$\boxed{\begin{aligned} E(Y^L) &= \alpha(1+r)\left[E\left(X\wedge \tfrac{m}{1+r}\right) - E\left(X \wedge \tfrac{d}{1+r}\right)\right] \\ E[(Y^P)^k] &= \frac{E[(Y^L)^k]}{S_X\left(\frac{d}{1+r}\right)} \\ &\Updownarrow \\ E[(Y^L)^k] &= E[(Y^P)^k]\,S_X\left(\tfrac{d}{1+r}\right) \end{aligned}}$$

or use $X^* = (1+r)X$, $\theta^* = (1+r)\theta$

+ $E(Y^L) = \alpha[E(X^*\wedge m) - E(X^*\wedge d)]$

use w/ scale distributions

→ Example → $S(x) = 1 - \frac{x}{200}$, $0 \le x \le 200$

→ $X \sim \text{Uniform}(0, 200)$

$d = 20$, $m = 100$ ⇒ $u = 100 - 20 = 80$

$V(Y^P) = E[(Y^P)^2] - E(Y^P)^2$

★ Solve for Y^L + convert

$= \dfrac{E[(Y^L)^2]}{S(d)} - \left(\dfrac{E(Y^L)}{S(d)}\right)^2$

$= \dfrac{4053.333}{0.9} - \left(\dfrac{56}{0.9}\right)^2$

$= 632.0988$

$$\boxed{\to E[(Y^L)^k] = \int_d^m (x-d)^k f(x)\,dx + u\,S(m)}$$

→ $Y^L = \begin{cases} 0 & x<20 \\ x-20 & 20<x<100 \\ 80 & x \ge 100 \end{cases}$

→ $E[(Y^L)^2] = \int_{20}^{100} (x-20)^2 \frac{1}{200}\,dx + 80^2 S(100)$

$= \langle \ldots \rangle$

$= 4053.333$

→ $E(Y^L) = \int_{20}^{100} (x-20)\frac{1}{200}\,dx + 80\,S(100)$

$= \langle \ldots \rangle$

$= 56$

→ Assignment

→ Q1) losses $X \sim \text{Pareto}(\alpha = 2, \theta = 1600)$

$u = 2457$, $r = 0.05$

$\%\Delta = \dfrac{E[(1+r)X\wedge 2457] - E(X\wedge 2457)}{E(X\wedge 2457)} = \dfrac{997.266 - 968.991}{968.991} = 0.029695$

→ $E[X\wedge 2457] = \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{u+\theta}\right)^{\alpha-1}\right]$

$= \frac{1600}{2-1}\left[1 - \left(\frac{1600}{2457+1600}\right)^{2-1}\right]$

$= 968.991$

→ $E[1.05X\wedge 2457]$ = ⟨same as above except w/ 1680 for 1600⟩

$\sim \text{Pareto}(\alpha = 2, \theta = 1.05(1600) = 1680)$ $= 997.266$

→ Q2 → losses $X \sim \text{Pareto}(\alpha = 4, \theta = 1500)$, $r = 0.20$

$E[((1+r)X - 100)_+] = E[(X^* - 100)_+]$, $X^* \sim \text{Pareto}(\alpha = 4, \theta = 1.2(1500))$

$= E(X^*) - E[X^*\wedge 100]$

$= \frac{\theta^*}{\alpha-1} - \frac{\theta^*}{\alpha-1}\left[1 - \left(\frac{\theta^*}{d+\theta}\right)^{\alpha-1}\right]$

$= \frac{1800}{4-1} - \frac{1800}{4-1}\left[1 - \left(\frac{1800}{100+1800}\right)^{4-1}\right]$

$= 510.1618$