correct = $\frac{37}{45} = 82.2\%$

think right, but wrong!

$\frac{29}{45} = 64\%$ → Difficulty = 5.7

w/ guesses

w/o guesses = $\frac{26}{45} = 57.8\%$

Prediction Result

✓ ✓ 1) $x \sim$ uniform $(-1,1)$ → $F(x) = \frac{x+1}{2}$

Potential U → $F_U(u) = P(U \le u) = P\left(\frac{x+1}{2} \le u\right)$

$= F_X(2u-1) \Rightarrow = \frac{(2u-1)+1}{2} = u$ ✓

→ $F_U(u) = P(U \le u) = P(|x| \le u)$

$= P(-u \le x \le u)$

$= F_X(u) - F_X(-u) \Rightarrow = \frac{u+1}{2} - \frac{-u+1}{2} = \frac{2u}{2} = u$ ✓

→ $F_U(u) = P(U \le u) = P(x^2 \le u)$

$= F_X(\sqrt{u}) \rightarrow \frac{\sqrt{u}+1}{2} \ne u$ ✗

✗ ✗ 2) $X \sim$ Pareto$(\alpha=3, \theta)$

$CTE_{0.95} = 10000 \Rightarrow 10000 = \pi_q + \frac{E(X) - E(X \wedge \pi_q)}{1-q}$

$= E(X \mid \cancel{X > \pi_q})$ = $E(X - \pi_q \mid X > \pi_q) + \pi_q$

$\sim$ Pareto$(\alpha, \theta + \pi_q)$, $\pi_q = 1.7144\theta$

$= \frac{\theta + \pi_q}{3-1} + \pi_q$

$= \frac{2.7144\theta}{2} + 1.7144\theta$

$\Rightarrow \theta = 3255.632$

$0.95 = 1 - \left(\frac{\theta}{x+\theta}\right)^3$

$\frac{\theta}{\theta+x} = (0.05)^{1/3}$

$\theta = (0.05)^{1/3}(\theta+x)$

$\frac{(1-0.05^{1/3})\theta}{0.05^{1/3}} = x$

$x = 1.7144\theta$

✓ ✗ 3) $N \sim PP(\lambda = 20/\text{hour})$

$P(E) = 0.3$, $P(L) = 0.7$ } thinning

↳ 18min ↳ 30min

X_1 → first train

X_2 → first express train

$E(X_1 - X_2) = E(X_1) - E(X_2)$

$= 32.4 - 34.2857$

$= -1.8857$

$N_E \sim PP(\lambda = 6/\text{hour}) \Rightarrow T_E \sim \text{Exp}(\lambda = 6)$

$N_L \sim PP(\lambda = 14/\text{hour}) \Rightarrow T_L \sim \text{Exp}(\lambda = 14)$

$X_1 \Rightarrow \overset{E}{\frac{\lambda_E}{\lambda_E+\lambda_L}}\left[E(T_E) + 18\right] \overset{OR}{+} \overset{L}{\frac{\lambda_L}{\lambda_E+\lambda_L}}\left[E(T_L) + 30\right]$

choose express [waiting time (hrs) + travel time (min)]

$= \frac{6}{6+14}\left[10 + 18\right] + \frac{14}{6+14}\left[\frac{60}{14} + 30\right]$

$= 8.4 + 24$ double counted wait time

$= 32.4$

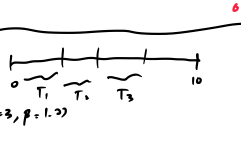
overthought → Not hard

$X_2 \Rightarrow$ ~~L~~ E

$E(L) + 30$

$\cancel{\frac{60}{14} + 30} = 34.2857$

✓ ✗ 4) $N(t) \sim PP(\lambda = 1.2)$



unordered times of past events ~ uniform $[0, t_n]$

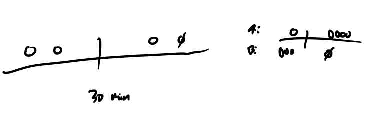
⇒ Find $X_{(j)}$ of interest ↳ out of $n-1$

$T_1 + T_2 + T_3 \sim$ Gamma$(\alpha=3, \beta=1.2)$

$E(T_1+T_2+T_3) = \frac{\alpha}{\beta} = \frac{3}{1.2} = 2.5$

given < 10 ~~→ assuming~~ irrelevant

✓ ✓ 5) $N(t) \sim PP(\lambda = 3/30\text{min})$



x	f(x)	$N(t)$
1	0.65	⇒ $N_1 \sim PP(\lambda = 1.95)$
5	0.2	⇒ $N_5 \sim PP(\lambda = 0.6)$
10	0.15	⇒ $N_{10} \sim PP(\lambda = 0.45)$

$P(N_{10}(1) \ge 1 \cap N_{10}(2) \ge 2) = \underset{P(A)}{(1 - P(N_{10}(1)=0))}$ ~~$(\ldots(1) \ge 1)$~~ $\overset{P(B)}{+}$

$= P(N_{10}(1)=1) * P(N_{10}(2) - N_{10}(1) \ge 1) + P(N_{10}(1) \ge 2)$

$= 1 - P(\ldots=0)$

$= (e^{-0.45}\,0.45)(1 - e^{-0.45}) + (1 - e^{-0.45} - e^{-0.45}\,0.45)$

$= 0.1039 + 0.0754 = 0.1794$

✓ ✓ 6) $A \sim PP\begin{cases} \lambda = 5/\text{hr} & \text{(half day)} \\ \lambda = 2/\text{hr} & \text{(half day)} \end{cases}$

$T \sim PP(\lambda = 1.5)$

Compound Poisson Process ⇒ $S = \sum_{i=1}^{A(24)} T_i$

$E(S(t)) = E(A)\,E(T) = 24\frac{1}{2}[5+2](1.5) = 126$

$V(S(t)) = E(A)\,E(T^2) = 24(\frac{1}{2})[5+2](1.5^2+1.5) = 315$

$V(T) = E(T^2) - (E(T))^2$

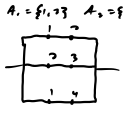
$\lambda = E(T^2) - \lambda^2$

$P(S > 150) \approx 0.088$

↳ same answer w/ continuity correction $P(S \ge 150.5)$

✓ ✓ 7) minimal path sets

$A_1 = \{1,2\}$ $A_2 = \{2,3\}$ $A_3 = \{1,4\}$



$\phi(x) = \max(\min(x_1,x_2), \min(x_2,x_3), \min(x_1,x_4))$

$= \max(x_1x_2, x_2x_3, x_1x_4)$

$= 1 - (1-x_1x_2)(1-x_2x_3)(1-x_1x_4)$

$= 1 - [(1 - x_2x_3 - x_1x_2 + x_1x_2x_3)(1 - x_1x_4)]$

$= 1 - [1 - x_1x_4 - x_2x_3 + x_1x_2x_3x_4 - x_1x_2 + x_1x_2x_4 + x_1x_2x_3 - x_1x_2x_3x_4]$

$= x_1x_4 + x_2x_3 - \cancel{x_1x_2x_3x_4} + x_1x_2 - x_1x_2x_4 - x_1x_2x_3 + \cancel{x_1x_2x_3x_4}$

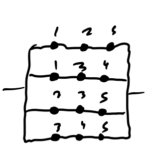
$= x_1x_4 + x_2x_3 + x_1x_2 - x_1x_2x_4 - x_1x_2x_3$

$= p_1p_4 + p_2p_3 + p_1p_2 - p_1p_2^2 - p_1^2p_2$

$= 3p_1p_2 - p_1p_2^2 - p_1^2p_2$

$= 0.87$

✓ ✓ 8) minimal path sets



1 element →

2 element → ~~{1,2}~~, {1,3}, ~~{1,4}~~, {1,5}

{2,3}, {2,4}, ~~{2,5}~~, ~~{3,4}~~

{3,5}, {4,5} ↳ not minimal b/c system functions if all else are on

minimal cut sets ⇒ assume all others on, turn any on & functions

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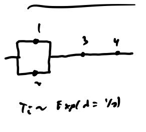
✓ ✓ 9) minimal cut sets



$T_i \sim \text{Exp}(\lambda = 1/2)$

functioning after 1 year → large + and → $P\{\max(T_1,T_2) \ge 1) \cap (T_3 \ge 1) \cap (T_4 \ge 1)\}$

$= P(\max(T_1,T_2) \ge 1)\,[e^{-1/2}]^2$

↳ $P(Y \le y) = P(\max(T_1,T_2) \le y)$

$= P(\text{both}(T_1,T_2) \le y)$

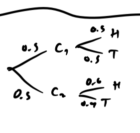
$= (F_T(y))^2$

$= (1 - e^{-y/2})^2$

$= (1 - (1 - e^{-1/2})^2)\,[e^{-1/2}]^2$

$= 0.3109$

✓ ✗ 10)



gambler's ruin

$P(H) = 0.55$, $P(T) = 0.45$ } →

$P(\text{winning 20 units}) = \frac{1 - (\frac{q}{p})^i}{1 - (\frac{q}{p})^j}$ → start, → stop

$= \frac{1 - (\frac{0.45}{0.55})^5}{1 - (\frac{0.45}{0.55})^{20}}$

$= 0.645$

Coin only chosen once !!!

✓ ✓ 11) Exp steps $= \frac{1}{\pi_0}$ → excel work

$= \frac{1}{0.03636}$

$= 275$

✓ ✓ 12) E(premium) = E(Benefit)

① ②

① → P

② → $\overset{\text{die 1}}{1000v\,q_{65}} + \overset{\text{die 2}}{1000v^2\,{}_1p_{65}\,q_{66}} + \overset{\text{die 3}}{1000v^3\,{}_2p_{65}\,q_{67}} + \overset{\text{not die}}{Pv^3\,{}_3p_{65}}$

↳ 0.15, ↳ 0.85, ↳ 0.2, ↳ 0.85(0.8), ↳ 0.75, ↳ 0.85(0.8)(0.75)

$P = 404.543 + 0.383P$

$\Rightarrow P = 655.407$

✗ ✓ 13) estimate $= \frac{1}{n}\sum_{i=1}^{n} K_i(x)$

↳ draw each kernel + find area below F(x)

↳ centered at x_i

✓ ✓ 14) $X \sim$ uniform$(0,\theta)$

$\hat{\theta} = \frac{2}{n}\sum x_i = 2\bar{x}$

$V(\hat{\theta}) = V(2\bar{x}) = 4V(\bar{x}) = 4\frac{V(x)}{n} = \frac{4}{n}\left(\frac{\theta^2}{12}\right) = \frac{\theta^2}{3n}$

✗ ✓ 15) $f(y) = \prod_{i=1}^{n} \frac{2y_i}{\theta} e^{-(y_i^2/\theta)}$

$= \left(\frac{2}{\theta}\right)^n \prod y_i\, e^{-\frac{1}{\theta}\sum x^2}$

$= h(y)\; g(t(y), \theta)$

exp → $f(y) = \prod \lambda e^{-\lambda y_i}$

$= \lambda^n e^{-\lambda \sum y_i}$

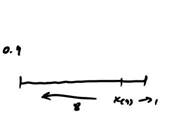
✓ ✓ 16) (qualitative)

✓ ✓ 17) $X_i \overset{iid}{\sim}$ Uniform$(0,\theta)$, $i = 1,\ldots,10$

H_0: $\theta = 1$

H_a: $\theta = 1.1$

RR : $X_{(9)} > 0.9$



$f_{X_{(9)}}(x) = \frac{10!}{8!1!1!}(F_X(x))^8 f(x)(1 - F(x))$

$= 90\left(\frac{x}{\theta}\right)^8\left(\frac{1}{\theta}\right)\left(\frac{\theta - x}{\theta}\right)$

$= \frac{90}{\theta^{10}}x^8(\theta - x)$

$= \frac{90x^8}{\theta^9} - \frac{90}{\theta^{10}}x^9 \Rightarrow F(x) = \int_0^x f(t)\,dt$

$= \frac{10t^9}{\theta^9} - \frac{9}{\theta^{10}}t^{10}\Big|_0^x$

$= \frac{10x^9}{\theta^9} - \frac{9}{\theta^{10}}t^{10}$

$\alpha = P(X_{(9)} > 0.9 \mid \theta = 1) = 1 - F_{X_{(9)}}(0.9)$

$= 1 - \left[\frac{10(0.9^9)}{1^9} - \frac{9}{1^{10}}0.9^{10}\right]$

$= 0.2639$

$\beta = P(X_{(9)} < 0.9 \mid \theta = 1.1)$

$= \frac{10(0.9^9)}{1.1^9} - \frac{9}{1.1^{10}}0.9^{10}$

$= 0.432$

✓ ✗ 18) UMP is test w/ ratio of likelihoods (probabilities) is the smallest

↳ RR = {x = 5}

$\beta_1 = P(x = 2,4,7 \mid \theta = 3) = 0.1 + 0.06 + 0.08 = \cancel{0.75}$ 0.84

$\beta_2 = 1 - P(x=5 \mid \theta = 3) = 1 - 0.08 = 0.92$ ↗ |difference| = 0.17

↳ keep selecting until get α = 0.05

~ ✗ 19)

↳ assuming ~~$\hat{\theta}_{MLE} = \bar{x}$~~ = 26 } Find MLE first

i	x	fitted cdf $\hat{F}_x(x_{(i)})$	empirical cdf $F_n(x_{(i)})$	d_i
1	4	0.4337	0.2	0.4337
2	6	0.6387	0.4	0.4387 → = T.S.
3	7	0.7043	0.6	0.7043
4	10	0.8231	0.8	0.2231
5	11	0.8477	1	0.1527

Correct

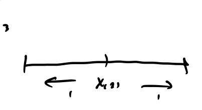
✓ ✗ 20) 95% CI = $\bar{x} \pm \cancel{1.96}\frac{s_x}{\sqrt{n}}$

$= 77.833 \pm \cancel{1.96}\frac{14.783}{\sqrt{6}}$ → $t_{5,0.025}$

$= [62.003, 93.663]$

✓ ✓ 21) $X_i \overset{iid}{\sim} \text{Exp}(\lambda = 1/1000)$, $i = 1,2,3$

$P(X_{(2)} < 2000)$



$f_Y(y) = \frac{3!}{1!1!1!}[F(x)\,f(x)\,(1 - F(x))]$

$= 6\left[1 - e^{-x/1000}\right]\frac{1}{1000}e^{-x/1000}\left[e^{-x/1000}\right]$

$= \frac{6}{1000}\left[1 - e^{-x/1000}\right]e^{-x/500}$

$= \frac{3}{500}\left[e^{-x/500} - e^{-3x/1000}\right]$

$F_{X_{(2)}}(2000) = \int_0^{2000} f_{X_{(2)}}(x)\,dx$

$= \frac{3}{500}\left[-500e^{-x/500} + \frac{1000}{3}e^{-3x/1000}\right]_0^{2000}$

$= \frac{3}{500}\left[-500e^{-\frac{2000}{500}} + \frac{1000}{3}e^{-\frac{3(2000)}{1000}} + 500 - \frac{1000}{3}\right]$

$= 0.95$

cdf of $Y = X_{(2)}$ → $P(Y \le y)$

$= P(X_{(2)} \le y)$

$= P(\text{atleast } 2\ X_i \le y)$

↳ $Y_i \sim$ Binomial$(n=3, p = P(X_i \le y))$ ↳ y = 2000

$P(Y \ge 2) = \langle \ldots \rangle$

✓ ✓ 22)

(qualitative)

✓ ✓ 23)

going to zero

{1,1,1,1,1} + {23, 23, 23, 23, 23}

$Y_{(3)} = \frac{Y_{(3)} + Y_{(4)}}{2}$

✓ ✓ 24)

< excel work >

✓ ✓ 25) $SE(\hat{\beta}_0) = SD(\hat{\beta}_0) = 2.004$

✓ ✓ 26)

$\hat{\beta} = (x'x)^{-1}x'y$ ⇒ excel work

$y = X\hat{\beta}$

↳ = $\begin{bmatrix} 1.355 \\ 0.038 \\ 1.374 \end{bmatrix}$ ✓

⇒ $R^2 = 1$ $(\hat{y} = y)$

✓ ✓ 27) $F = 3.75 = \frac{MS_{Treat}}{MSE} = \frac{606.69}{MSE} = \frac{SS_{Treat}/2}{SSE/n-3}$

$MS_{Treat} = 606.69$, df = 2

$SS_{To} = 2670.75$

⇒ $SS_{Treat} = 1213.38$

⇒ $SSE = 1456.87$

⇒ $3.75 = \frac{606.69(n-3)}{(1456.87)/(n-3)}$

$3.75 = \frac{606.69n - 606.69(3)}{1456.87}$

⇒ n = 12

✓ ✓ 28)

Source	df	SS
Mean	1	76246
Treat	6	x
Resid	22	y
Total	29	98125

⇒ includes mean ⇒ total df = n

① x + y = 21879

$F = 3.3669 = \frac{x/6}{y/22}$ ⇒ assuming $F = \frac{MS_{Treat}}{MS_{Error}}$ → w/ intercept SS taken out

② $\frac{3.3669y}{22} = x/6$

$0.9182y = x$

⇒ $1.9182y = 21879$

$y = 11405.735$

✓ ✗ 29) $VIF = \frac{1}{1 - R_j^2}$ = 0 ⇒ VIF = 1 ↳ Think!!

$\hat{y} = X\hat{\beta}$

$= X(X'X)^{-1}X'y$

$= Hy$

↳ $\sum \text{diag}(H) = p+1$

~ ✗ 30) $D_i = \frac{DFFITS^2}{MSE}$ → guessing Cook's distance is "standardized" DFFITS

$\frac{e_i^2h_i}{MSE(p+1)(1-h_i)^2} = \frac{DFFITS^2}{\cancel{MSE}\ p+1}$

$\langle \ldots \rangle =$

$0.1981 = \frac{DFFITS^2}{MSE}$

DFFITS = −3.1777

↳ sign of e_i

$\hat{Y}_i = 52.261$

Cook's distance $= \frac{e_i^2h_i}{MSE(p+1)(1-h_i)^2}$

$= \frac{DFFITS^2}{p+1}$

⇒ DFFITS $= \sqrt{d_i(p+1)}$ ↳ sign of residual

✓ ✓ 31) Bootstrap estimates are biased, but $SE(\beta)$ is accurate

(qualitative)

✓ ✓ 32) (qualitative)

✓ ✗ 33) $R^2_{adj} = \left(\frac{n-p}{n-1}\right)R^2$ → $R^2_{adj} = 1 - \left(\frac{n-1}{n-p}\right)\frac{SSE}{SSTO}$

< excel work > complement & inflate decrease

✓ ✗ 34)

(qualitative)

~ ✗ 35) (qualitative)

✓ ✓ 36) unconstrained model → $\sum_{j=1}^{2}\hat{\beta}_j^2 = 50.09$

Constrained model → $50.09 > 2.1^2 + x^2$

$6.759 > x$

✓ ✓ 37) (qualitative)

~ ✓ 38)

(qualitative)

✓ ✓ 39) (qualitative)

✗ ✗ 40) guessing decrease the prob be $x^T\beta$ is less

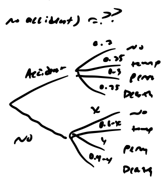
→ solve for parameters

~~proportional odds cumulative logit model ≠ regular logit model~~

✗ ✓ 41) P(death | accident) = 0.25 ⇒ P(Death | Accident) = 0.25

P(none or temp | no accident) = 0.6 ⇒ P(Perm or death | no accident) = 0.4

P(Perm | no accident) = ??



✓ ✓ 42) adding constraints ⇒ less model df

✗ ✗ 43) $\left(88.71 = 10 + \hat{\beta}_{11}(4.5) + \hat{\beta}_{21}(4.5^2)\right)\frac{5.7}{4.5}$

$\begin{cases} 103.910 = 10 + \hat{\beta}_{11}(5.7) + \hat{\beta}_{21}(5.7)^2 \\ 112.4167 = 12.667 + 5.7\hat{\beta}_{11} + 25.65\hat{\beta}_{21} \end{cases}$

$-9.5067 = -2.667 + 6.84\hat{\beta}_{21}$

$\hat{\beta}_{21} = -1$

⇒ $\hat{\beta}_{11} = 22$

$\left(101.214 = 370 + \hat{\beta}_{12}(26.3) + \hat{\beta}_{22}(26.3^2)\right)\frac{27}{26.3}$

$\begin{cases} 105.4 = 370 + \hat{\beta}_{12}(27) + \hat{\beta}_{22}(27^2) \\ 103.9029 = 379.848 + 27\hat{\beta}_{12} + 710.1\hat{\beta}_{22} \end{cases}$

$-1.492 = -9.848 + 18.9\hat{\beta}_{22}$

$\hat{\beta}_{22} = 0.6$

⇒ $\hat{\beta}_{21} = -36$

$Y = 10 + 22x - x^2$ ← when ~~equal?~~ first derivatives are equal!! → $Y = 370 - 36x + 0.6x^2$

$10 + 22x - x^2 = 370 - 36x + 0.6x^2$

$0 = 360 - 48x + 1.6x^2$

$x = \frac{48 \pm \sqrt{(-48)^2 - 4(1.6)(360)}}{2(360)}$

$= 0.0867$

✓ ✗ 44) (qualitative)

✗ ✗ 45) $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1x + \hat{\beta}_2x^2$

use matrices or linest (y_s, x) ↓ from $\hat{y}_s$ ↘ design matrix

① $4.2 = \hat{\beta}_0 + \hat{\beta}_1 3 + \hat{\beta}_2 9$

② $8.3 = \hat{\beta}_0 + \hat{\beta}_1 6 + \hat{\beta}_2 36$

③ $9.4 = \hat{\beta}_0 + \hat{\beta}_1 9 + \hat{\beta}_2 81$

① $\hat{\beta}_0 = 4.2 - 3\hat{\beta}_1 - 9\hat{\beta}_2$

② → $8.3 = \langle \ldots \rangle + 6\hat{\beta}_1 + 36\hat{\beta}_2$

$0 = -4.1 + 3\hat{\beta}_1 + 25\hat{\beta}_2$

$\hat{\beta}_1 = \frac{4.1 - 25\hat{\beta}_2}{3}$

get rid of variables, don't substitute!

②-③ $-1.1 = -3\hat{\beta}_1 - 45\hat{\beta}_2$

$-1.1 = 4.1 - 25\hat{\beta}_2 - 45\hat{\beta}_2$

$-5.2 = -70\hat{\beta}_2$

$0.074 = \hat{\beta}_2$

⇒ $\hat{\beta}_1 = 0.7476$

⇒ $\hat{\beta}_0 = 1.2886$

} @ x = 10, $\hat{y} = 1.2886 \cancel{+ 0.7476(10)} + 0.074(10^2)$

educated guess going to get round values for coefficients likely