1.4.4 → Properties of Poisson Process

→ Thinning

→ If a Poisson process can be broken down into two or more disjoint subprocesses, then these disjoint subprocesses are also Poisson processes w/ proportional rate functions. This process is called thinning.

→ Let $N \sim PP(\lambda(t))$ & $N_1, N_2, \ldots, N_n$ be disjoint subprocesses of N w/ proportions $\pi_1, \pi_2, \ldots, \pi_n$, respectively ($\pi_1 + \pi_2 + \cdots + \pi_n = 1$).

☆ ⇒ $N_1, N_2, \ldots, N_n$ are ⊥ PP w/ rate functions $\pi_1\lambda(t), \pi_2\lambda(t), \ldots, \pi_n\lambda(t)$, respectively

→ If N is a homogeneous PP & the proportions $\pi_1, \pi_2, \ldots, \pi_n$ are constant, the disjoint subprocesses $N_1, N_2, \ldots, N_n$ will also be homogeneous PPs. But if not constant, then non-homogeneous

→ Examples → Taxis arrive according to a PP w/ rate $\lambda(t) = 3t$

→ owners are registered to own countries

$P(A) = 0.3$, $P(B) = 0.45$, $P(C) = 0.25$

→ by $t=2$ → $\lambda(t) = \int_0^2 3t\,dt = 6$

→ $N_A(2) \sim Pois(\lambda = 0.3(6) = 1.8)$

$N_B(2) \sim Pois(\lambda = 0.45(6) = 2.7)$

$N_C(2) \sim Pois(\lambda = 0.25(6) = 1.5)$

→ $P(N_B(2) \ge 2) = 1 - P(N_B(2) \le 1) = 1 - \sum_{i=0}^{1} \frac{e^{-2.7}2.7^i}{i!} = 0.7513$

→ $P(N_A(2)=3, N_C(2)=2) = P(N_A(2)=3)\,P(N_C(2)=2)$ <b/c ⊥>

$= \frac{e^{-1.8}1.8^3}{3!}\left(\frac{e^{-1.5}1.5^2}{2!}\right) = 0.0403$

→ $N(t) \sim PP(\lambda = 100 \text{ per shift})$, t = hours past 8am

Spending $ = $X \sim Pareto(\alpha = 3, \theta = 300)$ → High $ spender if $X > 400$

$N_H(t)$ = # high $ spenders = disjoint subprocess of N

→ $P(X > 400) = 1 - F_X(400) = 1 - \left[1 - \left(\frac{300}{400+300}\right)^3\right] = 0.0787$

→ # high $ spenders between 8am to 10am is a Poisson RV w/ mean

$\lambda = \underbrace{100\left(\frac{2}{12}\right)}_{\text{rate over interval } (0,\, 2/12]}(0.0787) = 1.3120$

↳ π_1 → high spending & non-high spending are the subprocess (not each hour)

⇒ $P(N_H(\frac{2}{12}) = 2) = \frac{e^{-1.312}1.312^2}{2!} = 0.2318$

→ Superposition

→ The opposite of thinning a Poisson process is superposition. The superposition of a PP is the sum of 2 or more PPs (recall $\sum Pois \sim Pois$)

→ Let $N_1, N_2, \ldots, N_n$ be ⊥ PPs w/ respective rate functions $\lambda_1(t), \lambda_2(t), \ldots, \lambda_n(t)$.

☆ ⇒ $N_1 + N_2 + \cdots + N_n \sim PP(\lambda = \lambda_1(t) + \lambda_2(t) + \cdots + \lambda_n(t))$

→ Example → Taxis arrive at a $PP(\lambda(t) = 3t)$

→ Again countries → $P(A) = 0.3$, $P(B) = 0.45$, $P(C) = 0.25$

From earlier $\lambda = \int_0^2 \lambda(t)\,dt = 6$ ⇒ $\lambda_A = 1.8$, $\lambda_B = 2.7$, $\lambda_C = 1.5$

→ $P(N_{A,C}(2) = 3) = \frac{e^{-3.3}3.3^3}{3!} = 0.2209$

↳ $\sim PP(\lambda = \lambda_A + \lambda_C = 1.8 + 1.5 = 3.3)$

→ Joint probabilities

→ Assume we have 2 ⊥ PPs N_1 & N_2 w/ rates λ_1 & λ_2. We can determine the prob of observing one event from N_1 before observing one event from N_2.

→ Denote times to the first event for each process as $T_{1,1}$ & $T_{2,1}$. Since both of these are exponential

$$\boxed{P(T_{1,1} < T_{2,1}) = \frac{\lambda_1}{\lambda_1 + \lambda_2}}$$

→ We can generalize this to determine the prob of observing n events from the first process before observing m events from the second process (doesn't matter when m^{th} event from PP_2 occurs, as long as it is after n^{th} event from PP_1).

⇒ use negative binomial dist to calc the prob of observing at most $m-1$ events from PP_2 before the n^{th} event from PP_1. Each event from PP_1 has prob $\frac{\lambda_1}{\lambda_1+\lambda_2}$ ⇒ prob of event from $PP_2 = \frac{\lambda_2}{\lambda_1+\lambda_2}$

$$\boxed{P(T_{1,n} < T_{2,m}) = \sum_{i=0}^{m-1}\binom{n+i-1}{n-1}\underbrace{\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^n}_{\text{fixed } n \text{ successes}}\underbrace{\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^i}_{\sum_i \text{ possible scenarios (\# of failures)}}}$$

OR

→ use binomial dist → out of the first $n+m-1$ events, there needs to be at least n events from PP_1 & at most $m-1$ events from PP_2 (this guarantees n^{th} from PP_1 is before m^{th} from PP_2).

$$\boxed{P(T_{1,n} < T_{2,m}) = \sum_{i=n}^{n+m-1}\binom{n+m-1}{i}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^i\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{(n+m-1)-i}}$$

→ Examples

→ Time until next bus arrives $\sim Exp(\theta = 0.25)$

→ 25% of buses are express & 75% are local

→ J & R ride bus to work. J takes first bus (takes 20 min to get to office) while R takes the first express bus (takes 10 min to get to office)

→ R arrives 4 min after J

→ $E(\text{R to get to office}) = E(T_R + \overset{\text{travel time}}{\tfrac{1}{6}}) = 1 + \tfrac{1}{6} \text{ hours} = 70 \text{ min}$ (T_R ↓ ~ Exp(1 hour))

overall → $\theta = 0.25 \Rightarrow \underset{\text{rate}}{\lambda} = 4$

→ 25% of buses are express ⇒ $4(0.25) = 1$ rate of express

→ $P[(T_R + \tfrac{1}{6}) - (T_J + \tfrac{1}{3}) \ge \tfrac{1}{12}] = P[T_R \ge T_J + \tfrac{1}{4}]$

$T_R \sim Exp(\lambda = 1)$; $T_J \sim Exp(\lambda = 3)$, $= 0.75(4)$

$= P(\underset{A}{\{T_R \ge T_J\}} \cap \underset{B}{\{T_R \ge T_J + \tfrac{1}{4}\}})$ ⇒ $P(A \cap B) = P(A)P(B|A)$

$= P(T_R \ge T_J) \cdot P(T_R \ge T_J + \tfrac{1}{4} \mid T_R \ge T_J)$

$= \frac{\lambda_J}{\lambda_J + \lambda_R} \cdot P(T_R > \tfrac{1}{4})$ (by memoryless property)

$= \frac{3}{3+1}\, e^{-1(1/4)}$

$= 0.5841$

→ Same scenario

→ P(4 local before 2 express)

→ $P(\underset{\text{local}}{T_J} \le \underset{\text{express}}{T_R}) = \frac{3}{3+1} = 0.75$

→ $P(T_{J,4} \le T_{R,2})$ = → Binomial (out of first $4+2-1 = 5$ buses, need ≥ 4 local)

$\sum_{i=4}^{5}\binom{5}{i}0.75^i(0.25)^{5-i} = 0.6328$

→ ~NB (at most 1 express before 4^{th} local)

$= \sum_{i=0}^{1} \sim NB(r=4, p=0.75) \overset{\checkmark}{=} 0.6328$

→ Assignment

→ Q1) $N \sim PP$ → N_T & N_F: $\lambda = 3$ / day

N_M, N_W, & N_{Th}: $\lambda = 4$ / day

N_{Sa} & N_{Su}: $\lambda = 1$ / day

$P(N_{M-Su}(7) = 18) = \frac{e^{-20}20^{18}}{18!} = 0.08439$

↳ $\sim Pois(\lambda = \sum \lambda_i)$, $\lambda = 20$

→ Q2) $N(t) \sim PP(\lambda = 5/\text{year})$

$P(\underset{\text{claim size}}{X} < 100000) = 0.9$ → $N_H(t)$ = High dollar claims

→ $P(N_H(3) = 0) = \frac{e^{-1.5}1.5^0}{0!} = 0.2231$

↳ $\sim Pois(\lambda = \lambda \cdot t \cdot P(X > 100000))$, $\lambda = 5 \cdot 3 \cdot 0.1 = 1.5$

→ Q3) $N \sim PP(\lambda = 5/\text{hour})$ → $P(\text{Local}) = 0.2$, $P(E) = 0.8$

$P(T_E < T_L) = \frac{4}{4+1} = 0.8$ $\lambda_L = 0.2(5)$ $\lambda_E = 0.8(5)$

$P(T_{E,2} < T_{L,2})$ = → Binomial (out of $2+2-1$ buses, need ≥ 2 express)

$\sum_{i=2}^{3}\binom{3}{i}0.8^i\,0.2^{3-i} = 0.896$

→ ~NB (at most 1 express before 2^{nd} local)

$\sum_{i=0}^{1} \sim NB(r=2, p=0.8) = 0.896$

1.4.5 → Mixture of Poisson Process

→ overview → Thus far, $\lambda(t)$ has been either a constant or a function of time. What if the rate function is a RV?

⇒ mixture of Poisson processes, which creates a new process (note that it is not a PP) → Only $N|\Lambda$ is Poisson, not unconditional N

→ Let $N|\Lambda$ be a Poisson RV w/ rate Λ, where Λ is a RV

⇒ the unconditional dist of N has the following properties

→ The pmf of N is:

$P(N=n) = E_\Lambda[P(N=n \mid \Lambda = \lambda)]$

$= E_\Lambda\left[\frac{e^{-\Lambda}\Lambda^n}{n!}\right]$ → sum/integrate the conditional pmf/pdf over all possible values of Λ

Discrete Λ: $= \sum_{\lambda=0}^{\infty}\frac{e^{-\lambda}\lambda^n}{n!}p_\Lambda(\lambda)$

Continuous Λ: $= \int_0^\infty \frac{e^{-\lambda}\lambda^n}{n!}f_\Lambda(\lambda)\,d\lambda$

→ Mean & variance

→ $E(N) = E_\Lambda[E_N(N|\Lambda)] = E_\Lambda(\Lambda)$

→ $V(N) = E_\Lambda[V(N|\Lambda)] + V_\Lambda[E(N|\Lambda)] = E_\Lambda(\Lambda) + V_\Lambda(\Lambda)$

→ Examples

→ Annual # claims $N \sim Pois(\Lambda)$

↳ $p(\lambda) = \begin{cases} 0.8 & \lambda = 2 \\ 0.2 & \lambda = 6 \end{cases}$

→ $P(N=5) = P(N=5 \cap \Lambda = 2) + P(N=5 \cap \Lambda = 6)$

$= P(N=5 \mid \Lambda=2)\cdot P(\Lambda=2) + P(N=5 \mid \Lambda=6)\cdot P(\Lambda=6)$

$= \frac{e^{-2}2^5}{5!}(0.8) + \frac{e^{-6}6^5}{5!}(0.2)$

$= 0.0616$

→ $V(N) = E_\Lambda[V(\underset{\sim Pois}{N|\Lambda})] + V_\Lambda[E(N|\Lambda)]$ ⇒ $V(\cdot) = \Lambda$ ↳ also a RV ↳ same logic

$= E_\Lambda(\Lambda) + V_\Lambda(\Lambda)$

$= \sum \lambda p(\lambda) + \sum(\lambda - 2.8)^2 p(\lambda)$

$= 2.8 + 2.56$

$= 5.36$

→ Annual # claims $N|\Lambda \sim Pois(\Lambda)$ & $\Lambda \sim Exp(\lambda)$

$P(N=0) = \int_0^\infty \frac{e^{-\lambda}\lambda^0}{0!}\lambda e^{-\lambda}\,d\lambda$

☆ $= \int_0^\infty \lambda e^{-2\lambda}\,d\lambda$ $u = \lambda$, $dv = e^{-2\lambda}$; $du = 1$, $v = -\frac{1}{2}e^{-2\lambda}$

$= \left[-\frac{1}{2}\lambda e^{-2\lambda} + \int \frac{1}{2}e^{-2\lambda}\right]_0^\infty$

$= -\frac{1}{2}\lambda e^{-2\lambda} - \frac{1}{4}e^{-2\lambda}\Big|_0^\infty$

$= -e^{-2\lambda}\left(\frac{\lambda}{2} + \frac{1}{4}\right)\Big|_0^\infty$

$= 0 + \frac{1}{4}$

$= 0.25$

OR

☆ $= \int_0^\infty \lambda e^{-2\lambda}\,d\lambda$

$= \frac{\Gamma(2)}{2^2}\int_0^\infty \cancel{\frac{2^2}{\Gamma(2)}\lambda e^{-2\lambda}}\,d\lambda$ $\sim Gamma(\alpha=2, \beta=2)$

$= \frac{1}{4}$

→ Poisson & Gamma

→ There is a special case where the rate function of a Poisson process, Λ, follows a gamma dist

→ Let $N|\Lambda \sim Poisson(\Lambda)$, $\Lambda \sim Gamma(\alpha, \theta)$

⇒ Poisson−Gamma mixture ⇒ $N \sim NB(r = \alpha, \beta = \theta)$

→ Redoing last example → $N|\Lambda \sim Pois(\Lambda)$ & $\Lambda \sim Gamma(\alpha=2, \theta=1)$ $\frac{1}{1^2\Gamma(2)}\lambda^1 e^{-\lambda} \overset{\checkmark}{=} \lambda e^{-\lambda}$

⇒ $N \sim NB(r=2, \beta=1)$

$P(N=0) = (1+\beta)^{-r} = (1+1)^{-2} = 1/4$

→ Another example → # claims $N|\Lambda \sim Pois(\Lambda)$, $\Lambda \sim Gamma(\alpha=2, \theta=50)$

→ $P(N=5) = \frac{r(r+1)\cdots(r+k-1)\beta^k}{k!(1+\beta)^{r+k}} = \frac{2\cdot3\cdot4\cdot5\cdot6\,(50^5)}{5!(1+50)^{2+5}} \approx 0.0021$

↳ $\sim NB(r=2, \beta=50)$

→ $V(N) = r\beta(1+\beta) = 2(50)(1+50) = 5100$

→ Assignment

→ Q1) → $N|\Lambda \sim Pois(\Lambda)$, $\Lambda \sim Geo(\text{mean} = 1 \Rightarrow \beta = 1)$

$P(N=0) = \sum_{\lambda=0}^{\infty}\frac{e^{-\lambda}\lambda^0}{0!}\cdot\frac{1^\lambda}{(1+1)^{\lambda+1}}$

$= \sum_{\lambda=0}^{\infty}\frac{e^{-\lambda}}{2^{\lambda+1}}$

$= \frac{1}{2}\sum_{\lambda=0}^{\infty}(2e)^{-\lambda}$

$= (\ldots)$ series formula   ☆ use scratch pad (Excel) to approximate series using first few terms

$\overset{\checkmark}{\approx} 0.6127$

→ Q2) $N|\Lambda \sim Pois(\Lambda)$, $\Lambda \sim$ uniform → $E(\Lambda) = 2$, $V(\Lambda) = 4/3$

→ $\frac{a+b}{2} = 2$ → $a = 4 - b$

$\frac{(b-a)^2}{12} = 4/3$ → $\frac{(b-(4-b))^2}{12} = 4/3$

$4b^2 - 16b + 16 = 48/3$

$b - 4 = 0$

$b = 4 \Rightarrow a = 0$

$P(N=0) = \int_0^4 \frac{e^{-\lambda}\lambda^0}{0!}\cdot\frac{1}{4}\,d\lambda$

$= \frac{1}{4}\int_0^4 e^{-\lambda}\,d\lambda$

$= -\frac{1}{4}e^{-\lambda}\Big|_0^4$

$= -\frac{1}{4}(e^{-4} - 1)$

$= 0.2454$

1.4.6 → Compound Poisson Process

→ overview → when claims occur at a Poisson rate & all claim amounts are iid, then the aggregate amount is a Compound Poisson Process

→ Let N be a PP & $X_1, X_2, \ldots, X_{N(t)}$ be iid RVs. Then

$$\boxed{S(t) = \sum_{i=1}^{N(t)} X_i}$$

is a compound Poisson process w/ the following properties

- → $S(0) = 0$
- → If $N(t) = 0$ ⇒ $S(t) = 0$
- → $E[S(t)] = E[N(t)]\cdot E(X)$
- → $V[S(t)] = E[N(t)]\cdot E(X^2)$

Derivations

→ $E[S(t)] = E\left[\sum_{i=1}^{N(t)} X_i\right]$

$= E_{N(t)}\left[E_X\left[\sum_{i=1}^{N(t)} X_i \mid N(t)\right]\right]$

$= E_{N(t)}\left[\sum_{i=1}^{N(t)} E(X_i|N(t))\right]$

$= E_{N(t)}[N(t)E(X)]$

$\overset{\checkmark}{=} E(X)E[N(t)]$

↗ 2 RVs ⇒ need double expectation theorems

$V(S(t)) = E_{N(t)}\left[V_X\left[\sum_{i=1}^{N(t)} X_i \mid N(t)\right]\right] + V_{N(t)}\left[E\left[\sum_{i=1}^{N(t)} X_i \mid N(t)\right]\right]$

$= E_{N(t)}\left[\sum_{i=1}^{N(t)} V(X_i|N(t))\right] + V_{N(t)}\left[\sum_{i=1}^{N(t)} E(X_i|N(t))\right]$

$= E_{N(t)}[N(t)V(X_i)] + V_{N(t)}[N(t)E(X_i)]$

$= V(X_i)E[N(t)] + (E(X_i))^2\underbrace{V(N(t))}_{= E(N(t)) \text{ b/c } N(t) \sim PP}$

$= E[N(t)][V(X) + (E(X))^2]$

$\overset{\checkmark}{=} E[N(t)][E(X^2)]$

→ Distribution of $S(t)$ → Difficult to determine. However, b/c it is the sum of iid RVs, we can use the CLT to estimate probabilities of events

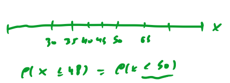
→ $S(t) \overset{\text{approx}}{\sim} N(E[S(t)], V[S(t)])$

⇒ $P(S(t) \le s) \approx \Phi\left[\frac{s - E[S(t)]}{\sqrt{V[S(t)]}}\right]$

*Using continuity correction if claim amounts are discrete

↳ ☆ Always use this for CLT if discrete dists

→ be careful if domain ≠ 0, 1, 2, 3, ...



$P(X \le 48) = P(X < 50)$ (← next possible value)

$\approx P(S \le 47.5)$ ↳ midpoint of gap ↳ normal approx

$P(X > 50) = 1 - P(X < 65)$

$= 1 - P(S \le 57.5)$

→ Examples

→ # claims $N(t) \sim PP(\lambda = 1000/\text{year})$ & $X_i \overset{iid}{\sim} N(\mu = 500, \sigma^2 = 2000)$

$P(S(1) > 520000) = 1 - \Phi\left[\frac{520000 - 500{,}000}{\sqrt{252{,}000{,}000}}\right] = 1 - \text{norm.dist}(x: 520000, \text{mean}: 500{,}000, \text{standard\_dev} = \text{sqrt}(252{,}000{,}000), \text{cumulative} = \text{TRUE}) \approx 0.1038$

↳ $S(1) \sim N\left(\mu = E[N(1)]E(X) = 1000(500) = 500{,}000;\ \sigma^2 = V(N(1))E(X^2) = V(N(1))[V(X) + (E(X))^2] = 1000[2000 + 500^2] = 252{,}000{,}000\right)$

→ $N(t) \sim PP(\lambda(t) = 4t^2)$ & $X_i = \begin{cases} 0.3 & x = 250 \\ 0.6 & x = 500 \\ 0.1 & x = 1000 \end{cases}$

→ $P(S(6) - S(3) < 100{,}000) \approx \text{norm.dist}(x = 99875, \text{mean} = 119700, \text{standard\_dev} = \text{sqrt}(67{,}725{,}000), \text{cumulative} = \text{TRUE}) \approx 0.008$

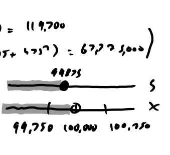
→ $N(6) - N(3) \sim Pois\left(\lambda = \int_3^6 4t^2\,dt = 252\right)$

$E(X) = \sum x p(x) = 475$

$V(X) = \sum(x - 475)^2 p(x) = 43125$

⇒ $S(t) \overset{\text{approx}}{\sim} N\left(\mu = 252(475) = 119{,}700;\ \sigma^2 = 252(43125 + 475^2) = 67{,}725{,}000\right)$

→ Continuity correction →



→ # customers $N(t) \sim PP(\lambda = 100/\text{shift})$, money spent $X_i \overset{iid}{\sim} Gamma(\alpha = 20, \theta)$

$P(S(1) > 295000) = 0.0951$ → $1 - P(Z < z_{1-0.0951}) = 0.0951$

↳ norm.inv(probability = 1−0.0951, mean = 0, standard_dev = 1)

$S(1) \overset{\text{approx}}{\sim} N\left(\mu = 100(20)\theta,\ \sigma^2 = 100[20\theta^2 + 20^2\theta^2]\right)$

⇒ $\frac{295{,}000 - 100(20)\theta}{\sqrt{100[20\theta^2 + 20^2\theta^2]}} = 1.3094$

$\frac{\cancel{\theta}\left(\frac{295{,}000}{\theta} - 100(20)\right)}{\cancel{\theta}\sqrt{100(20 + 20^2)}} =$

$\frac{295{,}000}{\theta} - 100(20) = 1.3094\sqrt{100(20 + 20^2)}$

$\frac{295{,}000}{1.3094\sqrt{100(20+20^2)} + 100(20)} = \theta$

$130.0506 = \theta$

→ Sums of Compound Poisson Processes

→ Let $N_1 \sim PP(\lambda_1)$ & $X_{1,1}, X_{1,2}, \ldots, X_{1,N_1(t)} \overset{iid}{\sim} f_1(x)$ such that

$S_1(t) = \sum_{i=1}^{N_1(t)} X_{1,i}$

is a compound Poisson process. And let $N_2(t) \sim PP(\lambda_2)$ w/ $X_{2,1}, X_{2,2}, \ldots, X_{2,N_2(t)} \overset{iid}{\sim} f_2(x)$ such that

$S_2(t) = \sum_{j=1}^{N_2(t)} X_{2,j}$

is a compound Poisson process.

→ Then $S(t) = S_1(t) + S_2(t)$ is also a compound Poisson process w/ rate

☆ $\lambda = \lambda_1 + \lambda_2$

w/ a discrete mixture severity. Specifically the severity probability function is a weighted average of $f_1(x)$ & $f_2(x)$ w/ respective weights $\frac{\lambda_1}{\lambda_1+\lambda_2}$ & $\frac{\lambda_2}{\lambda_1+\lambda_2}$

→ This can be easily extended to the sum of more than 2 CPPs

→ Example → # Auto claims $N_A(t) \sim PP(\lambda = 400/\text{year})$, $X_A \overset{iid}{\sim} Exp(\theta = 1000)$

Homeowners claims $N_H(t) \sim PP(\lambda = 150/\text{year})$, $X_H \overset{iid}{\sim} N(\mu = 2000, \sigma^2 = 500)$

→ $E(S(t)) = E(N(t))\,E(X) = 550\left(\frac{8}{11}E(X_A) + \frac{3}{11}E(X_H)\right)$

$= 550\left[\frac{8}{11}(1000) + \frac{3}{11}(2000)\right]$

$= 700{,}000$

↳ $S_1(t) + S_2(t)$

$N(t) \sim PP(\lambda = \lambda_A + \lambda_H = 400 + 150 = 550)$

$X = \frac{\lambda_1}{\lambda_1+\lambda_2}f_{X_A}(x) + \frac{\lambda_2}{\lambda_1+\lambda_2}f_{X_H}(x)$

$= \frac{400}{400+150}f_{X_A}(x) + \frac{150}{400+150}f_{X_H}(x)$

$= \frac{8}{11}f_{X_A}(x) + \frac{3}{11}f_{X_H}(x)$

→ Assignment

→ Q1) # withdrawals $N(t) \sim PP(\lambda = 100/\text{day})$, $X = \begin{cases} 0.25 & x = 20 \\ 0.50 & x = 40 \\ 0.10 & x = 50 \\ 0.15 & x = 100 \end{cases}$

$SD[S(t)] = \sqrt{V(S(t))} = \sqrt{265000} = 514.781$

↳ $= V(N(t))E(X^2)$

$= 100(V(X) + (E(X))^2)$

$= 100(25^2 + 45^2)$