CA – Quiz 1

Stats

Finish Quiz

You are given three normal distributions with the same mean and varying variances, $\frac{1}{\theta}, \frac{1}{2\theta}, \frac{1}{4\theta}$.

You select one random draw from each of the distributions respectively: 0, 4, 8

The probability density of the normal distribution is:

$$f\left(x \mid \mu, \sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Determine the Maximum Likelihood Estimate of θ.

Incorrect Answer

31% A Less than 0.04

37% At least 0.04, but less than 0.06

13% C At least 0.06, but less than 0.08

6% At least 0.08, but less than 0.10

13% E At least 0.10

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Claim sizes (in thousands), X, follow a distribution with the following probability density function:

$$f(x) = \frac{\alpha}{(x+0.5)^{\alpha+1}}, \quad x > 0.5$$

You wish to estimate α using the method of moments. You are given the following sample claim sizes:

$$568 \quad 930 \quad 1{,}092 \quad 1{,}142 \quad 1{,}268$$

Calculate the estimate of α.

Incorrect Answer

37% A Less than 1.5

18% B At least 1.5, but less than 2.5

40% C At least 2.5, but less than 3.5

3% D At least 3.5, but less than 4.5

2% E At least 4.5

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You are given the following information:

- $Y_1, Y_2, \ldots, Y_7$ is a random sample of size 7 where the observations are independent and identically distributed from an exponential distribution with mean θ.
- The mean of the sample is 400.

Calculate the minimum variance unbiased estimate of θ^2.

No Answer Provided

27% Less than 150,000

47% B At least 150,000, but less than 175,000

20% C At least 175,000, but less than 200,000

3% D At least 200,000, but less than 225,000

3% E At least 225,000

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Let 150, 204, 310, 480, 500 be a random sample from the density function given by:

$$f(y \mid \alpha, \theta) = \begin{cases} \dfrac{1}{\Gamma(\alpha)\theta^{\alpha}} y^{\alpha-1} e^{-y/\theta}, & \text{for } y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

- $\alpha = 20$
- The coefficient of variation (CV) is the standard deviation divided by the mean.
- θ is estimated using maximum likelihood.

Calculate the CV of the maximum likelihood estimator.

Incorrect Answer

5% A Less than 0.05

27% At least 0.05, but less than 0.15

61% C At least 0.15, but less than 0.25

3% D At least 0.25, but less than 0.35

4% At least 0.35

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Let $X_1, X_2, \ldots, X_8$ be a random sample from a distribution with the following density function:

$$f(x) = \frac{1}{2\theta^3} x^2 e^{-x/\theta}$$

You are given:

$$\sum_{i=1}^{8} X_i = 277 \qquad \sum_{i=1}^{8} X_i^2 = 10{,}585$$

Calculate the standard error of the estimate of θ using the minimum variance unbiased estimator of θ.

13% A Less than 2

40% ✔ At least 2, but less than 4

23% C At least 4, but less than 6

12% D At least 6, but less than 8

11% E At least 8

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You are given the following information:

- A random variable, X, follows a two-parameter Pareto distribution with parameters α = 5 and θ unknown.
- A random sample of 15 independent observations is taken.
- θ is estimated as $c\bar{X}$, where c is a constant.

Calculate the value of c that minimizes the mean squared error of the estimator of θ.

12% A Less than 1.0

16% B At least 1.0, but less than 2.0

12% C At least 2.0, but less than 3.0

41% ✔ At least 3.0, but less than 4.0

19% E At least 4.0

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Losses occur independently with the probabilities as described below:

Loss Size	Probability
0	25%
2	60%
6	15%

Using a sample of size 2, the variance of the individual losses is estimated using the sample variance given by:

$$\sum_{i=1}^{2}(x_i - \bar{x})^2$$

Calculate the mean square error of this estimator.

Incorrect Answer

16% A Less than 8.0

26% B At least 8.0, but less than 16.0

21% At least 16.0, but less than 24.0

27% At least 24.0, but less than 32.0

9% E At least 32.0

Finish Quiz

An actuary obtained the following random sample from an exponential distribution with mean θ :

$$7.96 \quad 23.65 \quad 5.84 \quad 3.38 \quad 2.35$$

Calculate the minimum variance unbiased estimate of θ^2.

2% A Less than 60

17% ✔ At least 60, but less than 65

8% C At least 65, but less than 70

57% D At least 70, but less than 75

16% E At least 75

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Let $X_1, X_2, ..., X_{20}$ be a random sample from a normal distribution with mean μ and variance 4.

You are given:

$$\sum_{i=1}^{20} X_i = 395 \qquad \sum_{i=1}^{20} X_i^2 = 7{,}871$$

Calculate the standard error of the estimate of μ using the minimum variance unbiased estimator of μ.

8% A. Less than 0.40

17% B. At least 0.40, but less than 0.42

20% C. At least 0.42, but less than 0.44

45% D. At least 0.44, but less than 0.46 ✓

9% E. At least 0.46

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Suppose that an urn contains ten balls, some of which are red, while the remainder are black. You want to test the null hypothesis that no more than three of the balls are red against the alternative hypothesis that the urn contains more than three red balls.

To perform this test, you pull three balls from the urn, without replacement, and note their colors. You will reject the null hypothesis if at least two of the balls pulled are red.

Calculate the significance level of this hypothesis test.

✖ Incorrect Answer

7% ✖ Less than 0.05

19% B At least 0.05, but less than 0.10

11% C At least 0.10, but less than 0.15

29% ✔ At least 0.15, but less than 0.20

35% E At least 0.20

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You are an actuary whose company sells personal auto insurance in two states. You wish to test the null hypothesis of standard deviations of claim sizes in the two states being equal against the alternative hypothesis that one standard deviation is larger.

You take random samples of size 6 and 12 from the two populations and compute unbiased sample standard deviations of S_1 and S_2, respectively.

Using an F-test, and defining the statistic $W = S_1/S_2$, calculate the length of the interval of values of W for which the null hypothesis is not rejected, if the significance level is 0.02.

Assume that claim amounts in each state are normally distributed, and that the rejection region has equal probabilities in each tail.

Incorrect Answer

24% Less than 2.5

10% B At least 2.5, but less than 3.0

8% C At least 3.0, but less than 3.5

7% D At least 3.5, but less than 4.0

52% At least 4.0

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You are given the information below:

- $X_1, \ldots, X_{11}$ is a random sample from a normal distribution and the sample mean is 10, the sample variance is 4, and the sample size is 11.
- $Y_1, \ldots, Y_{11}$ is a random sample from a normal distribution and the sample mean is 9, the sample variance is 12, and the sample size is 11.
- $H_0 : \mu_X = \mu_Y$
- $H_1 : \mu_Y > \mu_X$
- Assume that the underlying variance of the two normally distributed random variables is equal.
- Let t be the sample t-statistic used to test the difference of the means from two normally distributed random samples.
- Let T be the critical value at $\alpha = 0.05$.

Calculate the absolute value of the difference between the sample t-statistic and the critical value at $\alpha = 0.05$, $|t - T|$.

Incorrect Answer

3% A Less than 0.75

4% B At least 0.75, but less than 0.85

39% At least 0.85, but less than 0.95

9% D At least 0.95, but less than 1.05

44% At least 1.05

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An insurer is estimating the impact of a loss mitigation program. They ran an experiment to evaluate the severities of five losses before and after the program was instituted:

	A	B	C	D	E
Original Severity	400	800	1,200	2,000	5,000
New Severity	280	500	1,235	1,600	4,800

A paired t-test with the following hypotheses was used to evaluate the effectiveness of this program:

- H_0 : The program had no impact on losses.
- H_1 : The program was able to reduce losses.

Calculate the smallest significance level at which one would reject the null hypothesis.

Incorrect Answer

8% A Less than 1.0%

5% B At least 1.0%, but less than 2.5%

37% C At least 2.5%, but less than 5.0%

34% D At least 5.0%, but less than 10.0%

15% E At least 10.0%

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You are given:

- 100 decks of 52 cards are combined and shuffled to put the cards in a random order.
- Each deck contains only red and black cards.
- The first 4 cards are revealed to be all red cards.
- H_0 : 50% of the cards are red.
- H_A : More than 50% of the cards are red.

Calculate the minimum significance level for which H_0 can be rejected.

11% A Less than 0.005

8% B At least 0.005, but less than 0.010

38% C At least 0.010, but less than 0.050

35% ✔ At least 0.050, but less than 0.100

8% E At least 0.100

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You are given the following information about samples from two populations:

- A random sample of size 8, $X_1, \ldots, X_8$, was drawn from a normal population with unknown mean μ_1 and unknown variance σ_1^2.

$$s_1^2 = \frac{\sum (x_i - \bar{x})^2}{7}$$

- A random sample of size 10, $Y_1, \ldots, Y_{10}$, was drawn from a normal population with unknown mean μ_2 and unknown variance σ_2^2.

$$s_2^2 = \frac{\sum (y_i - \bar{y})^2}{9}$$

You want to test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 < \sigma_2^2$ at a 5% significance level by using the statistic $\frac{s_1^2}{s_2^2}$.

Calculate the boundary of the best critical region for the test.

No Answer Provided

39% ✔ Less than 1.5

4% B At least 1.5, but less than 3.0

54% C At least 3.0, but less than 4.5

Finish Quiz

You have been given the following information to compare two hypotheses using the Neyman-Pearson lemma:

- X_i follows an exponential distribution where $f(x) = \dfrac{e^{-x/\lambda}}{\lambda}, x > 0$.
- $H_0 : \lambda = 1$
- $H_1 : \lambda = 2$
- The numerator of the likelihood ratio test will hold the results for H_0 and the denominator will hold the results H_1.
- The test will be based on a random sample of size 100 with $\bar{X} = 1.5$.

Calculate the lower limit of the critical region value for the likelihood ratio test described above so that the significance level will be 5%.

Incorrect Answer

14% Less than 1.1

38% At least 1.1, but less than 1.2

15% C At least 1.2, but less than 1.3

12% D At least 1.3, but less than 1.4

21% E At least 1.4

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X is a single observation from the probability density function:

$$f(x) = 2\theta x + 1 - \theta, \quad \text{for } 0 < x < 1$$

You are testing the hypothesis:

$$H_0 : \theta = 0 \quad \text{vs.} \quad H_1 : \theta = 1$$

For the most powerful test of significance of size α, determine the critical region for which you would reject the null hypothesis.

8% A $x < \alpha$

27% B $x > \alpha$

10% C $x < 1 - \alpha$

40% D $x > 1 - \alpha$

15% E $x < \alpha/2$

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Finish Quiz

You are given a random sample $X_1, \ldots, X_{10}$ from a normal distribution with an unknown mean μ and a standard deviation of 10. You wish to perform the hypothesis test:

$$H_0 : \mu = 50 \quad \text{vs.} \quad H_1 : \mu < 50$$

You use the rejection region $RR : \left\{\sum_{i=1}^{10} X_i < 450\right\}$.

Consider the following statements regarding this hypothesis test:

I. The probability of a Type I error is less than 0.1.

II. If $\mu = 42$, the probability of a Type II error is less than 0.2.

III. This is the uniformly most powerful test of these hypotheses at its significance level.

Determine which of the above statements is/are true.

Incorrect Answer

7% A None of them are true.

48% B I and II only

12% C I and III only

7% D II and III only

27% E The correct answer is not given by (A), (B), (C), or (D).

Finish Quiz

Let X be a random variable from a beta distribution with $\theta = 1$, and $a = b$. A single observation from this distribution is being used to test

$$H_0 : a = b = 1 \quad \text{vs.} \quad H_A : a = b = 2$$

For the most powerful test of significance level 0.05, determine the critical region for which you would reject the null hypothesis.

✖ Incorrect Answer

6% A $x > 0.05$

35% B $x > 0.95$

24% C $x < 0.05$ or $x > 0.95$

11% ✖ $0.45 < x < 0.55$

25% ✔ $0.475 < x < 0.525$

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Finish Quiz

Loss amounts are subject to an ordinary deductible of 50 and a maximum covered loss of 400. Loss amounts below the deductible are left-truncated whereas loss amounts above the maximum covered loss are right-censored.

You observe the following 5 loss amounts, where + indicates that the value is right-censored:

$$100 \quad 215 \quad 350 \quad 400^{+} \quad 400^{+}$$

Then, using maximum likelihood estimator, you fit the ground-up loss distribution with an exponential distribution.

Using the Kolmogorov-Smirnov test, you test the null hypothesis that the ground-up distribution is an exponential distribution.

Calculate the value of the Kolmogorov-Smirnov test statistic.

Incorrect Answer

1% A Less than 0.10

4% B At least 0.10, but less than 0.12

43% At least 0.12, but less than 0.14

11% D At least 0.14, but less than 0.16

41% At least 0.16

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