2.1)

✓ a) < EDA on computer >

b) → 2 sample t-test

$\rightarrow Y_{jk} \sim N(\mu_j, \sigma^2)$

$H_0: \mu_1 = \mu_2 = \mu$, μ_1 = treatment, μ_2 = control

$H_1: \mu_1 \neq \mu_2$

$$\rightarrow t^* = \frac{\bar{Y}_1 - \bar{Y}_2 - 0}{\sigma_{\bar{Y}_1 - \bar{Y}_2}} = \frac{4.86 - 4.7265}{0.8280\sqrt{\frac{1}{20} + \frac{1}{20}}} \approx 0.5099$$

↳ pooled

⇒ assuming equal variance $\frac{S_1^2}{S_2^2} \overset{\checkmark}{\leq} 2$

$$= s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\hookrightarrow = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(20-1)\,0.7909^2 + (20-1)\,0.8635^2}{20+20-2}} \approx 0.8280$$

$$\rightarrow 95\%\ CI = (\bar{Y}_1 - \bar{Y}_2) \pm t_{n_1+n_2-2,\ 0.025}\ \sigma_{\bar{Y}_1 - \bar{Y}_2}$$

$$= 0.1335 \pm 2.024\,(0.2618)$$

$$= [-0.396,\ 0.6634] \ni 0 \Rightarrow \text{not significant}$$

c) $\rightarrow H_0: E(Y_{jk}) = \mu \rightarrow Y_{jk} \sim N(\mu, \sigma^2)$

$H_1: E(Y_{jk}) = \mu_j \rightarrow Y_{jk} \sim N(\mu_j, \sigma^2)$

→ MLEs →

$$L_0 = f(\mu|y) = \prod_{j=1}^{2}\prod_{k=1}^{20} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(Y_{jk}-\mu)^2}$$

$$= (2\pi\sigma^2)^{-40/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{j=1}^{2}\sum_{k=1}^{20}(Y_{jk}-\mu)^2\right\}$$

$$l_0 = \ln[L_0] = -\frac{40}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_j\sum_k(Y_{jk}-\mu)^2$$

$$l_0' = \frac{1}{\sigma^2}\sum_j\sum_k(Y_{jk}-\mu)$$

$0 = $ < ... >

$$= \sum_j\sum_k Y_{jk} - 40\mu$$

$$\boxed{\hat{\mu}_{MLE} = \frac{\sum_j\sum_k Y_{jk}}{40}}$$

$$L_1 = f(\mu_j|y) = \prod_{j=1}^{2}\prod_{k=1}^{20} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(Y_{jk}-\mu_j)^2}$$

$$= (2\pi\sigma^2)^{-40/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{j=1}^{2}\sum_{k=1}^{20}(Y_{jk}-\mu_j)^2\right\}$$

$$l_1 = -\frac{40}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{j=1}^{2}\sum_{k=1}^{20}(Y_{jk}-\mu_j)^2$$

$$\frac{dl}{d\mu_1} = -\frac{1}{\sigma^2}\sum_{k=1}^{20}(Y_{1k}-\mu_1)$$

$0 = $ < ... >

$$= \sum_{k=1}^{20} Y_{1k} - 20\mu_1$$

$$\boxed{\hat{\mu}_{1,MLE} = \frac{\sum_{k=1}^{20} Y_{1k}}{20}} \text{, likewise } \boxed{\hat{\mu}_{2,MLE} = \frac{\sum_{k=1}^{20} Y_{2k}}{20}}$$

LSEs →

$$S_0 = \sum_{j=1}^{2}\sum_{k=1}^{20}(Y_{jk}-\mu)^2$$

$$S_0' = -2\sum_{j=1}^{2}\sum_{k=1}^{20}(Y_{jk}-\mu)$$

$0 = $ < ... >

$$= \sum_j\sum_k Y_{jk} - 40\mu$$

$$\boxed{\hat{\mu}_{LSE} = \frac{\sum_j\sum_k Y_{jk}}{40}}$$

$$\rightarrow S_1 = \sum_j\sum_k (Y_{jk}-\mu_j)^2$$

$$\frac{\partial S_1}{\partial\mu_1} = -2\sum_{k=1}^{20}(Y_{1k}-\mu_1)$$

$0 = $ < ... >

$$\boxed{\hat{\mu}_{1,LSE} = \frac{\sum_k Y_{1k}}{20}} \text{ likewise } \boxed{\hat{\mu}_{2,LSE} = \frac{\sum_k Y_{2k}}{20}}$$

using data →

parameter	estimate
μ	4.79325
μ_1	4.86
μ_2	4.7265

d) using data → $\hat{S}_0 = $ < ... r ... > $= 26.232$

$\hat{S}_1 = $ < ... r ... > $= 26.053$

e) show $\frac{1}{\sigma^2}\hat{S}_1 = \frac{1}{\sigma^2}\sum_j\sum_k(Y_{jk}-\mu_j)^2 - \frac{20}{\sigma^2}\sum_k(\bar{Y}_j - \mu_j)^2$

? (based on 1.4, which was ?)

show if H_1 true, $\frac{1}{\sigma^2}\hat{S}^2 \sim \chi^2_{38}$

$$\rightarrow \frac{1}{\sigma^2}\hat{S}_1 = \sum_j\sum_k\underbrace{\left(\frac{\overset{N(\mu_j,\sigma^2)}{Y_{jk}} - \mu_j}{\sigma}\right)^2}_{\text{< from ex 1.4 c) }} -$$

$$= \left[\frac{40-2}{\sigma^2}S^2 + \cancel{\frac{20}{\sigma^2}\sum_k(\bar{Y}_j - \mu_j)^2}\right] - \cancel{\frac{20}{\sigma^2}\sum_k(\bar{Y}_j-\mu_j)^2}$$

$$\overset{\checkmark}{\sim} \chi^2_{38}$$

→ similarly, if H_0 is true → $\frac{1}{\sigma^2}\hat{S}_0 \sim \chi^2_{39}$

f) Not exactly sure how, but... understand result :)

$$\frac{\hat{S}_0 - \hat{S}_1}{\hat{S}_1/38} \sim F(1, 38)$$

$$\Rightarrow F = \frac{\chi^2_1/1}{\chi^2_{n-2}/n-2} = \frac{SSE_{reduced} - SSE_{full}}{SS_{full}/df_{full}} \sim F(\text{difference in df between reduced \& full models},\ \text{df reduced model})$$

↓ in SLR

reduction in SSE of full model relative to full model

= difference in # parameters estimated

mean top > mean bottom if H_1

$$F^* = \frac{E(MSR)}{E(MSE)} = \frac{E(SSR/1)}{E(SSE/n-2)} = \frac{\sigma^2 + \beta_1^2 S_{xx}}{\sigma^2}$$

→ $d = ncp$

g) $\frac{\hat{S}_0 - \hat{S}_1}{\hat{S}_1/38} = 0.2599$

h) $t^{*2} = F \Rightarrow$ confirmed w/ $0.5099^2 = 0.2599$

i) < on computer >

2.2

✓ a) $Y_{jk} \sim N(\mu_j, \sigma^2)$

unpaired t-test

$H_0: \mu_1 = \mu_2 = \mu$, μ_1 = Before, μ_2 = After

$H_1: \mu_1 \neq \mu_2$

$$t = \frac{\bar{Y}_1 - \bar{Y}_2 - 0}{\sigma_{\bar{Y}_1 - \bar{Y}_2}} = \frac{103.245 - 100.6}{13.0064\sqrt{\frac{1}{20}+\frac{1}{20}}} = 0.6431$$

$$\hookrightarrow = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$\hookrightarrow \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(20-1)\,13.5173^2 + (20-1)\,12.4745^2}{20+20-2}}$$

$$= 13.0064$$

b) Paired t-test

$\rightarrow D_k = Y_{1k} - Y_{2k} \rightarrow D_k \sim N(\mu_D, \sigma_D^2)$

$H_0: \mu_D = 0$, $\mu_D = \mu_1 - \mu_2$

$H_1: \mu_D \neq 0$

$$\rightarrow t^* = \frac{\bar{D} - 0}{S_D/\sqrt{n_D}} = \frac{2.645 - 0}{4.1167/\sqrt{20}} = 2.873$$ } significant

$$t_{crit} = |t_{(19,\ 0.025)}| = |qt(0.025, 19)| = |2.093|$$

$\alpha = 0.05$

c) different conclusions!

d) $Y_{jk} \overset{iid}{\sim} N(\mu_j, \sigma^2)$ in unpaired t-test

$D_k \overset{iid}{\sim} N(\mu_D, \sigma_D^2)$ in paired t-test

2.3

✗ ? skipping → based on previous didn't know

0

2.4

✓

X	Y
.	.
.	.
.	.
.	.
.	.
.	.

$\rightarrow E(Y) = \ln(\beta_0 + \beta_1 X + \beta_2 X^2)$

$\rightarrow g(E(Y)) = e^{E(Y)} = e^{\ln(\beta_0 + \beta_1 X + \beta_2 X^2)} = \beta_0 + \beta_1 X + \beta_2 X^2$

$$\rightarrow Y = \begin{bmatrix} 3.15 \\ 4.85 \\ 6.50 \\ 7.20 \\ 8.25 \\ 16.50 \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} \quad X = \begin{bmatrix} 1 & 1.0 & 1.0^2 \\ 1 & 1.2 & 1.2^2 \\ 1 & 1.4 & 1.4^2 \\ 1 & 1.6 & 1.6^2 \\ 1 & 1.8 & 1.8^2 \\ 1 & 2.0 & 2.0^2 \end{bmatrix}$$

2.5

$E(Y_{jk}) = \mu_{jk} + \mu + \alpha_j + \beta_j$ $\quad Y_{jk} \sim N(\mu_{jk}, \sigma^2)$

$j = 1, 2$

$k = 1, 2, 3$

$$E(Y) = X\beta = \begin{array}{c} \begin{array}{cccccc} \mu & \alpha_1 & \alpha_2 & \beta_1 & \beta_2 & \beta_3 \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & 1 & 0 & -1 \\ 1 & 1 & -1 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

$\underbrace{\hphantom{000000}}_{?}$