1.4 → Distributions related to the Normal dist

(only writing new stuff)

→ χ^2_n

→ Non central chi-squared dist

→ $z_i \overset{iid}{\sim} N(0,1)$

$Y_i = z_i + \mu_i$ ↳ at least one non-zero

$\Rightarrow \sum Y_i^2 = \sum (z_i + \mu_i)^2 = \sum z_i^2 + 2\sum z_i\mu_i + \sum \mu_i^2$

→ mean $= n + \lambda > n$ = mean of $\sum z_i^2 \sim \chi^2_n$

variance $= 2n + 4\lambda > 2n$ = variance χ^2_n

→ $\lambda = \sum \mu_i^2$

$\Rightarrow \chi^2_{n,\lambda}$ = noncentral chi-squared dist

→ $\underline{Y} = [Y_1, \dots, Y_n]^T \sim MVN(\mu, V)$ ↳ non-singular w/ inverse V^{-1}

↳ not necessarily $\perp$

→ $(\underline{Y} - \underline{\mu})^T \underline{V^{-1}} (\underline{Y} - \underline{\mu}) \sim \chi^2_n$

center scale square

→ More generally → if $Y \sim MVN(\mu, V)$

→ $Y^T V^{-1} Y \sim \chi^2_{n,\lambda}$ ↳ $\lambda = \mu^T V^{-1} \mu$

→ $\sum \chi^2(n_i, \lambda_i) = \chi^2(\sum n_i, \sum \lambda_i)$

→ $F_{n,m}$

→ Non central F dist

$$\frac{\chi^2_{n,\lambda}/n}{\chi^2_m/m} \sim F_{n,m,\lambda} \quad \text{↳ } \lambda = \mu^T V^{-1} \mu$$

↓

mean → mean noncentral w/ equivalent df