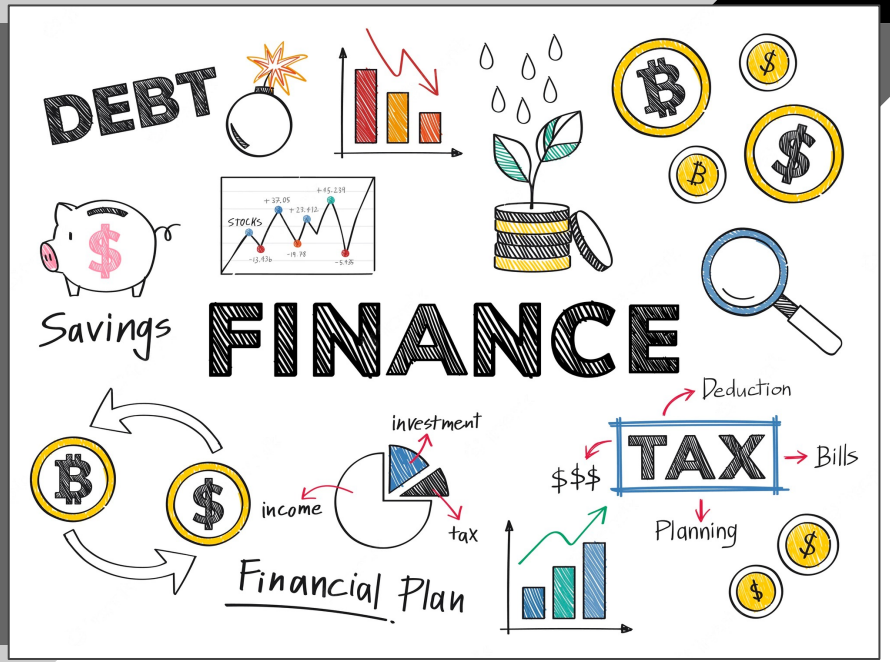
# 6.1
# Understanding Interest



# Goals for the Day

# Some Terms We Need

- **Principal** – the sum of money on which interest is charged (the initial investment)
- **Interest rate** – the amount charged to the borrower expressed as a percentage of the principal OR the amount you earn on an investment (still as a percentage)
- **Annual Percentage Rate (APR)** – the yearly interest rate that is charged for borrowing

# Simple Interest

- Interest calculated only based on the principal
  - I = interest
  - P = Principal
  - r = Rate (as a decimal)
  - t = Time (years)



$$I = Prt$$

## Example

**How much money will I have if... And how much interest will I earn if...**

I invest $500 at 10% APR for 8 years (with simple interest).

$$I = Prt$$
$$= 500 * 0.10 * 8 = \$400$$

$$Total\text{: } A = P + I$$
$$= 500 + 400 = \$900$$

## Example

**How much money will I have if... And how much interest will I earn if...**

I invest $500 at 10% APR for 6 months (with simple interest).

$$I = Prt$$

$$= 500 * 0.10 * \frac{6}{12} = \$25$$

$$Total\text{: } A = P + I$$

$$= 500 + 25 = \$525$$

# Compound Interest

- Interest calculated based on principal and accrued interest
  - A = Total (future) amount
  - n = # of compound intervals

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

(*n* values)

**Table 1: Compounding Intervals**

| Compounding | Number per Year |
|---|---|
| Annually | 1 |
| Semiannually | 2 |
| Quarterly | 4 |
| Monthly | 12 |
| Weekly | 52 |
| Daily | 365 |

# Compound Interest

## Example

**How much money will I have if... And how much interest will I earn if...**

I invests $500 at 10% APR for 8 years compounded monthly? Quarterly?

$$A_{monthly} = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 500\left(1 + \frac{0.1}{12}\right)^{12*8}$$

$$\approx \$1{,}109.09$$

$$A_{quarterly} = 500\left(1 + \frac{0.1}{4}\right)^{4*8}$$

$$\approx \$1{,}101.88$$

What do you notice about the relationship between *n* and *A*?

$$I_{monthly} = A - P$$

$$= 1{,}109.09 - 500 = 609.09$$

$$I_{quarterly} = A - P$$

$$= 1{,}101.88 - 500 = 601.88$$

# Continuously Compounded Interest

- The smaller the compounding interval, the more money earned.
- Based on a continuous compounding of the interest (literally every moment of every day)
  - e = the irrational constant

$$A = Pe^{rt}$$

# Compound Interest

## Example

**How much money will I have if... And how much interest will I earn if...**

I invests $500 at 10% APR for 8 years, compounded continuously?

$$A = Pe^{rt} = 500e^{0.1*8}$$

$$\approx \$1{,}112.77$$

# 2

# Annual Percentage Yield

# Annual Percentage Yield (APY)

- When compounding interest, the rate of interest earned is not the same as the stated APR.
  - Why is this true?
- Annual Percentage Yield (APY) is the effective annual interest rate

$$APY = \left[\left(1 + \frac{r}{n}\right)^{n} - 1\right] * 100\%$$

# Annual Percentage Yield (APY)

## Example

Revisit my investment of $500 at 10% APR for 8 years. What is his APY if this investment is compounded monthly?

$$APY_{monthly} = \left[\left(1 + \frac{r}{n}\right)^{n} - 1\right] * 100\% = \left[\left(1 + \frac{0.1}{12}\right)^{12} - 1\right] * 100\%$$

$$= 10.47\%$$

# 3

# Examples

## Example #1

Suppose Gavin wants to borrow $200 for five weeks. The amount of interest he ends up paying is $20 per $100 borrowed. What is the APR at which Gavin is borrowing money?

208%

# Example #2

Suppose that Casey deposited $13,000 for eight years at 4% APR. How much interest did Casey earn if the interest is compounded weekly? Round your answer to the nearest cent.

$4,900.46