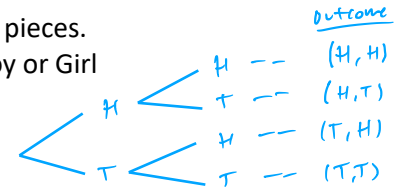
10.1 Introduction to Probability – Overview

Definitions / Key Ideas

Experiment (trial) – The process by which a random observation / outcome is generated.
- Example: Flip a coin, roll a die, gender of a child

Outcome – Any possible individual observation of that experiment, like the smallest pieces.
- Example: Flip a coin once, Heads or Tails; Rolling a die, 1 2 3 … 6; Gender, Boy or Girl



Sample Space, *S* – The set of all possible outcomes of an experiment.
- Example: Flip a coin twice, S = {HH, HT, TH, TT}

Tree diagram – Uses branches to indicate all possible outcomes at each stage for an experiment.
- Each path of branches in a tree diagram indicates a single possible outcome for the experiment.

Event, *E* – Any collection of possible outcomes of an experiment. In other words, any subset of S.
- Example: Flip a coin twice: A = HH, A = HH or HT, A = {TT, HT, TH}, etc.

Example: Roll a pair of 4-sided die, we are interested in the sum of the two dice. Using the proper notation:

Write the sample space: $S = \{2, 3, 4, 5, 6, 7, 8\}$

Write the event of a number less than 6: $E = \{2, 4, 6, 8\}$

Write the event of an even number: $E = \{2, 3, 4, 5\}$

Two Ways to Calculate Probability Probability = Likelihood of an event occurring $0 \leq \text{Probability} \leq 1$

NEVER occurs ALWAYS occurs

1) Classical Probability (Theoretical)
(if all outcomes are equally likely)

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space} = \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

- Examples: Suppose we are randomly selecting a single card from a standard 52-card deck.

a) Find the probability of a red card.

$P(\text{Red}) = \frac{\#\text{ successes}}{\#\text{ possible}} = \frac{26}{52} = 0.5$

b) Find the probability of a King.

$P(\text{King}) = \frac{\#\text{ king}}{\text{total }\#\text{ cards}} = \frac{4}{52} = \frac{1}{13} = 0.0769$

c) Find the probability of a Heart or a 10.

$P(\text{Heart or 10}) = \frac{\#\text{ hearts or 10}}{\text{total }\#\text{ cards}} = \frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

d) Find the probability of a card that is not a Club.

$P(\text{Not Club}) = \frac{\#\text{ not club}}{\text{total}} = \frac{39}{52} = \frac{52-13}{52} = \frac{3}{4}$

2) Empirical Probability (Data)
(if all outcomes are based on an experiment)

$$P(Event) = \frac{Number\ of\ times\ the\ event\ occurs}{Number\ of\ times\ the\ experiment\ is\ performed} = \frac{Number\ of\ successes}{Number\ of\ trials}$$

- Examples: Suppose we collected data on majors of MATH 125 students and are randomly selecting a single student.

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20
Total	76

a) Find the probability the student is a Math major.

$P(math) = \frac{\#\ math}{\#\ students} = \frac{23}{76}$

b) Find the student is not a Math major.

$P(not\ math) = \frac{\#\ not\ math}{\#\ students} = \frac{15+18+20}{76} = \frac{76-23}{76} = \frac{53}{76}$

c) Find the probability the student is an Art or Chemistry major.

$P(Art\ or\ Chem) = \frac{\#\ Art\ or\ Chem}{\#\ students} = \frac{18+15}{76} = \frac{33}{76}$

d) Find the probability the student is a Math, Chemistry or English major.

$P(math, Chem\ or\ English) = \frac{\#\ successes}{\#\ possibilities} = \frac{23+15+20}{76} = \frac{58}{76}$

Example: An experiment is performed where a fair 4-sided die is rolled and then a fair 3-color spinner is spun. The possible outcomes for each event are 1, 2, 3, and 4 for the 4-sided die and red (R), blue (B), and yellow (Y) for the 3-color spinner.

a) Identify the sample space for this experiment.

$S = \{1Y, 1R, 1B, 2Y, 2R, 2B, 3Y, 3R, 3B, 4Y, 4R, 4B\}$

b) Find the probability of rolling a 3.

$P(3) = \frac{\#\ include\ 3}{\#\ total\ outcomes} = \frac{3}{12}$

c) Find the probability of an even number and R.

$P(Even\ \&\ R) = \frac{\#\ Even\ \&\ R}{\#\ possibilities}$

$= \frac{2}{12} \rightarrow 2R, 4R$

d) Is this a classical or empirical probability?

Classical (not based on data)