

7.2 Counting Our Way to Probabilities





Goals for the Day

1

Fundamental
Counting
Principle

2

Factorials

3

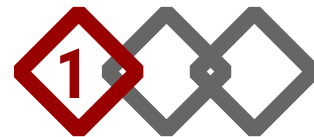
Permutations
and
Combinations

1

Fundamental Counting Principle



The Fundamental Counting Principle



- The Fundamental Counting Principle: If a job consists of n separate tasks, the first of which can be done k_1 ways, the second k_2 ways, and so on, then the total job can be done in $k_1 * k_2 * \dots * k_n$ ways.

Task 1	Task 2	...	Task n	Total Outcomes
k_1	k_2	...	k_n	$k_1 \times k_2 \times \dots \times k_n$

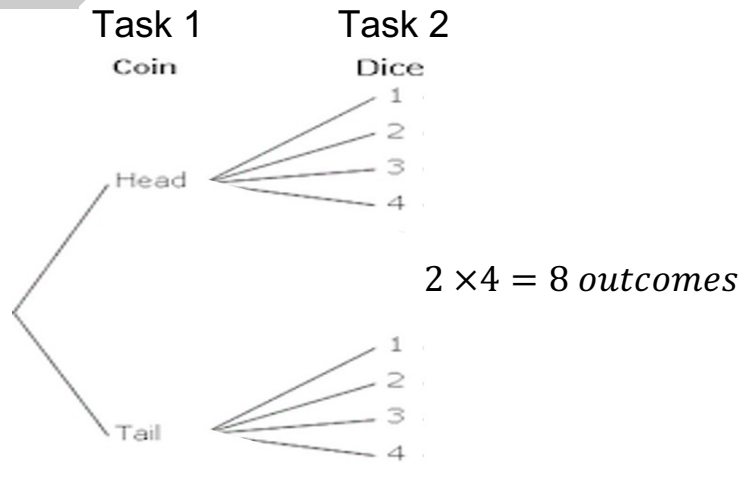
- You can find the total number of outcomes by multiplying the number of options together.



Determining Number of Outcomes



- Example: Flip a coin and roll a 4-sided die.
How many total outcomes are there?

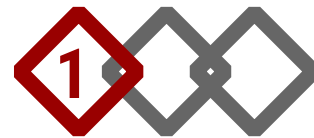


- Example: Sally has 6 pairs of socks, 4 shorts, 5 shirts and 3 sunglasses. How many ways can she get dressed?

$$\begin{array}{ccccccc} 6 & \times & 4 & \times & 5 & \times & 3 \\ \hline \text{Socks} & & \text{Shorts} & & \text{Shirts} & & \text{Sunglasses} \end{array} = 360 \text{ total outcomes}$$



Replacement



■ **With or without replacement:** We need to take into account whether or not objects can be repeated in our calculations.

■ Examples:

a) How many passwords can you make if it requires 4 digits? With replacement

$$\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 10^4 = 10,000 \text{ passwords}$$

b) How many passwords can you make if it requires 4 digits, but you cannot repeat digits? Without replacement

$$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} = 5,040$$

2

Factorials



Factorials



- $n!$ (read “n factorial”) is the product of all numbers less than and including n

$$n! = n(n - 1)(n - 2) \dots (3)(2)(1)$$

- Example: For the 9 starting players on a baseball team, how many different batting orders are possible?
Without replacement

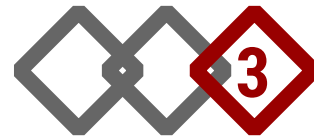
$$\underline{9} \times \underline{8} \times \underline{7} \times \dots \times \underline{1} = 9!$$

3

Permutations and Combinations



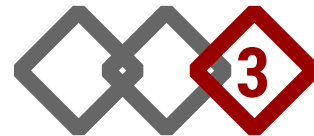
Selecting From a Group



- Often useful to count the number of ways to choose objects from a group (without replacement)
 - ▷ “Selecting r objects from a total of n objects” $r \leq n$
- Two methods
 - ▷ Permutation – order matters (e.g., first place, second place, ...)
 - ▷ Combinations – order doesn't matter (e.g., being picked for a team)



Permutations



■ Order matters

- ▷ When you are selected is important; position has meaning

■ Example

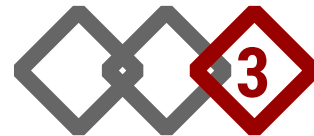
- ▷ Ranking favorite movies

$${}_nP_r = P\binom{n}{r} = P(n, r) = \frac{n!}{(n-r)!}$$

Total # How many selecting



Combinations



■ Order does not matter

- ▷ When you are selected is unimportant; only matters that you were selected

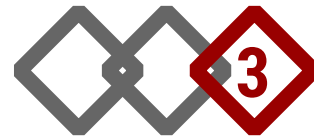
■ Example

- ▷ Picking pizza toppings

$${}_nC_r = \binom{n}{r} = C(n, r) = \frac{n!}{\textcircled{r!}(n-r)!}$$



Permutations and Combinations



Examples

- a) There are 8 runners in a race. How many ways can they place 1st, 2nd, and 3rd?

1st 2nd 3rd

Order matters → Permutation

$${}_8P_3 = 336$$

- b) Out of 12 students, how many ways can we select a committee of 4 students?

Order doesn't matter → Combination

$${}_{12}C_4 = 495$$

- c) We are forming a committee, and we need to select a president, vice president, and secretary. If there are 10 members, how many ways can this be done?

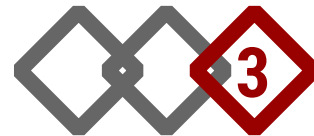
Pres VP Sec

Order matters → Permutation

$${}_{10}P_3 = 720$$



Permutations with Repeated Objects

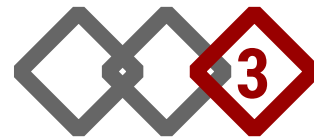


Permutations with Repeated Objects: Counting the number of distinct ways we can arrange all n objects when some of the objects are the same (repeated, specifically k_1 are alike, k_2 are alike, and so on).

$$\frac{n!}{(k_1!)(k_2!) \dots (k_p!)}, \text{ where } k_1 + k_2 + \dots + k_p = n$$



Permutations and Combinations



Example

Harmony was born on 05/19/1991. How many eight-digit codes could she make using the digits in her birthday?

8 total digits

9 ----> 3 times
1 ----> 3 times
5 ----> 1
0 ----> 1

$$\frac{8!}{3! 3! 1! 1!}$$

$$= 1,120$$