

7.5 Expected Value – Overview

Expected Value

Definition: The **expected value** of an event X is the long term average (the value we would expect to happen if we performed the experiment many, many times).

How to calculate it:

- In words: Multiply each outcome (x value) by its probability and add them together.
- Formula:

$$E(X) = x_1 P(X_1) + x_2 P(X_2) + \dots + x_n P(X_n),$$

where x_i is the i^{th} outcome and $P(x_i)$ is the probability of x_i .

Example 1: In soccer, you earn a certain number of points based on the result of a game. This is shown in the table below. Calculate the expected value of the number of points earned for a single game.

X	$P(X)$
Win = 3	0.3
Tie = 1	0.5
Loss = 0	0.2

$$E(X) = 3(0.3) + 1(0.5) + 0(0.2)$$

$\downarrow = 1.4 \text{ points (long term avg)}$

Sum of Expected Values: To find the combined expected value of multiple events, we can simply add the individual expected values.

$$E(X \text{ or } Y) = E(X) + E(Y)$$

Example 1 (continued): Find the expected value for the total number of points earned in a season if the season has 12 games.

$\rightarrow X_1 = \text{game 1}$
 $X_2 = \text{game 2}$
 \vdots
 $X_{12} = \text{Game 12}$

$$\rightarrow E(\text{Total points}) = E(X_1) + E(X_2) + \dots + E(X_{12})$$

$\rightarrow \text{Same Exp Value for each game } X$

$$= 1.4 + 1.4 + \dots + 1.4$$

$$\downarrow = 12(1.4) = 16.8 \text{ points}$$

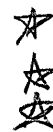
Example 2: Jim likes to day-trade on the Internet. On a good day, he averages a \$1400 gain. On a bad day, he averages a \$900 loss. Suppose that he has good days 30% of the time, bad days 50% of the time, and the rest of the time he breaks even. What is the expected value for one day of Jim's day-trading hobby?

$$\hookrightarrow = \$0$$

① Table

X	$P(X)$
1400	0.3
0	0.2 = 1 - (0.3 + 0.5)
-900	0.5

\rightarrow decimals
 \star check \rightarrow Total $P(X) = 1$
 complement



Strategy: First make table

- Think about possible X values
- THEN the probabilities

② Expected value

$$\rightarrow E(X) = 1400(0.3) + 0(0.2) + (-900)(0.5)$$

$$\downarrow = -\$30$$

Example 3: Suppose that you and a friend are playing cards and decide to make a bet. If your friend draws two hearts in succession from a standard deck of 52 cards without replacing the first card, you give him \$10. Otherwise, he pays you \$20. If the same bet was made 15 times, how much would you expect to win or lose? Round your answer to the nearest cent, if necessary.

① Table

	X	P(X)
two hearts w/o rep	-10	$\frac{1}{17}$
otherwise	20	$\frac{16}{17} = 1 - \frac{1}{17}$
		Sum (total) = 1

★ → First solve $E(X)$ for one bet
→ then consider 15 bets

$$P(2 \text{ Hearts w/o rep}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17} \text{ (Direct)}$$

$$\downarrow = \frac{\text{Hearts}}{52C_2} = \frac{13C_2}{1326} = \frac{78}{1326} = \frac{1}{17} \text{ (counting)}$$

complement

② Expected value

$$E(X) = -10\left(\frac{1}{17}\right) + 20\left(\frac{16}{17}\right)$$

$$\downarrow \approx 18.26$$

③ Combined Exp value → $E(15 \text{ bets}) = 15 E(X)$

$$= 15(18.26)$$

$$\downarrow = 273.90$$

order doesn't matter
⇒ combinations nCr

num = selecting from Hearts only
denom = selecting from ALL

Odds

Definition: Odds are another way to express probability.

- We can express this as a ratio (fraction) of probabilities.
- odds + probability are NOT interchangeable terms

Odds in favor of an event A:

$$\text{odds} = \frac{P(A)}{P(A^c)} = \frac{P(\text{win})}{P(\text{loss})}$$

Odds against an event A:

$$\text{odds} = \frac{P(A^c)}{P(A)} = \frac{P(\text{loss})}{P(\text{win})}$$

Notation: Odds are generally written as a ratio of two integers, such as 5:1, which is read "5 to 1".

Example 4: Suppose the probability of a soccer team winning a playoff game is 0.20. What are the odds of winning? Express your answer in the form a:b.

$$\rightarrow P(\text{win}) = 0.2 = \frac{1}{5}$$

$$P(\text{loss}) = 1 - 0.2 = 0.8 = \frac{4}{5}$$

Formula way

$$\rightarrow \text{odds} = \frac{P(\text{win})}{P(\text{loss})} = \frac{1/5}{4/5} = \frac{1}{4} \rightarrow 1:4$$



Strategy: First write the probability as a fraction

OR $P(\text{win}) = \frac{1}{5} \rightarrow 1 \text{ "parts" win}$
 4 "parts" loss

$$\text{odds} = 1:4$$

Example 5: If the odds on a bet are 18:1 against, what is the probability of winning? Express your answer as a fraction.

against

$$P(\text{loss}) = \frac{18}{18+1} = \frac{18}{19} \rightarrow P(\text{win}) = 1 - P(\text{loss})$$

(complement)

$$= 1 - \frac{18}{19} = \frac{1}{19}$$

★ **Strategy:** To convert from odds a probability $a:b \rightarrow P(A) = \frac{a}{a+b}$