10.6 Expected Value – Overview

Expected Value

Definition: The **expected value** of an event *X* is the long term average (the value we would expect to happen if we performed the experiment many, many times).

How to calculate it:

- In words: Multiply each outcome (*x* value) by its probability and add them together.
- Formula:

"weighted average" ←
$$\boxed{E(X) = x_1 P(X_1) + x_2 P(X_2) + \cdots + x_n P(X_n),}$$

where x_i is the i^{th} outcome and $P(x_i)$ is the probability of x_i.

Example 1: In soccer, you earn a certain number of points based on the result of a game. This is shown in the table below. Calculate the expected value of the number of points earned for a single game.

X	*P(X)*
Win = 3	0.3
Tie = 1	0.5
Loss = 0	0.2

$E(X) = 3(0.3) + 1(0.5) + 0(0.2)$

$\downarrow = 1.4$ points (long term average)

Sum of Expected Values: To find the combined expected value of multiple events, we can simply add the individual expected values.

$$\boxed{E(X \text{ or } Y) = E(X) + E(Y)}$$

Example 1 (continued): Find the expected value for the total number of points earned in a season if the season has 12 games.

X_1 = Game 1, ..., X_{12} = Game 12

$E(\text{Total Points}) = E(X_1) + E(X_2) + \cdots + E(X_{12})$ ⟶ Same $E(X)$ for each

$= 1.4 + 1.4 + \cdots + 1.4$

$\downarrow$

$= 12(1.4) = 16.8$ points

Example 2: Jim likes to day-trade on the Internet. On a good day, he averages a $1400 gain. On a bad day, he averages a $900 loss. Suppose that he has good days 30% of the time, bad days 50% of the time, and the rest of the time he breaks even. What is the expected value for one day of Jim's day-trading hobby?

↳ $0

Strategy: First make table
- Think about possible X values
- THEN the probabilities

X	P(X)	E(X)	Overall E(X)
+1400	0.3	1400(0.3) = 420	420 + 0 + −450 = $\boxed{-30}$
0	0.2	0(0.2) = 0	
−900	0.5	−900(0.5) = −450	

OR

Just using formula (weighted average)

$1400(0.3) + 0(0.2) + (-900)(0.5) = -\$30$

Example 3: Suppose that you and a friend are playing cards and decide to make a bet. If your friend draws two hearts in succession from a standard deck of 52 cards without replacing the first card, you give him $10. Otherwise, he pays you $20. If the same bet was made 15 times, how much would you expect to win or lose? Round your answer to the nearest cent, if necessary.

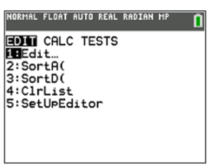
X	$P(X)$	$E(X)$	1 round $E(x)$	15 rounds $E(x)$
-10 (2 hearts)	$\frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$ OR $\frac{13C2}{52C2}$	$-10\left(\frac{1}{17}\right) = -\frac{10}{17}$	$-\frac{10}{17} + \frac{320}{17}$ $\approx \$18.26$	$15(18.26)$ $= \$273.90$
$+20$ (not that)	$1 - \frac{1}{17} = \frac{16}{17}$	$20\left(\frac{16}{17}\right) = \frac{320}{17}$		

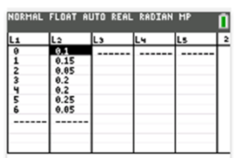
Example 4 (Calculator Fun Sess): Calculate the expected value of the scenario:

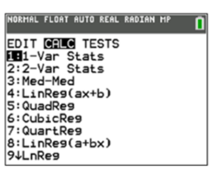
Steps for the TI-83/84	Steps for the TI-30XS MultiView	Steps for the TI-30 XIIS
1. Enter data: STAT → Edit →Enter X values in L_1 and probabilities in L_2 2. Calculate: STAT → CALC → 1-Var Stats List: L_1. FreqList: L_2. Calculate!	1. Data → Enter X values in L_1 and probabilities in L_2 2. 2nd → stat → 1-Var Stats a) DATA: L1 b) FRQ: L2 c) CALC	1. 2nd → STAT → 1-VAR (Enter) 2. DATA X_1 = # (scroll down) FRQ = $P(X_1)$ X_2 = # (scroll down) FRQ = $P(X_2)$ … 3. STATVAR (scroll across) 4. To exit this menu: 2nd → EXIT STAT → Y

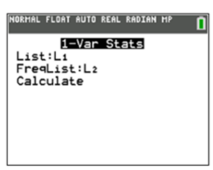
Inputs | **Results**

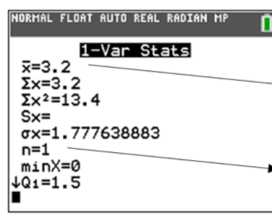
x	$P(X_i)$
0	0.10
1	0.15
2	0.05
3	0.20
4	0.20
5	0.25
6	0.05











Expected Value

If typed in probabilities correctly → Sum(Probs) = 1

Example 5: A typical three-reel mechanical slot machine has different payoffs determined by the number and position of various pictures. Suppose the payoff (in dollars) has the probability distribution given in the table below. Find the expected payout.

Center Pay line	3 7's	3 bars	3 plums	3 bells	3 oranges	3 cherries	2 cherries	1 cherry	
x	500	100	50	20	10	5	2	1	0
P(x)	$\frac{1}{8000}$	$\frac{1}{8000}$	$\frac{9}{8000}$	$\frac{48}{8000}$	$\frac{64}{8000}$	$\frac{30}{8000}$	$\frac{530}{8000}$	$\frac{3120}{8000}$	$\frac{4197}{8000}$

< calculator steps > → $E(\text{payoff}) = \$0.8725$

OR $\sum x P(x_i) = 500\left(\frac{1}{8000}\right) + \cdots + 0\left(\frac{4197}{8000}\right) = \$0.8725$