11.4 The Normal Distribution

Goals for the Day

1. Normal Distribution Properties
2. Empirical Rule
3. Z-scores and Finding Probabilities
4. Examples

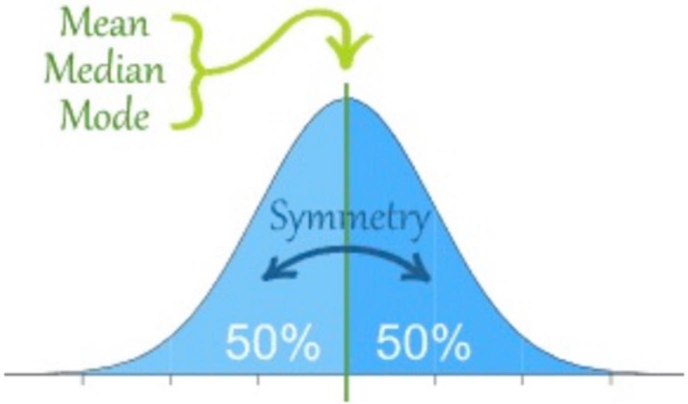
1

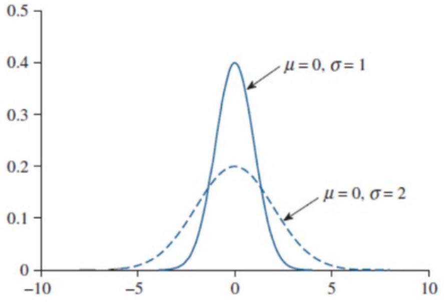
Normal Distribution Properties

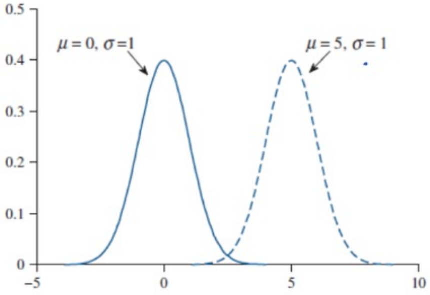
Normal Distribution Properties

PARANORMAL DISTRIBUTION

- It's a symmetric, unimodal and bell-shaped distribution
  → which implies mean = median = mode.
- Total area under curve (probability) is equal to 1 = 100%.
- Completely described by its mean μ (location) and standard deviation σ (spread).







2

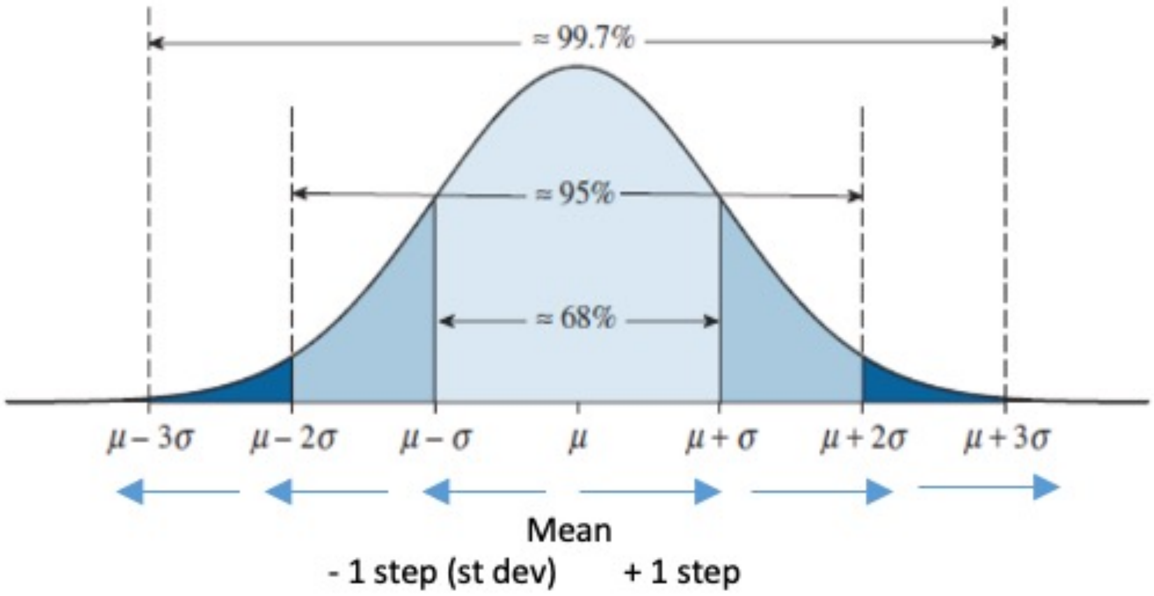
Empirical Rule

Empirical Rule (68 – 95 – 99.7 Rule)

- When data is approximately bell shaped, the standard deviation allows us to make fairly accurate approximations about the locations of our data values.

  - 68% of the data lies within 1 standard deviation of the mean.
  - 95% of the data lies within 2 standard deviations of the mean.
  - 99.7% of the data lies within 3 standard deviations of the mean.

Empirical Rule (68 – 95 – 99.7 Rule)



- We can use these breakdowns to find probabilities within certain intervals.

Empirical Rule Examples

Example 1: Suppose that diameters of a new species of apple have a bell-shaped distribution with a mean of 7 cm and a standard deviation of 0.5 cm. Using the empirical rule, find the following percentages of apples with diameters that are:

a) Between 5.5 cm and 8.5 cm

99.7%

b) More than 7.5 cm

16%

c) No more than 5.5 cm

0.15%

Step 1
Draw and label curve

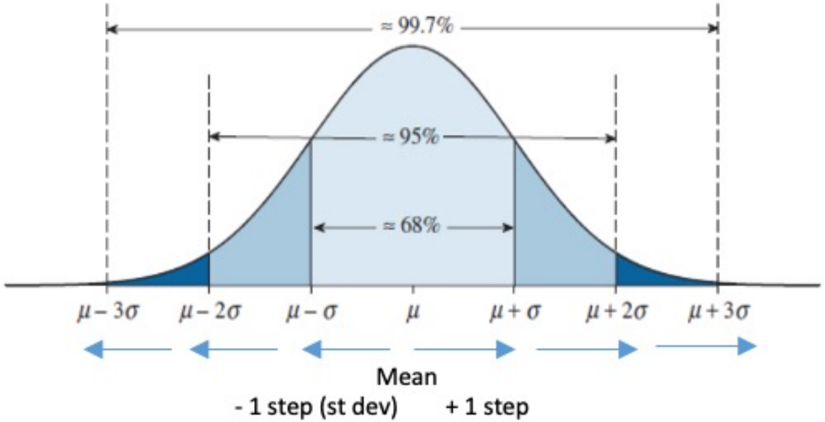
Step 2
Shade area of interest

3

Z-scores and Finding Probabilities

How We Use the Normal Curve

- The normal distribution allows us to find any probability, not just for points that lie exactly 1, 2, or 3 standard deviations ("steps") away from the mean like with the empirical rule!



Z-scores

- Z-scores ("Standard" scores in Hawkes Certify)
- Definition: A **z-score** standardizes observations based on the mean (center) and standard deviation (spread) of the distribution
  - Allows for comparisons on different scales.
  - Ex) ACT vs SAT

Formula: $z = \frac{x-\mu}{\sigma} = \frac{x-\bar{x}}{s} = \frac{obs - mean}{st\ dev}$

Z-scores

- Interpretation: A z-score tells us how many standard deviations an observation is away from the mean.

STEPS

– below ← Mean → + above

Z-scores Examples

- **Example 2**: For each data set with the stated μ and σ, find the standard score (z score) corresponding to the given observation, x.

a) $\mu = 8, \sigma = 3, x = 17$

$$z = \frac{17 \ - 8}{3} = 3$$

Above mean

b) $\mu = 100, \sigma = 16, x = 80$

$$z = \frac{80 \ - 100}{16} = -1.25$$

Below mean

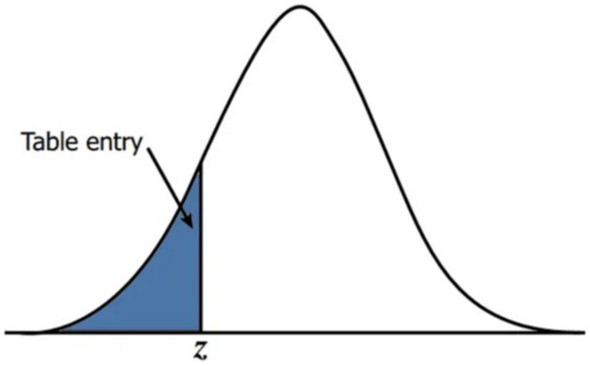
c) Which observation is further from the mean relatively?

(a) because z = 3 is a “larger” value

Finding probabilities based on the Normal Distribution

- Handout: Normal Distribution Table
  - Use the handout to <u>convert z-scores</u> to <u>percentiles</u> ("left probabilities").
  - ALWAYS gives probability LESS THAN Z: P(Z < z).

"LEFT"



Different Types of Probabilities

3

- Left Probability = Table (directly)
- Right Probability = 1 – Left (table)
- Between Probability = Left Z_2 – Left Z_1
- Outside Probability = Left Z_1 + Right Z_2

Draw, Label and Shade curve

4

Examples

Empirical Rule Examples

Example 3: Suppose that IQ scores have a bell-shaped distribution with a mean of 105 and a standard deviation of 15. Using the empirical rule answer the following questions:

Step 1
Draw and label curve

Step 2
Shade area of interest

a) What percentage of IQ scores are greater than 75?

97.5%

b) Between which two values do the middle 68% of IQ scores fall between?

(90, 120)

Empirical Rule Examples

Example 4: Suppose there is a new breed of giant cats, whose weights are normally distributed with an average of 100 pounds and a standard deviation of 15 lbs. You would like to own a smaller version of this type of cat, specifically between 59 lbs and 69 lbs.

What is the probability you can find a cat between these two weights.

$$P(59 \leq X \leq 69) =$$
$$P(-2.73 \leq Z \leq -2.07) =$$
$$0.016$$