10.3 Probability of Single Events

Goals for the Day

1. Odds
2. Complements
3. Calculating More Probabilities
4. Examples

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Odds

What are odds?

- Odds: Another way to express probability
  - But not interchangeable with probability
- Usually expressed as a ratio
  - “a:b for” or “a:b against”
  - Can be expressed as a fraction of probabilities

How can we calculate odds?

- Odds in favor of event A
  - "Odds for"

$$Odds = \frac{P(A)}{P(A^C)} = \frac{P(Win)}{P(Lose)}$$

- Odds against event A
  - "Odds against"

$$Odds = \frac{P(A^C)}{P(A)} = \frac{P(Lose)}{P(Win)}$$

Odds

<u>Example</u>

Suppose the probability of a soccer team winning a playoff game is 0.20. What are the odds of winning? Express your answer in the form a:b.

$$P(W) = 0.20 = \frac{1}{5}$$

1 "part" winning

4 "parts" losing

$$Odds\ Winning = 1:4$$

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Complements

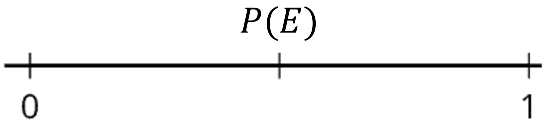
Probability Review

Probability

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space}$$

$$= \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

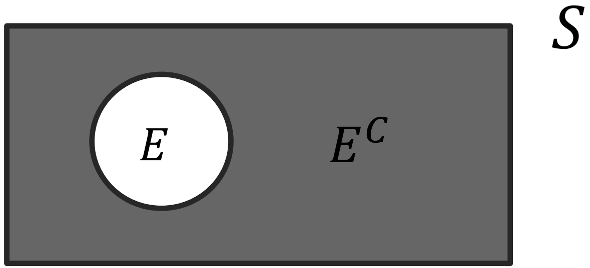
$$0 \leq Probability \leq 1$$



Complements

Opposite

- **Complement of an Event**: Consists of all outcomes in the sample space that are *not* in event E. E^C, $\bar{E}$, E'



- Complement Rules of Probability:

1) $P(E) + P(E^C) = 1$ 2) $P(E) = 1 - P(E^C)$ 3) $P(E^C) = 1 - P(E)$

Complement Examples

- Example: Suppose we are randomly selecting a single card from a standard 52-card deck.

a) Find the probability of a diamond.

$$P(Diamond) = \frac{13}{52} = \frac{1}{4}$$

b) Find the probability of not a diamond.

$$P(Not\ Diamond) = \frac{39}{52} = 1 - \frac{1}{4} = \frac{3}{4}$$

Directly Using complements

Complement Examples

- Example: Suppose we collected data on majors of MATH 125 students and are randomly selecting a single student.

a) Find the probability that the student is NOT an Art major.

$$\underset{Art^C}{P(Not\ Art)} = \quad 1 - P(Art) = 1 - \frac{18}{76} = \frac{58}{76}$$

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20

b) Find the probability that the student is NOT an English nor Chemistry major.

$$P(Not\ English\ nor\ Chem) = 1 - \frac{20+15}{76} = \frac{23+18}{76} = \frac{41}{76}$$

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Calculating More Probabilities

Calculating (Harder) Probabilities

- Example 1: Liam and Michael are going to play video games this afternoon. Together, they have 41 video games. If they decide to randomly choose two video games, what is the probability that the two they choose will consist of each of their favorite video games? Assume they have different favorites.

Two Approaches: 1) Direct way 2) Counting methods

$$** \; P(Event) = \frac{\# \; Successes}{\# \; Possibilities}$$

Direct Way

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Example 1: Liam and Michael are going to play video games this afternoon. Together, they have 41 video games. If they decide to randomly choose two video games, what is the probability that the two they choose will consist of each of their favorite video games? Assume they have different favorites.

$$\frac{2}{41} \times \frac{1}{40} = \frac{2}{1640} = \frac{1}{820}$$

1st Choice 2nd Choice

Without replacement

$$** \; P(Event) = \frac{\#\ Successes}{\#\ Possibilities}$$

Counting Methods Way

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$$Prob = \frac{①}{②} = \frac{1}{820}$$

$$** \; P(Event) = \frac{\#\, Successes}{\#\, Possibilities}$$

** Solve numerator and denominator separately

② Denominator → "No Condition"

Total ways to select 2 from 41

${}_{n}C_{r}$ → Combinations (Order doesn't matter)

${}_{41}C_{2} = 820$

① Numerator → "Is a Condition"

Only selecting from favorites

${}_{n}C_{r}$ → ${}_{2}C_{2} = 1$

Direct Way

- Example 2: A box of jerseys for a pick-up game of basketball contains 8 extra-large jerseys, 7 large jerseys, and 5 medium jerseys. If you are first to the box and grab 3 jerseys, what is the probability that you randomly grab 3 extra-large jerseys.

20 Total
8 XL

$$\frac{8}{20} \times \frac{7}{19} \times \frac{6}{18} = \frac{14}{285}$$

1st Selection　　2nd　　3rd

$$** \ P(Event) = \frac{\#\ Successes}{\#\ Possibilities}$$

Counting Methods Way

$$Prob = \frac{1}{2} = \frac{56}{1140} = \frac{14}{285}$$

1 Selecting 3 XL jerseys

$$_{8}C_{3} = 56$$

2 Selecting 3 jerseys from all jerseys

$$_{20}C_{3} = 1140$$

$$** \; P(Event) = \frac{\#\ Successes}{\#\ Possibilities}$$

** Solve numerator and denominator separately

Another Example

Example 3: There are 11 balls numbered 1 through 11 placed in a bucket. What is the probability of reaching into the bucket and randomly drawing two balls numbered 1 and 4 without replacement, in that order?

$$** \; P(Event) = \frac{\# \; Successes}{\# \; Possibilities}$$

<u>Direct Way</u>

$$\underset{\#1}{\underline{\frac{1}{11}}} \times \underset{\#4}{\underline{\frac{1}{10}}} = \frac{1}{110}$$

<u>Counting Way</u>

$$P(1,4) = \frac{1}{{}_{11}P_2} = \frac{1}{110}$$

Numerator → Only 1 way to get a 1, then a 4

1 2 3 4 5 6 7 8 9 10 11

First Second

★ Order matters! → ${}_nP_r$

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Examples

Example

- Example 4: Julia sets up a passcode on her smart phone, which allows only six-digit codes. A spy sneaks a look at Julia's smart phone and sees her fingerprints on the screen over six numbers. What is the probability the spy is able to unlock the smart phone on his first try?

Direct Way: $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{720}$

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Counting Way: $P(Correct\ Password) = \frac{1}{{}_6P_6} = \frac{1}{6!} = \frac{1}{720}$

Numerator: Only 1 correct password

Denominator: 6 #s to choose from (6 fingerprints) and Ordering all 6 → r = 6

Odds

Example 5

If the odds on a bet are 18:1 against, what is the probability of winning?

Strategy: To convert from odds to a probability $a:b \rightarrow P(A) = \frac{a}{a+b}$

$$Odds\ against = 18:1 \rightarrow \ P(Loss) = \frac{18}{18+1} = \frac{18}{19}$$

$$P(Win) = 1 - P(Win^C) = 1 - \frac{18}{19} = \frac{1}{19}$$

= Loss