11.3 Describing and Analyzing Data

Goals for the Day

1. Measures of Center
2. Measures of Spread
3. Using your Calculator
4. Measures of Relative Position

1

Measures of Center

Mean (Average Value)

- Simple, arithmetic <u>average</u> of the data.
  - Sum all numbers and divide by the sample size (n).
- Same calculation for the population and sample mean (just different notation).
  - Sample mean = $\bar{x}$ (pronounced "x-bar")
  - Population mean = μ (Greek letter mu)
- Mean is NOT a resistant measure.
  - This means it is heavily affected by outliers.

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Example 1 – Mean

Data: 1, 5, 2, 9, 3

$$\bar{x} = \frac{1 + 5 + 2 + 9 + 3}{5} = 4$$

Now change 3 to 30

New $\bar{x} = 9.4$

Median (Middle Value)

- The middle value in an ordered list.
- Median IS a resistant measure.
  - NOT affected by outliers.

Example 2 – Median

Case 1 – Odd *n*

7 Obs: 10, 5, 6, 1, 3, 9, 8

Sorted: ~~1~~, ~~3~~, ~~5~~, 6, ~~8~~, ~~9~~, ~~10~~

$$Med = 6$$

Case 2 – Even *n*

8 Obs: 10, 5, 6, 1, 3, 9, 8, 3

Sorted: ~~1~~, ~~3~~, ~~3~~, 5, 6, ~~8~~, ~~9~~, ~~10~~

$$Med = \frac{5+6}{2} = 5.5$$

Mode (Most Common Value)

- The most frequently occurring value(s).
  - Unimodal data has one mode.
  - Bimodal data has 2 modes.
  - Multimodal data has more than 2 modes.
  - Can be no modes (every value is distinct).
- This is the only measure of center that can be used with categorical data.
  - Ex) Most common favorite color (can't average this)

Example 2 – Median

<u>Case 2</u>

8 Obs: 10, 5, 6, 1, 3, 9, 8, 3

Mode = 3 (twice)

2

Measures of Spread

Range

- Range = Max - Min
- Gives idea of the entire "range" of values, how much distance do they span in total.
- Ex) Case 2: Range = 10 – 1 = 9

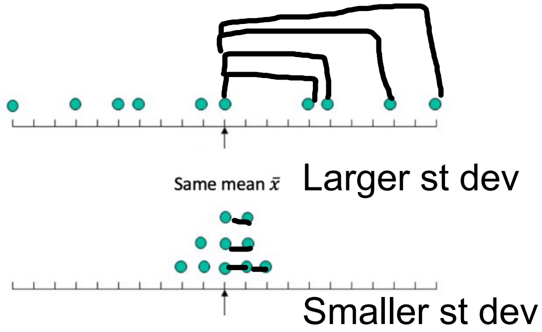
Example 2 – Median

<u>Case 2</u>

8 Obs: 10, 5, 6, 1, 3, 9, 8, 3

Standard Deviation

- Complex formula that measures the <u>average distance</u> that each <u>data point is from the mean</u>.



<u>Sample</u> Standard Deviation = S $= \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

<u>Population</u> Standard Deviation = $\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}}$

(Greek letter sigma)

3

Using your Calculator

Using Your Calculator

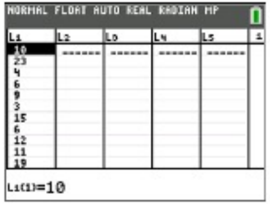
Steps for the TI-83/84	Steps for the TI-30XS MultiView	Steps for the TI-30 XIIS
1. Enter data: STAT → Edit → Enter data in L_1 *(Demo dataset: 10, 23, 4, 6, 9, 3, 15, 6)* 2. Calculate: STAT → CALC → 1-Var Stats a) List is L_1. b) Leave FreqList blank. c) Calculate!	1. Data → Enter data in L_1 2. 2^{nd} → stat → 1-Var Stats a) DATA: L1 b) FRQ: ONE c) CALC	1. 2^{nd} → STAT → 1-VAR (Enter) 2. DATA X_1 = # (scroll down) FRQ = 1 (for ALL Xs, scroll down) X_2 = # (scroll down) … 3. STATVAR (scroll across) 4. To exit this menu: 2^{nd} → EXIT STAT → Y

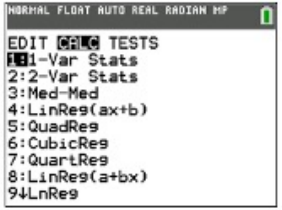
Does not give median

Inputs

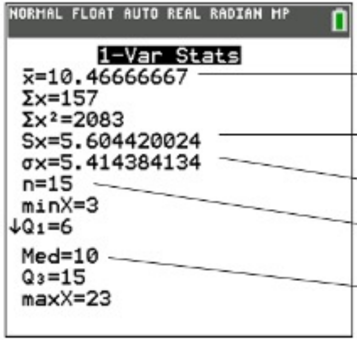
Data here:

10, 23, 4, 6, 9,
3, 15, 6, 12, 11,
19, 10, 6, 8, 15





Results



Mean
Sample st dev
Population st dev
Sample size
Median

Using Your Calculator Example

Example 3: Find the mean, median, mode and sample standard deviation of the following dataset.

- Data (7 obs): 35, 70, 31, 37, 65, 38, 38

Results

Mean $\bar{x} = 44.86$

Sample st dev: $S_x = 15.72$

Pop st dev $\sigma_x = 14.55$

Med = 38

Other Considerations

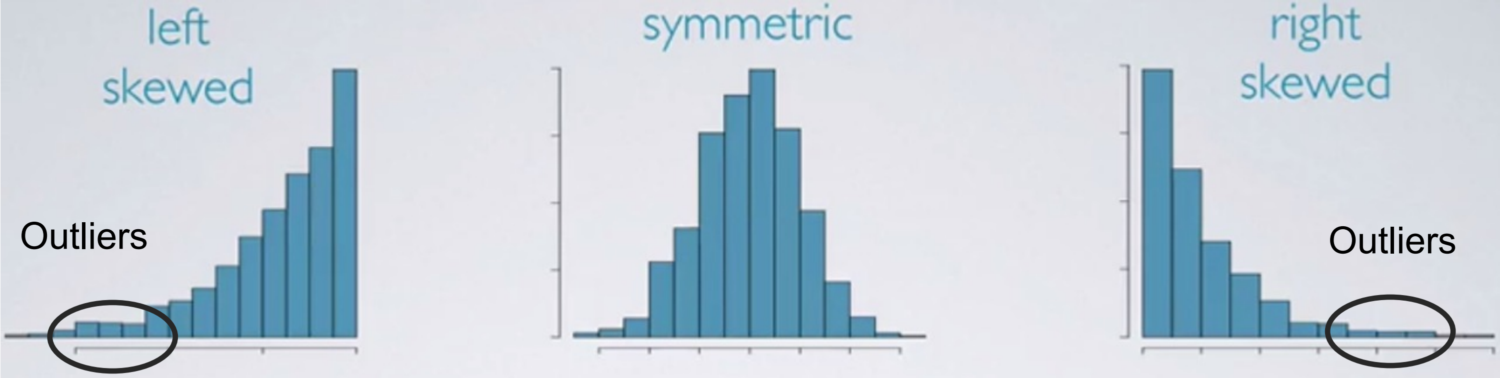
- Outliers
  - Data values that are extreme when compared to the rest of the data.
  - Can significantly impact measures of center and spread.

Example 3:
- Data (7 obs): 31, 35, 37, 38, 38, 65, 70

Outliers

Other Considerations

Types of Distributions



Best measure of center: Median | Mean | Median

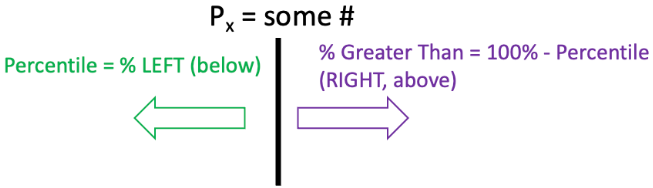
4

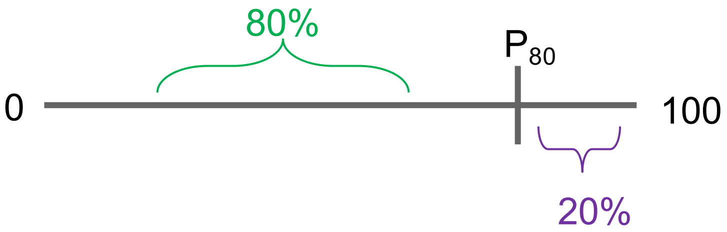
Measures of Relative Position

Percentiles

- A **percentile** tells you the percent of observations/individuals you are higher than.
  - Interpreting example: You are told you scored in the 90^{th} percentile on GRE. This means you have a score that is higher than 90% of all others that took the test.
  - Range from 0^{th} to 100^{th} percentile!
  - There is complement aspect to percentiles as well; for example, if you are the 80^{th} percentile, there is 20% greater than you!

- Best way to remember!

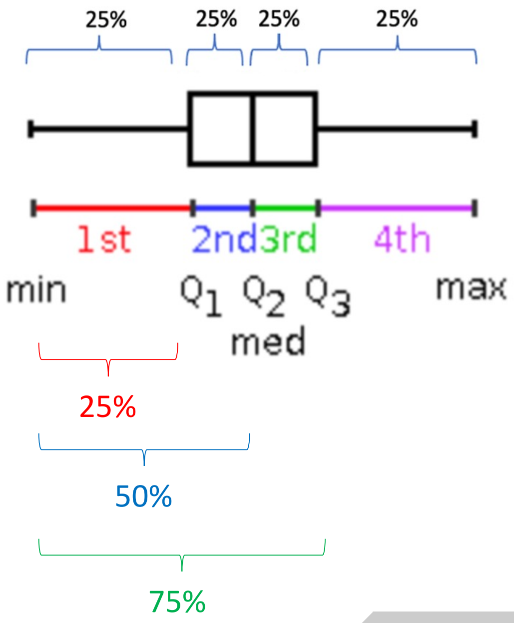




  - Notation: X^{th} Percentile = P_x

5-Number Summary and Boxplots

- Quartiles are specific percentiles.
  - Q_1 is the 25th Percentile.
  - Q_3 is the 75th Percentile.
  - Q_2 is the 50th Percentile = Median.
- Inner Quartile Range (IQR)
  - Another measure of variation, less informative than the standard deviation.
  - Uses <u>quartiles</u> to measure how far data is spread out around the <u>median</u>. Specifically, it measures the range of the middle 50% of the data
    - $IQR = Q_3 - Q_1$
  - Visualized very well in boxplots! It is the length of the box!
- 5-number summary
  - Min, Q_1, Med, Q_3, Max → Points of a boxplot



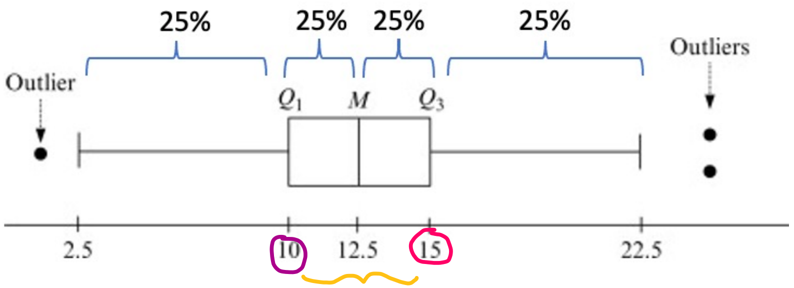
5-Number Summary and Boxplots

1st half

2nd half

25% 25% 25% 25%

1,1,2,2,3,(Q1)3,4,4,5,5,(M=Q2)6,6,7,7,8,(Q3)8,9,9,10,10

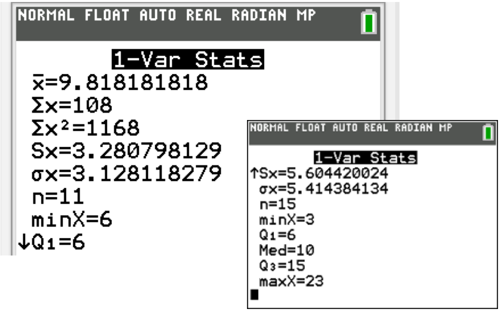


IQR = Q3 – Q1 = 15 – 10 = 5

Examples

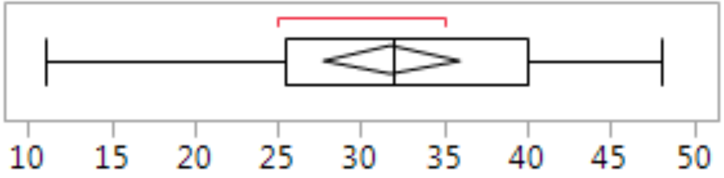
Example 4:

a) Using this output from a 1-Var Stat, what is the IQR?



$IQR\ = 9$

b) Find the IQR from this boxplot.



$IQR\ = 15$

Examples

Example 5:

a) Calculate the 5-number summary of the following dataset (20 numbers):

38, 33, 5, 5, 47, 29, 24, 42, 3, 18, 30, 46, 25, 44, 40, 42, 39, 44, 29, 13

<u>5-number summary</u>

$Min = 3$

$Q_1 = 21$

$Med = 31.5$

$Q_3 = 42$

$Max = 47$

b) Draw a boxplot based on the 5-number summary from (a).