8.5 Linear Regression

Goals for the Day

1 Scatterplots

2 Correlation

3 Using Your Calculator

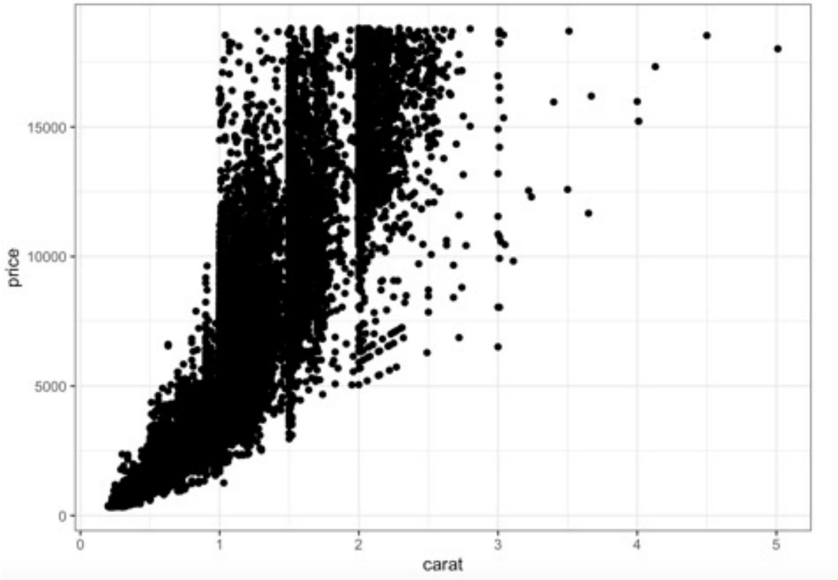
4 Regression

1

Scatterplots

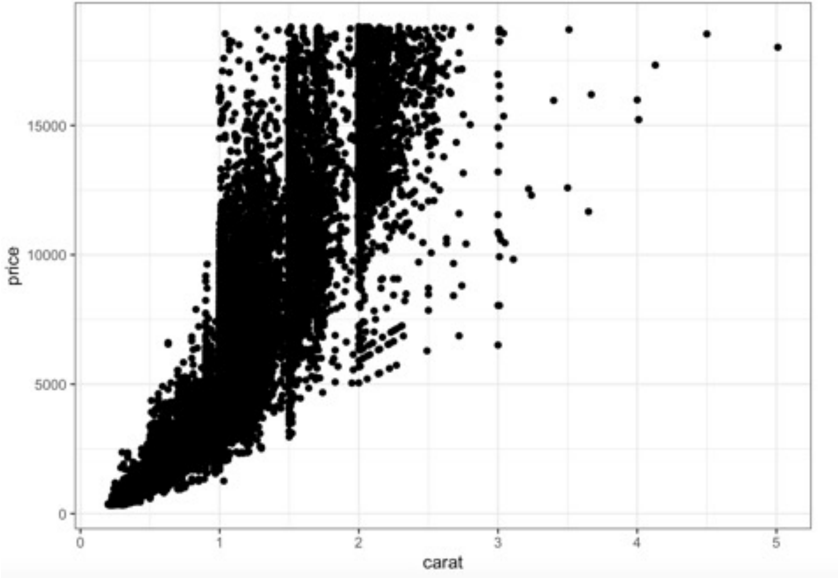
Scatterplots

- Displays the relationship between **two quantitative** variables measured on the same individuals.
- Useful to determine if an **association** exists!
  - So is there a <u>pattern</u> where some values of one variable tend to occur with some values of the other variable.
  - Ex) Smaller carat diamonds tend to have lower prices, and as the carat increases prices tend to increase as well.



Scatterplots

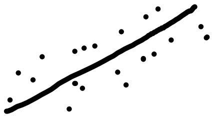
- Setup of axes
  - The <u>explanatory</u> (independent) variable goes on the X (<u>horizontal</u>) axis.
  - The <u>response</u> (dependent) variable goes on the Y (<u>vertical</u>) axis.
  - Ex) How large a diamond is impacts how much it costs → Carat = X; Price = Y.



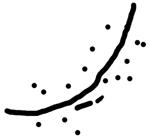
Interpreting Scatterplots

- Interpreting a scatterplot (what we are looking for in a scatterplot)

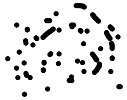
- **Form** (pattern of the dots)
  - Linear → Points follow a general linear trend; Straight line.



  - Curved → Points show some evidence of curvature; NOT a straight line.



  - Random scatter → No pattern, points are just scattered about randomly kinda like a cloud of points.



Interpreting Scatterplots

- Interpreting a scatterplot (what we are looking for in a scatterplot)

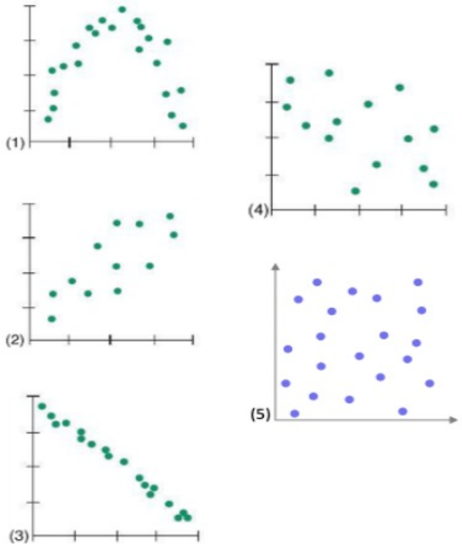
- **Direction** (of the association; only applies to linear relationships)
  - Positive → Upward trend.
  - Negative → Downward trend.
  - No Association → There is no pattern or general trend (corresponds to random scatter).

Interpreting Scatterplots

- Interpreting a scatterplot (what we are looking for in a scatterplot)

- **Strength** (how strong the association is; how well the data fits the pattern; only applying this to linear relationships)

Interpreting Scatterplots Example



Example

	Form	Direction	Strength
(1)	Curved	N/A (+/-)	N/A (strong)
(2)	Linear	Positive	Moderate
(3)	Linear	Negative	Strong
(4)	Roughly linear	Negative	Weak
(5)	Random scatter	No association	Very weak

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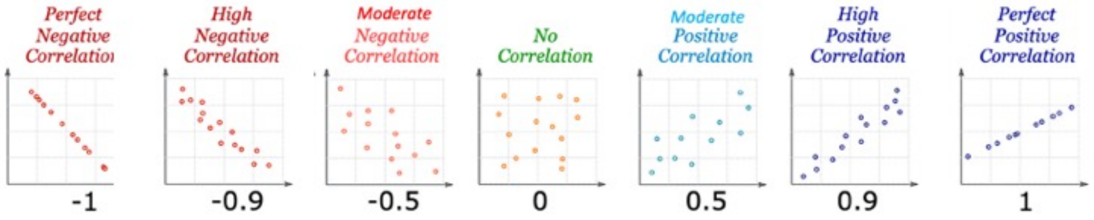
Correlation

Correlation

- The **correlation** (*r*) is an index that expresses the direction and strength of the relationship.
  - It combines both of these aspects into a single number measure.
  - Often referred to as the correlation coefficient (or Pearson's correlation).
- Interpreting correlation
  - Sign = Direction
  - Absolute value |r| = Strength



Correlation

- Properties of Correlation
  - Scale goes from -1 to 1 → $-1 \leq r \leq 1$
  - Only applies to LINEAR relationships.
  - r has no units and is the same regardless of which variable is X or Y.
  - Does NOT imply a cause-and-effect relationship.
    - Ex) Ice cream sales and shark attacks have a strong positive correlation.

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Using Your Calculator

Using Your Calculator

Using TI-83/84 (and TI-30 XS MultiView / XIIS) to calculate correlation (and regression line).

Steps for the TI-83/84	Steps for the TI-30XS MultiView	Steps for the TI-30 XIIS
1. Enter data: STAT → Edit → Enter X data in L_1 Enter Y data in L_2 2. Calculate: STAT → CALC → LinReg(ax+b) a) XList: L_1. b) YList: L_2. c) Rest leave blank. d) Calculate!	1. Data → Enter X data in L_1 Enter Y data in L_2 2. 2^{nd} → stat → 2-Var Stats a) *x*DATA: L1 b) *y*DATA: L2 c) CALC	1. 2^{nd} → STAT → 2-VAR (Enter) 2. DATA X_1 = # (scroll down) Y_1 = # (scroll down) … (repeat for all data points) 3. STATVAR (scroll across) 4. To exit this menu: 2^{nd} → EXIT STAT → Y

****** *One time setup for TI-83/84: 2^{nd} → Catalog → DiagnosticOn → Enter*

Using Your Calculator

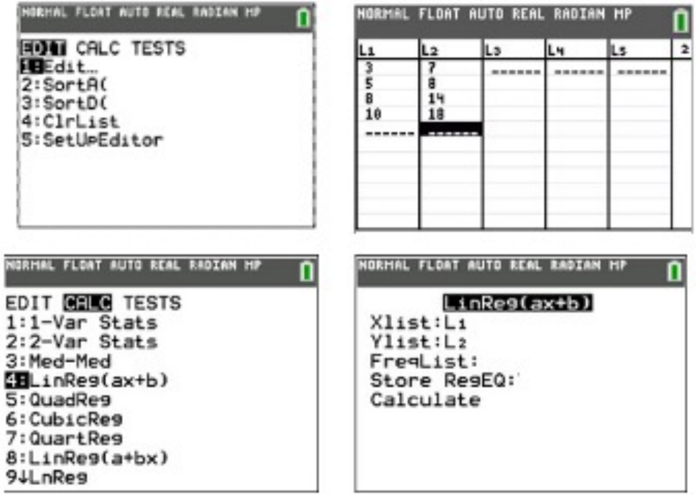
Demo dataset

n = 4

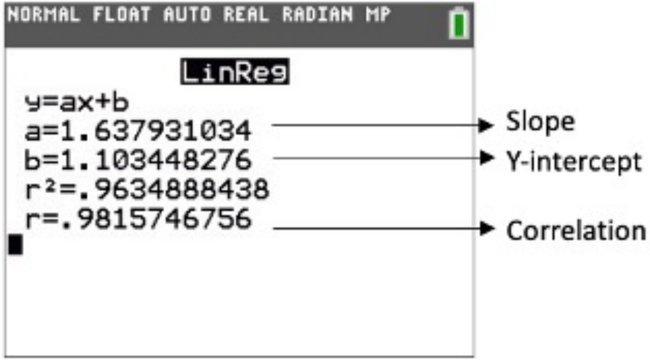
X	Y
3	7
5	8
8	14
10	18

L_1 L_2

Inputs



Results

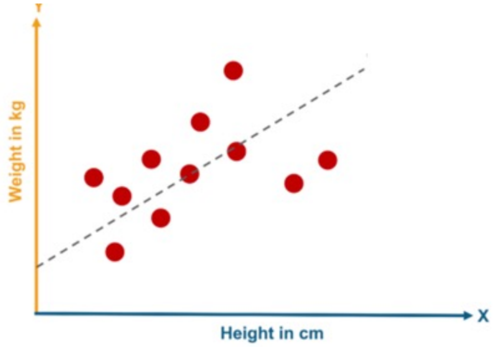


4

Regression

Regression

- Ultimately, we want to determine if we can use a straight line to model the relationship between two variables → If so, we can use that model to make predictions!
  - This process is called **Linear Regression**.



Regression

- Step 1 → Determine if there is a **significant correlation** (**linear relationship**).

a) Calculate correlation.

b) Compare it to the **Table of Critical Values or the Pearson Correlation Coefficient** to see if it is statistically significant.

- Match the sample size n and the Level of significance →(Probability our claims about the data are wrong) to the specific problem.
- If |r| > Critical Value (CV) → r is statistically significant (unlikely to have occurred by chance).
- Demo ex) n = 4, α = 0.05 → r = 0.982 > 0.950 = CV

→ Significant ✅ → Can make predictions ✅

Critical Values of the Pearson Correlation Coefficient		
n	$\alpha = 0.05$	$\alpha = 0.01$
4	0.950	0.990
5	0.878	0.959
6	0.811	0.917
7	0.754	0.875

Regression

- Step 2 → Once we have a significant correlation, we can find the **regression line**.
  - Linear equation that fits our data best (aka ‘line of best fit’).
    - It is IMPORTANT to get the X and Y variables correct!
  - Our calculator gives us our equation!

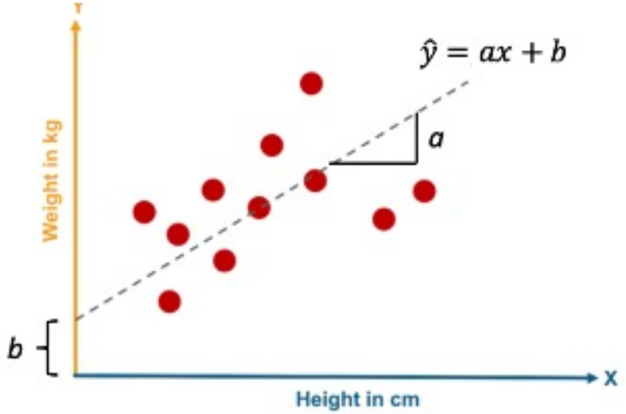
Regression

Equation

- Here is the form of our linear equation (written in slope-intercept form):

$$\hat{y} = ax + b \qquad \text{calculator}$$
$$= b_0 + b_1 x \qquad \text{Hawkes}$$

- x = Value of the explanatory variable
- $\hat{y}$ = Predicted value of the response variable for the given x
- a (or b_1) = Slope
  - It measures the direction and steepness of the line
- b (or b_0) = Y intercept
  - It is the location where the regression line crosses the Y-axis (value of Y when X = 0)



Regression

- Step 3 → Make **predictions** using the regression line.
  - We can think of our regression line, and specifically $\hat{y}$, as <u>predicted values of *Y*</u> for <u>all *X* values</u> *in the X range of our sample data*!
  - Calculating these is simple:
    - Just plug in the new X value to our equation and this will give us the predicted Y.
    - Demo example) Predict Y for X = 7.

$$\hat{y} = 1.638x + 1.103 \rightarrow \hat{y} = 1.638(7) + 1.103 = 12.569$$

ax + b

Full Example

Hours Spent on Homework	41	20	34	43	9	20	54	52	10	21
Grade on Test	79	63	76	100	55	82	95	80	60	80

a) Calculate the correlation for the dataset above and determine if it is statistically significant at a level of significance of $\alpha = 0.05$.

b) If appropriate, determine the regression equation.

c) If a student spends 35 hours on homework, make a prediction for their grade on the test.

d) If a student spends 50 hours on homework, make a prediction for their grade on the test.

a) r = 0.779 > 0.632 → Significant

b) Appropriate → $\hat{y} = 0.676x + 56.549$

c) $\hat{y} = 80.119$

d) $\hat{y} = 90.259$