7.3 Using Counting Methods to Find Probabilities – Overview

Definitions / Key Ideas

Probability review:

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space} = \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

$$0 \leq P(Event) \leq 1$$

Can't occur   Always

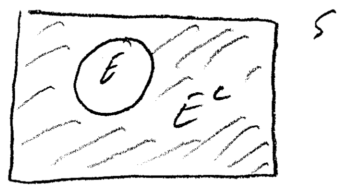
P(E)

0   1

Compliment of an Event: Consists of all outcomes in the sample space that are *not* in event E.

E^c, $\bar{E}$, E'

not E



Compliment rules of probability:

1) $P(E) + P(E^C) = 1$
2) $P(E) = 1 - P(E^C)$
3) $P(E^C) = 1 - P(E)$

Example 1: Suppose we are randomly selecting a single card from a standard 52-card deck.

a) Find the probability of a diamond.

$P(\text{diamond}) = \frac{13}{52} = \frac{1}{4}$

b) Find the probability of not a diamond.

$P(\text{not diamond}) = \underbrace{\frac{39}{52}}_{\text{directly}} = \underbrace{1 - \frac{1}{4}}_{\text{using complements}} = \frac{3}{4}$

Example 2: Suppose we collected data on majors of MATH 125 students and are randomly selecting a single student.

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20
Total	76

a) Find the probability the student is not an Art major.

$P(\text{not Art}) = 1 - P(\text{Art})$
Art^c
$= 1 - \frac{18}{76} = \frac{58}{76}$

(OR $P(\text{not Art}) = \frac{\text{Math + Chem + Eng}}{\text{total}} = \frac{23+15+20}{76} = \frac{58}{76}$)

b) Find the probability of the student is not an English or Chemistry major.

$P(\text{Not Eng or Chem}) = 1 - \frac{20+15}{76} = \frac{23+18}{76} = \frac{41}{76}$

Calculating (harder) probabilities

TWO Approaches: 1) Direct way 2) Counting methods

Example 1: Liam and Michael are going to play video games this afternoon. Together, they have 41 video games. If they decide to randomly choose two video games, what is the probability that the two they choose will consist of each of their favorite video games? Assume they have different favorites.

** $P(Event) = \frac{\#\,Successes}{\#\,Possibilities}$

** Solve numerator and denominator separately

Direct way

$\frac{2}{41} \times \frac{1}{40} = \frac{2}{1640} = \frac{1}{820}$

1st choice 2nd choice

without replacement

Counting

$Prob = \frac{①}{②} = \frac{1}{820}$

① numerator → "IS a condition"

Only selecting from favorites $_nC_r$ → $_2C_2 = 1$

② Denominator → "No condition"

Total ways to select 2 from 41 nCr → combinations (order doesn't matter)

$_{41}C_2 = 820$

Example 2: A box of jerseys for a pick-up game of basketball contains 8 extra-large jerseys, 7 large jerseys, and 5 medium jerseys. If you are first to the box and grab 3 jerseys, what is the probability that you randomly grab 3 extra-large jerseys.

Direct way

20 Total

8 XL

$\frac{8}{20} \times \frac{7}{19} \times \frac{6}{18} = \frac{14}{285}$

1st selection 2nd 3rd

Counting

$Prob = \frac{①}{②} = \frac{56}{1140} = \frac{14}{285}$

① selecting 3 XL jerseys

$_8C_3 = 56$

② Selecting 3 jerseys from all jerseys

$_{20}C_3 = 1140$

$$P(Event) = \frac{\#\,Successes}{\#\,Possibilities}$$

Example 3: There are 11 balls numbered 1 through 11 placed in a bucket. What is the probability of reaching into the bucket and randomly drawing two balls numbered 1 and 4 without replacement, in that order?

Counting

$$P(1,4) = \frac{1}{{}_{11}P_2} = \frac{1}{110}$$

numerator → only 1 way to get a 1, then a 4

1 2 3 4 5 6 7 8 9 10 11

first second

★ order matters ⟹ ${}_nP_r$

Selecting 2 from 11

Direct

$$\underbrace{\frac{1}{11}}_{\#1} \times \underbrace{\frac{1}{10}}_{\#4} = \frac{1}{110}$$

Example 4: Julia sets up a passcode on her smart phone, which allows only six-digit codes. A spy sneaks a look at Julia's smart phone and sees her fingerprints on the screen over six numbers. What is the probability the spy is able to unlock the smart phone on his first try?

only 1 success (correct password)

Counting

$$P(\text{correct password}) = \frac{1}{{}_6P_6} = \frac{1}{720}$$

${}_6P_6 = 6!$

↳ 6 #s to choose from (6 fingerprints) + ordering all 6 #s, $n = 6$, $r = 6$

Direct

$$\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{720}$$

6 digit code ⟹ 6 slots → only 1 correct # for each slot + have 6 #s to choose from first

→ then without replacement because fingerprints showed used all 6 #s