9.3 Saving Money – Overview

Definitions / Key Ideas

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

FV   Think: principal = PV

Future value (FV, A) – The value of your current investment at some future time (What we've been calculating with our compound interest formula).

Know: How much I have to invest now (P)

Want to know: How much I will have later (FV, A)

Present Value Formula

$$PV = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Present value (PV) – The amount you need to invest now in order to reach a desired future value amount.

Ex) I want to have $20,000 in 10 years. If I can get an 8% APR compounded semiannually, how large must a one-time investment right now be?

Know: How much I want to have (A)

Want to know: How much I need to invest now (PV)

$$PV = \frac{20{,}000}{\left(1 + \frac{0.08}{2}\right)^{2(10)}} \approx \$9{,}127.74$$

Annuity – Making repeated, regular payments into an account that earns (compounded) interest.

Annuity Formula for Finding Future Value

$$FV = PMT \cdot \frac{\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}$$

Total Deposited = PMT × # payments ($n*t$)

$= 200(12)(15)$

$\boxed{= \$36{,}000}$

Ex) I deposit $200 every month into a savings account earning 5% APR. How much money will be in the account after 15 years? How much will I contribute overall?

Know: How much I deposit regularly (PMT)

Want to know: How much I will have later (FV)

$$FV = 200\,\frac{\left[\left(1 + \frac{0.05}{12}\right)^{12(15)} - 1\right]}{\left(\frac{0.05}{12}\right)} \approx \$53{,}457.79$$

Annuity Formula for Finding Payment Amounts

$$PMT = FV \cdot \frac{\left(\frac{r}{n}\right)}{\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}$$

Ex) I want to have $50,000 in 20 years. How much do I need to deposit every month into a savings account earning 7% APR?

Know: How much I want to have (FV)

Want to know: How much I need to deposit regularly (PMT)

$$PMT = 50{,}000\,\frac{\left(\frac{0.07}{12}\right)}{\left[\left(1 + \frac{0.07}{12}\right)^{12(20)} - 1\right]} \approx \$95.98$$

Examples

$PV = ??$

Example 1: Calculate the amount Audrey needs to invest now in one lump sum in order to have $100,000 after 18 years with an APR of 7% compounded monthly. Round your answer to the nearest cent, if necessary. FV

$$PV = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{100{,}000}{\left(1+\frac{0.07}{12}\right)^{12(18)}} = \boxed{\$28{,}469.43}$$

Example 2: Drake starts an IRA (Individual Retirement Account) at the age of 22 to save for retirement. He deposits $400 each month. The IRA has an average annual interest rate of 7% compounded monthly. How much money will he have saved when he retires at the age of 65? Round your answer to the nearest cent, if necessary.

PMT

$FV = ??$

Annuity ★

$$FV = PMT\,\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\left(\frac{r}{n}\right)} = 400\,\frac{\left[\left(1+\frac{0.07}{12}\right)^{12(43)}-1\right]}{\left(\frac{0.07}{12}\right)} \approx \boxed{\$1{,}310{,}451.88}$$

$$t = 65 - 22 = 43$$

Example 3: Jacob deposits $203.77 each month into an annuity account for his child's college fund in order to accumulate a future value of $60,000 in 18 years. How much of the $60,000 will Jacob deposit into the account in total, (1) (2) and how much will be interest he has earned? Round your answers to the nearest cent, if necessary.

(1) Total Deposited = PMT * # of payments

$= 203.77\ (12 \times 18)$

$= \boxed{\$44{,}014.32}$

(2) Interest earned = FV − Total deposited

$= 60{,}000 - 44{,}014.32 = \boxed{\$15{,}985.68}$

Example 4: Devon deposits a fixed amount monthly into an annuity account for his child's college fund. He wishes to accumulate a future value of $65,000 in 17 years. Assuming an APR of 3.6% compounded monthly, how much of the $65,000 will Devon deposit into the account in total, and how much will be interest he has earned? Round your answers to the nearest cent, if necessary.

$$PMT = FV\,\frac{\left(\frac{r}{n}\right)}{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]} = 65{,}000\,\frac{\left(\frac{0.036}{12}\right)}{\left[\left(1+\frac{0.036}{12}\right)^{12(17)}-1\right]} \approx \$231.47$$

Total Deposited = PMT * # of payments

$= 231.47\ (12 \times 17)$

$= \boxed{\$47{,}219.88}$

Interest earned = FV − Total Deposited

$= 65{,}000 - 47{,}219.88$

$= \$17{,}780.12$