7.3 Using Counting Methods to find Probabilities
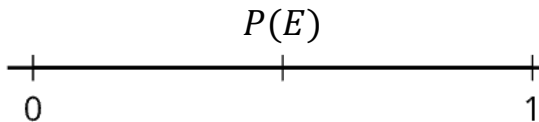
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Complements

Probability

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space}$$

$$= \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

$$0 \leq Probability \leq 1$$

$P(E)$

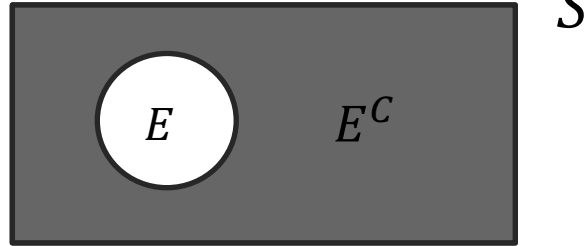0   1

Opposite

■ **Complement of an Event**: Consists of all outcomes in the sample space that are _not_ in event E. $E^C, \quad \bar{E}, \quad E'$
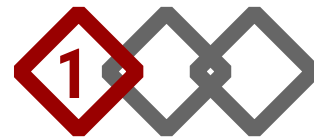
S

E E^C

■ Complement Rules of Probability:

1) $P(E) + P(E^C) = 1$ 2) $P(E) = 1 - P(E^C)$ 3) $P(E^C) = 1 - P(E)$

Example: Suppose we are randomly selecting a single card from a standard 52-card deck.

a) Find the probability of a diamond.
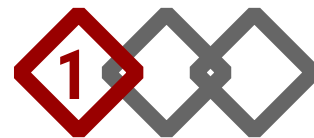
$$P(Diamond) = \frac{13}{52} = \frac{1}{4}$$

b) Find the probability of not a diamond.

$$P(Not\ Diamond) = \frac{39}{52} = 1 - \frac{1}{4} = \frac{3}{4}$$

Directly Using complements

▰ Example: Suppose we collected data on majors of MATH 125 students and are randomly selecting a single student.

a) Find the probability that the student is NOT an Art major.

$$P(Not\ Art) = 1 - P(Art) = 1 - \frac{18}{76} = \frac{58}{76}$$
$$Art^C$$

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20

b) Find the probability that the student is NOT an English nor Chemistry major.

$$P(Not\ English\ nor\ Chem) = 1 - \frac{20 + 15}{76} = \frac{23 + 18}{76} = \frac{41}{76}$$

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Calculating More Probabilities

Example 1: Liam and Michael are going to play video games this afternoon. Together, they have 41 video games. If they decide to randomly choose two video games, what is the probability that the two they choose will consist of each of their favorite video games? Assume they have different favorites.

Two Approaches: 1) Direct way   2) Counting methods

$$** \; P(Event) = \frac{\# \; Successes}{\# \; Possibilities}$$

Example 1: Liam and Michael are going to play video games this afternoon. Together, they have 41 video games. If they decide to randomly choose two video games, what is the probability that the two they choose will consist of each of their favorite video games? Assume they have different favorites.

$$\frac{2}{41} \quad \times \quad \frac{1}{40} \quad = \quad \frac{2}{1640} = \frac{1}{820}$$

1st Choice   2nd Choice

Without replacement

$$** P(Event) = \frac{\#\,Successes}{\#\,Possibilities}$$

$$Prob = \frac{①}{②} = \frac{1}{820}$$

$$** P(Event) = \frac{\#\ Successes}{\#\ Possibilities}$$

** Solve numerator and denominator separately

② Denominator → "No Condition"

Total ways to select 2 from 41 $_nC_r$ → Combinations
(Order doesn't matter)

$$_{41}C_2 = 820$$

① Numerator → "Is a Condition"

Only selecting from favorites

$$_nC_r →\ _2C_2 = 1$$

Example 2: A box of jerseys for a pick-up game of basketball contains 8 extra-large jerseys, 7 large jerseys, and 5 medium jerseys. If you are first to the box and grab 3 jerseys, what is the probability that you randomly grab 3 extra-large jerseys.

20 Total
8 XL

$$\frac{8}{20} \quad \text{x} \quad \frac{7}{19} \quad \text{x} \quad \frac{6}{18} \quad = \quad \frac{14}{285}$$

1st Selection   2nd   3rd

$$** P(Event) = \frac{\# \, Successes}{\# \, Possibilities}$$

$$Prob = \frac{①}{②} = \frac{56}{1140} = \frac{14}{285}$$

$$** P(Event) = \frac{\#\,Successes}{\#\,Possibilities}$$

** Solve numerator and denominator separately

① Selecting 3 XL jerseys

$$_8C_3 = 56$$

② Selecting 3 jerseys from all jerseys

$$_{20}C_3 = 1140$$

Example 3: There are 11 balls numbered 1 through 11 placed in a bucket. What is the probability of reaching into the bucket and randomly drawing two balls numbered 1 and 4 without replacement, in that order?

$$** P(Event) = \frac{\# \, Successes}{\# \, Possibilities}$$

Direct Way

$$\frac{1}{11} \times \frac{1}{10} = \frac{1}{110}$$

#1   #4

Counting Way

$$P(1,4) = \frac{1}{_{11}P_2} = \frac{1}{110}$$

Numerator → Only 1 way to get a 1, then a 4

Order matters! → $_nP_r$

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Examples

Example

3

Example 4: Julia sets up a passcode on her smart phone, which allows only six-digit codes. A spy sneaks a look at Julia's smart phone and sees her fingerprints on the screen over six numbers. What is the probability the spy is able to unlock the smart phone on his first try?

Direct Way: $\dfrac{1}{6} \times \dfrac{1}{5} \times \dfrac{1}{4} \times \dfrac{1}{3} \times \dfrac{1}{2} \times \dfrac{1}{1} = \dfrac{1}{720}$

Counting Way: $P(Correct\ Password) = \dfrac{1}{{}_6P_6} = \dfrac{1}{6!} = \dfrac{1}{720}$

Numerator: Only 1 correct password

Denominator: 6 #s to choose from (6 fingerprints) and Ordering all 6 → r = 6