★ No formula sheet on Test

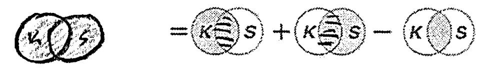
7.4 Addition and Multiplication Rules of Probability – Overview

Addition Rules

Addition Rule for Probability: Consider two events A and B. The probability of A or B occurring is

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$

P(king or spade) = P(king) + P(spade) − P(king and spade)



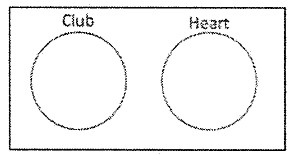
(double counted) intersection

Mutually Exclusive Events: Two events are considered to be mutually exclusive if they have no outcomes in common.

Addition Rule for Mutually Exclusive Events:

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

P(Club or Heart) = P(Club) + P(Heart)



No overlap

$P(A \text{ and } B) = 0$

Example 1: Suppose we collected data from MATH 125 students about their major and attendance record. Then we randomly selecting a single student. Assume no double majors.

	Statistics	Art	Chemistry	*Total*
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

a) Find the probability the student is a Statistics major.

$P(\text{stats}) = \frac{150}{435}$

b) Find the probability the student has Good attendance.

$P(\text{Good}) = \frac{140}{435}$

c) Find the probability the student is a Statistics major and has Good attendance.

$P(\text{stats and Good}) = \frac{20}{435}$

d) Find the probability the student is a Statistics major or has Good attendance.

$P(\text{stats or Good}) = \frac{150}{435} + \frac{140}{435} - \frac{20}{435} = \frac{270}{435} = \frac{18}{29}$

e) Find the probability the student is a Chemistry major or has Poor attendance.

$P(\text{Chem or Poor}) = \frac{180 + 75 - 30}{435} = \frac{225}{435} = \frac{15}{29}$

f) Find the probability the student is an Art major or a Chemistry major.

$P(\text{Art or Chem}) = \frac{105 + 180 - 0}{435} = \frac{285}{435}$

No overlap (mutually exclusive)

"B given A"

Conditional Probability

The conditional probability of Event B, given that Event A has already occurred is written as: P(B | A)

- Event A is the "additional information" that we know, so we can restrict what we are looking at if we have a table. Then we are interested in Event B..

Example 2:

	Statistics	Art	Chemistry	*Total*
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

→ goes second   → goes first

a) Given the student has Perfect attendance, find the probability they are a Chemistry major.

$$P(\text{Chem} \mid \text{Perfect}) = \frac{80}{220}$$

b) Find the probability the student has Perfect attendance given they are a Chemistry major.

$$P(\text{Perfect} \mid \text{Chem}) = \frac{80}{180}$$

c) Given the student is an Art major, find the probability they have Poor attendance.

$$P(\text{Poor} \mid \text{Art}) = \frac{15}{105}$$

Example 3: A swim team consists of 6 boys and 4 girls. A relay team of 4 swimmers is chosen at random from the team members. What is the probability that 2 boys are selected for the relay team given that the first two selections were girls?

Conditional   ↳ additional info

$\underline{G}$ $\underline{G}$ $\underline{\quad}$ $\underline{\quad}$ (Remaining spots)

8 total
6 Boys
2 Girls

$P(\text{next 2 Boys}) = ??$

Direct way

$$\underbrace{\frac{6}{8}}_{\text{Boy 1}} \times \underbrace{\frac{5}{7}}_{\text{Boy 2}} = \frac{30}{56} = \frac{15}{28}$$

Counting way

Order doesn't matter → $_nC_r$

$$\frac{\text{Successes}}{\text{Possibilities}} = \frac{_6C_2}{_8C_2} = \frac{15}{28}$$

Independent IF
→ with replacement
→ unrelated experiments

Multiplication Rules

No impact

Independent Events: The result of one event does not influence the probability of the other.

Dependent Events: The result of one event does influence the probability of the other.

Multiplication Rule for Independent Events: Consider two independent events A and B.
The probability of A and B occurring is:

$$\boxed{P(A \text{ and } B) = P(A) \times P(B)}$$

Independent

Example 2: Three cards are drawn with replacement from a standard deck of 52 cards.
Find the probability that the first card will be a diamond, the second card will be a red card, and the third card will be a queen. ↳ Q ↳ D ↳ R

$$P(D \text{ and } R \text{ and } Q) = \underbrace{\frac{13}{52}}_{P(D)} \times \underbrace{\frac{26}{52}}_{P(R)} \times \underbrace{\frac{4}{52}}_{P(Q)} = \frac{4}{416} = \frac{1}{104}$$

Multiplication Rule for Dependent Events: Consider two dependent events A and B.
The probability of A and B occurring is:

$P(A \text{ and } B) = P(A) \times P(B \mid A)$

"Both events occurred" = "A occurred, then B occurred later"

Dependent

Example 3: If you are dealt two cards from a standard 52 card deck without replacement. Find the probability of getting a 10 of hearts and then a red card.

$$P(10H \text{ and } Red) = \underbrace{\frac{1}{52}}_{P(10H)} \times \underbrace{\frac{25}{51}}_{P(Red \mid 10H)} = \frac{25}{2652}$$

Bayes Theorem: Useful for converting from one conditional probability to another (Example 10 in Learn)

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)},$$ when P(B) > 0.