# Chapter 10 Probability – (Study) Formula Sheet

## 10.1 – Introduction to Probability

Probability:

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space} = \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

$$0 \le P(Event) \le 1$$

Sample space:

- All possible outcomes
- Ex: Rolling a die, $S = \{1, 2, 3, 4, 5, 6\}$

## 10.2 – Counting Outcomes

Fundamental Counting Principle:

- Total number of ways a job can be done
- Just multiply the number of ways to do each individual task)

| Task 1 | Task 2 | ⋯ | Task $n$ | Total Outcomes |
|---|---|---|---|---|
| $k_1$ | $k_2$ | ⋯ | $k_n$ | $k_1 \times k_2 \times \cdots \times k_n$ |

Ex: Total # of ways to get dressed with 3 pairs of socks, 4 pairs of pants, 2 belts, and 3 ties

$\underset{socks}{\underline{3}} \times \underset{pants}{\underline{4}} \times \underset{belts}{\underline{2}} \times \underset{ties}{\underline{3}} = 72$ total ways

Factorial:

- $n! = n(n-1)(n-2)\cdots(3)(2)(1)$

Combinations and Permutations:

- How many ways to "select r objects from a total of n objects" (without replacement).
- Combinations: nCr → Order **does NOT** matter
  - Ex: Selecting a committee
- Permutations: nPr → Order **DOES** matter (meaning to the "slots")
  - Ex: Selecting a President, Vice President and Secretary

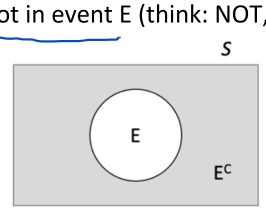
## 10.3 – Probability of Single Events

Odds for $= \frac{P(Win)}{P(Lose)}$ Odds against $= \frac{P(Lose)}{P(Win)}$

- To convert from probability to odds: $P(A) \to a : b$, First write the probabilities as fractions
- To convert from odds to a probability: $a : b \to P(A) = \frac{a}{a+b}$
-

Complements:

- $E^C$ → All outcomes in the sample space that are not in event E (think: NOT, opposite, take it out)
- Complement rules of probability
  1) $P(E) + P(E^C) = 1$
  2) $P(E) = 1 - P(E^C)$
  3) $P(E^C) = 1 - P(E)$

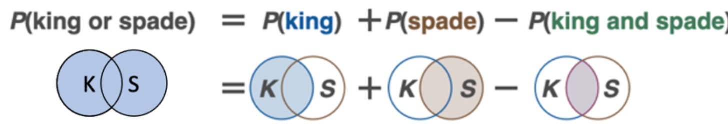



Calculating (harder) probabilities:

- TWO Approaches: Ex: Select 3 Hearts from 52 cards without replacement
  - Direct way $\underset{13}{\underline{\frac{13}{52}}} \times \underset{H}{\underline{\frac{12}{51}}} \times \underset{H}{\underline{\frac{11}{50}}}$ $P(Event) = \frac{\#\ Successes}{\#\ Possibilities}$
  - Counting methods $\frac{{}_{13}C_3 \text{ Hearts ONLY}}{{}_{52}C_3 \text{ ALL}}$ Solve numerator and denominator separately

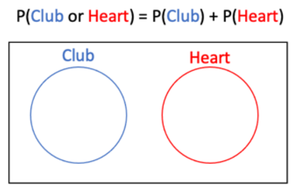
order doesn't matter
$\Rightarrow nCr$

## 10.4 – Addition and Multiplication Rules of Probability




Addition rules:

- The probability of A or B occurring
  *P(A or B) = P(A) + P(B) – P(A and B)*




- Mutually Exclusive Events:
  - No outcomes in common (no overlap)

- Addition Rule for Mutually Exclusive Events:
  - *P(A or B) = P(A) + P(B)*

Conditional Probability:

| | Stats | Art | Total |
|---|---|---|---|
| Perfect | 100 | 40 | 140 |
| Good | 20 | 50 | 70 |
| Total | 120 | 90 | 210 |

- P(B | A) = The conditional probability of Event B, given that Event A has already occurred
- Event A is the “additional info” (GOES SECOND); then we are interested in Event B (GOES FIRST)

Ex) Find probability a student is a stats major given they have Good attendance

$$P(\text{Stats} \mid \text{Good}) = \frac{20}{70}$$

Multiplication Rules:

- Independent Events:
  - The result of one event does not influence the probability of the other
  - With replacement, unrelated experiments

- Dependent Events:
  - The result of one event does influence the probability of the other
  - Without replacement

- Multiplication Rule for Independent Events:
  - The probability of A and B occurring is:
    *P(A and B) = P(A) x P(B)*

- Multiplication Rule for Dependent Events:
  - The probability of A and B occurring is:
    *P(A and B) = P(A) x P(B | A)*        *“Both events occurred” = “A occurred, then B occurred later”*

## 10.5 – Expected Value

| X | P(x) | E(x) | Overall E(x) |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

Expected value: $E(X) = x_1\,P(X_1) + x_2P(X_2) + \cdots + x_nP(X_n)$

- Steps to find:
  1) Make a table; 2) Think about X values first; 3) Then find probabilities; 4) Then calculate E(X)

Sum of Expected Values:

- To find the combined expected value of multiple events, just add the individual expected values.
- $E(X\ or\ Y) = E(X) + E(Y)$