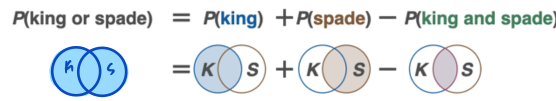
★ No formula sheet on Test :(

10.4 Addition and Multiplication Rules of Probability – Overview

Addition Rules

Addition Rule for Probability: Consider two events A and B. The probability of A or B occurring is

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$



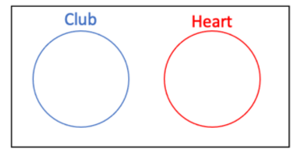
Double Counted Intersection

Mutually Exclusive Events: Two events are considered to be mutually exclusive if they have no outcomes in common.

Addition Rule for Mutually Exclusive Events:

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

P(Club or Heart) = P(Club) + P(Heart)



No overlap → $P(A \text{ and } B) = 0$

Example 1: Suppose we collected data from MATH 125 students about their major and attendance record. Then we randomly selecting a single student. Assume no double majors.

	Statistics	Art	Chemistry	*Total*
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

a) Find the probability the student is a Statistics major.

$P(\text{stats}) = \frac{150}{435}$

b) Find the probability the student has Good attendance.

$P(\text{Good}) = \frac{140}{435}$

c) Find the probability the student is a Statistics major and has Good attendance.

$P(\text{stats and Good}) = \frac{20}{435}$

d) Find the probability the student is a Statistics major or has Good attendance.

$P(\text{stats or Good}) = \frac{150}{435} + \frac{140}{435} - \frac{20}{435} = \frac{270}{435}$

e) Find the probability the student is a Chemistry major or has Poor attendance.

$P(\text{Chem or Poor}) = \frac{180 + 75 - 30}{435} = \frac{225}{435} = \frac{15}{29}$

f) Find the probability the student is an Art major or a Chemistry major.

$P(\text{Art or Chem}) = \frac{105 + 180 - 0}{435} = \frac{285}{435}$

No Overlap (mutually exclusive)

Conditional Probability

"B given A"

The conditional probability of Event B, given that Event A has already occurred is written as: P(B | A)

- Event A is the "additional information" that we know, so we can restrict what we are looking at if we have a table. Then we are interested in Event B.

Example 2:

	Statistics	**Art**	**Chemistry**	***Total***
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

goes second → goes first

a) Given the student has Perfect attendance, find the probability they are a Chemistry major.

$P(\text{chem} \mid \text{perfect}) = \frac{80}{220}$

b) Find the probability the student has Perfect attendance given they are a Chemistry major.

$P(\text{Perfect} \mid \text{Chem}) = \frac{80}{180}$

c) Given the student is an Art major, find the probability they have Poor attendance.

$P(\text{Poor} \mid \text{Art}) = \frac{15}{105}$

Example 3: A swim team consists of 6 boys and 4 girls. A relay team of 4 swimmers is chosen at random from the team members. What is the probability that 2 boys are selected for the relay team given that the first two selections were girls?

↳ Conditional ↳ additional info

6 6 (remaining spots)

$P(\text{next 2 Boys}) = ??$

8 Total
6 Boys
2 Girls

Direct

$\underset{\text{Boy 1}}{\frac{6}{8}} \times \underset{\text{Boy 2}}{\frac{5}{7}} = \frac{30}{56} = \frac{15}{28}$

Counting

order doesn't matter → $_nC_r$

$\text{Prob} = \frac{\text{Successes}}{\text{Possibilities}} = \frac{_6C_2}{_8C_2} = \frac{15}{28}$

Independent IF
→ with replacement
→ Unrelated experiments

Multiplication Rules

No impact

Independent Events: The result of one event does not influence the probability of the other.

Dependent Events: The result of one event does influence the probability of the other.

Multiplication Rule for Independent Events: Consider two independent events A and B.
The probability of A and B occurring is:

$\boxed{P(A \text{ and } B) = P(A) \times P(B)}$

Independent

Example 4: Three cards are drawn with replacement from a standard deck of 52 cards.
Find the probability that the first card will be a diamond, the second card will be a red card, and the third card will be a queen.

$$P(D + R + Q) = \underbrace{\frac{13}{52}}_{P(D)} \times \underbrace{\frac{26}{52}}_{P(R)} \times \underbrace{\frac{4}{52}}_{P(Q)} = \frac{4}{416} = \frac{1}{104}$$

Multiplication Rule for Dependent Events: Consider two dependent events A and B.
The probability of A and B occurring is:

$\boxed{P(A \text{ and } B) = P(A) \times P(B \mid A)}$

"Both events occurred" = "A occurred, then B occurred later"

Dependent

Example 5: If you are dealt two cards from a standard 52 card deck without replacement. Find the probability of getting a 10 of hearts and then a red card.

$$P(10H + \text{Red}) = \underbrace{\frac{1}{52}}_{P(10H)} \times \underbrace{\frac{25}{51}}_{P(\text{Red} \mid 10H)} = \frac{25}{2652}$$

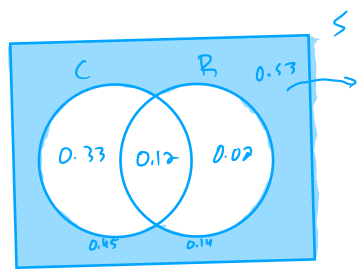
Example 6: Suppose that the probability of randomly selecting a cookie with chocolate chips out of a variety tin is 0.45 and the probability of selecting a cookie with raisins is 0.14. If the probability of selecting a cookie with raisins and chocolate chips is 0.12. What is the probability that the cookie chosen has neither raisins nor chocolate chips?

$P(C) = 0.45$

$P(R) = 0.14$

$P(C \text{ and } R) = 0.12$

$P(\text{Not } C \text{ and Not } R) = ??$



$1 - [0.33 + 0.12 + 0.02] = \boxed{0.53}$

★ Label overlap first

OR

$P(\text{outside}) = 1 - P(C \text{ or } R)$

$= 1 - [0.45 + 0.14 - 0.12]$

$= 1 - 0.47$

$= 0.53$ ✓