

9.2 Understanding Interest – Overview

Definitions / Key Ideas

Interest – Amount charged for borrowing money or earned from investing

(n values)

Principal – Initial investment or loan amount.

Annual Percentage Rate (APR) – Yearly interest rate (normally given as percentage per year).

Simple Interest – Only calculated on the principal.

Compound Interest – Calculated on principal and accrued interest.

Continuously Compounded Interest – Interest is compounded continuously.

Annual Percentage Yield (APY) – Effective annual interest rate (accounts for compounding).

Table 1: Compounding Intervals

| Compounding | Number per Year |
|--------------|-----------------|
| Annually | 1 |
| Semiannually | 2 |
| Quarterly | 4 |
| Monthly | 12 |
| Weekly | 52 |
| Daily | 365 |

Formulas and Examples

1. Simple Interest

$$I = P \cdot r \cdot t$$

\downarrow Interest
 \downarrow Principal
 \downarrow rate (decimal)
 \downarrow time (Years)

2. Compound Interest (regular)

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

\downarrow Future value (Total Value)
 \downarrow # of compounding periods

3. Compound Interest (continuous)

$$A = Pe^{rt}$$

How much interest will I earn if...

Ex: I invest \$500 at 10% APR with simple interest for 8 years? For 6 months?

$$I = 500 (0.10) 8$$

$\downarrow = \$400$ in interest

$$A = P + I$$

$\downarrow = 500 + 400 = \900

$$I = 500 (0.10) \left(\frac{6}{12} \right)$$

$\downarrow = \$25$ in interest

$$A = P + I$$

$\downarrow = 500 + 25 = \525

Ex: I invest \$500 at 10% APR for 8 years, compounded monthly? Quarterly?

$$A = 500 \left(1 + \frac{0.10}{12} \right)^{12(8)}$$

$\downarrow = 500 (1.008333)^{96}$

$\downarrow = \$1109.09$

$$I = A - P$$

$\downarrow = 1109.09 - 500$

$\downarrow = \$609.09$ in interest

↳ same process, but change to $n = 4$

$\Rightarrow A = \$1101.88$

Ex: I invest \$500 at 10% APR for 8 years with continuous compounding?

$$A = 500 e^{0.10(8)}$$

$\downarrow \approx \$1112.77$

$$I = A - P$$

$\downarrow = 1112.77 - 500$

$\downarrow = \$612.77$ in interest

4. Annual Percentage Yield

Ex: What is the Annual Percentage Yield (APY) for example 2? ^{Monthly example}

$$APY = \left[\left(1 + \frac{r}{n} \right)^n - 1 \right] \times 100 \%$$

$$APY_{\text{monthly}} = \left[\left(1 + \frac{0.10}{12} \right)^{12} - 1 \right] \times 100 \%$$

$\downarrow \approx 10.47 \%$

★ Calculator tips ★

→ recommend typing equation ALL IN ONE STEP → Type like how it is written

Examples

Example 1: Suppose you wish to borrow \$200 for five weeks and the amount of interest you must pay is \$20 per \$100 borrowed. What is the APR at which you are borrowing money?

Simple Interest

$$I = P \cdot r \cdot t$$

$$I = \frac{\$20}{\$100} \Rightarrow \$40 \text{ for } \$200 \text{ borrowed}$$

$$P = \$200$$

$$r = ?? \text{ (This is what we are looking for)}$$

$$t = \frac{5 \text{ weeks}}{52 \text{ weeks}} = \frac{5}{52}$$

$$\frac{40}{200} = \frac{200 \cdot r \left(\frac{5}{52} \right)}{200}$$

$$\frac{52}{5} \times \frac{1}{5} = r \frac{5}{52} \times \frac{52}{5}$$

$$r = \frac{52}{25} = 2.08 \times 100\%$$

$$= 208\%$$

→ You are borrowing at an APR of 208%

Example 2: Suppose that \$13,000 is deposited for eight years at 4% APR. Calculate the interest earned if interest is compounded weekly. Round your answer to the nearest cent.

★ Compound interest

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$\rightarrow A = 13,000 \left(1 + \frac{0.04}{52} \right)^{52(8)}$$

$$A = ??$$

$$\downarrow = \$17,900.46$$

It type
calc all in
one step

$$P = \$13,000$$

$$I = A - P$$

$$r = 4\% = 0.04$$

$$= 17,900.46 - 13,000$$

$$n = 52 \text{ (weekly)}$$

$$\downarrow = \$4,900.46 \rightarrow \text{you earned } \approx \$4,900.46 \text{ in interest}$$

$$t = 8 \text{ (years)}$$

→ If do calculations in steps

$$A = 13,000 \left(1 + \frac{0.04}{52} \right)^{52(8)}$$

$$\downarrow \approx 13,000 (1 + 0.000769)^{416}$$

$$\downarrow \approx \$17,898.74$$

$$I = A - P$$

$$= 17,898.74 - 13,000$$

$$\downarrow \approx \$4,898.74$$

(slightly different than previous answer due to rounding)