Chapter 10 Probability – (Study) Formula Sheet

10.1 – Introduction to Probability

Probability:

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space} = \frac{Number\ of\ successes}{Number\ of\ possibilities}$$
$$0 \le P(Event) \le 1$$

Sample space:

- All possible outcomes
- Ex: Rolling a die, *S* = {1, 2, 3, 4, 5, 6}

10.2 – Counting Outcomes

Fundamental Counting Principle:

- Total number of ways a job can be done
- Just multiply the number of ways to do each individual task)

Task 1	Task 2	⋯	Task *n*	Total Outcomes
k_1	k_2	⋯	k_n	$k_1 \times k_2 \times \cdots \times k_n$

Ex: Total # of ways to get dressed with 3 pairs of socks, 4 pairs of pants, 2 belts, and 3 ties

$\underset{\text{socks}}{3} \times \underset{\text{pants}}{4} \times \underset{\text{belts}}{2} \times \underset{\text{ties}}{3} = 72$ total ways

Factorial:

- $n! = n(n-1)(n-2)\cdots(3)(2)(1)$

Combinations and Permutations:

- How many ways to "select r objects from a total of n objects" (without replacement).
- Combinations: nCr → Order **does NOT** matter
  - Ex: Selecting a committee
- Permutations: nPr → Order **DOES** matter (meaning to the "slots")
  - Ex: Selecting a President, Vice President and Secretary

10.3 – Probability of Single Events

Odds for $= \frac{P(Win)}{P(Lose)}$ Odds against $= \frac{P(Lose)}{P(Win)}$

- To convert from probability to odds: $P(A) \rightarrow a:b$, First write the probabilities as fractions
- To convert from odds to a probability: $a:b \rightarrow P(A) = \frac{a}{a+b}$
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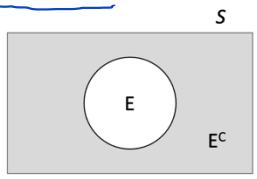
Complements:

- E^C → All outcomes in the sample space that are not in event E (think: NOT, opposite, take it out)
- Complement rules of probability
  1) $P(E) + P(E^C) = 1$
  2) $P(E) = 1 - P(E^C)$
  3) $P(E^C) = 1 - P(E)$



Calculating (harder) probabilities:

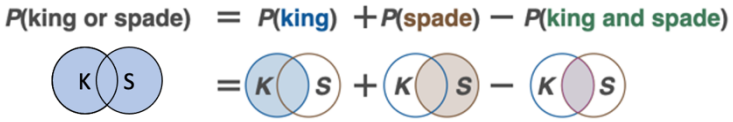
- TWO Approaches: Ex: Select 3 Hearts from 52 cards without replacement
  - Direct way $\underset{13}{\frac{13}{52}} \times \underset{H}{\frac{12}{51}} \times \underset{H}{\frac{11}{50}}$ $P(Event) = \frac{\#\ Successes}{\#\ Possibilities}$
  - Counting methods $\frac{(13)C_3 \text{ Hearts ONLY}}{(52)C_3 \text{ ALL}}$ Solve numerator and denominator separately

order doesn't matter ⇒ nCr

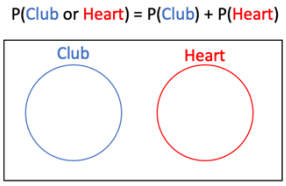
10.4 – Addition and Multiplication Rules of Probability

Addition rules:

- The probability of A or B occurring
  P(A or B) = P(A) + P(B) – P(A and B)



- Mutually Exclusive Events:
  - No outcomes in common (no overlap)
- Addition Rule for Mutually Exclusive Events:
  - *P(A or B) = P(A) + P(B)*



Conditional Probability:

	Stats	Art	Total
Perfect	100	40	140
Good	20	50	70
Total	120	90	210

- P(B | A) = The conditional probability of Event B, given that Event A has already occurred
- Event A is the "additional info" (GOES SECOND); then we are interested in Event B (GOES FIRST)

Ex) Find probability a student is a stats major given they have Good attendance

$P(\text{stats} \mid \text{Good}) = \frac{20}{70}$

Multiplication Rules:

- Independent Events:
  - The result of one event does not influence the probability of the other
  - With replacement, unrelated experiments
- Dependent Events:
  - The result of one event does influence the probability of the other
  - Without replacement
- Multiplication Rule for Independent Events:
  - The probability of A and B occurring is:
    P(A and B) = P(A) x P(B)
- Multiplication Rule for Dependent Events:
  - The probability of A and B occurring is:
    P(A and B) = P(A) x P(B | A) *"Both events occurred" = "A occurred, then B occurred later"*

10.5 – Expected Value

Expected value: $E(X) = x_1 P(X_1) + x_2 P(X_2) + \cdots + x_n P(X_n)$

X	P(x)	E(x)	Overall E(x)

- Steps to find:
  1) Make a table; 2) Think about X values first; 3) Then find probabilities; 4) Then calculate E(X)

Sum of Expected Values:

- To find the combined expected value of multiple events, just add the individual expected values.
- $E(X \text{ or } Y) = E(X) + E(Y)$