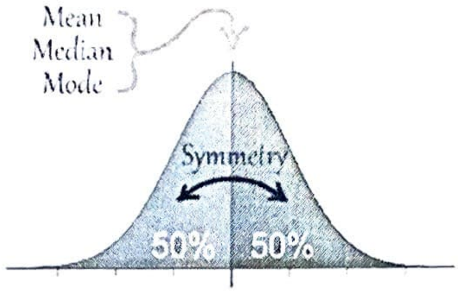
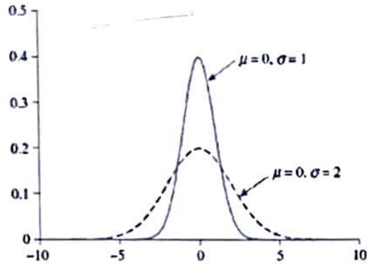
8.4 The Normal Distribution – Overview

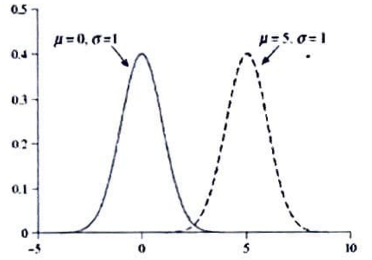
PARANORMAL DISTRIBUTION

Normal Distribution Properties



- It's a symmetric, unimodal, bell-shaped distribution
  ⇒ which implies mean = median = mode.
- Total area under curve (probability) is equal to 1 = 100%.
- Completely described by its mean μ (location) and standard deviation σ (spread).





- The normal distribution allows us to find any probability, not just for points that lie exactly 1, 2, or 3 standard deviations ("steps") away from the mean like with the empirical rule!

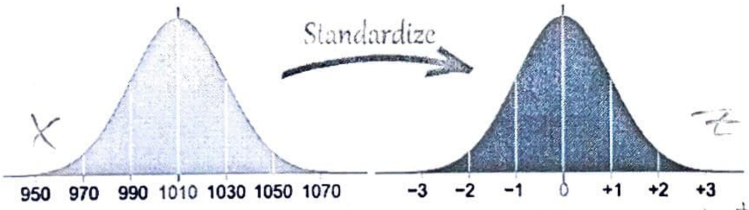
Z-scores ("Standard" scores in Hawkes Certify)

- Definition: A **z-score** standardizes observations based on the mean (center) and standard deviation (spread) of the distribution.
  - Allows for comparisons on different scales.

    ex) ACT vs SAT

Formula: $z = \frac{x-\mu}{\sigma} = \frac{x-\bar{x}}{s} = \frac{obs - mean}{st\ dev}$

- Interpretation:
  - A **z-score** tells us how many standard deviations an observation is away from the mean.
  - The unit of a **z-score** is standard deviations.



−below ← Mean → + above

Example 1) For each data set with the stated μ and σ, find the standard score (z score) corresponding to the given observation, x.

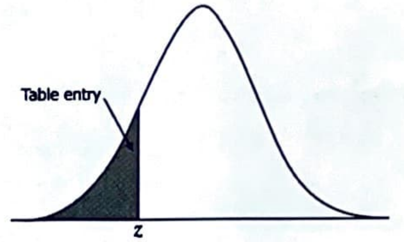
$z = \frac{x-\mu}{\sigma}$

a) $\mu = 8$, $\sigma = 3$, $x = 17$ → $z = \frac{(17-8)}{3} = 3$, above mean

b) $\mu = 100$, $\sigma = 16$, $x = 80$ → $z = \frac{80-100}{16} = -1.25$, below mean

c) Which observation is further from the mean relatively? (a) $z=3$ because "larger" value

Finding probabilities based on the Normal Distribution

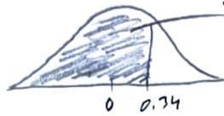


- Handout: Normal Distribution Table
  - Use the handout to convert z-scores to percentiles ("left probabilities").
  - ★ ALWAYS gives probability LESS THAN Z: P(Z < z). "Left"

- Different types of probabilities
  - Left probability = Table (directly)

★ Draw, label + shade curve

  - Example: Find the total area under the standard normal curve (probability or percentage) to the left of z = 0.34.

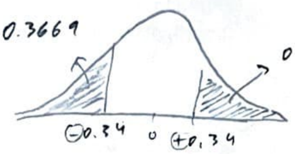


Left = 0.6331

  - Right probability = 1 − Left (table)
    - Examples: Find the probability to the right of z = 0.34.

Symmetric

Right = 1 − Left
= 1 − 0.6331
= 0.3669

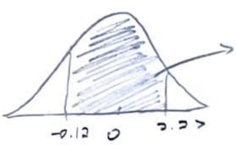


Left −z = Right +z

Find the probability to the right of z = -1.2.

Right = 1 − Left
= 1 − 0.1151
= 0.8849

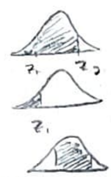
  - Between probability = Left z_2 − Left z_1
    - Example: Find the probability between z_1 = -0.12 and z_2 = 2.27.



Between = Left z_2 − Left z_1
= 0.9884 − 0.4522
= 0.5362

why this works

Left z_2
(minus) −
Left z_1
=
Between



  - Outside probability = Left z_1 + Right z_2
    - Example: Find the probability to the left of z_1 = -0.12 and to the right of z_2 = 2.27.

Outside = Left z_1 + Right z_2
= 0.4522 + 0.0116
= 0.4638

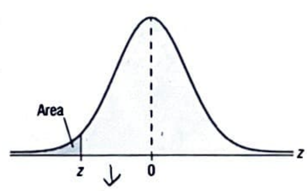
OR

Outside = 1 − Between

A Standard Normal Distribution

Numerical entries represent the probability that a standard normal random variable is between −∞ and z.

P(Z < −3.12) = 0.0009



z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
−3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
−3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
−3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
−3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
−3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
−2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
−2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026