10.6 Expected Value

Goals for the Day

1

Expected Value

2

Examples + Calculator Fun Sess

1

Expected Value

Expected Value

- Definition: The **Expected value** of event *X* is the value we would expect to happen if we performed an experiment many, many times.
- A long-term average

- How to calculate it:
  - Multiple each outcome (*x* value) by its probability and add them together

"Weighted average" ⟵ $$E(X) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_n P(x_n),$$

where x_i is the i^{th} outcome, and $P(x_i)$ is the probability of x_i.

Example 1

In soccer, you earn a certain number of points based on the result of a game, as shown in the table below. Calculate the expected value of the number of points earned for a single game.

X	P(X)
Win (3 points)	0.3
Tie (1 point)	0.5
Loss (No points)	0.2

$$E(X) = 3(0.3) + 1(0.5) + 0(0.2) = 1.4 \text{ points}$$

(Long term average)

Sum of Expected Values

- **Sum of Expected Values**: To find the combined expected value of multiple events, add the individual expected values of each event.

$$E(X \text{ or } Y) = E(X) + E(Y)$$

Example 1 (Continued)

Find the expected value for the total number of points earned in a season if the season has 12 games.

$$X_1 = Game\ 1, \ldots, X_{12} = Game\ 12$$

$$E(Total\ Points) = E(X_1) + E(X_2) + \ldots + E(X_{12})$$

$$= 1.4 + 1.4 + \ldots + 1.4$$

Same E(X) for each

$$= 12(1.4) = 16.8\ Points$$

Expected Value

Strategy
1) Think about the possible X values
2) THEN the probabilities

Example 2

Jim likes to day trade on the Internet. On a good day, he averages a $1400 gain. On a bad day, he averages a $900 loss. Suppose that he has good days 30% of the time, bad days 50% of the time, and the rest of the time, he breaks even ($0 gain). What is the expected value for one day of Jim's day-trading hobby? (Hint: Fill in the table to help solve the problem.)

X	***P(X)***	***E(X)***	***Overall E(X)***

Expected Value

Example 3

Suppose that you and a friend are playing cards and decide to make a bet. If your friend draws two hearts in a row from a standard deck of 52 cards without replacing the first card, you give him $10. Otherwise, he pays you $20. If the same bet was made 15 times, how much would you expect to win or lose? Round your answer to the nearest cent, if necessary.

X	***P(X)***	***E(X)***	***1 round E(X)***	***15 rounds E(X)***

2

Examples

Examples - Calculator Fun Sess

Example 4

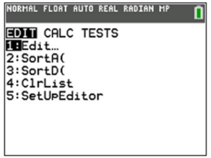
Calculate the expected value of the scenario:

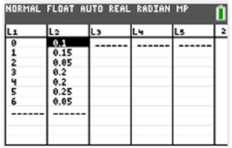
Steps for the TI-83/84	Steps for the TI-30XS MultiView
1. Enter data: STAT → Edit →Enter X values in L_1 and probabilities in L_2 2. Calculate: STAT → CALC → 1-Var Stats List: L_1. FreqList: L_2. Calculate!	1. Data → Enter X values in L_1 and probabilities in L_2 2. 2^{nd} → stat → 1-Var Stats a) DATA: L1 b) FRQ: L2 c) CALC

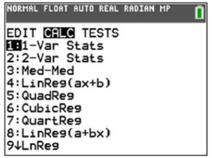
** Note: Unable to do this on the TI-30 XIIS

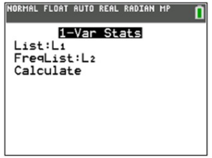
Inputs

x	$P(X_i)$
0	0.10
1	0.15
2	0.05
3	0.20
4	0.20
5	0.25
6	0.05

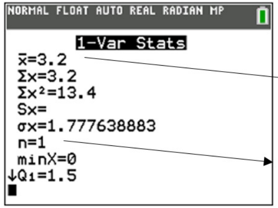








Results



Expected Value

If typed in probabilities correctly → Sum(Probs) = 1

Example 5

A typical three-reel mechanical slot machine has different payoffs determined by the number and position of various pictures. Suppose the payoff (in dollars) has the probability distribution given in the table below.

Center Pay line	3 7's	3 bars	3 plums	3 bells	3 oranges	3 cherries	2 cherries	1 cherry	
x	500	100	50	20	10	5	2	1	0
P(x)	$\frac{1}{8000}$	$\frac{1}{8000}$	$\frac{9}{8000}$	$\frac{48}{8000}$	$\frac{64}{8000}$	$\frac{30}{8000}$	$\frac{530}{8000}$	$\frac{3120}{8000}$	$\frac{4197}{8000}$

Find the expected payoff.

$$E(Payoff) = \$0.8725$$