6.3 Borrowing Money

Goals for the Day

1 Credit Cards

2 Loans

3 Amortization Schedules

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Credit Cards

Paying off credit cards

- Number of Fixed Payments Required to Pay Off Credit Cart Debt
  - R = # of payments
  - A = Future value (loan amount)

$$R = \frac{-\log\left[1 - \frac{r}{n}\left(\frac{A}{PMT}\right)\right]}{\log\left(1 + \frac{r}{n}\right)}$$

Credit Card Payments

<u>Example</u>

Franz wants to buy a new computer that costs $2200 using a credit card that has an APR of 19.99%. How long will it take him to pay off the computer if he makes regular monthly payments of $40? How much will he pay overall for the computer?

$$R = \frac{-\log\left[1 - \frac{r}{n}\left(\frac{A}{PMT}\right)\right]}{\log\left(1 + \frac{r}{n}\right)} = \frac{-\log\left[1 - \frac{0.1999}{12}\left(\frac{2200}{40}\right)\right]}{\log\left(1 + \frac{0.1999}{12}\right)} \approx 150.08$$ (151) payments

$$Total\ paid = PMT * \#\ payments = 40 * 151 = \$6{,}040$$

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Loans

Fixed Installment Loans

- **Fixed installment loans (present value annuity)**: Receive money now, in the present, and use the regular payments to pay off the future value of the loan (principal and interest).
- **Down payments** are often required on large loans (house, car, etc.). These reduce the principal of the loan, and the amount that remains is *financed* (borrowed with interest).
- Monthly Payment Formula for Fixed Installment Loans

$$P = Price - Down\ Payment$$

Principal that is financed

$$PMT = \frac{\left(P \cdot \frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}$$

Example

Owen wants to buy a new car for $34,000 (including taxes and fees). He chooses to make a down payment of 20% and wants to finance the remainder. If Owen can get an APR of 3.99% for a 72-month loan, what is the amount of his monthly payment?

$$PMT = \frac{\left(P \cdot \frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]} = \frac{\left(27{,}200 \cdot \frac{0.0399}{12}\right)}{\left[1 - \left(1 + \frac{0.0399}{12}\right)^{-12 \cdot 6}\right]}$$

$$\approx \$425.43$$

$$P = Price \; - Down\ payment$$

$$= 34{,}000 - (34{,}000 * 0.20)$$
$$= 34{,}000 - 6{,}800$$
$$= \$27{,}200$$

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Amortization Schedules

Home Mortgages – Maximum Purchase Price

- **Amortization schedule**: Payments on loans such as mortgages are portioned out between interest and principal. To show you this breakdown over time, lenders provide loan amortization schedules.
- When the monthly schedule is made, the interest is computed using the simple interest formula $I = Prt$.

$$I = 190{,}000 * 0.035 * \frac{1}{12} = \$554.17$$

$$\begin{aligned} Principal &= Payment \ - Interest \\ &= \ 853.18 - 554.1 \\ &= \$299.01 \end{aligned}$$

Loan amount: $190,000 @ 3.5% APR

Payment Number	Payment	Principal	Interest	Balance
1	($853.18)	($299.02)	($554.17)	$189,700.98
2	($853.18)	($299.89)	($553.29)	$189,401.09
3	($853.18)	($300.77)	($552.42)	$189,100.33
4	($853.18)	($301.64)	($551.54)	$188,798.68
5	($853.18)	($302.52)	($550.66)	$188,496.16
6	($853.18)	($303.40)	($549.78)	$188,192.76

Home Mortgage

Example

Find the mortgage balance after the first three payments on a 30-year $180,000 mortgage that was financed at an APR of 5.25% and has a monthly payment of $993.97.

Payment Number	Interest Payment	Principal Payment	Mortgage Balance
1	$787.50	$206.47	$179,793.53
2	$786.60	$207.37	$179,586.16
3	$785.69	$208.28	$179,377.88

$$New\ Balance = Old\ Balance\ - Principal\ Payment$$

$$Balance\ 1 = \ 180{,}000.00 - 206.47\ = \$179{,}793.53$$

$$Balance\ 2 = \ 179{,}793.53 - 207.37\ \ = \$179{,}586.16$$

$$Balance\ 3 = \ 179{,}586.16 - 208.28\ = \$179{,}377.88$$

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Examples

Example #1

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Natalie bought a new car for $26,000. She paid a 10% down payment and financed the remaining balance for 36 months with an APR of 4.8%. Assuming she made monthly payments, determine the total cost of Natalie's car. Round your answer to the nearest cent. Then, determine how much interest she paid.

Total cost = $27,771.92
Interest = $1,771.92

Example #2

Jake bought several concert tickets for a total of $900. He used a credit card that has an APR of 17.77%. How much will he pay in total to pay off the purchases if he makes monthly payments of $30? Round the number of monthly payments up to the nearest whole number. Round your final answer to the nearest whole number, if necessary.

40 payments
$1200