10.2 Counting Outcomes

Goals for the Day

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Fundamental Counting Principle

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Factorials

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Permutations and Combinations

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Fundamental Counting Principle

The Fundamental Counting Principle

- The Fundamental Counting Principle: If a job consists of *n* separate tasks, the first of which can be done k_1 ways, the second k_2 ways, and so on, then the total job can be done in $k_1 * k_2 * \cdots * k_n$ ways.

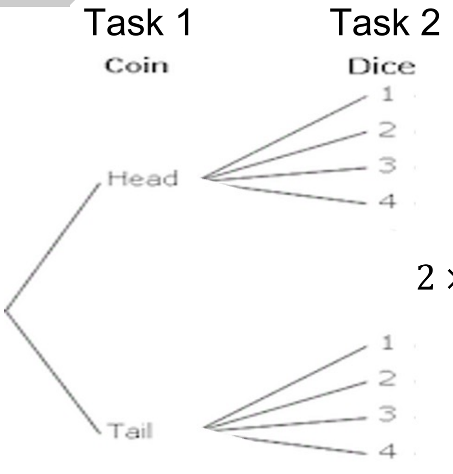
Task 1	Task 2	⋯	Task *n*	Total Outcomes
k_1	k_2	⋯	k_n	$k_1 \times k_2 \times \cdots \times k_n$

- You can find the total number of outcomes by multiplying the number of options together.

Determining Number of Outcomes

- Example: Flip a coin and roll a 4-sided die.

  How many total outcomes are there?



$$2 \times 4 = 8\ outcomes$$

- Example: Sally has 6 pairs of socks, 4 shorts, 5 shirts and 3 sunglasses. How many ways can she get dressed?

$$\underset{Socks}{\underline{6}} \times \underset{Shorts}{\underline{4}} \times \underset{Shirts}{\underline{5}} \times \underset{Sunglasses}{\underline{3}} = 360 \text{ total outcomes}$$

Replacement

- **With or without replacement**: We need to take into account whether or not objects can be repeated in our calculations.

- Examples:

a) How many passwords can you make if it requires 4 digits? With replacement

$$\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 10^4 = 10{,}000 \text{ passwords}$$

Without replacement

b) How many passwords can you make if it requires 4 digits, but you cannot repeat digits?

$$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} = 5{,}040$$

2

Factorials

Factorials

- *n!* (read “n factorial”) is the product of all numbers less than and including n

$$n! = n(n-1)(n-2)\dots(3)(2)(1)$$

- Example: For the 9 starting players on a baseball team, how many different batting orders are possible?

Without replacement

$$\underline{\ 9\ } \times \underline{\ 8\ } \times \underline{\ 7\ } \times \dots \times \underline{\ 1\ } = 9!$$

3

Permutations and Combinations

Selecting From a Group

- Often useful to count the number of ways to choose objects from a group (without replacement)
  - "Selecting r objects from a total of n objects" $r \leq n$
- Two methods
  - Permutation – order matters (e.g., first place, second place, …)
  - Combinations – order doesn't matter (e.g., being picked for a team)

Permutations

- <u>Order matters</u>
  - <u>*When*</u> you are selected is important; **position has meaning**
- Example
  - Ranking favorite movies

$$\underbrace{{}_{n}}_{\text{Total \#}}P_{\underbrace{r}_{\text{How many selecting}}} = P\binom{n}{r} = P(n,r) = \frac{n!}{(n-r)!}$$

Combinations

- Order does not matter
  - When you are selected is unimportant; only matters *that* you were selected
- Example
  - Picking pizza toppings

$$ {}_nC_r = \binom{n}{r} = C(n,r) = \frac{n!}{r!(n-r)!} $$

Permutations and Combinations

Examples

a) There are 8 runners in a race. How many ways can they place 1st, 2nd, and 3rd?

_____ _____ _____
1st 2nd 3rd

Order matters → Permutation $_8P_3 = 336$

b) Out of 12 students, how many ways can we select a committee of 4 students?

Order doesn’t matter → Combination $_{12}C_4 = 495$

c) We are forming a committee, and we need to select a president, vice president, and secretary. If there are 10 members, how many ways can this be done?

Order matters → Permutation $_{10}P_3 = 720$

_____ _____ _____
Pres VP Sec

Permutations with Repeated Objects

- **Permutations with Repeated Objects:** Counting the number of distinct ways we can arrange all *n* objects when some of the objects are the same (repeated, specifically k_1 are alike, k_2 are alike, and so on).

$$\frac{n!}{(k_1!)(k_2!)\ldots(k_p!)}, \text{where } k_1 + k_2 + \cdots + k_p = n$$

Example

Harmony was born on 05/19/1991. How many eight-digit codes could she make using the digits in her birthday?

8 total digits

9 ----> 3 times
1 ----> 3 times
5 ----> 1
0 ----> 1

$$\frac{8!}{3!\ 3!\ 1!\ 1!} = 1{,}120$$