

9.2 Understanding Interest





Goals for the Day

1

Interest

2

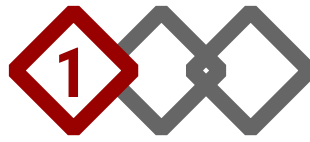
Annual
Percentage
Yield

3

Examples



Some Terms We Need

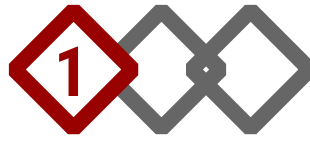


- **Principal** – the sum of money on which interest is charged (the initial investment)
- **Interest rate** – the amount charged to the borrower expressed as a percentage of the principal OR the amount you earn on an investment (still as a percentage)
- **Annual Percentage Rate (APR)** – the yearly interest rate that is charged for borrowing





Simple Interest



■ Interest calculated only based on the principal

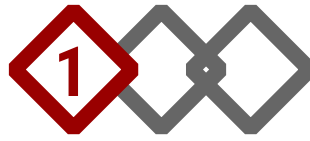
- ▷ I = interest
- ▷ P = Principal
- ▷ r = Rate (as a decimal)
- ▷ t = Time (years)



$$I = Prt$$



Simple Interest



Example

Hannah invests \$500 at 10% APR for 8 years (with simple interest). How much money will she have at the end of the investment (8 years)?

$$I = Prt$$

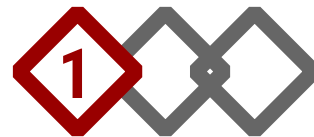
$$= 500 * 0.10 * 8 = \$400$$

$$\text{Total: } A = P + I$$

$$= 500 + 400 = \$900$$



Simple Interest



Example

After 6 months, Hannah needed her money for school, so she withdrew her money. How much interest did she earn?

$$I = Prt$$

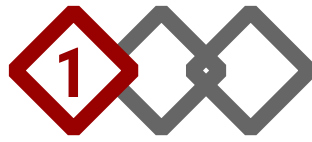
$$= 500 * 0.10 * \frac{6}{12} = \$25$$

$$\text{Total: } A = P + I$$

$$= 500 + 25 = \$525$$



Compound Interest



(n values)

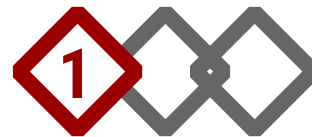
Table 1: Compounding Intervals

Compounding	Number per Year
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$



Compound Interest



Example

Elias invests \$500 at 10% APR for 8 years. How much money will he have at the end of the investing period (8 years) if his interest is compounded monthly? Quarterly?

$$\begin{aligned}A_{\text{monthly}} &= P \left(1 + \frac{r}{n}\right)^{nt} \\&= 500 \left(1 + \frac{0.1}{12}\right)^{12 \cdot 8} \\&\approx \$1,109.09\end{aligned}$$

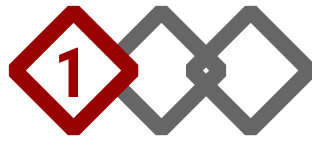
$$\begin{aligned}A_{\text{quarterly}} &= 500 \left(1 + \frac{0.1}{4}\right)^{4 \cdot 8} \\&\approx \$1,101.88\end{aligned}$$

$$\begin{aligned}I_{\text{quarterly}} &= A - P \\&= 1,101.88 - 500 = 601.88\end{aligned}$$

What do you notice about the relationship between n and A ?



Continuously Compounded Interest



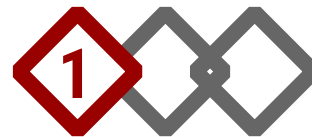
- The smaller the compounding interval, the more money earned.
- Based on a continuous compounding of the interest (literally every moment of every day)
 - ▷ e = the irrational constant

$$A = Pe^{rt}$$





Compound Interest



Example

Amya invests \$500 at 10% APR for 8 years. How much money will he have at the end of the investing period (8 years) if his interest is compounded continuously?

$$\begin{aligned} A &= Pe^{rt} = 500e^{0.1 \cdot 8} \\ &\approx \$1,112.77 \end{aligned}$$

2

Annual Percentage Yield



Annual Percentage Yield (APY)



- When compounding interest, the rate of interest earned is not the same as the stated APR.
 - ▷ Why is this true?
- Annual Percentage Yield (APY) is the effective annual interest rate

$$APY = \left[\left(1 + \frac{r}{n} \right)^n - 1 \right] * 100\%$$



Annual Percentage Yield (APY)



Example

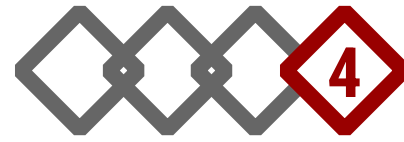
Revisit Elias' investments (\$500 at 10% APR for 8 years). What is his APY if his investment is compounded monthly?

$$\begin{aligned} APY_{monthly} &= \left[\left(1 + \frac{r}{n} \right)^n - 1 \right] * 100\% = \left[\left(1 + \frac{0.1}{12} \right)^{12} - 1 \right] * 100\% \\ &= 10.47\% \end{aligned}$$

3

Examples

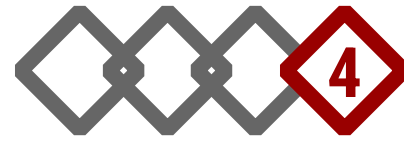
Example #1



Suppose Gavin wants to borrow \$200 for five weeks. The amount of interest he ends up paying is \$20 per \$100 borrowed. What is the APR at which Gavin is borrowing money?

208%

Example #2



Suppose that Casey deposited \$13,000 for eight years at 4% APR. How much interest did Casey earn if the interest is compounded weekly? Round your answer to the nearest cent.

\$4,900.46