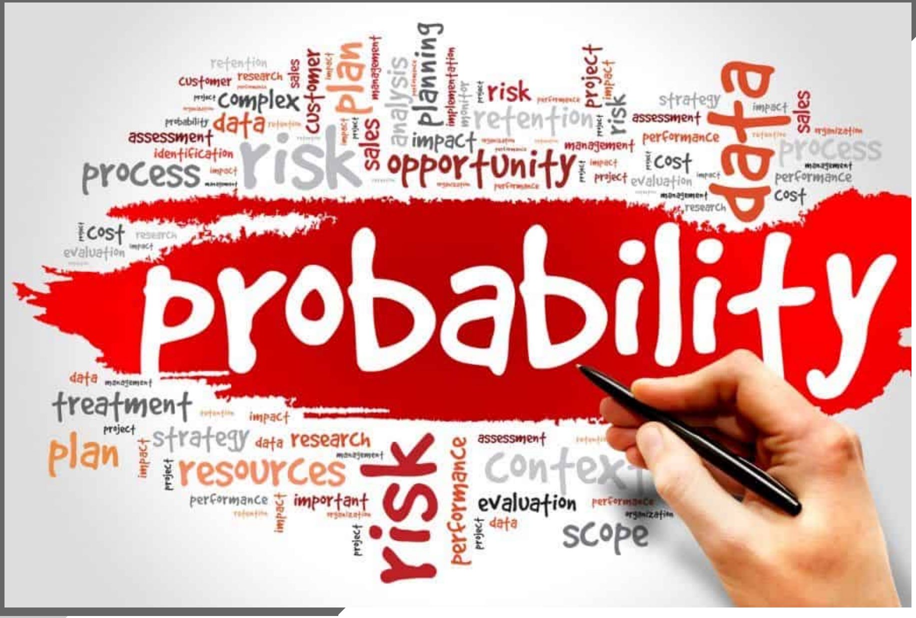
7.5 Expected Value



Goals for the Day

1

Expected Value

2

Odds

1

Expected Value

Expected Value

- Definition: The **Expected value** of event X is the value we would expect to happen if we performed an experiment many, many times.
- A long-term average

- How to calculate it:
  - Multiple each outcome (x value) by its probability and add them together

$$E(X) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_n P(x_n),$$

where x_i is the i^{th} outcome, and $P(x_i)$ is the probability of x_i.

Expected Value

Example

In soccer, you earn a certain number of points based on the result of a game, as shown in the table below. Calculate the expected value of the number of points earned for a single game.

X	P(X)
Win (3 points)	0.3
Tie (1 point)	0.5
Loss (No points)	0.2

$$E(X) = 3(0.3) + 1(0.5) + 0(0.2) = 1.4 \text{ points}$$

(Long term average)

Sum of Expected Values

- **Sum of Expected Values**: To find the combined expected value of multiple events, add the individual expected values of each event.

$$E(X\ or\ Y) = E(X) + E(Y)$$

Sum of Expected Values

Example (Revisited)

Find the expected value for the total number of points earned in a season if the season has 12 games.

$$X_1 = Game\ 1, \dots, X_{12} = Game\ 12$$

$$E(Total\ Points) = E(X_1) + E(X_2) + \dots + E(X_{12})$$

$$= 1.4 + 1.4 + \dots + 1.4$$

Same E(X) for each

$$= 12(1.4) = 16.8\ Points$$

Expected Value

Strategy
1) Think about the possible X values
2) THEN the probabilities

Example

Jim likes to day trade on the Internet. On a good day, he averages a $1400 gain. On a bad day, he averages a $900 loss. Suppose that he has good days 30% of the time, bad days 50% of the time, and the rest of the time, he breaks even ($0 gain). What is the expected value for one day of Jim's day-trading hobby? (Hint: Fill in the table to help solve the problem.)

X	***P(X)***	***E(X)***	***Overall E(X)***

Expected Value

Example

Suppose that you and a friend are playing cards and decide to make a bet. If your friend draws two hearts in a row from a standard deck of 52 cards without replacing the first card, you give him $10. Otherwise, he pays you $20. If the same bet was made 15 times, how much would you expect to win or lose? Round your answer to the nearest cent, if necessary.

X	***P(X)***	***E(X)***	***1 round E(X)***	***15 rounds E(X)***

2

Odds

What are odds?

- **Odds**: Another way to express probability
  - But not interchangeable with probability
- Usually expressed as a ratio
  - “a:b for” or “a:b against”
  - Can be expressed as a fraction of probabilities

How can we calculate odds?

- Odds in favor of event A
  - “Odds for”

$$Odds = \frac{P(A)}{P(A^C)} = \frac{P(Win)}{P(Lose)}$$

- Odds against event A
  - “Odds against”

$$Odds = \frac{P(A^C)}{P(A)} = \frac{P(Lose)}{P(Win)}$$

Odds

Example

Suppose the probability of a soccer team winning a playoff game is 0.20. What are the odds of winning? Express your answer in the form a:b.

$$P(W) = 0.20 = \frac{1}{5}$$

1 “part” winning

4 “parts” losing

$$Odds\ Winning = 1:4$$

Odds

Example

If the odds on a bet are 18:1 against, what is the probability of winning?

Strategy: To convert from odds to a probability $a\!:b \rightarrow P(A) = \frac{a}{a+b}$

$$Odds\ against = 18\!:1 \rightarrow P(Loss) = \frac{18}{18+1} = \frac{18}{19}$$

$$P(Win) = 1 - P(Win^C) = 1 - \frac{18}{19} = \frac{1}{19}$$

= Loss