7.5 Expected Value – Overview

Expected Value

Definition: The **expected value** of an event X is the long term average (the value we would expect to happen if we performed the experiment many, many times).

How to calculate it:

- In words: Multiply each outcome (x value) by its probability and add them together.
- Formula:

$$E(X) = x_1 P(X_1) + x_2 P(X_2) + \cdots + x_n P(X_n),$$

where x_i is the i^{th} outcome and $P(x_i)$ is the probability of x_i.

Example 1: In soccer, you earn a certain number of points based on the result of a game. This is shown in the table below. Calculate the expected value of the number of points earned for a single game.

X	*P(X)*
Win = 3	0.3
Tie = 1	0.5
Loss = 0	0.2

$E(X) = 3(0.3) + 1(0.5) + 0(0.2)$

$= 1.4$ points (long term avg)

Sum of Expected Values: To find the combined expected value of multiple events, we can simply add the individual expected values.

$$E(X \text{ or } Y) = E(X) + E(Y)$$

Example 1 (continued): Find the expected value for the total number of points earned in a season if the season has 12 games.

→ X_1 = game 1
X_2 = game 2
⋮
X_{12} = game 12

→ $E(\text{Total points}) = E(X_1) + E(X_2) + \cdots + E(X_{12})$

→ Same Exp value for each game X

$= 1.4 + 1.4 + \cdots + 1.4$

$= 12(1.4) = 16.8$ points

Example 2: Jim likes to day-trade on the Internet. On a good day, he averages a $1400 gain. On a bad day, he averages a $900 loss. Suppose that he has good days 30% of the time, bad days 50% of the time, and the rest of the time he breaks even. What is the expected value for one day of Jim's day-trading hobby?

↳ = $0

Strategy: First make table
- Think about possible X values
- THEN the probabilities

① Table

X	P(X)
1400	0.3
0	0.2 = 1 − (0.3 + 0.5) complement
−900	0.5

★ decimals

★ Check → Total P(X) = 1 ✓

② Expected value → $E(X) = 1400(0.3) + 0(0.2) + (-900)\,0.5$

$= -\$30$

Example 3: Suppose that you and a friend are playing cards and decide to make a bet. If your friend draws two hearts in succession from a standard deck of 52 cards without replacing the first card, you give him $10. Otherwise, he pays you $20. If the same bet was made 15 times, how much would you expect to win or lose? Round your answer to the nearest cent, if necessary.

① Table

	x	p(x)
two hearts w/o rep	−10	1/17
otherwise	20	16/17 = 1 − 1/17 (complement)

★ Sum (total) = 1

★ → First solve E(X) for one bet
→ then consider 15 bets

$P(\text{2 Hearts w/o rep}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$ (Direct)

$= \frac{{}_{13}C_2}{{}_{52}C_2} = \frac{78}{1326} = \frac{1}{17}$ (counting)

Hearts / ALL

order doesn't matter ⇒ combinations ${}_nC_r$

num / denom = selecting from Hearts only / selecting from ALL

② Expected Value

$E(X) = -10(1/17) + 20(16/17)$

$\approx \$18.26$

③ Combined Exp Value → $E(15 \text{ bets}) = 15\,E(X)$

$= 15(18.26)$

$= \$273.90$

Odds

Definition: **Odds** are another way to express probability.

- We can express this as a ratio (fraction) of probabilities.
- odds ≠ probability are NOT interchangeable terms

Odds in favor of an event *A*:

$$\text{odds} = \frac{P(A)}{P(A^c)} = \frac{P(\text{win})}{P(\text{loss})}$$

Odds against an event *A*:

$$\text{Odds} = \frac{P(A^c)}{P(A)} = \frac{P(\text{loss})}{P(\text{win})}$$

Notation: Odds are generally written as a ratio of two integers, such as 5:1, which is read "5 to 1".

Example 4: Suppose the probability of a soccer team winning a playoff game is 0.20. What are the odds of winning? Express your answer in the form a:b.

★ *Strategy*: First write the probability as a fraction

→ $P(\text{win}) = 0.2 = 1/5$

$P(\text{loss}) = 1 - 0.8 = 0.2 = 4/5$

OR $P(\text{win}) = \frac{1}{5}$ → 1 "parts" win, 4 "parts" loss

Formula way → $\text{odds} = \frac{P(\text{win})}{P(\text{loss})} = \frac{1/5}{4/5} = \frac{1}{4} \longrightarrow 1:4$

odds = 1:4

Example 5: If the odds on a bet are 18:1 against, what is the probability of winning? Express your answer as a fraction.

★ *Strategy*: To convert from odds a probability $a:b \rightarrow P(A) = \frac{a}{a+b}$

against

$P(\text{loss}) = \frac{18}{18+1} = \frac{18}{19}$ → $P(\text{win}) = 1 - P(\text{win}^c)$ (complement)

$= \text{loss}$

$= 1 - 18/19$

$= 1/19$