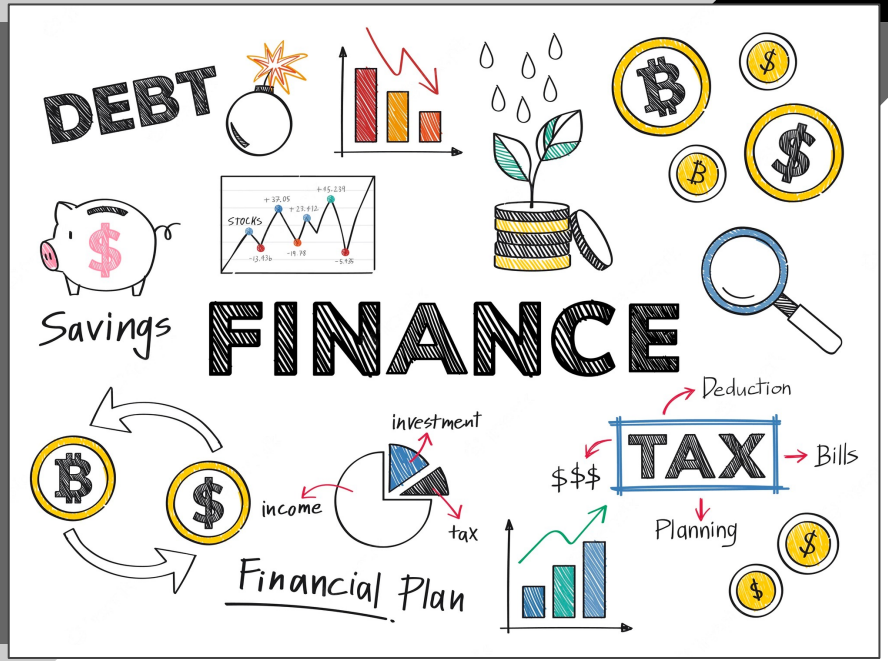
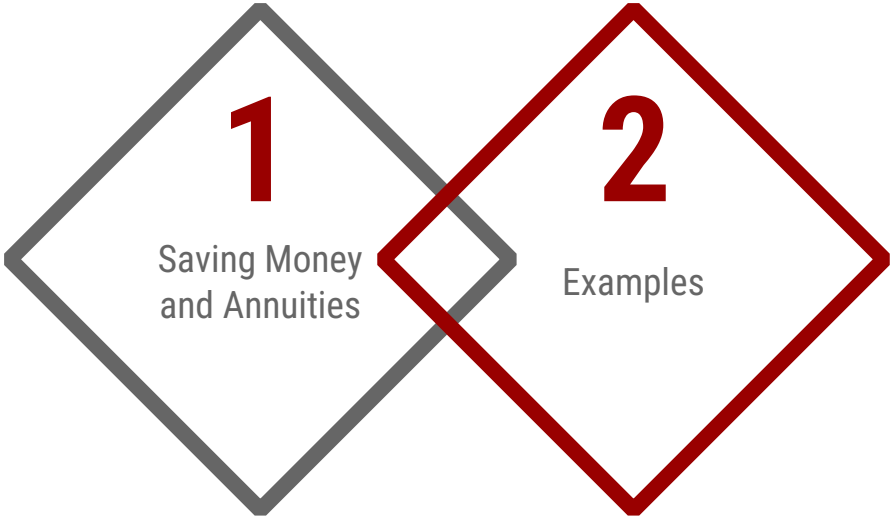
9.3 Saving Money



Goals for the Day



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Saving Money and Annuities

What is future value (FV or A)?

- Amount of money at some future time

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = FV

Think: Principal = Present Value PV

Know:
Initial investment (P)

Want to know:
How much will I have later (FV, A)

What is present value?

- Amount of principal needed now in order to reach a future value amount

$$PV = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Know:
How much will I have later (A)

Want to know:
Initial investment (PV)

Present Value

Example

Lyric wants to have $20,000 in 10 years. If she can get an 8% APR compounded semiannually, how much money does she need to invest right now to reach her goal?

$$PV = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{20{,}000}{\left(1+\frac{0.08}{2}\right)^{2*10}} \approx \$9{,}127.74$$

What is an annuity?

- Making <u>regular, repeated payments</u> into an account (or taken out of an account) over time
- Annuities earn compound interest

Finding Future Value of an Annuity

$$FV = PMT\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\left(\frac{r}{n}\right)}$$

Know:
Regular payment amount (PMT)

Want to know:
How much will I have later (FV)

Annuities – Finding Future Value

Example

Regina deposits $200 every month into a savings account earning 5% APR.

How much money will ***be in her account*** after 15 years?

$$FV = PMT\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\left(\frac{r}{n}\right)} = 200 * \frac{\left[\left(1+\frac{0.05}{12}\right)^{12*15}-1\right]}{\left(\frac{0.05}{12}\right)} \approx \$53{,}457.79$$

Annuities – Finding Future Value

Example

Continuing the previous example.

How much money will ***Regina contribute*** to her account over 15 years?

$$Total\ Deposited = PMT * \#\ of\ payments \quad \rightarrow n * t$$

$$= 200 * 12 * 15 = \$36{,}000$$

Finding Payments from an Annuity

$$PMT = FV \frac{\left(\frac{r}{n}\right)}{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}$$

Know:
How much I will have later (FV)

Want to know:
Regular payment amount (PMT)

Annuities – Finding Payments

Example

Anna wants to have $50,000 in 20 years. How much does she need to deposit each month into a savings account earning 7% APR?

$$PMT = FV\frac{\left(\frac{r}{n}\right)}{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]} = 50{,}000 * \frac{\left(\frac{0.07}{12}\right)}{\left[\left(1+\frac{0.07}{12}\right)^{12*20}-1\right]} \approx \$95.98$$

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Examples

Example #1

Calculate the amount of money Audrey needs to invest right now (in one lump sum) in order to have $100,000 after 18 years with an APR of 7% compounded monthly. Round your answer to the nearest cent, if necessary.

$28,469.43

Example #2

Drake starts an IRA (Individual Retirement Account) at the age of 22 to save for retirement. He deposits $400 each month. The IRA has an average annual interest rate of 7% compounded monthly. How much money will he have saved when he retires at the age of 65? Round your answer to the nearest cent, if necessary.

$1,310,451.88

Example #3

Jacob deposits $203.77 each month into an annuity account for his child's college fund in order to accumulate a future value of $60,000 in 18 years. How much of the $60,000 will Jacob deposit into the account in total, and how much will be interest he has earned? Round your answers to the nearest cent, if necessary.

Total invested = $44,014.32
Interest earned = $15,985.68

Example #4

Devon deposits a fixed amount monthly into an annuity account for his child's college fund. He wishes to accumulate $65,000 in 17 years. Assuming an APR of 3.6% compounded monthly, how much of the $65,000 will Devon deposit into the account in total, and how much will be interest he has earned? Round your answers to the nearest cent, if necessary.

PMT = $231.47 (rounded to nearest cent)
Total invested = $47,219.88
Interest earned = $17,780.12