8.4 The Normal Distribution

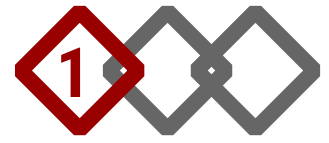1 — Normal Distribution Properties

2 — Z-scores

3 — Finding probabilities

1

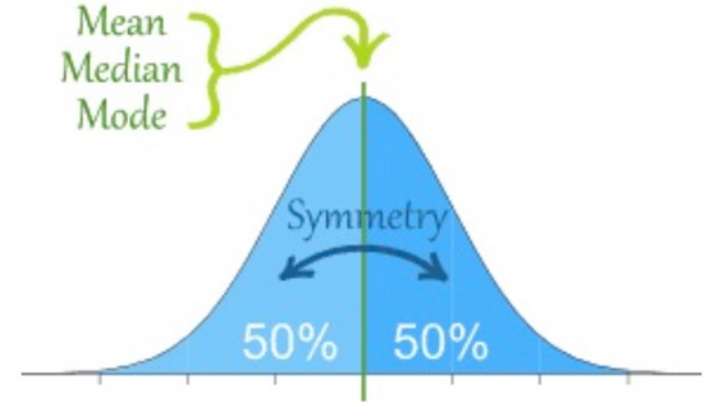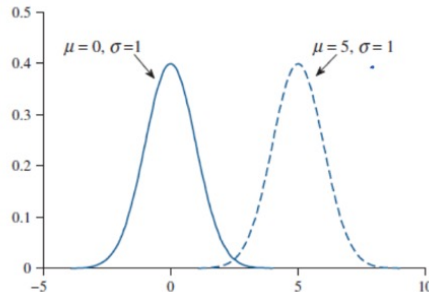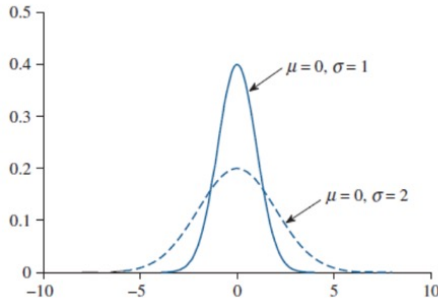# Normal Distribution Properties

Normal Distribution Properties

PARANORMAL DISTRIBUTION

- It's a symmetric, unimodal and <u>bell-shaped</u> distribution
  → which implies mean = median = mode.
- Total area under curve (<u>probability</u>) is equal to <u>1 = 100%.</u>
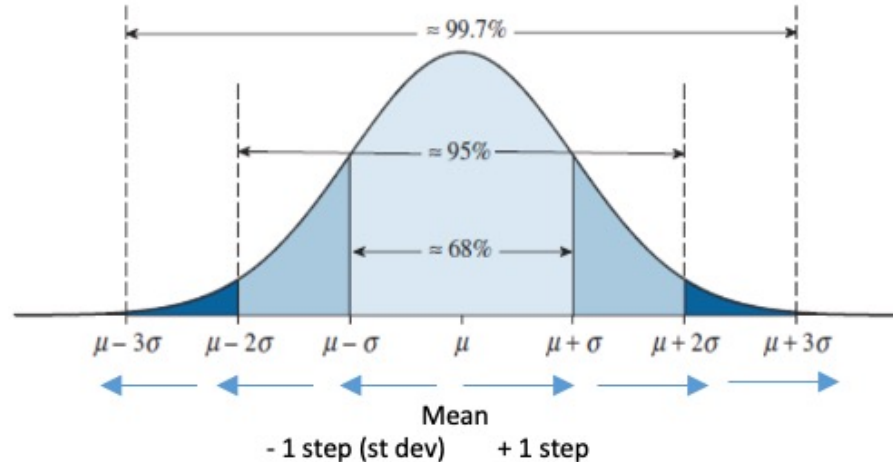- Completely described by its mean μ (location) and standard deviation σ (spread).

Mean
Median
Mode

Symmetry

50%   50%

$\mu = 0, \sigma = 1$

$\mu = 0, \sigma = 2$

$\mu = 0, \sigma = 1$ $\mu = 5, \sigma = 1$

The normal distribution allows us to find any probability, not just for points that lie exactly 1, 2, or 3 standard deviations ("steps") away from the mean like with the empirical rule!

2

Z-scores

Z-scores ("Standard" scores in Hawkes Certify)

Definition: A **z-score** <u>standardizes</u> observations based on the <u>mean</u> (center) and <u>standard deviation</u> (spread) of the distribution

▷ Allows for comparisons on different scales.

▷ Ex) ACT vs SAT

Formula: $z = \dfrac{x - \mu}{\sigma} = \dfrac{x - \bar{x}}{s} = \dfrac{obs - mean}{st\ dev}$

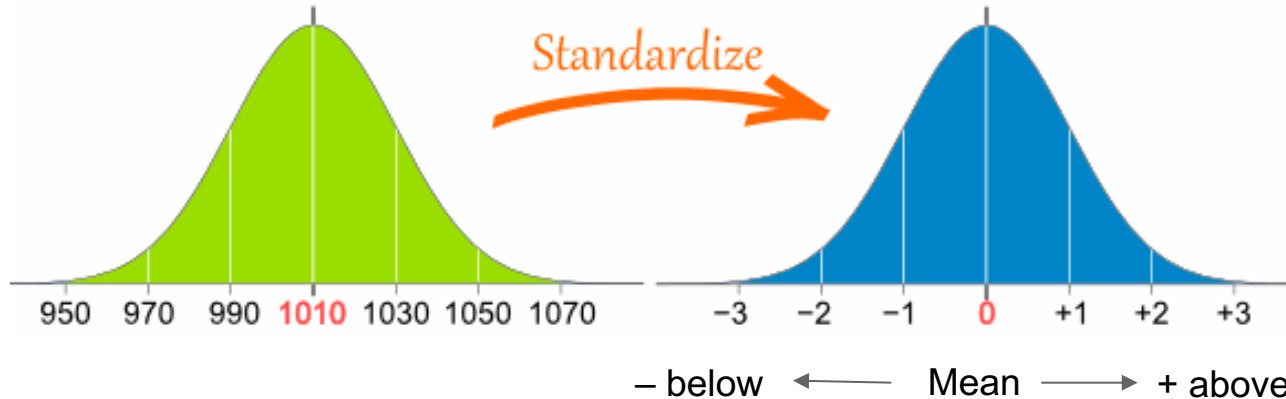Interpretation: A z-score tells us <u>how many standard deviations</u> an observation is <u>away from the mean</u>.

STEPS

Standardize

950 970 990 **1010** 1030 1050 1070

−3 −2 −1 **0** +1 +2 +3

− below ← Mean → + above

Example 1) For each data set with the stated μ and σ, find the standard score (z score) corresponding to the given observation, x.

a) $\mu = 8, \sigma = 3, x = 17$

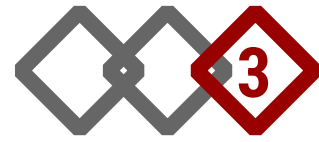$$z = \frac{17 - 8}{3} = 3$$ Above mean

b) $\mu = 100, \sigma = 16, x = 80$

$$z = \frac{80 - 100}{16} = -1.25$$ Below mean

c) Which observation is further from the mean relatively?

(a) because z = 3 is a "larger" value

3

Finding Probabilities
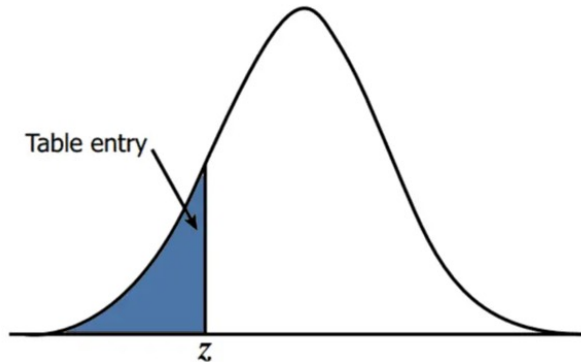
Finding probabilities based on the Normal Distribution

Handout: Normal Distribution Table

➤ Use the handout to <u>convert z-scores</u> to <u>percentiles</u> ("left probabilities").
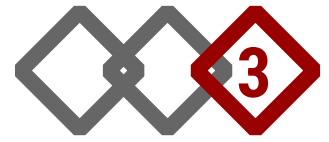
ALWAYS gives probability LESS THAN Z: P(Z < z).

"LEFT"

Table entry

z

Different Types of Probabilities

Draw, Label and Shade curve

Left Probability = Table (directly)

Right Probability = 1 − Left (table)

Between Probability = Left Z_2 − Left Z_1

Outside Probability = Left Z_1 + Right Z_2