9.4 Borrowing Money

Goals for the Day

1 Credit Cards

2 Loans

3 Mortgages

4 Examples

1

Credit Cards

Number of Fixed Payments Required to Pay Off Credit Cart Debt

➤ R = # of payments

➤ A = Future value (loan amount)

$$R = \frac{-\log\left[1 - \frac{r}{n}\left(\frac{A}{PMT}\right)\right]}{\log\left(1 + \frac{r}{n}\right)}$$

Example

Franz wants to buy a new computer that costs $2200 using a credit card that has an APR of 19.99%. How long will it take him to pay off the computer if he makes regular monthly payments of $40? How much will he pay overall for the computer?

$$R = \frac{-\log\left[1 - \frac{r}{n}\left(\frac{A}{PMT}\right)\right]}{\log\left(1 + \frac{r}{n}\right)} = \frac{-\log\left[1 - \frac{0.1999}{12}\left(\frac{2200}{40}\right)\right]}{\log\left(1 + \frac{0.1999}{12}\right)} \approx 150.08 \text{ (151) payments}$$

$$Total\ paid = PMT * \#\ payments = 40 * 151 = \$6{,}040$$

2

Loans

- Fixed installment loans (present value annuity): Receive money now, in the present, and use the regular payments to pay off the future value of the loan (principal and interest).

- Down payments are often required on large loans (house, car, etc.). These reduce the principal of the loan, and the amount that remains is *financed* (borrowed with interest).

- Monthly Payment Formula for Fixed Installment Loans

$$P = Price - Down\ Payment$$

Principal that is financed

$$PMT = \frac{\left(P \cdot \frac{r}{n} \right)}{\left[1 - \left(1 + \frac{r}{n} \right)^{-nt} \right]}$$

Example

Owen wants to buy a new car for $34,000 (including taxes and fees). He chooses to make a down payment of 20% and wants to finance the remainder. If Owen can get an APR of 3.99% for a 72-month loan, what is the amount of his monthly payment?

$$PMT = \frac{\left(P \cdot \frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]} \quad = \frac{\left(27{,}200 \cdot \frac{0.0399}{12}\right)}{\left[1 - \left(1 + \frac{0.0399}{12}\right)^{-12 \cdot 6}\right]}$$

$$\approx \$425.43$$

$$P = Price \ - Down\ payment$$

$$= 34{,}000 - (34{,}000 * 0.20)$$
$$= 34{,}000 - 6{,}800$$
$$= \$27{,}200$$

3

Mortgages

If we want to stay within the recommended monthly mortgage payment (25% of your monthly take-home pay), we can use this formula to find the most house you can afford.

$$Max\ Purchase\ Price\ = PMT \cdot \frac{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)}$$

Home Mortgage

Example

Alaina wants to buy a house, but she doesn't know how much house she can afford. Her take-home pay is $3220 per month, and she doesn't want to spend more than the recommended 25% of her take-home pay. She can get a 3.37% APR compounded monthly. If she takes a 30-year mortgage, what is the maximum purchase price she can afford?
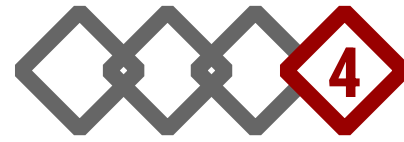
$$Max\ Purchase\ Price = PMT \cdot \frac{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)}$$

$$= 805 \cdot \frac{\left[1 - \left(1 + \frac{0.0337}{12}\right)^{-12 \cdot 30}\right]}{\left(\frac{0.0337}{12}\right)} \approx \$182,201.11$$

$$PMT = 0.25(3,220) = \$805$$

4

Examples

Example #1

4

Natalie bought a new car for $26,000. She paid a 10% down payment and financed the remaining balance for 36 months with an APR of 4.8%. Assuming she made monthly payments, determine the total cost of Natalie's car. Round your answer to the nearest cent. Then, determine how much interest she paid.

Total cost = $27,771.92
Interest = $1,771.92

Example #2
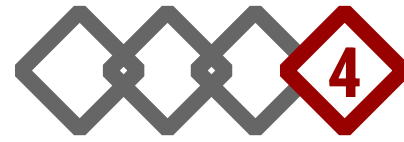
Jake bought several concert tickets for a total of $900. He used a credit card that has an APR of 17.77%. How much will he pay in total to pay off the purchases if he makes monthly payments of $30? Round the number of monthly payments up to the nearest whole number. Round your final answer to the nearest whole number, if necessary.

40 payments
$1200