

## 6.3 Borrowing Money – Overview

### Key Ideas / Formulas

**Paying off credit cards** – Use the equation below to find how long it takes to pay off a credit card.

Number of Fixed Payments Required to Pay Off Credit Card Debt

$$R = \frac{-\log \left[ 1 - \frac{r}{n} \left( \frac{A}{\text{PMT}} \right) \right]}{\log \left( 1 + \frac{r}{n} \right)}$$

*# of payments* (pointing to R)      *Future Value (Loan Amount)* (pointing to A)

**Ex:** How long will it take to pay off a \$2200 purchase on a credit card with an APR of 19.99% with \$40 monthly payments?

$$R = \frac{-\log \left[ 1 - \frac{0.1999}{12} \left( \frac{2200}{40} \right) \right]}{\log \left( 1 + \frac{0.1999}{12} \right)} \approx 150.08 \rightarrow 151 \text{ Monthly payments (round up)}$$

**Fixed installment loans** (present value annuity) – Receive money now, in the present, and use the regular payments to pay off the future value of the loan (principal and interest).

**Down payments** – Down payments are often required on large loans (house, car, etc.). These reduce the principal of the loan, and the amount that remains is *financed* (borrowed with interest).

Monthly Payment Formula for Fixed Installment Loans

$$\text{PMT} = \frac{\left( P \cdot \frac{r}{n} \right)}{\left[ 1 - \left( 1 + \frac{r}{n} \right)^{-nt} \right]}$$

*Principal (that is financed)* (pointing to P)

**Ex:** What is the monthly payment for an auto loan if the purchase price is \$34,000 with a 20% down payment and a 3.99% APR on a 72-month loan?

$$P = 34,000 - (34,000 \times 0.2) = \$27,200$$

$$\text{PMT} = \frac{27,200 \left( \frac{0.0399}{12} \right)}{\left[ 1 - \left( 1 + \frac{0.0399}{12} \right)^{-72(1)} \right]} \approx \$425.43$$

**Amortization schedule** – Payments on loans such as mortgages are portioned out between interest and principal. To show you this breakdown over time, lenders provide loan amortization schedules.

**Ex:** Find the mortgage balance after the first three payments on a 30-year \$180,000 mortgage that was financed at an APR of 5.25% and has a monthly payment of \$993.97.

Payment Number	Interest Payment	Principal Payment	Mortgage Balance
1	\$787.50	\$206.47	\$179,793.53
2	\$786.60	\$207.37	\$179,586.16
3	\$785.69	\$208.28	\$179,377.88

*New Balance = Old Balance – Principal Payment*

$$\text{Balance 1} = 180,000 - 206.47 = \$179,793.53$$

$$\text{Balance 2} = 179,793.53 - 207.37 = \$179,586.16$$

$$\text{Balance 3} = 179,586.16 - 208.28 = \$179,377.88$$

## Examples

**Example 1:** Natalie bought a new car for \$26,000. She paid a 10% down payment and financed the remaining balance for 36 months with an APR of 4.8%. Assuming she made monthly payments, determine the total cost of Natalie's car. Round your answer to the nearest cent, if necessary. Then, determine how much interest Natalie paid.

① Setup

$$\begin{aligned} \rightarrow \text{Down payment} &= \text{Price} \times \frac{\%}{100} = 26,000 \times 0.1 = \$2,600 \\ \rightarrow P &= \text{Price} - \text{Down payment} = 26,000 - 2,600 = \$23,400 \\ &\hookrightarrow \text{principal that is financed} \end{aligned}$$

② PMT

$$\rightarrow PMT = \frac{P \left( \frac{r}{n} \right)}{\left[ 1 - \left( 1 + \frac{r}{n} \right)^{-nt} \right]} = \frac{23,400 \left( \frac{0.048}{12} \right)}{\left[ 1 - \left( 1 + \frac{0.048}{12} \right)^{-12(3)} \right]} \approx \$699.22/\text{month}$$

③ Total cost

$$\begin{aligned} \rightarrow \text{Total cost} &= PMT \times \# \text{ payments} + \text{down payment} \\ &\downarrow \\ &= 699.22 \times 36 + 2600 = \$27,771.92 \end{aligned}$$

④ Interest

$$\rightarrow \text{Interest} = \text{Total cost} - \text{Price} = 27,771.92 - 26,000 = \$1,771.92$$

**Example 2:** Jake bought several concert tickets for a total of \$900. He used a credit card that has an APR of 17.77%. How much will he pay in total to pay off the purchases if he makes monthly payments of \$30? Round the number of monthly payments up to the nearest whole number. Round your final answer to the nearest whole number, if necessary.

$$B = \frac{-\log \left[ 1 - \frac{r}{n} \left( \frac{A}{PMT} \right) \right]}{\log \left( 1 + \frac{r}{n} \right)} = \frac{-\log \left[ 1 - \frac{0.1777}{12} \left( \frac{900}{30} \right) \right]}{\log \left( 1 + \frac{0.1777}{12} \right)} \approx 39.96 \rightarrow 40 \text{ payments}$$

$$\text{Total cost} = PMT \times \# \text{ payments} = \$30 \times 40 = \$1200$$