9.4 Borrowing Money – Overview

Key Ideas / Formulas

Paying off credit cards – Use the equation below to find how long it takes to pay off a credit card.

Number of Fixed Payments Required to Pay Off Credit Card Debt

# of payments

$$R = \frac{-\log\left[1 - \frac{r}{n}\left(\frac{A}{\text{PMT}}\right)\right]}{\log\left(1 + \frac{r}{n}\right)}$$

A: future value (loan amount)

EX: How long will it take to pay off a \$2200 purchase on a credit card with an APR of 19.99% with \$40 monthly payments?

$$R = \frac{-\log\left[1 - \frac{0.1999}{12}\left(\frac{2200}{40}\right)\right]}{\log\left(1 + \frac{0.1999}{12}\right)} \approx 150.08 \rightarrow \text{round up} \rightarrow \boxed{151 \text{ monthly payments}}$$

Fixed installment loans (present value annuity) – Receive money now, in the present, and use the regular payments to pay off the future value of the loan (principal and interest).

Down payments – Down payments are often required on large loans (house, car, etc.). These reduce the principal of the loan, and the amount that remains is *financed* (borrowed with interest).

Monthly Payment Formula for Fixed Installment Loans

$$\text{PMT} = \frac{\left(P \cdot \frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}$$

P: principal (that is financed)

EX: What is the monthly payment for an auto loan if the purchase price is \$34,000 with a 20% down payment and a 3.99% APR on a 72-month loan?

$$P = 34{,}000 - (34{,}000 * 0.2) = \$27{,}200$$

$$\text{PMT} = \frac{27{,}200\left(\frac{0.0399}{12}\right)}{\left[1 - \left(1 + \frac{0.0399}{12}\right)^{-72}\right]} \approx \boxed{\$425.43}$$

Mortgage payments – If we want to stay within the recommended monthly mortgage payment (25% of your monthly take-home pay), we can use this formula to find the most house you can afford.

Maximum Purchase Price

$$\text{maximum purchase price} = \text{PMT} \cdot \frac{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)}$$

EX: What is the maximum purchase price of a home if your monthly take-home pay is \$3220 and you can get a 3.37% APR on a 30-year mortgage?

$$\text{PMT} = 0.25(3220) = \$805$$

$$\text{Max purchase price} = 805\left[\frac{1 - \left(1 + \frac{0.037}{12}\right)^{-12(30)}}{\left(\frac{0.037}{12}\right)}\right] \approx \boxed{\$192{,}202.02}$$

Examples

Example 1: Natalie bought a new car for $26,000. She paid a 10% down payment and financed the remaining balance for 36 months with an APR of 4.8%. Assuming she made monthly payments, determine the total cost of Natalie's car. Round your answer to the nearest cent, if necessary. Then, determine how much interest Natalie paid.

① Setup

→ Down payment = Price * $\frac{\%}{100}$ Down payment = 26,000 * 0.1 = $2,600

→ P = Price − Down payment = 26,000 − 2600 = $23,400

↳ principal that is financed

② PMT

→ $PMT = \frac{P(\frac{r}{n})}{[1-(1+\frac{r}{n})^{-nt}]} = \frac{23,400\left(\frac{0.048}{12}\right)}{\left[1-\left(1+\frac{0.048}{12}\right)^{-3(12)}\right]} = \$699.12$ / month

③ Total cost

→ Total cost = PMT * # of payments + down payment

= 699.12 * 36 + 2600 = $25,771.92

④ Interest

→ Interest = Total cost − Price = 25,771.92 − 26,000 = $−1771.92

Example 2: Jake bought several concert tickets for a total of $900. He used a credit card that has an APR of 17.77%. How much will he pay in total to pay off the purchases if he makes monthly payments of $30? Round the number of monthly payments up to the nearest whole number. Round your final answer to the nearest whole number, if necessary.

$$R = \frac{-\log\left[1-\frac{r}{n}\left(\frac{A}{PMT}\right)\right]}{\log\left(1+\frac{r}{n}\right)} = \frac{-\log\left[1-\frac{0.1777}{12}\left(\frac{900}{30}\right)\right]}{\log\left(1+\frac{0.1777}{12}\right)} \approx 39.96$$

→ 40 payments

Total cost = PMT * # of payments = $30 * 40 = $1200