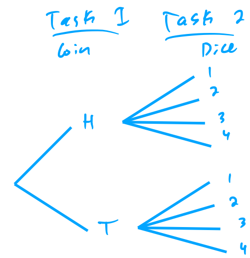
10.2 Counting Outcomes – Overview

Definitions / Key Ideas

Fundamental Counting Principle: If a job consists of *n* separate tasks, the first of which can be done k_1 ways, the second k_2 ways and so on, then the total job can be done $k_1 \times k_2 \times \cdots \times k_n$ ways.

Task 1	Task 2	⋯	Task *n*	Total Outcomes
k_1	k_2	⋯	k_n	$k_1 \times k_2 \times \cdots \times k_n$



$2 \times 4 = 8$ outcomes

Example: Flip a coin and roll a 4-sided die.
How many total outcomes are there?

Example: Sally has 6 pairs of socks, 4 shorts, 5 shirts, and 3 sunglasses. How many ways can she get dressed?

$\underset{\text{socks}}{\underline{6}} \times \underset{\text{shorts}}{\underline{4}} \times \underset{\text{shirts}}{\underline{5}} \times \underset{\text{sunglasses}}{\underline{3}} = 360$ total ways

With or without replacement: We need to into account take whether or not objects can be repeated in our calculations.

Examples: a) How many passwords can you make if it requires 4 digits? with replacement

$\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 10^4 = 10{,}000$ passwords

b) How many passwords can you make if it requires 4 digits, but you cannot repeat digits? without replacement

$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} = 5040$ passwords

Factorials: *n!* (read "n factorial") is the product of all numbers less than and including *n*

$$\boxed{n! = n(n-1)(n-2)\cdots(3)(2)(1)}$$

Example: For the 9 starting players on a baseball team, how many different batting orders are there?

$\underline{9} \times \underline{8} \times \underline{7} \times \cdots \times \underline{1} = 9! = 362{,}880$

without replacement

Combinations and Permutations

- We often want to be able to count the number of ways that we can choose members from a group of objects (without replacement).
  - "Selecting *r* objects from a total of *n* objects".

    $r \le n$
- There are two methods to count in these scenarios, the only difference is if order matters or order doesn't matter.

Permutations: Order matters. When you are selected is important. Your position has meaning.

$${}_nP_r = P\binom{n}{r} = P(n,r) = \boxed{\frac{n!}{(n-r)!}}$$

Total # how many selecting

Combinations: Order does not matter. It does not matter when you are selected, only if you were selected.

$${}_nC_r = \binom{n}{r} = C(n,r) = \boxed{\frac{n!}{r!(n-r)!}}$$

Examples: Decide if we should use permutations or combinations to count the total number of outcomes (possible ways to select our group). Then count the number of outcomes.

a) There are 8 runners in a race. How many ways can they place 1st, 2nd, and 3rd?

Permutation → order matters

$_8P_3 = 336$

Meaning to each slot → $\underline{1^{st}}$ $\underline{2^{nd}}$ $\underline{3^{rd}}$

b) Out of 12 students, how many ways can we select a committee of four students?

Combinations → order doesn't matter

(only interested in selected vs not selected)

$_{12}C_4 = 495$

c) We are forming a committee and we need to select a president, vice president and secretary. How many ways can this be done if there are 10 members?

Permutation → order matters

$_{10}P_3 = 720$

Matters if Pres vs VP vs Sec → $\underline{Pres}$ $\underline{VP}$ $\underline{Sec}$

Permutations with Repeated Objects: Counting the number of distinct ways we can arrange all *n* objects when some of the objects are the same (repeated, specifically k_1 are alike, k_2 are alike, and so on).

$$\frac{n!}{(k_1!)(k_2!)\ldots(k_p!)}, \qquad \text{where } k_1 + k_2 + \ldots + k_p = n$$

Example: Harmony was born on 05/19/1991. How many eight-digit codes could she make using the digits in her birthday?

8 total digits

9 → 3 times

1 → 3 times

5 → 1

0 → 1

$\frac{8!}{3!\,3!\,1!\,1!} = 1{,}120$