

11.4 The Normal Distribution





Goals for the Day

1

Normal
Distribution
Properties

2

Empirical Rule

3

Z-scores and
Finding
Probabilities

4

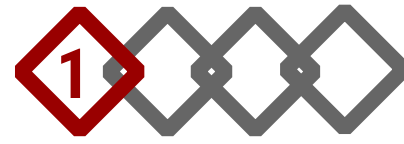
Examples

1

Normal Distribution Properties

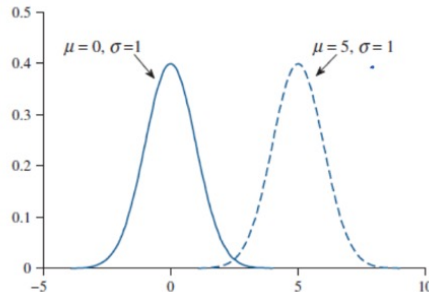
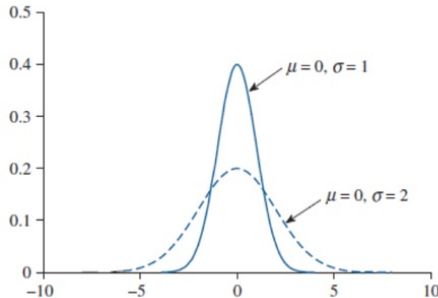
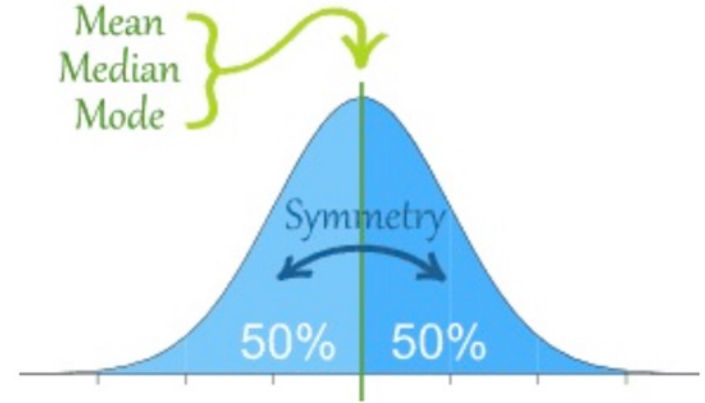


Normal Distribution Properties



PARANORMAL DISTRIBUTION

- It's a symmetric, unimodal and bell-shaped distribution
→ which implies mean = median = mode.
- Total area under curve (probability) is equal to 1 = 100%.
- Completely described by its mean μ (location) and standard deviation σ (spread).



2

Empirical Rule

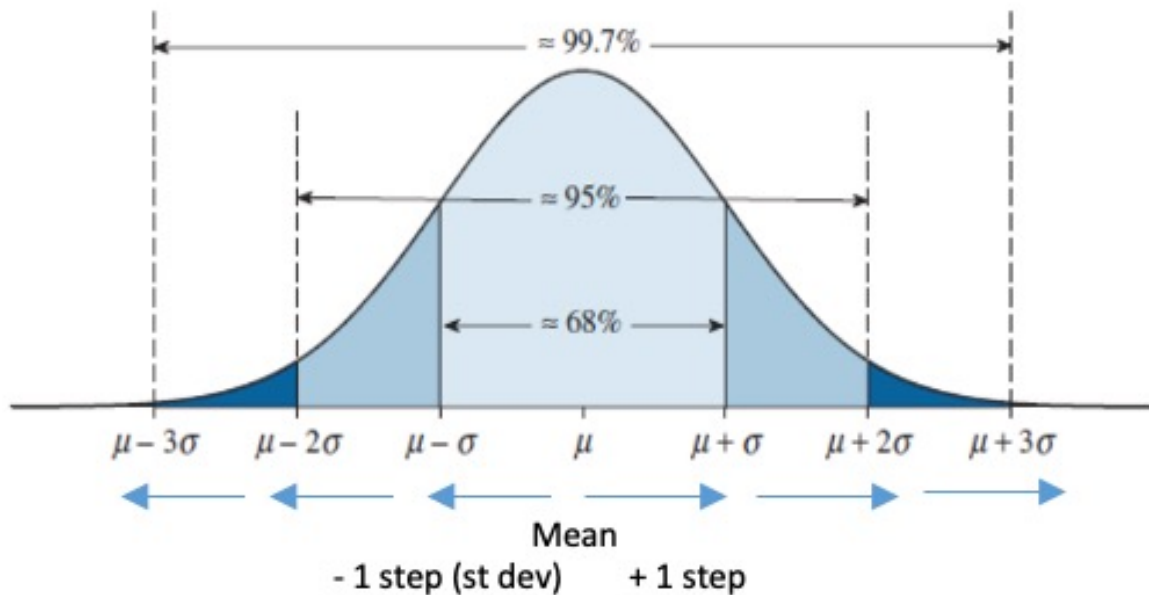


Empirical Rule (68 – 95 – 99.7 Rule)

When data is approximately bell shaped, the standard deviation allows us to make fairly accurate approximations about the locations of our data values.

- ▷ 68% of the data lies within 1 standard deviation of the mean.
- ▷ 95% of the data lies within 2 standard deviations of the mean.
- ▷ 99.7% of the data lies within 3 standard deviations of the mean.

Empirical Rule (68 – 95 – 99.7 Rule)



- We can use these breakdowns to find probabilities within certain intervals.

Empirical Rule Examples

Example 1: Suppose that diameters of a new species of apple have a bell-shaped distribution with a mean of 7 cm and a standard deviation of 0.5 cm. Using the empirical rule, find the following percentages of apples with diameters that are:

a) Between 5.5 cm and 8.5 cm

99.7%

b) More than 7.5 cm

16%

c) No more than 5.5 cm

0.5%

Step 1

Draw and label curve

Step 2

Shade area of interest

3

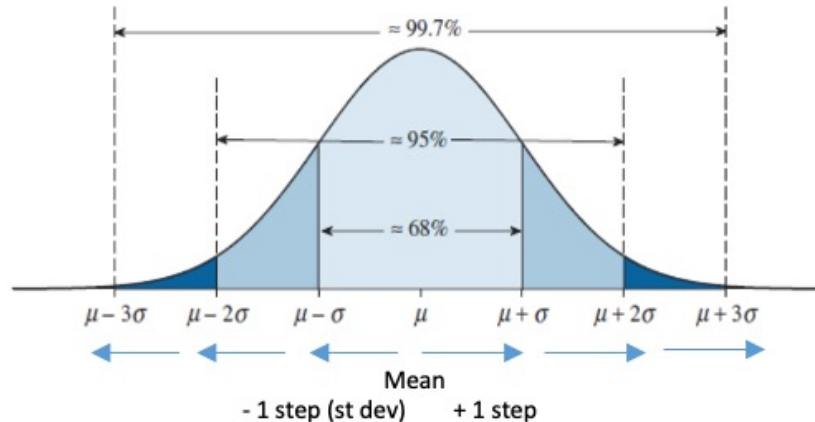
Z-scores and Finding Probabilities



How We Use the Normal Curve



The normal distribution allows us to find any probability, not just for points that lie exactly 1, 2, or 3 standard deviations (“steps”) away from the mean like with the empirical rule!





Z-scores



- Z-scores (“Standard” scores in Hawkes Certify)
- Definition: A **z-score** standardizes observations based on the mean (center) and standard deviation (spread) of the distribution
 - ▷ Allows for comparisons on different scales.
 - ▷ Ex) ACT vs SAT

$$\text{Formula: } z = \frac{x - \mu}{\sigma} = \frac{x - \bar{x}}{s} = \frac{\text{obs} - \text{mean}}{\text{st dev}}$$

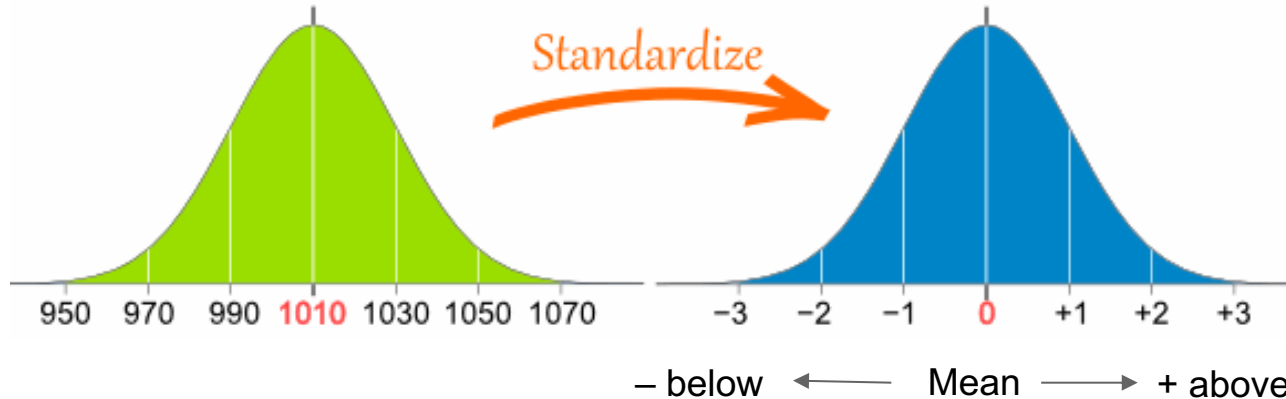


Z-scores



■ Interpretation: A z-score tells us how many standard deviations an observation is away from the mean.

STEPS





Z-scores Examples



Example 2: For each data set with the stated μ and σ , find the standard score (z score) corresponding to the given observation, x .

a) $\mu = 8, \sigma = 3, x = 17$

$$z = \frac{17 - 8}{3} = 3 \quad \text{Above mean}$$

b) $\mu = 100, \sigma = 16, x = 80$

$$z = \frac{80 - 100}{16} = -1.25 \quad \text{Below mean}$$

c) Which observation is further from the mean relatively?

(a) because $z = 3$ is a “larger” value



Finding probabilities based on the Normal Distribution

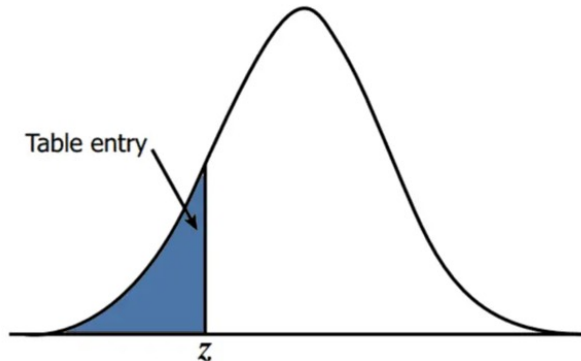
■ Handout: Normal Distribution Table

- ▷ Use the handout to convert z-scores to percentiles (“left probabilities”).



ALWAYS gives probability LESS THAN Z: $P(Z < z)$.

“LEFT”



Different Types of Probabilities



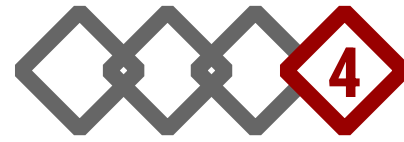
Draw, Label and
Shade curve

- Left Probability = Table (directly)
- Right Probability = $1 - \text{Left (table)}$
- Between Probability = $\text{Left } Z_2 - \text{Left } Z_1$
- Outside Probability = $\text{Left } Z_1 + \text{Right } Z_2$

4

Examples

Empirical Rule Examples



Example 3: Suppose that IQ scores have a bell-shaped distribution with a mean of 105 and a standard deviation of 15. Using the empirical rule answer the following questions:

a) What percentage of IQ scores are greater than 75?

Step 1

Draw and label curve

Step 2

Shade area of interest

97.5%

b) Between which two values do the middle 68% of IQ scores fall between?

(90, 120)

Empirical Rule Examples

Example 4: Suppose there is a new breed of giant cats, whose weights are normally distributed with an average of 100 pounds and a standard deviation of 15 lbs. You would like to own a smaller version of this type of cat, specifically between 59 lbs and 69 lbs.

What is the probability you can find a cat between these two weights.

$$\begin{aligned}P(59 \leq X \leq 69) &= \\P(-2.73 \leq Z \leq -2.07) &= \\0.016\end{aligned}$$