7.2 Counting Our Way to Probabilities – Overview

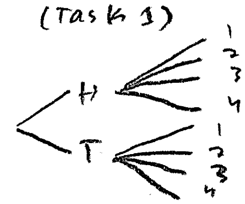
Definitions / Key Ideas

Fundamental Counting Principle: If a job consists of n separate tasks, the first of which can be done k_1 ways, the second k_2 ways and so on, them the total job can be done $k_1 \times k_2 \times \cdots \times k_n$ ways.

Task 1	Task 2	⋯	Task n	Total Outcomes
k_1	k_2	⋯	k_n	$k_1 \times k_2 \times \cdots \times k_n$

Flip a coin + roll a die (4)

(Task 1) (Task 2)



$2 \times 4 = 8$ total outcomes

Example: Sally has 6 pairs of socks, 4 shorts, 5 shirts, and 3 sunglasses. How many ways can she get dressed?

$\underset{\text{socks}}{\underline{6}} \times \underset{\text{shorts}}{\underline{4}} \times \underset{\text{shirts}}{\underline{5}} \times \underset{\text{sunglasses}}{\underline{3}} = 360$ total ways

With or without replacement: We need to take whether or not objects can be repeated in our calculations.

Examples: a) How many passwords can you make if it requires 4 digits? with replacement

$\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 10^4 = 10{,}000$ passwords

b) How many passwords can you make if it requires 4 digits, but you cannot repeat digits? without replacement

$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} = 5040$ passwords

Factorials: In general, *n!* (read "n factorial") is the product of all the positive integers less than or equal to n, where n is a positive integer.

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Example: For the 9 starting players on a baseball team, how many different batting orders are there?

$\underline{9} \times \underline{8} \times \underline{7} \times \cdots \times \underline{1} = 9!$

without replacement

Combinations and Permutations

- We often want to be able to count the number of ways that we can choose members from a group of objects (without repetition).
  - "Selecting r objects from a total of n objects".

    $r \leq n$

- There are two methods to count in these scenarios, the only difference is if order matters or order doesn't matter.

Permutations: Order matters. When you are selected is important. Your position has meaning.

$$\underbrace{}_{\text{total \#}}{}_nP_{\underbrace{r}_{\text{how many selecting}}} = P\binom{n}{r} = P(n,r) = \boxed{\frac{n!}{(n-r)!}}$$

Combinations: Order does not matter. It does not matter when you are selected, only if you were selected.

$${}_nC_r = \binom{n}{r} = C(n,r) = \boxed{\frac{n!}{r!(n-r)!}}$$

Examples: Decide if we should use permutations or combinations to count the total number of outcomes (possible ways to select our group). Then count the number of outcomes.

a) There are 8 runners in a race. How many ways can they place 1st, 2nd, and 3rd?

Permutation → order matters

${}_8P_3 = 336$

meaning to each slot

$\underline{\quad}_{1^{st}}\ \underline{\quad}_{2^{nd}}\ \underline{\quad}_{3^{rd}}$

b) Out of 12 students, how many ways can we select a committee of four students?

Combinations → order doesn't matter

only interested in selected vs not selected

${}_{12}C_4 = 495$

c) We are forming a committee and we need to select a president, vice president and secretary. How many ways can this be done if there are 10 members?

Permutations → order matters

${}_{10}P_3 = 720$

matters if Pres vs VP vs Sec

$\underline{\quad}_{Pres}\ \underline{\quad}_{VP}\ \underline{\quad}_{Sec}$

Permutations with Repeated Objects: Counting the number of distinct ways we can arrange all n objects when some of the objects are the same (repeated, specifically k_1 are alike, k_2 are alike, and so on).

$$\frac{n!}{(k_1!)(k_2!)\dots(k_p!)}, \quad \text{where } k_1 + k_2 + \ldots + k_p = n$$

Example: Harmony was born on 05/19/1991. How many eight-digit codes could she make using the digits in her birthday? 8 total digits

9 → 3 times

1 → 3 times

5 → 1

0 → 1

$$\frac{8!}{(3!)(3!)(1!)(1!)} = 1120$$