7.1 Introduction to Probability – Overview

Definitions / Key Ideas

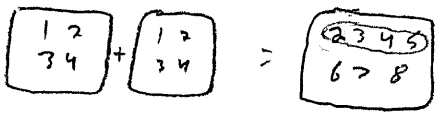
Experiment (trial) – The process by which a random observation / outcome is generated.
- Example: Flip a coin, roll a die, gender of a child

Outcome – Any possible individual observation of that experiment, like the smallest pieces.
- Example: Flip a coin once, Heads or Tails; Rolling a die, 1 2 3 … 6; Gender, Boy or Girl

Sample Space, *S* – The set of all possible outcomes of an experiment.
- Example: Flip a coin twice, S = {HH, HT, TH, TT}

Event, *E* – Any collection of possible outcomes of an experiment. In other words, any subset of S.
- Example: Flip a coin twice: A = HH, A = HH or HT, A = {TT, HT, TH}, etc.

Example: Roll a pair of 4-sided die, we are interested in the sum of the two die.



$S = \{2, 3, 4, 5, 6, 7, 8\}$

6 sample space

Events

E = less than 6 $= \{1, 2, \ldots, 5\}$

E = even $= \{2, 4, 6, 8\}$

Two Ways to Calculate Probability

Probability = likelihood of event occurring

$0 \leq$ Probability ≤ 1

↓ NEVER occurs

↓ ALWAYS occurs

1) Classical Probability (theoretical)
(if all outcomes are equally likely)

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space} = \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

- Examples: Suppose we are randomly selecting a single card from a standard 52-card deck.

a) Find the probability of a red card.

$$P(red) = \frac{\#\ successes}{\#\ possible} = \frac{26}{52} = 0.5$$

b) Find the probability of a King.

$$P(King) = \frac{\#\ King}{total\ \#\ cards} = \frac{4}{52} = 0.0769 \rightarrow \frac{1}{13}$$

c) Find the probability of a Heart or a 10.

$$P(heart\ or\ 10) = \frac{\#\ heart\ or\ 10}{total\ \#\ cards} = \frac{13 + 3}{52} = \frac{4}{13}$$

d) Find the probability of a card that is not a Club.

$$P(not\ club) = \frac{\#\ not\ club}{total} = \frac{39}{52} = \frac{52 - 13}{52} = \frac{3}{4}$$

2) Empirical Probability (Data)
(if all outcomes are based on an experiment)

$$P(Event) = \frac{Number\ of\ times\ the\ event\ occurs}{Number\ of\ times\ the\ experiment\ is\ performed} = \frac{Number\ of\ successes}{Number\ of\ trials}$$

- Examples: Suppose we collected data on majors of MATH 125 students and are randomly selecting a single student.

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20
total	76

a) Find the probability the student is a Math major.

$P(\text{math}) = \frac{\#\ \text{math}}{\#\ \text{students}} = \frac{23}{76}$

b) Find the student is not a Math major.

$P(\text{not math}) = \frac{\#\ \text{not math}}{\#\ \text{students}} = \frac{15+18+20}{76} = \frac{76-23}{76} = \frac{53}{76}$

c) Find the probability the student is an Art or Chemistry major.

$P(\text{Art or Chem}) = \frac{\#\ \text{Art or Chem}}{\#\ \text{students}} = \frac{18+15}{76} = \frac{33}{76}$

d) Find the probability the student is a Math, Chemistry or English major.

$P(\text{Math, Chem or English}) = \frac{\#\ \text{successes}}{\#\ \text{possibilities}} = \frac{23+15+20}{76} = \frac{58}{76}$

Example: An experiment is performed where a fair 4-sided die is rolled and then a fair 3-color spinner is spun. The possible outcomes for each event are 1, 2, 3, and 4 for the 4-sided die and red (R), blue (B), and yellow (Y) for the 3-color spinner.

a) Identify the sample space for this experiment.

$S = \{1Y, 1R, 1B, 2Y, 2R, 2B, 3Y, 3R, 3B, 4Y, 4R, 4B\}$

b) Find the probability of rolling a 3.

$P(3) = \frac{\#\ \text{include 3}}{\#\ \text{total outcomes}} = \frac{3}{12}$

c) Find the probability of an even number and R.

$P(\text{Even + R}) = \frac{\#\ \text{Even + R}}{\#\ \text{possibilities}} = \frac{2}{12}$ → 2R, 4R

d) Is this a classical or empirical probability?

classical (not based on data)