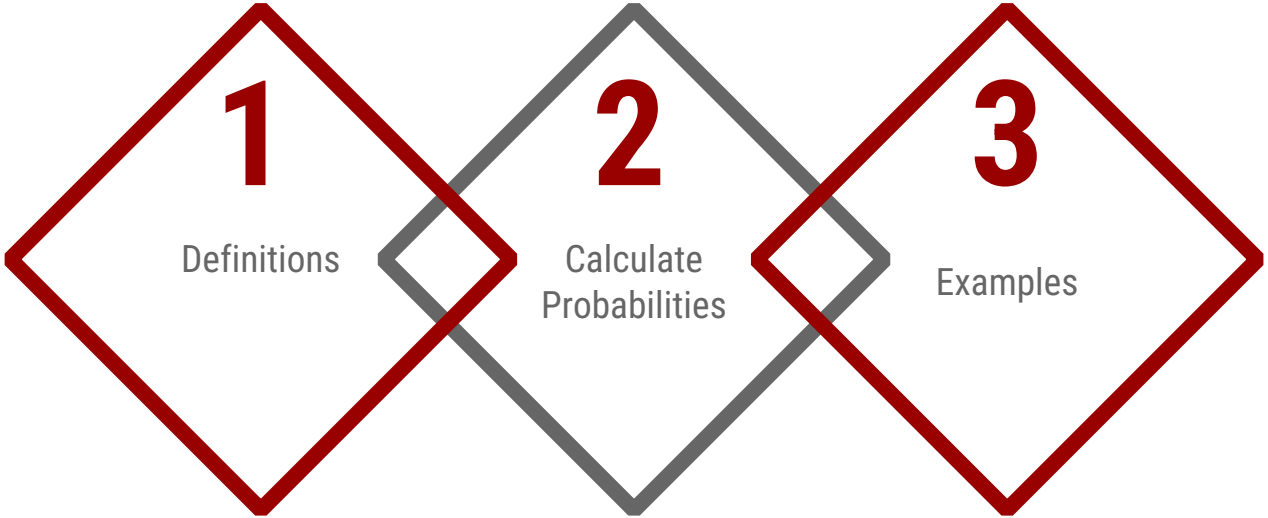
10.1 Intro to Probability

Goals for the Day



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Definitions

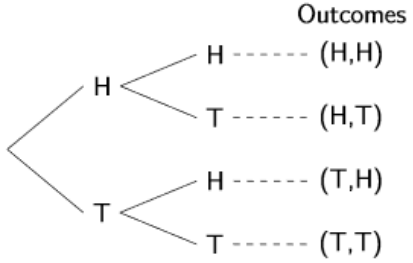
Definitions

- An **experiment** or **trial** is the process by which a random observation or outcome is generated.
  - Ex: Flipping a coin, rolling a die, determining the sex of a baby
- An **outcome** is any possible individual observation of that experiment
  - Ex: If you flip a coin, you have two possible outcomes: heads, or tails. Heads is an outcome, and tails is another outcome.

Definitions

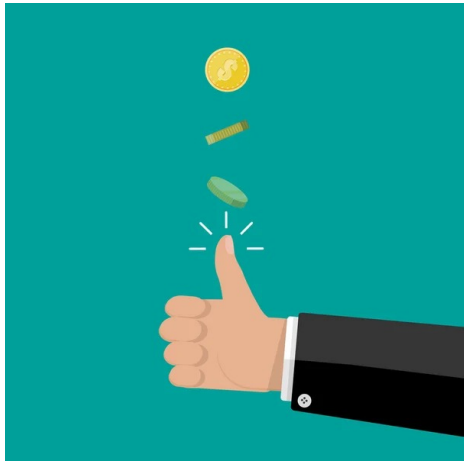
- Sample space *(S)*, which is the set of all possible outcomes of an experiment/trial.
  - EX: If you flip a coin twice, $S = \{HH, HT, TH, TT\}$.

- A tree diagram uses branches to indicate all possible outcomes at each stage for an experiment.
  - Each path of branches in a tree diagram indicates a single possible outcome for the experiment.



Definitions

- An **event *(E)***, is any collection of possible outcomes of an experiment (i.e., any subset of *S*).
  - Ex: If you flip a coin twice, you might be interested in getting two heads, which would be notated as $A = \{HH\}$.
  - Ex: …getting at least one head, which is $A = \{HH, HT, TH\}$.



Example

- You are playing a game in which you roll a pair of 4-sided die. To determine your next move, you need to know the sum of the two die.

Using proper notation:

- Write the sample space: $S = \{2,3,4,5,6,7,8\}$
- Write the event of rolling a sum that is even: $E = \{2,4,6,8\}$
- Write the event of rolling a sum that less than 6: $E = \{2,3,4,5\}$

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Calculating Probabilities

Probability

- Probability = likelihood of event occurring
  - $0 \leq Probability \leq 1$
  - 0 = NEVER occurs; 1 = ALWAYS occurs
  - The probability of an event *A* happening is notated *P(A)*

Classical (Theoretical) Probability

- If all outcomes are equally likely,

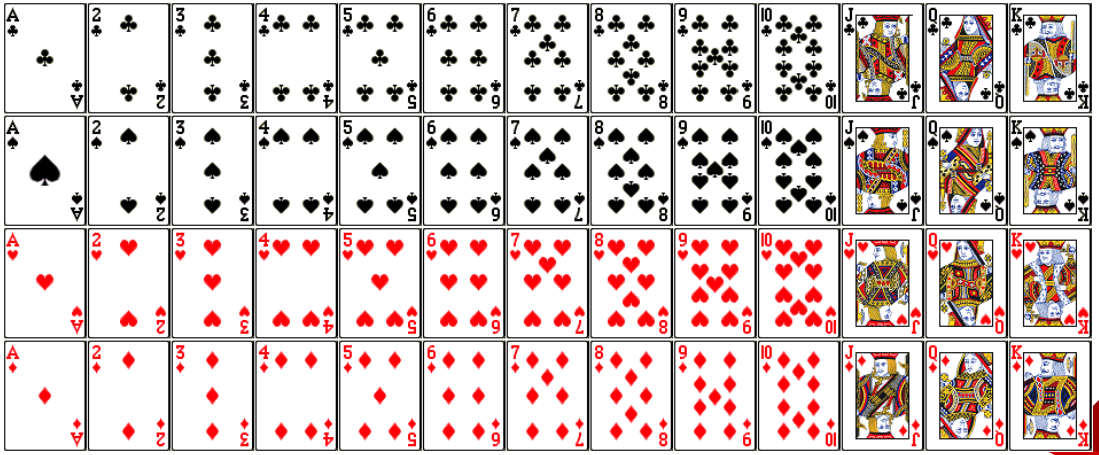
$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space}$$

$$= \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

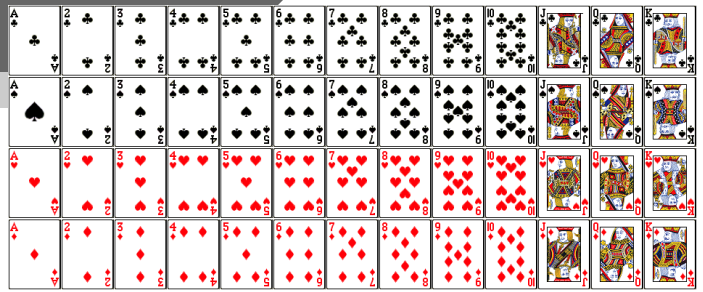
Classical (Theoretical) Probability

Example

- Suppose you are randomly selecting cards from a standard 52-card deck of playing cards.



Classical (Theoretical) Probability



Examples

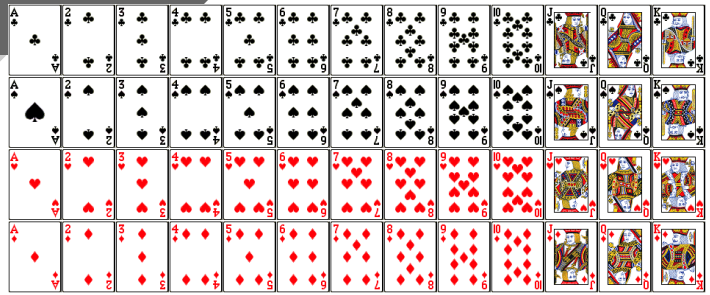
- Find the probability of drawing a red card.

$$P(Red) = \frac{\#\ Successes}{\#\ Possible} = \frac{26}{52} = \frac{1}{2} = 0.5$$

- Find the probability of a King.

$$P(King) = \frac{\#\ Kings}{Total\ \#\ of\ cards} = \frac{4}{52} = \frac{1}{13} \approx 0.077$$

Classical (Theoretical) Probability



Examples

- Find the probability of drawing a Heart or a 10.

$$P(Heart\ or\ 10) = \frac{\#\ Hearts\ or\ 10s}{Total\ \#\ of\ cards} = \frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$$

- Find the probability of drawing a card that is not a Club.

$$P(NOT\ a\ club) = \frac{\#\ Not\ Club}{Total} = \frac{39}{52} = \frac{52-13}{52} = \frac{3}{4}$$

Empirical Probability

Empirical probability

- Based on experiments (DATA).

$$P(Event) = \frac{Number\ of\ times\ the\ event\ occurs}{Number\ of\ times\ experiment\ is\ performed}$$

$$= \frac{Number\ of\ successes}{Number\ of\ trials}$$

Empirical Probability

Examples

- Suppose we collected data on MATH 125 students and are selecting a single student randomly.
- Find the probability the student is a Math major.

$$P(Math) = \frac{\#\ Math}{\#\ Students} = \frac{23}{76}$$

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20

Empirical Probability

Examples

- Find the probability the student is not a Math major.

$$P(Not\ Math) = \frac{\#\ Not\ Math}{\#\ Students} = \frac{15 + 18 + 20}{76} = \frac{76 - 23}{76} = \frac{53}{76}$$

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20

Empirical Probability

Examples

- Find the probability the student is an Art or Chemistry major.

$$P(Art\ or\ Chem) = \frac{\#\ Art\ or\ Chem}{\#\ Students} = \frac{18 + 15}{76} = \frac{33}{76}$$

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20

Empirical Probability

Examples

- Find the probability the student is a Math, Chemistry, or English major.

$$P(Math, Chem, or\ English) = \frac{\#\ Successes}{\#\ Possibilities} = \frac{23 + 15 + 20}{76} = \frac{58}{76}$$

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20

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Examples

Example

An experiment is performed where a fair 4-sided die is rolled, then a fair 3-color spinner is spun. The possible outcomes for each event are 1, 2, 3, and 4 for the 4-sided die and red (R), blue (B), and yellow (Y) for the 3-color spinner.

a) Identify the sample space for this experiment.

b) Find the probability of rolling a 3

c) Find the probability of rolling an even number AND spinning red.

d) Determine if this is classical or empirical probability.

$a)\ S = \{1R, 1B, 1Y, 2R, 2B, 2Y, 3R, 3B, 3Y, 4R, 4B, 4Y\}$

$b)\ P(3) = \frac{3}{12}$ $\quad$ $c)\ P(Even\ AND\ R) = \frac{2}{12}$

d) Classical (Theoretical) Probability