7.4 Addition and Multiplication Rules of Probability

Goals for the Day

1 Addition Rules

2 Conditional Probability

3 Multiplication Rules

Bad news: No formula sheet on Exam ☹ → Focus on the PROCESS!!!

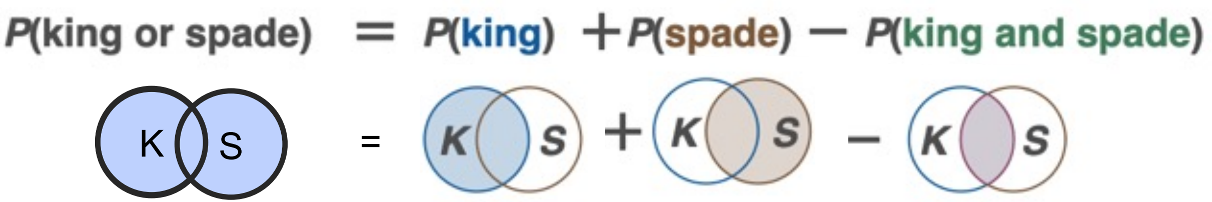
1

Addition Rules

Addition Rules

- **Addition Rule for Probability**: Consider two events A and B.
  The probability of <u>A or B</u> occurring is:

$$P(A \; or \; B) = P(A) + P(B) - P(A \; and \; B)$$



Double counted intersection

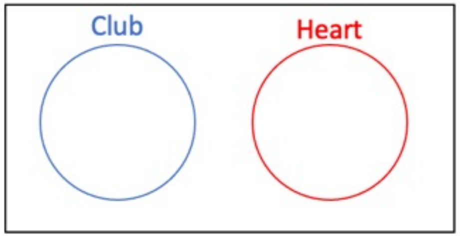
Mutually Exclusive Events

- **Mutually Exclusive Events**: Two events are considered to be mutually exclusive if they have no outcomes in common.

Addition Rule for Mutually Exclusive Events:

$$P(A \text{ or } B) = P(A) + P(B)$$

P(Club or Heart) = P(Club) + P(Heart)



No Overlap → $P(A \text{ and } B) = 0$

Addition Rule

Example

Suppose we collected data from MATH 125 students about their majors and attendance records and recorded the data in the table below. Then we randomly selected a single student. For this example, assume that no students are double majors.

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

Addition Rule

Example

a) What is the probability that the student is a Statistics major?

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

$$P(Statistics) = \frac{\#\ of\ statistics}{\#\ of\ students} = \frac{150}{\mathbf{435}}$$

Addition Rule

Example

b) What is the probability that the student has Good attendance?

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

$$P(Good) = \frac{\boxed{140}}{\mathbf{435}}$$

Addition Rule

Example

c) What is the probability that the student is a Statistics major AND has Good attendance?

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

$$P(Stats\ AND\ Good) = \frac{\#\ of\ stats\ and\ good}{\#\ of\ students} = \boxed{\frac{20}{\mathbf{435}}}$$

Addition Rule

Example

d) What is the probability that the student is a Statistics major OR has Good attendance?

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

$$P(Stats\ OR\ Good) = \frac{\boxed{150}}{435} + \frac{\boxed{140}}{435} - \frac{\boxed{20}}{435} = \frac{270}{435}$$

Addition Rule

Example

e) What is the probability that the student is a Chemistry major OR has Poor attendance?

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

$$P(Chem\ OR\ Poor) = \frac{\boxed{180} \boxed{+75} \boxed{-30}}{\mathbf{435}} = \frac{225}{435}$$

Addition Rule

Example

f) What is the probability that the student is a Art major OR has Chemistry?

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

$$P(Art\ OR\ Chem) = \frac{\boxed{105}\ \boxed{+\ 180}\ - 0}{\mathbf{435}} = \frac{285}{435}$$

No overlap → Mutually exclusive!

2

Conditional Probability

Conditional Probability

- The conditional probability of B, given event A has already occurred is written as

$$P(B|A) = \text{“P(B given A)”}$$

- Event A is the “additional information” that we know, so we can restrict what we are looking at if we have a table. Then we are interested in Event B.

Conditional Probability

Goes second

Goes first

Example 2

a) Given the student has Perfect attendance, find the probability they are a Chemistry major

$$P(Chemistry|Perfect) = \frac{\boxed{80}}{\boxed{220}}$$

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

Conditional Probability

Example 2

b) Find the probability the student has Perfect attendance, given they are a Chemistry major.

$$P(Perfect|Chem) = \frac{\boxed{80}}{\boxed{180}}$$

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

Conditional Probability

Example 2

c) Given the student is an Art major, find the probability they have Poor attendance.

$$P(Poor|Art) = \frac{\boxed{15}}{\boxed{105}}$$

	Statistics	Art	Chemistry	Total
Perfect	100	40	80	220
Good	20	50	70	140
Poor	30	15	30	75
Total	150	105	180	435

Another Example

Example 3: A swim team consists of 6 boys and 4 girls. A relay team of 4 swimmers is chosen at random from the team members. What is the probability that 2 boys are selected for the relay team given that the first two selections were girls?

Conditional probability

Additional info

G G ___ ___

$P(Next\ 2\ Boys) = ??$

8 Total
6 Boys
2 Girls

Direct Way

$$\frac{6}{8} \times \frac{5}{7} = \frac{30}{56} = \frac{15}{28}$$

Boy 1 Boy 2

Counting Way

Order doesn't matter → ${}_nC_r$

$$\frac{Successes}{Possibilities} = \frac{{}_6C_2}{{}_8C_2} = \frac{15}{28}$$

3

Multiplication Rules

Different Types of Events

No impact

- Independent events – The result of one does not influence the probability of the other.

Independent IF

- With replacement
- Unrelated experiments

- Dependent events – The result of one does influence the probability of the other.

Independent Events

- Multiplication Rule for Independent Events
  - Probability of A AND B happening

$$P(A \text{ and } B) = P(A) \times P(B)$$

Independent Events

Independent

Example 2

Three cards are drawn with replacement from a standard deck of 52 cards. Find the probability that the first card will be a diamond, the second card will be a red card, and the third card will be a queen.

$$P(D\ and\ R\ and\ Q) = \underbrace{\frac{13}{52}\times\frac{26}{52}\times\frac{4}{52}}_{P(D)\times P(R)\times P(Q)} = \frac{4}{416} = \frac{1}{104}$$

Dependent Events

- Multiplication Rule for Dependent Events
  - Probability of A AND B happening

$$P(A\ and\ B) = P(A) \times P(B|A)$$

  - “Both events occurred” = “A occurred, then B occurred”

Dependent

Example 3

If you are dealt two cards from a standard 52-card deck without replacement, find the probability of getting a 10 of hearts and then a red card.

$$P(10H \ and \ Red) = \frac{1}{52} \times \frac{25}{51} = \frac{25}{2652}$$

$$P(10H) \times P(Red \mid 10H)$$

One More Thing

- Bayes' Theorem – helpful when trying to find $P(A|B)$ when you know $P(B|A)$, as in Example 10 from the 7.4 Learn

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}, \text{ when } P(B) > 0$$

Additional Example: The Probability of A OR B

- In a standard deck of cards, what is the probability of drawing a face card OR a red card?
  - A (face card): 12 face cards (3 in each of 4 suits)
  - B (red card): 26 red cards (13 in 2 suits)
  - Don't forget the **overlap**: A and B (red face cards): 6 red face cards (3 in 2 suits)

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B) \rightarrow P(Face\ or\ Red) = P(Face) + P(Red) - P(Face\ and\ Red)$$

$$= \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{12 + 26 - 6}{52} = \frac{32}{52} = \frac{8}{13} \approx 0.615385$$