10.3 Probability of Single Events – Overview

Definitions / Key Ideas

Odds

Definition**: Odds** are another way to express probability.

- We can express this as a ratio (fraction) of probabilities.
- Odds and Probability are NOT interchangeable terms.

Odds in favor of an event *A*:

$$Odds = \frac{P(A)}{P(A^C)} = \frac{P(win)}{P(lose)}$$

Odds against an event *A*:

$$Odds = \frac{P(A^C)}{P(A)} = \frac{P(lose)}{P(win)}$$

Notation: Odds are generally written as a ratio of two integers, such as 5:1, which is read "5 to 1".

Example: Suppose the probability of a soccer team winning a playoff game is 0.20. What are the odds of winning? Express your answer in the form a:b.

Strategy: First write the probability as a fraction THEN think about parts

$P(win) = 0.2 = \frac{1}{5}$ → 1 "parts" win → 4 "parts" loss

⟹ odds = 1 : 4

Probability review:

$$P(Event) = \frac{Number\ of\ outcomes\ in\ the\ event}{Number\ of\ outcomes\ in\ the\ sample\ space} = \frac{Number\ of\ successes}{Number\ of\ possibilities}$$

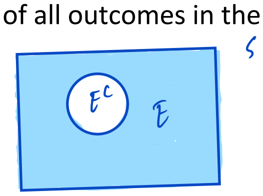
$$0 \leq P(Event) \leq 1$$

⟹ 0 —— P(E) —— 1

☆ opposite

Complement of an Event: Consists of all outcomes in the sample space that are *not* in event E.

E^C, $\bar{E}$, E'



Complement Rules of Probability:

1) $P(E) + P(E^C) = 1$ 2) $P(E) = 1 - P(E^C)$ 3) $P(E^C) = 1 - P(E)$

Example 1: Suppose we are randomly selecting a single card from a standard 52-card deck.

a) Find the probability of a diamond.

$P(Diamond) = \frac{13}{52} = \frac{1}{4}$

b) Find the probability of not a diamond.

$P(Not\ Diamond) = \underbrace{\frac{39}{52}}_{directly} = \underbrace{1 - \frac{1}{4}}_{using\ complements} = \frac{3}{4}$

Example 2: Suppose we collected data on majors of MATH 125 students and are randomly selecting a single student.

Major	Number of Students
Math	23
Chemistry	15
Art	18
English	20

Total 76

a) Find the probability that the student is NOT an Art major.

$P(\text{Not Art}) = 1 - P(\text{Art})$

Art^c

$= 1 - \frac{18}{76} = \frac{58}{76}$

b) Find the probability that the student is NOT an English nor Chemistry major.

$P(\text{Not English nor Chem}) = 1 - \frac{20+15}{76} = \frac{23+18}{76} = \frac{41}{76}$

Calculating More Probabilities

Two Approaches: 1) Direct way 2) Counting methods

Example 1: Liam and Michael are going to play video games this afternoon. Together, they have 41 video games. If they decide to randomly choose two video games, what is the probability that the two they choose will consist of each of their favorite video games? Assume they have different favorites.

$$** \; P(Event) = \frac{\#\,Successes}{\#\,Possibilities}$$

** Solve numerator and denominator separately

Direct

$\frac{2}{41} \times \frac{1}{40} = \frac{2}{1640} = \frac{1}{820}$

1st choice 2nd choice

without replacement

Counting Method

$\rightarrow \text{Prob} = \frac{1}{2} = \frac{1}{820}$

1 Numerator → "Is a condition"

only selecting from favorites, nCr → $_2C_2 = 1$

2 Denominator → "No condition"

Total ways to select 2 from 41, nCr → Combinations (order doesn't matter)

$_{41}C_2 = 820$

Example 2: A box of jerseys for a pick-up game of basketball contains 8 extra-large jerseys, 7 large jerseys, and 5 medium jerseys. If you are first to the box and grab 3 jerseys, what is the probability that you randomly grab 3 extra-large jerseys.

Direct

20 Total
8 XL

$$\underbrace{\frac{8}{20}}_{1^{st}\text{ selection}} \times \underbrace{\frac{7}{19}}_{2^{nd}} \times \underbrace{\frac{6}{18}}_{3^{rd}} = \frac{14}{285}$$

Counting Method

$$\rightarrow \text{Prob} = \frac{①}{②} = \frac{56}{1140} = \frac{14}{285}$$

① Selecting 3 XL Jerseys

$$_8C_3 = 56$$

② Selecting 3 Jerseys from ALL Jerseys

$$_{20}C_3 = 1140$$

$$P(Event) = \frac{\#\ Successes}{\#\ Possibilities}$$

Example 3: There are 11 balls numbered 1 through 11 placed in a bucket. What is the probability of reaching into the bucket and randomly drawing two balls numbered 1 and 4 without replacement, in that order?

Direct

$$\underbrace{\frac{1}{11}}_{\#1} \times \underbrace{\frac{1}{10}}_{\#4} = \frac{1}{110}$$

Counting

$$P(1,4) = \frac{1}{_{11}P_2} = \frac{1}{110}$$

① only 1 way to get a 1, then a 4

1 (first) 2 3 4 (second) 5 6 7 8 9 10 11

② ★ order matters ⟹ $_nP_r$

Selecting 2 from 11

Example 4: Julia sets up a passcode on her smart phone, which allows only six-digit codes. A spy sneaks a look at Julia's smart phone and sees her fingerprints on the screen over six numbers. What is the probability the spy is able to unlock the smart phone on his first try?

Direct

$$\underline{\frac{1}{6}} \times \underline{\frac{1}{5}} \times \underline{\frac{1}{4}} \times \underline{\frac{1}{3}} \times \underline{\frac{1}{2}} \times \underline{\frac{1}{1}} = \frac{1}{720}$$

6 digit code → only 1 correct # for each slot + have 6 #s to choose from

→ then without replacement because fingerprints showed used all 6 #s

Counting

$$P(\text{correct password}) = \frac{1}{{}_6P_6}$$

1 → only 1 success (correct password)

${}_6P_6$ → 6 #s to choose from → $n=6$ (6 fingerprints) + ordering all 6 #s → $r=6$

$= 6!$

Example 5: If the odds on a bet are 18:1 against, what is the probability of winning? Express your answer as a fraction.

Strategy: To convert from odds to a probability $a:b \rightarrow P(A) = \frac{a}{a+b}$

odds against = 18 : 1 ⟶ odds for = 1 : 18 ⟹ $P(\text{win}) = \frac{1}{18+1} = \frac{1}{19}$

flip

OR

Against

$$P(\text{loss}) = \frac{18}{18+1} = \frac{18}{19}$$

⟶

Complement

$$P(\text{win}) = 1 - P(\text{lose})$$

$$= 1 - \frac{18}{19}$$

$$= \frac{1}{19}$$