**Diagram

Description automatically generated with low confidence11.4 The Normal Distribution – Overview**

**Diagram

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**Normal Distribution Properties**

* It’s a symmetric, unimodal and bell-shaped distribution

⇒ which implies mean = median = mode.

* Total area under curve (probability) is equal to 1 = 100%.
* Completely described by its mean 𝜇 (location) and standard deviation 𝞂 (spread).

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**Empirical Rule (68 – 95 – 99.7 Rule)**

* When data is approximately bell shaped, the standard deviation allows us to make fairly accurate approximations about the locations of our data values.

\_\_\_\_\_ of the data lies within 1 standard deviation of the mean.

\_\_\_\_\_ of the data lies within 2 standard deviations of the mean.

Chart, histogram

Description automatically generated \_\_\_\_\_ of the data lies within 3 standard deviations of the mean.

Mean

- 1 step (st dev) + 1 step

* We can use these breakdowns to find probabilities within certain intervals.

**Example 1**: Suppose that diameters of a new species of apple have a bell-shaped distribution with a mean of 7 cm and a standard deviation of 0.5 cm. Using the empirical rule, find the following percentages of apples with diameters that are:

Step 1

Draw and label curve

Step 2

Shade area of interest

a) Between 5.5 cm and 8.5 cm

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b) More than 7.5 cm.

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c) No more than 5.5 cm.

**Normal Distribution – Again**

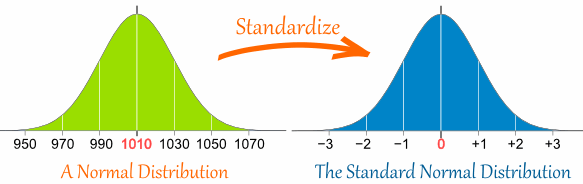
* The normal distribution allows us to find any probability, not just for points that lie exactly 1, 2, or 3 standard deviations (“steps”) away from the mean like with the empirical rule!

**Z-scores** (“Standard” scores in Hawkes Certify)

* Definition: A **z-score** standardizes observations based on the mean (center) and standard deviation (spread) of the distribution.

Formula:

* + Allows for comparisons on different scales.
  + Ex) ACT vs SAT
* Interpretation:
  + A **z-score** tells us how many standard deviations an observation is away from the mean.
  + The unit of a **z-score** is standard deviations.



*Z*

*X*

**Example 2**:For each data set with the stated and , find the standard score (z score) corresponding to the given observation, .

1. Which observation is further from the mean relatively?

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**Finding probabilities based on the Normal Distribution**

* Handout: Normal Distribution Table
  + Use the handout to convert z-scores to percentiles (“left probabilities”).
  + ALWAYS gives probability LESS THAN Z: P(Z < z).
* Different types of probabilities
  + Left probability

Draw, Label and Shade curve

* + - Example: Find the total area under the standard normal curve (probability or percentage) to the left of z = 0.34.

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* + Right probability
    - Examples: Find the probability to the right of z = 0.34.

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Find the probability to the right of z = -1.2.

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* + Between probability

Why this works

* + - Chart, line chart

      Description automatically generatedExample: Find the probability between z1 = -0.12 and z2 = 2.27.

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* + Outside probability
    - Example: Find the probability to the left of z1 = -0.12 and to the right of z2 = 2.27.

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**Examples**

**Example 3**: Suppose that IQ scores have a bell-shaped distribution with a mean of 105 and a standard deviation of 15. Using the empirical rule answer the following questions:

a) What percentage of IQ scores are greater than 75?

b) Between which two values do the middle 68% of IQ scores fall between?

**Example 4**: Suppose there is a new breed of giant cats, whose weights are normally distributed with an average of 100 pounds and a standard deviation of 15 lbs. You would like to own a smaller version of this type of cat, specifically between 59 lbs and 69 lbs. What is the probability you can find a cat between these two weights.