

Name:

MATH 320: Homework 9

Due _____: Turn in a hard copy, neat and stapled.

1. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the pmf

$$f(x) = \frac{5-x}{10}, \quad x = 1, 2, 3, 4.$$

If the patient is to receive \$350 from an insurance company for each of the first two days in the hospital and \$150 for each day after the first two days, what is the expected total payment for the hospitalization?

2. A tour operator has a tour bus that can accommodate 16 tourists. The operator knows that tourists may not show up, so he sells 17 tickets. The probability that an individual tourist will not show up is 0.04, independent of all other tourists.

Each ticket cost \$40, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat it is not available, the tour operator has to pay \$80 (ticket cost + \$40 penalty) to the tourist.

What is the expected revenue of the tour operator?

Hint: Define the random variable R for the revenue, and think of it as a piecewise function with two cases.

3. Let X have pmf

$$f(x) = \frac{2x-1}{16} \quad x = 1, 2, 3, 4$$

(a) Find the variance of $Y = X + 4$.

(b) Find the standard deviation of $Z = 2X$.

4. A probability distribution of the claim sizes for an auto insurance policy are given in the table below:

Claim size (x)	Probability $f(x)$
15	0.10
20	0.30
25	0.15
30	0.25
35	0.20

Find the percentage of claims that are within one standard deviation of the mean claim size.

5. A recent study indicates that the annual cost of maintaining and repairing a mountain bike averages \$150 with a standard deviation \$25.

Suppose a tax of 20% is introduced on all items associated with the maintenance and repair of mountain bikes (i.e. everything is made 20% more expensive). Calculate the new variance of the annual cost of maintaining and repairing a mountain bike.

6. A salesperson can contact either one or two customers per day with probability $1/4$ and $3/4$, respectively. Each contact will result in either no sale or a \$10,000 sale, with the probabilities 0.8 and 0.2, respectively.

(a) Find the pmf for the *number of sales*.

Hint: Think about this scenario using a tree diagram.

(b) Find the expected value and variance of the *number of sales*.

(c) Use your answers from part (b) to find the mean and variance of the *total dollar amount of the sales*.

7. Let random variable X have the following density function:

$$f(x) = \begin{cases} 25x & 0 \leq x \leq 0.20 \\ 1.5625(1-x) & 0.2 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(0.15 \leq X \leq 0.55)$.

(b) Find the median m .

8. Let random variable X have the following density function:

$$f(x) = \begin{cases} e^x & 0 \leq x \leq \ln(2) \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the cdf $F_X(x)$.

(b) Find the IQR of X .

9. Let random variable X have the following density function:

$$f(x) = \begin{cases} \frac{|x|}{10} & -2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the expected value of X .

10. The loss due to water damage for a home is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x) & 0 \leq x \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate the variance of the loss.

Select answers

1. $E(\text{Total payment}) = 620$
2. $E(\text{Revenue}) \approx 640.03$
3. (a) $V(Y) \approx 0.859$
(b) $SD(Z) \approx 1.854$
4. Prob = 0.7
5. $V(\text{Cost}) = 900$
6. (a)
(b) $E(\text{Sales}) = 0.35$ and $V(\text{Sales}) = 0.2875$
(c) $V(\text{Money}) \approx 5361.90$
7. (a) Prob ≈ 0.5606
(b)
8. (a)
(b) $IQR \approx 0.3365$
9. $E(X) \approx 1.867$
10. $V(X) \approx 22.22$