Name:

MATH 320: Homework 11

Due ____________________: Turn in a hard copy, neat and stapled.

1. A professor gives an exam where the time limit is 60 min and the anticipated minimum time to students to take the exam is 35 min. Assume that the random variable T for the time it takes a student to finish the exam is uniformly distributed over $[35, 60]$.
   (a) Find the probability that it takes a student between 40 and 55 min to finish the exam.
   (b) Find the $E(T)$ and $SD(T)$.
   (c) Find the time T that 75% of students will be finished with the exam.
   (d) Suppose you and your friend are taking the exam (independently). Find the probability that both of you are finished within 50 min.

2. Using the definition of expected value $E(X) = \int_{-\infty}^{\infty} xf(x)\,\mathrm{d}x$, show that if $T \sim \text{Exponential}\,(\lambda)$, then $E(T) = 1/\lambda$.

3. (a) Let $T \sim \text{Exponential}\,(\lambda = 5)$. Find $P[T < E(T)]$.
   (b) Let $T \sim \text{Exponential}\,(\lambda = 0.15)$. Find $P[T < E(T)]$.
   (c) Let $T \sim \text{Exponential}\,(\lambda)$. Find $P[T < E(T)]$.

4. If $T \sim \text{Exponential}\,(\lambda)$, find the median of T.

5. Researchers at a medical facility have discovered a virus whose mean time from initial infection to symptoms appearing is 30 days. Assume this time T has an exponential distribution.

   HINT: Be careful with the parameter.
   (a) Find the probability that a patient who has just been infected will show symptoms in 25 days.
   (b) Find the probability that a patient who has just been infected will not show symptoms for at least 33 days.
   (c) Given that an infected patient has been without symptoms for the first 14 days, find the probability they will not show symptoms for at least the next 7 days.

6. Suppose hurricanes in Florida occur following a Poisson process at a rate of 0.02 per year, and the waiting time T between hurricanes can be modeled with an exponential random variable.
   (a) Find the probability that the first hurricane happens before year 5.
   (b) Find the probability that the first hurricane happens between years 7 and 10.
   (c) Let W be the waiting time from the start of observation until the third hurricane. Find $E(W)$ and $V(W)$.

7. A gamma distribution has a mean of 9 and a variance of 16. Find α and β for this distribution.

8. Let $X \sim \text{Gamma}\,(\alpha = 2, \beta = 1/2)$. Find $P(X < 3)$.

9. If a number is selected at random from the interval $[0, 100]$, its value has a uniform distribution over that interval. Let S be the random random variable for the sum of 45 numbers selected at random from $[0, 100]$. Find $P(2050 < S < 2650)$.

10. A charity receives 2000 contributions. Contributions are assumed to be independent and identically distributed with mean \$3500 and standard deviation \$300. Calculate the approximate 95^{th} percentile for this distribution for the total contributions received.

11. The total claim amount for a health insurance policy follows a distribution with density function

$$f(x) = \frac{1}{700}\mathrm{e}^{-x/700} \qquad x \geq 0.$$

The premium (cost of the policy for policyholders) is set at 50 over the expected total claim amount.

If 100 policies are sold, find the approximate probability that the insurance company will have claims (total losses on policies) exceeding the premiums collected (total money earned from policyholder payments). Assume all policies are independent and have identical claim amount distributions.

Select answers

1. (a) Prob $= 0.6$
   (b)
   (c) 53.75 min
   (d) Prob $= 0.36$
2.
3. (a)
   (b)
   (c) Prob ≈ 0.632
4. $m \approx 0.693/\lambda$
5. (a) Prob ≈ 0.565
   (b) Prob ≈ 0.333
   (c) Prob ≈ 0.7918
6. (a) Prob ≈ 0.095
   (b) Prob ≈ 0.0506
   (c) $E(W) = 150$
7. $\alpha \approx 5.0625$ and $\beta \approx 0.5625$
8. Prob ≈ 0.442
9. Prob ≈ 0.8297 using calc or Prob ≈ 0.8293 using table
10. $\approx 7{,}022{,}068.03$ using calc or 7,022,137.03 using table
11. Prob ≈ 0.2375 using calc or Prob ≈ 0.2389 using table