Name:

MATH 320: In-Class 4

Answer all questions. Show your work where necessary.

1. Four cards are to be dealt successively at random and without replacement from an ordinary deck of playing cards. What is the probability of receiving in order a spade, heart, spade, and a club?

2. In an experiment of tossing a single fair coin three times, what is the probability of getting exactly 2 tails, given that you get at least one tails?

3. If two events, A and B, are such that $P(A) = 0.4$, $P(B) = 0.25$ and $P(A \cap B) = 0.1$, find the following:
   (a) $P(A \mid B)$

   (b) $P(B \mid A)$

   (c) $P(A \mid A \cup B)$

   (d) $P(A \mid A \cap B)$

   (e) $P(A \cap B \mid A \cup B)$

4. (a) A woman has two children. She tells you that at least one of them is a boy. What is the probability that both children are boys? Assume $P(\text{Boy}) = 0.5$.

(b) Same question as part (a), but now assume $P(\text{Boy}) = 0.47$.

5. The number of injury claims per month is modeled by a random variable N with $P(N = n) = \frac{1}{(n+1)(n+2)}$, where $n \geq 0$.

   Determine the probability of exactly one claim during the particular month, given there have been at most two claims.

6. Monty Hall problem: In the game show "Let's Make a Deal", a contestant is presented with 3 doors. There is a prize behind one of the doors, and the host of the show knows which one. When the contestant makes a choice of door, at least one of the other doors will not have a prize, and the host will open a door (one not chosen by the contestant) with no prize. The contestant is given the option to change his choice after the host shows the door without a prize.

   (a) If the contestant switches doors, what is the probability that they get the door with the prize?

   (b) If the contestant does NOT switch doors, what is the probability that they get the door with the prize?