Name:

MATH 320: In-Class 7-8

Answer all questions. Show your work where necessary.

1. Toss a fair coin until the first head occurs. Let X be the number of tosses until the first head occurs.
   (a) Find the sample space S and the range of X, $\mathcal{X}$.

   (b) Define the random variable X using a figure. It should include S, $\mathcal{X}$ and arrows.

2. Consider the experiment of tossing a fair coin three times. Define the random variable X as the number of heads observed.
   (a) Write the pmf $f_X(x)$ as a piecewise function.
   (b) Write the cdf $F_X(x)$ as a piecewise function.

   (c) Plot both the pmf and cdf.

3. A random variable X has the cdf:

$$F_X(x) = P(X \leq x) = \begin{cases} 0 & x < -1 \\ 0.2 & -1 \leq x < 0 \\ 0.7 & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

   (a) Is X a discrete or continuous random variable? Why?

   (b) Find $P(-1 < X \leq 1)$
   (c) Find the pmf $f_X(x)$.

4. Let $f(x) = \frac{1}{6}(x+1), \quad 1 < x < c$. Find the constant c so that $f(x)$ is a valid pdf.

5. Let $F_X(x) = 1 - \mathrm{e}^{-5x}, \quad 0 \leq x < \infty$.
   (a) Find $P(X < 3)$.

   (b) Find the pdf $f_X(x)$.

   (c) Find $P(X < 3)$ using the pdf AND THEN find $P(X \geq 3)$.

   (d) Find $P(1 \leq X \leq 5)$ using the pdf AND THEN using the cdf.

6. Let $f(x) = 1.5x + 0.25, \quad 0 \leq x \leq 1$.
   (a) Find $P(X \leq 0.5)$ and $P(X \geq 0.75)$ using areas of shapes.

   (b) Find the cdf $F(x)$.