Name:

MATH 320: Homework 10

Due ____________________: Turn in a hard copy, neat and stapled.

1. For a binomial random variable X with $n = 2$ and $P(\text{Success}) = p$, show using the definitions that
   (a) $E(X) = 2p$ (i.e. use $E(X) = \sum xf(x)$)
   (b) $V(X) = 2p(1-p) = 2pq$ (You may use the alternate form of variance)

2. A contestant on a game show selects a ball from a basket containing 25 balls numbered from 1 to 25. Their prize is $850 times the number of the ball selected. Find the mean and standard deviation of the amount they win.

3. A nutrition company receives 2/5 of its supplement shipments from company X and the remainder of its shipments from other companies. Each shipment contains a very large number of supplement bottles.

   For Company X's shipments, 7% of the bottles are mislabelled. For every other company, 12% of the bottles are mislabelled.

   The nutrition company inspects 25 randomly selected bottles from a single shipment and finds that one bottle is mislabelled. Find the probability that the shipment came from Company X.

4. A telemarketer makes successful calls with probability 0.27. Their shift ends when they make 4 sales. Find the following:
   (a) The probability that the 4^{th} sale will be on the 13^{th} call.
   (b) The probability it will take more than 5 calls to make the 4 sales.

5. Suppose now that each sale made by the person in problem 4 is for $300. Find the mean number of total calls they will have to make to reach $2400.

6. In a shipment of lightbulbs 120 lightbulbs, there are 12 defective ones. An inspector randomly selects 15 bulbs. Let X represent the number of defective light bulbs selected by the inspector.
   (a) Find the pmf of X.
   (b) Find the probability less than 3 defective light bulbs are found.
   (c) Find the probability at least one defective light bulb is found.
   (d) Find the expected value and standard deviation of the number of defective light bulbs found.

7. Claims filed in a year by a policyholder of an insurance company have a Poisson distribution with $\lambda = 0.6$. The number of claims filed by two different policyholders are independent events.
   (a) If two policyholders are selected at random, find the probability that each of them will file one claim during the year.
   (b) Find the probability that at least one of them will file zero claims.

8. An actuary has discovered that policyholders are four times as likely to file two claims as to file three claims. If the number of claims filed as a Poisson distribution, find the variance of the number of claims filed.

Select answers

1. (a)
   (b)
2. $SD(\text{Money}) \approx \$6,129.44$
3. Prob ≈ 0.5943
4. (a) Prob ≈ 0.0688
   (b) Prob ≈ 0.9792
5. Exp Value ≈ 29.63
6. (a) Prob ≈ 0.8269
   (b) Prob ≈ 0.8149
   (c) $SD \approx 1.0914$
   (d)
7. (a) Prob ≈ 0.1084
   (b) Prob ≈ 0.7964
8.