MATH 320: Probability
Assessments

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Contents

1 In-Class Assignments

Name:

MATH 320: In-Class 1

Answer all questions. Show your work where necessary.

1. A computer runs a part of a bigger machine. If the computer or the part fails, then the machine does not work. Possible outcomes, where each is an ordered pair (such as (C_f, P_f)) involve the following:

C_f: Computer fails	C_w: Computer works as expected
P_f: Part fails	P_w: Part works as expected

   Write the outcomes in each of the following events:

   (a) Sample space $S =$

   (b) Part fails $A =$

   (c) Machine fails $F =$

   (d) $\sim F =$

2. An experiment consist of tossing a coin and then rolling a 6-sided die. An outcome is an ordered pair, such as $(H, 3)$. Let A be event that the coin shows heads and B be the event the number on the die is greater than four. List the outcomes in each of the following:

   (a) Sample space $S =$

   (b) $B =$

   (c) $A \cap B =$

   (d) $\sim(A \cap B) =$

   (e) $\sim(A \cup B) =$

3. In an experiment of tossing two 4-sided dice, we have the following events:
   - E = sum of dice is 4
   - F = both dice show the same number
   - G = both die show a number ≥ 3

   List the outcomes in each of the following events:

   (a) $E \cap F =$

   (b) $E \cup F =$

   (c) $E \cup (F \cap G) =$

   (d) $E \cap (F \cup G) =$

Name:

MATH 320: In-Class 2

Answer all questions. Show your work where necessary.

1. If A and B are two sets, draw Venn diagrams to verify the distributive laws (i.e. draw one for each side of the equation and confirm they are equivalent):
   (a) $A \cup (B \cap C) =$

   (b) $A \cap (B \cup C) =$

2. An insurance agent sells two types of insurance, life and health. Of his clients, 38 have life policies, 29 have health policies and 21 have both. How many clients does he have?

3. A stockbroker has 95 clients who own either stocks or bonds. If 67 own stocks and 52 own bonds, how many own both stocks and bonds?

4. When purchasing a car, you have 4 choices of body styles, 15 color combinations and 6 accessory packages. How many ways can you select your car?

5. Suppose a password has 7 characters and is not case-sensitive.
   (a) If letters or numbers may be used, how many different passwords can be made?

   (b) If the first 4 characters must be letters and the next 3 characters must be digits, how many different passwords can be made?

6. For the 9 starting players on a baseball team, how many different batting orders are there?

7. There are seven different faculty members in a college's English department. Four members are to be selected for four different committee chairs. Find the number of ways faculty members can be assigned to chairs using:

   (a) Sampling without replacement

   (b) Sampling with replacement

8. How many 5 card (poker) hands are possible form a deck of 52 cards?

9. A class has 15 boys and 13 girls.

   (a) In how many ways can the teacher select 4 boys and 5 girls for a field trip?

   (b) In how many ways can the teacher select either 4 or 5 boys and the remaining field trip members are girls?

Name:

MATH 320: In-Class 3

Answer all questions. Show your work where necessary.

1. Liam and Michael are going to play video games this afternoon. Together, they have 41 video games. If they decide to randomly choose two video games, what is the probability that the two they choose will consist of each of their favorite video games? Assume they have different favorites.

   (a) Solve this using counting tools; (b) Solve this the "direct way".

2. A high school is forming a club, and they would like to have each class represented. There are 30 freshman, 55 sophomores, 45 juniors and 40 seniors, totaling 170 students. Find the probability the club, which will have 20 members, includes exactly 4 freshman, 6 sophomores, 5 juniors and 5 seniors.

3. Suppose a committee of 5 is being formed randomly from people in the College of Sciences and Humanities. There are 4 deans, 40 faculty, and 10 staff.

   (a) What is the probability that none of the committee members are deans?

   Solve this using counting tools; Solve this the "direct way".

   (b) What is the probability at least one dean is on the committee?

   (c) What is the probability at least one staff member is on the committee?

   (d) What is the probability exactly two staff members are on the committee? Solve this both ways like part (a).

4. Suppose there are 3 sets of balls numbered 1 through 15 in a bag. If 3 balls are randomly chosen, without replacement, what is the probability that the balls have the same number on them?

5. From a deck of 52 ordinary playing cards, find the following probabilities.
   (a) Select a hand of five cards of all spades?

   (b) Select a hand of at least 4 cards that are spades?

6. If a pair of dice is rolled, find the probability that the sum of the two dice is less than or equal to 5.

7. Four people are doing an ESP (extrasensory perception) experiment. Each one is asked to guess a number between 1 and 10. What is the probability that no two of the four people guess the same number?

8. In a large lecture class, when students study for the final it is found that 22% of them go to office hours and tutoring, whereas 12% do neither of these. If the probability that a student goes to tutoring is 40%, find the probability that a student goes to office hours.

Name:

MATH 320: In-Class 4

Answer all questions. Show your work where necessary.

1. Four cards are to be dealt successively at random and without replacement from an ordinary deck of playing cards. What is the probability of receiving in order a spade, heart, spade, and a club?

2. In an experiment of tossing a single fair coin three times, what is the probability of getting exactly 2 tails, given that you get at least one tails?

3. If two events, A and B, are such that $P(A) = 0.4$, $P(B) = 0.25$ and $P(A \cap B) = 0.1$, find the following:
   (a) $P(A \mid B)$

   (b) $P(B \mid A)$

   (c) $P(A \mid A \cup B)$

   (d) $P(A \mid A \cap B)$

   (e) $P(A \cap B \mid A \cup B)$

4. (a) A woman has two children. She tells you that at least one of them is a boy. What is the probability that both children are boys? Assume $P(\text{Boy}) = 0.5$.

(b) Same question as part (a), but now assume $P(\text{Boy}) = 0.47$.

5. The number of injury claims per month is modeled by a random variable N with $P(N = n) = \frac{1}{(n+1)(n+2)}$, where $n \geq 0$.

   Determine the probability of exactly one claim during the particular month, given there have been at most two claims.

6. Monty Hall problem: In the game show "Let's Make a Deal", a contestant is presented with 3 doors. There is a prize behind one of the doors, and the host of the show knows which one. When the contestant makes a choice of door, at least one of the other doors will not have a prize, and the host will open a door (one not chosen by the contestant) with no prize. The contestant is given the option to change his choice after the host shows the door without a prize.

   (a) If the contestant switches doors, what is the probability that they get the door with the prize?

   (b) If the contestant does NOT switch doors, what is the probability that they get the door with the prize?

Name:

MATH 320: In-Class 5

Answer all questions. Show your work where necessary.

1. A student has three questions left on an exam. Each question is multiple choice with 4 options. If the student didn't study and randomly guesses for each of the remaining questions, find the following:
   (a) The probability that the student gets only the third question correct.

   (b) The probability that the student gets the first or the third question correct.

2. Two cards are drawn from a standard deck with replacement. Let $A1$ be the event the first card is an ace and $A2$ be the event the second card is an ace. Show that $A1$ and $A2$ are independent.

3. A company specializes in coaching people to pass a major professional examination. The company had helped 200 people last year. Their pass rates, based on type of the student, are shown in the following contingency table.

   Show if the type of student and pass / fail are independent using two different ways.

	Student	Professional	Total
Pass	48	72	120
Fail	50	30	80
Total	98	102	200

4. If A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.2$, find the following:
   (a) $P(A \cup B)$.

   (b) $P(\sim A \cap \sim B)$.

   (c) $P(\sim A \cup \sim B)$.

5. Three inspectors look at a critical component of a product. Their probabilities of detecting a defect are 0.90, 0.92 and 0.95, respectively. Let I_j be the event that inspector j finds the defect, $j = 1, 2, 3$. Assuming mutual independence, find the following probabilities.

   (a) At least one inspector detecting the defect.

   (b) Only one inspector detects the defect.

   (c) Exactly two inspector detects the defect.

6. *(Challenge!)* An insurer offers a health plan to employees of a large company, where employees may choose *exactly two* of the supplementary coverages: A, B, or C, or no supplementary coverage. The proportions of employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12, respectively.

   Find the probability that a randomly selected employee will choose no supplementary coverage.

Name:

MATH 320: In-Class 6

Answer all questions. Show your work where necessary.

1. The manufacturing company has a fabrication plant and an assembly line. The fabrication plant has 65% of the employees and the assembly line has 35%. During the past year 25% of the workers in the fabrication plant sustained injuries and 15% of the assembly line workers had injuries.
   (a) What percentage of all workers had injuries in this period? Draw a tree diagram to help.

   (b) If an employee had an injury, what is the probability that they worked on the assembly line?

2. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew next year. Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy and 15% of policyholders have both an auto and a homeowners policy.
   (a) What percentage of policyholders will renew at least one policy next year?

   (b) What percentage of policyholders will not renew at least one policy next year?

   (c) Given that a customer renews, what is the probability they have only an auto policy?

   (d) Given that a customer does not renew, what is the probability they have only an auto policy?

3. A corporate team has three managers, M_1, M_2 and M_3, who train 50%, 30% and 20% of the employees, respectively. Further, 15% of the employees trained by M_1 make mistakes on a certain process, 22% for M_2 and 18% for M_3. Find the probability that an employee makes a mistake.

4. Consider the game of Three. You shuffle a deck of three cards: 1, 2, 3. You draw cards without replacement until your total is 3 or more. You win if your total is 3.

   Let C_i denote the event that card C is the ith card drawn. For example, 3_2 is the event that the 3 was the second card drawn. Given that you win, find the probability that the Card 3 is drawn.

5. Suppose we have two coins. We know one of the two coins is biased and comes up heads with probability 3/4, we will call this Coin 1. And we know the other coin is fair and comes up heads with probability 1/2, we will call this Coin 2. However, we don't actually know which coin is which. Suppose you pick a coin randomly and flip it.

   (a) What is the probability that the outcome of the coin flip is a head?
   HINT: Think carefully about the stages of the tree diagram.

   (b) Given that the outcome of the flip is a head, what is the probability that you picked up the biased coin?

6. The probability that a randomly chosen make has a circulation problem is 0.3. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem.

   What is the conditional probability that a male has a circulation problem, given that he is a smoker?

Name:

MATH 320: In-Class 7-8

Answer all questions. Show your work where necessary.

1. Toss a fair coin until the first head occurs. Let X be the number of tosses until the first head occurs.
   (a) Find the sample space S and the range of X, $\mathcal{X}$.

   (b) Define the random variable X using a figure. It should include S, $\mathcal{X}$ and arrows.

2. Consider the experiment of tossing a fair coin three times. Define the random variable X as the number of heads observed.
   (a) Write the pmf $f_X(x)$ as a piecewise function.
   (b) Write the cdf $F_X(x)$ as a piecewise function.

   (c) Plot both the pmf and cdf.

3. A random variable X has the cdf:

$$F_X(x) = P(X \leq x) = \begin{cases} 0 & x < -1 \\ 0.2 & -1 \leq x < 0 \\ 0.7 & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

   (a) Is X a discrete or continuous random variable? Why?

   (b) Find $P(-1 < X \leq 1)$
   (c) Find the pmf $f_X(x)$.

4. Let $f(x) = \frac{1}{6}(x+1), \quad 1 < x < c$. Find the constant c so that $f(x)$ is a valid pdf.

5. Let $F_X(x) = 1 - \mathrm{e}^{-5x}, \quad 0 \leq x < \infty$.

   (a) Find $P(X < 3)$.

   (b) Find the pdf $f_X(x)$.

   (c) Find $P(X < 3)$ using the pdf AND THEN find $P(X \geq 3)$.

   (d) Find $P(1 \leq X \leq 5)$ using the pdf AND THEN using the cdf.

6. Let $f(x) = 1.5x + 0.25, \quad 0 \leq x \leq 1$.

   (a) Find $P(X \leq 0.5)$ and $P(X \geq 0.75)$ using areas of shapes.

   (b) Find the cdf $F(x)$.

Name:

MATH 320: In-Class 9

Answer all questions. Show your work where necessary.

1. A game is played where a fair six-sided die is first rolled. You receive a payout in the following manner:
   - If 1, 2 or 3 is rolled the game pays 1 dollar.
   - If a 4 or 5 is rolled the game pays 2 dollars.
   - If a 6 is rolled the game pays 3 dollars.

   (a) Find the expected payout of this game by hand.

   (b) Find the variance of the payout of this game by hand and by calculator (confirm the expected value too).

2. Given pmf table for X below. Let $Y = g(X) = \sqrt{X} + 3$.

x	0	16	25
$f(x)$	0.5	0.32	0.18

   (a) Find $E(Y)$ by hand.

   (b) Find $SD(Y)$ using your calculator.

3. Let X have the following cdf:

$$F(x) = \begin{cases} \frac{1}{2}x^2 & 0 \le x \le 1 \\ 1 - \frac{1}{2x^2} & 1 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the difference between the 70^{th} percentile and the 30^{th} percentile.

(b) Find the mode of this distribution.

(c) Find $E(X)$.

(d) Find the expected value of $g(X) = 15X - 3$.

Name:

MATH 320: In-Class 10

Assume that 18% of people are left handed. Answer the following questions based on each described experiment.

1. Suppose we select 9 people at random. Let X be the number of lefties selected out of the 9 people.

   (a) Find the probability there are exactly 4 lefties.

   (b) Find the probability there are at least 6 lefties.

   (c) Find the probability there are is a majority of righties.

   (d) How many lefties do you expect in the group? With what standard deviation?

   (e) Suppose there are less than 4 lefties in the group. Find the probability there are exactly 2 lefties?

2. Suppose we select people at random until the first lefty is selected. Let Y be the number of people selected in order to select the first lefty.

   (a) Find the probability the first lefty is the 5^{th} person.

   (b) Find the probability the first lefty is before the 7^{th} person.

   (c) Find the probability the first lefty is the 3^{rd} through 6^{th} person.

   (d) How many people do you expect to select until the first lefty? With what standard deviation?

   (e) Suppose the first 5 people were righties, find the probability the first 9 people are righties.

Name:

MATH 320: In-Class 10-2

1. Consider the experiment of drawing from a deck of cards with replacement.

   (a) What is the probability that the third heart appears on the tenth draw?

   (b) What is the probability that the third heart appears before the seventh draw?

   (c) What is the mean number of cards drawn to get the fifth red card?

2. (a) In a hospital there are 120 patients, 10 of whom have a particular disease. If a doctor is assigned 6 patients, what is the probability they receive more than 2 of these patients?

   (b) In a different hospital there are 30 patients, 13 of whom have a particular disease. If another doctor is assigned 22 patients, what is the probability they receive no more than 8 of these patients?

3. An insurance company has 5,000 policyholders who have had policies for at least 10 years. Over this period there have been a total of 12,200 claims on these policies. Assuming a Poisson distribution for these claims, answer each of the following questions:

   (a) What is λ, the average number of claims per policy per year?

   (b) What is the probability that a policyholder will file less than 2 claims in a year?

   (c) If all claims are for \$1,000, what is the mean and variance for the claim amount for a policyholder in a year?

4. A company prices its hurricane insurance using the following assumptions:

   (i) In any calendar year, there can be at most one hurricane.

   (ii) In any calendar year, the probability of a hurricane in 0.05.

   (iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

   Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

Name:

MATH 320: In-Class 11

1. Let $T \sim \text{Uniform}(0, 120)$.

   (a) Find $P(60 < T < 75)$.

   (b) Find $P(T > 50 \mid T > 30)$.

   (c) Let x be any real number in the interval $[30, 120]$. Find $P(T > x \mid T > 30)$.

   (d) Using your answer from part c, can we say anything about this function of x? (i.e. does the function match the cdf or survival of a particular distribution)?

2. Let $T \sim \text{Exponential}(\lambda = 1/3)$.

   (a) Find $P(T < 6)$.

(b) Find $P(T > 8)$.

(c) Use the cdf to find $P(2 < T < 5)$.

(d) Use the memoryless property to find $P(T > 8 \mid T > 3)$.

(e) Use the memoryless property to find $P(T < 9 \mid T > 4)$.

3. Let $T \sim \text{Exponential}\,(\lambda)$ with median equal to 5. Find $P(T > 4)$.

4. Let X have an exponential distribution with E(X) = 5. Find the 30th percentile of X.

Name:

MATH 320: In-Class 11-2

1. A electronics website receives orders from all over the world at a mean rate of 150 per hour according to the Poisson process. Let X be the waiting time in minutes until the fifth order.

   (a) Give the pdf of X.

   (b) Find the mean and variance of X.

2. Let $X \sim \text{Gamma}\,(\alpha = 3, \beta = 1/2)$. Find the expected value of $Y = 30X + 6X^2$.

3. Find the following probabilities:

   (a) $P(Z < 1.12)$.

   (b) $P(Z > -0.34)$.

   (c) If $X \sim \text{Normal}\,(\mu = 17, \sigma = 3)$, $P(13 \leq X < 22)$.

4. The lifetimes of light bulbs produced by a company are normally distributed with mean 150 hours and standard deviation 12.5 hours.

   (a) Find the probability that a bulb will last at least 140 hours.

   (b) If 4 new bulbs are installed at the same time, find the probability at least 3 of them will last more than 140 hours.

   (c) If a pack of 32 light bulbs is installed, find the probability the *combined lifetime* will be less than 4720 hours.

5. If $X \sim \text{Normal}\,(\mu = 5.2, \sigma = 0.8)$ and $Y = \mathrm{e}^X \Longrightarrow Y \sim \text{Lognormal}$, find $P(100 \leq Y \leq 500)$.

6. If $X \sim \text{Normal}\,(\mu = 0.10, \sigma = 0.03)$ and $Y = 100\mathrm{e}^X \Longrightarrow Y \sim \text{Lognormal}$, find $P(Y \leq 107.50)$.

7. If $X \sim \text{Beta}\,(\alpha = 2, \beta = 3)$.

   (a) Find $E(X)$ and $V(X)$.

   (b) Find $P(X < 0.3)$.

Name:

MATH 320: In-Class 12

1. Suppose X has the following pmf:

x	4	5	6	7
$f(x)$	0.17	0.53	0.22	0.08

   (a) Find the mgf $M_X(t)$.

   (b) Find $E(X)$ using $M_X(t)$.

   (c) Find $V(X)$ using $M_X(t)$.

2. If $X \sim \text{Gamma}\,(\alpha = 4, \beta = 0.75)$, find $M_X(t)$.

3. If $X \sim \text{Exponential}\,(\lambda = 3)$, use $M_X(t)$ to show that $E(X) = 1/3$.

4. If $X \sim \text{Normal}\,(\mu = -10, \sigma = 2)$, find $M_X(t)$.

5. Using the normal distribution from problem 4, use $M_{aX+b}(t) = \mathrm{e}^{tb} M_X(at)$ to find the mgf of $Y = -3X$. What is the distribution of Y?

Name:

MATH 320: In-Class 13

1. Suppose losses for a single insurance policy $X \sim \text{Uniform}\,(a = 0, b = 100)$. If there is a deductible of 10 and a cap of 40, find the expected amount of a single claim for this policy.

2. Let $X \sim \text{Uniform}\,(a = 1, b = 5)$ and $Y = -X$. Find the cdf of Y, $F_Y(y)$.

3. Let $X \sim \text{Normal}\,(\mu = 10, \sigma^2 = 4)$ and $Y = e^X$. Find the pdf of Y, $f_Y(y)$, using the pdf method. Compare the result to the pdf of $Y \sim \text{Lognormal}$ pdf.

4. Let $f(x) = 1.5x + 0.25 \quad 0 \le x \le 1$ and $Y = \ln(X)$.

   (a) Find the cdf of Y, $F_Y(y)$.

   (b) Find the pdf using the cdf method.

5. Let $X \sim \text{Geometric}\,(p = 0.3)$ and $Y = \sqrt{X}$. Find the pdf of Y, $f_Y(y)$.

2 Homework

Name:

MATH 320: Homework 0

Due ____________________: Turn in a hard copy, neat and stapled.

This homework is designed as a calculus review of the concepts we will use later in the semester. You will be expected to solve all parts of these questions by hand, showing all work and without the aide of technology.

1. Solve the following derivatives:
   (a) $\frac{\mathrm{d}}{\mathrm{d}x} 14x^7$
   (b) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{-3} + \frac{1}{7}x^2\right]$
   (c) $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{7x}$
   (d) $\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{e}^{6x}$
   (e) $\frac{\mathrm{d}}{\mathrm{d}x} \ln(5x)$
   (f) $\frac{\mathrm{d}}{\mathrm{d}x} 8x^2\mathrm{e}^{2x}$
   (g) $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[\frac{1}{3}x^6 + 4\mathrm{e}^{-3x}\right]$

2. Solve the following integrals. Make sure to show work; can check against answers provided:
   (a) $\displaystyle\int_0^{1.5} \frac{1}{2}\,\mathrm{d}x$
   (b) $\displaystyle\int_0^1 3x^2\,\mathrm{d}x$
   (c) $\displaystyle\int_0^1 \frac{3}{14}\left[x^2 + x\right]\mathrm{d}x$
   (d) $\displaystyle\int_1^\infty 3\mathrm{e}^{-3x}\,\mathrm{d}x$
   (e) $\displaystyle\int_0^2 0.25\,x\,\mathrm{e}^{-0.5x}\,\mathrm{d}x$
   (f) $\displaystyle\int_0^{0.5}\int_0^1 3x\,\mathrm{d}y\,\mathrm{d}x$

Integral answers:

(a) 0.75 (b) 1 (c) 5/28 (d) ≈ 0.05 (e) ≈ 0.264 (f) 0.375

Name:

MATH 320: Homework 1-2

Due ___________________: Turn in a hard copy, neat and stapled.

1. If A and B are two sets, draw Venn diagrams to verify the following:
   (a) $A = (A \cap B) \cup (A \cap \sim B)$.
   (b) If $B \subset A$, $A = B \cup (A \cap \sim B)$.

2. Refer to #1. Use the identities $A = A \cap S$ and $S = B \cup \sim B$ and a distributive law to prove that:

   (*Note*: "prove" means to try and rewrite the right-hand side of the equation to be equal to the left-hand side)

   (a) $A = (A \cap B) \cup (A \cap \sim B)$.
   (b) If $B \subset A$, $A = B \cup (A \cap \sim B)$.

3. A company has 152 employees. There are 94 who have been with the company more than 10 years and 62 of those are college graduates. There are 41 who do not have college degrees and have been with the company less than 10 years. How many employees are college graduates?

4. A school wants to know surveys their students about which sports they play, if any. Responses show that 26 play soccer, 32 play basketball, 23 play volleyball, 14 play soccer and basketball, 11 play soccer and volleyball, 9 play basketball and volleyball, 5 play all three and 40 don't play any sports. How many students are there?

5. An experiment has two stages. First stage consist of drawing a card from a standard deck. If the card is red, the second stage consist of tossing a coin. If the card is black, the second stage consist of rolling a die. How many outcomes are possible?

6. (*Challenge!*) Seven people are to be seated in a row of seven chairs. In how many ways can these people be seated in two of them insist on sitting next to each other?

7. (a) How many different six-digit telephone numbers can be formed if the first digit cannot be nine?
   (b) How many different six-digit telephone numbers can be formed if the first digit cannot be nine AND you cannot reuse digits?

8. A small town has 50 voters who are registered to vote on a ballot issue on whether a new park should be built. Of these 50 voters, 35 support building the new park. A construction company is going to take a poll of 5 of these 50 voters at random.
   (a) How many different selections of 5 voters are possible from these 50 voters?
   (b) How many of the selections in part (a) include 3 or more voters who support building the new park?

9. (a) A company has 12 analysts. It has a major project which has been divided into 3 sub projects, and assigns 4 analyst to each task. In how many ways can this be done?
   (b) Suppose that in part (a), the company divides the 12 analyst into 3 teams of 4 each, AND each team works on the whole project. In how many ways can this be done?

Select answers

1. (a)
   (b)
2. (a)
   (b)
3. 79
4. 92
5. 208
6. 1440
7. (a) 900,000
   (b) 136,080
8. (a) 2,118,760
   (b) 1,797,257
9. (a) 34,650
   (b) 5775

Name:

MATH 320: Homework 3

Due ____________________: Turn in a hard copy, neat and stapled.

1. A computer company has a shipment of 60 computer components of which 12 are defective. If 8 components are chosen at random to be tested, what is the probability that:
   (a) All are good?
   (b) 5 are good and 3 are defective?
   (c) At least 6 are defective?

2. 12 people, 6 men and 6 women, are to be seated in a row of 12 chairs. What is the probability that the men and women end up in alternate chairs?

3. An auto insurance company finds in the past 10 years 25% of its policyholders have filed liability claims, 33% have filed comprehensive claims, and 18% have filed both types of claims. What is the probability that a policyholder chosen at random has not filed a claim of either kind?

4. Prove $P(A \cap B) \geq P(A) + P(B) - 1$.

   Note that this theorem allows us to place a lower bound on the probability of simultaneous events (intersection) in terms of the probabilities of the individual events.

5. You are given $P(A \cup B) = 0.6$ and $P(A \cup {\sim}B) = 0.85$. Determine $P(A)$ using each of the following methods:
   (a) Venn Diagram.
   (b) Set theory and probability theorems.

6. If 5 cards are dealt from a deck of 52 ordinary playing cards, find the probability of:
   (a) A “full house”. Note that a fill house contains three matching cards of one rank and two matching cards of another rank. *HINT: Think of selecting the rank and the suit as separate tasks.*
   (b) A hand of one pair. Note that one pair contains two cards of the same rank and three cards of three other ranks.

Select answers

1. (a) Prob ≈ 0.147
   (b) Prob ≈ 0.147
   (c) Prob ≈ 0.809
2. Prob ≈ 0.0022
3. Prob $= 0.6$
4.
5. Prob $= 0.45$
6. (a) Prob ≈ 0.0014
   (b) Prob ≈ 0.4226

Name:

MATH 320: Homework 4-5

Due ____________________: Turn in a hard copy, neat and stapled.

1. A swim team consists of 6 boys and 4 girls. A relay team of 4 swimmers is chosen at random from the team members. What is the probability that 2 boys are selected for the relay team given that the first 2 selections were girls?

2. An actuary is studying the prevalence of three health risk factors, denoted A, B and C, within a population of women. For each of the three factors, the probability is 0.12 that a women in the population has only this risk factor (and no others). For any two of the three risk factors, the probability is 0.15 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is 1/3.

   (a) Find the probability that a woman has all three risk factors.

   (b) Find the probability that a woman has none of the three risk factors, given that she does not have risk factor A.

3. If A and B are independent events, prove $\sim A$ and $\sim B$ are also independent.

4. For the experiment of tossing a single fair coin 3 times, let E be the event the first toss is a head and F be the event 2 heads and 1 tail are tossed. Show if E and F are independent.

5. A jar contains 10 marbles: 4 red and 6 blue. A second jar contains 16 red marbles and an unknown number of blue marbles. A single marble is drawn from each jar.

   (a) Suppose the probability that both marbles are red is 0.256. Calculate the number of blue marbles in the second jar.

   (b) Now suppose the probability that both marbles are the same color is 0.44. Calculate the number of blue marbles in the second jar.

6. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

   Calculate the probability that a claim submitted to the insurance company includes operating room charges.

1. Prob ≈ 0.536
2. (a) Prob $= 0.075$
   (b) Prob ≈ 0.228
3.
4.
5. (a) 4
   (b) 9
6. Prob $= 0.4$

Name:

MATH 320: Homework 8

Due ____________________: Turn in a hard copy, neat and stapled.

1. 5 cards are face down in a row on a table. Exactly one of them is an ace. You turn the cards over one at a time, moving from left to right. Let X be the random variable for the number of cards turned *before* an ace is turned over.

   Find the pmf for X. You can write it as a piecewise function or as a table.

2. Let X be the number of claims on an auto insurance policy having pmf

$$f(x) = \begin{cases} 0.9 & x = 0 \\ \frac{c}{x} & x = 1, 2, 3, 4, 5, 6 \end{cases}$$

   where c is a constant.

   (a) Determine the value of c that makes $f(x)$ a valid pmf.

   (b) Write the cdf $F(x)$ as a piecewise function (round to 4 decimals).

3. Let $f(x) = \dfrac{7}{256}x^6, \quad -c < x < c$. Find the constant c so that $f(x)$ is a valid pdf.

4. The lifetime of a machine part has a continuous distribution on the interval $(0, 10)$ with probability density function $f(x)$, where $f(x)$ is *proportional* to $(10 + x)^{-2}$.

   Calculate the probability that the lifetime of the machine part is less than 5.

5. Let X have the following density function:

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 1/x^3 & 1 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the cdf $F(x)$.

   (b) Plot the cdf $F(x)$.

   (c) Use the cdf $F(x)$ to find $P(0.5 \le X \le 5)$.

6. The loss due to water damage for a home is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20 - x) & 0 \le x \le 20 \\ 0 & \text{elsewhere} \end{cases}$$

   Given that the water loss exceeds 6, calculate the probability that it exceeds 15.

Select answers

1.
2. (a) $c \approx 0.0408$

   (b)
3. $c = 2$
4. Prob ≈ 0.667
5. (a)

   (b)

   (c) Prob $= 0.855$
6. Prob ≈ 0.1276

Name:

MATH 320: Homework 9

Due ____________________: Turn in a hard copy, neat and stapled.

1. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the pmf
$$f(x) = \frac{5-x}{10}, \quad x = 1, 2, 3, 4.$$
If the patient is to receive \$350 from an insurance company for each of the first two days in the hospital and \$150 for each day after the first two days, what is the expected total payment for the hospitalization?

2. A tour operator has a tour bus that can accommodate 16 tourists. The operator knows that tourists may not show up, so he sells 17 tickets. The probability that an individual tourist will not show up is 0.04, independent of all other tourists.

   Each ticket cost \$40, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat it is not available, the tour operator has to pay \$80 (ticket cost + \$40 penalty) to the tourist.

   What is the expected revenue of the tour operator?

   Hint: Define the random variable R for the revenue, and think of it as a piecewise function with two cases.

3. Let X have pmf
$$f(x) = \frac{2x-1}{16} \quad x = 1, 2, 3, 4$$
   (a) Find the variance of $Y = X + 4$.

   (b) Find the standard deviation of $Z = 2X$.

4. A probability distribution of the claim sizes for an auto insurance policy are given in the table below:

Claim size (x)	Probability $f(x)$
15	0.10
20	0.30
25	0.15
30	0.25
35	0.20

   Find the percentage of claims that are within one standard deviation of the mean claim size.

5. A recent study indicates that the annual cost of maintaining and repairing a mountain bike averages \$150 with a standard deviation \$25.

   Suppose a tax of 20% is introduced on all items associated with the maintenance and repair of mountain bikes (i.e. everything is made 20% more expensive). Calculate the new variance of the annual cost of maintaining and repairing a mountain bike.

6. A salesperson can contact either one or two customers per day with probability 1/4 and 3/4, respectively. Each contact will result in either no sale or a \$10,000 sale, with the probabilities 0.8 and 0.2, respectively.

   (a) Find the pmf for the *number of sales*.

   Hint: Think about this scenario using a tree diagram.

   (b) Find the expected value and variance of the *number of sales*.

   (c) Use your answers from part (b) to find the mean and variance of the *total dollar amount of the sales*.

7. Let random variable X have the following density function:

$$f(x) = \begin{cases} 25x & 0 \leq x \leq 0.20 \\ 1.5625(1-x) & 0.2 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find $P(0.15 \leq X \leq 0.55)$.

   (b) Find the median m.

8. Let random variable X have the following density function:

$$f(x) = \begin{cases} \mathrm{e}^{x} & 0 \leq x \leq \ln(2) \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the cdf $F_X(x)$.

   (b) Find the IQR of X.

9. Let random variable X have the following density function:

$$f(x) = \begin{cases} \dfrac{|x|}{10} & -2 \leq x \leq 4 \\ \\ 0 & \text{otherwise} \end{cases}$$

   Find the expected value of X.

10. The loss due to water damage for a home is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x) & 0 \leq x \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$

   Calculate the variance of the loss.

Select answers

1. $E(\text{Total payment}) = 620$
2. $E(\text{Revenue}) \approx 640.03$
3. (a) $V(Y) \approx 0.859$
   (b) $SD(Z) \approx 1.854$
4. $\text{Prob} = 0.7$
5. $V(\text{Cost} = 900$
6. (a)
   (b) $E(\text{Sales}) = 0.35$ and $V(\text{Sales}) = 0.2875$
   (c) $V(\text{Money}) \approx 5361.90$
7. (a) $\text{Prob} \approx 0.5606$
   (b)
8. (a)
   (b) $IQR \approx 0.3365$
9. $E(X) \approx 1.867$
10. $V(X) \approx 22.22$

Name:

MATH 320: Homework 10

Due ___________________: Turn in a hard copy, neat and stapled.

1. For a binomial random variable X with $n = 2$ and $P(\text{Success}) = p$, show using the definitions that
   (a) $E(X) = 2p$ (i.e. use $E(X) = \sum xf(x)$)
   (b) $V(X) = 2p(1 - p) = 2pq$ (You may use the alternate form of variance)

2. A contestant on a game show selects a ball from a basket containing 25 balls numbered from 1 to 25. Their prize is \$850 times the number of the ball selected. Find the mean and standard deviation of the amount they win.

3. A nutrition company receives 2/5 of its supplement shipments from company X and the remainder of its shipments from other companies. Each shipment contains a very large number of supplement bottles.

   For Company X's shipments, 7% of the bottles are mislabelled. For every other company, 12% of the bottles are mislabelled.

   The nutrition company inspects 25 randomly selected bottles from a single shipment and finds that one bottle is mislabelled. Find the probability that the shipment came from Company X.

4. A telemarketer makes successful calls with probability 0.27. Their shift ends when they make 4 sales. Find the following:
   (a) The probability that the 4^{th} sale will be on the 13^{th} call.
   (b) The probability it will take more than 5 calls to make the 4 sales.

5. Suppose now that each sale made by the person in problem 4 is for \$300. Find the mean number of total calls they will have to make to reach \$2400.

6. In a shipment of lightbulbs 120 lightbulbs, there are 12 defective ones. An inspector randomly selects 15 bulbs. Let X represent the number of defective light bulbs selected by the inspector.
   (a) Find the pmf of X.
   (b) Find the probability less than 3 defective light bulbs are found.
   (c) Find the probability at least one defective light bulb is found.
   (d) Find the expected value and standard deviation of the number of defective light bulbs found.

7. Claims filed in a year by a policyholder of an insurance company have a Poisson distribution with $\lambda = 0.6$. The number of claims filed by two different policyholders are independent events.
   (a) If two policyholders are selected at random, find the probability that each of them will file one claim during the year.
   (b) Find the probability that at least one of them will file zero claims.

8. An actuary has discovered that policyholders are four times as likely to file two claims as to file three claims. If the number of claims filed as a Poisson distribution, find the variance of the number of claims filed.

Select answers

1. (a)
   (b)
2. $SD(\text{Money}) \approx \$6,129.44$
3. Prob ≈ 0.5943
4. (a) Prob ≈ 0.0688
   (b) Prob ≈ 0.9792
5. Exp Value ≈ 29.63
6. (a) Prob ≈ 0.8269
   (b) Prob ≈ 0.8149
   (c) $SD \approx 1.0914$
   (d)
7. (a) Prob ≈ 0.1084
   (b) Prob ≈ 0.7964
8.

Name:

MATH 320: Homework 11

Due __________________: Turn in a hard copy, neat and stapled.

1. A professor gives an exam where the time limit is 60 min and the anticipated minimum time to students to take the exam is 35 min. Assume that the random variable T for the time it takes a student to finish the exam is uniformly distributed over $[35, 60]$.

   (a) Find the probability that it takes a student between 40 and 55 min to finish the exam.

   (b) Find the $E(T)$ and $SD(T)$.

   (c) Find the time T that 75% of students will be finished with the exam.

   (d) Suppose you and your friend are taking the exam (independently). Find the probability that both of you are finished within 50 min.

2. Using the definition of expected value $E(X) = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x$, show that if $T \sim \text{Exponential}\,(\lambda)$, then $E(T) = 1/\lambda$.

3. (a) Let $T \sim \text{Exponential}\,(\lambda = 5)$. Find $P[T < E(T)]$.

   (b) Let $T \sim \text{Exponential}\,(\lambda = 0.15)$. Find $P[T < E(T)]$.

   (c) Let $T \sim \text{Exponential}\,(\lambda)$. Find $P[T < E(T)]$.

4. If $T \sim \text{Exponential}\,(\lambda)$, find the median of T.

5. Researchers at a medical facility have discovered a virus whose mean time from initial infection to symptoms appearing is 30 days. Assume this time T has an exponential distribution.

   HINT: Be careful with the parameter.

   (a) Find the probability that a patient who has just been infected will show symptoms in 25 days.

   (b) Find the probability that a patient who has just been infected will not show symptoms for at least 33 days.

   (c) Given that an infected patient has been without symptoms for the first 14 days, find the probability they will not show symptoms for at least the next 7 days.

6. Suppose hurricanes in Florida occur following a Poisson process at a rate of 0.02 per year, and the waiting time T between hurricanes can be modeled with an exponential random variable.

   (a) Find the probability that the first hurricane happens before year 5.

   (b) Find the probability that the first hurricane happens between years 7 and 10.

   (c) Let W be the waiting time from the start of observation until the third hurricane. Find $E(W)$ and $V(W)$.

7. A gamma distribution has a mean of 9 and a variance of 16. Find α and β for this distribution.

8. Let $X \sim \text{Gamma}\,(\alpha = 2, \beta = 1/2)$. Find $P(X < 3)$.

9. If a number is selected at random from the interval $[0, 100]$, its value has a uniform distribution over that interval. Let S be the random random variable for the sum of 45 numbers selected at random from $[0, 100]$. Find $P(2050 < S < 2650)$.

10. A charity receives 2000 contributions. Contributions are assumed to be independent and identically distributed with mean \$3500 and standard deviation \$300. Calculate the approximate 95^{th} percentile for this distribution for the total contributions received.

11. The total claim amount for a health insurance policy follows a distribution with density function

$$f(x) = \frac{1}{700}\mathrm{e}^{-x/700} \qquad x \geq 0.$$

The premium (cost of the policy for policyholders) is set at 50 over the expected total claim amount.

If 100 policies are sold, find the approximate probability that the insurance company will have claims (total losses on policies) exceeding the premiums collected (total money earned from policyholder payments). Assume all policies are independent and have identical claim amount distributions.

Select answers

1. (a) Prob = 0.6
   (b)
   (c) 53.75 min
   (d) Prob = 0.36
2.
3. (a)
   (b)
   (c) Prob ≈ 0.632
4. $m \approx 0.693/\lambda$
5. (a) Prob ≈ 0.565
   (b) Prob ≈ 0.333
   (c) Prob ≈ 0.7918
6. (a) Prob ≈ 0.095
   (b) Prob ≈ 0.0506
   (c) $E(W) = 150$
7. $\alpha \approx 5.0625$ and $\beta \approx 0.5625$
8. Prob ≈ 0.442
9. Prob ≈ 0.8297 using calc or Prob ≈ 0.8293 using table
10. $\approx 7,022,068.03$ using calc or 7,022,137.03 using table
11. Prob ≈ 0.2375 using calc or Prob ≈ 0.2389 using table

Name:

MATH 320: Homework 13

Due ____________________: Turn in a hard copy, neat and stapled.

1. Assume the amount of a single loss for an insurance policy has the density function $f(x) = 0.05e^{-0.05x}$, for $x > 0$.

   (a) Suppose this policy has a \$5 per claim deductible. Find the expected amount of a single claim for this policy.

   (b) Now suppose there is a payment cap of \$30 (and no deductible). Find the expected amount of a single claim for this policy.

2. An insurance policy is written to cover a loss, X, where $X \sim \text{Uniform}\,(a = 0, b = 1000)$.

   At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

3. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is uniformly distributed on the interval $[0, 80]$ years.

   Since the device will not be monitored during its first 10 years of service, the time to discovery of its failure is $X = \max(T, 10)$ (i.e. whichever X takes the value of whichever is greater for that particular x point).

   (a) Write X as a piecewise function of T.

   (b) Find $E(X)$.

4. Let $X \sim \text{Exponential}\,(\lambda = 0.5)$ and $Y = 1/X$. Assume $x > 0$ for this problem.

   (a) Find $F_Y(y)$.

   (b) Find $f_Y(y)$.

   (c) Find $P(1 \leq Y \leq 2)$.

5. An investment account earns an annual interest rate R that follows a uniform distribution on the interval $(0.05, 0.08)$. The value of a 10,000 initial investment in this account after one year is given by $V = 10,000e^R$.

   (a) Find $F_V(v)$.

   (b) Find $P(V \geq 10,500)$.

Select answers

1. (a) Exp value $= 15.576$

   (b) Exp value $= 15.537$

2. $d = 500$

3. (a)

   (b) $E(X) = 40.625$

4. (a)

   (b)

   (c) Prob ≈ 0.1723

5. (a)

   (b) Prob ≈ 0.7802

3 Finals Reviews

Name:

MATH 320: Review Part 1

1. Passwords for your iPhone require 6 characters.

   (a) If the first three must be digits 0-9 and the last three must be lowercase letters a-z, how many different passwords can you make?

   (b) Using the same scenario from part (a), now suppose you cannot reuse digits / letters. How many different passwords can you make?

   (c) What is the probability your friend guesses your password correctly using the scenario from part (b).

2. There is a group of 145 supporters representing England, Argentina or a different country for the world cup.

   (a) Suppose 50 people support only England, 35 people support England and Argentina, and 20 support a different country. How many people support England or Argentina?

   (b) Now suppose 85 people support a different country. How many people support England or Argentina?

   (c) Suppose 40 people support England. 75 people support Argentina and of those 25 support England. How many support a different country?

3. There are two events: A = watching Brazil vs Croatia and B = watching Portugal vs Morocco. Let $P(A) = 0.6$, $P(B) = 0.35$ and $P(A \cap B) = 0.12$. Find the following:

   (a) $P(A \mid B) =$

   (b) $P(B \mid A =$

   (c) $P(A \mid A \cup B) =$

   (d) $P(\sim B \mid \sim A) =$

   (e) Suppose now A and B are independent and we do not know $P(B)$ (we still know all other information in the setup). Find $P(B)$.

4. There are 8 teams left in the world cup.

   (a) How many different ways can the top 4 teams finish?

   (b) 4 teams move onto the next round, how many different ways can these 4 teams be selected?

   (c) What is the probability Morocco and Netherlands are selected to move onto the next round?

   (d) What is the probability Morocco or Netherlands are selected to move onto the next round?

5. Suppose you roll an 8-sided die. Let $A =$ odd numbers and $B =$ numbers less than or equal to 4. Show if A and B are independent or not mathematically using TWO different ways.

6. France scores 0 goals in 20% of their games, one goal in 50% of their games, and 2 goals in 30% of their games. When they do not score, there is a 20% chance of winning; when they score one goal, there is a 40% chance of winning; when they score two goals, there is a 80% chance of winning. Assume they can only win or lose (no ties).

   (a) What is the probability France wins?

   (b) What is the probability France loses?

   (c) If France wins, what is the probability that they scored one goal?

   (d) If France loses, what is the probability that they scored?

Name:

MATH 320: Review Part 2

1. Below is the pmf of X, the number of goals scored by Brazil.

x	0	1	2	3
$f(x)$	0.26	0.37	0.22	0.15

(a) Find $P(X > 1)$ and $P(X \leq 2)$.

(b) Find $E(X)$ and $V(X)$.

(c) Suppose players get paid \$1,000 for each game and an additional \$100 bonus for every goal the team scores. Let Y be the random variable for the total amount of money a single player receives for a particular game. Find $E(Y)$ and $SD(Y)$.

(d) Write the cdf of Y as a piecewise function.

(e) Plot the cdf of Y.

2. Let X have the following pmf:

$$f(x) = \begin{cases} 0.2 & x = 3, 4, 5, 6, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $E(X)$ and $V(X)$.

(b) If $Y = -0.25X + 1$, find $SD(Y)$.

3. Suppose X has the following density function with constant c: $f_X(x) = cx^2, \qquad -1 < x < 1.$

(a) Find the constant c so that $f_X(x)$ is a valid pdf.

(b) Find $E(X)$ and $V(X)$.

(c) Find the cdf of X, $F_X(x)$.

(d) Find $P(-0.1 < X < 0.3)$.

(e) Find $P(X > 0.2 \mid X < 0.4)$.

(f) Find the median m of X.

(g) Find the cutoff for the upper 20^{th} percent of X.

(h) Let $Y = X^3$. Find $E(Y)$.

(i) Find $P(Y < 0.5)$

Name:

MATH 320: Review Part 3

1. A player from Spain takes penalty kicks until they score. The probability of scoring from each penalty kick is 0.62. Assume successive attempts are independent. Let X represent the number of attempts for the first score.

   (a) Find the distribution of X.

   (b) Find the probability the first score is on the 5^{th} attempt.

   (c) Find the probability it takes more than 3 attempts for the first score.

   (d) Find the probability it takes less than 5 attempts for the first score.

   (e) Find the probability the first score is on the 4^{th} through 7^{th} attempt.

   (f) Find $E(X)$ and $SD(X)$.

   (g) Now suppose this player will take penalty kicks until they score three times. Let Y represent the number of attempts to score three times. Find the distribution of Y.

   (h) Find the probability it takes more than 6 attempts to score three times.

(i) Find the probability it takes less than or equal to 4 attempts to score three times.

2. There are 11 players on the field for a team. Let X be the number of players that are wearing Nike boots. Suppose there is a 0.55 probability that each player is wearing Nike boots and that each player's choice is independent of their teammates.

   (a) Find the distribution of X.

   (b) Find the probability more than 6 players are wearing Nike's.

   (c) Find the probability less or equal to 4 players are not wearing Nike's.

   (d) Find the mgf of X.

   (e) Suppose you made an obscure bet where you win \$30 if exactly one player is wearing Nike's and lose \$5 if not. Find the expected value of this bet.

3. Suppose 150 fans are waiting outside the stadium in Qatar, 20 of which support Portugal and the rest support Morocco. 25 fans are to be randomly selected for a special seating arrangement in the front row. Let X represent the number of Portuguese fans selected.

   (a) Find the distribution of X.

   (b) Find the probability there is exactly one Portuguese fan selected.

   (c) Find the probability there is at least two Portuguese fans selected.

   (d) Find the probability that the majority of fans selected support Morocco.

4. Suppose goals in soccer matches occur following a Poisson process and the average number of goals per 90 min is 3.5. Let X be the number of goals in a 90 min interval.

   (a) Find the probability a game (which is 90 min long) has exactly 3 goals.

   (b) Find the probability a game has more than 2 goals.

   (c) Suppose 3 games are played on a particular day. Find the probability less than 8 goals are scored in the three games.

5. Games at the previous world cup in Russia averaged approximately 2.6 goals per 90 min (which is $2.6/90 \approx 0.03$ goals per min). Lets assume goals occur following a Poisson process with the given rate. Let T be the waiting time (in minutes) from the start of the match until the first goal.

   Contextually, there is an upper limit for time of a game, but for problem solving assume $t \to \infty$.

   (a) Find the distribution of T.

   (b) Find the probability the first goal occurs before the 20^{th} minute.

   (c) Find the probability the first goal occurs after the 80^{th} minute.

   (d) Find the probability the first goal occurs between the 40^{th} and 50^{th} minute.

   (e) Suppose W is the waiting time from the start of the match until the 3^{rd} goal of a match. Find the distribution of W. Assume goals occur independently.

   (f) Find $E(W)$, $V(W)$ and $M_W(t)$.

   (g) The stress levels of fans can be modeled as a function of the waiting time until the first goal. Let the stress levels $A = 0.25T^2$.

   Find the cdf of A, $F_A(a)$.

(h) Find the pdf of A, $f_A(a)$.

6. The density function for the time T in minutes from the start of the match until Messi scores a goal can be given by:

$$f_T(t) = \frac{1}{90}, \qquad 0 < t < 90$$

   (a) Find the probability Messi scores a goal in the first 15 min of a particular game.

   (b) Find the probability Messi scores a goal in the last 28 min of a particular game.

   (c) If the score at halftime (after 45 min) of a particular game is 0-0, find the probability Messi scores in the first 20 min of the second half.

7. Recall the problem where we modeled the number of players out of 11 that were wearing Nike boots. Now suppose we are considering both teams and all players on the bench, for a total of 52. Let the number of players wearing Nike boots $S \sim \text{Binomial}\,(n = 52, p = 0.55)$.

   (a) Write S as a sum of *iid* random variables Y_i.

   (b) Find the approximate distribution of S.

(c) Find the approximate probability less than 30 players are wearing Nike boots.

(d) Find the approximate probability more than 40 players are wearing Nike boots.

(e) Find the approximate probability between 25 and 35 players are wearing Nike boots.

(f) Find the IQR of S.

8. If $M_X(t) = 0.4\mathrm{e}^{-3t} + 0.25\mathrm{e}^{t} + 0.35\mathrm{e}^{2t}$. Find $E(X)$.