

Where does data come from

Definitions

- An **experiment** is the process by which an observation/outcome is made, which cannot be predicted with certainty (outcomes are random).
- An **outcome** of an experiment is any possible observation of that experiment (often called sample points).

Examples: Write the set of all possible outcomes for the following experiments.

1. Sampling students and computing the average number of study hours each day:
2. Roll die, record number that appears:
3. Roll die, record first role that a one appears:

What is probability and how to calculate it

Intuitively, probability is the likelihood of something occurring.

One approach (simplest case)

- **Probability by counting equally likely outcomes**

Probability of an event =

– Example: Flip a fair coin, $P(\text{Heads}) =$

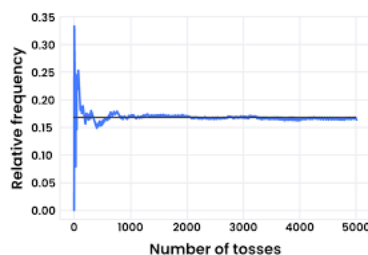
- Events are not always equally likely, so cannot determine probabilities by counting. But there is a simple way to estimate that probability.

Another approach

- **Empirical probability** (based on collected data).
- Relative frequency estimate of the probability of an event

Probability of an event =

- These are two ways of looking at probability.
 - If outcomes are equally likely, *relative frequency* \approx *counting* for a very large number of trials (e.g. if we simulated rolling a fair die 10,000 times, $\frac{\#6s}{10,000} \approx \frac{1}{6}$).



Third approach

- **Subjective probability** is asking a well-informed person for his/her personal estimate of the probability of an event (relying on experience and personal recollections of relative frequencies in the past).
- The rest of this chapter will be building in more precise mathematical framework for probability.
- Counting will play a big role, but keep in mind that in practice many probability numbers actually used in calculations may come from relative frequencies or subjective estimates.

Set Theory

The mathematical basis of probability is sets. Set theory is useful here to provide a precise language for dealing with the outcomes in a probability experiment.

Definitions

- A **set** is a collection of objects (such as the numbers 1, 2, 3, 4, 5, 6).
 - The objects are called **elements** of the set. _____ of an experiment correspond to _____ in a set.

- Writing sets: **Notation is important!**

We use capital letters to denote sets such as A , B , C .

Can list the elements in braces if only a few, or use set-builder notation for large or infinite sets.

- Example: All positive numbers can be written as

" $A = \text{the set of all } x\text{'s such that (condition) } x > 0$ "

- **Subset** $A \subset B$ means that _____ in A is also an element of B .
- The **sample space** S (aka outcome space) is the set of _____ outcomes of an experiment.
 - There are different types of sample spaces.
 - Countable or uncountable and if numeric: discrete or continuous.
- An **event** is a collection of possible outcomes of an experiment, that is, any _____ of S .
 1. Examples: Roll die, record the number that appears.
 - Define notation:
 - Show event:

An event occurs when at least one element in the event has occurred.

2. Is S an event?

- The **null (empty) set** is the set containing _____.

Basic operations (algebra of sets)

- **Union:**

The set of elements that belong to _____

- **Intersection:**

The set of elements that belong to _____

- **Complement:**

The set of elements (in the sample space) that are _____

So _____ all elements of A from the original sample space S .

- Example: Let $S = \{1, 2, 3, 4, 5, 6\}$ and $A_1 = \{1\}$, $A_2 = \{2, 3, 4\}$, $A_3 = \{4, 5, 6\}$. Find each of the following events:

$$A_1 \cup A_2 =$$

$$A_2 \cup A_3 =$$

$$A_1 \cap A_2 =$$

$$A_2 \cap A_3 =$$

$$\sim A_2 =$$

$$\sim(A_1 \cup A_2) =$$

Set identities

- These laws help simplify (rewrite) events that are stated verbally or in set notation when solving problems.

- **Commutative Law:** Reordering Order doesn't matter

$$A \cup B =$$

$$A \cap B =$$

- **Associative Law:**

Changing location of parentheses Order of operations with () doesn't matter

$$(A \cup B) \cup C =$$

$$(A \cap B) \cap C =$$

- **Distributive Law:** Distribute set operation Distribute to items in ()

$$A \cap (B \cup C) =$$

$$A \cup (B \cap C) =$$

- **De Morgan's Law:** Distributing complement (flip everything)

$$\sim(A \cup B) =$$

$$\sim(A \cap B) =$$

Relationships among sets

- Definitions

– Two events are **mutually exclusive** (or **disjoint**) if _____.

– The events A_1, A_2, \dots are **pairwise mutually exclusive** if _____ is mutually exclusive.

Formally, the condition is stated as: if $A_i \cap A_j = \underline{\hspace{1cm}}$ for all $i \neq j$.

– Events A_1, \dots, A_k are **exhaustive** if when combined, they form _____.

Formally, this is written as: $\bigcup_{i=1}^k A_i = A_1 \cup \dots \cup A_k =$

– Events A_1, \dots, A_k form a **partition** of S if they are _____ and _____ mutually exclusive.

- Examples: Given the sample space and events below, classify the relationship between events.

$$S = \{1, 2, 3, 4, 5, 6\} \quad \text{and} \quad A_1 = \{1\}, A_2 = \{2, 3, 4\}, A_3 = \{4, 5, 6\}, A_4 = \{5, 6\}.$$

(a) A_1 and A_2 are _____

(b) A_2 and A_3 are _____

(c) A_1, A_2, A_3 are _____

but are _____

(d) A_1, A_2, A_4 are _____

To check:

1. $A_1 \cup A_2 \cup A_4 = \{1\} \cup \{2, 3, 4\} \cup \{5, 6\} =$

2. $A_1 \cap A_2 = A_1 \cap A_4 = A_2 \cap A_4 =$