Name:

MATH 321: Homework 4

Due ____________________: Turn in a hard copy, neat and stapled.

1. Let $X \sim \text{Gamma}(\alpha, \beta)$. Find the MMEs for α and β.

2. Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Poisson}(\lambda)$. Find the MLE for $P(X = 0)$.

3. Let $X_1, \ldots, X_n$ be a random sample from $f(x \mid \theta) = \theta x^{\theta - 1} \quad 0 < x < 1,\ 0 < \theta < \infty$.
   (a) Find the MLE for θ.
   (b) Find the MME for θ.

4. Let $X_1, \ldots, X_4 \overset{iid}{\sim}$ (continuous) $\text{Uniform}(0, \theta), \quad 0 < \theta < \infty$.
   (a) Find the MLE for θ. We are going to "logic" our way to this MLE (without taking any derivatives).

   Find the likelihood function like usual. Then think about what the range of θ must be if we actually have collected data (say $\mathbf{x} = \{0.5, 0.1, 2, 3\}$). Then sketch a plot of the likelihood, and it should be clear what the MLE is :)

   (b) Show that $\hat{\theta}_{MLE}$ is a biased estimator of θ.
   (c) Find an unbiased estimator of θ as a function of $\hat{\theta}_{MLE}$.

Select answers

1. $\hat{\alpha} = \dfrac{\bar{X}^2}{v}$ and $\hat{\beta} = \dfrac{\bar{X}}{v}$
2. $MLE = \mathrm{e}^{-\bar{X}}$
3. (a) $MLE = \dfrac{-n}{\sum \ln(X_i)}$
   (b) $MME = \dfrac{\bar{X}}{1 - \bar{X}}$
4. (a)
   (b)
   (c)