5.5-1. Let $X_1, X_2, \dots, X_{16}$ be a random sample from a normal distribution $N(\mu = 77, \sigma = 25)$. Compute

(a) $P(77 < X_1 < 79.5)$. **(b)** $P(74.2 < \overline{X} < 78.4)$.

(c) Find $P\left(-0.1 < \frac{\bar{X}-77}{25/\sqrt{16}} < 2\right)$

(d) Find $P\left(-0.1 < \frac{\bar{X}-77}{S/\sqrt{16}} < 2\right)$

(e) Find $P\left(\frac{15}{625} S^2 < 10\right)$

(f) Find $P(S^2 < 1000)$

2) Let $X_1, \dots, X_4$ be a random sample from

$$f(x) = \frac{1}{30}(2x+1), 0 < x < 5$$

a) Find the cdf and pdf of $X_{(1)}$

b) Find $P(X_{(1)} < 1)$ using the cdf and $\mathrm{E}(X_{(1)})$ (use integral calculator)

c) Find $P(X_{(1)} < 1)$ using the pdf and the integral calculator (answer should match (b))

d) Find the cdf and pdf of $X_{(4)}$

e) Find $P(X_{(4)} > 4)$ and $\mathrm{E}(X_{(4)})$

f) Find the cdf and pdf of $X_{(2)}$

g) Find $P(1 \le X_{(2)} \le 3)$ and $\mathrm{E}(X_{(2)})$

9.74 Let $Y_1, Y, \dots, Y_n$ be a random sample of size n from the following pdf:

$$f(y \mid \theta) = \begin{cases} \left(\frac{2}{\theta^2}\right)(\theta - y), & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the method of moments estimator (MME) for θ

(b) If $\boldsymbol{x} = \{1.5,\ 0.2,\ 3.1,\ 7,\ 1.5,\ 2.25,\ 6,\ 8.2\}$, calculate the point estimate for $\hat{\theta}$

6.4-1. Let $X_1, X_2, \dots, X_n$ be a random sample from $Normal(\mu, \sigma^2 = 36)$. We are going to show that $\bar{\boldsymbol{X}}$ is the MLE for μ.

(a) Find the likelihood function $L(\mu \mid \boldsymbol{x})$ and the log-likelihood function $\ell(\mu)$

(b) Optimize the log-likelihood function

(c) Check the second derivative to confirm global max at $\hat{\mu} = \bar{X}$

6.4-10. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a geometric distribution for which p is the probability of success.

Use the alternate form of the geometric distribution that counts the number of failures before the first success:

$f(y_i \mid p) = (1-p)^{y_i} p,\ y_i = 0, 1, 2, \dots$

$E(Y) = \frac{1-p}{p}$

(a) Use the method of moments to find a point estimate for p.

(b) Find the MLE for p (skip the second derivative check)

(c) If $\boldsymbol{y} = \{1, 4, 2, 8, 9, 4, 10\}$, calculate the point estimate for the MME and MLE of p

6.4-9. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution whose pdf is $f(x;\theta) = (1/\theta)e^{-x/\theta}$, $0 < x < \infty$, $0 < \theta < \infty$.

(This is the scale parametrization of the exp dist)

(a) Find the MLE for θ

(b) Show that $\hat{\theta}$ is an unbiased estimator of θ (Can use integral calculator for $E(X)$)

(c) Find the MLE for $V(X) = \theta^2$

(d) Calculate the MLE for $V(X)$ if an observed random sample of $n = 5$ is $\boldsymbol{x} = \{3.5,\ 8.1,\ 0.9,\ 4.4,\ 0.5\}$

Answers

5.5-1

a) 0.0398
b) 0.2615
c) 0.5171
d) 0.5072
e) 0.1803
f) 0.9349

2

a)
b) P() = 0.2412, E() = 1.7991
c)
d)
e) P() = 0.8025, E() = 4.3947
f)
g) P() = 0.1781, E() = 3.704

9.74

a)
b) 11.156

6.4-1

6.4-10

a)
b)
c) MME = 0.1555 = MLE

6.4-9

a)
b)
c)
d) 12.1104