Name:

MATH 321: In-Class 4

1. Let $X_1, \ldots, X_m \overset{iid}{\sim} \text{Binomial}\,(n = 5, p)$.
   (a) Find the method of moments estimator for p.

   (b) Show that $\hat{p}_{MME}$ is an unbiased estimator.

2. Let $X_1, \ldots, X_n$ be a random sample from $f(x \mid \theta) = (\theta + 1)x^{\theta}, \quad 0 < x < 1, \theta > -1$.
   Find the MME of θ.

3. Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Bernoulli}\,(p)$. We are going to find the maximum likelihood estimator for p.
   (a) Find the likelihood function and log-likelihood function for p.

   (b) Optimize the log-likelihood function and solve for $\hat{p}$.

(c) Perform second derivative test to confirm if $\hat{p}$ is the MLE for p.

(d) Suppose we collected a random sample of size $n = 8$ and $\mathbf{x} = \{0, 1, 1, 1, 0, 1, 0, 0\}$. Compute $\hat{p}_{MLE}$.

(e) Now find the MLE for $V(X) = p(1 - p)$.