Name:

MATH 321: Homework 14

Due _____: Turn in a hard copy, neat and stapled.

1. A fair coin is tossed. If heads is tossed then one fair 4-sided die is thrown and if tails is tossed two fair 4-sided dice are thrown. Let $X = 1$ for heads and $X = 2$ for tails and let Y be the total number of dots on the dice.
   (a) Plot the range of the joint pmf of (X, Y), then find the corresponding joint probabilities.
   (b) Find the following probabilities: $P(X = Y)$, $P(2X < Y)$, and $P(X + Y \leq 7)$.
   (c) Find the marginal pmfs of X and Y, $f_X(x)$ and $f_Y(y)$, respectively.
   (d) Find the following probabilities: $P(X = 1)$ and $P(3 \leq Y \leq 5)$.

2. A basketball team has 3 players from Ohio, 5 from Indiana and 2 from Kentucky. Two of these players are selected at random for an interview. Let X be the random variable for the number of players from Ohio chosen and let Y be the random variable for the number of players from Indiana chosen.
   (a) Construct the joint pmf table for (X, Y).
   (b) Let $g_1(X, Y) = 2X$, $g_2(X, Y) = Y^2$ and $g_3(X, Y) = XY$.
   Find the expected values of each $g_i(X, Y)$, $i = 1, 2, 3$.

3. A home insurance company separates its claims into two parts: losses due to wind damage and losses due to water damage. If X is the random variable for losses due to wind damage and Y is the random variable for losses due to water damage,
$$f(x, y) = \frac{30 - x - y}{1875} \quad \text{for } 0 \leq x \leq 5, 0 \leq y \leq 25$$
   (a) If a claim is filed after a storm, find the probability that there is more loss due to water damage than wind damage.
   (b) Find the expected value of the total loss for a claim, i.e. wind damage plus water damage.

4. Let (X, Y) be a bivariate continuous random vector with joint pdf
$$f(x, y) = 2x \quad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1$$
   Find $P(X^2 < Y < X)$.

Select answers

1. (a)
   (b) $P(X + Y \leq 7) = 0.8125$
   (c) $P(3 \leq Y \leq 5) = 0.53125$
2. (a)
   (b) $E[g_3(X, Y)] = 1/3$
3. (a) Prob ≈ 0.8333
   (b) $E(X + Y) \approx 11.3889$
4. $P(X^2 < Y < X) = 1/6$