Name:

MATH 321: Homework 16

Due _____: Turn in a hard copy, neat and stapled.

1. An insurance company sells two types of auto insurance policies: Basic and Deluxe.

   The time until the next Basic Policy (X) claim is an exponential random variable with mean two days. The time until the next Deluxe Policy claim (Y) is an independent exponential random variable with mean three days.

   (a) Find the density function needed to solve $P(Y < X)$.

   (b) Calculate the probability from part (a).

   (c) Find the variance of the combined waiting time for the two different policy types $X + Y$.

2. Let $f(x, y) = \dfrac{4(1 - xy)}{3}$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

   (a) Show if X and Y are independent or dependent.

   (b) Find $\text{Cov}(X, Y)$.

3. A joint density function is given by

$$f(x, y) = kx \quad 0 < x < 1, 0 < y < 1$$

   where k is a constant.

   (a) Find $\text{Corr}(X, Y)$.

   (b) Find $f(x \mid y)$.

4. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a month. The following information is known about X and Y.

$$E(X) = 50 \quad V(X) = 50 \quad E(Y) = 20 \quad V(Y) = 30 \quad \text{Cov}(X, Y) = 10$$

   (a) Find $\text{Corr}(X, Y)$.

   (b) Let X' and Y' now represent the number of minutes spent watching movies and sporting events respectively. Find $\text{Cov}(X', Y')$.

   (c) Find $\text{Corr}(X', Y')$.

   (d) Find $V(X' + Y')$.

Select answers

1. (a)
   (b) $P(Y < X) = 0.4$
   (c) $V(X + Y) = 13$
2. (a)
   (b) $\text{Cov}(X, Y) = -1/81$
3. (a)
   (b)
4. (a) $\text{Corr}(X, Y) \approx 0.2582$
   (b) $\text{Cov}(X', Y') = 36,000$
   (c)
   (d) $V(X' + Y') = 360,000$