Distributions

Discrete Distributions	
Discrete uniform (N_0, N_1)	
Pmf	$P(X = x \mid N_0, N_1) = \frac{1}{N_1 - N_0 + 1}; \qquad x = N_0, \ldots, N_1; \qquad N_0 \leq N_1$
Mean and Variance	$E(X) = \frac{N_0 + N_1}{2}, \qquad V(X) = \frac{(N_1 - N_0 + 1)^2 - 1}{12}$
Mgf	$M_X(t) = \frac{1}{N_1 - N_0 + 1} \sum_{x=N_0}^{N_1} \mathrm{e}^{tx}$
Notes	
Bernoulli (p)	
Pmf	$P(X = x \mid p) = p^x (1-p)^{1-x}; \qquad x = 0, 1; \qquad 0 < \mathrm{p} < 1$
Mean and Variance	$E(X) = p, \qquad V(X) = p(1-p) = pq$
Mgf	$M_X(t) = (1-p) + p\mathrm{e}^t = q + p\mathrm{e}^t$
Notes	Special case of binomial with $n = 1$.
Binomial (n, p)	
Pmf	$P(X = x \mid n, p) = \binom{n}{x} p^x (1-p)^{n-x}; \qquad x = 0, 1, \ldots, n; \qquad 0 < p < 1$
Mean and Variance	$E(X) = np, \qquad V(X) = np(1-p) = npq$
Mgf	$M_X(t) = (q + p\mathrm{e}^t)^n$
Notes	Sum of *iid* bernoulli RVs.
Geometric (p)	
Pmf	$P(X = x \mid p) = q^{x-1} p; \qquad x = 1, 2, \ldots; \qquad 0 < p < 1$
Cdf	$F_X(x \mid p) = 1 - q^x$
Mean and Variance	$E(X) = \frac{1}{p}, \qquad V(X) = \frac{1-p}{p^2} = \frac{q}{p^2}$
Mgf	$M_X(t) = \frac{p\mathrm{e}^t}{1 - q\mathrm{e}^t}; \qquad t < -\ln(q)$
Notes	Special case of negative binomial with $r = 1$. * See other geometric probabilities. Alternate form $Y = X - 1$. This distribution is *memoryless*: $P(X > s \mid X > t) = P(X > s - t); \qquad s > t$.

Negative binomial (r, p)	
Pmf	$P(X = x \mid r, p) = P(X = x \mid r, p) = \binom{x-1}{r-1} p^r q^{x-r}; \qquad x = r, r+1, \ldots; \qquad 0 < p < 1$
Mean and Variance	$E(X) = \frac{r}{p}, \qquad V(X) = \frac{r(1-p)}{p^2} = \frac{rq}{p^2}$
Mgf	$M_X(t) = \left[\frac{pe^t}{1-qe^t}\right]^r; \qquad t < -\ln(q)$
Notes	Sum of *iid* geometric RVs.
Hypergeometric (N, M, K)	
Pmf	$P(X = x \mid r, p) = P(X = x \mid N, M, K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}; \qquad x = 0, 1, \ldots, \min(M, K)$
Mean and Variance	$E(X) = K\left(\frac{M}{N}\right), \qquad V(X) = K\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-K}{N-1}\right)$
Mgf	
Notes	If do not require $M \geq K$, $\mathcal{X} = \{\max(0, K+M-N), \ldots, \min(M, K)\}$, mean and variance converge to that of binomial $(n = K, p = M/K)$ when $N \to \infty$.
Poisson (λ)	
Pmf	$P(X = x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0, 1, 2, \ldots; \qquad \lambda > 0$
Mean and Variance	$E(X) = \lambda, \qquad V(X) = \lambda$
Mgf	$M_X(t) = e^{\lambda(e^t - 1)}$
Notes	If $X_i \overset{\perp\!\!\!\perp}{\sim}$ Poisson (λ_i), then $\sum X_i \sim \text{Poisson}(\lambda = \sum \lambda_i)$.

Other geometric probabilities

- Let $X \sim \text{Geometric}(p)$.

$$
\begin{aligned}
P(X < \infty) &= 1 \\
P(X > x) &= q^x \\
P(X \geq x) &= q^{x-1} \\
P(a < X \leq b) &= q^a - q^b \\
P(a \leq X \leq b) &= q^{a-1} - q^b
\end{aligned}
$$

Continuous Distributions

Continuous uniform (a, b)

Pdf	$f(x \mid a,b) = \frac{1}{b-a}, \qquad a \le x \le b; \qquad a,b \in \mathbb{R}, \qquad a \le b$
Cdf	$F(x) = \frac{x-a}{b-a} \qquad a \le x \le b$
Survival	$S(t) = \frac{b-t}{b-a} \qquad a \le t \le b \qquad$ if $T \sim \text{Uniform}\,(a,b)$
Mean and Variance	$E(X) = \frac{a+b}{2}; \qquad V(X) = \frac{(b-a)^2}{12}$
Mgf	$M_X(t) = \frac{\mathrm{e}^{tb} - \mathrm{e}^{ta}}{t(b-a)} \qquad t \neq 0$
Notes	

Exponential (λ)

Pdf	$f(t \mid \lambda) = \lambda \mathrm{e}^{-\lambda t}, \qquad t \ge 0; \qquad \lambda > 0$
Cdf	$F(t) = 1 - \mathrm{e}^{-\lambda t} \qquad t \ge 0$
Survival	$S(t) = \mathrm{e}^{-\lambda t} \qquad t \ge 0$
Mean and Variance	$E(X) = \frac{1}{\lambda}; \qquad V(X) = \frac{1}{\lambda^2}$
Mgf	$M_X(t) = \frac{\beta}{\beta - t} \qquad t < \beta; \qquad$ if $T \sim \text{Exp}\,(\beta)$
Notes	Special case of gamma with $\alpha = 1, \beta$. This distribution is *memoryless*: $P(T > a+b \mid T > a) = P(T > b); \qquad a,b > 0.$ Alternate parameterization is with scale $\theta = 1/\lambda$.

Gamma (α, β)

Pdf	$f(x \mid \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\, x^{\alpha-1}\, \mathrm{e}^{-\beta x}, \qquad x \ge 0; \qquad \alpha, \beta > 0$
Cdf	N/A
Mean and Variance	$E(X) = \frac{\alpha}{\beta} \qquad V(X) = \frac{\alpha}{\beta^2}$
Mgf	$M_X(t) = \left(\frac{\beta}{\beta - t}\right)^\alpha \qquad t < \beta$
Notes	Sum of *iid* exponential RVs. A special case is exponential $(\alpha = 1, \beta)$. Alternate parameterization is with scale $\theta = 1/\beta$.

Normal (μ, σ^2)

Pdf	$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\, \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \qquad -\infty < x < \infty; \qquad -\infty < \mu < \infty, \quad \sigma > 0$
Cdf	N/A
Mean and Variance	$E(X) = \mu, \qquad V(X) = \sigma^2$
Mgf	$M_X(t) = \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right]$
Notes	Special case: Standard normal $Z \sim \text{Normal}\,(\mu = 0, \sigma^2 = 1)$.

Lognormal (μ, σ^2)

Pdf	$f(y \mid \mu, \sigma^2) = \frac{1}{y\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\ln(y)-\mu)^2}{2\sigma^2}\right]; \qquad y \geq 0; \qquad -\infty < \mu < \infty; \qquad \sigma > 0$
Mean and Variance	$E(Y) = \mathrm{e}^{\mu + \frac{\sigma^2}{2}}, \qquad V(Y) = \mathrm{e}^{2\mu+\sigma^2}(\mathrm{e}^{\sigma^2} - 1)$
Mgf	
Notes	If $Y \sim$ Lognormal $\Longrightarrow \ln(Y) \sim$ Normal (μ, σ^2); equivalently, if $X \sim$ Normal (μ, σ^2) and $Y = \mathrm{e}^X \Longrightarrow Y \sim$ Lognormal. μ and σ^2 represent the mean and variance of the normal random variable X which appears in the exponent.

Beta (α, β)

Pdf	$f(x \mid \alpha, \beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}; \qquad 0 \leq x \leq 1; \qquad \alpha, \beta > 0$
Mean and Variance	$E(X) = \frac{\alpha}{\alpha+\beta}, \qquad V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Mgf	
Notes	$B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,\mathrm{d}x = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

Chi-square, $\boldsymbol{\chi^2}$ (r)

Pdf	$f(x \mid r) = \frac{1}{\Gamma(\frac{r}{2})2^{r/2}} x^{\frac{r}{2}-1} \mathrm{e}^{-\frac{x}{2}}, \qquad x \geq 0; \qquad r = 0.5, 1, 1.5, 2, \ldots$
Cdf	N/A
Mean and Variance	$E(X) = r, \qquad V(X) = 2r$
Mgf	$M_X(t) = \left(\frac{\theta}{\theta - 2t}\right)^{r/2} \qquad t < 1/2$
Notes	Special case of (scale) gamma with $\alpha = r/2, \theta = 2$.

$\boldsymbol{t}$ (r)

Pdf	$f(t \mid r) = f_T(t) = \frac{\Gamma(\frac{r+1}{2})}{\frac{1}{\sqrt{r\pi}}\Gamma(\frac{r}{2})}\left(\frac{1}{(1+t^2/r)^{(r+1)/2}}\right), \qquad -\infty < t < \infty$
Cdf	N/A
Mean and Variance	$E(T) = 0 \quad \text{if } r > 1, \qquad V(X) = \frac{r}{r-2} \quad \text{if } r > 2$
Mgf	N/A
Notes	See derivation notes above.

$\boldsymbol{F}$ (r_1, r_2)

Notes	See derivation notes above.