4.1-3. Let the joint pmf of X and Y be defined by

$$f(x,y) = \frac{x+y}{32}, \qquad x = 1,2, \quad y = 1,2,3,4.$$

(a) Find $f_X(x)$, the marginal pmf of X.

(b) Find $f_Y(y)$, the marginal pmf of Y.

(c) Find $P(X > Y)$.

(d) Find $P(Y = 2X)$.

(e) Find $P(X + Y = 3)$.

(f) Find $P(X \leq 3 - Y)$.

(g) Are X and Y independent or dependent? Why or why not?

(h) Find the means and the variances of X and Y.

(i) Find Cov(X,Y)

(j) Find Cov(6X, –Y + 1)

(k) Find Corr(X,Y)

(l) Find E(Y | X = 2) and P(X = 1 | Y = 1)

4.4-18. Let $f(x,y) = 1/8, 0 \leq y \leq 4, y \leq x \leq y + 2$, be the joint pdf of X and Y.

(a) Sketch the region for which $f(x,y) > 0$.

(b) Find $f_Y(y)$, the marginal pdf of Y.

(c) Find P(X > 3, Y > 3)

(d) P(Y < 2)

Answers

4.1-3

(a) (2x + 5)/16

(b) (2y + 3)/32

(c) 3/32

(d) 9/32

(e) 6/32

(f) 8/32

(g) Dependent

(h) $E(X) = 1.5625, V(X) \approx 0.2461$
$E(Y) = 2.8125, V(Y) \approx 1.1523$

(i) -0.0195

(j) ≈ 0.117

(k) ≈ -0.0366

(l) E(Y | X = 2) = 2.778, P(X = 1 | Y = 1) = 2/5

4.4-18

(a) Looks like a slanty rectangle

(b) 1/4

(c) 1/4

(d) 0.5