Name:

MATH 321: In-Class 2

1. Suppose $X_1, X_2 \overset{iid}{\sim} f(x) = 3x^2 \quad 0 < x < 1$.
   (a) Find the survival function of $X_{(1)} = min(X_1, X_2)$.

   (b) Find the cdf of $X_{(1)} = min(X_1, X_2)$.

   (c) Find the cdf of $X_{(2)} = max(X_1, X_2)$.

2. Let X_1, X_2, X_3 be a random sample from Exponential ($\lambda = 2$).
   (a) Find the cdf of $X_{(3)} = max(X_1, X_2, X_3)$.

   (b) Find the pdf of $X_{(3)} = max(X_1, X_2, X_3)$ by taking the derivative of part (a).

   (c) Find the pdf of $X_{(3)} = max(X_1, X_2, X_3)$ using the pdf theorem (answer should match (b)).

(d) Find the cdf of $X_{(1)} = min(X_1, X_2, X_3)$. *HINT: Can start with the cdf written as a probability statement and then think about it with logic to continue (and use a complement).*

(e) Find $P(X_{(1)} < 1.5)$ and $P(X_{(3)} < 1.5)$.

(f) Find the cdf and the pdf of the sample median $X_{(2)}$ using the theorems.

3. Use R to create a qqplot using the following steps:

   (a) Generate (and save) a random sample of size $n = 100$ from $X \sim t\,(2)$.

   (b) Run the following two lines:

   qqnorm(< your sample >)

   qqline(< your sample >)

   (c) Roughly sketch the result, which is visually testing whether a t distribution matches the characteristics of a normal distribution.

   Is there a pattern? Draw / trace it. What does this pattern tell you about the t-distribution?