Name:

MATH 321: In-Class 1

1. Suppose $X_i \overset{iid}{\sim} \text{Poisson}\,(\lambda = 5)$, for $i = 1, \ldots, 30$.

   (a) Find the joint pmf $f(x_1, \ldots, x_{30})$.

   (b) Let $\bar{X} = \dfrac{1}{30}\displaystyle\sum_{i=1}^{30} X_i$. Find $E(\bar{X})$ and $V(\bar{X})$.

   (c) Find the distribution of $\bar{X}$.

2. Let $Z_1, \ldots, Z_9$ be independent. Also assume $Z_i \sim \text{Normal}\,(0, 1)$ for $i = 1, \ldots, 9$.

   (a) If $Y_1 = Z_1^2 + \ldots + Z_9^2$ find $P(Y_1 < 15)$.

   (b) If $Y_2 = Z_3^2 + \ldots + Z_9^2$ find $P(2 < Y_2 < 13)$.

   (c) If $Y_3 = Z_3^2 + \ldots + Z_5^2$ find $P(Y_3 > 12)$.

3. (a) Suppose $X \sim \text{Normal}\,(\mu = 100, \sigma = 25)$. Find the 90^{th} percentile of X (i.e. find $x_{0.9}$ such that $P(X < x_{0.9}) = 0.90$).

   HINT: Use Z-table "backwards" or invNorm() on graphing calculator.

(b) Now use qnorm() in R to find the 90^{th} percentile of X. Answer should match part (a).

(c) Suppose $Y \sim \chi^2(15)$. Find the 30^{th} percentile of Y.

(d) Suppose $T \sim \mathrm{t}(10)$. Find the *upper* 25^{th} percentile of T.

4. Let $X_1, \ldots, X_{20}$ be a random sample from Normal($\mu = 30, \sigma^2 = 100$) and let $\bar{X}$ be the sample mean and S be the sample standard deviation.

   (a) Find $P(-2 < \dfrac{\bar{X} - 30}{S/\sqrt{20}} < 2)$.

   (b) Find $P(-2 < \dfrac{\bar{X} - 30}{10/\sqrt{20}} < 2)$.