Name:

MATH 321: Review Part 2

1. Let X_1 and X_2 be independent random variables where $X_1 \sim \text{Normal}\,(\mu = 1, \sigma = 2)$, and $X_2 \sim \text{Normal}\,(\mu = 5, \sigma = 3)$.

   Find $P(3X_1 - X_2 < 2)$.

2. Let $X_{(1)}, \dots, X_{(8)}$ be the order statistics of a random sample of size $n = 8$ from a random variable X with pdf

$$f(x) = \frac{3}{4}(x^2 + 1), \quad 0 \le x \le 1$$

   (a) Find the cdf of X, $F_X(x)$, and $P(X \le 0.25)$.

   (b) Find the cdfs $F_{X_{(1)}}(x)$ and $F_{X_{(8)}}(x)$.

   (c) Find the pdfs $f_{X_{(1)}}(x)$ and $f_{X_{(8)}}(x)$.

(d) Find the cdf $F_{X_{(6)}}(x)$

(e) Find $P(X_{(6)} \leq 0.25)$.

(f) Find the pdf $f_{X_{(5)}}(x)$.

(g) Write the integrals to find $E(X_{(5)})$ and $P(X_{(5)} \geq 0.75)$

3. Let $X_1, \dots, X_n$ be a random sample from $f(x \mid \theta) = \theta x, \quad 0 < x < 1,\ \theta > 0$.

   (a) Find the MME of θ.

   (b) Show if $\hat{\theta}_{MME}$ is unbiased.

4. Let $X_1, \dots, X_n$ be a random sample from $f(x \mid \theta) = (\theta + 1)x^{\theta}, \quad 0 < x < 1,\ \theta > -1$.

   (a) Find the MLE of θ.

   (b) Find the MLE of $E(X) = \dfrac{\theta + 1}{\theta + 2}$.

5. Let $X_1, \ldots, X_{35}$ be a random sample from a Geometric $(p = 0.45)$ experiment.

   (a) Let Y_1 be the average number of trials to get the first success in the $n = 35$ runs of this experiment, i.e. $Y_1 = \bar{X}$. Find the distribution of Y_1.

   (b) Let Y_2 be the total number of runs (across all trials) to get the first success in $n = 35$ trials of this experiment, i.e. $Y_2 = \sum_{i=1}^{35} X_i$. Find the distribution of Y_2.

6. Let $X_1, \ldots, X_{40}$ be a random sample from a $f(x)$, where $\mu = 15$ and σ^2 is unknown. Let $\bar{X}$ be the sample mean and S be the sample standard deviation.

   (a) Find the distribution of $\bar{X}$.

   (b) Find the distribution of $\dfrac{\bar{X} - 15}{S/\sqrt{40}}$.

   (c) Suppose we know the population variance $\sigma^2 = 4$. Find the distribution of $\dfrac{\bar{X} - 15}{2/\sqrt{40}}$.