	Homework #1 Solutions
(0,12)	By the chain rule for total derintives
	$D(G \circ F)(x) = DG(F(x)) \circ DF(x)$
	In matrix form, we can write this as
	In matrix form, we can write this as $ \left[\frac{\partial (G \cdot F)^{2}(x)}{\partial x^{2}}(x)\right] = \left[\frac{\partial G^{2}(F(x))}{\partial y^{2}}\right] \left[\frac{\partial F^{k}(x)}{\partial x^{2}}(x)\right] $
	Sina (GoF) = GOF, the (i, j) the entry of
	Knis matrix equation says
	Muis matrix equation says $\frac{\partial G' \circ F}{\partial x^{i}}(x) = \sum_{k=1}^{\infty} \frac{\partial G'}{\partial y^{k}} \left(F(x) \right) \frac{\partial F^{k}}{\partial x^{i}}(x)$
	8x' k=1
	b) The chain rule says that if F and G are differentiable
	then so is GoF. By induction, if F and G are
	Ck, so is GoF. Therefore if Fand G are smooth,
	then GoF is Ck for all k30, and hence smooth
C. 39)	Polar coordinates: The map F: (0,00) x (-17, 17) -> 12°
·	defined by
	$F(r, \theta) = (r \cos \theta, r \sin \theta)$
	is Co, and a bijection from (0,00) x (-17,77) to
	$\mathbb{R}^2 - \{(x, 0) \mid x \leq 0\}$. Its Jacobian is $\mathbb{R}^2 - \{(x, 0) \mid x \leq 0\}$. Its Jacobian is
`	I SIND Y CASO J
	which has determinant r x0, so by Cor C.36 it is a
-	distromorphism (Its inverse map is
i i	$F'(x,y) = \left(\sqrt{x^2 + y^2}, \frac{1}{\sqrt{109}}\left(\frac{x + \sqrt{3}}{\sqrt{x^2 + y^2}}\right)\right)$
	I for an appropriately chosen branch of the log truction on
	(-) = (Re(2) < 0, Im(2) = 0 }

Spherical coordinates: The map $G(\rho, \Psi, \theta) = (\rho \sin \Psi \cos \theta, \rho \sin \Psi \sin \theta, \rho \cos \Psi)$ is Con and a bijection from $G(\rho, \Psi, \theta) : \rho > 0, 0 < \Psi < \Pi, -\Pi < \theta < \Pi^{2}$ onto its image. Its Jacobian is $Sin \Psi \cos \theta \rho \cos \Psi \cos \theta -\rho \sin \Psi \sin \theta$ $Sin \Psi \sin \theta \rho \cos \Psi \sin \theta \rho \sin \Psi \cos \theta$ $\cos \Psi -\rho \sin \Psi 0$
G(p, f, 0) = (p sin f cos 0, p sin f sin 0, p cos f) is Con and a bijection from S(p, f, 0): p>0, 0< f< T, -T< 0< Tf onto its image. Its Jacobian is Sinf cos 0 p cos f cos 0 - p sin f sin 0 J= sin f sin 0 p cos f sin 0 p sin f cos 0 cos f - p sin f
is Command a bijection from \[\left(\rho, \phi \theta) : \rho > 0, 0 < \ph < \pi, -\pi < 0 < \pi \right] \[\text{onto its image. Its Jacobian is} \] \[\text{Sin P cos 9} \text{p cos P cos 9} - \rho \text{sin 9} \\ \text{J = sin Y sin 0} \text{p cos P sin P cos 0} \] \[\text{cos P} - \text{p sin P} \]
onto its image. Its Jacobian is [sint cos 9 prost cos 0 - psint sin 0] To sint sin 0 prost sin 0 psint cos 0] cos 4 - psint 0
onto its image. Its Jacobian is [sint cos 0 p cos 4 cos 0 - p sint sin 0] T= sint sin 0 p cos 4 sin 0 p sin 4 cos 0 cos 4 - p sin 4
J= sint cos 9 p cos 4 cos 0 - p sint sin 0 J= sint sin 0 p cos 4 sin 0 p sin 4 cos 0 cos 4 - p sin 4 D
J= siny sind p cosy sind p siny coso cosy - p siny
L cos P - psin P D J
which has determinant print to on the given
domain. So by Cor. C.36 it is a diffeomorphism.
Additional) Le can write [gu gin [b']
Problem] q(v, v): [a' an]
gn gn Ib
= the Aquib
Under the given change of basis we have
Under the given change of basis, we have $\widetilde{a} = R^{-1}\widetilde{a}$ $\widetilde{b} = R^{-1}\widetilde{b}$
n a Rã. To RÃ
The matrix \overrightarrow{A}_g should satisfy
$g(\overline{v}, \overline{z}) = \overline{a}^{t} \overline{A}_{y} \overline{b} = \overline{a}^{t} A_{y} \overline{b}$
= ãt RtA, Rã

9	for all victors & E. Therefore
	Ã: RtA.R.
	for all victors & & Therefore \$\bar{A}_g = R^t A_g R_g\$ Contrast with a linear transformation T:V->V
77.7	which would transform as
	$\widetilde{A}_{r} = R' A_{r} R_{r}$
THE PARTY OF THE P	So no, a metric is not the same kind of object
7.2277	as a linear transformation, because they transform
	differently under a charge of basis for V.
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