MATH 120 PRACTICE MIDTERM

Give complete proofs unless otherwise indicated. Each question is worth 6 points. They are not in order of difficulty.

1. For this question, give answers only.
(a) Find the order of the element $(12)(13)(14)$ in S_4.
(b) Which of the following groups are isomorphic: D_6, S_3, $\mathbb{Z}/6$?
(c) Which of the following is true? (i) For all subsets A of a group G, the centralizer $C_G(A)$ is always contained in the normalizer $N_G(A)$. (ii) For all subsets A of a group G, the centralizer $C_G(A)$ always contains in the normalizer $N_G(A)$. (iii) Neither (i) nor (ii) is true.

2. Prove that the subgroup generated by 9 and 6 in $\mathbb{Z}/(24\mathbb{Z})$ is cyclic, and find its order.

3. Determine whether the following subsets of the group $GL_n(\mathbb{R})$ are subgroups. (Here $GL_n(\mathbb{R})$ is the set of invertible $n \times n$ matrices with real entries.)
(a) those elements with rational entries.
(b) those elements with integer entries.

4. Suppose $\phi : G \to H$ is a group homomorphism. Suppose $H' < H$ is a subgroup of H. Show that $\phi^{-1}(H')$ (those elements of G mapped by H into H") is a subgroup of G.

5. Show that $\mathbb{Z}/7\mathbb{Z}$, the integers modulo 7, form a field (with the usual addition and multiplication).

6. Show that there is only one group homomorphism from $\mathbb{Q}$ to $\mathbb{R}$ sending 1 to 2. (The operation for both $\mathbb{Q}$ and $\mathbb{R}$ is addition.)

Date: October 4, 2010.