ACTIONS AND SUBGROUPS

COLTON GRAINGER (MATH 6130 ALGEBRA)

3. Assignment due 2018-09-19

3.1. **[1, No. 2.1.8].** Let H and K be subgroups of G. We have that $H \cup K$ is a subgroup if and only if either $H \subset K$ or $K \subset H$.

3.2. **[1, No. 2.1.9].** Let $G = GL_n(\mathbf{F})$ where $\mathbf{F}$ is an field. We define the *special linear group*

$$SL_n(\mathbf{F}) = \{A \in GL_n(\mathbf{F}) : \det(A) = 1\}.$$

We have that $SL_n(\mathbf{F}) \leq GL_n(\mathbf{F})$.

3.3. **[1, No. 2.1.14].** The set $\{x \in D_{2n} : x^2 = 1\}$ is not a subgroup of D_{2n} (where $n \geq 3$).

3.4. **[1, No. 2.2.6].** Let H be a subgroup of the group G.

(a) $H \leq N_G(H)$. We show that this is not necessarily true if H is not a subgroup.

(b) $H \leq C_G(H)$ if and only if H is abelian.

3.5. **[1, No. 2.2.10].** Let H be a subgroup of order 2 in G. Then $N_G(H) = C_G(H)$.

If $N_G(H) = G$ then $H \leq Z(G)$.

3.6. **[1, No. 2.2.12].** Let R be the set of all polynomials with integer coefficients in the independent variables $\{x_j\}_1^4$. That is, members of R are finite sums of elements of the form $ax_1^{r_1}x_2^{r_2}x_3^{r_3}x_4^{r_4}$ where $a \in \mathbf{Z}$ and $r_j \in \mathbf{Z}_{\geq 0}$.

Each $\sigma \in S_4$ gives a permutation of $\{x_1, x_2, x_3, x_4\}$ by defining $\sigma \cdot x_j = x_{\sigma(j)}$. This extends naturally to a map from R to R by defining

$$\sigma \cdot p(x_1, x_2, x_3, x_4) = p(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}, x_{\sigma(4)})$$

for all $p(x_1, x_2, x_3, x_4) \in R$ (that is, σ simply permutes the indices of the variables).

(a) Let $p = p(x_1, x_2, x_3, x_4)$ be the polynomial

$$12x_1^5x_2^7x_4 - 18x_2^3x_3 + 11x_1^6x_2x_3^3x_4^{23}$$

3.7. **[1, No. 2.3.25].**

3.8. **[1, No. 2.4.3].**

3.9. **[1, No. 2.4.12].**

3.10. **[1, No. 2.4.15].**

3.11. **[1, No. 2.4.16].**

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3.12. **Maximal subgroups in a finite group.** A finite group with no more that two maximal subgroups is cyclic.

References

[1] D. S. Dummit and R. M. Foote, *Abstract algebra*, 3rd ed. Hardcover; Prentice Hall, 2004 [Online]. Available: http://www.worldcat.org/isbn/0471433349