MATH 120 PRACTICE MIDTERM

1. *(6 points)*

(a) What is the order of A_4?
(b) How many rotations of the cube have order exactly 2 (i.e. if you do them twice, you get the identity, but they are not the identity)? Possible hint: we have seen that the group of rotations of the cube is isomorphic to S_4.
(c) Which of the following is true?
 (i) For all subsets A of a group G, the centralizer $C_G(A)$ is always contained in the normalizer $N_G(A)$.
 (ii) For all subsets A of a group G, the centralizer $C_G(A)$ always contains in the normalizer $N_G(A)$.
 (iii) Neither (i) nor (ii) is true.

2. *(6 points)* Suppose G acts on a set A, and a and $b \in A$ are in the same orbit of G. Show that G_a, the stabilizer of a, is conjugate to G_b, the stabilizer of b.

3. *(6 points)* Suppose $H \leq K \leq G$. Prove that $|G : H| = |G : K| \cdot |K : H|$. (Do not assume the groups are finite!)

4. *(6 points)* Let G be any group.

(a) Prove that the map $G \to G$ defined by $g \mapsto g^2$ is a homomorphism if and only if G is abelian.
(b) If G is abelian and finite show that this map is an isomorphism if and only if G has odd order.

5. *(6 points)* Suppose G is group of order p^2q, where p and q are distinct prime numbers. Show that G is not simple. (This is in the text, so of course do not just quote the text!)

6. *(6 points)* Suppose that H_1 and H_2 are groups of finite index in G. Show that $H_1 \cap H_2$ is also of finite index in G.

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