MATH 6140: Second midterm examination. Wednesday, 21 March 2007.

Put **your name** on each answer sheet. Answer **all three** questions.

Show your working in full.

Formula sheets, calculators, notes and books are not permitted.

1. Let V be a 5-dimensional vector space over a field F. Prove (quoting any standard results that you need) that the exterior algebra $\bigwedge(V)$ has dimension 32 as an F-vector space.

2. Let
$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$
be elements of $M_2(\mathbb{Q})$. Prove that there is no 2×2 invertible matrix P with entries from $\mathbb{Q}$ such that $P^{-1}AP = B$. Is there a 2×2 invertible matrix with entries from $\mathbb{C}$ such that $P^{-1}AP = B$?

3. Let $F = \mathbb{Q}$ and $K = \mathbb{Q}(\sqrt{5}, i)$ be subfields of $\mathbb{C}$. Find the degree of K over F, and write down a basis for K as an F-vector space. Is K algebraic over F?