1. Using the mean value theorem, prove that if a function f is differentiable on (a, b) and $f'(x) = 0$ for all $x \in (a, b)$, then f is a constant function on (a, b).

2. Taylor series practice: Compute the Taylor series (around $x = 0$, expressed in closed summation form) for the polynomial $f(x) = e^{-x}$. (Do the whole Taylor series process for this. Afterwards, see if you can come up with an easy way to come up with this Taylor series given the Taylor series for e^x, which you might have memorized.)

3. Use the remainder formula (Taylor's Theorem with Remainder) to prove that $e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + ...$ (Do this without explicitly referencing the Taylor series for e^x.)

4. Calculate the derivative of inverse of $f(x)$ in the following cases, you may assume the result from proof 8.2, and that the inverse functions are differentiable.

   (a) $f(x) = cos(x)$

   (b) $f(x) = tan(x)$

   (c) $f(x) = e^x$

5. Use Taylor's theorem with remainder and the function $f(x) = \sqrt{1 + x}$ to show that

$$\lim_{n \to \infty} \sqrt{n + \sqrt{n}} - \sqrt{n} = \frac{1}{2} \tag{1}$$

   (Source:mathcs.org)