1. Affine approximation and Newton's Method

   (a) Beginning with the formula for affine approximation (from Week 9), derive the formula for Newton's Method (want to approximate the zeroes of a function)

   (b) After a magical Boston snowfall, you and your pset buddies decide to have a snowball fight. Your team wants to practice throwing snowballs for x hours and construct a snowball-fight strategy plan for y hours. You are subject to the following constraints: First, you can prepare only for a total of 8 hours (so $x + y = 8$). Second, you decide that practicing throwing is more important than strategizing, so you create the constraint $x + y^3 = 27$ so that you'll spend a maximum of 3 hours strategizing. You guess that an approximate solution to these equations is $x = 6$, $y = 2$. Using **one iteration** of Newton's Method, improve your estimate.

2. Non-differentiable functions. Consider the function f given by:

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} 0 & \text{if } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \frac{xy^2}{x^2+y^2} & \text{otherwise} \end{cases}$$

   (a) Using the definition of the directional derivative (from last week), calculate $\nabla_{\vec{e_1}} f \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\nabla_{\vec{e_2}} f \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

   (b) If the derivative exists, what should the directional derivative along $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ be? Why?

   (c) Calculate the directional derivative from (b). What can you conclude and why?

3. What is a manifold? A highly non-rigorous way to describe it is that "a manifold is subset of $\mathbb{R}^n$ where, if you zoom in far enough, it looks flat." Let's try it out!

4. Ways to describe manifolds (k-dimensional manifold in $\mathbb{R}^n$)

   (a) **Graph-making function:** the *graph* $\tilde{g}$ of a graph-making function g that takes in k active variables and outputs $n - k$ passive variables

   (b) **Parametrization:** a map $\gamma : \mathbb{R}^k \to \mathbb{R}^n$ that takes in k parameters and returns a point on the manifold in n-dimensional space

   (c) **Locus function:** a map $F : \mathbb{R}^n \to \mathbb{R}^{n-k}$ that imposes constraints (lowering your degrees of freedom). A manifold is specified by the set of points hwere these functions are 0

5. True/False

   (a) Using enough iterations of Newton's mthod, any initial guess $\vec{x_0}$ will let you approximate some zero of a function f.

   (b) Any finite collection of points in $\mathbb{R}^2$ is a smooth manifold.

   (c) For any functions $f, g : \mathbb{R}^m \to \mathbb{R}^n$, $[D(f \circ g)(t)] = [D(f(g(t)))]$.

   (d) If all partials of a function $f : \mathbb{R}^m \to \mathbb{R}^n$ are continuous at a point $\vec{x}$, then f is differentiable at $\vec{x}$.