1. Road map of the module
2. Heine-Borel hip hip hooray
   (a) What is an open cover of a set S? What is a finite subcover?
   (b) Rather than hire a normal company to do your cake decoration, you decide to hire Heine-Borel Cake Decorators to frost your one-dimensional birthday cake, which extends across the interval [0,1]. HBCD offers you a few different cake-decorating plans, but you're worried that they might not finish frosting the cake in time.
      i. Plan 1: During hour -1, your cake will be frosted on the interval $\left(\frac{1}{3}, 1\right]$. During hour 0, your cake will be frosted on $[0, 0.00001)$. During hour i after that, your cake will be frosted on the open interval $\left(\frac{1}{2^{i+1}}, \frac{1}{2^i}\right)$. Does this cake-frosting scheme form an open cover of your birthday cake? (Will everything get frosted?)
      ii. Plan 2: During hour -1, your cake will be frosted on the interval $\left(\frac{1}{3}, 1\right]$. During hour 0, your cake will be frosted on $[0, 0.00001)$. During hour i after that, your cake will be frosted on the open interval $\left(\frac{1}{2^{i+2}}, \frac{1}{2^i}\right)$. The way that this plan is set up, you'd have to wait infinitely long for the frosting to finish. Why don't you have to wait that long?
      iii. HBCD convinces you that your cake would look better if they only frosted on (0,1). How could they change plan 2 in this case to make you wait an infinite length of time before your cake is ready?
3. Affine approximation: Consider the function $f\begin{pmatrix} x \\ y \end{pmatrix} = x^4y^2$. Calculate the derivative of this function, evaluate it at and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and then use this to approximate $f\begin{pmatrix} 0.95 \\ 2.05 \end{pmatrix}$.
4. True/false
   (a) For any collection of sets S_n where $S_{n+1} \subseteq S_n$, $\cap_{i=1}^{\infty} S_n$ is nonempty
   (b) In $\mathbb{R}^n$, any convergent sequence is Cauchy.
   (c) In $\mathbb{R}^n$, any Cauchy sequence is convergent.
   (d) If a set $S \subseteq \mathbb{R}^n$ is not compact, then no open cover has a finite subcover.
   (e) In $\mathbb{R}^2$, if both $\lim_{x\to 0} f\begin{pmatrix} x \\ 0 \end{pmatrix}$ and $\lim_{y\to 0} f\begin{pmatrix} 0 \\ y \end{pmatrix}$ exist, then $\lim_{\vec{x}\to 0} f(\vec{x})$ exists.
   (f) If $\nabla_{\vec{e_1}} f(\vec{a}) = 1$ and $\nabla_{\vec{e_2}} f(\vec{a}) = 2$, then $\nabla_{\vec{e_1}+\vec{e_2}} f(\vec{a}) = 3$.
5. If time: Using the pigeonhole principle, prove that given any five points on a sphere, there is a closed hemisphere containing at least four of them.