1. Terminology
   (a) Linear independence: a set of vectors $\{\vec{v_1}, ..., \vec{v_n}\}$ is linearly independent if
   $$\sum_{i=1}^{n} a_i \vec{v_i} = 0 \leftrightarrow \forall i[a_i = 0]$$
   (b) Subspace: a subspace $S \subset V$ is a space that is closed under addition and scalar multiplication—i.e. if $x, y \in S$, so is $ax + by$ for any $a, b \in \mathbb{R}$
      i. Is the set of points (x, y) s.t. $x+y = 0$ a subspace? Prove that it is, or give a counterexample.
      ii. Is the set of points (x, y) s.t. $x+y = 1$ a subspace? Prove that it is, or give a counterexample.
      iii. Is the set of points (x, y) s.t. $y = \sin(x)$ a subspace? Prove that it is, or give a counterexample.
   (c) Span: the span of a set of vectors is the space of all vectors that can be expressed as linear combinations of those vectors, i.e. the set of all $\vec{w}$ s.t. $\exists[a_1, ..., a_n]$ s.t. $\sum_{i=1}^{n} a_i \vec{v_i} = \vec{w}$
   (d) Basis: a basis for a vector space V is a set of linearly independent vectors that span V
   (e) Dimension of a vector space: the number of vectors in any basis for that space (this well-defined!)
   (f) Orthonormal basis: a basis in which all the vectors are unit vectors, and each basis vector is orthogonal to all the other basis vectors
   (g) Image: the subspace of vectors that are possible outputs of a matrix T (a subspace of the **codomain**)
   (h) Rank: the rank of a matrix is the dimension of the image
      i. Can calculate the rank as either the number of independent rows or the number of independent columns! ($rank(A) = rank(A^T)$)
   (i) Kernel: the "zero space" of a matrix, the subspace of vectors that T maps to the zero vector (a subspace of the **domain**)
   (j) Nullity: the dimension of the kernel
      i. THE KERNEL IS NEVER EMPTY!

2. Find a basis for the image and kernel of matrix A below.
$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 0 \\ 3 & 6 & 0 & 3 & -3 \\ 0 & 0 & 2 & 2 & 2 \end{bmatrix}$$

3. True/false about images and kernels and ranks and functions
   (a) If A is an $n \times n$ matrix and $A\vec{x} = \vec{0}$, then $x = \vec{0}$.
   (b) If $A\vec{v} = A\vec{w}$, then $\vec{v} - \vec{w} \in ker(A)$.
   (c) If $m > n$, a function $f : \mathbb{R}^m \to \mathbb{R}^n$ cannot be one-to-one.
   (d) If $n > m$, a function $f : \mathbb{R}^m \to \mathbb{R}^n$ cannot be onto.
   (e) A function $f : \mathbb{R}^m \to \mathbb{R}^n$ is onto $\mathbb{R}^n$ if every vector in $\mathbb{R}^m$ maps onto some vector in $\mathbb{R}^n$.
   (f) All functions $f : \mathbb{R}^m \to \mathbb{R}^n$ map linearly independent vectors in $\mathbb{R}^m$ to linearly independent vectors in $\mathbb{R}^n$.
   (g) There exists a 2×2 matrix A such that $rank(A) = 0$.

(h) Let A and B be $n \times n$ matrices. If $\vec{v}$ is in $ker(B)$, then $\vec{v}$ is in $ker(AB)$.

(i) Let A and B be $n \times n$ matrices. If $\vec{v}$ is in $ker(A)$, then $\vec{v}$ is in $ker(AB)$.

(j) If a square matrix has two equal rows, then it is not invertible.

4. Using elementary matrices, find a vector not in the span of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

5. Walk through how Gram-Schmidt works in the 2-dimensional case