

## SOLUTION SET (FUNCTIONS AND LIMITS)

COLTON GRAINGER (MATH 1300)

- Consider the function  $f(x) := x^2 + 1$ . What is the polynomial describing  $f(f(x))$ ?

- (A)  $x^2 + 2$
- (B)  $x^4 + x^2 + 1$
- (C)  $x^4 + x^2 + 2$
- (D)  $x^4 + 2x^2 + 1$
- (E)  $x^4 + 2x^2 + 2$

*Answer:* Option (E). Explanation from Vipul Naik:

We have  $f(f(x)) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 1 + 1$ , which simplifies to option (E).

Option (A) is  $(x^2 + 1) + 1 = x^2 + 2$ . The error here is is not squaring  $x^2 + 1$ .

Option (D) is  $(x^2 + 1)^2 = x^4 + 2x^2 + 1$ . The error here is in forgetting to add 1.

Options (B) and (C) are like (D) and (E), with an error in the coefficient of  $x^2$ .

- If  $f(g(x)) = 5$  and  $f(x) = x + 3$  for all real  $x$ , then  $g(x) =$

- (A)  $x - 3$
- (B)  $3 - x$
- (C)  $\frac{5}{x+3}$
- (D) 2
- (E) 8

*Answer:* (D)  $g(x) = 2$ . Explanation:

With the constant function  $g(x) = 2$ , we evaluate  $f(g(x)) = f(2) = 5$ .

- For all positive functions  $f$  and  $g$  of the real variable  $x$ , let  $\sim$  be a relation defined by

$$f \sim g \text{ if and only if } \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1.$$

Which of the following is NOT a consequence of  $f \sim g$ ?

- (A)  $f^2 \sim g^2$
- (B)  $\sqrt{f} \sim \sqrt{g}$
- (C)  $e^f \sim e^g$
- (D)  $f + g \sim 2g$
- (E)  $g \sim f$

*Answer:* (C)  $e^f \sim e^g$ . Explanation from Charlie Rambo:

Let's find a counter example to  $e^f \sim e^g$ . Consider  $f(x) = x$  and  $g(x) = x + 1$ . Clearly  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1$ . But

$$\lim_{x \rightarrow \infty} \frac{e^x}{e^{x+1}} = \lim_{x \rightarrow \infty} \frac{1}{e} = \frac{1}{e} \neq 1.$$

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*Repo:* <https://github.com/coltongrainger/pro19ta>.

0.1. **References.**

- Vipul Naik, *Math 152 Week 1*. <https://vipulnaik.com/math-152/>.
- GRE Mathematics Test Form GR0568 and Form GR9367.