HOMEWORK 11 SELECTED SOLUTIONS

COLTON GRAINGER

(Prob 1) Is the limit of this quotient an indeterminate form? Yes. Is the denominator zero as x goes to infinity? No. See section 4.5. Apply L'Hopital's rule.

(Prob 3.a) Never true. The function has no global maximum value. It's strictly monotonically increasing on an open interval. Suppose for contradiction we had a maximum value. Do you see how you could find a value which is greater than the maximum value? This leads to a contradiction. So the function can have no maximum value.

(Prob 3.b) Sometimes it's true. I'm giving you full credit for either an argument involving a function that is weakly monotonically decreasing on a closed interval, or a function that is strictly monotonically increasing on the same interval.

(Prob 3.c) Never true. Consider the right endpoint. At this point the function achieves a global maximum value. (Careful! It's not a point where the first derivative is equal to 0.) Perhaps check with yourself: do you see at least two methods for determining that the function achieves a global maximum value at the right endpoint?

(Prob 4) I wrote up the solution as SAGE code.

Here's the permalink: `https://sagecell.sagemath.org/?q=osxqgn`

Input:

```
# HW11 problem 4
# declare w for width as a variable
var('w')

# here's the cost C as function of width w
# (assume w is a positive real number)
C(w) = 180/w + 20*w^2

# here's the first derivative of cost with respect to width
Cprime = diff(C,w)
print("first derivative", Cprime)

# we differentiated to minimize C, so find critical points
# verify! by! hand!
critpts = solve(Cprime == 0, w)

# take the real root (complex roots are not in the domain)
print("critical point", critpts[2])

# verify that C achieves a minimum value with second derivative test
Cdoubleprime = diff(Cprime, w)
```

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```
print("second deriv greater than 0?", Cdoubleprime(critpts[2].rhs()), "Yes.")

# return the minimum cost
print("minimum cost?", n(C(critpts[2].rhs())))

# a plot of the cost function
plot(C(w), (w, 1, 5), color="green") + point((critpts[2].rhs(), C(critpts[2].rhs())))
```

Output:

```
('first derivative', w |--> 40*w - 180/w^2)
('critical point', w == 1/2*9^(1/3)*2^(2/3))
('second deriv greater than 0?', 120, 'Yes.')
('minimum cost?', 163.540853354893)
```