SOLUTIONS TO ATTENDANCE QUIZ (WEEK 8)

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This was an **ungraded** quiz given on 2019-03-07. I didn't collect statistics, but here's my impression (on a 0 to 4 scale with 4/4 high).

problem number	actual class performance	problem difficulty
1	2/4	2/4
2	2/4	2/4
3	1/4	3/4
4	2/4	4/4

Definitions

A **function** is a rule f that assigns to each element of a set A to *exactly one* element of a set B. We write

$$f\colon A \to B$$

to display all the ingredients together. The set A is called the **domain** of f, and the set B is called the **codomain** (or **range**) of f. We often represent the "data" of a function $f\colon A \to B$ by plotting its **graph**. For example, the graph of the function $\arctan\colon \mathbf{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$ is the set of *coordinate pairs*

$$\{(t, \arctan(t)) : \text{ for all } t \text{ in the domain } \mathbf{R}\}.$$

For the following questions, you may find it helpful to look at plots of the graphs of *arctangent*, *arcsine*, and *arccosine* on the back of this sheet.

Multiple choice

1. arcsin assigns elements of the set [?] to the set [?]

   (A) $[-1, 1] \to [0, \pi]$
   (B) $[-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$
   (C) $[-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$
   (D) All of the above
   (E) None of the above

   Correct answer.[1] (B) $[-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$. Recall that, to construct an inverse function for the sin function, we need an interval for which the sin function is increasing from -1 to 1 or decreasing from -1 to 1. There are several choices, but we stick with the interval $[-\pi/2, \pi/2]$, which we call the *principal branch* of the sin function. The function $\arcsin\colon [-1, 1] \to [-\pi/2, \pi/2]$ sends a to the unique $\theta \in [-\pi/2, \pi/2]$ such that $\sin a = \theta$.

2. arccos assigns elements of the set [?] to the set [?]

   (A) $[-1, 1] \to [0, \pi]$
   (B) $[-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$
   (C) $[-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$
   (D) All of the above

Date: 2019-03-06.
Git Repo: `https://github.com/coltongrainger/fy19ta`.

[1] From Vipul Naik's Math 153 course notes. See `http://files.vipulnaik.com/math-153-sequence/inversetrig.pdf`.

(E) None of the above

Correct answer. (A) $[-1, 1] \to [0, \pi]$. See plot.

3. One of these functions has a horizontal asymptote as $x \to +\infty$ and a horizontal asymptote as $x \to -\infty$, with the limiting values for $+\infty$ and $-\infty$ being *different*. Identify the function.

   (A) $f(x) := \ln |x|$.
   (B) $f(x) := \arctan x$.
   (C) $f(x) := e^{-x}$.
   (D) $f(x) := e^{-x^2}$.

   Correct answer. (B) Only $\arctan x$ has distinct horizontal asymptotes of the four functions.

   Other choices. One should rule out all of the *even* functions immediately. (A) is false because, for example, $\lim_{x \to +\infty} \ln x = \infty$. In other words, the inverse of the natural log, $\exp\{x\}$, is defined and *finite* for all real numbers. It follows that $\ln |x|$ (because $\ln |-x| = \ln |x|$ is an *even* function) has no horizontal asymptotes. (C) is false because $\exp\{-x\}$ only has a horizontal asymptote as $x \to +\infty$, but not as $x \to -\infty$. (D) belongs to a family of functions called *Gaussian* functions, most used for *probability distributions*. $\exp\{-x^2\}$ has two identical horizontal asymptotes $y = 0$.

True or False

4. Recall that an **open interval** (a, b) is the set of real numbers $\{x : a < x < b\}$. Extending this notion, we say that a subset O of $\mathbf{R}$ is **open** if

   - for each point $x \in O$,
   - there's an open interval (a, b) such that
     - the point x is an element of (a, b), and
     - the set (a, b) is contained in O.

   TRUE or FALSE: The domain of arctan is open.

   Correct answer. TRUE. The set of real numbers $\mathbf{R}$ is open in itself. In other words, at any point x in $\mathbf{R}$, there's just enough "breathing room" to fit an open interval (a, b) such that $a < x < b$.

   Other answer. Because $\mathbf{R}$ has no "boundaries", it is just not possible to find a point x in $\mathbf{R}$ that doesn't have any "wiggle room".

Graphs

The **Cartesian plane**, denoted $\mathbf{R}^2$, is the set of coordinate pairs

$$\{(x, y) : \text{ for all real numbers } x \text{ and } y\}.$$

So, here are graphs of the inverse trigonometric functions. They are subsets of the Cartesian plane!

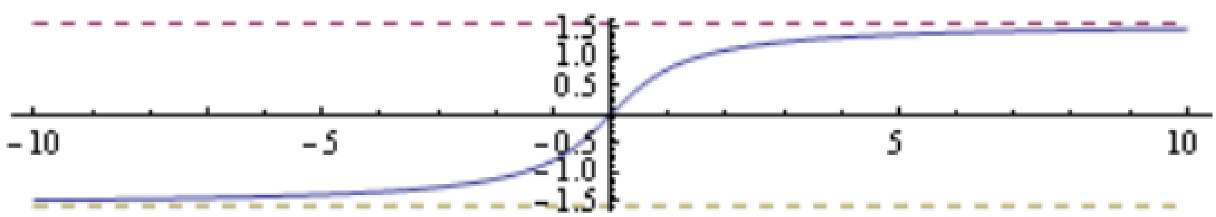

Figure 1. Arctangent

References

- Vipul Naik made the plots and wrote question 3. See `https://vipulnaik.com/math-152/`.
- I am borrowing *cuisine* as a quiz theme from Hiro Lee Tanaka.

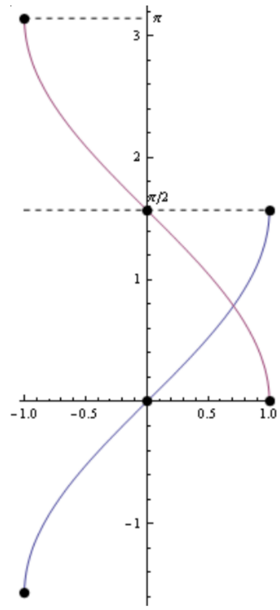

FIGURE 2. Arcsine and Arcosine