MORE LIMITS (ATTENDANCE QUIZ)

COLTON GRAINGER (MATH 1300)

Print your **full name** and **three digit section number** in the top right corner, attempt the problems, and return this page to me. You have about 1 minute per question. You are free to discuss these questions with others while making your attempt.

- We call a function f left continuous on an open interval I if, for all $a \in I$, $\lim_{x \to a^-} f(x) = f(a)$. Which of the following is an example of a function that is left continuous but not continuous on $(0, 1)$?

  (A) $f(x) := \begin{cases} x, & 0 < x \le 1/2 \\ 2x, & 1/2 < x < 1 \end{cases}$
  (B) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x, & 1/2 \le x < 1 \end{cases}$
  (C) $f(x) := \begin{cases} x, & 0 < x \le 1/2 \\ 2x - (1/2), & 1/2 < x < 1 \end{cases}$
  (D) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x - (1/2), & 1/2 \le x < 1 \end{cases}$
  (E) All of the above

- Suppose f and g are functions $(0, 1)$ to $(0, 1)$ that are both left continuous on $(0, 1)$. Which of the following is *not* guaranteed to be left continuous on $(0, 1)$? Last year's performance: $4/13$ correct

  (A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
  (B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
  (C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
  (D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
  (E) None of the above, i.e., they are all guaranteed to be left continuous functions

- Consider the function
  $$f(x) := \begin{cases} x, & x \text{ rational} \\ 1/x, & x \text{ irrational} \end{cases}$$
  What is the set of all points at which f is continuous?

  (A) $\{0, 1\}$
  (B) $\{-1, 1\}$
  (C) $\{-1, 0\}$
  (D) $\{-1, 0, 1\}$
  (E) f is continuous everywhere

- Define the base e of the "natural" exponential function. Hint: The derivative of every exponential function of the form $f(x) := a^x$ with $a > 0$ is equal to a multiple of itself $f'(x) = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = a^x \lim_{h \to 0} \frac{a^h - 1}{h}$.

  (A) $e = \lim_{h \to 0} e^h$
  (B) e is the number that satisfies $\log(1) = e$
  (C) $e = \lim_{h \to 0} \frac{e^h}{h}$
  (D) e is the number that satisfies $e^{x+y} = e^x e^y$ for all $x, y \in \mathbf{R}$
  (E) e is the number that satisfies $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

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Repo: `https://github.com/coltongrainger/pro19ta`.