DERIVATIVE OF ARCCOSINE

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We find the derivative of $\theta = \arccos x = \cos^{-1} x$ with respect to x.

Simplify the equation by taking the cosine of both sides:

$$\begin{aligned} \theta &= \cos^{-1} x \\ \cos\theta &= x \end{aligned}$$

Now take the derivative of both sides of the equation with respect to x and solve for $\frac{d\theta}{dx}$. We appeal to the *chain rule* to change the variable of differentiation.

$$\begin{aligned} \cos\theta &= x \\ \frac{d}{dx}[\cos\theta] &= \frac{d}{dx}[x] \\ \frac{d}{d\theta}[\cos\theta]\frac{d\theta}{dx} &= 1 \\ (-\sin\theta)\cdot\frac{d\theta}{dx} &= 1 \\ \frac{d\theta}{dx} &= \frac{-1}{\sin\theta} \end{aligned}$$

We want to rewrite this in terms of $x = \cos\theta$. How? Recall

$$\begin{aligned} \cos^2\theta + \sin^2\theta &= 1 \\ \sin^2\theta &= 1 - \cos^2\theta \\ \sin\theta &= \sqrt{1-\cos^2\theta} \\ &\text{(Note } \sin\theta \geq 0 \text{ on the range of } \theta = \cos^{-1} x\text{).} \end{aligned}$$

Substitution implies

$$\begin{aligned} \frac{d\theta}{dx} &= \frac{-1}{\sin\theta} \\ &= \frac{-1}{\sqrt{1-\cos^2\theta}} \\ &= \frac{-1}{\sqrt{1-x^2}} \end{aligned}$$

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We made a choice between a positive a negative square root when solving for $\sin\theta$. We choose the positive root because we usually define $\cos^{-1} x$ to have outputs between 0 and π, and the sine function is positive (or zero) on this interval.

1. References

Jerison, David. "Session 15: Implicit Differentiation and Inverse Functions, Part B: Implicit Differentiation and Inverse Functions, MIT OpenCourseWare"[1]. Retrieved October 5, 2018.

[1] https://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/1.-differentiation/part-b-implicit-differentiation-and-inverse-functions/session-15-implicit-differentiation-and-inverse-functions/