ATTENDANCE QUIZ (WEEK 10)

COLTON GRAINGER

Your name (print clearly in capital letters): ____

This is an **ungraded** quiz that will count for attendance; it is due by the end of recitation.

Definitions and comments

1. A **zero** (or a **root**) of a function f is

$$\text{a point } r \text{ such that } f(r) = 0.$$

   i. The term *root* is traditionally[1] used for the study of polynomial functions.
   ii. For example, if r is a root of the polynomial function p (that is, if $p(r) = 0$), then

$$p(x) = (x - r)q(x) \quad \text{for some polynomial function } q.$$

2. A **critical point** of a function f is

$$\text{a point } a \text{ such that } f'(a) = 0.$$

   i. The number $f(a)$ itself is called a **critical value** of f.

3. A **point of local extremum** of a function f is

$$\text{a point of either local maximum or minimum.}$$

   i. If b is a point of local extremum, we say $f(b)$ is a **local extreme value**.

4. An **inflection point** of a function f is

$$\text{a point } c \text{ such that the tangent line to } f \text{ at } (c, f(c)) \text{ crosses the graph of } f.$$

   i. In order for c to be an inflection point of a function f, it is necessary that f'' should have different signs to the left and right of c.
   ii. For example (see figure), $\sqrt{1/3}$ and $-\sqrt{1/3}$ are inflection points of $f(x) := 1/(1 + x^2)$.

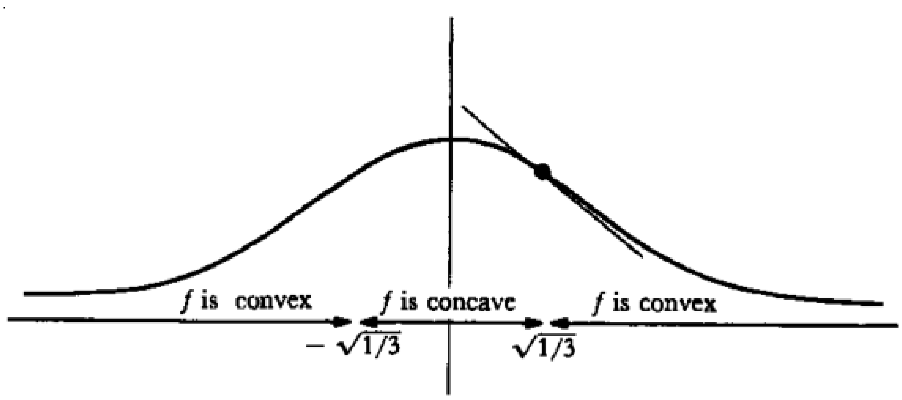

Figure 1. Inflection points

Date: 2019-03-17.

[1]"The **roote quadrat** of the whole number, is the desired distance or line Hypothenusal." (Digges, *Pantom*, 1571).

Multiple Choice

1. Consider the function $f\colon \mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^3(x-1)^4(x-2)^2$. Which of the following **is true**?

   (A) 0, 1, and 2 are all critical points and all of them are points of local extrema.
   (B) 0, 1, and 2 are all critical points, but only 0 is a point of local extremum.
   (C) 0, 1, and 2 are all critical points, but only 1 and 2 are points of local extrema.
   (D) 0, 1, and 2 are all critical points, and none of them is a point of local extremum.
   (E) 1 and 2 are the only critical points.

   Your answer: ______________________________

2. Say $f\colon \mathbf{R} \to \mathbf{R}$ is a differentiable function. What information **can not** be determined from **only** the first derivative f'?

   (A) roots of the function
   (B) critical points
   (C) points of inflection
   (D) local extreme values
   (E) neither (D) nor (A)

   Your answer: ______________________________

3. Say $f\colon \mathbf{R} \to \mathbf{R}$ is a twice-differentiable function. What information **can** be determined from **only** the second derivative f''?

   (A) intervals where the function is positive or negative
   (B) intervals where the function increases and decreases
   (C) intervals where the function is concave up and concave down
   (D) y-values of horizontal asymptotes
   (E) all of the above

   Your answer: ______________________________

4. Suppose f and g are continuously differentiable functions on $\mathbf{R}$. Suppose f and g are both concave up. Which of the following is **always true**?

   (A) $f+g$ is concave up.
   (B) $f-g$ is concave up.
   (C) $f \cdot g$ is concave up.
   (D) $f \circ g$ is concave up.
   (E) All of the above.

   Your answer: ______________________________

References

The definitions are from Spivak [1]. The questions are adapted from Naik's Math-152 notes [2].

[1] M. Spivak, *Calculus*, 3rd ed. Publish or Perish, Inc., 1994.

[2] V. Naik, "Math 152 Course Notes," 2012.