HW6

Thursday, February 28, 2019 9:27 AM

2. (a) If $F(x) = f(x)g(x)$, where f and g have derivatives of all orders, show that

$$F'' = f''g + 2f'g' + fg''.$$

(b) Find similar formulas for F''' and $F^{(4)}$.

(c) Guess a formula for $F^{(n)}$.

(a) Product rule: $F'(x) = f'(x)g(x) + f(x)g'(x)$.

Again...

$$F''(x) = f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x)$$

(b) I'll omit the x so things are easier to read:

$$F''' = f'''g + 3f''g' + 3f'g'' + fg'''$$

$$F^{(4)} = f^{(4)}g + 4f'''g' + 6f''g'' + 4f'g''' + g^{(4)}$$

Notice that $\longrightarrow$ shows which term contributes to which pair of terms when applying the product rule.

Compare to Pascal's Δ:

~ Think of why Product rule copies the rule for Pascal's Δ:

Each # is the sum of the 2 numbers above it.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

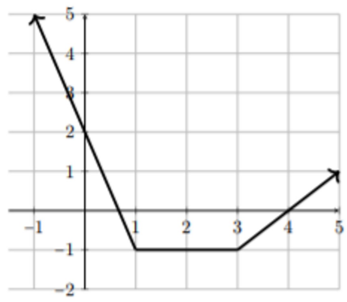
$$F^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} g^{(n-k)}$$

↳ Binomial coefficients, compare ↑

3. A table of values for the functions $f(x)$ and $f'(x)$ and a graph of the piecewise linear function $g(x)$ are shown below.

x	$f(x)$	$f'(x)$
-1	11	-7
0	2	-2
1	-2	5
2	9	3
3	0	4
4	1	2

Parts b, c, e are all just done by product, quotient, & chain rule.



$g(x)$

(a) Given $h(x) = f(x)g(x)$, find $h'(1)$.

(b) Given $p(x) = \dfrac{f(x)}{g(x)}$, find $p'(2)$.

(c) Given $q(x) = \dfrac{g(x)}{f(x)}$, find $q'(2)$.

(d) Given $q(x) = \dfrac{f(x)}{g(x)}$, find $q'(3)$.

(e) $l(x) = \dfrac{g(x)}{\sqrt{x}}$, find $l'(4)$.

(a) $h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(1) = f'(1)g(1) + f(1)g'(1)$ Undefined, but need to check from both sides

To the left of 1, g' is -3 (slope on graph).
So from left,
$h'(1) = 5\cdot(-1) + (-2)(-3) = -5+6 = 1$

To right of 1, g' is 0 (slope on graph)
so $h'(1) = 5\cdot(-1) + (-2)0 = -5$

not equal, therefore $h'(1)$ is undefined.

(b) $q'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$q'(3) = \dfrac{f'(3)g(3) - f(3)g'(3)}{(g(x))^2}$ undef, but check both sides.

see slope of g

$(g(x))^2$

On left of 3, g' is 0 so from left: ← see slope of g

$g'(3) = \frac{4(-1) - 0 \cdot 0}{(-1)^2} = \frac{-4}{1} = -4$

On right of 3, g' is 1, so from right, ← again by slope

$g'(3) = \frac{4(-1) - 0 \cdot 1}{(-1)^2} = \frac{-4}{1} = -4$

Therefore, $g'(3) = -4$.

6. A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

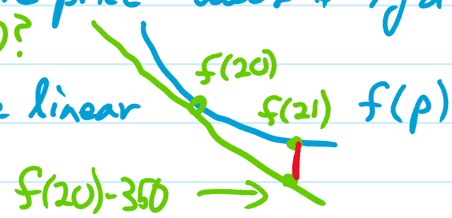
(a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?

(b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

(a) $f(20) = 10000$ means when price/yard of fabric is \$20/yd., the manufacturer will sell 10,000 yards of fabric.
$f'(20) = -350$ says that if the price goes up to \$21/yd (a \$1 increase) then about 350 fewer yards of fabric are sold than when the price was \$20/yd.

Why only ≈ 350?

Because derivatives give linear approximations →



$f(21) \approx f(20) - 350$.

You could also say that $f(19) \approx f(20) + 350$, and adjust the explanation accordingly.

(b) $R(p) = p\,f(p)$, $\quad R'(p) = pf'(p) + f(p)$

Note $p' \neq 0$, $p' = 1$. Think $d/dx\,(x) = 1$. p is the variable here, not a constant!

$R'(20) = 20 \cdot (-350) + 10000$
$= -7000 + 10000$

$$R'(20) = 350(-20)+10000$$
$$= -7000+10000$$
$$= 3000$$

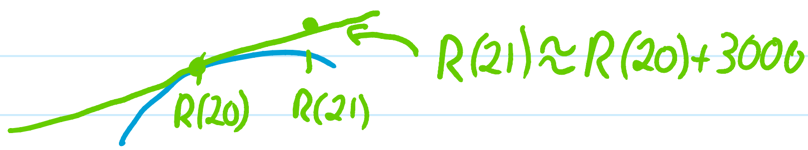
What are the units of $R'(p)$? The units of R are dollars, and the units of p are also dollars. Thus $R'(p)$ measures

change in Revenue (\$) / change in price (\$).

It is hard to give meaning to "instantaneous rate of change" with regards to money, so we should describe the rate of change like so:

Since $R'(20)=3000$, the manufacturer can expect approximately \$3000 more revenue at a price of \$21/yd than at a price of \$20/yd.

Visually:



Again, we can only say approximately since the derivative lets us compute a linear approximation. It is very important to say this in your solution, and interpretations like "instantaneous rate of change" do not suffice here since we can't attribute a meaningful interpretation to that phrase in this context.