INTEGRATION (ATTENDANCE QUIZ)

COLTON GRAINGER (MATH 1300)

Print your **full name** and **three digit section number** in the top right corner and attempt each problem. You are free to discuss these questions with others while making your attempt. (Questions due to Vipul Naik [1].)

1. Consider the function(s) $[0,1] \to \mathbf{R}$. **Identify the function**s for which the integral (using upper sums and lower sums) is not defined.
   (A) $f_1(x) := \begin{cases} 0, & 0 \leqslant x < 1/2 \\ 1, & 1/2 \leqslant x \leqslant 1 \end{cases}.$
   (B) $f_2(x) := \begin{cases} 0, & x \neq 0 \text{ and } 1/x \text{ is an integer} \\ 1, & \text{otherwise} \end{cases}.$
   (C) $f_3(x) := \begin{cases} 0, & x \text{ rational} \\ 1, & x \text{ irrational} \end{cases}$
   (D) All of the above
   (E) None of the above

2. Suppose $a < b$. Recall that a *regular partition* into n parts of $[a,b]$ is a partition $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$ where $x_i - x_{i-1} = (b-a)/n$ for all $1 \leqslant i \leqslant n$. A partition P_1 is said to be a *finer partition* than a partition P_2 if the set of points of P_1 contains the set of points of P_2. Which of the following is a **necessary and sufficient condition** for the regular partition into m parts to be a *finer partition* than the regular partition into n parts? (Note: We'll assume that any partition is finer than itself).
   (A) $m \leqslant n$
   (B) $n \leqslant m$
   (C) m divides n (i.e., n is a multiple of m)
   (D) n divides m (i.e., m is a multiple of n)
   (E) m is a power of n

3. *Consumption smoothing (or, advice for Thanksgiving)*: A certain measure of happiness is found to be a logarithmic function of consumption, i.e., the happiness level H of a person is found to be of the form $H = a + b \ln C$ where C is the person's current consumption level, and a and b are positive constants independent of the consumption level. The person has a certain total consumption C_{tot} to be split within two years, year 1 and year 2, i.e., $C_{tot} = C_1 + C_2$. Thus, the person's happiness level in year 1 is $H_1 = a + b \ln C_1$ and the person's happiness level in year 2 is $H_2 = a + b \ln C_2$. How would the person choose to split consumption between the two years to maximize average happiness across the years?
   (A) All the consumption in either one year
   (B) Equal amount of consumption in the two years
   (C) Consume twice as much in one year as in the other year
   (D) Consumption in the two years is in the ratio $a : b$

[1] V. Naik, "Math 152 Course Notes" [Online]. Available: https://vipulnaik.com/math-152/

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