Notes for linearization sample teaching wrkshp. 2.5 min

2018-10-01 ~~2018-01-17 #2~~ ~~2.5 min~~

GOALS

- approximate e
- define linearization
- find a root

Q| what's $\frac{d}{dx}[e^x]$?

EX| $e^x = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$

$= e^x \lim_{h\to 0} \frac{e^h - 1}{h}$

Hence e is the number s.th.

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

Now when h is small, we write

$$\frac{e^h - 1}{h} \approx 1 \quad \text{or} \quad e^h \approx h + 1$$

and say that $h+1$ approximates e^h.

Say that $h = 1/4$. Then

$$e^{1/4} \approx 1 + 1/4$$

hence

$$e \approx (5/4)^4 = \frac{625}{256}.$$

SETUP| Let $f: X \to Y$ be a ~~function~~ differentiable function.

Suppose ~~or~~ $f'(x_0)$ is known at some point x_0 in the domain X.

Then for x close to x_0,

(f is ~~as~~ and) $f(x)$ is close to $f(x_0)$

and thus

~~DEF~~ $f'(x_0) = \lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

$$\approx \frac{f(x) - f(x_0)}{x - x_0}.$$

DEF| The linearization of f at x_0 is (the ~~line~~ tangent line)

$$y = f'(x)(x - x_0) + y_0$$

and we say y approximates $f(x)$.

5 min

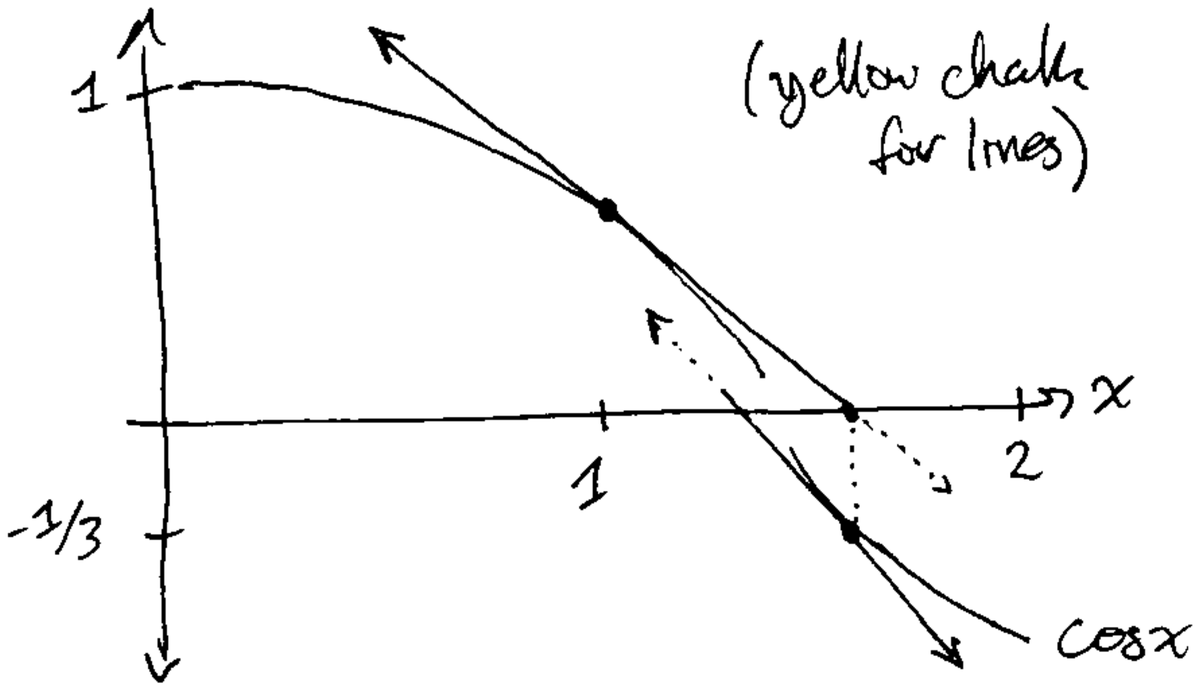
EX] Consider the f^n $\cos x$ on $[0,2]$ ✓

~~[0,2]. How cos~~

~~How cos cos x~~

Now $\cos 0 = 1$ and $\cos 2 < -\frac{1}{3}$

So (by the IVP) $\cos x$ intersects the ~~ax~~ x-axis somewhere in $(0,2)$



Linearizing $\cos x$ at 1, we have

$$y = -\sin(1)(x+1) + \cos 1$$

which intersects the x axis when

$$x = \frac{\cos 1}{\sin 1} + 1 \approx 1.64$$

Again, linearizing $\cos x$ at 1.64, we've

$$y = -\sin(1.64)(x+1.64) + \cos 1.64$$

conclusion

which has a root at

$$x = \frac{\cos 1.64}{\sin 1.64} + 1.64 \approx 1.57.$$

Then ~~check~~ test $\cos 1.57 \approx 0$

as $1.57 \approx \pi/2$, as desired.

Microteaching Peer Feedback Form

Name: Kristina

Teacher's name: Colton

1. What were the learning objectives? In what ways were they achieved or not quite achieved?

- approx e ✓ → 625/256
- def linearization ✓
- find a root

10 mins

compare to what students might already know

2. What are your observations about organization and boardwork?

— Spacing & writing is clear, easy to follow.

— Organization matches w Colton's pacing/speaking

3. How do you feel as a student in the class?

- Being called on kept me focused. (Before & After)

- Difference quotient → linearization (used prior knowledge)

4. What type of questions did the instructor ask?

"What is a root" – RECALL

"$\frac{d}{dx}(e^x) = e^x$"

MAKING CONNECTIONS: (using "e" example to def. linearization.)

5. What was the best / most interesting part?

Humor: "What's a small number?

1/10^6 ⟶ 1/4 ☺ "

LOL

Microteaching Peer Feedback Form

Name: Chris

Teacher's name: Colton

1. What were the learning objectives? In what ways were they achieved or not quite achieved?

Our goal was to approximate e, define linearization. He walked through the process of approximating e, which led to the definition, which was nice. The way it was laid out seems like we may use this process to approximate numbers, losing that we can use it to approx. functions near a point.

2. What are your observations about organization and boardwork?

Very clear handwriting, perhaps include more algebraic steps. Should be careful using notation $f: x \Rightarrow y$ and terminology codomain they might not have seen. Should write "when $x \sim x_0$" or something in the setup portion, and "$y_0 = f(x_0)$". Maybe draw some pictures.

3. How do you feel as a student in the class?

I feel that I would enjoy having Colton lecture, the motivation gives a good sense of why it works.

4. What type of questions did the instructor ask?

He asked many lower level questions, i.e. asking for a correct response. Also "How do we evaluate ..." was a good slightly higher level question.

5. What was the best / most interesting part?

The first example linearizing e^x was an interesting motivator, we sort of stumbled upon the linearization by walking through the def. of derivative.

Microteaching Peer Feedback Form

Name: _____

Teacher's name: _____

1. What were the learning objectives? In what ways were they achieved or not quite achieved?

2. What are your observations about organization and boardwork?

3. How do you feel as a student in the class?

4. What type of questions did the instructor ask?

5. What was the best / most interesting part?

Microteaching Peer Feedback Form

Name: Shen Lu

Teacher's name: Colton

1. What were the learning objectives? In what ways were they achieved or not quite achieved?

- There are three learning goals, clearly spelled out at the beginning
- $\boxed{e^h \approx h+1}$ was not clear to me, without a picture, or written explanation.

2. What are your observations about organization and boardwork?

- Very organized, flows from left to right, with clear labels

3. How do you feel as a student in the class?

- Engaged, but with some fear of being called on
- ~~Hi~~ I wasn't sure how/when to ask a question when I was confused.

4. What type of questions did the instructor ask?

- (past) Content knowledge

5. What was the best / most interesting part?

- Using definition of derivative to teach linearization
- clear labels, very nice boardwork!

Microteaching Peer Feedback Form

Name: ERIK KNUTSEN

Teacher's name: COLTON

1. What were the learning objectives? In what ways were they achieved or not quite achieved?

Learning Objectives: See an example, define the concept, apply it.

Laying out the structure of your lecture helped me to make sense of where we are in the lecture and how what we are doing at any given point fits with the big picture.

2. What are your observations about organization and boardwork?

Boardwork was nicely organized except that the setup started at the bottom and continued above, it might be nice to have the setup start at the top of the board and the Defn started at the bottom corner of that board (but it might be better to give it more pride of place) and finished on the opposite top corner. I think the central definitions should be kept together.

3. How do you feel as a student in the class?

I feel a little scared that you will call on me. I take time to absorb information sometimes and am not always prepared to give an answer right on the fly.

in this case I was obvious because it was hard to focus on the math we were doing and the critical evaluation of your teaching.

4. What type of questions did the instructor ask?

What is a root? - Good to ask reminder questions.

It was good to call out names, but I think it might be good to make eye contact when you do. (When you first called on Shen it took him a second to realize you were calling on him.)

5. What was the best / most interesting part?

Your example with e worked for me, it motivated the definition and it illuminated a mysterious and interesting number we see a lot in calculus.

Microteaching Peer Feedback Form

Name: Peter

Teacher's name: Colton

1. What were the learning objectives? In what ways were they achieved or not quite achieved?

Approx e → done

define limit → done

find roots → run out of time

2. What are your observations about organization and boardwork?

Your writing is nice and legible.

Try to keep definitions and such on one board as much as possible.

3. How do you feel as a student in the class?

When I got confused on algebraic set, it would have helped if you took a second to explain things in more detail

4. What type of questions did the instructor ask?

There were a lot of ~~level~~ level 0-1 questions but not a lot of level 2-3 questions.

5. What was the best / most interesting part?

I enjoyed the derivation of the approximation of $e^{1/2}$

Microteaching Peer Feedback Form

Name: Mateo Muro

Teacher's name: Colton Grainger

1. What were the learning objectives? In what ways were they achieved or not quite achieved?
   - approximate e → felt natural but relied on kids knowing $d/dx[e^x]$
   - define linearization → The defn was wordy, lost the intuition
   - find roots

2. What are your observations about organization and boardwork?

   very neat handwriting, organized, allowed me to follow the whole time

3. How do you feel as a student in the class?

   The questions were good to feel like Colton was actually teaching me.

4. What type of questions did the instructor ask?

   Some knowledge check but also some "what should we do next" to make us feel a part of the process.

5. What was the best / most interesting part?

   Approximation of e is cool because e is so mysterious.

Microteaching Peer Feedback Form

Name: Ian Miller

Teacher's name: Colton

1. What were the learning objectives? In what ways were they achieved or not quite achieved?

2. What are your observations about organization and boardwork?

- Large clear handwriting helped make the presentation easier to follow.
- You mention "when h is small" for part of the setup, but would be helpful to see or at least hear important information like that repeated when you state that h!| "approximates" e^h.

3. How do you feel as a student in the class?

The example of approximating e was nice, but if it was a first time seeing linearization it would be hard to follow at the quick pace. Maybe a picture could help?

4. What type of questions did the instructor ask?

A lot of fill in the blank questions, I think to make sure people were following along. This seems like a good strategy to keep people focused while maintaining momentum in a fast lecture.

5. What was the best / most interesting part?

Seeing applications of linearization to approximating interesting numbers/values.