

SOLUTION SET (FUNCTIONS AND LIMITS)

COLTON GRAINGER (MATH 1300)

- Consider the function $f(x) := x^2 + 1$. What is the polynomial describing $f(f(x))$?
 - $x^2 + 2$
 - $x^4 + x^2 + 1$
 - $x^4 + x^2 + 2$
 - $x^4 + 2x^2 + 1$
 - $x^4 + 2x^2 + 2$

Answer: Option (E). Explanation from Vipul Naik:

We have $f(f(x)) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 1 + 1$, which simplifies to option (E).

Option (A) is $(x^2 + 1) + 1 = x^2 + 2$. The error here is is not squaring $x^2 + 1$.

Option (D) is $(x^2 + 1)^2 = x^4 + 2x^2 + 1$. The error here is in forgetting to add 1.

Options (B) and (C) are like (D) and (E), with an error in the coefficient of x^2 .

- If $f(g(x)) = 5$ and $f(x) = x + 3$ for all real x , then $g(x) =$
 - $x - 3$
 - $3 - x$
 - $\frac{5}{x+3}$
 - 2
 - 8

Answer: $g(x) = 2$. Explanation:

With the constant function $g(x) = 2$, we evaluate $f(g(x)) = f(2) = 5$.

- For all positive functions f and g of the real variable x , let \sim be a relation defined by

$$f \sim g \text{ if and only if } \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1.$$

Which of the following is NOT a consequence of $f \sim g$?

- $f^2 \sim g^2$
- $\sqrt{f} \sim \sqrt{g}$
- $e^f \sim e^g$

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Repo: <https://github.com/coltongrainger/pro19ta>.

d. $f + g \sim 2g$

e. $g \sim f$

Answer: $e^f \sim e^g$. Explanation from Charlie Rambo:

Let's find a counter example to $e^f \sim e^g$. Consider $f(x) = x$ and $g(x) = x + 1$.

Clearly $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1$. But

$$\lim_{x \rightarrow \infty} \frac{e^x}{e^{x+1}} = \lim_{x \rightarrow \infty} \frac{1}{e} = \frac{1}{e} \neq 1.$$

REFERENCES

- Vipul Naik, *Math 152 Week 1*. <https://vipulnaik.com/math-152/>.
- GRE Mathematics Test Form GR0568 and Form GR9367.