CONNECTEDNESS

COLTON GRAINGER (TOPOLOGY MATH 6210)

4. Assignment due 2018-10-08

4.1. **[1, No. 23.1].** Let $\mathscr{T}$ and $\mathscr{T}'$ be two topologies on X. If $\mathscr{T} \supset \mathscr{T}'$, what does connectedness of X in one topology imply about connectedness in the other?

4.2. **[1, No. 23.4].** If X is an infinite set, it is connected in the finite complement topology.

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4.3. **[1, No. 23.5].** If X has the discrete topology, then X is totally disconnected. The converse does not hold.

4.4. **[1, No. 23.7].** $\mathbf{R}_\ell$ is a totally disconnected space.

4.5. **[1, No. 23.11].** Let $p\colon X \to Y$ be a quotient map. If each set $p^{-1}(\{y\})$ is connected, and if Y is connected, then X is connected.

4.6. **[1, No. 24.1].** Let $f\colon S^1 \to \mathbf{R}$ be a continuous map. We exhibit a point x of S^1 such that $f(x) = f(-x)$.

4.7. **[1, No. 24.2].** Let $f\colon X \to X$ be a continuous transformation of X. If $X = [0,1]$, then there's a point such that $f(x) = x$. There are continuous transformations of $[0,1)$ and $(0,1)$ without fixed points.

4.8. **[1, No. 24.7].**

(a) Let X and Y be ordered sets in the order topology. If $f\colon X \to Y$ is order preserving and surjective, then f is a homeomorphism.

(b) Let $X = Y = \mathbf{R}_{\geq 0}$. Given any positive integer n, the function $f\colon X \to Y$ defined by $f(x) := x^n$ is order preserving and surjective. Moreover, f has a continuous inverse $f^{-1}\colon Y \to X$ given by $f^{-1}(y) := \sqrt[n]{y}$.

4.9. **[1, No. 24.10].** If U is an *open* connected subspace of $\mathbf{R}^2$, then U is path connected.

4.10. **[1, No. 25.2].**

(a) What are the components and path components of $\mathbf{R}^\omega$ (in the product topology)?

(b) Let $\mathbf{R}^\omega$ have the uniform topology. Now x and y lie in the same component of $\mathbf{R}^\omega$ if and only if the sequence $x - y = (x_1 - y_1, x_2 - y_2, \ldots)$ is bounded.

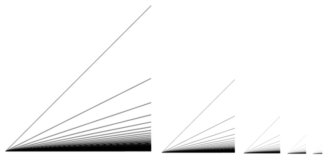
(c) Suppose $\mathbf{R}^\omega$ has the box topology. Then x and y lie in the same component of $\mathbf{R}^\omega$ if and only if the sequence $x - y$ is eventually zero.

4.11. **[1, No. 25.4].** Let X be locally path connected. Then every connected open set in X is path connected.

4.12. **[1, No. 25.5].** Let X denote the rational points of the interval $[0,1] \times 0$ of $\mathbf{R}^2$. Let T denote the union of all line segments joining the point $p = 0 \times 1$ to points of X.

(a) T is path connected—but only locally connected at the point p.
(b) We exhibit a subset of $\mathbf{R}^2$ that is path connected but is locally connected at none of it's points.

4.13. **[1, No. 25.7].** The closed infinite broom X is not locally connected at the point at the endpoint p, but is weakly locally connected at p.



References

[1] J. R. Munkres, *Topology*, 2nd ed. Hardcover; Prentice Hall, Inc., 2000 [Online]. Available: http://www.worldcat.org/isbn/0131816292