TODO AND TO-REVISE

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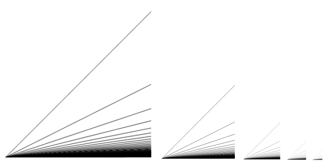
0.1. **Quotienting out K in the K-topology [1, No. 22.6].** Let Y be the quotient space obtained from $\mathbf{R}_K$ by collapsing the set K to point, with $p\colon \mathbf{R}_K \to Y$ as the corresponding quotient map.

(a) Y is T_1 but not Hausdorff.
(b) The map $p \times p\colon \mathbf{R}_K \times \mathbf{R}_K \to Y \times Y$ is not a quotient map.[1]

0.2. **[1, No. 25.5].** Let X denote the rational points of the interval $[0, 1] \times 0$ of $\mathbf{R}^2$. Let T denote the union of all line segments joining the point $p = 0 \times 1$ to points of X.

(a) T is path connected—but only locally connected at the point p.
(b) We exhibit a subset of $\mathbf{R}^2$ that is path connected but is locally connected at none of it's points.

0.3. **[1, No. 25.7].** The closed infinite broom X is not locally connected at the point at the endpoint p, but is weakly locally connected at p.



0.4. **2018-10-10 midterm problem 2.**

(a) Let $\mathscr{N} = \{1/n : n \in \mathbf{N}\} \subset \mathbf{R}$ denote the set of all reciprocals of natural numbers. Let
$$\mathscr{B} = \{(a, b) : a, b \in \mathbf{R}\} \cup \{(a, b) \setminus \mathscr{N} : a, b \in \mathbf{R}\} \cup \{\mathbf{R}, \varnothing\}.$$
Now $\mathscr{B}$ generates a topology on $\mathbf{R}$, denote that topology $\mathscr{T}_{\mathscr{B}}$.
   (i) What is the closure of $\mathscr{N}$ in $\mathscr{T}_{\mathscr{B}}$?
   (ii) Is every closed set of the topological space $(\mathbf{R}, \mathscr{T}_{\mathscr{B}})$ closed in the standard topology on $\mathbf{R}$?

(b) Is every closed set of the standard topology on $\mathbf{R}$ also closed in the finite complement topology on $\mathbf{R}$?

References

[1] J. R. Munkres, *Topology*, 2nd ed. Hardcover; Prentice Hall, Inc., 2000 [Online]. Available: `http://www.worldcat.org/isbn/0131816292`

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[1] The diagonal is not closed in $Y \times Y$, but its inverse image is closed in $\mathbf{R}_K \times \mathbf{R}_K$.