COUNTABILITY, METRIZABILITY, INTRO TO HOMOTOPY

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6. Assignment due 2018-11-02

6.1. **[1, No. 30.4].** Every compact metrizable space X has a countable basis.[1]

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[1]Hint: let $\mathscr{A}_n$ be a finite covering of X by $1/n$-balls

6.2. **[1, No. 30.6].** The lower limit topology $\mathbf{R}_\ell$ is not metrizable. The ordered square I_o^2 is not metrizable.

6.3. **[1, No. 30.13].** If X has a countable dense subset, every collection of disjoint open sets in X is countable.

6.4. **[1, No. 31.5].** If Y is Hausdorff and $f, g\colon X \to Y$ are continuous maps, then $\{x : f(x) = g(x)\}$ is closed in X.

6.5. **[1, No. 31.6].** If X is normal, and $p\colon X \to Y$ is a closed continuous surjective map, then Y is normal.[2]

[2]Hint: if U is an open set containing $p^{-1}(\{y\})$, there's a neighborhood W of y such that $p^{-1}(W) \subset U$.

6.6. **[1, No. 51.3].** Let $[X, Y]$ denote the set of homotopy classes of maps of X into Y. A space X is said to be *contractible* if the identity map $i_X : X \to X$ is nulhomotopic.

(a) The interval I and the real line $\mathbf{R}$ are both contractible.
(b) A contractible space is path connected.
(c) If Y is contractible, then for any X, the set $[X, Y]$ has a single element.
(d) If X is contractible and Y is path connected, then $[X, Y]$ has a single element.

6.7. **[1, No. 52.4].** Let $A \subset X$; suppose $r \colon X \to A$ is a continuous map such that $r(a) = a$ for each $a \in A$. The map r is called a retraction of X onto A. If $a_0 \in A$, then $r_* \colon \pi_1(X, a_0) \to \pi_1(A, a_0)$ is surjective.

References

[1] J. R. Munkres, *Topology*, 2nd ed. Hardcover; Prentice Hall, Inc., 2000 [Online]. Available: http://www.worldcat.org/isbn/0131816292