Chapter 4 Elementary Probability Theory

Probability

A numerical measure between zero and one that describes the likelihood an event will occur.

Notation/Shorthand

A = event that it rains today at 2pm
$P(A) = 30\%$

The probability that it rains today at 2pm is 30%

IMPORTANT For any event A, P(A) must be one of the following:

- decimal or fraction between 0 and 1
- percentage between 0% and 100%

RECALL: Convert $0.23 = 23\%$ and $8\% = 0.08$

- $P(A) = 0$, then we say "event A is certain to NOT occur."
- $P(A) = 1$, then we say "event A is certain to occur."

VOCAB Read in the text for "sample space", "event", "simple event", and "statistical experiment".

Probability Assignment

1) use intuition/past experience to estimate
2) use data/relative frequency → Law of Large Numbers says as n gets large, relative freq. approaches theoretical probability!

$$P(A) = \frac{\text{frequency of event } A}{n \text{ total observations}}$$

3) if all outcomes equally likely

$$P(A) = \frac{\text{number of outcomes favorable for event } A}{\text{total possible outcomes}}$$

Example)

2) From a random sample of 500 students, 375 wore glasses.

A = student wears glasses , $P(A) = \frac{375}{500}$

3) You roll a fair die once.

A = roll an even number , $P(A) = \frac{3}{6}$

Example)

You want to consider the experiment of flipping a fair coin 3 times.

Key: H = heads , T = tails

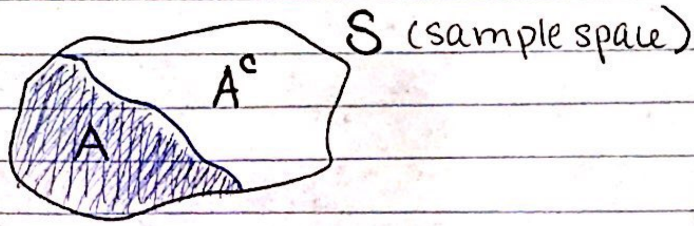
Sample Space = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Event	Probability
A = flip atleast 2 heads = {HHH, HHT, HTH, THH}	$P(A) = 4/8$
B = flip at least 2 tails = {TTT, TTH, THT, HTT}	$P(B) = 4/8$
A and B = flip atleast 2 heads and atleast 2 tails = { }	$P(A \text{ and } B) = 0$

→ these events are "mutually exclusive" they can't happen at the same time

Complements:

Recall that a sample space is all the possible outcomes.



An event, A, and its complement, A^c, together form the entire sample space, S.

$P(S) = 1$

$P(A) + P(A^c) = 1$

$P(A) = 1 - P(A^c)$

$P(A^c) = 1 - P(A)$

Example) If $P(A) = 0.34$, then $P(A^c) = 0.66$.

Example) With a fair 6 sided die. If event A is rolling a 4, then its complement A^c is rolling a 1, 2, 3, 5, or 6!

Compound Events:

For any event A, B;

$$P(A \text{ and } B) = \begin{cases} P(A)P(B|A) \\ P(B)P(A|B) \end{cases}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

For mutually exclusive A, B;

$$P(A \text{ and } B) = 0$$
$$P(A \text{ or } B) = P(A) + P(B)$$

For independent events A, B;

$$P(A \text{ and } B) = P(A)P(B)$$
$$P(A \text{ or } B) = P(A) + P(B) - P(A)P(B)$$

Conditional Probability:

$P(A|B)$ = "Probability of event A, given that event B is known/has happened."

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \quad (\text{assuming } P(B) \neq 0)$$

* If events A, B are independent, the conditional probabilities become:

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

Example) Suppose $P(A) = 0.2$ and $P(B) = 0.4$.

a) Suppose $P(A|B) = 0.1$. Are A and B independent?

$P(A|B) = 0.1 \neq 0.2 = P(A)$ so they are not independent.

b) Compute $P(A \text{ and } B)$.

From (a) we know A, B are not independent.
$P(A \text{ and } B) = P(B)P(A|B) = (0.4)(0.1) = 0.04$

c) Compute $P(A \text{ or } B)$.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.2 + 0.4 - 0.04$
$= 0.56$

d) Compute $P(A^c)$ and $P(B^c)$.

$P(A^c) = 1 - P(A) = 1 - 0.2 = 0.8$
$P(B^c) = 1 - P(B) = 1 - 0.4 = 0.6$

Example)

A survey of 138 students yielded the following data:

	Eye Color				
	Hazel	Brown	Blue	Green	Total
Math Major	7	30	10	15	62
NonMath Major	22	34	11	9	76
Total	29	64	21	24	138

major

Tip: If totals are not provided, I usually fill them in.

a) Find the probability that a student has blue eyes.

$$P(\text{Blue Eyes}) = \frac{21}{138}$$

b) Find the probability that a student has green eyes and brown eyes.

$$P(\text{Green Eyes and Brown Eyes}) = 0$$ * mutually exclusive

c) Find the probability that a student is a math major.

$$P(\text{Math Major}) = \frac{62}{138}$$

d) Find the probability that a student is a math major and has blue eyes.

$$P(\text{Math Major and Blue Eyes}) = \frac{10}{138}$$

Tip: "Given math major" indicates a restriction to that row only.

e) Given that a student is a math major, find the probability that they have blue eyes.

$$P(\text{Blue Eyes} \mid \text{Math Major}) = \frac{10}{62}$$