TO THE STUDENT

As you start each topic in this book, you will no doubt wonder and ask "Just why should I study this subject and what use does it have in applications?" There is a story about a young mathematics instructor who asked an older professor "What do you say when students ask about the practical applications of some mathematical topic?" The experienced professor said "I tell them!" This text tries to follow that advice. However, you must on your part be reasonable in your request. It is not possible in one book or course to cover both the mathematical methods and very many detailed applications of them. You will have to be content with some information as to the areas of application of each topic and some of the simpler applications. In your later courses, you will then use these techniques in more advanced applications. At that point you can concentrate on the physical application instead of being distracted by learning new mathematical methods.

One point about your study of this material cannot be emphasized too strongly: To use mathematics effectively in applications, you need not just knowledge but *skill*. Skill can be obtained only through practice. You can obtain a certain superficial *knowledge* of mathematics by listening to lectures, but you cannot obtain *skill* this way. How many students have I heard say "It looks so easy when you do it," or "I understand it but I can't do the problems!" Such statements show lack of practice and consequent lack of skill. The *only* way to develop the skill necessary to use this material in your later courses is to practice by solving many problems. Always study with pencil and paper at hand. Don't just read through a solved problem—try to do it yourself! Then solve some similar ones from the problem set for that section,

trying to choose the most appropriate method from the solved examples. See the Answers to Selected Problems and check your answers to any problems listed there. If you meet an unfamiliar term, look for it in the Index (or in a dictionary if it is nontechnical).

My students tell me that one of my most frequent comments to them is "You're working too hard." There is no merit in spending hours producing a solution to a problem that can be done by a better method in a few minutes. Please ignore anyone who disparages problem-solving techniques as "tricks" or "shortcuts." You will find that the more able you are to choose effective methods of solving problems in your science courses, the easier it will be for you to master new material. But this means practice, practice, practice! The *only* way to learn to solve problems is to solve problems. In this text, you will find both drill problems and harder, more challenging problems. You should not feel satisfied with your study of a chapter until you can solve a reasonable number of these problems.

You may be thinking "I don't really need to study this—my computer will solve all these problems for me." Now Computer Algebra Systems are wonderful—as you know, they save you a lot of laborious calculation and quickly plot graphs which clarify a problem. But a computer is a tool; *you* are the one in charge. A very perceptive student recently said to me (about the use of a computer for a special project): "*First* you learn how to do it; *then* you see what the computer can do to make it easier." Quite so! A very effective way to study a new technique is to do some simple problems by hand in order to understand the process, and compare your results with a computer solution. You will then be better able to use the method to set up and solve similar more complicated applied problems in your advanced courses. So, in one problem set after another, I will remind you that the point of solving some simple problems is not to get an answer (which a computer will easily supply) but rather to learn the ideas and techniques which will be so useful in your later courses.

M. L. B.

TI-84: CALCULATING SUMMARY STATISTICS

Use the `STAT`, `CALC`, `1-Var Stats` command to find summary statistics such as mean, standard deviation, and quartiles.

1. Enter the data as described previously.
2. Press `STAT`.
3. Right arrow to `CALC`.
4. Choose `1:1-Var Stats`.
5. Enter `L1` (i.e. `2ND 1`) for List. If the data is in a list other than `L1`, type the name of that list.
6. Leave `FreqList` blank.
7. Choose `Calculate` and hit `ENTER`.

TI-83: Do steps 1-4, then type `L1` (i.e. `2nd 1`) or the list's name and hit `ENTER`.

Calculating the summary statistics will return the following information. It will be necessary to hit the down arrow to see all of the summary statistics.

$\bar{x}$	Mean	n	Sample size or # of data points
Σx	Sum of all the data values	minX	Minimum
Σx^2	Sum of all the squared data values	Q_1	First quartile
Sx	Sample standard deviation	Med	Median
σx	Population standard deviation	maxX	Maximum

TI-83/84: DRAWING A BOX PLOT

1. Enter the data to be graphed as described previously.
2. Hit `2ND Y=` (i.e. `STAT PLOT`).
3. Hit `ENTER` (to choose the first plot).
4. Hit `ENTER` to choose `ON`.
5. Down arrow and then right arrow three times to select box plot with outliers.
6. Down arrow again and make `Xlist: L1` and `Freq: 1`.
7. Choose `ZOOM` and then `9:ZoomStat` to get a good viewing window.

3.3.3 Calculator: binomial probabilities

TI-83/84: COMPUTING THE BINOMIAL COEFFICIENT $\binom{n}{x}$

Use `MATH`, `PRB`, `nCr` to evaluate n choose r. Here r and x are different letters for the same quantity.

1. Type the value of n.
2. Select `MATH`.
3. Right arrow to `PRB`.
4. Choose `3:nCr`.
5. Type the value of x.
6. Hit `ENTER`.

Example: `5 nCr 3` means 5 choose 3.

CASIO FX-9750GII: COMPUTING THE BINOMIAL COEFFICIENT $\binom{n}{x}$

1. Navigate to the `RUN-MAT` section (hit `MENU`, then hit `1`).
2. Enter a value for n.
3. Go to `CATALOG` (hit buttons `SHIFT` and then `7`).
4. Type `C` (hit the `ln` button), then navigate down to the bolded `C` and hit `EXE`.
5. Enter the value of x. Example of what it should look like: `7C3`.
6. Hit `EXE`.

TI-84: COMPUTING THE BINOMIAL FORMULA, $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Use `2ND VARS`, `binompdf` to evaluate the probability of *exactly* x occurrences out of n independent trials of an event with probability p.

1. Select `2ND VARS` (i.e. `DISTR`)
2. Choose `A:binompdf` (use the down arrow to scroll down).
3. Let `trials` be n.
4. Let `p` be p
5. Let `x value` be x.
6. Select `Paste` and hit `ENTER`.

TI-83: Do step 1, choose `0:binompdf`, then enter n, p, and x separated by commas: `binompdf(n, p, x)`. Then hit `ENTER`.

TI-84: COMPUTING $P(X \le x) = \binom{n}{0}p^0(1-p)^{n-0} + \ldots + \binom{n}{x}p^x(1-p)^{n-x}$

Use `2ND VARS`, `binomcdf` to evaluate the cumulative probability of *at most* x occurrences out of n independent trials of an event with probability p.

1. Select `2ND VARS` (i.e. `DISTR`)
2. Choose `B:binomcdf` (use the down arrow).
3. Let `trials` be n.
4. Let `p` be p
5. Let `x value` be x.
6. Select `Paste` and hit `ENTER`.

TI-83: Do steps 1-2, then enter the values for n, p, and x separated by commas as follows: `binomcdf(n, p, x)`. Then hit `ENTER`.

CASIO FX-9750GII: BINOMIAL CALCULATIONS

1. Navigate to `STAT` (`MENU`, then hit `2`).
2. Select `DIST` (`F5`), and then `BINM` (`F5`).
3. Choose whether to calculate the binomial distribution for a specific number of successes, $P(X = k)$, or for a range $P(X \le k)$ of values (0 successes, 1 success, ..., x successes).
   - For a specific number of successes, choose `Bpd` (`F1`).
   - To consider the range 0, 1, ..., x successes, choose `Bcd`(`F1`).
4. If needed, set `Data` to `Variable` (`Var` option, which is `F2`).
5. Enter the value for `x` (x), `Numtrial` (n), and `p` (probability of a success).
6. Hit `EXE`.

G

GUIDED PRACTICE 3.71

Find the number of ways of arranging 3 blue marbles and 2 red marbles.[55]

G

GUIDED PRACTICE 3.72

There are 13 marbles in a bag. 4 are blue and 9 are red. Randomly draw 5 marbles *with replacement*. Find the probability you get exactly 3 blue marbles.[56]

G

GUIDED PRACTICE 3.73

There are 13 marbles in a bag. 4 are blue and 9 are red. Randomly draw 5 marbles *with replacement*. Find the probability you get *at most* 3 blue marbles (i.e. less than or equal to 3 blue marbles).[57]

[55] Here $n = 5$ and $x = 3$. Doing 5 `nCr` 3 gives the number of combinations as 10.
[56] Here, $n = 5$, $p = 4/13$, and $x = 3$, so set `trials` $= 5$, `p` $= 4/13$ and `x value` $= 3$. The probability is 0.1396.
[57] Similarly, set `trials` $= 5$, `p` $= 4/13$ and `x value` $= 3$. The cumulative probability is 0.9662.

4.1.5 Calculator: finding normal probabilities

TI-84: FINDING AREA UNDER THE NORMAL CURVE

Use 2ND VARS, normalcdf to find an area/proportion/probability between two Z-scores or to the left or right of a Z-score.

1. Choose 2ND VARS (i.e. DISTR).
2. Choose 2:normalcdf.
3. Enter the lower (left) Z-score and the upper (right) Z-score.
   - If finding just a lower tail area, set lower to -5.
   - If finding just an upper tail area, set upper to 5.
4. Leave μ as 0 and σ as 1.
5. Down arrow, choose Paste, and hit ENTER.

TI-83: Do steps 1-2, then enter the lower bound and upper bound separated by a comma, e.g. normalcdf(2, 5), and hit ENTER.

CASIO FX-9750GII: FINDING AREA UNDER THE NORMAL CURVE

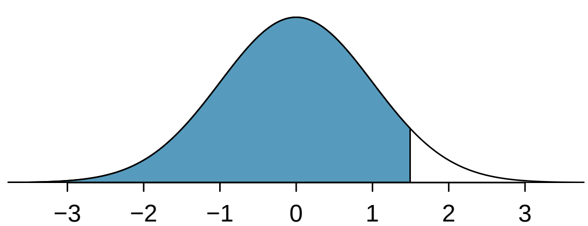
1. Navigate to STAT (MENU, then hit 2).
2. Select DIST (F5), then NORM (F1), and then Ncd (F2).
3. If needed, set Data to Variable (Var option, which is F2).
4. Enter the Lower Z-score and the Upper Z-score. Set σ to 1 and μ to 0.
   - If finding just a lower tail area, set Lower to -5.
   - For an upper tail area, set Upper to 5.
5. Hit EXE, which will return the area probability (p) along with the Z-scores for the lower and upper bounds.

E

EXAMPLE 4.11

Use a calculator to determine what percentile corresponds to a Z-score of 1.5.

Always first sketch a graph:[7]



To find an area under the normal curve using a calculator, first identify a lower bound and an upper bound. Theoretically, we want all of the area to the left of 1.5, so the left endpoint should be -∞. However, the area under the curve is nearly negligible when Z is smaller than -4, so we will use -5 as the lower bound when not given a lower bound (any other negative number smaller than -5 will also work). Using a lower bound of -5 and an upper bound of 1.5, we get $P(Z < 1.5) = 0.933$.

[7] normalcdf gives the result without drawing the graph. To draw the graph, do 2nd VARS, DRAW, 1:ShadeNorm. However, beware of errors caused by other plots that might interfere with this plot.

G

GUIDED PRACTICE 4.12

Find the area under the normal curve to right of $Z = 2$. [8]

G

GUIDED PRACTICE 4.13

Find the area under the normal curve between -1.5 and 1.5. [9]

TI-84: FIND A Z-SCORE THAT CORRESPONDS TO A PERCENTILE

Use `2ND VARS`, `invNorm` to find the Z-score that corresponds to a given percentile.

1. Choose `2ND VARS` (i.e. `DISTR`).
2. Choose `3:invNorm`.
3. Let `Area` be the percentile as a decimal (the area to the left of desired Z-score).
4. Leave μ as `0` and σ as `1`.
5. Down arrow, choose `Paste`, and hit `ENTER`.

TI-83: Do steps 1-2, then enter the percentile as a decimal, e.g. `invNorm(.40)`, then hit `ENTER`.

CASIO FX-9750GII: FIND A Z-SCORE THAT CORRESPONDS TO A PERCENTILE

1. Navigate to `STAT` (`MENU`, then hit `2`).
2. Select `DIST` (`F5`), then `NORM` (`F1`), and then `InvN` (`F3`).
3. If needed, set `Data` to `Variable` (`Var` option, which is `F2`).
4. Decide which tail area to use (`Tail`), the tail area (`Area`), and then enter the σ and μ values.
5. Hit `EXE`.

E

EXAMPLE 4.14

Use a calculator to find the Z-score that corresponds to the 40th percentile.

Letting Area be 0.40, a calculator gives -0.253. This means that $Z = -0.253$ corresponds to the 40th percentile, that is, $P(Z < -0.253) = 0.40$.

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GUIDED PRACTICE 4.15

Find the Z-score such that 20 percent of the area is to the right of that Z-score.[10]

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[8] Now we want to shade to the right. Therefore our lower bound will be 2 and the upper bound will be +5 (or a number bigger than 5) to get $P(Z > 2) = 0.023$.

[9] Here we are given both the lower and the upper bound. Lower bound is -1.5 and upper bound is 1.5. The area under the normal curve between -1.5 and 1.5 = $P(-1.5 < Z < 1.5) = 0.866$.

[10] If 20% of the area is the right, then 80% of the area is to the left. Letting area be 0.80, we get $Z = 0.841$.