MATH 3430-02 EXAM II GUIDE

This is a brief guide for your exam II.

The range is **Lectures [6-1]-[10-3]**. Corresponding to our book, these are mainly Sec. 2.4-2.12. We didn't cover 2.6.2, 2.8.2 and 2.8.3. In 2.8.1, we only discussed Euler equations.

Here are some sample questions (in addition to those on our quizzes):

1. Find a particular solution of the equation

$$y'' - 3y' - 4y = 2\sin(t).$$

2. Use the method of variation of parameters to find a particular solution of the equation

$$y'' - y' - 2y = 2e^{-t}.$$

3. Consider the initial value problem

$$y' + 2y = t, \qquad y(0) = 1.$$

Solve this problem using power series $y(t) = \sum_{n=0}^{\infty} a_n t^n$. (Find the recurrence relation for a_n).

4. Solve the following initial value problem using the power series method:

$$y'' + y' - ty = 0, \qquad y(0) = 1, \quad y'(0) = 2.$$

Find a recurrence relation for the coefficients. Also write down the first 5 nonzero terms in your series solution.

5. Find the general power series solution (centered at $t_0 = 0$) of the equation

$$(1 - t^2)y'' - y = 0.$$

Write down a recurrence relation of the coefficients as well as the first 4 nonzero terms in your solution. What is the radius of convergence of such a series solution?

6. For the equation

$$t^2 y'' + 4ty' + 2y = e^t.$$

First find a basis for the homogeneous solutions; then use the method of variation of parameters to write down a formula for a particular solution.

7. Suppose that a homogeneous linear second order ODE has fundamental solutions u_1, u_2. There is a way to find the wronskian $W = W(u_1, u_2)$ without solving for u_1, u_2 explicitly. Here is an example you'll work on. Consider the equation

$$(1+e^t)y'' - e^t y' + y = 0.$$

Let u_1, u_2 be its fundamental solutions.

(1) Write down the expression of W in terms of u_1, u_2, u_1' and u_2'.

(2) Express u_i'' $(i = 1, 2)$ in terms of t, u_i and u_i'.

(3) By computing and simplifying $\frac{\mathrm{d}W}{\mathrm{d}t}$, find a first order ODE that W satisfies.

(4) Solve the ODE you found in step (3) for W.

8. Consider the following integro-differential equation with an initial value condition:

$$y'(t) + \int_0^t e^{-2(t-\tau)} y(\tau) d\tau = \delta(t-1), \qquad y(0) = 0.$$

Use the method of Laplace transform to find the solution $y(t)$.

9. Solve the following initial value problem using the Laplace tranform:

$$y'' + 2y' + 4y = 1 + (t-1)H_1(t), \qquad y(0) = 1, \quad y'(0) = 2.$$

10. Solve the initial value problem using Laplace transform

$$y'' + y = \sin(t) - 2\delta(t-4), \qquad y(0) = 0, \quad y'(0) = 0.$$

11. Suppose that $f(t)$ is a function defined on $[0, \infty)$ satisfying

$$\mathcal{L}\{f(t)\} = \frac{s^3}{s^4+1},$$

$$f(0) = 1, \quad f'(0) = 0, \quad f(\sqrt{2}\pi) = -\cosh(\pi).$$

(Note: $\cosh(x) = (e^x + e^{-x})/2$.)

(1) Find $\mathcal{L}\{tf(t)\}$. (2) Find $\mathcal{L}\{f''(t)\}$. (3) Find $\mathcal{L}\{e^{-t}f(t)\}$.

(4) Let $g(t) = \mathcal{L}^{-1}\left\{e^{-3s}\frac{s^3}{s^4+1}\right\}$. Find $g(1)$.

(5) Find $\mathcal{L}\{\delta(t-\sqrt{2}\pi)f(t)\}$.