Math 3430-02 Spring 2019
Sample Final Exam

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: ________________________ Signature: ________________________

1. True or False? Explain.

(1) The equation $(x^2 + y)y' = y \cos x + x^2$ is linear.

(2) The equation $y' = \dfrac{3x - 2019y}{2019x - 15y}$ would become exact after multiplying an integrating factor $\mu(x, y)$.

(3) If u_1, u_2 are two solutions of the linear second order equation

$$u'' + e^x u' + (x^2 - 1)u = \cos(x),$$

then $u_1 + u_2$ is also a solution of the same equation.

(4) The general solutions of the third order equation $y''' - 2y'' + y' = 0$ are

$$y = C_1 e^x + C_2 x e^x.$$

2. Find a constant γ such that the following first order ODE becomes exact; then solve the corresponding initial value problem.

$$\left(3x^2y^2 - y\cos(xy) - 2x\right) + \left(\gamma x^3 y - x\cos(xy) - 2y\right)\frac{dy}{dx} = 0, \quad y(1) = \pi.$$

3. Solve the initial value problem

$$y' + \frac{y}{x} = \sin x, \quad y(\pi) = 1.$$

4. Knowing that x is a solution of the equation

$$y'' - \frac{x}{x-1}y' + \frac{1}{x-1}y = 0,$$

use *Reduction of order* to find the general solution of the equation above. (Hint: If you are not sure of how to integrate an expression, think of the quotient rule.)

5. Use Euler's Method (with $\Delta t = 1$) to approximate $y(3)$ for $y(t)$ satisfying

$$y' = t(1 + y^2), \qquad y(0) = 0.$$

6. Consider the ODE

$$y'' + (1 - x^2)y' + xy = 0.$$

Let $\sum_{n=0}^{\infty} a_n x^n$ be its series solution centered at $x = 0$.

Find the *recurrence relation* between the coefficients a_n.

7. Consider the following integro-differential equation with an initial value condition:

$$y'(t) + \int_0^t e^{-2(t-\tau)} y(\tau) d\tau = \delta(t-1), \qquad y(0) = 0.$$

Use the method of Laplace transform to find the solution $y(t)$.

8. Find the general solutions for the homogeneous 1-st order linear system

$$\mathbf{y}' = A\mathbf{y},$$

where

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 4 & 1 \\ 0 & 3 & 2 \end{pmatrix}.$$

9. A 1-st order system $\mathbf{y}' = A\mathbf{y}$ has the general solutions:

$$\mathbf{y}(t) = C_1 e^{2t} \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + C_2 e^{2t} \left(\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} \right) + C_3 e^{-3t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Find A.