In-class Worksheet 10

1. The college physical education department offered an advanced first aid course last semester. The scores on the comprehensive final exam were normally distributed, and the z scores for some of the students are shown below:

$$\{\text{Robert}, 1.10\}\ \{\text{Juan}, 1.70\}\ \{\text{Susan}, -2.00\}\ \{\text{Joel}, 0.00\}\ \{\text{Jan}, -0.80\}\ \{\text{Linda}, 1.60\}$$

If the mean score was $\mu = 150$ with standard deviation $\sigma = 20$, what was the final exam score for each student?

Robert: $z = 1.10$, so the raw score is $1.10 \cdot 20 + 150 = 172$.
Juan: $z = 1.70$, so the raw score is $1.70 \cdot 20 + 150 = 184$.
Susan: $z = -2.00$, so the raw score is $-2.00 \cdot 20 + 150 = 110$.
Joel: $z = 0.00$, so the raw score is $0.00 \cdot 20 + 150 = 150$.
Jan: $z = -0.80$, so the raw score is $-0.80 \cdot 20 + 150 = 134$.
Linda: $z = 1.60$, so the raw score is $1.60 \cdot 20 + 150 = 182$.

2. Let z be a random variable with a *standard* normal distribution. Find the indicated probability.

(a) $P(z \geq 0)$
This is simply 50% because $z = 0$ sits at the mean, and 50% of the area is above the mean.

(b) $P(z \leq 3.20)$
`normalcdf(-1E99,3.20,0,1)` $= 0.9993127979 \approx 99.93\%$

(c) $P(z \geq -1.50)$
`normalcdf(-1.50,1E99,0,1)` $= 0.9331927713 \approx 93.32\%$

(d) $P(-1.78 \leq z \leq 1.40)$
`normalcdf(-1.78,1.40,0,1)` $= 0.8817053583 \approx 88.17\%$

3. Let x be a random variable with a normal distribution with mean $\mu = 15$ and standard deviation $\sigma = 4$. Find the indicated probability.

(a) $P(x \leq 12)$
`normalcdf(-1E99,12,15,4)` $= 0.2266272794 \approx 22.66\%$

(b) $P(x > 10)$
`normalcdf(10,1E99,15,4)` $= 0.894350161 \approx 89.44\%$

(c) $P(15 \leq x \leq 20)$
`normalcdf(15,20,15,4)` $= 0.3943501605 \approx 39.44\%$

(d) $P(8 < x < 22)$
`normalcdf(8,22,15,4)` $= 0.9198817729 \approx 91.99\%$

4. Find the z-value described.

   (a) Find z such that 5.2% of the standard normal curve lies to the left of z.
   This is a left-tail area, so $z =$ `invNorm(0.052, 0, 1) = -1.625763385`.

   (b) Find z such that 5% of the standard normal curve lies to the right of z.
   Since this is a right-tail area, we can first translate this to a left-tail area by equating "5% to the right of z" with "95% to the left of z."

   Then $z =$ `invNorm(0.95,0,1)` $= 1.644853626$.

   Note that alternatively, symmetry can be used with `invNorm(0.05,0,1) = -1.644853626` to conclude $z = 1.644853626$.

   (c) Find z such that 95% of the standard normal curve lies between $-z$ and $+z$.
   To use `invNorm`, we must first translate this to a left-tail area problem. If 95% of the area lies *between* $-z$ and $+z$, then we can equate this to 97.5% lies to the left of $+z$. Then $z =$ `invNorm(0.975, 0, 1)` $= 1.959963986$, and $-z = -1.959963986$.

5. Suppose that the scores for a Chemistry midterm had a normal distribution with mean $\mu = 71.4$ and a standard deviation of $\sigma = 8.3$.

   (a) Find the exam score for a student that scored in the 80^{th} percentile.
   The 80^{th} percentile refers to the score where 80% of the class earned that score or below. That is, the left-tail area related to this score is 80%. So, $x =$ `invNorm(0.80, 71.4, 8.3)` $= 78.38545624 \approx 78.4$.

   (b) Find the exam score for a student that scored in the 25^{th} percentile.
   The 25^{th} percentile refers to the score where 25% of the class earned that score or below. That is, the left-tail area related to this score is 25%. So, $x =$ `invNorm(0.25, 71.4, 8.3)` $= 65.80173508 \approx 65.80$.

   (c) Find the exam score for a student who score lower than 60% of the class.
   Scoring lower than 60% of the class implies that the right-tail area related to this score is 60%. However, since the `invNorm` function requires the left-tail area as its input, we must first translate this problem to the left-tail area of 40%. Then, $x =$ `invNorm(0.40, 71.4, 8.3)` $= 69.29721906 \approx 69.30$.

6. Suppose the distribution of heights of American men (20 years of age and older) is approximately normal with a mean of 69.4 inches and a standard deviation of 3 inches.

   (a) What is the z value for a height of 6 feet?
   First we convert 6 feet to 72 inches. Then $z = \frac{72-69.4}{3} = 0.86666667$.

   (b) What percentage of American men (20 years of age and older) are shorter than 6 feet?
   We can use `normalcdf`.

   $$P(z < 0.86666667) = \texttt{normalcdf(-1E99, 0.86666667, 0, 1)} = 0.8069377087 \approx 80.69\%$$

   (c) What is the z value for a height of 6 feet 4 inches?
   First we convert 6 feet 4 inches to 76 inches. Then $z = \frac{76-69.4}{3} = 2.2$.

   (d) What percentage of American men (20 years of age and older) are taller than 6 feet 4 inches?
   We can use `normalcdf`.

   $$P(z > 2.2) = \texttt{normalcdf(2.2, 1E99, 0, 1)} = 0.0139033989 \approx 1.39\%$$

7. Accrotime is a manufacturer of quartz crystal watches. Accrotime researchers have shown that the watches have an average life of 28 months before certain electronic components deteriorate, causing the watch to become unreliable. The standard deviation of watch lifetimes is 5 months, and the distribution of lifetimes is normal.

   (a) If Accrotime guarantees a full refund on any defective watch for 2 years, what percentage of total production will the company expect to replace?
   First we need to make sure all time intervals are measured in the same units. So, we are looking for the percentage of watches that are expected to fail in 24 months or fewer.
   $P(x \leq 24) = \texttt{normalcdf(-1E99, 24, 28, 5)} = 0.2118553337$. This means that the company should expect to replace about 21.19% of its watches.

   (b) If Accrotime does not want to make refunds on more than 12% of the watches it makes, how long should the guarantee period be (to the nearest month)?
   Certainly, it is shorter than 24 months, since that led to 21.19% replaced. To determine a more precise number of months, we can use `invNorm` where we are looking for a left-tailed area equal to 0.12.
   `invNorm(.12, 28, 5)` = 22.12506604. This implies that the guarantee period should be 22 months.