SELF-ASSESSMENT 05

(your name)
(executive summary)
(normalized participation score)
(normalized reading score)
(normalized problem set score)

8. Statistics

A **probability space** $(\Omega, \mathcal{A}, \mathrm{P})$ consists of a set Ω, a **σ-algebra** $\mathcal{A}$ and a **probability measure** P. A σ-algebra is a collection of subset of Ω which contains the empty set and which is closed under the operations of taking complements, countable unions and countable intersections. The function P on $\mathcal{A}$ takes values in the interval $[0,1]$, satisfies $\mathrm{P}[\Omega] = 1$ and $\mathrm{P}[\bigcup_{A \in S} A] = \sum_{A \in S} \mathrm{P}[A]$ for any finite or countable set $S \subset \mathcal{A}$ of pairwise disjoint sets. The elements in $\mathcal{A}$ are called **events**. Given two events A, B where B satisfies $\mathrm{P}[B] > 0$, one can define the **conditional probability** $\mathrm{P}[A|B] = \mathrm{P}[A \cap B]/\mathrm{P}[B]$. Bayes theorem states:

Theorem: $\mathrm{P}[A|B] = \mathrm{P}[B|A]\mathrm{P}[A]/\mathrm{P}[B]$

3

FUNDAMENTAL THEOREMS

The setup stated the **Kolmogorov axioms** by Andrey Kolmogorov who wrote in 1933 the "Grundbegriffe der Wahrscheinlichkeit" [209] based on measure theory built by Emile Borel and Henry Lebesgue. For history, see [291], who report that "Kolmogorov sat down to write the Grundbegriffe, in a rented cottage on the Klyaz'ma River in November 1932". Bayes theorem is more like a fantastically clever definition and not really a theorem. There is nothing to prove as multiplying with $\mathrm{P}[B]$ gives $\mathrm{P}[A \cap B]$ on both sides. It essentially restates that $A \cap B = B \cap A$, the Abelian property of the product in the ring $\mathcal{A}$. More general is the statement that if $A_1, \dots, A_n$ is a disjoint set of events whose union is Ω, then $\mathrm{P}[A_i|B] = \mathrm{P}[B|A_i]\mathrm{P}[A_i]/(\sum_j \mathrm{P}[B|A_j]\mathrm{P}[A_j]$. Bayes theorem was first proven in 1763 by Thomas Bayes. It is by some considered to the theory of probability what the Pythagoras theorem is to geometry. If one measures the ratio applicability over the difficulty of proof, then this theorem even beats Pythagoras, as no proof is required. Similarly as "a+(b+c)=(a+b)+c", also Bayes theorem is essentially a definition but less intuitive as "Monty Hall" illustrates [279]. See [199].

1. Review of the last 3 weeks

```
03M
 2019-blitzstein-sec-1-1-why-study-probability.pdf
 JIBLM
 lec
 scratch
03W
 r_code_from_r_sections.txt
03Z
 2019-09-13-.png
 chebyshev-theorem-worksheet.pdf
 dearborn-elementary-probability-theory.pdf
 numerical-summaries-worksheet.pdf
 presentation-groups.md
 presentation-groups.pdf
 presentation-instructions.md
 presentation-instructions.pdf
 section-4-1-brase-and-brase-lecture-notes.pdf
 self-assessment.md
 self-assessment.pdf
 summary-statistics.md
 summary-statistics.pdf
04M
 2010-chances-are-edit.md
 2010-chances-are.pdf
 2019-09-15.pdf
 2019-09-15.tex
 relative-freq.jpg
04W
 2019-09-17.pdf
 2019-09-17-probability-notions.png
 2019-09-17.tex
 ch_probability
 quiz-04.md
04Z
 quiz-04.md
 quiz-04.pdf
05M
 2008-brase-and-brase-understandable-statistics-ch-4-and-5-review.pdf
 2019-09-23-guided-notes.md
 2019-09-23-guided-notes.pdf
 2019-09-23-pset-week-05.pdf
05W
 2019-09-25-group-problems-conditional-prob-and-binomial-distr.pdf
05Z
 2014-samaniego-discrete-probability-models-sections-1-and-2.pdf
 quiz-05.md
 quiz-05.pdf
```

6. "Theory week"

Reading list.

```
06M
 2019-openintro-statistics-distributions-of-random-variables-binomial.pdf
06W
 2019-openintro-statistics-distributions-of-random-variables-normal.pdf
06Z
 2019-blitzstein-hwang-recap-random-variables-with-R-code.pdf
 2019-openintro-statistics-distributions-of-random-variables-chapter-4-exercises.pdf
```

6.1. Sep 30.

- Lecture 06M: Probability distributions I
- Problem set: Expected values, variances, binomial distributions

6.2. Oct 2.

- Lecture 06W: Probability distributions I: Normal distributions
- Problem set: normal distributions, Chebyshev's inequality

... some **phase transition** appears. Computing these values is often important. The concept of **maximizing entropy** explains many things like why the Gaussian distribution is fundamental as it maximizes entropy. Measures maximizing entropy are often special and often **equilibrium measures**. This is a central topic in statistical mechanics [282, 283]. In combinatorial topology, the **upper bound theorem** was a milestone. It was long a conjecture of Peter McMullen and then proven by Richard Stanley that **cyclic polytopes** maximize the volume in the class of polytopes with a given number of vertices. Fundamental area also some **inequalities** [131] like the **Cauchy-Schwarz inequality** $|a \cdot b| \leq |a||b|$, the **Chebyshev inequality** $\mathrm{P}[|X - [\mathrm{E}[X]| \geq |a|] \leq \mathrm{Var}[X]/a^2$. In complex analysis, the **Hadamard three circle theorem** is important as gives bounds between the maximum of $|f|$ for a holomorphic function f defined on an annulus given by two concentric circles. Often inequalities are more fundamental and powerful than equalities because they are more widely used. Related to inequalities are **embedding theorems** like **Sobolev embedding theorems**. For more inequalities, see [53]. Apropos embedding, there are the important Whitney or Nash embedding theorems which are appealing. ...

6.3. Oct 4.

- Research peer-group rosters by 8am.
- Quiz 06: Uniform, Normal, and Binomial Distributions
- Lecture 06F: Introduction to R.
- Individual presentation problems due by 11am.

7. "Practice week"

7.1. **Oct 7.**

- Introduction to R.
- Project proposals due.

7.2. **Oct 9.**

- Review I

7.3. **Oct 11.**

- Review II

8. "Midterms week"

8.1. **study materials.**

```
midterm01
 2019-10-16-midterm-1-syllabus.pdf
 2019-10-16-midterm-1-syllabus.txt
 2019-os4-cumulative-review-highlights.pdf
 2019-os4-cumulative-review-TI-84-functions.pdf
 2019-os4-cumulative-review-week-01-sampling-to-week-06-rvs-and-distributions.pdf
```

8.2. **Oct 14.**

- Review III
- Submissions for project drafts open.
- *Final drafts due by 12:00am, midnight, the morning of Wed Oct 16.*

8.3. **Oct 16.**

- *Projects final submission deadline, 2019-09-26-00:00 (Mountain Time), midnight between evening 25th and morning of 26th.*
- *In-class written midterm, 7:55am–8:55am*

9. Questions

What else should be prepared for?

`Reply below. Thanks!`	Requests?	Comments?	Suggestions?