MATH 3430-02 WEEK 8-2

Key Words: Series solutions II: Undetermined coefficients, shifting of summation index.

The method of undetermined coefficients (series method) can be made more efficient with the use of shifting of summation indices, which I'll explain below.

Q1. Write the first 5 terms of the summation:

$$\sum_{n=0}^{\infty}(2n+1)x^{n+2}.$$

Q2. Write the first 5 terms in the summation:

$$\sum_{n=2}^{\infty}(2n-3)x^{n}.$$

Q3. How do you think the power series in **Q1** and **Q2** are related? Can you justify your claim?

Q4. Now rewrite the following summation in a form where the (general) power of x is n.

$$\sum_{n=1}^{\infty}(3n+2)x^{n-1}.$$

Q5. Rewrite the following expression as a few terms plus a single infinite summation, where, in the infinite summation, the power of x is n.

$$\sum_{n=1}^{\infty}(2n-1)x^n + \sum_{n=3}^{\infty}(n+3)x^n.$$

It is not a formal terminology, but let's say that an infinite summation (power series) is in 'standard form' if the power of x in it is n.

Q6. Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Put $y''(x)$ in the standard form.

Q7. Assuming a solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ for the equation

$$y'' - xy' + x^2 y = 0,$$

find relations satisfied by all a_n.

Such relations are called the **recurrence relations** of a_n. When, in the equation $y'' + p(x)y' + q(x) = 0$, $p(x), q(x)$ are polynomials, we can always find recurrence relations between the a_n.

Q8. Solve the equation

$$y'' - xy = 0$$

using the method of undetermined coefficients. In particular, derive a recurrence relation between all the coefficients of x^n.