

MATH 3430-02 QUIZ 3

Name: _____

Approximate $y(1)$ of the following initial value problem using Euler's Method for $n = 2$.

$$\frac{dy}{dt} = \frac{1}{1 + t^2 + y^2}, \quad y(0) = 0.$$

Below a table is provided to help you. (*I recommend first putting the formulae for y_k and y'_k as well as the values of t_k , Δt in the respective entries.*)

k	t_k	$y_k =$	$y'_k =$	Δt (or h)
0				
1				
2				

Ans: $y_2 =$ _____.

Now consider the choice:

$$a = 1, \quad b = 1,$$

and the expressions

$$M = \max_{(t,y) \in \mathcal{R}} |f(t,y)|, \quad \alpha = \min \left\{ a, \frac{b}{M} \right\}.$$

Use this information (with suitable calculations) to explain why it is reasonable to use Euler's Method to approximate $y(1)$. (Hint: What guarantees that a solution exists at $t = 1$ and is unique?)

Is it reasonable to use Euler's Method to approximate $y(2019)$? (Hint: Does M depend on the choice of a, b ? How can you choose a, b to obtain a larger α ?)