

## 5.1 Discrete Probability Distributions

1. Refer to the “sum of two dice” exercise as an example of such a probability distribution.
2. Discuss **expected value** and **standard deviation** of a probability distribution. Using the example of the two dice is helpful. Note that  $E(X) = 7$ ,  $\text{Var}(X) = \frac{105}{18} \approx 5.83$ , and  $\sigma \approx 2.415$ .
  - Point how the formulas are like the formulas we already know for mean, variance and standard deviation but also what makes them different.

$$E(X) = \mu = \sum xP(X = x)$$

$$\text{Var}(x) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2 = E(X^2) - E(x)^2.$$

- These last two formulas for variance are not in the book, but are easier to use if someone were to compute them by hand.
  - Note that 1-Var Stats can still get the job done for us, like a weighted average or frequency table. Use the probabilities in the “frequency table”.
3. It is worth pointing out, just for completeness, that  $E(X)$  is a linear function, meaning  $E(\alpha X + \beta Y) = \alpha E(X) + \beta E(Y)$ . Also,  $\text{Var}(\alpha X) = \alpha^2 \text{Var}(X)$ . It is not true that  $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$  in general, but it is true when  $X$  and  $Y$  are independent. Covariance is somewhat talked about in this class when we talk about linear correlation. Maybe it is worth mentioning off-hand that  $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$  whenever  $X$  and  $Y$  are uncorrelated.

## 5.2 & 5.3 Binomial Distribution

1. Describe the features of a binomial experiment. Begin with the example of flipping a coin 10 times in a row and counting the number of heads (which we consider a success).
  - A **fixed number**  $n$  of trials.
  - Each trial is independent of all others.
  - Each trial has two outcomes: a success (with probability  $p$ ) and failure (with probability  $1 - p = q$ ).
  - The goal is to count the number of successes  $r$  in  $n$  trials.
2. Present the formula,  $P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r} = \binom{n}{r} p^r q^{n-r}$ .

Because we didn't cover Section 4.3 about counting, the reference to  $C_{n,r} = \binom{n}{r}$  will be to state that it is the number of ways that the  $r$  successes could have fallen in those  $n$  trials. I do list all the ways to get 2 heads in 4 flips, just to emphasize the point.

3. Note that the TI calcs have `binompdf` and `binomcdf`. Refer them to the screencasts if they don't know how to use those functions already, but make a point to review them right before the worksheet next Wednesday.

Specifically (on Wednesday) review the syntax and application of `binompdf(n,p,r)` which computes the probability of EXACTLY  $r$  successes out of  $n$  trials,  $P(X = r)$ , while `binomcdf(n,p,r)` computes the probability of at most  $r$  success out of  $n$  trials,  $P(X \leq r)$ .

4. Discuss how we can compute the probability of at least  $r$  successes (say) when neither function is explicitly designed to do that by using compliments. Namely,  $P(X > r) = 1 - P(X \leq r)$ .

Drawing a chart and emphasizing the events and their compliments is very helpful. This is one of the more confusing topics for the students.

5. A quick note of the formulas for expected value and standard deviation for a binomially distributed random variable. They won't use 1-Var Stats, most likely, because the formulas are so much simpler.

$$E(X) = np$$

$$Var(x) = npq = np(1 - p)$$