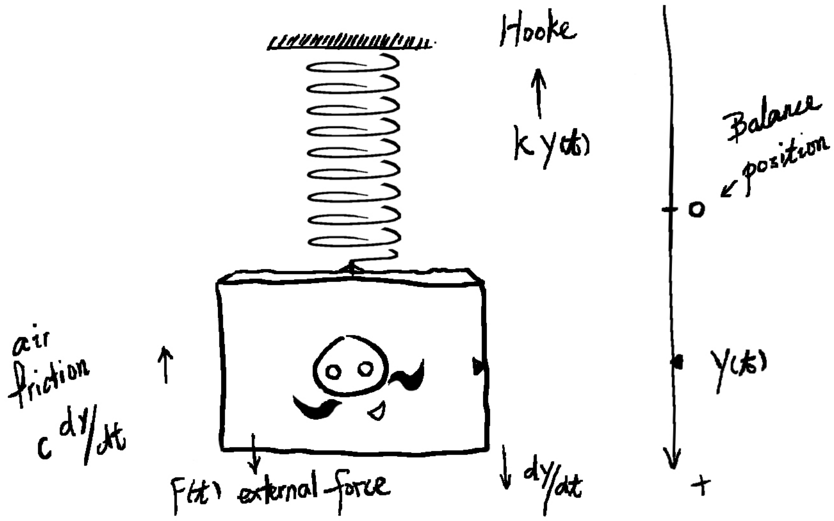
MATH 3430-02 WEEK 7-2

Key Words: Mass on a spring.

This lecture we study an application of constant coefficient 2nd order ODEs.

By 'a mass on a spring' we mean a mechanical system illustrated by the following picture:



The total force applied to the 'mass' is

The accelaration of the mass is

By Newton's second law of motion,

Thus the equation of motion of a 'mass on a spring' is a constant coefficient 2nd order ODE. Note that all parameters m, c, k are nonnegative. (In particular, it is natural to assume m, k to be positive.)

Several terminologies will bring us convenience:

The system is said to be

i. forced if $F(t) \neq 0$;
ii. free if $F(t) = 0$;
iii. damped if $c > 0$;
iv. undamped if $c = 0$.

1. Free, undamped

The equation looks like

$$my'' + ky = 0.$$

Describe all the solutions. (The solutions can be put in the form $y(t) = A\cos(\omega_0 t - \phi)$, where A is the amplitude, ω_0 is called the 'natural frequency' of the system.)

2. Free, damped

The equation looks like

$$my'' + cy' + ky = 0.$$

Describe all the solutions. (Depending on the sign of $c^2 - 4mk$, we encounter cases of 'underdamped', 'overdamped', 'critically damped'. I'll explain.)

3. Forced, undamped

When the force is of the kind: $F(t) = a\cos\omega t$, the equation looks like

$$my'' + ky = a\cos\omega t.$$

What are the possible solutions? (The answer depends on whether $\omega = \sqrt{k/m}$.)

4. Forced, damped

Again, consider the case when $F(t) = a\cos\omega t$. The equation looks like

$$my'' + cy' + ky = a\cos\omega t. \quad (c > 0)$$

Describe all solutions. (In this case, there will be the notion of a 'gain' and a 'phase shift' that is intrinsic to the system. I'll explain.)