Math 3430-02 Spring 2019
Exam I

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: Solutions   Signature: d / dt

Instructions:

- Notes, books, calculators or computers are not allowed in this exam.
- Make sure you show the work that leads to your answer to receive full credit. If you are using a theorem or a fact to draw some conclusions, quote the result.
- This test contains **5 pages** and **5 questions**. You have **50 minutes** to answer all the questions.

Good Luck !

Question	Score
1	/ 20
2	/ 20
3	/ 20
4	/ 20
5	/ 20
Total	/100

1. Classify the following 1-st order ODEs. (Check all that apply.)

ODE	Linear	Separable	Exact
$-\cos^2 y + (t^2+1)\frac{dy}{dt} = 0$		✓	
$(\cos t + e^y - y) + (te^y - t)\frac{dy}{dt} = 0$			✓
$y\cos t + t^2\frac{dy}{dt} = 0$	✓	✓	
$t^2 + (y+1)\frac{dy}{dt} = 0$		✓	✓

I'm leaving some space for you to calculate a bit, if needed. I'll not read your calculation, but only your 'checked answers' above.

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2. Solve the following initial value problem. You may leave your answer in an implicit form.

$$\underbrace{(y\cos t + e^{2y})}_{M} + \underbrace{(\sin t + 2te^{2y} + 2y)}_{N}\frac{dy}{dt} = 0, \qquad y(0) = 1.$$

$N_t = \cos t + 2e^{2y} = M_y$ $\quad\therefore$ Exact.

$F(t,y) = \int M\,dt + h(y) = y\sin t + e^{2y}t + h(y)$

It turns out $\quad F_y = \sin t + 2e^{2y}t + h'(y)$

$\underset{\text{(should)}}{=} \sin t + 2te^{2y} + 2y = N$

$\therefore$ can choose $h(y) = y^2$. $\Rightarrow F(t,y) = y\sin t + e^{2y}t + y^2$

But $y(0)=1 \quad \therefore \quad F(t,y) = 1$

$\Rightarrow$ Sol: $\boxed{y\sin t + e^{2y}t + y^2 = 1}$

3. Given that $y_1(t) = t^{-1}$ is a solution of the second order equation

$$t^2y'' + 3ty' + y = 0, \qquad t > 0,$$

use the method of *reduction of order* to find all solutions of this equation. (You should justify that the two fundamental solutions you've found are in fact linearly independent.)

Useful formula:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{(y_1(t))^2 \exp(\int p(t)\mathrm{d}t)}.$$

$y_1(t) = \frac{1}{t}$

$p(t) = \frac{1}{1/3t}$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{1}{\left(\frac{1}{t}\right)^2 e^{3\ln t}} = \frac{t^2}{t^3} = \frac{1}{t}$$

$\therefore$ can take $v(t) = \ln t$.

all solutions : $y(t) = t^{-1}(C_1 + C_2 \ln t)$.

4. Consider the inhomogeneous ODE

$$y'' - 4y' + 4y = e^{2t}.$$

It has a particular solution of the form

$$y_p(t) = At^2 e^{2t},$$

where A is a constant to be determined.

Find A, then write down the general solutions of the original inhomogeneous equation.

Substituting $y_p(t)$ into the ODE, we obtain :

$$(2A + 8At + 4At^2)\, e^{2t} - 4\,(2At + 2At^2)\, e^{2t} + 4Ae^{2t} \cdot t^2$$

$$= 2Ae^{2t} \underset{\text{(should)}}{=} e^{2t}$$

$$\therefore \quad A = 1/2.$$

By $p(r) = r^2 - 4r + 4$ roots $r = 2$ mult $= 2$

$$y_h = C_1 e^{2t} + C_2 t e^{2t}.$$

Conclusion:

$$y(t) = \left(C_1 + C_2 t + \frac{1}{2} t^2\right) e^{2t}.$$

5. True or False? Explain.

(1) Applying Euler's method to the initial value problem

$$y' = y^2 - 4, \qquad y(1) = 2$$

always yields accurate solutions.

True. Since $y(1) = 2$, each y' computed is 0, thus each y approximated is 2.

(note: $y(t) = 2$ is a ‹the› solution.)

(2) If u_1, u_2 are two solutions of the linear second order equation

$$u'' + e^t u' + (t^2 - 1)u = \cos t,$$

then $u_1 + u_2$ is also a solution of the same equation.

False. Equation is linear but in-homogeneous.

(3) The general solutions of the second order equation $y'' - 2y' + y = 0$ are

$$y = Ce^t.$$

False. Missing te^t's.

(4) Every first order ODE of the form $y' = f(y)$ is linear, where $f(y)$ is an arbitrary function in y.

False. e.g. $y' = y^2$ is not linear.