

Math 3430-02 Spring 2019

Exam II—Supplementary Problems

Due Monday Apr. 22, 2019

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: _____

Signature: _____

(Each question below is worth 20 points. Staple this sheet at the top when turning in.)

1. Consider the following initial value problem:

$$e^t y'' - ty' + 3y = 0, \quad y(0) = 1, \quad y'(0) = -1.$$

- (1) Use the method of **successive differentiation** to find $y''(0)$ and $y'''(0)$.
(2) Write down the first four terms (i.e., up to the x^3 -term) in the series solution to the initial value problem above.
2. Use the method of **undetermined coefficients** to solve (i.e., finding a recurrence relation for the power series solution of the form $\sum_{k=0}^{\infty} a_k t^k$)

$$\begin{cases} y'' - 2ty' + y = 0, \\ y(0) = 1, \quad y'(0) = 2. \end{cases}$$

3. Find examples for the following if they exist, or explain if they do not.

- (1) An **initial value problem** whose solution is $y(x) = e^t * \sin(t)$, assuming part of the initial values being $y'(0) = 0$, where ‘*’ stands for ‘convolution’.
(2) A **second order ODE** whose general solutions are $C_1 t^{-3} + C_2 t^{-3} \ln t$ ($t > 0$), where C_1 and C_2 are arbitrary constants.

4. Solve the following initial value problem for $t > 0$ using the Laplace transform.

$$y'' + 2y' + 4y = f(t), \quad f(t) = \begin{cases} 1, & t < 1, \\ t, & t \geq 1. \end{cases}$$

with initial conditions

$$y(0) = 1, \quad y'(0) = 2.$$

5. Find an initial value problem of the form

$$y'' + Ay' + By = f(t), \quad y(0) = a, \quad y'(0) = b \quad (t > 0)$$

which has the solution

$$y(t) = 3e^{-t} \cos(t) + u_3(t)e^{-(t-3)} \sin(t-3) - 2 \int_0^t e^{\tau} \sin(t-\tau) d\tau.$$

(In other words, the goal is to find constants A, B, a, b and a function $f(t)$.)