2.

Worksheet: Testing the Mean

(1) If the P-value in a statistical test is less that or equal to the level of significance for the test, do we reject or fail to reject H_0? Does this imply that there IS or IS NOT enough evidence in the data (and the test being used) to justify the rejection of H_0?

When the P-value is less than or equal to the level of significance, we REJECT the null hypothesis H_0. This means that there IS enough evidence in the data to justify the rejection of H_0 and choose the alternate hypothesis H_1...although it is NOT proof that H_1 is true beyond all doubt.

(2) *Weatherwise* magazine is published in association with the American Meteorological Society. Volume 46, Number 6 has a rating system to classify Nor'easter storms that frequently hit New England states and can cause much damage near the ocean coast. A *severe* storm has an average peak wave height of 16.4 feet for waves hitting the shore. Suppose that a Nor'easter is in progress at the severe storm class rating.

(a) Let us say that we want to set up a statistical test to see if the wave action (i.e., height) is dying down or getting worse. What would be the null hypothesis regarding average wave height?
$H_0 : \mu = 16.4$ feet.

(b) If you wanted to test the hypothesis that the storm is getting worse, what would you use for the alternate hypothesis?
$H_1 : \mu > 16.4$ feet.

(c) If you wanted to test the hypothesis that the waves are dying down, what would you use for the alternate hypothesis?
$H_1 : \mu < 16.4$ feet.

(d) Suppose you do not know whether the storm is getting worse or dying out. You just want to test the hypothesis that the average wave height is *different* (either higher or lower) from the severe storm class rating. What would you use for the alternate hypothesis?
$H_1 : \mu \neq 16.4$ feet.

(e) For each of the tests in parts (b), (c), and (d), would the area corresponding to the P-value be on the left, on the right, or on both sides of the mean? Explain your answer in each case.
(b) Right; (c) Left; (d) Both Sides. That is, for (b), we use a right-tailed test; for (c), we use a left-tailed test; and for (d), we use a two-tailed test.

(3) Gentle Ben is a Morgan horse at a Colorado dude ranch. The mean glucose level for horses should be $\mu = 85$ mg/100 ml (Reference: *Merck Veterinary Manual*). Over the past 8 weeks, a veterinarian took weekly glucose readings from this horse (in mg/100 ml) and found the sample mean $\bar{x} = 93.8$ mg/100 ml. Do the data indicate that Gentle Ben has an overall average glucose level higher than 85 mg/100 ml?

(a) State the appropriate null and alternate hypothesis for this test. Is this a left-tailed, right-tailed, or two-tailed test?
$H_0 : \mu = 85$mg/100ml $H_1 : \mu > 85$mg/100/ml This is a right-tailed test.

(b) If we assume that x has a normal distribution and that we know from past experience that $\sigma = 12.5$, then the corresponding P-value is about 0.0232. Verify this is correct using the appropriate statistical test. At the $\alpha = 0.05$ level, do these data indicate that Gentle Ben has an overall average glucose level higher than 85 mg/100 ml? Explain.
Using `Z-Test`, one verifies that the P-value is about 0.0232. Since $P \leq \alpha$, at the 0.05 significance level we choose to REJECT the null hypothesis. In other words, the data suggests (but do NOT prove) that Gentle Ben has an overall average glucose level higher than 85mg/100ml.

(4) The price-to-earnings (P/E) ratio is an important tool in financial work. A recent copy of the *Wall Street Journal* indicated that the P/E ratio of the entire S&P 500 stock index is $\mu = 19$. A random sample of 14 large U.S. banks (J.P. Morgan, Bank of America, and others) had a sample mean of $\bar{x} \approx 17.1$. Do these data indicate that the P/E ratio of all U.S. bank stocks is less than 19?

(a) State the appropriate null and alternate hypothesis for this test. Is this a left-tailed, right-tailed, or two-tailed test?
$H_0 : \mu = 19$ $H_1 : \mu < 19$ This is a left-tailed test.

(b) If we assume that x has a normal distribution and that the sample standard deviation is $s = 4.52$, then the corresponding P-value is about 0.0699. Verify this is correct using the appropriate statistical test. At the $\alpha = 0.05$ level, do these data indicate that the P/E ratio of all U.S. bank stocks is less than 19?
Using `T-Test`, one verifies that the P-value is about 0.0699. Since $P > \alpha$, at the 0.05 significance level, we FAIL TO REJECT the null hypothesis. In other words, the data is not strong enough to suggest that the P/E ration of all U.S. bank stocks is less than 19.

(5) *USA Today* reported that the state with the longest mean life span is Hawaii, where the population mean life span is 77 years. A random sample of 20 obituary notices in the *Honolulu Advertizer* gave the following information about life span (in years) of Honolulu residents:

72, 68, 81, 93, 56, 19, 78, 94, 83, 84 77, 69, 85, 97, 75, 71, 86, 47, 66, 27

Assuming that the life span in Honolulu is approximately normal distributed, does this information indicate that the population mean life span for Honolulu residents is less than 77 years? Use a 5% level of significance.

(a) State the null and alternate hypothesis.
$H_0 : \mu = 77$ $H_1 : \mu < 77$

(b) What sampling distribution should be used? Explain.
Because the population standard deviation is not known for this data, the Student's t distribution with $d.f. = 19$ is the more appropriate distribution. Note, the information is provided that x is approximately normally distributed, so there is not a concern about the small sample size.

(c) Is this a right-tailed, a left-tailed, or two-tailed test? Find the P-value.
Because $H_1 : \mu < 77$, this is a left-tailed test. Using `T-Test` with `Inpt: Data` and the above values entered in list L_1, $\mu_0 : 77$, `Freq: 1`, and $\mu :< \mu_0$, we get $P = 0.1200213854$.

(d) Will you reject or fail to reject the null hypothesis? Explain and interpret this conclusion.
Because the α-level was set at $\alpha = 0.05$, $P > \alpha$. Therefore, we fail to reject the null hypothesis. That is, at the 5% level, the evidence is not strong enough to conclude that the population mean life span is less that 77 years.

(6) *Weatherize* is a magazine published by the American Meteorological Society. One issue gives a rating system used to classify Nor'easter storms that frequently hit New England and can cause much damage near the ocean. A severe storm has an average peak wave height of $\mu = 16.4$ feet for waves hitting the shore. Suppose that a Nor'easter is in progress at the severe storm class rating. Peak wave heights are usually measured from land (using binoculars) off fixed cement piers. Suppose that a reading of 36 waves showed an average wave height of $\bar{x} = 17.3$ feet. Previous studies of severe storms indicate that $\sigma = 3.5$ feet. Does this information suggest that the storm is (perhaps temporarily) increasing above the severe rating? Use $\alpha = 0.01$. (Note that although this problem has not itemized out the steps, like the previous problems on this worksheet, a complete solution will include all such steps.)

$H_0 : \mu = 16.4$ $H_1 : \mu > 16.4$ Because $n = 36 > 30$ and σ is known, we can use the standard normal distribution with a right-tailed test. Using `Z-Test` with $\mu_0 = 16.4$, $\sigma = 3.5$, $\bar{x} = 17.3$, $n = 36$, and $\mu > \mu_0$, we get $z = 1.542857143$ and $P = 0.0614327356$. At an α-level of 0.01, we fail to reject the null hypothesis. That is, at the 1% level, there is insufficient evidence to support the claim that the storm is increasing above the severe rating.

3.

Worksheet: Testing the Proportion

(1) Women athletes at the University of Colorado, Boulder, have a long-term graduation rate of 67% (Source: *Chronicle of Higher Education*). Over the past several years, a random sample of 38 women athletes at the school showed that 21 eventually graduated. Does this indicate that 67% is now an overestimate for the population proportion of women athletes who graduate from the University of Colorado, Boulder? Use a 5% level of significance.

(a) State the null and alternate hypothesis.
$H_0 : p = 0.67$ $H_1 : p < 0.67$

(b) What sampling distribution should be used? Explain.
Because we considering a proportion of the population and both $np = (38) * (.67) = 25.46 > 5$ and $nq = (38) * (.33) = 12.54 > 5$, we should use the normal distribution as an approximation of the binomial distribution.

(c) Is this a right-tailed, a left-tailed, or two-tailed test? Find the P-value.
Because $H_1 : p < 0.67$, this is a left-tailed test. Using `1-PropZTest` with $p_0 : 0.67$, $x : 21$, $n : 38$, and `prop` $< p_0$, we get $P = 0.061941703$.

(d) Will you reject or fail to reject the null hypothesis? Explain and interpret this conclusion.
Because the α-level was set at $\alpha = 0.05$, $P > \alpha$. Therefore, we fail to reject the null hypothesis. That is, at the 5% level, the evidence is not strong enough to reject the statement that the graduation rate is 67%.

(2) A large survey of countries, including the United States, China, Russia, France, Turkey, Kenya, and others, indicated that most people prefer the color blue. In fact, about 24% of the population claim blue as their favorite color. (Reference: Study by J.Bunge and A.Freeman-Gallant, Statistics Center, Cornell University). Suppose a random sample of $n = 56$ college students were surveyed and $r = 12$ of them said that blue is their favorite color. Does this information imply that the color preference of all college students is different (either way) from that of the general population? Use $\alpha = 0.05$.

(a) State the null and alternate hypothesis.
$H_0 : p = 0.24$ $H_1 : p \neq 0.24$

(b) What sampling distribution should be used? Explain.
Because we considering a proportion of the population and both $np = (56) * (.24) = 13.44 > 5$ and $nq = (56) * (.76) = 42.56 > 5$, we should use the normal distribution as an approximation of the binomial distribution.

(c) Is this a right-tailed, a left-tailed, or two-tailed test? Find the P-value.
Because $H_1 : p \neq 0.24$, this is a two-tailed test. Using `1-PropZTest` with $p_0 : 0.24$, $x : 12$, $n : 56$, and `prop` $\neq p_0$, we get $z = -0.4505635569$ and $P = 0.6523041776$.

(d) Will you reject or fail to reject the null hypothesis? Explain and interpret this conclusion.
Because the α-level was set at $\alpha = 0.05$, $P > \alpha$. Therefore, we fail to reject the null hypothesis. That is, at the 5% level, the evidence is not strong enough to support the claim that the color preference of all college students is different from that of the general population.

(3) *Symposium* is part of a larger work referred to as Plato's *Dialogues*. Wishart and Leach (in *Computer Studies of Humanities and Verbal Behavior*, Vol. 3, pp. 90-99) found that about 21.4% of five-syllable sequences in *Symposium* of the type in which four are short and one is long. Suppose an antiques store in Athens has a very old manuscript that the owner claims is part of Plato's *Dialogues*. A random sample of 493 five-syllable sequences from this manuscript showed that 136 we of the type four short and one long. Do the data indicate that the population proportion of this type of five-syllable sequence is higher than that found in Plato's *Symposium*? Use $\alpha = 1\%$. (Make sure that you are showing all relevant steps to the hypothesis test.)

$H_0 : p = 0.214$ $H_1 : p > 0.214$ Because $np = 493(0.214) = 105.502 > 5$ and $nq = 493(0.786) = 387.498 > 5$, we should use the normal distribution as an approximation to the binomial distribution.

This is a right-tailed test and using `1-PropZTest` with $p_0 : 0.214$, $x : 136$, $n : 493$, and `prop` $> p_0$, we get $z = 3.349112535$ and $P = 0.00040540943$.

Because $P < \alpha$, we reject the null hypothesis. That is, at the 1% level, the evidence is strong enough to support the claim that the population proportion of this type of five-syllable sequence is higher than that found in Plato's *Symposium*.

(4) Suppose a hypothesis test is executed and the P-value is found.

(a) If the P-value is such that you can reject H_0 at the 1% level of significance, can you always reject H_0 at the 5% level of significance too? Explain.
If $P < 0.01$, then it is also the case that $P < 0.05$. So, you will always to able to reject at the 5% level too. If the result is "rare enough" to satisfy the 1% level of significance, then it will also satisfy the 5% level which does not require as rare a result.

(b) If the P-value is such that you can reject H_0 at the 5% level of significance, can you always reject H_0 at the 1% level of significance too? Explain.
Just because $P < 0.05$, that does not necessarily mean that $P < 0.01$ as well. So, you may not be able to reject H_0 at the 1% level. In order to satisfy the 1% level of significance, the sample must be "more rare" than for the 5% level of significance.