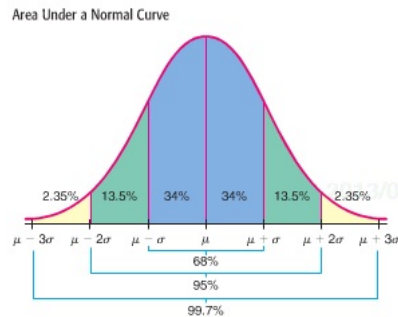


- The incubation time for Rhode Island Red chicks is normally distributed with a mean of  $\mu = 21$  days and a standard deviation of about  $\sigma = 1$  day. If 1000 eggs are being incubated, how many chicks do we expect to hatch
  - on day 21 or after?
  - between 20 and 21 days?
  - between 22 and 23 days?
  - between 19 and 22 days?
- The Empirical Rule tells us the percentage of data values that lie within certain intervals about the mean when we have a normal distribution as follows.



- According to Chebyshev's Theorem, what is the minimum percentage of data that must lie within the interval  $\mu - 2\sigma$  to  $\mu + 2\sigma$ ? How does this compare to the results for the Empirical Rule?
- The diagram for the Empirical Rule only accounts for 99.7% of the data values. Where do the remaining 0.3% of the data fall?

- (c) Keeping in mind that remaining 0.3% of the data, what percentage of the data values lie to the left of  $\mu - \sigma$ ?
- (d) Keeping in mind that remaining 0.3% of the data, what percentage of the data values lie to the left of  $\mu + 2\sigma$ ?
3. In preparation for Sections 6.2 and 6.3 (of Brase and Brase), suppose the mean of a normal distribution is  $\mu = 75$  with a standard deviation of  $\sigma = 8$ . For each of the following values of the random variable  $x$ , determine whether  $x$  is above or below the mean and then determine the number of standard deviations away from the mean that  $x$  lies. (Note that your second answer could be a fraction or decimal number.)
- (a)  $x = 83$
- (b)  $x = 51$
- (c)  $x = 79$
- (d)  $x = 65$
4. Hydraulic pressure in the main cylinder of the landing gear of a commercial jet is very important for a safe landing. In-flight landing tests show that the actual pressure in the main cylinders is variable (approximately normal) with a mean of 819 pounds per square inch and a standard deviation of 23 pounds per square inch, which are considered safe.

Two different planes were tested with 10 test-landings.

landing number for plane A	1	2	3	4	5	6	7	8	9	10
pressure	870	855	830	815	847	836	825	810	792	820
landing number for plane B	1	2	3	4	5	6	7	8	9	10
pressure	865	850	841	820	815	789	801	765	730	725

- (a) Is the pressure for Plane A “in control” or not? If not, describe the specific out-of-control signals present.
- (b) Is the pressure for Plane B “in control” or not? If not, describe the specific out-of-control signals present.
5. Let  $x$  be a random variable with a normal distribution with mean  $\mu = 15$  and standard deviation  $\sigma = 4$ . Find the indicated probability.
- (a)  $P(x \leq 12)$
- (b)  $P(x > 10)$
- (c)  $P(15 \leq x \leq 20)$
- (d)  $P(8 < x < 22)$
6. Find the  $z$ -value described.
- (a) Find  $z$  such that 5.2% of the standard normal curve lies to the left of  $z$ .
- (b) Find  $z$  such that 5% of the standard normal curve lies to the right of  $z$ .

- (c) Find  $z$  such that 95% of the standard normal curve lies between  $-z$  and  $+z$ .
7. Suppose that the scores for a Chemistry midterm had a normal distribution with mean  $\mu = 71.4$  and a standard deviation of  $\sigma = 8.3$ .
- (a) Find the exam score for a student that scored in the 80th percentile.
- (b) Find the exam score for a student that scored in the 25<sup>th</sup> percentile.
- (c) Find the exam score for a student who score lower than 60% of the class.
8. Suppose the distribution of heights of American men (20 years of age and older) is approximately normal with a mean of 69.4 inches and a standard deviation of 3 inches.
- (a) What is the  $z$  value for a height of 6 feet?
- (b) What percentage of American men (20 years of age and older) are shorter than 6 feet?
- (c) What is the  $z$  value for a height of 6 feet 4 inches?
- (d) What percentage of American men (20 years of age and older) are taller than 6 feet 4 inches?

9. Accrotime is a manufacturer of quartz crystal watches. Accrotime researchers have shown that the watches have an average life of 28 months before certain electronic components deteriorate, causing the watch to become unreliable. The standard deviation of watch lifetimes is 5 months, and the distribution of lifetimes is normal.
- (a) If Accrotime guarantees a full refund on any defective watch for 2 years, what percentage of total production will the company expect to replace?
- (b) If Accrotime does not want to make refunds on more than 12% of the watches it makes, how long should the guarantee period be (to the nearest month)?

This section of the problem set is an activity with the goal of simulating a sampling distribution of sample means. The first part of the activity is for each of you to generate a list of sample means. After the data is gathered, the second part directs you to use the results of the entire class to analyze the outcome.

10. Attached to this page is a copy of a random number table for the population of digits  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . Assuming that this random distribution is uniform:
- (a) What is the population mean of the digits?
- (b) What is the population standard deviation of the digits?
11. Repeat the following steps 10 times. In the end, you will have generated 10 different sample means. Please do NOT simply copy a classmate's values on this part, but please do your own work so that you really are generating your own list of sample means.
- (a) *Randomly* select a spot on the random number table.
- (b) Write down the next 10 digits listed from that spot horizontally (if you reach the end of a row, wrap around to the start of the next row to complete your list of 10).
- (c) Compute the mean of that sample of 10 random digits.

Sample	List of 10 digits	Sample mean
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

- (a) Looking at the list of sample means, there should be a variety of values, but they are not uniformly distributed. What is the shape of the distribution for the sample means?
- (b) What is the mean of the sample means?
- (c) What is the standard deviation of the sample means?
- (d) In Section 6.5, one of the most critical theorems in statistics is introduced; the Central Limit Theorem. Essentially it says that a sampling distribution for sample means (where the sample size for all samples is  $n$ ) will have a distribution close to a normal distribution. Further, the mean of the sample means will equal the population mean  $\mu$  and the standard deviation of the sample means will equal  $\frac{\sigma}{\sqrt{n}}$ . How did the simulation the class do match up to the theorem?
- Was the shape of the distribution of sample means approximately normal?

- ii. Was the mean of the sample means close the the population mean  $\mu$ ?
- iii. What was the sample size for our sample means?
- iv. Was the standard deviation of the sample means close to the value  $\frac{\sigma}{\sqrt{(n)}}$ ?

6 1 9 2 5	8 0 2 9 6	8 1 0 4 8	9 7 1 9 1	6 5 1 8 2	6 2 8 2 3	1 0 7 4 2	1 6 8 5 9
9 4 2 9 5	4 0 3 3 9	0 4 5 5 3	1 9 6 5 6	7 7 6 4 5	1 5 0 4 4	7 2 8 1 5	4 2 6 3 8
2 0 4 8 9	3 8 4 5 5	9 5 7 2 4	7 4 6 4 6	7 4 4 4 1	9 0 3 9 7	4 8 6 3 0	5 5 8 4 3
5 0 1 9 6	0 6 3 3 0	7 7 5 6 8	8 7 0 5 4	8 2 8 5 8	5 9 6 9 2	6 2 1 7 1	2 7 8 8 7
2 8 4 0 1	9 7 5 1 0	6 3 5 0 7	1 4 3 9 6	3 9 0 0 5	5 7 0 7 8	7 1 0 0 2	2 4 4 2 5
7 0 1 6 6	5 1 4 2 1	8 4 0 5 5	2 5 1 4 6	8 1 4 7 6	2 7 9 6 3	5 2 3 3 7	4 3 0 0 4
3 6 9 9 0	1 6 5 0 7	1 8 7 6 8	1 3 1 8 6	8 3 1 0 6	4 5 9 1 9	0 7 0 8 8	9 6 3 0 2
8 5 5 3 4	7 1 3 5 5	3 6 0 3 0	2 8 9 5 0	7 2 1 5 4	4 8 4 6 7	4 4 7 7 9	1 6 1 5 2
5 0 4 3 6	3 0 8 7 6	8 0 3 6 7	6 0 4 2 2	2 3 8 0 7	0 9 5 2 4	4 7 1 5 4	8 8 8 2 1
1 5 5 6 3	0 6 0 2 0	0 0 4 4 0	4 7 7 0 8	9 1 4 1 7	3 7 7 2 5	3 3 1 3 2	5 5 4 0 9
0 1 9 9 8	8 9 5 5 8	4 7 1 0 5	1 5 1 9 7	4 5 2 0 9	3 0 9 0 7	0 6 5 5 1	5 0 4 0 0
6 2 5 2 0	9 6 7 0 3	7 9 8 9 6	5 5 3 1 2	0 3 1 5 9	6 6 4 9 5	6 9 8 3 3	9 4 6 6 2
9 1 2 6 9	4 4 0 6 1	7 2 5 8 4	3 9 1 5 8	7 6 5 0 8	1 8 3 3 4	3 9 3 5 7	1 7 9 3 5
0 5 2 6 1	7 3 6 5 5	3 1 7 1 2	4 9 9 9 0	6 2 7 9 0	6 6 2 3 8	9 5 7 3 8	6 8 7 1 2
4 6 8 7 2	8 8 2 9 1	8 8 8 5 3	2 8 2 1 3	0 9 2 3 0	4 3 5 2 2	0 4 3 7 6	0 1 0 9 7
5 7 5 3 7	2 8 7 0 8	5 2 4 5 1	8 1 7 4 0	5 4 8 2 8	8 9 4 4 5	1 5 3 6 8	8 8 2 5 2
9 3 9 6 1	6 4 0 2 7	3 4 4 5 1	8 6 8 7 0	0 3 5 8 0	7 8 0 4 6	5 8 3 2 8	2 5 6 0 6
6 9 0 6 9	4 0 8 9 1	0 0 5 2 4	0 5 6 4 4	9 0 9 4 5	7 3 2 0 6	0 3 5 6 2	8 9 3 0 9
8 6 4 9 8	2 1 2 9 7	7 0 4 8 3	0 3 2 2 2	8 9 1 7 7	4 1 7 6 6	1 2 6 2 3	8 7 1 4 1
6 2 9 5 4	7 9 6 2 9	2 3 9 3 8	4 2 5 0 6	4 9 4 1 7	8 9 0 7 6	1 5 8 4 1	3 9 8 2 2
9 5 3 6 7	6 7 2 6 5	8 2 9 4 7	9 1 6 9 6	7 6 2 3 1	7 5 5 2 3	3 4 7 6 9	3 0 4 9 6
8 0 0 2 0	5 6 4 2 8	8 9 2 9 1	5 6 4 1 3	8 7 5 5 8	5 4 7 8 1	5 9 8 7 7	6 2 3 6 3
0 9 0 4 4	0 2 5 1 1	5 0 6 1 2	7 9 7 1 5	0 2 3 5 0	3 2 7 2 6	0 0 5 5 5	4 5 6 2 8
9 7 0 8 2	3 0 3 0 1	5 6 2 6 6	1 1 6 5 2	4 1 5 4 0	9 3 6 9 3	0 8 7 5 7	9 2 3 4 9
1 2 0 3 5	2 1 9 4 1	6 7 7 3 8	6 7 7 7 9	6 5 0 8 4	0 5 4 5 5	1 9 9 3 1	8 7 5 5 7
9 1 1 2 7	4 3 3 6 4	3 3 0 8 0	4 6 0 8 8	7 4 3 4 1	9 4 2 2 0	4 7 9 9 9	6 2 1 7 5
2 5 9 0 8	6 9 5 1 1	4 7 5 2 4	7 8 6 2 3	7 4 9 1 6	3 9 0 8 3	6 5 5 7 5	6 4 0 5 0
7 8 7 2 2	6 5 8 4 5	1 4 5 7 0	6 5 3 1 6	0 0 2 2 8	4 9 1 1 4	8 6 8 0 1	7 9 8 8 1
7 0 6 1 4	1 5 4 0 7	8 7 4 4 6	9 0 2 9 6	7 6 8 8 6	5 5 7 1 2	9 9 4 7 5	7 8 6 4 0
8 7 3 7 5	7 3 6 4 9	9 0 2 2 4	6 6 0 8 7	9 7 6 7 4	5 1 4 9 0	3 3 6 7 4	3 3 0 6 2
4 9 0 2 0	1 7 4 1 9	4 6 1 8 0	9 2 2 2 3	0 0 4 3 5	4 8 3 1 9	0 5 8 3 5	1 5 2 5 9
1 7 1 6 6	1 1 3 8 1	6 3 5 1 5	3 1 2 8 5	7 9 6 3 7	6 1 0 5 9	5 4 9 2 3	8 7 0 8 8
7 6 8 0 0	5 7 1 8 7	9 3 2 4 8	6 3 7 9 2	1 6 9 9 4	0 7 9 7 2	7 5 3 3 7	2 7 5 5 9
6 4 9 0 8	2 2 9 7 1	7 0 8 3 7	3 0 0 4 0	5 5 4 9 7	6 8 4 9 6	3 1 0 1 7	1 0 3 2 9
9 6 5 4 4	6 0 4 5 3	2 1 9 5 6	5 6 3 8 5	5 3 3 6 6	3 5 7 1 7	4 3 4 0 2	8 0 3 9 7
2 0 7 4 6	0 2 6 5 6	8 2 8 9 0	9 5 2 5 6	7 0 3 4 6	7 8 1 1 8	5 3 8 8 5	6 6 5 9 4
0 9 8 9 2	0 3 8 5 8	7 9 4 6 6	2 5 2 3 0	5 1 6 9 6	2 5 1 9 1	5 1 1 9 2	7 5 9 7 8
8 1 8 5 8	6 7 6 5 3	8 5 1 6 7	0 3 5 5 5	9 9 3 2 7	0 2 3 5 4	2 0 9 4 1	6 7 5 0 9
8 4 7 2 0	3 2 5 2 3	1 7 9 7 9	4 3 3 3 4	1 6 6 9 5	0 8 9 2 1	9 7 8 4 9	8 2 6 5 2
7 2 5 8 7	4 0 1 3 5	6 1 0 6 7	0 3 4 8 6	1 4 8 2 3	2 9 2 0 0	9 2 7 1 9	0 4 4 7 4
5 6 1 8 4	3 4 9 8 3	7 6 3 4 1	1 6 5 5 8	5 4 5 0 1	9 0 5 7 9	3 6 4 2 3	2 1 8 8 5
2 7 4 5 0	9 0 6 8 0	5 8 3 8 8	7 6 5 8 6	1 1 4 6 3	3 5 7 2 9	0 8 4 9 3	7 4 7 9 4
2 2 1 4 5	8 7 1 0 3	2 7 7 3 1	2 7 2 6 5	5 8 0 3 9	8 6 7 8 7	3 9 2 7 2	5 9 7 1 7
4 0 8 9 5	9 2 1 6 0	8 0 2 5 2	2 4 6 2 0	3 2 4 7 1	5 4 7 6 3	4 5 1 1 5	6 7 2 0 2
6 8 7 4 8	5 8 2 2 8	6 1 0 6 1	8 4 6 3 1	7 4 5 2 8	6 5 7 3 4	4 4 9 5 9	4 0 2 1 2
1 9 1 1 3	6 2 3 9 8	1 7 0 3 3	6 2 0 3 9	6 4 8 1 0	3 6 1 4 9	7 2 6 3 2	7 1 2 9 4
8 2 7 3 7	6 6 6 8 8	7 6 9 2 3	5 6 4 1 6	7 6 1 1 6	6 0 8 7 0	9 1 0 6 4	2 6 8 6 7
2 9 1 5 1	0 7 6 8 1	8 7 9 4 7	0 5 5 1 5	1 5 9 9 8	8 5 1 7 4	0 9 1 0 1	6 4 3 3 1
6 6 2 2 2	9 3 9 6 5	4 1 5 8 6	4 8 4 1 7	4 1 7 7 2	2 5 7 8 4	0 5 0 9 4	7 6 4 1 9
1 0 2 3 4	5 9 0 2 0	7 0 8 7 2	7 4 0 9 9	4 5 7 8 8	8 3 1 3 7	6 3 2 2 2	1 2 5 2 2



12. Suppose a population has a distribution with  $\mu = 72$  and  $\sigma = 8$ .
- (a) If we know nothing about the original population distribution and random samples of size  $n = 16$  are selected, why can't we say anything about the  $\bar{x}$  distribution of sample means?
  - (b) If we know that the original population distribution is normal, then what can we say about the  $\bar{x}$  distribution of random samples of  $n = 16$ . In this case, find  $P(68 \leq \bar{x} \leq 73)$ .
13. The heights of 18-year-old men are approximately normally distributed, with a mean of 68 inches and a standard deviation of 3 inches.
- (a) What is the probability that an 18-year-old man selected at random is between 67 and 69 inches tall?
  - (b) If a random sample of nine 18-year-old men are selected at random, what is the probability that the mean height of the sample  $\bar{x}$  is between 67 and 69 inches?
  - (c) Explain why the probability in part (b) is so much higher than the probability in part (a) even though they both refer to the same interval of heights (67 to 69 inches).
14. Let  $x$  be a random variable that represents white blood cell count per cubic milliliter of whole blood. Assume that  $x$  has a distribution that is approximately normal with a mean of  $\mu = 7500$  and estimated standard deviation of  $\sigma = 1750$ . A test result of  $x < 3500$  is an indication of leukopenia. This indicates bone marrow repression that may be the result of a viral infection.

- (a) What is the probability that, on a single test,  $x$  is less than 3500?
  - (b) Suppose a doctor uses the average  $\bar{x}$  for two tests taken about a week apart. What can we say about the probability distribution of  $\bar{x}$ ? What is the probability that  $\bar{x} < 3500$ ?
  - (c) Repeat part (b) but with  $n = 3$  tests.
  - (d) How did the probabilities change as  $n$  increased? What do such results imply about a patient that has  $\bar{x} < 3500$  for 3 those tests?
15. Assume that IQ scores are normally distributed, with a standard deviation of 15 points and a mean of 100 points. If 10 people are chosen at random, what is the probability that the sample mean of their IQ scores will not differ from the population mean by more than 2 points?
16. A large tank of fish from a hatchery is being delivered to a lake. The hatchery claims that the mean length of fish in the tank is 15 inches, and the standard deviation is 2 inches. A random sample of 36 fish is taken from the tank. Let  $\bar{x}$  be the mean length of the sample. What is the probability that  $\bar{x}$  is within 0.5 inches of the claimed population mean?