<u>In-class Worksheet 8</u>

1. Consider a binomial experiment with $n = 25$ trials and the probability of success on each trial is $p = 20\%$. Compute the indicated probability.

   (a) The probability of exactly 4 successes.
   $P(r = 4) = \texttt{binompdf}(25, 0.20, 4) = 0.18668105$

   (b) The probability of at most 4 successes.
   $P(r \leq 4) = \texttt{binomcdf}(25, 0.20, 4) = 0.42067431$

   (c) The probability of at least 4 successes.
   $P(r \geq 4) = 1 - \texttt{binomcdf}(25, 0.20, 3) = 0.76600674$

   (d) The probability of more than 4 successes.
   $P(r > 4) = 1 - \texttt{binomcdf}(25, 0.20, 4) = 0.57932569$

   (e) The probability of fewer than 4 successes.
   $P(r < 4) = \texttt{binomcdf}(25, 0.20, 3) = 0.23399326$

2. In a certain casino game, a player can *win*, *lose* or *tie* the dealer. In a single instance of the game, the probability that the player wins is 0.35, the probability that the player loses is 0.55, and the probability that the player ties is 0.10. Suppose the player participates in 10 games and the outcome of each game is independent of all other games.

   (a) Can we use the binomial experiment model to determine the probability of exactly 6 losses? If so, what are the relevant values of n, r, p, and q? If not, explain why.
   Since problem is posed as a simple distinction between *losses* (success in this case) and *not losses* (failure in this case), we can us the binomial experiment model. Specifically, $n = 10$ trials at the game, $r = 6$ losses, $p = 0.55$ the probability of a *loss*, and $q = 0.45$ the probability of *not loss*. Specifically, the answer is `binompdf(n,p,r) = 0.23836665`.

   (b) Can we use the binomial experiment model to determine the probability of 3 wins, 6 losses, and 1 tie? If so, what are the relevant values of n, r, p, and q? If not, explain why.
   In this case, we can not use the binomial experiment model, because the problem is asking us to distinguish between 3 distinct outcomes and the binomial experiment is exclusively for two outcomes: *success* and *failure*.

3. A research team at Cornell University conducted a study showing that approximately 10% of all businessmen who wear ties wear them so tightly that they actually reduce blood flow to the brain, diminishing cerebral functions (Source: *Chances: Risk and Odds in Everyday Life*, by James Burke). At a board meeting 20 businessmen, all of whom wear ties, what is the probability that

   (a) at least one tie is too tight? (That is, $P(r \geq 1)$.)
   This can be solve as a binomial experiment, where success is marked as wearing a tie to tightly. So, $p = 0.10$ is the probability of success. There are $n = 20$ trials for which we are trying to compute the probability of $r = 1$ or more.
   For this problem, the complement rule is relevant in that $P(r \geq 1) = 1 - P(r = 0)$.
   So, $P(r \geq 1) = 1 - P(r = 0) = 1 - \texttt{binompdf(20, 0.10, 0)} = 0.87842335$.

   (b) fewer than 6 ties are too tight? (That is, $P(r < 6)$.)
   In this case, we are looking for the probability of 6 or fewer successes out of 20 trials. So, we can use the cumulative probability function `binomcdf`. However, remember that this calculator function will compute "r or fewer"...so for FEWER THAN 6, we must use $r = 5$ as the argument in the function.
   Specifically, $P(r < 6) = \texttt{binomcdf(20, 0.10, 5)} = 0.98874687$.

4. The quality-control inspector of a production plant will reject an entire batch of syringes if two or more defective syringes are found in a random sample of eight syringes taken from the batch. Suppose that the batch actually contains 1% defective syringes.

   (a) Find the mean of the probability distribution, μ.

   $\mu = np = 8(0.01) = 0.08$

   (b) Find the standard deviation of the probability distribution, σ.

   $\sigma = \sqrt{npq} = \sqrt{8(0.01)(0.99)} = 0.2814249456$

   (c) What is the probability that the batch will be accepted?

   The batch will be rejected if two or more defective syringes are found in the random sample of eight. Equivalently, the batch of syringes will be ACCEPTED if fewer than 2 defective syringes are found. The probability is $P(r < 2) = P(r \leq 1) =$ `binomcdf(8, 0.01, 1)` $= 0.9973099223$. So, there is about a 99.73% probability that the batch will be accepted.

5. A large bank vault has several automatic burglar alarms. The probability is 0.55 that a single alarm will detect a burglar.

   (a) What is the minimum number of such alarms that should be installed to provide 99% certainty that a burglar trying to enter the vault is detected by at least one alarm?

   This is an example of a quota problem. Specifically, we are looking for the smallest value of n so that $P(r \geq 1)$ is 99% (or higher).

   Since $P(r \geq 1) = 1 - P(r = 0)$ and $P(r = 0) =$ `binompdf(n, 0.55, 0)`, we need to find n so that $1 -$ `binompdf(n, 0.55, 0)` $= 0.99$ (or higher).

   Using the `TABLE` function on the TI-84 with Y_1`=1-binompdf(X,0.55,0)`, we see that for $n = 5$, $Y_1 = 0.98155$ and for $n = 6$, $Y_1 = 0.9917$. Therefore, there must be a minimum of 6 alarms to reach that 99% certainty that at least one will detect the burglar.

   (b) If, in fact, the bank decides to install 9 such alarms, then what is the expected number of alarms that will detect a burglar?

   With $p = 0.55$ that each single alarm will detect a burglar and $n = 9$ alarms, the expected number of alarms that will detect a burglar is $np = 9(0.55) = 4.95$ alarms. So, on average about 5 alarms will detect the burglar.