MIDTERM 2 (INSTRUCTIONS AND PRACTICE PROBLEMS)

COLTON GRAINGER (MATH 2510-001)

1. Time and Date

Midterm 2 will be offered **8:00am to 8:50am on Wednesday November 20, 2019** in Muenzinger E064. **Please arrive by 7:55am** to give yourself plenty of time (e.g., the full 50 minutes) for the actual exam. Colton will be proctoring this exam.

2. Ground Rules

For this midterm, **you are allowed a personal calculator** (and you will need it!).

The midterm is closed book, closed note, and closed internet, excepting that **I will allow you to keep one half page of handwritten notes** (e.g., the front and back half of an 8.5 by 11 inch sheet of letter paper) on your desk during the midterm.

You do not need to bring your own scratch paper. I will supply blank scratch paper if you request it.

I expect you to legibly write your answers directly on the exam. When you are asked for a short answer, **I expect you to write in clear, grammatically correct, English sentences**. Please provide answers that include both **mathematical reasoning** and the **explicit name and argument**[1] of any relevant calculator function you used. (If you don't like this requirement, think of it like so: writing down your reasoning is a "safety" measure to guarantee that you'll at least receive partial credit on any problem.)

You will not need to successfully answer every midterm question to obtain an *A* in this class, but **you should try to write down what you know about each problem even if you cannot solve it.**

3. How to pass this midterm

Do what you can. I suggest that you review and **revise your own notes**.

What material will be examined?

> Expect to be tested on *Understandable Statistics*, Chapters 7 (confidence intervals) and 8 (hypothesis testing). Chapter 6 (normal distributions) is essential for both the construction of confidence intervals and hypothesis testing, so you might want to review this material as well.[2]

To get warmed up for the timed midterm, **try working out a few practice problems** (attached) by hand. Word problems are for math students what scales and arpeggios are for music students. Your goals in *reading* a word problem should be to *quickly* and *accurately* identify

1. what is *known*,
2. what is *unknown*,
3. what you *want to find*, and
4. any *mathematical relationships* between the known and unknown quantities.

Your goals in actually *solving* a word problem should be

1. to *prepare* the relevant mathematical information (see steps 1–4 for reading a word problem),
2. to *check* that your chosen statistical model is appropriate,
3. to *compute* the relevant statistics, and then
4. to *conclude* with an argument supporting your answer to the original problem.

Date: 2019-11-10.

[1] In both computer science and mathematics, the *argument of a function* is that function's "input".

[2] Yes, you should review the basics facts about normal distributions, even though they were already examined during midterm 1, okay? Recall that **your final exam will be cumulative**, so it would behoove you to use studying for midterm 2 as an opportunity to master Chapters 6–8.

SUMMARY

In this chapter, we examined properties and applications of the normal probability distribution.

Part I

- A normal probability distribution is a distribution of a continuous random variable. Normal distributions are bell-shaped and symmetric around the mean. The high point occurs over the mean, and most of the area occurs within 3 standard deviations of the mean. The mean and median are equal.
- The empirical rule for normal distributions gives areas within 1, 2, and 3 standard deviations of the mean. Approximately

  68% of the data lie within the interval $\mu \pm \sigma$
  95% of the data lie within the interval $\mu \pm 2\sigma$
  99.7% of the data lie within the interval $\mu \pm 3\sigma$
- For symmetric, bell-shaped distributions,

$$\text{standard deviation} \approx \frac{\text{range of data}}{4}$$

- A z score measures the number of standard deviations a raw score x lies from the mean.

$$z = \frac{x - \mu}{\sigma} \quad \text{and} \quad x = z\sigma + \mu$$

- For the standard normal distribution, $\mu = 0$ and $\sigma = 1$.
- Table 5 of Appendix II gives areas under a standard normal distribution that are to the left of a specified value of z.
- After raw scores x have been converted to z scores, the standard normal distribution table can be used to find probabilities associated with intervals of x values from any normal distribution.
- The inverse normal distribution is used to find z values associated with areas to the left of z. Table 5 of Appendix II can be used to find approximate z values associated with specific probabilities.
- Tools for assessing the normality of a data distribution include:

  Histogram of the data. A roughly bell-shaped histogram indicates normality.
  Presence of outliers. A limited number indicates normality.
  Skewness. For normality, Pearson's index is between -1 and 1.
  Normal quantile plot. For normality, points lie close to a straight line.
- Control charts are an important application of normal distributions.

Part II

- Sampling distributions give us the basis for inferential statistics. By studying the distribution of a sample statistic, we can learn about the corresponding population parameter.
- For random samples of size n, the $\bar{x}$ distribution is the sampling distribution for the sample mean of an x distribution with population mean μ and population standard deviation σ. If the x distribution is normal, then the corresponding $\bar{x}$ distribution is normal.

  By the central limit theorem, when n is sufficiently large ($n \geq 30$), the $\bar{x}$ distribution is approximately normal even if the original x distribution is not normal.

  In both cases,

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- For n binomial trials with probability of success p on each trial, the $\hat{p}$ distribution is the sampling distribution of the sample proportion of successes. When $np > 5$ and $nq > 5$, the $\hat{p}$ distribution is approximately normal with

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

- The binomial distribution can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$ provided

$$np > 5 \text{ and } nq > 5, \text{ with } q = 1 - p$$

and a continuity correction is made.

Data from many applications follow distributions that are approximately normal. We will see normal distributions used extensively in later chapters.

IMPORTANT WORDS & SYMBOLS

VIEWPOINT *Nenana Ice Classic*

The Nenana Ice Classic is a betting pool offering a large cash prize to the lucky winner who can guess the time, to the nearest minute, of the ice breakup on the Tanana River in the town of Nenana, Alaska. Official breakup time is defined as the time when the surging river dislodges a tripod on the ice. This breaks an attached line and stops a clock set to Yukon Standard Time. The event is so popular that the first state legislature of Alaska (1959) made the Nenana Ice Classic an official statewide lottery. Since 1918, the earliest breakup has been April 20, 1940, at 3:27 P.M., and the latest recorded breakup was May 20, 1964, at 11:41 A.M. Want to make a statistical guess predicting when the ice will break up? Breakup times from the years 1918 to 1996 are recorded in *The Alaska Almanac*, published by Alaska Northwest Books, Anchorage.

CHAPTER REVIEW PROBLEMS

1. **Statistical Literacy** Describe a normal probability distribution.

2. **Statistical Literacy** According to the empirical rule, approximately what percentage of the area under a normal distribution lies within 1 standard deviation of the mean? within 2 standard deviations? within 3 standard deviations?

3. **Statistical Literacy** Is a process in control if the corresponding control chart for data having a normal distribution shows a value beyond 3 standard deviations of the mean?

4. **Statistical Literacy** Can a normal distribution always be used to approximate a binomial distribution? Explain.

5. **Statistical Literacy** What characteristic of a normal quantile plot indicates that the data follow a distribution that is approximately normal?

Tables and art to accompany margin answers may be found in the back of the book.

1. Normal probability distributions are distributions of continuous random variables. They are symmetric about the mean and bell-shaped. Most of the data fall within 3 standard deviations of the mean. The mean and median are the same.
2. 68% within 1 standard deviation of μ; 95% within 2 standard deviations of μ; 99.7% within 3 standard deviations of μ.
3. No.
4. No, np and nq must both be greater than 5.

5. The points lie close to a straight line.
6. No, the probability is only about 2.5%.

6. **Statistical Literacy** For a normal distribution, is it likely that a data value selected at random is more than 2 standard deviations above the mean?

7. $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.

7. **Statistical Literacy** Give the formula for the *standard error* of the sample mean $\bar{x}$ distribution, based on samples of size n from a distribution with standard deviation σ.

8. $\sigma_{\hat{p}} = \sqrt{pq/n}$, where $q = 1 - p$.

8. **Statistical Literacy** Give the formula for the *standard error* of the sample proportion $\hat{p}$ distribution, based on n binomial trials with probability of success p on each trial.

9. (a) A normal distribution.
(b) The mean μ of the x distribution.
(c) $\sigma/\sqrt{n}$, where σ is the standard deviation of the x distribution.
(d) Approximately normal with the same mean, but the standard deviations will be $\sigma/\sqrt{50}$ and $\sigma/\sqrt{100}$, respectively.

9. **Critical Thinking** Let x be a random variable representing the amount of sleep each adult in New York City got last night. Consider a sampling distribution of sample means $\bar{x}$.
(a) As the sample size becomes increasingly large, what distribution does the $\bar{x}$ distribution approach?
(b) As the sample size becomes increasingly large, what value will the mean $\mu_{\bar{x}}$ of the $\bar{x}$ distribution approach?
(c) What value will the standard deviation $\sigma_{\bar{x}}$ of the sampling distribution approach?
(d) How do the two $\bar{x}$ distributions for sample size $n = 50$ and $n = 100$ compare?

10. All the $\bar{x}$ distribution will be normal with mean 15. The standard deviations will be 3/2, 3/4, and 3/10, respectively.

10. **Critical Thinking** If x has a normal distribution with mean $\mu = 15$ and standard deviation $\sigma = 3$, describe the distribution of $\bar{x}$ values for sample size n, where $n = 4$, $n = 16$ and $n = 100$. How do the $\bar{x}$ distributions compare for the various sample sizes?

11. (a) 0.9821.
(b) 0.3156.
(c) 0.2977.

11. **Basic Computation: Probability** Given that x is a normal variable with mean $\mu = 47$ and standard deviation $\sigma = 6.2$, find
(a) $P(x \leq 60)$ (b) $P(x \geq 50)$ (c) $P(50 \leq x \leq 60)$

12. (a) 0.7967.
(b) 0.9938.
(c) 0.2865.

12. **Basic Computation: Probability** Given that x is a normal variable with mean $\mu = 110$ and standard deviation $\sigma = 12$, find
(a) $P(x \leq 120)$ (b) $P(x \geq 80)$ (c) $P(108 \leq x \leq 117)$

13. 1.645.

13. **Basic Computation: Inverse Normal** Find z such that 5% of the area under the standard normal curve lies to the right of z.

14. ± 2.58.

14. **Basic Computation: Inverse Normal** Find z such that 99% of the area under the standard normal curve lies between $-z$ and z.

15. (a) 0.8665.
(b) 0.7330.

15. **Medical: Blood Type** Blood type AB is found in only 3% of the population (Reference: *Textbook of Medical Physiology,* by A. Guyton, M.D.). If 250 people are chosen at random, what is the probability that
(a) 5 or more will have this blood type?
(b) between 5 and 10 will have this blood type?

16. (a) 0.6103.
(b) 0.9573.
(c) 0.6872

16. **Customer Complaints: Time** The Customer Service Center in a large New York department store has determined that the amount of time spent with a customer about a complaint is normally distributed, with a mean of 9.3 minutes and a standard deviation of 2.5 minutes. What is the probability that for a randomly chosen customer with a complaint, the amount of time spent resolving the complaint will be
(a) less than 10 minutes?
(b) longer than 5 minutes?
(c) between 8 and 15 minutes?

17. (a) 0.0166.
(b) 0.9750.

17. **Recycling: Aluminum Cans** One environmental group did a study of recycling habits in a California community. It found that 70% of the aluminum cans sold in the area were recycled.
(a) If 400 cans are sold today, what is the probability that 300 or more will be recycled?

(b) Of the 400 cans sold, what is the probability that between 260 and 300 will be recycled?

18. (a) 0.4562.
(b) About 7 hours.

18. **Guarantee: Battery Life** Many people consider their smart phone to be essential! Communication, news, Internet, entertainment, photos, and just keeping current are all conveniently possible with a smart phone. However, the battery better be charged or the phone is useless. Battery life of course depends on the frequency, duration, and type of use. One study involving heavy use of the phones showed the mean of the battery life to be 10.75 hours with a standard deviation of 2.2 hours. Then the battery needs to be recharged. Assume the battery life between charges is normally distributed.
(a) Find the probability that with heavy use, the battery life exceeds 11 hours.
(b) **Inverse Normal Distribution** You are planning your recharging schedule so that the probability your phone will die is no more than 5%. After how many hours should you plan to recharge your phone?

19. (a) 0.9772.
(b) 17.3 hours.

19. **Guarantee: Package Delivery** Express Courier Service has found that the delivery time for packages is normally distributed, with mean 14 hours and standard deviation 2 hours.
(a) For a package selected at random, what is the probability that it will be delivered in 18 hours or less?
(b) **Inverse Normal Distribution** What should be the guaranteed delivery time on all packages in order to be 95% sure that the package will be delivered before this time? *Hint:* Note that 5% of the packages will be delivered at a time beyond the guaranteed time period.

20. (a) In control.
(b) Out-of-control signals I and III.


JSX80/iStock/Getty Images

20. **Control Chart: Landing Gear** Hydraulic pressure in the main cylinder of the landing gear of a commercial jet is very important for a safe landing. If the pressure is not high enough, the landing gear may not lower properly. If it is too high, the connectors in the hydraulic line may spring a leak.

In-flight landing tests show that the actual pressure in the main cylinders is a variable with mean 819 pounds per square inch and standard deviation 23 pounds per square inch. Assume that these values for the mean and standard deviation are considered safe values by engineers.
(a) For nine consecutive test landings, the pressure in the main cylinder is recorded as follows:

Landing number	1	2	3	4	5	6	7	8	9
Pressure	870	855	830	815	847	836	825	810	792

Make a control chart for the pressure in the main cylinder of the hydraulic landing gear, and plot the data on the control chart. Looking at the control chart, would you say the pressure is "in control" or "out of control"? Explain your answer. Identify any out-of-control signals by type (I, II, or III).
(b) For 10 consecutive test landings, the pressure was recorded on another plane as follows:

Landing number	1	2	3	4	5	6	7	8	9	10
Pressure	865	850	841	820	815	789	801	765	730	725

Make a control chart and plot the data on the chart. Would you say the pressure is "in control" or not? Explain your answer. Identify any out-of-control signals by type (I, II, or III).

21. (a) 0.2389.
(b) 0.0162.

21. **Job Interview: Length** The personnel office at a large electronics firm regularly schedules job interviews and maintains records of the interviews. From the past records, they have found that the length of a first interview is normally distributed, with mean $\mu = 35$ minutes and standard deviation $\sigma = 7$ minutes.
(a) What is the probability that a first interview will last 40 minutes or longer?
(b) Nine first interviews are usually scheduled per day. What is the probability that the average length of time for the nine interviews will be 40 minutes or longer?

22. (a) 0.2743.
(b) 0.0287.
(c) The standard deviation of $\bar{x}$ is smaller.

22. **Drugs: Effects** A new muscle relaxant is available. Researchers from the firm developing the relaxant have done studies that indicate that the time lapse between administration of the drug and beginning effects of the drug is normally distributed, with mean $\mu = 38$ minutes and standard deviation $\sigma = 5$ minutes.
(a) The drug is administered to one patient selected at random. What is the probability that the time it takes to go into effect is 35 minutes or less?
(b) The drug is administered to a random sample of 10 patients. What is the probability that the average time before it is effective for all 10 patients is 35 minutes or less?
(c) Comment on the differences of the results in parts (a) and (b).

23. 0.8164.

23. **Psychology: IQ Scores** Assume that IQ scores are normally distributed, with a standard deviation of 15 points and a mean of 100 points. If 100 people are chosen at random, what is the probability that the sample mean of IQ scores will not differ from the population mean by more than 2 points?

24. 0.8664.

24. **Hatchery Fish: Length** A large tank of fish from a hatchery is being delivered to a lake. The hatchery claims that the mean length of fish in the tank is 15 inches, and the standard deviation is 2 inches. A random sample of 36 fish is taken from the tank. Let $\bar{x}$ be the mean sample length of these fish. What is the probability that $\bar{x}$ is within 0.5 inch of the claimed population mean?

25. (a) Yes, np and nq both exceed 5.
(b) $\mu_{\hat{p}} = 0.4$; $\sigma_{\hat{p}} = 0.1$.

25. **Basic Computation: $\hat{p}$ Distribution** Suppose we have a binomial distribution with $n = 24$ trials and probability of success $p = 0.4$ on each trial. The sample proportion of successes is $\hat{p} = r/n$.
(a) Is it appropriate to approximate the $\hat{p}$ distribution with a normal distribution? Explain.
(b) What are the values of $\mu_{\hat{p}}$ and $\sigma_{\hat{p}}$?

26. (a) Yes, np and nq both exceed 5.
(b) $\mu_{\hat{p}} = 0.25$; $\sigma_{\hat{p}} = 0.05$.

26. **Green Behavior: Purchasing Habits** A recent Harris Poll on green behavior showed that 25% of adults often purchase used items instead of new ones. Consider a random sample of 75 adults. Let $\hat{p}$ be the sample proportion of adults who often purchase used instead of new items.
(a) Is it appropriate to approximate the $\hat{p}$ distribution with a normal distribution? Explain.
(b) What are the values of $\mu_{\hat{p}}$ and $\sigma_{\hat{p}}$?

SUMMARY

PART I

How do you get information about a population by looking at a random sample? One way is to use point estimates and confidence intervals.

- Point estimates and their corresponding parameters are

  $\bar{x}$ for μ
  $\hat{p}$ for p

- Confidence intervals are of the form

  point estimate $- E <$ parameter $<$ point estimate $+ E$

- E is the maximal margin of error. Specific values of E depend on the parameter, level of confidence, whether population standard deviations are known, sample size, and the shapes of the original population distributions.

  For μ: $E = z_c \dfrac{\sigma}{\sqrt{n}}$ when σ is known

  $E = t_c \dfrac{s}{\sqrt{n}}$ with $d.f. = n - 1$ when σ is unknown

  For p: $E = z_c \sqrt{\dfrac{\hat{p}(1 - \hat{p})}{n}}$ when $n\hat{p} > 5$ and $n\hat{q} > 5$

- Confidence intervals have an associated probability c called the confidence level. For a given sample size, the proportion of all corresponding confidence intervals that contain the parameter in question is c.

PART II

- For independent populations, the confidence intervals for the difference of two means or of two proportions follow the same format as confidence intervals for a single mean or single proportion.
- The maximal margin of error depends on the sampling distribution of the point estimate.

  $\bar{x}_1 - \bar{x}_2$ for $\mu_1 - \mu_1$
  $\hat{p}_1 - \hat{p}_2$ for $p_1 - p_2$

For $\mu_1 - \mu_2$: $E = z_c\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$: when σ_1 and σ_2 are known

$E = t_c\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ when σ_1 or σ_2 is unknown

with $d.f.$ = smaller of $n_1 - 1$ or $n_2 - 1$

Software uses Satterthwaite's approximation for $d.f.$

For $p_1 - p_2$: $E = z_c\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$ for sufficiently large n

IMPORTANT WORDS & SYMBOLS

PART I

SECTION 7.1

Point estimate for μ 361
Confidence level c 361
Critical values z_c 361
Maximal margin of error E 363
c confidence interval 364
Sample size for estimating μ 368

SECTION 7.2

Student's t distribution 374
Degrees of freedom $d.f.$ 374
Critical values t_c 375

SECTION 7.3

Point estimate for p, $\hat{p}$ 388
Confidence interval for p 389
Margin of error for polls 393
Sample size for estimating p 394

PART II

SECTION 7.4

Independent samples 401
Dependent samples 401
$\bar{x}_1 - \bar{x}_2$ sampling distribution 402
Confidence interval for $\mu_1 - \mu_2$ (σ_1 and σ_2 known) 402
Confidence interval for $\mu_1 - \mu_2$ (σ_1 and σ_2 unknown) 404
Confidence interval for $p_1 - p_2$ 409
Satterthwaite's Formula for $d.f.$ 421
Pooled standard deviation 422

CHAPTER REVIEW PROBLEMS

1. See text.
2. $\bar{x}$ = mean of the confidence interval endpoints = \$3.30; E = \$0.15.
3. (a) No, the probability that μ is in the interval is either 0 or 1.
   (b) Yes, 99% confidence intervals are constructed in such a way that 99% of all such confidence intervals based on random samples of the designated size will contain μ.
4. Interval for a mean; \$1549 to \$1592; \$1536 to \$1604.
5. Interval for a mean; 176.91 to 180.49.


Jeff Greenberg/AGE Fotostock

6. Sample size for a mean; 102.

1. **Statistical Literacy** In your own words, carefully explain the meanings of the following terms: *point estimate, critical value, maximal margin of error, confidence level,* and *confidence interval.*

2. **Critical Thinking** Suppose you are told that a 95% confidence interval for the average price of a gallon of regular gasoline in your state is from \$3.15 to \$3.45. Use the fact that the confidence interval for the mean has the form $\bar{x} - E$ to $\bar{x} + E$ to compute the sample mean and the maximal margin of error E.

3. **Critical Thinking** If you have a 99% confidence interval for μ based on a simple random sample,
   (a) is it correct to say that the *probability* that μ is in the specified interval is 99%? Explain.
   (b) is it correct to say that in the long run, if you computed many, many confidence intervals using the prescribed method, about 99% of such intervals would contain μ? Explain.

For Problems 4–19, categorize each problem according to the parameter being estimated: proportion p, mean μ, difference of means $\mu_1 - \mu_2$, or difference of proportions $p_1 - p_2$. Then solve the problem.

4. **Auto Insurance: Claims** Anystate Auto Insurance Company took a random sample of 370 insurance claims paid out during a 1-year period. The average claim paid was \$1570. Assume $\sigma = \$250$. Find 0.90 and 0.99 confidence intervals for the mean claim payment.

5. **Psychology: Closure** Three experiments investigating the relationship between need for cognitive closure and persuasion were reported in "Motivated Resistance and Openness to Persuasion in the Presence or Absence of Prior Information" by A. W. Kruglanski (*Journal of Personality and Social Psychology*, Vol. 65, No. 5, pp. 861–874). Part of the study involved administering a "need for closure scale" to a group of students enrolled in an introductory psychology course. The "need for closure scale" has scores ranging from 101 to 201. For the 73 students in the highest quartile of the distribution, the mean score was $\bar{x} = 178.70$. Assume a population standard deviation of $\sigma = 7.81$. These students were all classified as high on their need for closure. Assume that the 73 students represent a random sample of all students who are classified as high on their need for closure. Find a 95% confidence interval for the population mean score μ on the "need for closure scale" for all students with a high need for closure.

6. **Psychology: Closure** How large a sample is needed in Problem 5 if we wish to be 99% confident that the sample mean score is within 2 points of the population mean score for students who are high on the need for closure?

7. Interval for a mean.
(a) Use a calculator.
(b) 64.1 to 84.3.

7. **Archaeology: Excavations** The Wind Mountain archaeological site is located in southwestern New Mexico. Wind Mountain was home to an ancient culture of prehistoric Native Americans called Anasazi. A random sample of excavations at Wind Mountain gave the following depths (in centimeters) from present-day surface grade to the location of significant archaeological artifacts (Source: *Mimbres Mogollon Archaeology*, by A. Woosley and A. McIntyre, University of New Mexico Press).

85	45	120	80	75	55	65	60
65	95	90	70	75	65	68	

(a) Use a calculator with mean and sample standard deviation keys to verify that $\bar{x} \approx 74.2$ cm and $s \approx 18.3$ cm.

(b) Compute a 95% confidence interval for the mean depth μ at which archaeological artifacts from the Wind Mountain excavation site can be found.

8. Interval for a mean.
(a) Use a calculator.
(b) 14.27 to 17.33.

8. **Archaeology: Pottery** Shards of clay vessels were put together to reconstruct rim diameters of the original ceramic vessels found at the Wind Mountain archaeological site (see source in Problem 7). A random sample of ceramic vessels gave the following rim diameters (in centimeters):

15.9	13.4	22.1	12.7	13.1	19.6	11.7	13.5	17.7	18.1

(a) Use a calculator with mean and sample standard deviation keys to verify that $\bar{x} \approx 15.8$ cm and $s \approx 3.5$ cm.

(b) Compute an 80% confidence interval for the population mean μ of rim diameters for such ceramic vessels found at the Wind Mountain archaeological site.

9. Interval for a proportion; 0.50 to 0.54.

9. **Telephone Interviews: Survey** The National Study of the Changing Work Force conducted an extensive survey of 2958 wage and salaried workers on issues ranging from relationships with their bosses to household chores. The data were gathered through hour-long telephone interviews with a nationally representative sample (*The Wall Street Journal*). In response to the question "What does success mean to you?" 1538 responded, "Personal satisfaction from doing a good job." Let p be the population proportion of all wage and salaried workers who would respond the same way to the stated question. Find a 90% confidence interval for p.

10. Sample size for a proportion; 9589.

10. **Telephone Interviews: Survey** How large a sample is needed in Problem 9 if we wish to be 95% confident that the sample percentage of those equating success with personal satisfaction is within 1% of the population percentage? *Hint:* Use $p \approx 0.52$ as a preliminary estimate.

11. Interval for a proportion.
(a) $\hat{p} \approx 0.4072$.
(b) 0.333 to 0.482.

11. **Archaeology: Pottery** Three-circle, red-on-white is one distinctive pattern painted on ceramic vessels of the Anasazi period found at the Wind Mountain archaeological site (see source for Problem 7). At one excavation, a sample of 167 potsherds indicated that 68 were of the three-circle, red-on-white pattern.

(a) Find a point estimate $\hat{p}$ for the proportion of all ceramic potsherds at this site that are of the three-circle, red-on-white pattern.

(b) Compute a 95% confidence interval for the population proportion p of all ceramic potsherds with this distinctive pattern found at the site.

12. Sample size for a proportion; $n = 258$ total, or 91 more.

12. **Archaeology: Pottery** Consider the three-circle, red-on-white pattern discussed in Problem 11. How many ceramic potsherds must be found and identified if we are to be 95% confident that the sample proportion $\hat{p}$ of such potsherds is within 6% of the population proportion of three-circle, red-on-white patterns found at this excavation site? *Hint:* Use the results of Problem 11 as a preliminary estimate.

13. **Agriculture: Bell Peppers** The following data represent soil water content (percent water by volume) for independent random samples of soil taken from two experimental fields growing bell peppers (Reference: *Journal of*

Agricultural, Biological, and Environmental Statistics). *Note:* These data are also available for download at the Componion Sites for this Text.

Soil water content from field I: x_1; $n_1 = 72$

15.1	11.2	10.3	10.8	16.6	8.3	9.1	12.3	9.1	14.3
10.7	16.1	10.2	15.2	8.9	9.5	9.6	11.3	14.0	11.3
15.6	11.2	13.8	9.0	8.4	8.2	12.0	13.9	11.6	16.0
9.6	11.4	8.4	8.0	14.1	10.9	13.2	13.8	14.6	10.2
11.5	13.1	14.7	12.5	10.2	11.8	11.0	12.7	10.3	10.8
11.0	12.6	10.8	9.6	11.5	10.6	11.7	10.1	9.7	9.7
11.2	9.8	10.3	11.9	9.7	11.3	10.4	12.0	11.0	10.7
8.8	11.1								

Soil water content from field II: x_2; $n_2 = 80$

12.1	10.2	13.6	8.1	13.5	7.8	11.8	7.7	8.1	9.2
14.1	8.9	13.9	7.5	12.6	7.3	14.9	12.2	7.6	8.9
13.9	8.4	13.4	7.1	12.4	7.6	9.9	26.0	7.3	7.4
14.3	8.4	13.2	7.3	11.3	7.5	9.7	12.3	6.9	7.6
13.8	7.5	13.3	8.0	11.3	6.8	7.4	11.7	11.8	7.7
12.6	7.7	13.2	13.9	10.4	12.8	7.6	10.7	10.7	10.9
12.5	11.3	10.7	13.2	8.9	12.9	7.7	9.7	9.7	11.4
11.9	13.4	9.2	13.4	8.8	11.9	7.1	8.5	14.0	14.2

(a) Use a calculator with mean and standard deviation keys to verify that $\bar{x}_1 \approx 11.42$, $s_1 \approx 2.08$, $\bar{x}_2 \approx 10.65$, and $s_2 \approx 3.03$.
(b) Let μ_1 be the population mean for x_1 and let μ_2 be the population mean for x_2. Find a 95% confidence interval for $\mu_1 - \mu_2$.
(c) **Interpretation** Explain what the confidence interval means in the context of this problem. Does the interval consist of numbers that are all positive? all negative? of different signs? At the 95% level of confidence, is the population mean soil water content of the first field higher than that of the second field?
(d) Which distribution (standard normal or Student's *t*) did you use? Why? Do you need information about the soil water content distributions?

13. Difference of means.
(a) Use a calculator.
(b) *d.f.* ≈ 71; round down to *d.f.* ≈ 70; $E \approx 0.83$; interval from −0.06 to 1.6.
(c) Because the interval contains both positive and negative values, we cannot conclude at the 95% confidence level that there is any difference in soil water content between the two fields.
(d) Student's *t* distribution because σ_1 and σ_2 are unknown. Both samples are large, so no assumptions about the original distributions are needed.

14. **Stocks: Retail and Utility** How profitable are different sectors of the stock market? One way to answer such a question is to examine profit as a percentage of stockholder equity. A random sample of 32 retail stocks such as Toys "Я" Us, Best Buy, and Gap was studied for x_1, profit as a percentage of stockholder equity. The result was $\bar{x}_1 = 13.7$. A random sample of 34 utility (gas and electric) stocks such as Boston Edison, Wisconsin Energy, and Texas Utilities was studied for x_2, profit as a percentage of stockholder equity. The result was $\bar{x}_2 = 10.1$ (Source: *Fortune 500*, Vol. 135, No. 8). Assume that $\sigma_1 = 4.1$ and $\sigma_2 = 2.7$.
(a) Let μ_1 represent the population mean profit as a percentage of stockholder equity for retail stocks, and let μ_2 represent the population mean profit as a percentage of stockholder equity for utility stocks. Find a 95% confidence interval for $\mu_1 - \mu_2$.
(b) **Interpretation** Examine the confidence interval and explain what it means in the context of this problem. Does the interval consist of numbers that are all positive? all negative? of different signs? At the 95% level of confidence, does it appear that the profit as a percentage of stockholder equity for retail stocks is higher than that for utility stocks?

14. Difference of means.
(a) 1.91% to 5.29%.
(b) Yes, it appears that profit as a percentage of stockholder equity is higher for retail stocks.

15. Difference of means.
(a) $d.f. \approx 17$; $E \approx 2.5$; interval from 5.5 to 10.5 pounds.
(b) Yes, it appears that the average weight of adult male wolves from the Northwest Territories is greater.

16. Difference of means.
(a) $d.f. \approx 5$; $E \approx 0.9$; interval from 1.2 to 3 pups per den.
(b) Yes, it appears that the average litter size of wolf pups in Ontario is greater.

17. Difference of proportions.
(a) $\hat{p}_1 \approx 0.8495$; $\hat{p}_2 \approx 0.8916$; -0.1409 to 0.0567.
(b) No. We do not detect a difference in proportions at the 95% level.

18. Difference of two proportions.
(a) $\hat{p}_1 \approx 0.533$; $\hat{p}_2 \approx 0.5435$; -0.2027 to 0.1823.
(b) No, at the 90% confidence level, we do not detect any differences in the proportions.

15. **Wildlife: Wolves** A random sample of 18 adult male wolves from the Canadian Northwest Territories gave an average weight $\bar{x}_1 = 98$ pounds, with estimated sample standard deviation $s_1 = 6.5$ pounds. Another sample of 24 adult male wolves from Alaska gave an average weight $\bar{x}_2 = 90$ pounds, with estimated sample standard deviation $s_2 = 7.3$ pounds (Source: *The Wolf* by L. D. Mech, University of Minnesota Press).

(a) Let μ_1 represent the population mean weight of adult male wolves from the Northwest Territories, and let μ_2 represent the population mean weight of adult male wolves from Alaska. Find a 75% confidence interval for $\mu_1 - \mu_2$.

(b) **Interpretation** Examine the confidence interval and explain what it means in the context of this problem. Does the interval consist of numbers that are all positive? all negative? of different signs? At the 75% level of confidence, does it appear that the average weight of adult male wolves from the Northwest Territories is greater than that of the Alaska wolves?

16. **Wildlife: Wolves** A random sample of 17 wolf litters in Ontario, Canada, gave an average of $\bar{x}_1 = 4.9$ wolf pups per litter, with estimated sample standard deviation $s_1 = 1.0$. Another random sample of 6 wolf litters in Finland gave an average of $\bar{x}_2 = 2.8$ wolf pups per litter, with sample standard deviation $s_2 = 1.2$ (see source for Problem 15).

(a) Find an 85% confidence interval for $\mu_1 - \mu_2$, the difference in population mean litter size between Ontario and Finland.

(b) **Interpretation** Examine the confidence interval and explain what it means in the context of this problem. Does the interval consist of numbers that are all positive? all negative? of different signs? At the 85% level of confidence, does it appear that the average litter size of wolf pups in Ontario is greater than the average litter size in Finland?

17. **Survey Response: Validity** The book *Survey Responses: An Evaluation of Their Validity* by E. J. Wentland and K. Smith (Academic Press), includes studies reporting accuracy of answers to questions from surveys. A study by Locander et al. considered the question "Are you a registered voter?" Accuracy of response was confirmed by a check of city voting records. Two methods of survey were used: a face-to-face interview and a telephone interview. A random sample of 93 people were asked the voter registration question face-to-face. Seventy-nine respondents gave accurate answers (as verified by city records). Another random sample of 83 people were asked the same question during a telephone interview. Seventy-four respondents gave accurate answers. Assume the samples are representative of the general population.

(a) Let p_1 be the population proportion of all people who answer the voter registration question accurately during a face-to-face interview. Let p_2 be the population proportion of all people who answer the question accurately during a telephone interview. Find a 95% confidence interval for $p_1 - p_2$.

(b) **Interpretation** Does the interval contain numbers that are all positive? all negative? mixed? Comment on the meaning of the confidence interval in the context of this problem. At the 95% level, do you detect any difference in the proportion of accurate responses from face-to-face interviews compared with the proportion of accurate responses from telephone interviews?

18. **Survey Response: Validity** Locander et al. (see reference in Problem 17) also studied the accuracy of responses on questions involving more sensitive material than voter registration. From public records, individuals were identified as having been charged with drunken driving not less than 6 months or more than 12 months from the starting date of the

study. Two random samples from this group were studied. In the first sample of 30 individuals, the respondents were asked in a face-to-face interview if they had been charged with drunken driving in the last 12 months. Of these 30 people interviewed face-to-face, 16 answered the question accurately. The second random sample consisted of 46 people who had been charged with drunken driving. During a telephone interview, 25 of these responded accurately to the question asking if they had been charged with drunken driving during the past 12 months. Assume the samples are representative of all people recently charged with drunken driving.

(a) Let p_1 represent the population proportion of all people with recent charges of drunken driving who respond accurately to a face-to-face interview asking if they have been charged with drunken driving during the past 12 months. Let p_2 represent the population proportion of people who respond accurately to the question when it is asked in a telephone interview. Find a 90% confidence interval for $p_1 - p_2$.

(b) **Interpretation** Does the interval found in part (a) contain numbers that are all positive? all negative? mixed? Comment on the meaning of the confidence interval in the context of this problem. At the 90% level, do you detect any differences in the proportion of accurate responses to the question from face-to-face interviews as compared with the proportion of accurate responses from telephone interviews?

Problem 19 contains a good topic for class discussion.

19. (a) Events are independent; use multiplication rule. Then use complement rule.
(b) $c \approx 0.9487$.
(c) Answers vary.

19. **Expand Your Knowledge: Two Confidence Intervals** What happens if we want several confidence intervals to hold at the same time (concurrently)? Do we still have the same level of confidence we had for *each* individual interval?

(a) Suppose we have two independent random variables x_1 and x_2 with respective population means μ_1 and μ_2. Let us say that we use sample data to construct two 80% confidence intervals.

Confidence Interval	Confidence Level
$A_1 < \mu_1 < B_1$	0.80
$A_2 < \mu_2 < B_2$	0.80

Now, what is the probability that *both* intervals hold at the same time? Use methods of Section 4.2 to show that

$$P(A_1 < \mu_1 < B_1 \text{ and } A_2 < \mu_2 < B_2) = 0.64$$

Hint: You are combining independent events. If the confidence is 64% that both intervals hold concurrently, explain why the risk that at least one interval does not hold (i.e., fails) must be 36%.

(b) Suppose we want *both* intervals to hold with 90% confidence (i.e., only 10% risk level). How much confidence c should each interval have to achieve this combined level of confidence? (Assume that each interval has the same confidence level c.)

Hint: $P(A_1 < \mu_1 < B_1 \text{ and } A_2 < \mu_2 < B_2) = 0.90$

$$P(A_1 < \mu_1 < B_1) \times P(A_2 < \mu_2 < B_2) = 0.90$$
$$c \times c = 0.90$$

Now solve for c.

(c) If we want *both* intervals to hold at the 90% level of confidence, then the individual intervals must hold at a *higher* level of confidence. Write a brief but detailed explanation of how this could be of importance in a large, complex engineering design such as a rocket booster or a spacecraft.

SUMMARY

PART I

Hypothesis testing is a major component of inferential statistics. In hypothesis testing, we propose a specific value for the population parameter in question. Then we use sample data from a random sample and probability to determine whether or not to reject this specific value for the parameter.

Basic components of a hypothesis test are:

- The *null hypothesis* H_0 states that a parameter equals a specific value.
- The *alternate hypothesis* H_1 states that the parameter is greater than, less than, or simply not equal to the value specified in H_0.
- The *level of significance* α of the test is the probability of rejecting H_0 when it is true.
- The *sample test statistic* corresponding to the parameter in H_0 is computed from a random sample and converted to an appropriate sampling distribution.
- Assuming H_0 is true, the probability that a sample test statistic will take on a value as extreme as, or more extreme than, the observed sample test statistic is the *P-value* of the test. The *P*-value is computed by using the sample test statistic, the corresponding sampling distribution, H_0, and H_1.
- If *P*-value $\le \alpha$, we reject H_0. If *P*-value $> \alpha$, we fail to reject H_0.
- We say that sample data are *significant* if we can reject H_0.

An alternative way to conclude a test of hypotheses is to use critical regions based on the alternate hypothesis and α. Critical values z_0 are found in Table 5(c) of Appendix II. Critical values t_0 are found in Table 6 of Appendix II. If the sample test statistic falls beyond the critical values—that is, in the critical region—we reject H_0.

The methods of hypothesis testing are very general, and we will see them used again in later chapters. In the first part of this chapter, we looked at tests involving

- Parameter μ. Use standard normal or Student's *t* distribution. See procedure displays in Section 8.2.
- Parameter p. Use standard normal distribution. See procedure displays in Section 8.3.

PART II

In the second part of this chapter we looked at tests involving

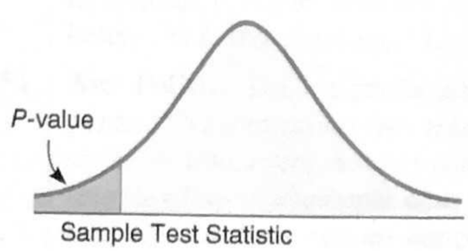
- Paired difference test for difference of means from dependent populations. Use Student's *t* distribution. See procedure displays in Section 8.4.
- Parameter $\mu_1 - \mu_2$ from independent populations. Use standard normal or Student's *t* distribution. See procedure displays in Section 8.5.
- Parameter $p_1 - p_2$ from independent populations. Use standard normal distribution. See procedure displays in Section 8.5.

FINDING THE *P*-VALUE CORRESPONDING TO A SAMPLE TEST STATISTIC

Use the appropriate sampling distribution as described in procedure displays for each of the various tests.

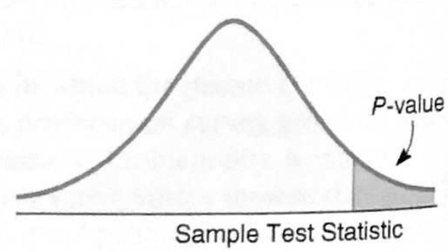
Left-Tailed Test

P-value = area to the left of the sample test statistic.



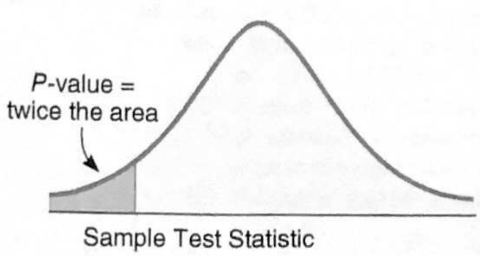
Right-Tailed Test

P-value = area to the right of the sample test statistic.

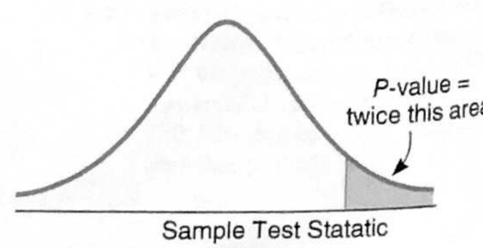


Two-Tailed Test

Sample test statistic lies to *left* of center
P-value = twice the area to the left of sample test statistic



Sample test statistic lies to *right* of center
P-value = twice area to the right of sample test statistic



Sampling Distributions for Inferences Regarding μ or p

Parameter	Condition	Sampling Distribution
μ	σ is known and x has a normal distribution or $n \geq 30$	Normal Distribution
μ	σ is not known and x has a normal or mound-shaped symmetric distribution or $n \geq 30$	Student's t Distribution with $d.f. = n - 1$
p	$np > 5$ and $n(1 - p) > 5$	Normal Distribution

IMPORTANT WORDS & SYMBOLS

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VIEWPOINT *Will It Rain?*

Do cloud-seed experiments ever work? If you seed the clouds, will it rain? If it does rain, who will benefit? Who will be displeased by the rain? If you seed the clouds and nothing happens, will taxpayers (who support the effort) complain or rejoice? Maybe this should be studied over a remote island—such as Tasmania (near Australia). Using what you already know about statistical testing, you can conduct your own tests, given the appropriate data. Remember, there are sociologic questions (pleased/displeased with result) as well as technical questions (number of inches of rain produced). For data regarding cloud-seeding experiments over Tasmania, visit the web site for the Carnegie Mellon University Data and Story Library (DASL). From the DASL site, look under Datasets for Cloud.

CHAPTER REVIEW PROBLEMS

Answers may vary due to rounding.

1. Look at the original x distribution. If it is normal or $n \geq 30$, and σ is known, use the standard normal distribution. If the x distribution is mound-shaped or $n \geq 30$, and σ is unknown, use the Student's t distribution. The $d.f.$ is determined by the application.
2. If a test is significant, we reject H_0. Results may or may not be important.
3. A larger sample size increases the $|z|$ or $|t|$ value of the sample test statistic.
4. A larger $|z|$ or $|t|$ value has a smaller corresponding P-value.
5. Single mean.
   (a) $\alpha = 0.05$; H_0: $\mu = 11.1$; H_1: $\mu \neq 11.1$.
   (b) Standard normal; $z = -3.00$.
   (c) P-value $= 0.0026$; on standard normal curve, shade area to the right of 3.00 and to the left of -3.00.
   (d) P-value of $0.0026 \leq 0.05$ for α; reject H_0.
   (e) At the 5% level of significance, the evidence is sufficient to say that the miles driven per vehicle in Chicago is different from the national average.
6. Single proportion.
   (a) $\alpha = 0.05$; H_0: $p = 0.35$; H_1: $p > 0.35$.
   (b) Standard normal; $z = 2.48$.
   (c) P-value $= 0.0066$; on standard normal curve, shade area to the right of 2.48.
   (d) P-value of $0.0066 \leq 0.05$ for α; reject H_0.
   (e) At the 5% level of significance, the evidence indicates that more than 35% of the students have jobs.
7. Single mean.
   (a) $\alpha = 0.01$; H_0: $\mu = 0.8$; H_1: $\mu > 0.8$.
   (b) Student's t, $d.f. = 8$; $t \approx 4.390$.
   (c) $0.0005 < P\text{-value} < 0.005$; on t graph, shade area to the right of 4.390. From TI-84, P-value ≈ 0.0012.
   (d) P-value interval ≤ 0.01 for α; reject H_0.
   (e) At the 1% level of significance, the evidence is sufficient to say that the Toylot claim of 0.8 A is too low.
8. Difference of means.
   (a) $\alpha = 0.01$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 > \mu_2$.
   (b) Student's t, $d.f. = 11$; $t \approx 2.986$.
   (c) $0.005 < P\text{-value} < 0.01$; on t graph, shade area to the right of 2.986. From TI-84, $d.f. \approx 21.95$; P-value ≈ 0.0034.
   (d) P-value interval ≤ 0.01 for α; reject H_0.
   (e) At the 1% level of significance, the evidence shows that the yellow paint has less visibility after 1 year.

1. **Statistical Literacy** When testing μ or the difference of means $\mu_1 - \mu_2$ from independent populations, how do we decide whether to use the standard normal distribution or a Student's t distribution?

2. **Statistical Literacy** What do we mean when we say a test is *significant*? Does this necessarily mean the results are important?

3. **Critical Thinking** All other conditions being equal, does a larger sample size increase or decrease the corresponding magnitude of the z or t value of the sample test statistic?

4. **Critical Thinking** All other conditions being equal, does a z or t value with larger magnitude have a larger or smaller corresponding P-value?

Before you solve each problem below, first categorize it by answering the following question: Are we testing a single mean, a difference of means, a paired difference, a single proportion, or a difference of proportions? Assume underlying population distributions are mound-shaped and symmetric for problems with small samples that involve testing a mean or difference of means. Then provide the following information for Problems 5–18.

(a) What is the level of significance? State the null and alternate hypotheses.
(b) **Check Requirements** What sampling distribution will you use? What assumptions are you making? Compute the sample test statistic and corresponding distribution value.
(c) Find (or estimate) the P-value. Sketch the sampling distribution and show the area corresponding to the P-value.
(d) Based on your answers in parts (a) to (c), will you reject or fail to reject the null hypothesis? Are the data statistically significant at level α?
(e) **Interpret** your conclusion in the context of the application.

Note: For degrees of freedom $d.f.$ not in the Student's t table, use the closest $d.f.$ that is *smaller*. In some situations, this choice of $d.f.$ may increase the P-value by a small amount and therefore produce a slightly more "conservative" answer. Answers may vary due to rounding.

5. **Vehicles: Mileage** Based on information in *Statistical Abstract of the United States* (116th edition), the average annual miles driven per vehicle in the United States is 11.1 thousand miles, with $\sigma \approx 600$ miles. Suppose that a random sample of 36 vehicles owned by residents of Chicago showed that the average mileage driven last year was 10.8 thousand miles. Does this indicate that the average miles driven per vehicle in Chicago is different from (higher or lower than) the national average? Use a 0.05 level of significance.

6. **Student Life: Employment** Professor Jennings claims that only 35% of the students at Flora College work while attending school. Dean Renata thinks that the professor has underestimated the number of students with part-time or full-time jobs. A random sample of 81 students shows that 39 have jobs. Do the data indicate that more than 35% of the students have jobs? (Use a 5% level of significance.)

7. **Toys: Electric Trains** The Toylot Company makes an electric train with a motor that it claims will draw an average of only 0.8 ampere (A) under a normal load. A sample of nine motors was tested, and it was found that the mean current was $\bar{x} = 1.4$ A, with a sample standard deviation of $s = 0.41$ A. Do the data indicate that the Toylot claim of 0.8 A is too low? (Use a 1% level of significance.)

8. **Highways: Reflective Paint** The highway department is testing two types of reflecting paint for concrete bridge end pillars. The two kinds of paint are alike in every respect except that one is orange and the other is yellow. The orange paint is applied to 12 bridges, and the yellow paint is applied to 12 bridges. After a period of 1 year, reflectometer readings were made on all these bridge end pillars. (A higher reading means better visibility.) For the orange paint, the

9. Single proportion.
(a) $\alpha = 0.01$; H_0: $p = 0.60$;
H_1: $p < 0.60$.
(b) Standard normal; $z = -3.01$.
(c) P-value $= 0.0013$; on standard normal curve, shade area to the left of -3.01.
(d) P-value of $0.0013 \le 0.01$ for α; reject H_0.
(e) At the 1% level of significance, the evidence is sufficient to show that the mortality rate has dropped.

10. Difference of means.
(a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$.
(b) Student's t, $d.f. = 50$; $t \approx -2.735$.
(c) $0.0010 < P\text{-value} < 0.010$; on t graph, shade area to the right of 2.735 and to the left of -2.735. From TI-84, $d.f. \approx 94.53$; P-value ≈ 0.0074.
(d) P-value interval ≤ 0.05 for α; reject H_0.
(e) At the 5% level of significance, the evidence shows there is a significant difference in average off-schedule times.

11. Single mean.
(a) $\alpha = 0.01$; H_0: $\mu = 40$; H_1: $\mu > 40$.
(b) Standard normal; $z = 3.34$.
(c) P-value $= 0.0004$; on standard normal curve, shade area to the right of 3.34.
(d) P-value of $0.0004 \le 0.01$ for α; reject H_0.
(e) At the 1% level of significance, the evidence is sufficient to say that the population average number of matches is larger than 40.

12. Difference of proportions.
(a) $\alpha = 0.05$; H_0: $p_1 = p_2$; H_1: $p_1 < p_2$.
(b) Standard normal; $z = -0.91$.
(c) P-value $= 0.1814$; on standard normal curve, shade area to the left of -0.91.
(d) P-value of $0.1814 > 0.05$ for α; fail to reject H_0.
(e) At the 5% level of significance, there is insufficient evidence to say that the population proportion of suburban residents subscribing to *Sporting News* is higher.

13. Difference of means.
(a) $\alpha = 0.05$; H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$.
(b) Student's t, $d.f. = 50$; $\bar{x}_1 - \bar{x}_2 = 0.3$ cm; $t \approx 1.808$.
(c) $0.050 < P\text{-value} < 0.100$; on t graph, shade area to the right of 1.808 and to the left of -1.808. From TI-84, $d.f. \approx 100.27$; P-value ≈ 0.0735.
(d) P-value interval > 0.05 for α; do not reject H_0.

mean reflectometer reading was $\bar{x}_1 = 9.4$, with standard deviation $s_1 = 2.1$. For the yellow paint, the mean was $\bar{x}_2 = 6.9$, with standard deviation $s_2 = 2.0$. Based on these data, can we conclude that the yellow paint has less visibility after 1 year? (Use a 1% level of significance.)

9. **Medical: Plasma Compress** A hospital reported that the normal death rate for patients with extensive burns (more than 40% of skin area) has been significantly reduced by the use of new fluid plasma compresses. Before the new treatment, the mortality rate for extensively burned patients was about 60%. Using the new compresses, the hospital found that only 40 of 90 patients with extensive burns died. Use a 1% level of significance to test the claim that the mortality rate has dropped.

10. **Bus Lines: Schedules** A comparison is made between two bus lines to determine if arrival times of their regular buses from Denver to Durango are off schedule by the same amount of time. For 51 randomly selected runs, bus line A was observed to be off schedule an average time of 53 minutes, with standard deviation 19 minutes. For 60 randomly selected runs, bus line B was observed to be off schedule an average of 62 minutes, with standard deviation 15 minutes. Do the data indicate a significant difference in average off-schedule times? Use a 5% level of significance.

11. **Matches: Number per Box** The Nero Match Company sells matchboxes that are supposed to have an average of 40 matches per box, with $\sigma = 9$. A random sample of 94 Nero matchboxes shows the average number of matches per box to be 43.1. Using a 1% level of significance, can you say that the average number of matches per box is more than 40?

12. **Magazines: Subscriptions** A study is made of residents in Phoenix and its suburbs concerning the proportion of residents who subscribe to *Sporting News*. A random sample of 88 urban residents showed that 12 subscribed, and a random sample of 97 suburban residents showed that 18 subscribed. Does this indicate that a higher proportion of suburban residents subscribe to *Sporting News*? (Use a 5% level of significance.)

13. **Archaeology: Arrowheads** The Wind Mountain archaeological site is in southwest New Mexico. Prehistoric Native Americans called Anasazi once lived and hunted small game in this region. A stemmed projectile point is an arrowhead that has a notch on each side of the base. Both stemmed and stemless projectile points were found at the Wind Mountain site. A random sample of $n_1 = 55$ stemmed projectile points showed the mean length to be $\bar{x}_1 = 3.0$ cm, with sample standard deviation $s_1 = 0.8$ cm. Another random sample of $n_2 = 51$ stemless projectile points showed the mean length to be $\bar{x}_2 = 2.7$ cm, with $s_2 = 0.9$ cm (Source: *Mimbres Mogollon Archaeology*, by A. I. Woosley and A. J. McIntyre, University of New Mexico Press). Do these data indicate a difference (either way) in the population mean length of the two types of projectile points? Use a 5% level of significance.

14. **Civil Service: College Degrees** The Congressional Budget Office reports that 36% of federal civilian employees have a bachelor's degree or higher (*The Wall Street Journal*). A random sample of 120 employees in the private sector showed that 33 have a bachelor's degree or higher. Does this indicate that the percentage of employees holding bachelor's degrees or higher in the private sector is less than that in the federal civilian sector? Use $\alpha = 0.05$.

15. **Vending Machines: Coffee** A machine in the student lounge dispenses coffee. The average cup of coffee is supposed to contain 7.0 ounces. Eight cups of coffee from this machine show the average content to be 7.3 ounces with a standard deviation of 0.5 ounce. Do you think that the machine has slipped out of adjustment and that the average amount of coffee per cup is different from 7 ounces? Use a 5% level of significance.

(e) At the 5% level of significance, the evidence is insufficient to indicate a difference in population mean length between the two types of projectile points.

14. Single proportion.
(a) $\alpha = 0.05$; $H_0: p = 0.36$; $H_1: p < 0.36$.
(b) Standard normal; $z = -1.94$.
(c) P-value $= 0.0262$; on standard normal curve, shade area to the left of -1.94.
(d) P-value of $0.0262 \le 0.05$ for α; reject H_0.
(e) At the 5% level of significance, the evidence is sufficient to show that the population percentage of employees holding bachelor's degrees or higher in the private sector is less than that in the federal civilian sector.

15. Single mean.
(a) $\alpha = 0.05$; $H_0: \mu = 7$ oz; $H_1: \mu \ne 7$ oz.
(b) Student's t, $d.f. = 7$; $t \approx 1.697$.
(c) $0.100 < P$-value < 0.150; on t graph, shade area to the right of 1.697 and to the left of -1.697. From TI-84, P-value ≈ 0.1335.
(d) P-value interval > 0.05 for α; do not reject H_0.
(e) At the 5% level of significance, the evidence is insufficient to show that the population mean amount of coffee per cup is different from 7 oz.

16. Paired difference test.
(a) $\alpha = 0.01$; $H_0: \mu_d = 0$; $H_1: \mu_d > 0$.
(b) Student's t, $d.f. = 5$; $\bar{d} = 9.833$; $t = 6.066$.
(c) $0.0005 < P$-value < 0.005; on t graph, shade area to the right of 6.066. From TI-84, P-value ≈ 0.0009.
(d) P-value interval ≤ 0.01 for α; reject H_0.
(e) At the 1% level of significance, there is sufficient evidence to indicate that the program of the experimental group promoted creative problem solving.

17. Paired difference test.
(a) $\alpha = 0.05$; $H_0: \mu_d = 0$; $H_1: \mu_d < 0$.
(b) Student's t, $d.f. = 4$; $\bar{d} \approx -4.94$; $t \approx -2.832$.
(c) $0.010 < P$-value < 0.025; on t graph, shade area to the left of -2.832. From TI-84, P-value ≈ 0.0236.
(d) P-value interval ≤ 0.05 for α; reject H_0.
(e) At the 5% level of significance, there is sufficient evidence to claim that the population average net sales improved.

16. **Psychology: Creative Thinking** Six sets of identical twins were randomly selected from a population of identical twins. One child was taken at random from each pair to form an experimental group. These children participated in a program designed to promote creative thinking. The other child from each pair was part of the control group that did not participate in the program to promote creative thinking. At the end of the program, a creative problem-solving test was given, with the results shown in the following table:

Twin pair	A	B	C	D	E	F
Experimental group	53	35	12	25	33	47
Control group	39	21	5	18	21	42

Higher scores indicate better performance in creative problem solving. Do the data support the claim that the program of the experimental group did promote creative problem solving? (Use $\alpha = 0.01$.)

17. **Marketing: Sporting Goods** A marketing consultant was hired to visit a random sample of five sporting goods stores across the state of California. Each store was part of a large franchise of sporting goods stores. The consultant taught the managers of each store better ways to advertise and display their goods. The net sales for 1 month before and 1 month after the consultant's visit were recorded as follows for each store (in thousands of dollars):

Store	1	2	3	4	5
Before visit	57.1	94.6	49.2	77.4	43.2
After visit	63.5	101.8	57.8	81.2	41.9

Do the data indicate that the average net sales improved? (Use $\alpha = 0.05$.)

18. **Sports Car: Fuel Injection** The manufacturer of a sports car claims that the fuel injection system lasts 48 months before it needs to be replaced. A consumer group tests this claim by surveying a random sample of 10 owners who had the fuel injection system replaced. The ages of the cars at the time of replacement were (in months):

29 42 49 48 53 46 30 51 42 52

i. Use your calculator to verify that the mean age of a car when the fuel injection system fails is $\bar{x} = 44.2$ months, with standard deviation $s \approx 8.61$ months.

ii. Test the claim that the fuel injection system lasts less than an average of 48 months before needing replacement. Use a 5% level of significance.

18. i. Use a calculator.
ii. Single mean.
(a) $\alpha = 0.05$; $H_0: \mu = 48$ months; $H_1: \mu < 48$ months.
(b) Student's t, $d.f. = 9$; $t = -1.396$.
(c) $0.075 < P$-value < 0.100; on t graph, shade area to the left of -1.396. From TI-84, P-value ≈ 0.0981.
(d) P-value interval > 0.05 for α; do not reject H_0.
(e) At the 5% level of significance, there is insufficient evidence to claim that the injection system lasts less than an average of 48 months.

4. Questions for Colton

If you have questions about any of these practice problems, please take a picture of this QR code and **vote for solutions you want to see**. I plan to spend class on Monday, November 18 to work out solutions (by hand and with the TI-84) based on your votes.

Figure 1. Voting link for solutions to midterm 2 practice problems: shoot me!

You can also email me a question or arrange office hours with me via
`https://go.oncehub.com/coltongrainger`.

Good luck on this exam, and have fun studying!