MATH 3430-02 WEEK 6-1

Key Words: Variation of Parameters.

Setting: Second order linear ODE, nonhomogeneous, homogeneous solutions known, need to find a particular solution.

Idea: It is again an idea of 'modifying' something known. In *reduction of order*, you've multiplied a homogeneous solution by a function $v(t)$ to seek for another solution. Here, in **variation of parameters**, suppose that two fundamental (basis) homogeneous solutions are known, you make a 'linear combination' with function coefficients in order to seek a nonhomogeneous solution.

Let's derive some formulae.

Consider the ODE:

$$y'' + p(t)y' + q(t)y = g(t).$$

Suppose that y_1, y_2 are fundamental homogeneous solutions.

We want to seek a particular solution of the form

$$u_1 y_1 + u_2 y_2.$$

Let's substitute this expression in the equation and see what we can get.

At this point, it may look difficult to solve for u_1, u_2 from the only equation given. However, the equation has some 'freedom' in it. We are allowed to impose some new condition on u_1, u_2 and still being able to obtain solutions.

In fact, we can force

$$u_1' y_1 + u_2' y_2$$

to be zero.

In doing this, the second derivatives of u_i as well as the terms involving $p(t)$ are gone. We are left with this single condition

$$u_1'y_1' + u_2'y_2' = g(t).$$

You should write the previous two equations in the matrix form:

$$\left(\qquad\qquad\qquad \right) \begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} 0 \\ g(t) \end{pmatrix}.$$

Who guarantees that you can find u_1', u_2', hence u_1, u_2, from this equation?

Now find u_1, u_2.

$$u_1 =$$

$$u_2 =$$

It follows that we have the formula

$$y_p(t) = u_1y_1 + u_2y_2 =$$

At this point, you should have a picture for solving 2nd order linear ODEs: (We'll draw a picture in class.)

Q1. Use the method of variation of parameters to find a particular solution of the ODE

$$y'' - 2y' - 3y = e^{3t}.$$

Q2. Find the general solutions of the equation

$$y'' - \frac{2t}{1+t^2}y' + \frac{2}{1+t^2}y = 1 + t^2.$$

You may notice an obvious homogeneous solution $y_1(t) = t$.