Math 3430-02 Spring 2019
Exam II

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: ________________________ Signature: ________________________

Instructions:

- Notes, books, calculators or computers are not allowed in this exam.
- Make sure you show the work that leads to your answer to receive full credit. If you are using a theorem or a fact to draw some conclusions, quote the result.
- This test contains **6 pages** and **4 questions**. You have **50 minutes** to answer all the questions.

Good Luck !

Question	**Score**
1	/ 25
2	/ 25
3	/ 25
4	/ 25
Total	/**100**

1. Consider the following 2nd order initial value problem:

$$y''y = (y')^2, \qquad y(0) = y'(0) = 1.$$

a. Let $y(x)$ be the solution of the system above. Use the method of **successive differentiation** to find $y''(0), y'''(0)$ and $y^{(4)}(0)$.

b. Write down the first **five** nonzero terms in the power series solution $y(x)$ (centered at $x = 0$).

c. Based on your answer in part **b**, do you have a guess of what $y(x)$ is? If so, verify your answer by substituting it in the original equation.

2. Consider the following 1st-order inhomogeneous initial value problem:

$$(t^2+1)y' + 3y = 1 + 2t, \qquad y(0) = 1.$$

Use the method of **undetermined coefficients** to find a series solution of this problem. In particular, write down a recurrence relation satisfied by the coefficients of the series.

3. Answer the following questions.

(a) What is a 2nd order ODE that has

$$y(x) = x^2 - 1 + C_1 e^{-2x} + C_2 e^{3x}$$

as its general solution?

(b) Compute the Laplace transform

$$\mathcal{L}\{H_3(t) e^{-t} \cos(t-3)\}.$$

(c) What is an example of an ODE for which the method of Laplace transform works is more suitable compared with other solving methods?

Now write down an ODE for which the method of Laplace transform may not work very well.

4. Use the method of **Laplace Transform** to solve the following 1st order system with initial values:

$$\begin{cases} \dfrac{\mathrm{d}y_1}{\mathrm{d}t} - y_2 = H_1(t)(t-1), \\ y_1 + \dfrac{\mathrm{d}y_2}{\mathrm{d}t} = 0, \\ y_1(0) = 0, \quad y_2(0) = 0. \end{cases}$$

Note: It may be useful to denote $Y_1(s) := \mathcal{L}\{y_1(t)\}$ and $Y_2(s) := \mathcal{L}\{y_2(t)\}$.

Table of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}, \qquad s > 0$
e^{at}	$\frac{1}{s-a}, \qquad s > a$
t^n ($n > 0$, integer)	$\frac{n!}{s^{n+1}}, \qquad s > 0$
$\sin at$	$\frac{a}{s^2 + a^2}, \qquad s > 0$
$\cos at$	$\frac{s}{s^2 + a^2}, \qquad s > 0$
$\sinh at$	$\frac{a}{s^2 - a^2}, \qquad s > 0$
$\cosh at$	$\frac{s}{s^2 - a^2}, \qquad s > 0$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \qquad s > a$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \qquad s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}, \qquad s > 0$
$e^{ct} f(t)$	$F(s-c)$
$u_c(t) f(t-c)$	$e^{-cs} F(s)$
$f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right), \qquad c > 0$
$\int_0^t f(t-\tau) g(\tau) d\tau$	$F(s)G(s)$
$\delta(t-c)$	e^{-cs}
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \ldots - f^{(n-1)}(0)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$