MATH 3430-02 WEEK 5-2

Key Words: Homogeneous linear 2nd order ODEs in general; Reduction of Order.

Review.
Q1. Find the general solution of the following ODEs:
(1) $y'' - 2y' + 3y = 0$;

(2) $y'' + 5y' + 4y = 0$;

(3) $y'' - 6y' + 9y = 0$.

Q2. Solve the initial value problem

$$y'' + 8y' + 16y = 0, \qquad y(0) = 1, \quad y'(0) = 2.$$

So we know how to solve ODEs of the form

$$y'' + ay' + by = 0,$$

where a, b are real constants.

How about the general homogeneous linear 2nd order ODEs

$$y'' + p(t)y' + q(t)y = 0?$$

Bad news: A first solution is often not easy to find.

Good news: Suppose you are given one solution, there is a method to find a second one that is linearly independent from the first, called *reduction of order*.

Q3. Suppose that you observed that $y(t) = t$ is a solution of the ODE below. Find all possible functions $v(t)$ such that $y_2(t) := v(t)y_1(t)$ is a solution of the same ODE.

$$t^2y'' - 2ty' + 2y = 0.$$

Q4. Find the general solution of the ODE below, given that $y(t) = 3t^2 - 1$ is a solution.

$$(1 - t^2)y'' - 2ty' + 6y = 0.$$

In general, given a homogeneous linear second order ODE

$$y'' + p(t)y' + q(t)y = 0,$$

suppose that $y_1(t)$ is a solution. We derive the condition on $v(t)$ such that $y_2(t) := v(t)y_1(t)$ remains a solution of the same ODE.

Q5. Replacing $y(t)$ by $v(t)y_1(t)$ in the ODE, then use the condition that $y_1(t)$ is already a solution, what equation in $v(t)$ do you obtain?

Q6. Notice that, in the equation for $v(t)$ you derived in **Q5**, $v(t)$ only appears in its derivatives. This suggests us to make the substitution:

$$u(t) := v'(t).$$

Therefore, we obtain the equation in $u(t)$:

This is a ____-order ________________ ODE, whose general solution is:

$$u(t) =$$

It follows that

$$v(t) =$$

and

$$y_2(t) =$$

Q7. Solve the following ODE, given that $y_1(t) = \sin(t)/\sqrt{t}$ is a solution.

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y = 0.$$