

MATH 3430-02 WEEK 10-2

Key Words: Equations with a discontinuous forcing term.

Q1. Consider the expression

$$ay'' + by' + cy$$

with the zero initial conditions $y(0) = y'(0) = 0$. Find the Laplace transform:

$$\mathcal{L}\{ay'' + by' + cy\}.$$

What do you observe?

Q2. Solve the initial value problem

$$2y'' + y' + 2y = g(t), \quad y(0) = y'(0) = 0,$$

where

$$g(t) = \begin{cases} 1, & 5 \leq t < 20, \\ 0, & \text{elsewhere.} \end{cases}$$

(The idea is first expressing $g(t)$ in a closed form using step functions then applying the Laplace transform to both sides of the ODE.)

Q3. Solve the initial value problem

$$y'' - 3y' + 2y = g(t), \quad y(0) = y'(0) = 0,$$

where

$$g(t) = \begin{cases} 1, & t \in [0, 1) \cup [2, 3) \cup [4, 5), \\ 0, & \text{elsewhere.} \end{cases}$$

Q4. Solve the initial value problem

$$y'' + 2y' + y = g(t), \quad y(0) = 1, \quad y'(0) = 0,$$

where

$$g(t) = \begin{cases} \sin 2t, & t \in [0, \pi/2), \\ 0, & \text{elsewhere.} \end{cases}$$