MATH 3430-02 WEEK 5-3

Key Words: Nonhomogeneou vs. homogeneous linear equations.

This lecture is essentially a review of the ideas in lecture [4-2]; the structure of the solution set of a linear (second order) ODE.

Q1. Find a solution of the form $y(t) = Ae^{2t}$ for the ODE

$$y'' + 2y' - 3y = 3e^{2t}.$$

Then find all the *homogeneous solutions*, where A is a constant you should determine. Finally, find the general solutions of the original ODE.

Q2. Find the general solutions of the ODE

$$y'' - 6y' + 9y = 2t.$$

You may guess that a particular solution is of the form $at + b$.

Q3. Find the solution of the initial value problem

$$y'' + 2y' + 4y = \cos t, \quad y(0) = 1, \quad y'(0) = 2.$$

Here what do you think could be a reasonable guess of a particular solution?

Q4. Find the general solution of the following initial value problem

$$y'' - 5y' + 4y = 2e^t.$$

Here what do you think could be a reasonable guess of a particular solution? What do you observe after trying? If you observe a problem, how to get around it?

Next time, we'll learn a more systematic way to 'guess' the form of a particular solution.

Q5. Suppose that a second order linear ODE has the following solutions:

$$y_1(t) = e^{3t} + 2t + 3, \quad y_2(t) = e^{2t} + e^{3t} + 2t + 3, \quad y_3(t) = e^{2t} + 2t + 3.$$

Find the general solutions of the ODE.

Q6. Suppose that a second order linear ODE has the following solutions:

$$y_1(t) = 1 + e^{t^2}, \quad y_2(t) = 1 + te^{t^2}, \quad y_3(t) = (1+t)e^{t^2} + 1.$$

Find the solution that satisfies the initial condition

$$y(0) = 1, \quad y'(0) = -1.$$