Week 05 group problems & study guide

2019-09-25

1. Consider the following events for a college student selected at random:

$$A = \text{student is female}$$
$$B = \text{student is majoring in business}$$

(a) Describe with an English phrase, what quantity each of the following expressions describes.

i. $P(A \mid B)$

ii. $P(A \text{ and } B)$

iii. $P(A \text{ or } B^c)$

iv. $P(A^c \text{ and } B)$

(b) Given the following contingency table, compute the value of each of the following expressions.

	MAJOR			
GENDER	Business	Biology	Psychology	Row Total
Female	30	47	50	127
Male	41	45	20	106
Column Total	71	92	70	233

i. $P(A \mid B)$

ii. $P(A \text{ and } B)$

iii. $P(A \text{ or } B^c)$

iv. $P(A^c \text{ and } B)$

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2. In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new automobile: the aggressive approach and the passive approach. For 1160 customers, the following record was kept.

	Sale	No Sale	Row Total
Aggressive	270	310	580
Passive	416	164	580
Column Total	686	474	1160

Suppose that a customer is selected at random from the 1160 participating customers. Let us use the following notation for events: A = the aggressive approach, Pa = the passive approach, S = sale, and N = no sale.

(a) Compute the probability that a sale was made to the customer, $P(S)$.

(b) Compute the probabilities $P(S \mid A)$ and $P(S \mid Pa)$.

(c) Are the events S = sale and Pa = passive approach independent? Explain.

(d) Compute the probability $P(A \text{ and } S)$.

(e) Compute the probability $P(A \text{ or } S)$.

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3. You draw two cards from a standard deck of 52 cards **without** replacing the first one before drawing the second.

   (a) Are the outcomes on the two cards independent? Why?

   (b) Find $P(3 \text{ on the 1st card } and 10 \text{ on the 2nd})$.

   (c) Find $P(10 \text{ on the 1st card } and 3 \text{ on the 2nd})$.

   (d) Find the probability of drawing a 10 *and* a 3 in either order.

4. Suppose you have a standard deck of 52 cards from which you are drawing cards at random (without replacement).

   (a) How many cards must you draw so that the probability of drawing 'at least one ace' is 1?

   (b) What is the probability that if you draw 5 cards that you have NO aces?

5. Suppose for two events A and B we know that $P(A) = 0.67$ and $P(B) = 0.5$.

   (a) Can events A and B be mutually exclusive? Explain.

   (b) If we further know that A and B are independent, compute $P(A \text{ and } B)$ and $P(A \text{ or } B)$.

4/8

1. An urn contains 6 white marbles and 4 black marbles. If two marbles are randomly selected from the urn without replacement. Let x be the random variable representing the number of white marbles selected.

   (a) What are the possible values of x?

   (b) Is x a continuous or discrete random variable?

   (c) Create a probability distribution table for x. (Do your probabilities add up to 1?)

x			
$P(x)$			

   (d) Determine the mean (or expected value) of x.

   (e) Determine the standard deviation of x.

2. A local grocery store has determined that (to the nearest minute) 15% of the customers take 1 minute to check out, 20% take 2 minutes to check out, 30% take 3 minutes to check out, 25% take 4 minutes to check out, and the rest take 5 minutes to check out.

   (a) What is the average number of minutes it takes for one of the customers of this store to check out?

   (b) If in a typical hour the store has 85 customers, how many check-out clerks would the store need to meet the demand?

5/8

3. *USA Today* reported that approximately 25% of all state prison inmates released on parole become repeat offenders while on parole. Suppose the parole board is examining five prisoners up for parole. Let x be the number of prisoners out of five on parole who become repeat offenders.

   (a) The probability distribution for x is shown here.

x	0	1	2	3	4	5
$P(x)$	0.237	0.395	0.264	0.088	0.015	0.001

   i. What is the probability that one or more of the five parolees will be repeat offenders? (That is, what is $P(x \geq 1)$?)

   ii. What is the probability that fewer than 3 will be repeat offenders? (That is, what is $P(x < 3)$?)

   iii. What is the expected number of repeat offenders out of five?

4. Sara is a 60-year-old Anglo female in reasonably good health. She wants to take out a $50,000 term (that is, straight death benefit) life insurance policy until she is 65. The policy will expire on her 65th birthday. (So, if she dies before her 65th birthday, the policy will pay out $50,000. Otherwise, it expires and pays out nothing.)

   The probability of death in a give year is provide by the Vital Statistics Section of the *Statistical Abstract of the United States.*

$x =$ age	60	61	62	63	64
Probability of death	0.00756	0.00825	0.00896	0.00965	0.01035

   (a) Sara is applying to Big Rock Insurance Company for her term insurance policy. What is the total expected cost to Big Rock Insurance over the years 60 through 64? (Again, in the case that Sara does not die, the cost to the insurance company is $0.)

   (b) If Big Rock Insurance Company charges $5000 for the policy, then what is the expected profit for this policy?

6/8

1. Consider a binomial experiment with $n = 25$ trials and the probability of success on each trial is $p = 20\%$. Compute the indicated probability.
   (a) The probability of exactly 4 successes.
   (b) The probability of at most 4 successes.
   (c) The probability of at least 4 successes.
   (d) The probability of more than 4 successes.
   (e) The probability of fewer than 4 successes.

2. In a certain casino game, a player can *win*, *lose* or *tie* the dealer. In a single instance of the game, the probability that the player wins is 0.35, the probability that the player loses is 0.55, and the probability that the player ties is 0.10. Suppose the player participates in 10 games and the outcome of each game is independent of all other games.
   (a) Can we use the binomial experiment model to determine the probability of exactly 6 losses? If so, what are the relevant values of n, r, p, and q? If not, explain why.
   (b) Can we use the binomial experiment model to determine the probability of 3 wins, 6 losses, and 1 tie? If so, what are the relevant values of n, r, p, and q? If not, explain why.

3. A research team at Cornell University conducted a study showing that approximately 10% of all businessmen who wear ties wear them so tightly that they actually reduce blood flow to the brain, diminishing cerebral functions (Source: *Chances: Risk and Odds in Everyday Life*, by James Burke). At a board meeting 20 businessmen, all of whom wear ties, what is the probability that
   (a) at least one tie is too tight? (That is, $P(r \geq 1)$.)
   (b) fewer than 6 ties are too tight? (That is, $P(r < 6)$.)

4. The quality-control inspector of a production plant will reject an entire batch of syringes if two or more defective syringes are found in a random sample of eight syringes taken from the batch. Suppose that the batch actually contains 1% defective syringes.

   (a) Find the mean of the probability distribution, μ.

   (b) Find the standard deviation of the probability distribution, σ.

   (c) What is the probability that the batch will be accepted?

5. A large bank vault has several automatic burglar alarms. The probability is 0.55 that a single alarm will detect a burglar.

   (a) What is the minimum number of such alarms that should be installed to provide 99% certainty that a burglar trying to enter the vault is detected by at least one alarm?

   (b) If, in fact, the bank decides to install 9 such alarms, then what is the expected number of alarms that will detect a burglar?

"upper Limit" of examinable content for quiz week 5

- check your solutions on trello
- do the quiz preps on trello
- show up on Friday well rested
- learn the Tao of TI-84

8/8

5.1 Discrete Probability Distributions

1. Refer to the "sum of two dice" exercise as an example of such a probability distribution.
2. Discuss **expected value** and **standard deviation** of a probability distribution. Using the example of the two dice is helpful. Note that $E(X) = 7$, $\mathrm{Var}(X) = \frac{105}{18} \approx 5.83$, and $\sigma \approx 2.415$.
   - Point how the formulas are like the formulas we already know for mean, variance and standard deviation but also what makes them different.

$$E(X) = \mu = \sum xP(X = x)$$

$$\mathrm{Var}(x) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2 = E(X^2) - E(x)^2.$$

   - These last two formulas for variance are not in the book, but are easier to use if someone were to compute them by hand.
   - Note that 1-Var Stats can still get the job done for us, like a weighted average or frequency table. Use the probabilities in the "frequency table".
3. It is worth pointing out, just for completeness, that $E(X)$ is a linear function, meaning $E(\alpha X + \beta Y) = \alpha E(X) + \beta E(Y)$. Also, $\mathrm{Var}(\alpha X) = \alpha^2 \mathrm{Var}(X)$. It is not true that $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ in general, but it is true when X and Y are independent. Covariance is somewhat talked about in this class when we talk about linear correlation. Maybe it is worth mentioning off-hand that $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ whenever X and Y are uncorrelated.

5.2 & 5.3 Binomial Distribution

1. Describe the features of a binomial experiment. Begin with the example of flipping a coin 10 times in a row and counting the number of heads (which we consider a success).
   - A **fixed number** n of trials.
   - Each trial is independent of all others.
   - Each trial has two outcomes: a success (with probability p) and failure (with probability $1-p = q$).
   - The goal is to count the number of successes r in n trials.
2. Present the formula, $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r} = \binom{n}{r} p^r q^{n-r}$.

   Because we didnt cover Section 4.3 about counting, the reference to $C_{n,r} = \binom{n}{r}$ will be to state that it is the number of ways that the r successes could have fallen in those n trials. I do list all the ways to get 2 heads in 4 flips, just to emphasize the point.
3. Note that the TI calcs have binompdf and binomcdf. Refer them to the screencasts if they dont know how to use those functions already, but make a point to review them right before the worksheet next Wednesday.

   Specifically (on Wednesday) review the syntax and application of `binompdf(n,p,r)` which computes the probability of EXACTLY r successes out of n trials, $P(X = r)$, while `binomcdf(n,p,r)` computes the probability of at most r success out of n trials, $P(X \leq r)$.
4. Discuss how we can compute the probability of at least r successes (say) when neither function is explicitly designed to do that by using compliments. Namely, $P(X > r) = 1 - P(X \leq r)$.

   Drawing a chart and emphasizing the events and their compliments is very helpful. This is one of the more confusing topics for the students.
5. A quick note of the formulas for expected value and standard deviation for a binomially distributed random variable. They wont use 1-Var Stats, most likely, because the formulas are so much simpler.

$$E(X) = np$$

$$Var(x) = npq = np(1-p)$$

solutions to Week 05 group problems 1/7

In-class Worksheet 6

1. Consider the following events for a college student selected at random:

$$A = \text{student is female}$$
$$B = \text{student is majoring in business}$$

(a) Describe with an English phrase, what quantity each of the following expressions describes.

i. $P(A \mid B)$
$P(A \mid B)$ is the probability that the student is a female given that we know they are majoring in business.

ii. $P(A \text{ and } B)$
$P(A \text{ and } B)$ is the probability that the student is both a female and majoring in business.

iii. $P(A \text{ or } B^c)$
$P(A \text{ or } B^c)$ is the probability that the student is either a female or is not majoring in business.

iv. $P(A^c \text{ and } B)$
$P(A^c \text{ and } B)$ is the probability that the student is a male business major.

(b) Given the following contingency table, compute the value of each of the following expressions.

	MAJOR			
GENDER	Business	Biology	Psychology	Row Total
Female	30	47	50	127
Male	41	45	20	106
Column Total	71	92	70	233

i. $P(A \mid B)$
$P(A \mid B)$ can be computed by looking at the females WITHIN the business majors and so

$$P(A \mid B) = \frac{30}{71}.$$

ii. $P(A \text{ and } B)$
$P(A \text{ and } B)$ can be computed by looking at the female business majors in the whole class,

$$P(A \text{ and } B) = \frac{30}{233}.$$

iii. $P(A \text{ or } B^c)$
$P(A \text{ or } B^c)$ can be computed by first counting all the females and then adding to that set any male non-business majors. In other words, count everyone EXCEPT the male business majors.

$$P(A \text{ or } B^C) = \frac{127+45+20}{233} = \frac{192}{233}.$$

iv. $P(A^c \text{ and } B)$
$P(A^c \text{ and } B)$ can be counted by finding the number of male business majors, so

$$P(A^c \text{ and } B) = \frac{41}{233}.$$

2/7

2. In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new automobile: the aggressive approach and the passive approach. For 1160 customers, the following record was kept.

	Sale	No Sale	Row Total
Aggressive	270	310	580
Passive	416	164	580
Column Total	686	474	1160

Suppose that a customer is selected at random from the 1160 participating customers. Let us use the following notation for events: A = the aggressive approach, Pa = the passive approach, S = sale, and N = no sale.

(a) Compute the probability that a sale was made to the customer, $P(S)$.

$P(S)$ can be computed by looking at the total sales and the total number of customers.

$$P(S) = \frac{686}{1160} \approx 59.1\%$$

(b) Compute the probabilities $P(S \mid A)$ and $P(S \mid Pa)$.

$P(S \mid A)$ can be computed by looking at the number of sales WITHIN the aggressive category, and $P(S \mid Pa)$ can be computed by looking at the number of sales WITHIN the passive category. $P(S \mid A) = \frac{270}{580} \approx 46.6\%$ and $P(S \mid Pa) = \frac{416}{580} \approx 71.7\%$

(c) Are the events S = sale and Pa = passive approach independent? Explain.

These events are not independent. If they were independent, then $P(S)$ would equal $P(S \mid Pa)$ because the fact that the passive approach was used would not change the probability of a sale. However, according to the probabilities computed above, they are not equal. So, apparently, the fact that a passive approach was used does affect the probability of a sale.

(d) Compute the probability $P(A \text{ and } S)$.

$P(A \text{ and } S)$ can be computed by counting all the aggressive approach sales in all the total sales.

$$P(A \text{ and } S) = \frac{270}{1160} \approx 23.3\%$$

(e) Compute the probability $P(A \text{ or } S)$.

$P(A \text{ or } S)$ can be computed by first counting all the aggressive approaches and then add to that the sales with the passive approach. In other words, count every customer EXCEPT the passive approach, no sales.

$$P(A \text{ or } S) = \frac{580 + 416}{1160} = \frac{996}{1160} \approx 85.9\%$$

3/7

3. You draw two cards from a standard deck of 52 cards **without** replacing the first one before drawing the second.

   (a) Are the outcomes on the two cards independent? Why?
   No because sampling without replacement always yields dependent events. This is because the denominator will always change in computing the probabilities.

   (b) Find $P(3 \text{ on the 1st card } and 10 \text{ on the 2nd})$.
   $= \frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}$.

   (c) Find $P(10 \text{ on the 1st card } and 3 \text{ on the 2nd})$.
   $= \frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}$.

   (d) Find the probability of drawing a 10 *and* a 3 in either order.
   By adding the previous two computations, we yield the appropriate answer of $\frac{8}{663}$.

4. Suppose you have a standard deck of 52 cards from which you are drawing cards at random (without replacement).

   (a) How many cards must you draw so that the probability of drawing 'at least one ace' is 1?
   Since there are 4 aces and 48 non-aces, we must have drawn 49 cards to be CERTAIN that we have drawn an ace. Drawing any fewer could still leave the 4 aces in the stack of untouched cards.

   (b) What is the probability that if you draw 5 cards that you have NO aces?
   One way to approach this is to think that the first card is NOT an ace, and then the second card is NOT an ace, and then the third card is NOT an ace...
   $\frac{48}{52} \times \frac{47}{51} \times \frac{46}{50} \times \frac{45}{49} \times \frac{44}{48} = 0.6588419983$.

5. Suppose for two events A and B we know that $P(A) = 0.67$ and $P(B) = 0.5$.

   (a) Can events A and B be mutually exclusive? Explain.
   Because $P(A) + P(B) > 1$, there must be some overlap in the events. That is, some outcomes are being double-counted. So, it is not possible for them to be mutually exclusive.

   (b) If we further know that A and B are independent, compute $P(A \text{ and } B)$ and $P(A \text{ or } B)$.
   If A and B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B) = 0.67 \cdot 0.5 = 0.335$.
   Then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.67 + 0.5 - 0.335 = 0.835$.
   Note that the 0.335 is representative of the overlap in this case.

4/7

In-class Worksheet 7

1. An urn contains 6 white marbles and 4 black marbles. If two marbles are randomly selected from the urn without replacement. Let x be the random variable representing the number of white marbles selected.

   (a) What are the possible values of x?
   $x = 0$, $x = 1$, $x = 2$

   (b) Is x a continuous or discrete random variable?
   Because the values of x are the result of a count and are whole number values, x is a discrete random variable.

   (c) Create a probability distribution table for x. (Do your probabilities add up to 1?)

x	0	1	2
$P(x)$	$\frac{2}{15}$	$\frac{8}{15}$	$\frac{1}{3}$

   (d) Determine the mean (or expected value) of x.
   $\mu = \sum xP(x) = 0 \cdot \frac{2}{15} + 1 \cdot \frac{8}{15} + 2 \cdot \frac{1}{3} = 1.2$

   (e) Determine the standard deviation of x.
   $\sigma = \sqrt{\sum(x-\mu)^2 P(x)}$. With L_1 as x and L_2 as $P(x)$, 1-Var Stats L_1, L_2 yields 0.6531972647.

2. A local grocery store has determined that (to the nearest minute) 15% of the customers take 1 minute to check out, 20% take 2 minutes to check out, 30% take 3 minutes to check out, 25% take 4 minutes to check out, and the rest take 5 minutes to check out.

   (a) What is the average number of minutes it takes for one of the customers of this store to check out?
   With L_1 as time (that is, $x = 1$, $x = 2$, $x = 3$, $x = 4$, and $x = 5$) and L_2 as probability (that is $P(1) = 0.15$, $P(2) = 0.20$, $P(3) = 0.30$, $P(4) = 0.25$, and $P(5) = 0.10$), 1-Var Stats L_1, L_2 yields $\bar{x} = 2.95$. So, on average it takes a customer 2.95 minutes to check out.

   (b) If in a typical hour the store has 85 customers, how many check-out clerks would the store need to meet the demand?
   If there are 85 customers, each taking 2.95 minutes to check out, then this requires $85 \cdot 2.95 = 250.75$ worker-minutes per hour. Since the greatest number of worker-minutes a single person can provide in one hour is 60, there would need to be a total of $\frac{250.75}{60} = 4.17916667$ workers per hour. Since we cannot have a fraction of a person, we will need 5 total checkout clerk bodies per hour, although not all 5 will have to be working for the entire time.

3. *USA Today* reported that approximately 25% of all state prison inmates released on parole become repeat offenders while on parole. Suppose the parole board is examining five prisoners up for parole. Let x be the number of prisoners out of five on parole who become repeat offenders.

   (a) The probability distribution for x is shown here.

x	0	1	2	3	4	5
$P(x)$	0.237	0.395	0.264	0.088	0.015	0.001

   i. What is the probability that one or more of the five parolees will be repeat offenders? (That is, what is $P(x \geq 1)$?)
   $P(x \geq 1) = 0.395 + 0.264 + 0.088 + 0.015 + 0.001 = 0.763$. So, there is a probability of 76.3% that one or more of the five parolees will be a repeat offender.

   ii. What is the probability that fewer than 3 will be repeat offenders? (That is, what is $P(x < 3)$?)
   $P(x < 3) = 0.237 + 0.395 + 0.264 = 0.896$. So, there is a probability of 89.6% that fewer than 3 will be repeat offenders.

   iii. What is the expected number of repeat offenders out of five?
   $\mu = \sum xP(x)$. With L_1 as x and L_2 as $P(x)$, 1-Var Stats yields $\bar{x} = 1.252$.

4. Sara is a 60-year-old Anglo female in reasonably good health. She wants to take out a $50,000 term (that is, straight death benefit) life insurance policy until she is 65. The policy will expire on her 65th birthday. (So, if she dies before her 65th birthday, the policy will pay out $50,000. Otherwise, it expires and pays out nothing.)

   The probability of death in a give year is provide by the Vital Statistics Section of the *Statistical Abstract of the United States*.

$x =$ age	60	61	62	63	64
Probability of death	0.00756	0.00825	0.00896	0.00965	0.01035

   (a) Sara is applying to Big Rock Insurance Company for her term insurance policy. What is the total expected cost to Big Rock Insurance over the years 60 through 64? (Again, in the case that Sara does not die, the cost to the insurance company is $0.)

   To compute the expected cost (value) we can use $\mu = \sum xP(x)$ where x is either $50000 or $0. Since multiplying by 0 yields 0, we really only need consider the cases when $x = 50000$.

   So, the expected cost is $50000 * 0.00756 + 50000 * 0.00825 + 50000 * 0.00965 + 50000 * 0.01035 = 2238.50$. That is, the expected cost to the insurance company is $2238.50.

   (b) If Big Rock Insurance Company charges $5000 for the policy, then what is the expected profit for this policy?

   Because the expected cost is $2238.50 and the known revenue is $5000, the expected profit is $5000 - 2238.50 = 2761.50$ dollars.

6/7

In-class Worksheet 8

1. Consider a binomial experiment with $n = 25$ trials and the probability of success on each trial is $p = 20\%$. Compute the indicated probability.
   (a) The probability of exactly 4 successes.
   $P(r = 4) = \texttt{binompdf}(25, 0.20, 4) = 0.18668105$
   (b) The probability of at most 4 successes.
   $P(r \leq 4) = \texttt{binomcdf}(25, 0.20, 4) = 0.42067431$
   (c) The probability of at least 4 successes.
   $P(r \geq 4) = 1 - \texttt{binomcdf}(25, 0.20, 3) = 0.76600674$
   (d) The probability of more than 4 successes.
   $P(r > 4) = 1 - \texttt{binomcdf}(25, 0.20, 4) = 0.57932569$
   (e) The probability of fewer than 4 successes.
   $P(r < 4) = \texttt{binomcdf}(25, 0.20, 3) = 0.23399326$

2. In a certain casino game, a player can *win*, *lose* or *tie* the dealer. In a single instance of the game, the probability that the player wins is 0.35, the probability that the player loses is 0.55, and the probability that the player ties is 0.10. Suppose the player participates in 10 games and the outcome of each game is independent of all other games.
   (a) Can we use the binomial experiment model to determine the probability of exactly 6 losses? If so, what are the relevant values of n, r, p, and q? If not, explain why.
   Since problem is posed as a simple distinction between *losses* (success in this case) and *not losses* (failure in this case), we can us the binomial experiment model. Specifically, $n = 10$ trials at the game, $r = 6$ losses, $p = 0.55$ the probability of a *loss*, and $q = 0.45$ the probability of *not loss*. Specifically, the answer is `binompdf(n,p,r) = 0.23836665`.
   (b) Can we use the binomial experiment model to determine the probability of 3 wins, 6 losses, and 1 tie? If so, what are the relevant values of n, r, p, and q? If not, explain why.
   In this case, we can not use the binomial experiment model, because the problem is asking us to distinguish between 3 distinct outcomes and the binomial experiment is exclusively for two outcomes: *success* and *failure*.

3. A research team at Cornell University conducted a study showing that approximately 10% of all businessmen who wear ties wear them so tightly that they actually reduce blood flow to the brain, diminishing cerebral functions (Source: *Chances: Risk and Odds in Everyday Life*, by James Burke). At a board meeting 20 businessmen, all of whom wear ties, what is the probability that
   (a) at least one tie is too tight? (That is, $P(r \geq 1)$.)
   This can be solve as a binomial experiment, where success is marked as wearing a tie to tightly. So, $p = 0.10$ is the probability of success. There are $n = 20$ trials for which we are trying to compute the probability of $r = 1$ or more.
   For this problem, the complement rule is relevant in that $P(r \geq 1) = 1 - P(r = 0)$.
   So, $P(r \geq 1) = 1 - P(r = 0) = 1 - \texttt{binompdf(20, 0.10, 0)} = 0.87842335$.
   (b) fewer than 6 ties are too tight? (That is, $P(r < 6)$.)
   In this case, we are looking for the probability of 6 or fewer successes out of 20 trials. So, we can use the cumulative probability function `binomcdf`. However, remember that this calculator function will compute "r or fewer"...so for FEWER THAN 6, we must use $r = 5$ as the argument in the function.
   Specifically, $P(r < 6) = \texttt{binomcdf(20, 0.10, 5)} = 0.98874687$.

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4. The quality-control inspector of a production plant will reject an entire batch of syringes if two or more defective syringes are found in a random sample of eight syringes taken from the batch. Suppose that the batch actually contains 1% defective syringes.

   (a) Find the mean of the probability distribution, μ.

   $\mu = np = 8(0.01) = 0.08$

   (b) Find the standard deviation of the probability distribution, σ.

   $\sigma = \sqrt{npq} = \sqrt{8(0.01)(0.99)} = 0.2814249456$

   (c) What is the probability that the batch will be accepted?

   The batch will be rejected if two or more defective syringes are found in the random sample of eight. Equivalently, the batch of syringes will be ACCEPTED if fewer than 2 defective syringes are found. The probability is $P(r < 2) = P(r \leq 1) =$ `binomcdf(8, 0.01, 1)` $= 0.9973099223$. So, there is about a 99.73% probability that the batch will be accepted.

5. A large bank vault has several automatic burglar alarms. The probability is 0.55 that a single alarm will detect a burglar.

   (a) What is the minimum number of such alarms that should be installed to provide 99% certainty that a burglar trying to enter the vault is detected by at least one alarm?

   This is an example of a quota problem. Specifically, we are looking for the smallest value of n so that $P(r \geq 1)$ is 99% (or higher).

   Since $P(r \geq 1) = 1 - P(r = 0)$ and $P(r = 0) =$ `binompdf(n, 0.55, 0)`, we need to find n so that $1 -$ `binompdf(n, 0.55, 0)` $= 0.99$ (or higher).

   Using the `TABLE` function on the TI-84 with Y_1`=1-binompdf(X,0.55,0)`, we see that for $n = 5$, $Y_1 = 0.98155$ and for $n = 6$, $Y_1 = 0.9917$. Therefore, there must be a minimum of 6 alarms to reach that 99% certainty that at least one will detect the burglar.

   (b) If, in fact, the bank decides to install 9 such alarms, then what is the expected number of alarms that will detect a burglar?

   With $p = 0.55$ that each single alarm will detect a burglar and $n = 9$ alarms, the expected number of alarms that will detect a burglar is $np = 9(0.55) = 4.95$ alarms. So, on average about 5 alarms will detect the burglar.