# Quiz Review Guide and Practice Problems Week 09

2019-10-24

NAME Colton Grainger #1

Section ~~001~~

Please make sure to show your work. If you use a function in your calculator to derive the answer, please write what was entered into your calculator and the output of the function that you used.

1) (1 point) Find the value of z such that 16% of the area under the standard normal distribution lies to the left of z. Round your answer to 3 decimal places.

Let $Z \sim N(\mu = 0, \sigma = 1)$.

Find $k$, a real number, such that $\mathbb{P}(Z \le k) = 0.16$.

(Either use a table for the normal distribution or invNorm)

2) The time that you wait at a red light at an intersection is normally distributed with a mean of 1.4 minutes and standard deviation 0.5 minutes.

Actually is "lognormal", but nevermind.

a.) (2 points) What is the probability that a single motorist chosen at random ~~wais~~ waits less than 1 minute at the intersection? Round your answer to 5 decimal places.

Let $W$ model the waiting time.

Then $W \sim N(\mu = 1.4, \sigma = 0.5)$.

Find $\mathbb{P}(W < 1)$. (use normcdf or a table of z-scores.)

b.) (2 points) What is the probability that 11 motorists chosen at random have a mean wait time of less than 1 minute at the intersection? Round your answer to 5 decimal places.

Let $W_1, W_2, \ldots, W_{11}$ be a random sample from the population distribution. Then each of $W_i$ (from $i=1$ up to $i=11$) is normally distributed $N(\mu = 1.4, \sigma = 0.5)$. We want to know the sampling distribution of the sample mean $\overline{W} = \frac{W_1 + \cdots + W_{11}}{11}$. Because a linear combination of normal r.v.s is again normal, $\overline{W}$ is determined by $\mu_{\overline{W}}$ and $\sigma_{\overline{W}}$ alone! Moreover, $E(\overline{W}) = \mu = 1.4$ and $\sqrt{Var(\overline{W})} = \frac{\sigma}{\sqrt{11}} = \frac{0.5}{\sqrt{11}}$, so $\overline{W} \sim N(\mu_{\overline{W}} = 1.4, \sigma_{\overline{W}} = 0.5/\sqrt{11})$. Find $\mathbb{P}(\overline{W} \le 1)$.

Notice that we don't need the Central Limit Theorem because the population is already normally distributed.

#2

3) It has been reported that coffee drinkers spend on average \$15.14 per week on coffee with a standard deviation of \$3.77.

(With $n=100$ the sample size much greater than 30)

a.) (1 point) What does the Central Limit Theorem allow you to conclude? Circle the best response.

$n=1$ and false $\sigma$ ~~(i)~~ The average amount that coffee drinkers spend on coffee per week is normally distributed with a mean of \$15.14 and standard deviation $\boxed{0.377}$.

$n=1$ and true $\sigma$ ~~(ii)~~ The average amount that coffee drinkers spend on coffee per week is normally distributed with a mean of \$15.14 and standard deviation $\boxed{3.77}$.

$n=100$ and true $\sigma$ (iii) The average amount that $\boxed{100}$ coffee drinkers spend on coffee per week is normally distributed with a mean of \$15.14 and standard deviation $\boxed{0.377}$. Justify.

$n=100$ and false $\sigma$ ~~(iv)~~ The average amount that $\boxed{100}$ coffee drinkers spend on coffee per week is normally distributed with a mean of \$15.14 and standard deviation $\boxed{3.77}$.

b.) (2 points) What is the probability that the average amount spent per week on coffee of 100 random coffee drinkers is between \$14 and \$16? Round your answer to 5 decimal places.

Find $\mathbb{P}(14 \le \bar{X} \le 16)$.

Because $X_1, \dots, X_{100}$ form a large random sample from an unknown population distribution with $\mu = 15.14$ and $\sigma = 3.77$ known, by the C.L.T. we know the sample mean $\bar{X} = \frac{X_1 + \dots + X_{100}}{100}$ is approximately normally distributed with mean $\mu_{\bar{X}} = 15.14$ and std dev. $\sigma_{\bar{X}} = \frac{3.77}{\sqrt{100}}$.

c.) (2 points) How would your answer from part b.) change if there were only 50 random coffee drinkers, instead of 100? Would your answer increase, ~~decrese~~ decrease, or stay the same? Explain.

Consider $Y = \frac{X_1 + \dots + X_{50}}{50}$.

Then $Y \sim N\left(\mu_Y = 15.14, \sigma_Y = \frac{3.77}{\sqrt{50}}\right)$.

What is the probability $\mathbb{P}(14 \le Y \le 16)$?

How does it compare to $\mathbb{P}(14 \le X \le 16)$?

# More practice with confidence intervals Week 09

#4

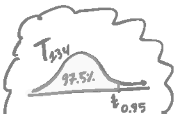
NAME: ____________________________________

Section: ________________

Show work on all problems. If you use a function in your calculator to derive an answer, please write what was entered into your calculator and the output of the function that you used.

1) The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component.

A (2 points) After observing 135 workers assembling the component, the manager calculates their mean assembly time as 16.0 minutes and the sample standard deviation as 3.7 minutes. Construct a 95% confidence interval for the mean assembly time based on this sample.




Find $t_{0.95}$ such that $\mathbb{P}(T_{134} \le t_{0.95}) = 0.975$, where $T_{134}$ is the random variable distributed according to Student's t-distr. with $n-1 = 134$ degrees of freedom. Then guess the pop$^{\underline{n}}$ mean assembly time $\mu$ is near the point estimate $\hat{\mu} := \bar{X} = 16$. Finally, compute the sample error $E = t_{0.95} \frac{S_x}{\sqrt{n}} = t_{0.95} \frac{3.7}{\sqrt{135}}$ you may be "95% confident" that the pop$^{\underline{n}}$ mean assembly time is within the interval $(16-E, 16+E)$.

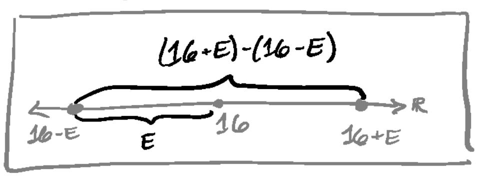
**Interpretation:** Once a sample of some given size $n$ is taken, we compute $\bar{x}$ and the above confidence interval endpoints turn into numbers. Once you have done this, there is no more probability involved. The true value of $\mu$ is either in that interval or it is not. The probability comes in from the sample. If you were to take another random sample of size $n$ from this normal "population", you would get a different value for $\bar{x}$, which would give you different endpoints for the confidence interval. A third sample gives you yet another $\bar{x}$ which gives you a different confidence interval again. Repeating many times, you will have that the true value of $\mu$ is captured between the endpoints 95% of the time.
an

Now please do all this on your calculator with $\boxed{\text{TInterval.}}$

B (1 point) What is the margin of error of the confidence interval?

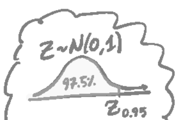
$$E = \frac{(16+E)-(16-E)}{2}.$$

Proof by picture:




Now please $\boxed{\text{numerically compute}}$ E.

C (3 points) Now assume that the population standard deviation of assembly time is 3.7 minutes. How many workers would the manager need to observe so that the margin of error is no more than 32 seconds with 95% confidence?




We know $\sigma$ the population std. deviation! Hoorah! So find $z_{0.95}$ such that for $Z \sim N(0,1)$, $\mathbb{P}(Z \le z_{0.95}) = 0.975$. Recall that the sampling error for a 95% (z-) confidence interval around a normally distributed population mean $\mu$ is $\boxed{E = z_{0.95} \frac{\sigma}{\sqrt{n}}}$.

Hence $\boxed{n = \left(\frac{z_{0.95}\,\sigma}{E}\right)^2}$. Lastly, evaluate $\boxed{E = 32\text{sec} = \frac{8}{15}\text{min},\ \sigma = 3.7,\ z_{0.95} \approx 1.96.}$



Please find n numerically.

D (2 points) Assume that the population standard deviation of assembly time is 3.7 minutes. The manager then observes 135 workers and calculates their mean assembly time as 16.0 minutes. Construct a 95% confidence interval for the mean assembly time based on this sample.

This is the same as part C, but with $\boxed{n = 135 \text{ given and } E \text{ unknown.}}$

Please numerically compute the confidence interval with Z Interval.

E (2 points) How does the confidence interval in part A differ from the confidence interval in part D? Explain why this difference occurs.

In part A the population std dev. $\sigma$ is unknown and we guess $\hat{\sigma} = S_{\bar{X}}$ as a point estimate for $\sigma$.

In part D, $\sigma$ is known, so we may apply the C.L.T. to study $\bar{X} \sim N(\mu = ?, \sigma = 3.7)$ (some normal distribution).

In part A we have $\bar{X}$ distributed according to Student's $t$ distribution with 134 degrees of freedom, whereas in part D $\bar{X}$ is normally distributed.

You should remember that all of Student's $t$ distributions have fatter tails than the normal distribution.

$\nu$ (Greek letter "nu") on the right is the degrees of freedom parameter. The content of the C.L.T. states that as $\nu \to \infty$, Student's $t$ distribution with $\nu$ degrees of freedom approaches the standard normal distribution, which is the curve corresponding to "$\nu = +\infty$."

Student's $t$ distribution