Chapter 4

Probability

4.1 Set Theory

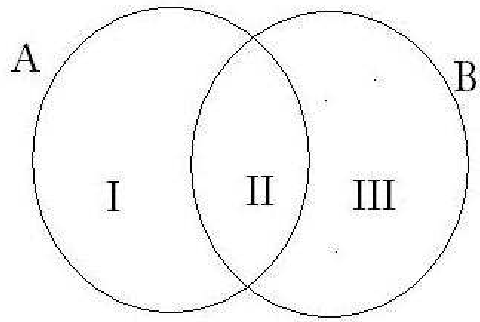
Here are some terminology and notation:

- $\emptyset$ denotes the empty set or the set that contains no elements. We also write this $\{\}$.
- A, B, and C denote sets.
- Ω denotes the universe of all possible elements in consideration.
- $\overline{A}$ denotes the set consisting of elements that are in Ω and not in the set A. We call this A *complement*.
- $A \cup B$ is the set consisting of all elements in the set A combined with all the elements in set B. We call this A *union* B.
- $A \cap B$ is the set that contains only the elements that are in both A and B. We call this A *intersect* B.
- We denote $A \subseteq B$ to say that "A is a subset of B". This means that every element of A is also an element of set B.
- We write $A - B$ to mean the set containing elements that are in A and not in B. Notice that $\overline{A} = \Omega - A$.
- We say that two sets are **disjoint** if they have no elements in common. In other words, A and B are disjoint if and only if $A \cap B = \emptyset$.

4.2 Exercises

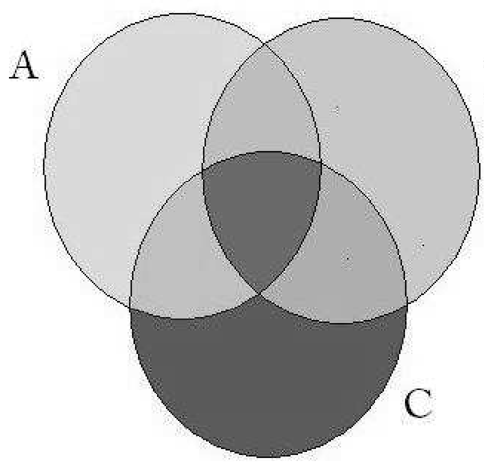
1. Consider the sets $A = \{1,2,3,4,5\}$ and $B = \{2,4,6,8,10\}$ where $\Omega = \{1,2,3,4,5,6,7,8,9,10\}$. Compute each of the following sets:
   (a) $A \cup B$
   (b) $A \cap B$
   (c) $\overline{A}$
   (d) $\overline{B}$
   (e) $B - A$
   (f) $A - B$
   (g) $\overline{A \cup B}$
   (h) $\overline{A} \cup \overline{B}$
   (i) $(B - \overline{A}) \cap \overline{(A \cap B)}$
2. Suppose $\Omega = \{$red, orange, yellow, green, blue, indigo, violet$\}, A = \{$red$\}$ and $B = \{$red, orange, blue$\}$. Compute a–i from question 1 for this example.
3. Write down an example of a specific Ω, A, B, and C so that $A \subseteq B$ and C is disjoint from both A and B.
4. Write down an example of a specific Ω, A, B, and C so that A, B, and C are all disjoint and $A \cup B \cup C = \Omega$.
5. (Extra Credit) Consider the sets $A = \emptyset$ and $B = \{\emptyset\}$ and $C = \{1, \emptyset\}$. Find each of the following sets, if it is possible. If it isn't, state why.
   (a) $C \cup B$
   (b) $A \cap B$
   (c) $\overline{A}$
   (d) $B - A$
   (e) $C - B$

Venn Diagrams



Set A corresponds to regions I and II.
Set B corresponds to regions II and III.
Set $A \cap B$ corresponds to region II.
Set B-A corresponds to region III.

Set $A \cup B$ corresponds to regions I, II, and III.



Set A is yellow which includes the regions that are yellow, green, brown and orange.

The set $A \cap B \cap C$ corresponds to the brown region.

The set $A \cap B$ corresponds to the brown and orange regions.

4.3 Venn Diagrams

A **Venn Diagram** is useful in illustrating sets and their relationships to each other. At the top of the page is an example of a Venn diagram with two sets. Below that is an example of a Venn diagram with three sets.

4.4 Exercises

1. Determine whether each of the following is true or false. If you say true, show that the Venn diagram of the left-hand side is the same as the Venn diagram of the right hand side. If you say false, come up with specific sets where the equality does not hold.[1, 2.1.1]

   (a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

   (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   (c) $\overline{A - B} = \overline{A} \cup B$

   (d) $A - \overline{A} = \emptyset$

(e) $\overline{A \cap \overline{A}} = \Omega$

(f) $A = (A \cap B) \cup (A \cap \overline{B})$

2. Let Ω be the set of all students currently enrolled in classes at Susquehanna University. Let A be the set of all students enrolled in intro stats this term, let B be the set of all students who play a varsity sport.

   Suppose there are 2,000 total students enrolled at SU and 120 are enrolled in a section of intro stats this term and 230 play a varsity sport. Note: These numbers are not the official counts.

   Interpret each of the sets below in terms of this example and, if possible, determine how many people are in each set.

   (a) A

   (b) B

   (c) Ω

   (d) $A \cap B$

   (e) $A \cup B$

   (f) $\overline{A}$

   (g) $\overline{B}$

   (h) $\overline{A \cup B}$

   (i) $\overline{A \cap B}$

3. Repeat number 2 with the added information that there are 102 students enrolled in Introductory Statistics this term who do not play a varsity sport.