MATH 3430-02 WEEK 9-1

Key Words: The Laplace transform.

The definition of a **Laplace transform** is simple: taking an *appropriate* function $f(t)$ as input, the output is

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\mathrm{d}t = \lim_{A\to\infty} \int_0^A e^{-st} f(t)\mathrm{d}t,$$

which is a function in s.

Note that $\mathcal{L}$ is linear in the sense that

$$\mathcal{L}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 \mathcal{L}\{f_1(t)\} + c_2 \mathcal{L}\{f_2(t)\}.$$

Q1. Find $\mathcal{L}\{1\}$.

Q2. Find $\mathcal{L}\{t\}$.

Q3. Find $\mathcal{L}\{t^n\}$ ($n > 0$).

Q4. Find $\mathcal{L}\{e^{at}\}$.

Q5. Find $\mathcal{L}\{\cos(bt)\}$ and $\mathcal{L}\{\sin(bt)\}$.

Q6. Express $\mathcal{L}\{y'(t)\}$ and $\mathcal{L}\{y''\}$ in terms of $Y(s) = \mathcal{L}\{y(t)\}$, $y(0)$ and possibly $y'(0)$.